

Demosaicking Multispectral Images by Sphere Packing Filter Design

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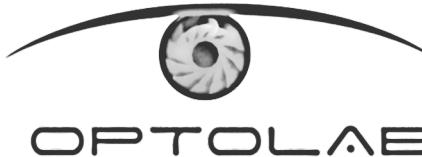
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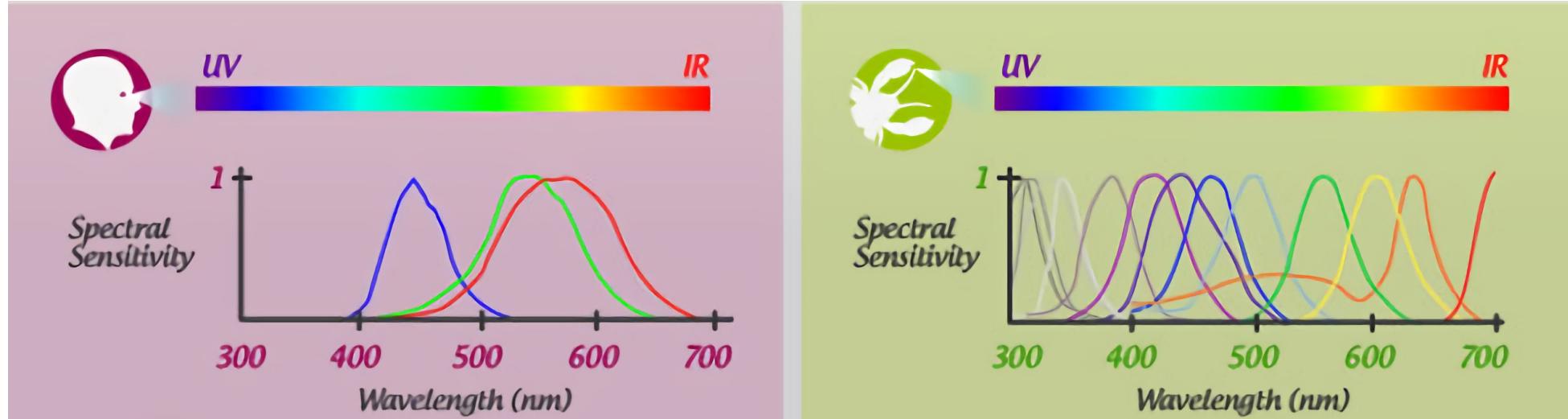


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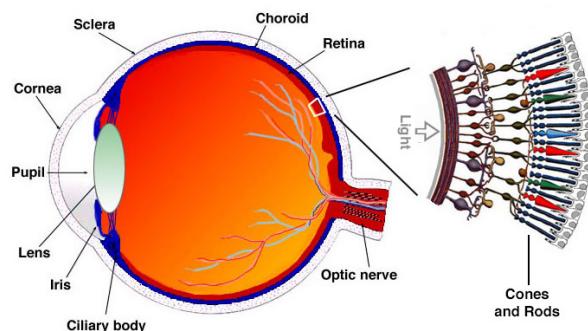
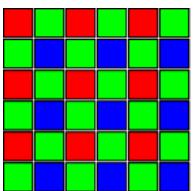


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Motivation

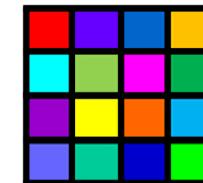


RGB



Human
3 Different types of Cones

MSI



Mantis Shrimp
12 Different types of Cones

Mathematical Model

Continuous acquisition of the grayscale mosaic

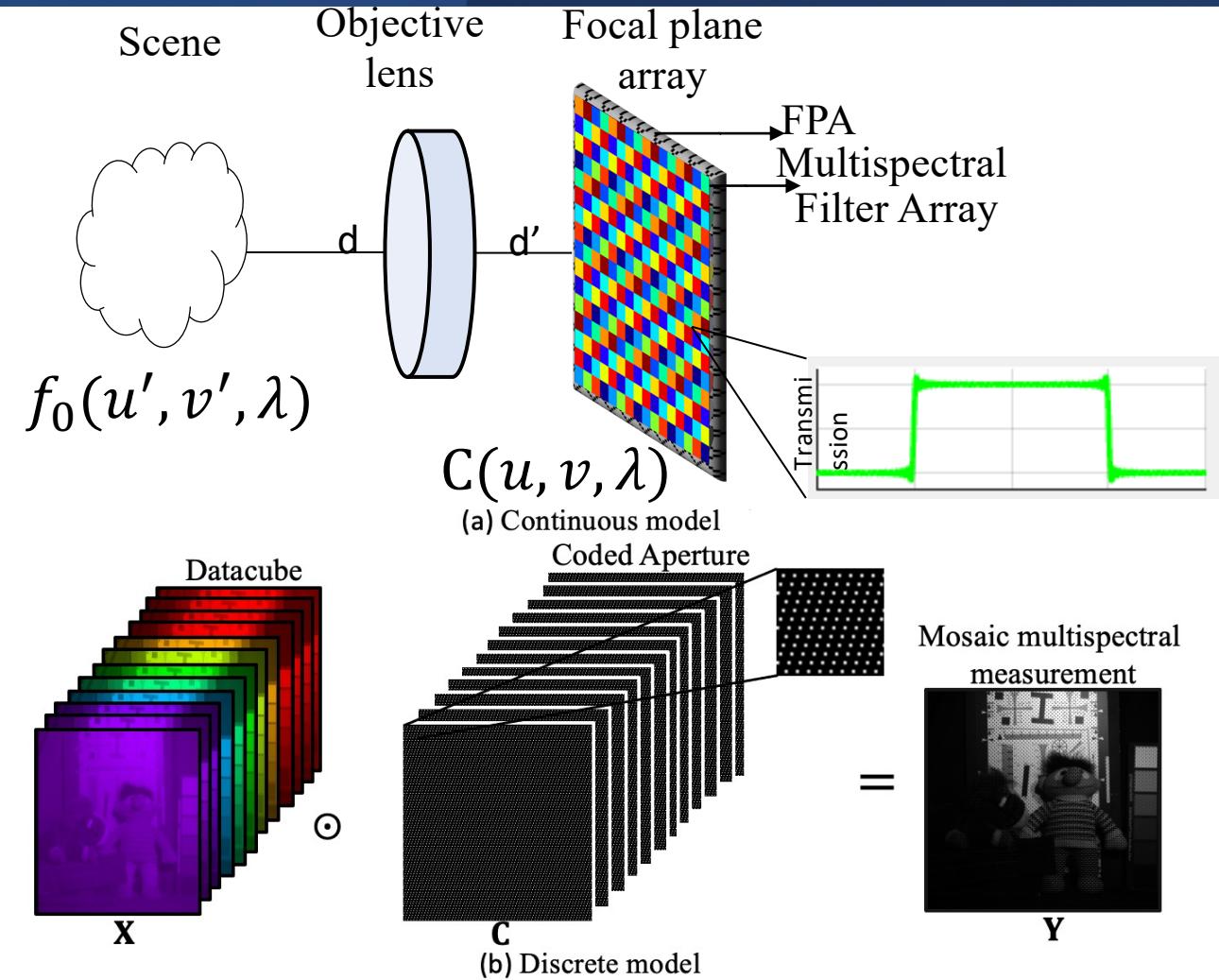
$$y(u, v) = \int_{\Lambda} C(u, v, \lambda) f_0(u', v', \lambda) d\lambda \quad (1)$$

Where C is coded aperture in focal plane and f_0 is the scene.

