Compressive spectral-video by optimal 3D-sphere packing

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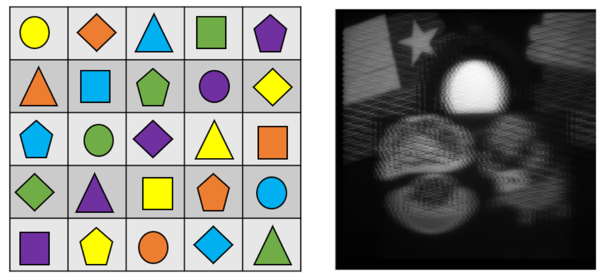
***Abstract*—This paper introduces a new method to capture compressive spectral video (SV) imaging by exploiting 3-dimensional sphere packing (SP) to design 4D-coded apertures. The proposed approach reconstructs the 4D-hypercube from a single snapshot. Simulations using SV demonstrate the performance of SP approach.**

***Index Terms*—3D-sphere packing, spectral-video, coded aperture design.**



(a) (b)

Fig. 1: (a) Example of the N^2 officers' problem, where $N = 5$ resulting in a 5×5 Graeco-latin square. (b) Compressive measurement used in the simulation.

I. Introduction

Sensing spectral video (SV) is a challenging problem that consists of optimally sampling 4-dimensional Euclidean space [1]. In contrast, conventional sensing approaches rely on scanning systems that are time-consuming in collecting the SV, *e.g.*, pushbroom architecture captures spectral images in aerial platforms such as satellites and drones. To address these drawbacks, compressive sensing has emerged as a sensing approach to capture multidimensional signals by collecting a single projection of the scene [2]. Thus, the underlying scene is recovered by solving the inverse problem. In this paper, we exploit the N^2 officers' problem and SP approach to design the coded aperture (CA)s in 4-dimensional Euclidean space. This paper is organized as follows in four sections. Section II gives an overview of the N^2 officers' problem, SP, and the proposed coding optimization. Section III discuses the results using the SV dataset and metrics. Finally, our conclusions are drawn in the final section.

II. Methods

Our 4D-CA design leverages the N^2 officers' problem and SP density to optimally design the entries of the patterns. The following subsections introduce the N^2 officers' problem and give an overview of SP.

A. N^2 officers' problem

The N^2 officers' problem is a Graeco-Latin square, where there are N^2 officers organized in N regiments and distributed in N ranks. Figure 1(a) depicts with different colors each of the regiments and with distinct shapes each of the ranks, such that there is only one officer of each rank per column and row. The following subsection details the SP that optimizes the 3D centers of spheres.

B. Sphere packing background

SP is a problem in discrete geometry, where N^2 balls are packed in a hypercube container of size $N \times N \times N$ [3]. The packing density is the fraction of the volume filled by the spheres inside the container. For 3D-equal-size spheres, the SP optimal density is $\eta_3 = \frac{\pi}{\sqrt{18}} = 0.74\ldots$, known as the Kepler conjecture [4]. In 2005, the proof of Kepler conjecture proposed by Hales was accepted [5], [6]. The theorem asserts that **Theorem 1:** *No packing of congruent balls in 3-dimensional Euclidean space has a density greater than that of face-centered cubic (FCC) lattice packing.* Recently, 3D-SP has been used to design temporal CAs in CMOS sensors [7] and to design MSFAs [8]. In our approach, we first design the temporal entries and then assign spectral filters to all the temporal entries.

C. 4D-coded aperture optimization

In our 4D-CA design, we design a kernel that is then replicated along the spatial resolution of the sensor. In detail, each shape of the N^2 officers' problem denotes a temporal frame, and the color of the shape determines a spectral filter. Figure 2 shows an example of the 4D-CAs. For the optimization of the frames and spectral filters, we use 3D-SP, separately designing the time and spectrum. First, the temporal entries are optimized [7], and then an MSFA is computed using 3D-SP [8]. Finally, the resulting 4D-CA sequentially merges the MSFA in the temporal entries of each frame.

