CSC 226 Assignment 3
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1. Proof by contradiction:
Assume there are 2 MST - MST_1 and MST_2. Let E be the set of edges in MST_2.
Consider MST_1, since it is a minimum spanning tree, adding an edge to it will creat an cycle. Let's add an $e \in E$ to MST_1 then there should be a cycle in MST_1. The new Tree T has one more edge than a minimum spanning. To make T a MST we need to remove the most expensive edge from the cycle. since all the edges has different weight. There is only one most expensive edge. If e is the most expensive edge then there is no mutiple MST, if e is not the most expensive edge then MST_1 was not a mimimum spanning tree.

2. Let V' be a subset of V in a weighted graph G(V,E) Let E_{mi} be the set of the lightest edge in the G Then there is a MST T for G that contains at least one of the edge from E_{mi}
Proof: w.t.s $\exists$mst s.t. at least one edge e from $E_{mi} \in$ MST T
Suppose there is an MST T containning none of those lightest edge.
Consider any e (s,u) there is some path from s to u in T, so that adding e to T forms a cycle. There must be some other edges in the cycle such as (s,v) $\rightarrow$ $w(s,u) < w(s,v)$
we remove (s,v) replace it with (s,u) forms a spanning tree with smaller total weight than T $\rightarrow$ contradicting our assumption that T is MST.

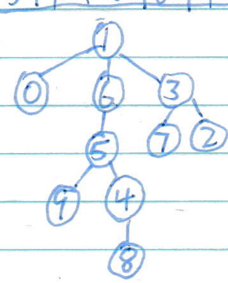
3. ① edges: ab, ad, bc, be weight: 7 Profile: 1,1,2,3
② edges: ad, bc, be, cd weight: 7 profile: 1,1,2,3
③ edges: ab, ad, bc, ce weight: 7 Profile: 1,1,2,3
④ edges: ab, ad, bc, de weight: 7 profile: 1,1,2,3
⑤ edges: ad, bc, cd, ce weight: 7 profile: 1,1,2,3
⑥ edges: ad, bc, cd, de weight: 7 profile: 1,1,2,3
Conjecture: The profiles in a MST should be the same as we only take those lightest edges

4.

	0	1	2	3	4	5	6	7	8	9
9 0	$\boxed{9}$	1	2	3	4	5	6	7	8	9
3 4	9	1	2	3	$\boxed{3}$	5	6	7	8	9
5 8	9	1	2	3	3	5	6	7	$\boxed{5}$	9
7 2	9	1	$\boxed{7}$	3	3	5	6	7	5	9
2 1	9	$\boxed{2}$	7	3	3	5	6	7	5	9
5 7	9	2	7	3	3	$\boxed{2}$	6	7	5	9
0 3	9	2	7	3	$\boxed{9}$	2	6	7	5	9
4 2	9	2	7	3	9	2	6	7	5	$\boxed{7}$

5.

0	1	2	3	4	5	6	7	8	9
1	1	3	1	5	6	1	3	4	5



no! since by using weighted quick-union the depth is at most logn, in this case the depth is 4 n is 10
$5 \leq \log_2 10$ is not true ($\log_2 10 \approx 3.32$).