The acquisition of the grayscale mosaic compressive multispectral (discrete) projection of L spectral bands is

$$\mathbf{Y} = \sum_{l=1}^L \mathbf{X}_l \odot \mathbf{C}_l + \boldsymbol{\Omega} \quad (2)$$

Where \mathbf{X}_l the individual grayscale image in the l^{th} wavelength



What is Sphere Packing?

2D



$$\text{Density} = \frac{\pi}{\sqrt{12}} \approx 0.9068$$

3D



$$\text{Density} = \frac{\pi}{\sqrt{18}} \approx 0.7404$$



History of Sphere Packing Problem



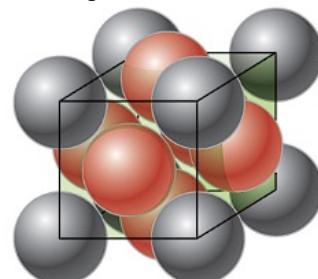
- 1611
- Johannes Kepler conjectured about the closest packing of equal spheres.
- He did not have a prove to the conjecture.



- 1998
- Thomas Hales provides the formal proof of Kepler's conjecture.
- But **eliminating all possible irregular arrangements** is very difficult, and this is what made the Kepler conjecture so hard to prove.



- 1831
- Carl Friedrich Gauss proved that the highest packing fraction that can be achieved by any packing of equal sphere.
- He proved that the Kepler conjecture is true if the spheres have to be arranged in a **regular lattice**.



Face-centered cubic structure



- 2017
- Maryna Viazovska solved sphere packing problem in **8-dimensions** [1] (E_8 lattice). And in collaboration with others **24-dimensions** [2] (Leech lattice).
- Winner of the Fields Medal 2022.

Theorem 1: No packing of congruent balls in Euclidean three space has density greater than that of the **face-centered cubic packing**, which corresponds to:

$$\rho = \frac{\pi}{3\sqrt{2}} \approx 0.7405$$

[1] M. S. Viazovska, "The sphere packing problem in dimension 8," *Annals of Mathematics*, vol. 185, no. 3, pp. 991-1015, 2017.

[2] H. Cohn, A. Kumar, S. D. Miller, D. Radchenko, and M. Viazovska, "The sphere packing problem in dimension 24," *Annals of Mathematics*, vol. 185, no. 3, pp. 1017-1033, 2017.

Proposed Multispectral Filter Array (MSFA)

The problem associated to the MSFA-sensing is

$$\mathbf{B} = (a \odot \mathbf{I} + b \odot \mathbf{J}) \mod L + 1 \quad (3)$$

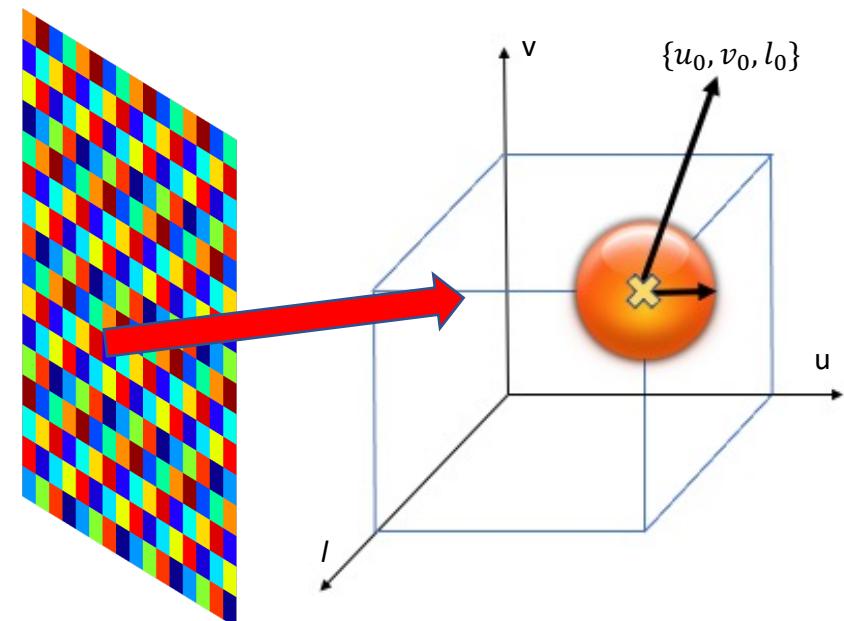
$\mathbf{I} = \mathbf{g}^T \otimes \mathbf{q}$, \mathbf{g} is a ones 1D-vector with length L , \mathbf{q} is equal to $[1, \dots, L]$ and $\mathbf{J} = \mathbf{I}^T$. a and b were calculated with the proposed algorithm in [3]

The positions of the MSFA-OSP are given by:

$$\mathbf{E} = \mathbf{A} \otimes \mathbf{B} \quad (4)$$

Where \mathbf{A} is a matrix of all ones such that $\mathbf{A} \in \mathbb{1}^{\alpha \times \beta}$, where $\alpha = \lfloor \frac{M}{L} \rfloor$ and $\beta = \lfloor \frac{N}{L} \rfloor$, M and N are number of pixels.

\odot denotes the Hadamard product and \otimes represents the Kronecker product.