D. Discrete sensing model

The corresponding discrete model is as follows:

$$\mathbf{Y} = \sum_{t=0}^{T-1} \sum_{k=0}^{K-1} \mathcal{X}_{(:,:,k,t)} \odot \mathcal{C}_{(:,:,k,t)} + \mathbf{\Omega}, \tag{1}$$

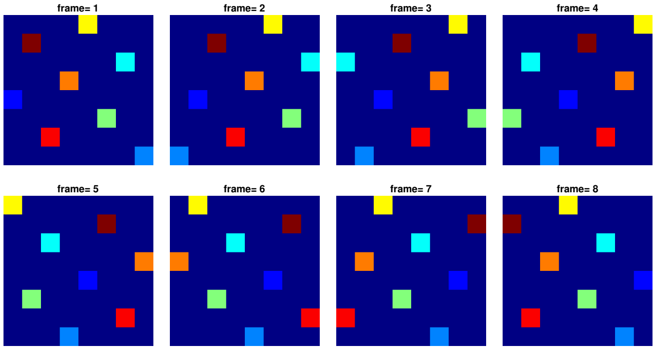


Fig. 2: Zoomed version of the 4D-CA design, where 8 frames with 8 filters are designed.

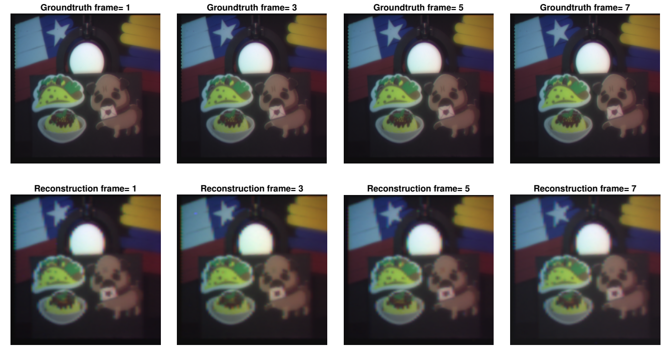


Fig. 3: Reconstruction quality comparison. The first row denotes the groundtruth of frames 1^{st}, 3^{rd}, 5^{th}, and 7^{th}. The second row shows the reconstruction quality of frames 1^{st}, 3^{rd}, 5^{th}, and 7^{th}.

where $\mathbf{Y} \in \mathbb{R}^{L\times M}$ denotes the 2D single compressive measurement, and $L \times M$ is the number of pixels of each frame, where the tensor $\mathcal{X}_{(:,:,k,t)} \in \mathbb{R}^{L\times M}$ denotes the SV, and the tensor $\mathcal{C}_{(:,:,k,t)} \in \mathbb{R}^{L\times M}$ represents the 4D-CA at the k^{th} band and at the t^{th} frame, respectively. The number of frames is T, and the number of bands is K. The additive noise is $\mathbf{\Omega}$. The discrete model is similar to the sensing model of coded aperture compressive temporal imaging (CACTI) presented in [9].

III. Simulations

To test our approach, we capture an SV scene in our laboratory, whose spatial resolution is 695×541 pixels; note that $L = 695$, $M = 541$ and $N^2 = K \times T$, where the spectral band is $K = 8$ and the number of frames is $T = 8$. A single compressive measurement was captured using the model in Eq. (1). The recovering algorithm is the scattered data interpolation method [10]. In addition, quantitative metrics are used to test the proposed approach. The spatial similarity is evaluated using the Peak-Signal-to-Noise Ratio (PSNR), and the Structural Similarity Index (SSIM); high values in both metrics indicate better spatial fidelity. Moreover, the spectral fidelity is measured using the Spectral Angle Mapper (SAM); lower values of the metric denote better spectral fidelity. The spatial fidelity of 4 out of 8 frames is depicted in Fig. 3, whose PSNR is 36.11 dB and SSIM is 0.94. The corresponding spectral similarity is 0.04. Our approach can recover a 4D-SV by collecting a single snapshot. Note a decrease in the resolution of the recovered image that occurs due to the sub-sampling of the approach. Specifically, the compression ratio of our approach is $\frac{1}{64}$, which explains the reduction in the resolution of the reconstruction.

IV. Conclusions

We introduce a novel 4D-coded aperture design by exploiting the N^2 officers' problem and 3D-sphere packing. Our approach optimally distributes spectral filters in the temporal frame entries. The simulation results show that the proposed approach recovers the SV with significant image quality. Future work will involve designing 4D-CA using 4D-SP and novel reconstruction methods involving deep learning. In particular, the 4D-SP approach involves a new mathematical model that leverages the best-known 4D-SP density, which corresponds to the $\mathbf{D}_4$ lattice, whose packing density is $\eta_4 = 0.61\ldots$. The optimal 4D-CA design could be applied to other domains, for instance, spatial-polarization sampling. Extending our approach requires studying the optimal design when the spectral-temporal signal is multiplexed.

Acknowledgment

This work has been supported by ANID ANILLOS ATE220022, ANID FONDECYT 1221883, and ANID FONDECYT Postdoctorado 3230489.

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