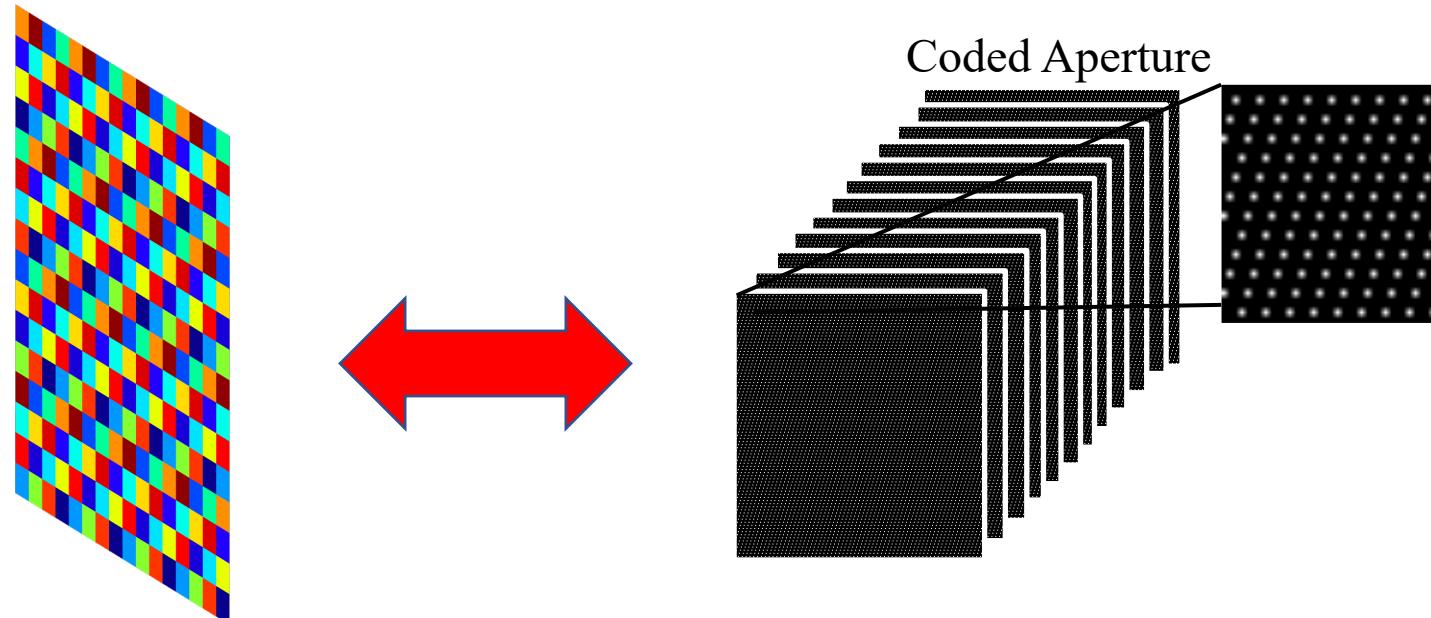


Proposed MSFA

The resulting positions of the MSFA can be expressed in binary coded aperture form:

$$C_{m,n,l} = \begin{cases} 1 & \text{if } l = E_{m,n} \\ 0 & \text{if } l \neq E_{m,n} \end{cases} \quad (5)$$

Where $m \in \{0, \dots, M - 1\}$, $n \in \{0, \dots, N - 1\}$, $l \in \{1, \dots, L\}$.



Sphere Packing Upper Bound

Optimal Sphere Packing

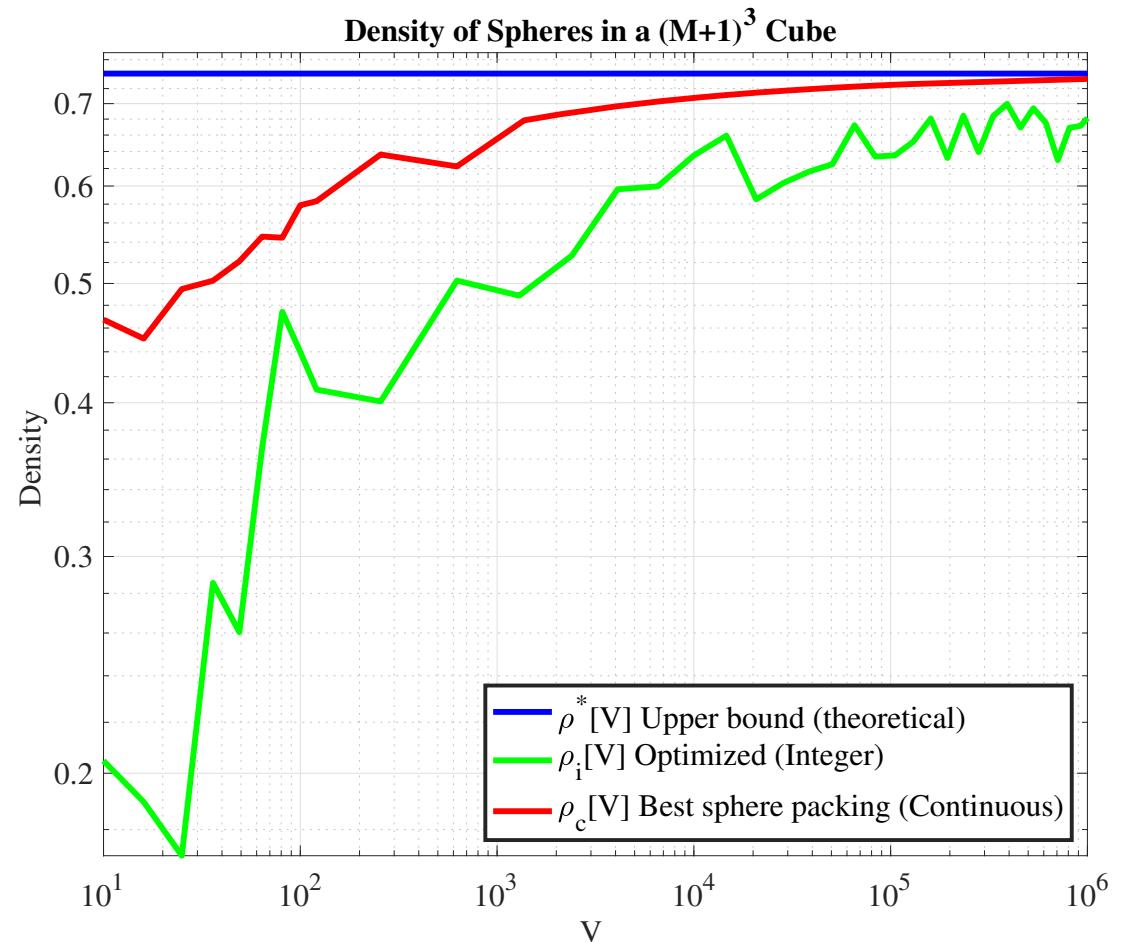
$$d^*(V) = \max \left(\min_{1 \leq k_1 < k_2 \leq V} D_{k_1, k_2} \right) \quad (6)$$

V is the number of the spheres.

D_{k_1, k_2} is the all pairwise distance matrix

Theoretical upper bound sphere packing density

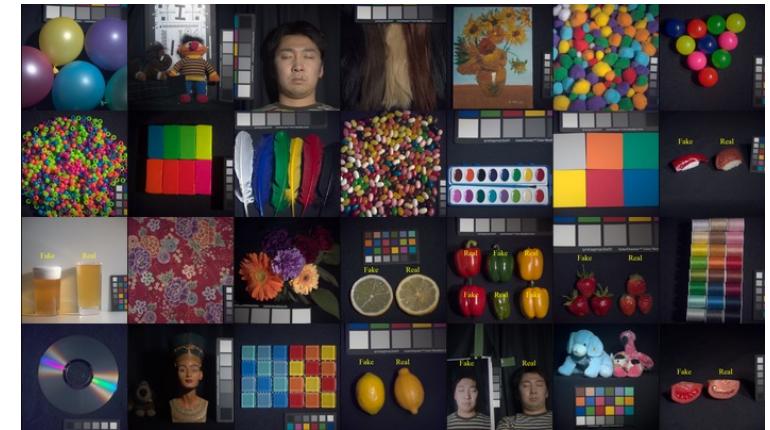
$$\rho^*(V) = 2 \sqrt[3]{\frac{(\sqrt{V} + 1)^3}{4V\sqrt{2}}} \quad (7)$$



Continuous model: O. Packomania, “Packings of equal spheres in fixed-sized containers with maximum packing density,” URL <http://www.packomania.com>, 2013.

Dataset and Network

- Use Cave Dataset [4]
 - 32 Scenes with 31 spectral bands and 512x512 pixels.
 - Scenes resized to 256x256 pixels and 16 spectral bands.
- State-Of-The-Art in Demosaicking Algorithms:
 - WB: Weighted bilinear [5]
 - itID: Iterative intensity difference [6]
 - itNCD: Iterative nearby channel difference [6]
- TRevSCI-net [7] (3D-CNN for tensor completion) was training with 10560 cubes.
 - Name: Tensor reversible snapshot compressive imaging.
 - 80% train and 20% validation.
 - L1 cost function



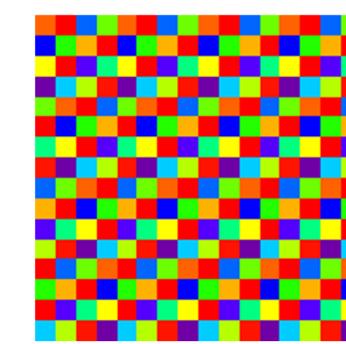
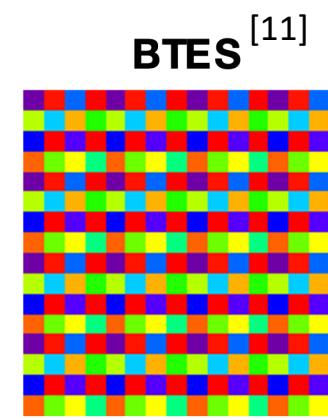
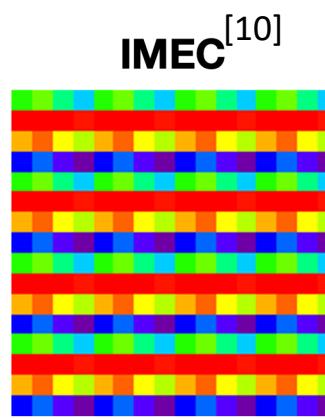
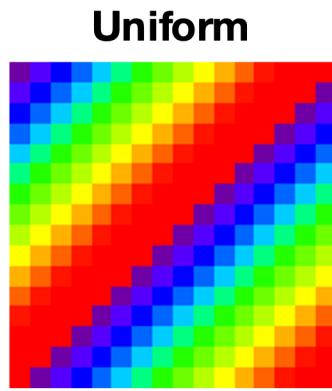
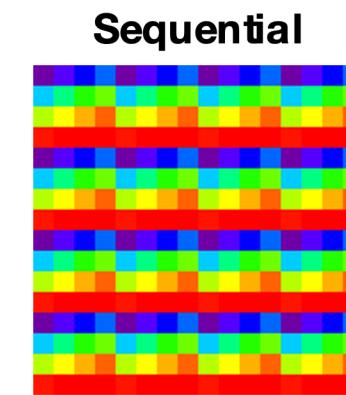
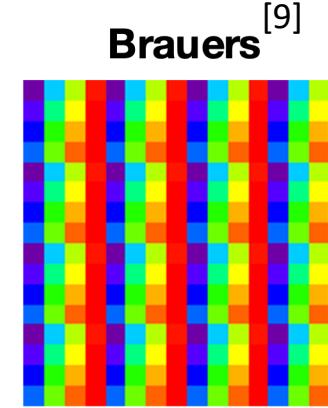
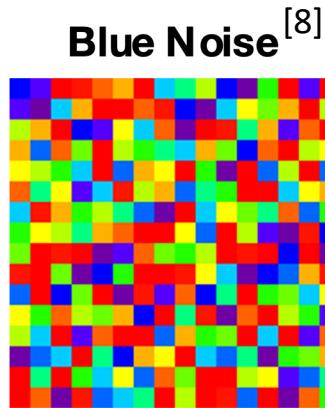
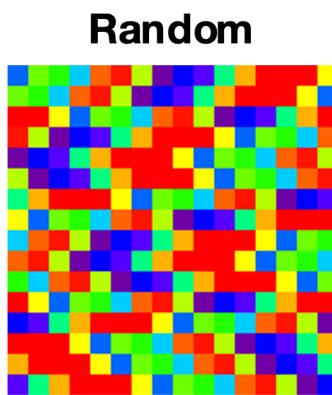
[4] F. Yasuma, T. Mitsunaga, D. Iso, and S. K. Nayar, “Generalized assorted pixel camera: Postcapture control of resolution, dynamic range, and spectrum”, IEEE Transactions on Image Processing, 2010.

[5] J. Brauers and T. Aach, “A color filter array based multispectral camera”, Workshop Farbbildverarbeitung, Oct 2006.

[6] S. Mihoubi, O. Lossen, B. Mathon, and L. Macaire, “Multispectral demosaicing using intensity in edge-sensing and iterative difference-based methods”, in 2016 12th International Conference on Signal-Image Technology Internet-Based Systems (SITIS), 2016.

[8] Z. Cheng, B. Chen, G. Liu, H. Zhang, R. Lu, Z. Wang, and X. Yuan, “Memory-Efficient Network for Large-scale Video Compressive Sensing”, Proceedings of the IEEE Computer Society Conference on CVPR, 2021.

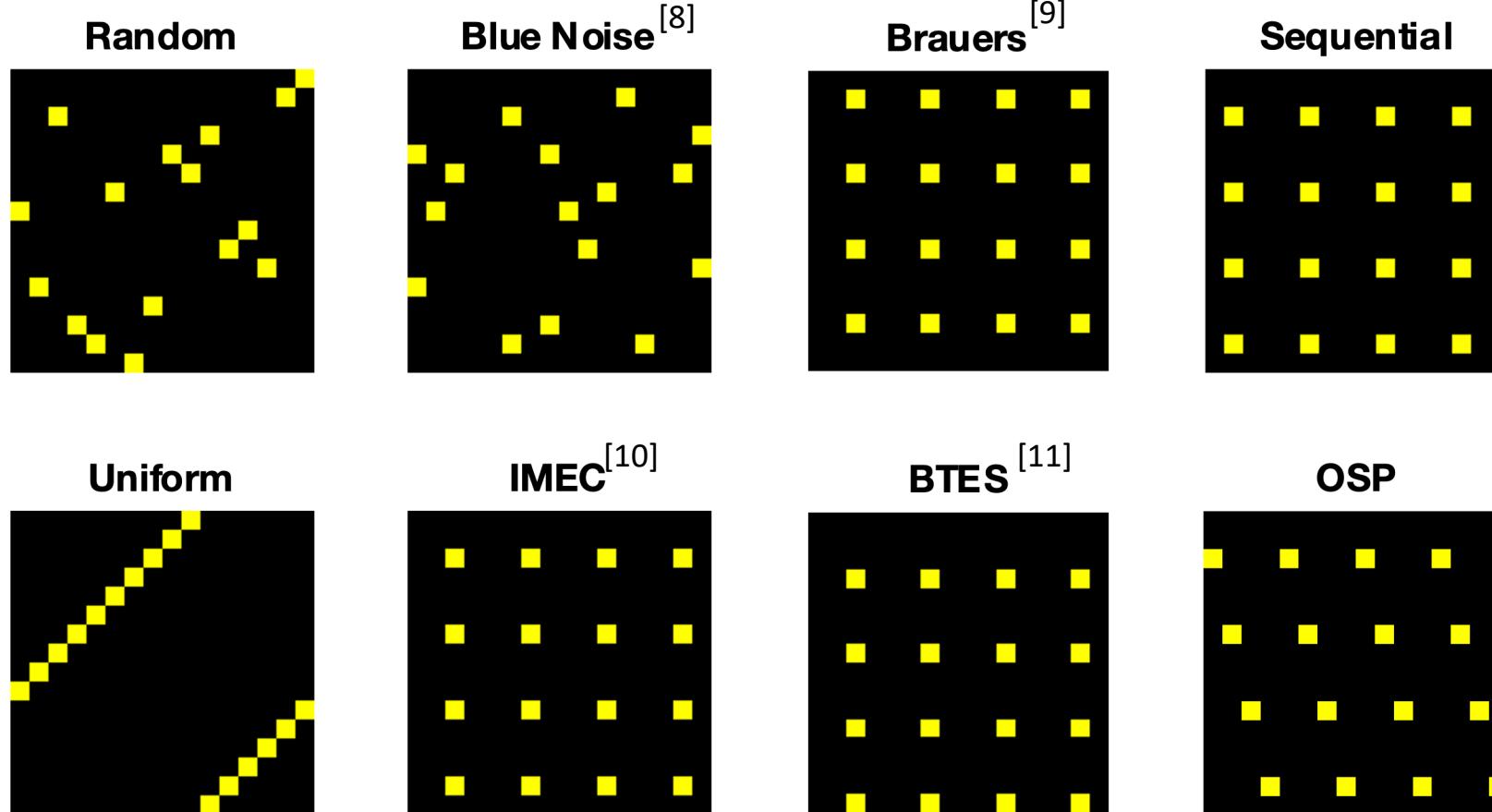
MSFA patterns



*Blue Noise is the only irregular lattice included in the comparison but satisfies the restricted isometry property (RIP).

- [8] C. V. Correa, H. Arguello, and G. R. Arce, "Spatiotemporal blue noise coded aperture design for multi-shot compressive spectral imaging", Journal of the Optical Society of America A, Dec 2016.
- [9] J. Brauers and T. Aach, "A color filter array based multispectral camera", Workshop Farbbildverarbeitung, Oct 2006.
- [10] B. Geelen, N. Tack, and A. Lambrechts, "A compact snapshot multispectral imager with a monolithically integrated per-pixel filter mosaic", Advanced Fabrication Technologies for Micro/Nano Optics and Photonics VII, 2014.
- [11] L. Miao, H. Qi, R. Ramanath, and W. Snyder, "Binary tree-based generic demosaicking algorithm for multispectral filter arrays", IEEE Transactions on Image Processing, 2006.

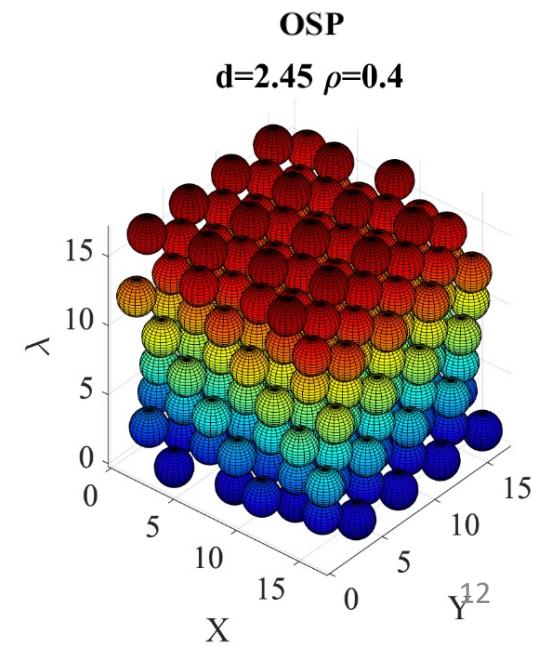
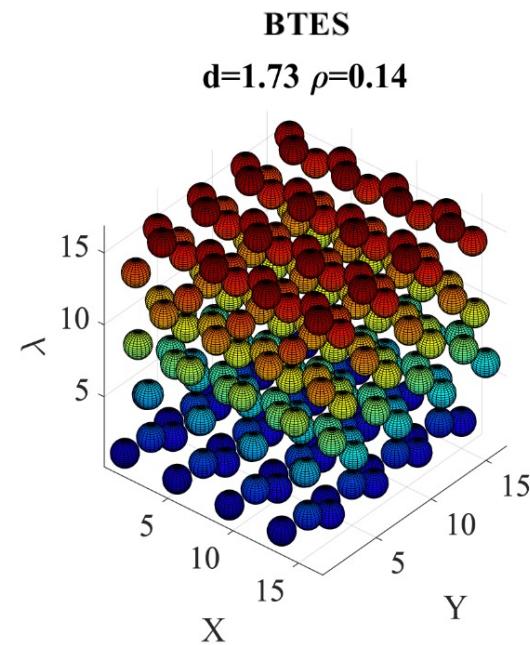
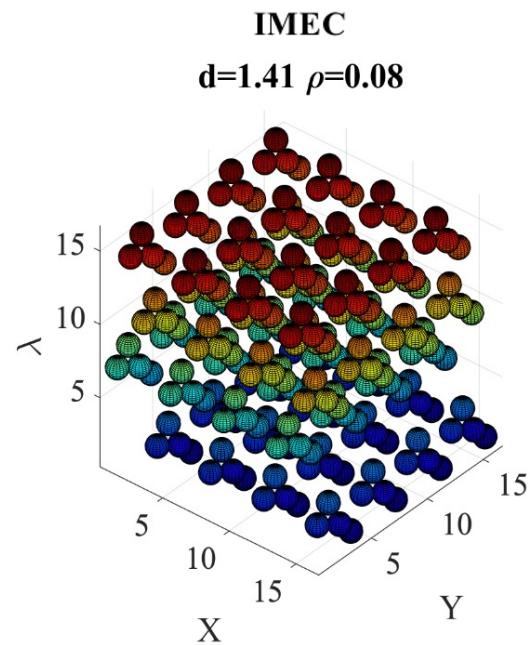
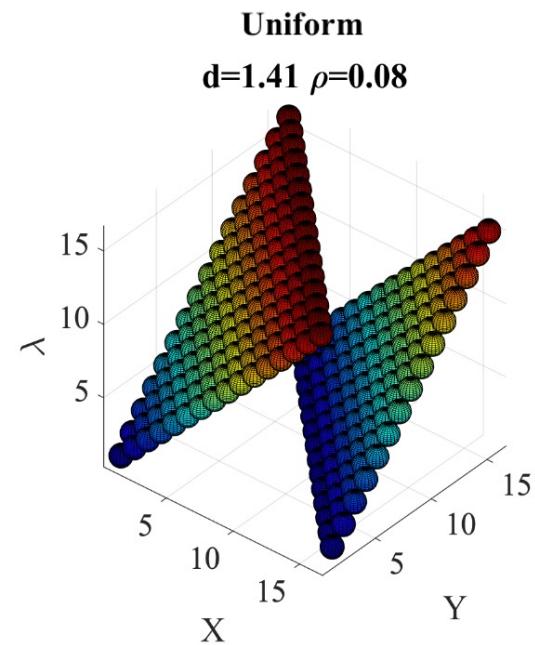
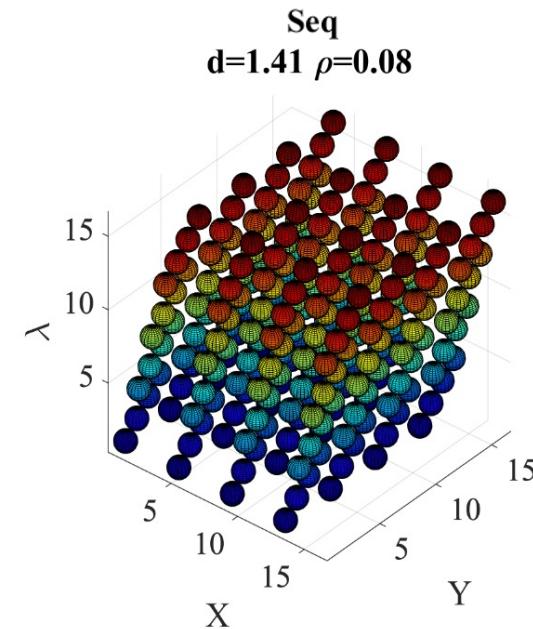
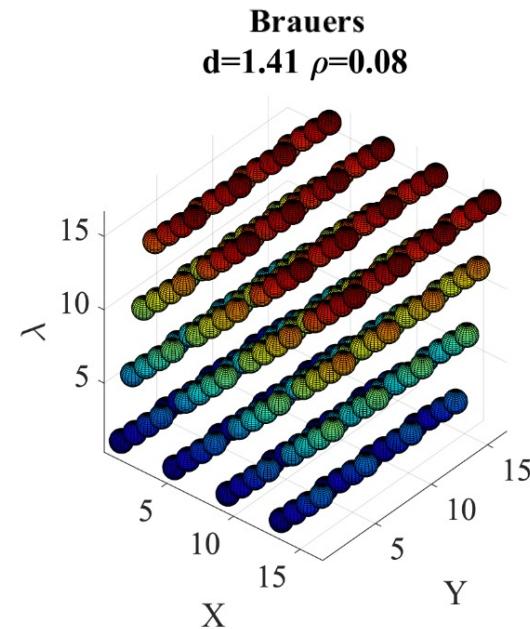
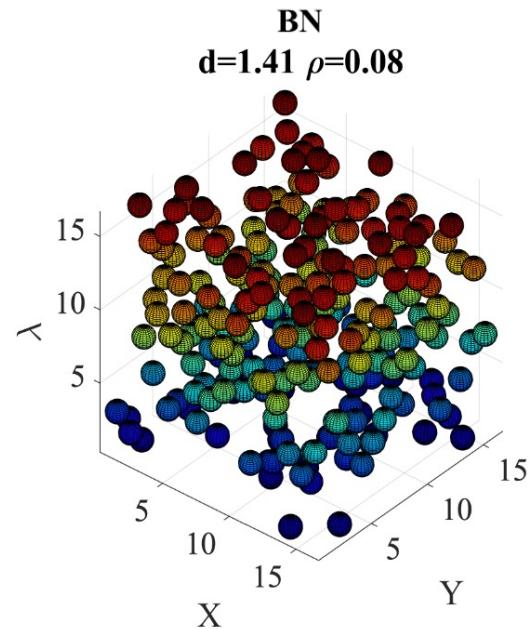
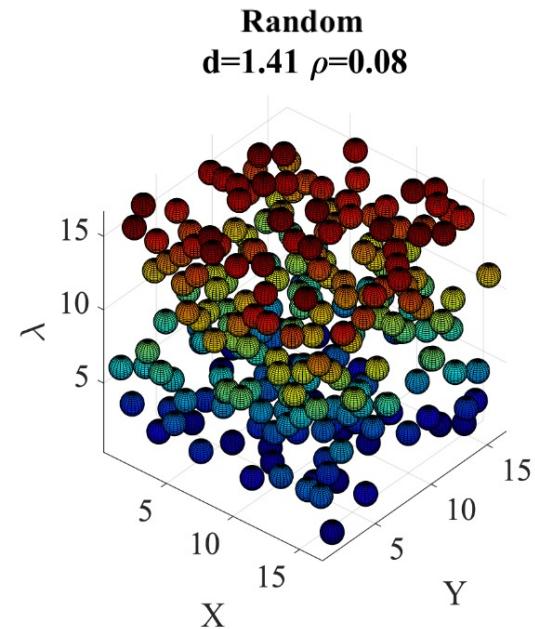
MSFA patterns



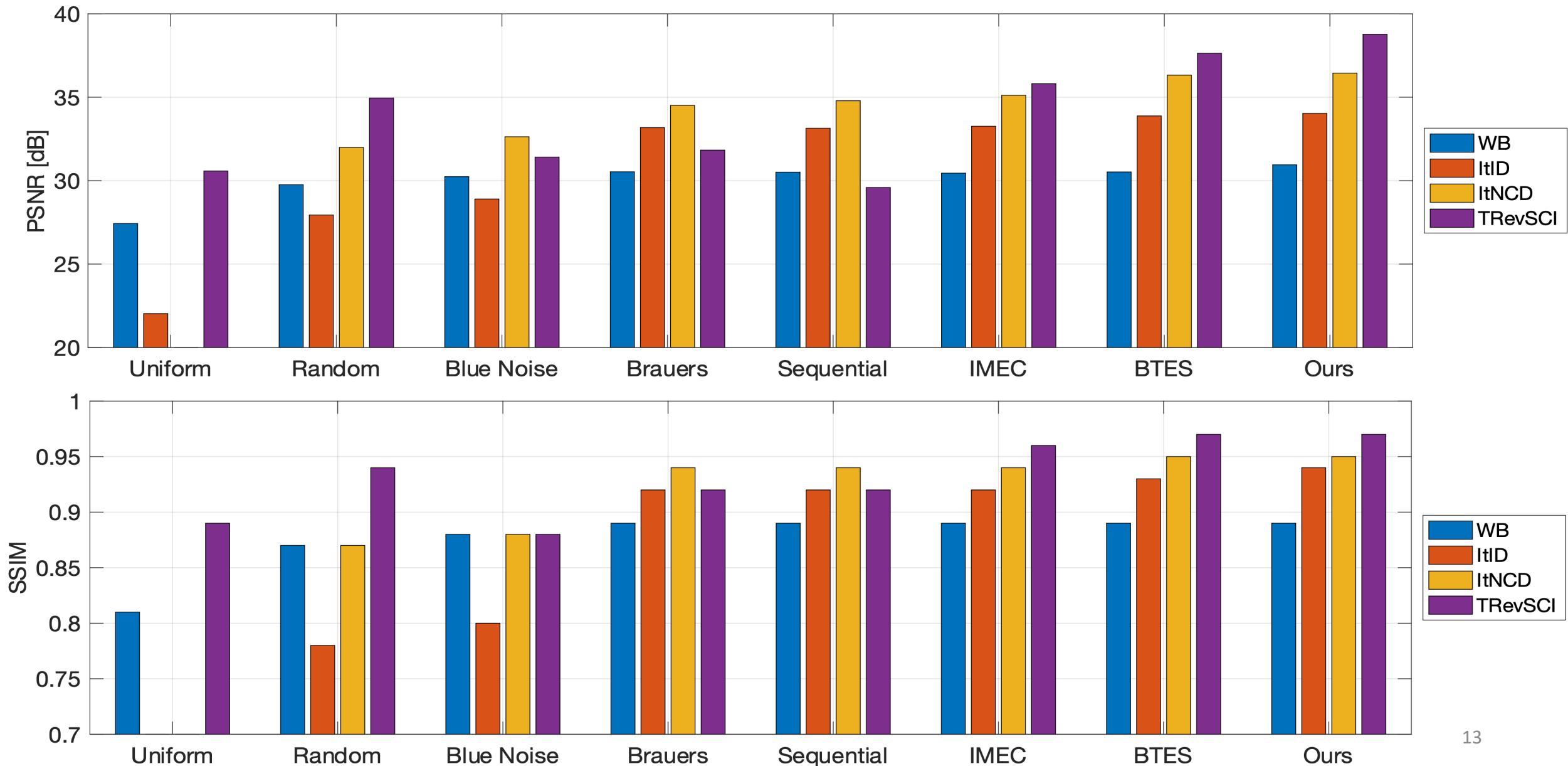
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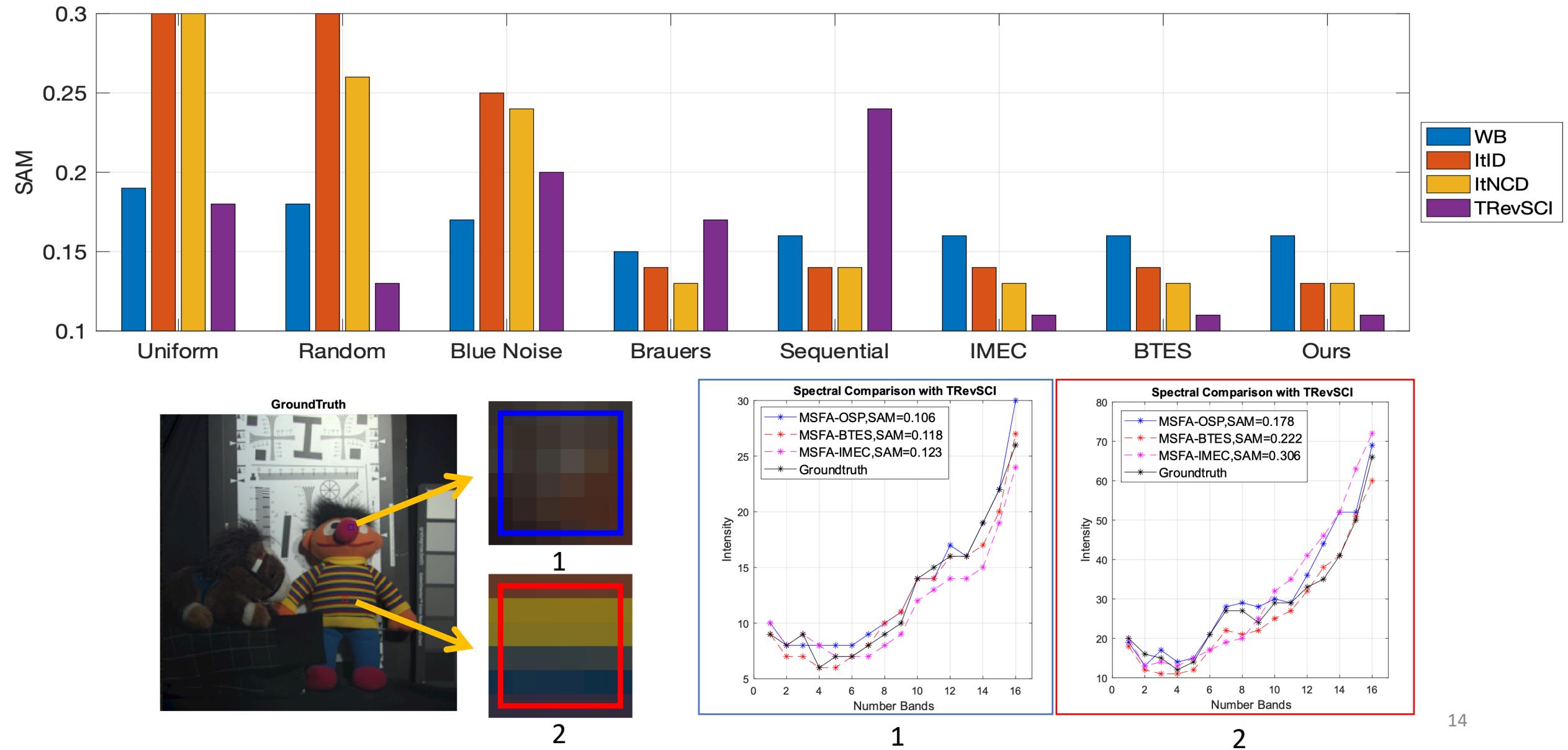
Density Comparison



Results: Cave Dataset

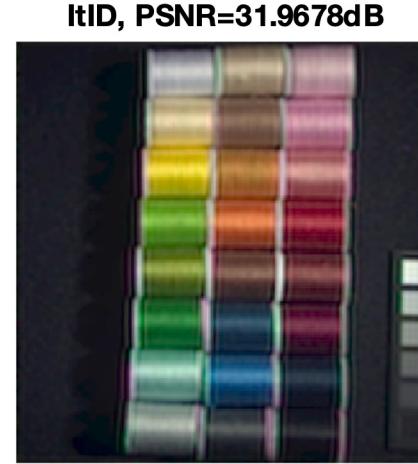
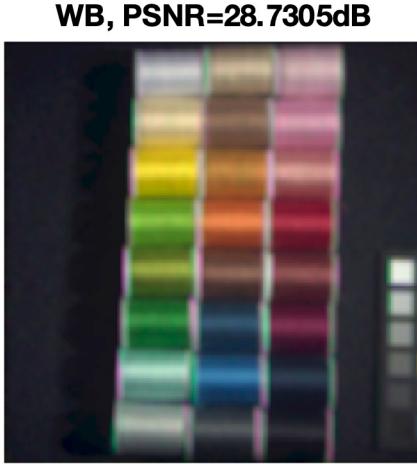


Results: Cave Dataset

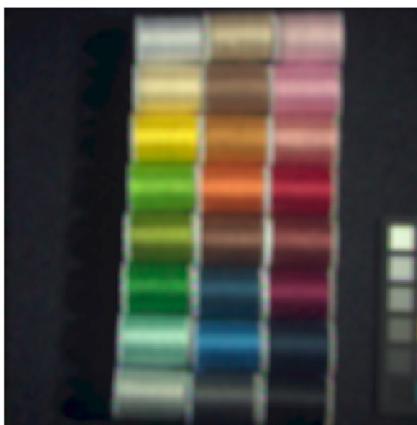


Comparison: RGB

MSFA-BTES

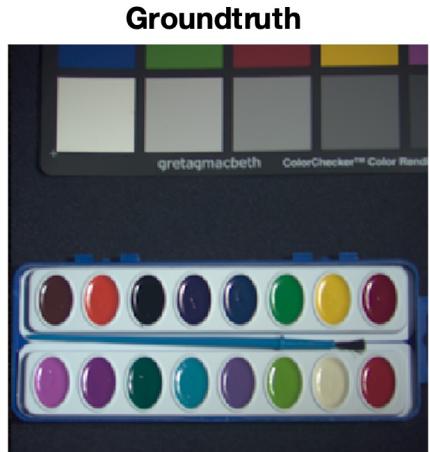


MSFA-OSP

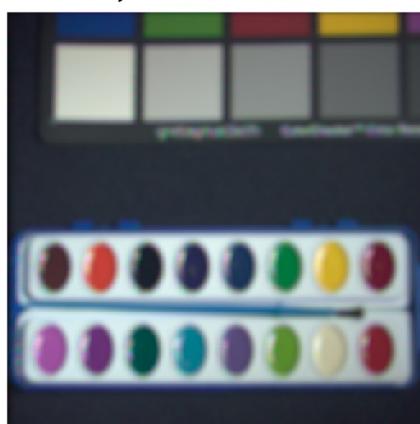


Comparison: RGB

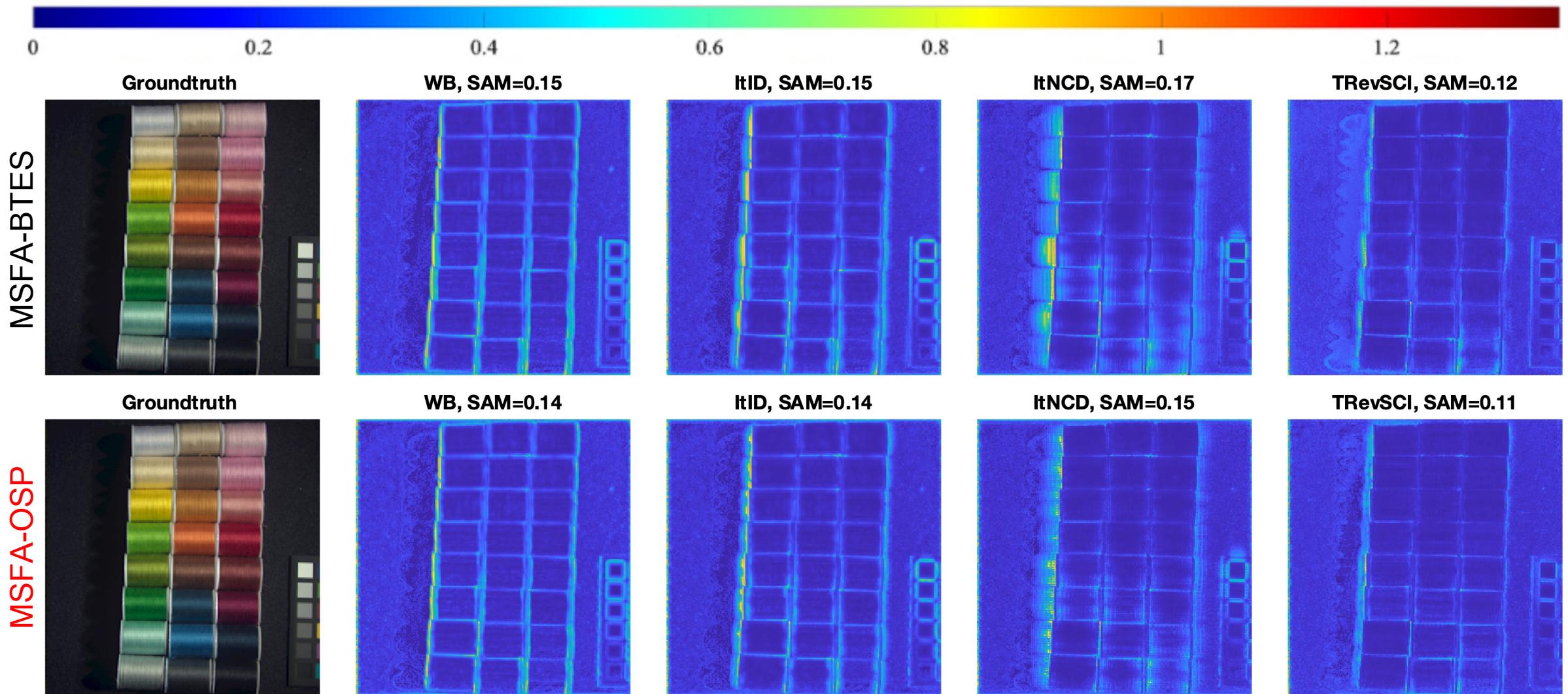
MSFA-BTES



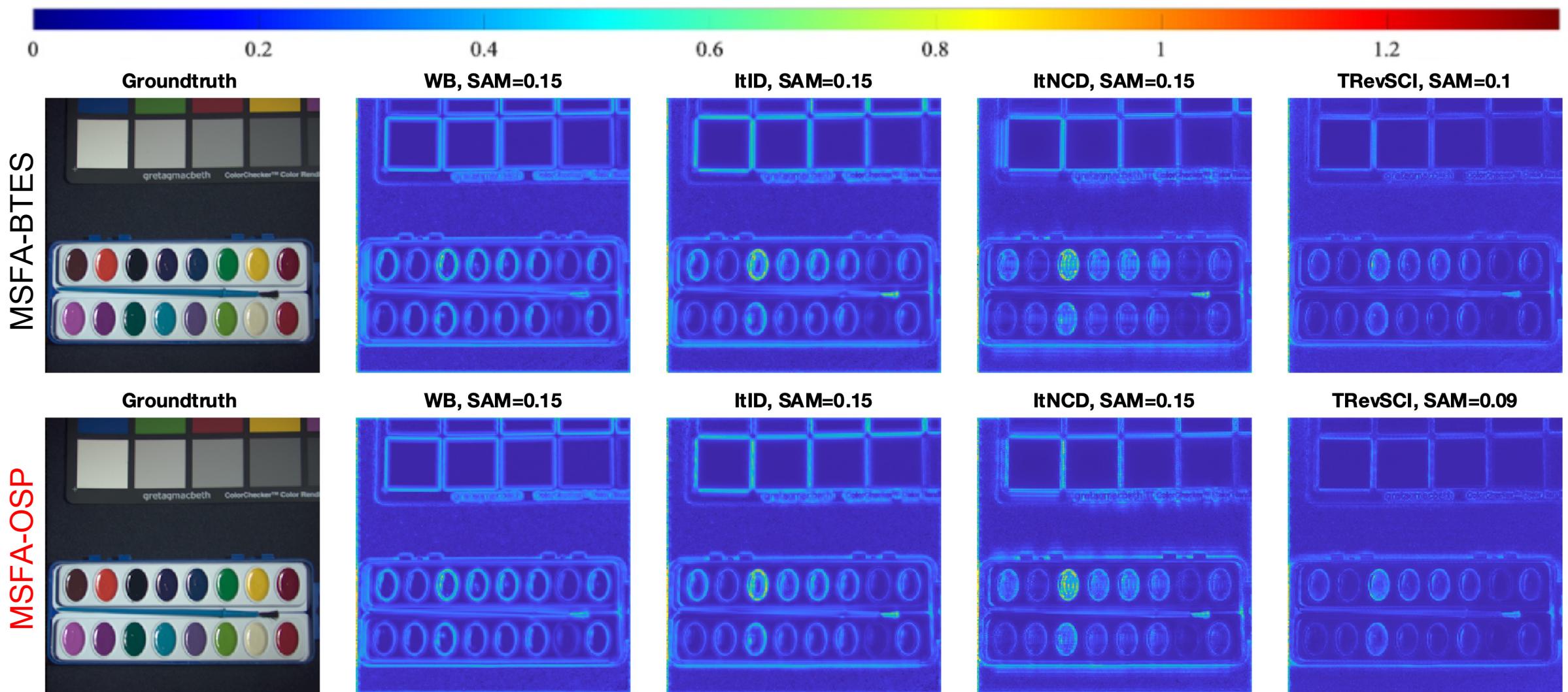
MSFA-OSP



Comparison: SAM

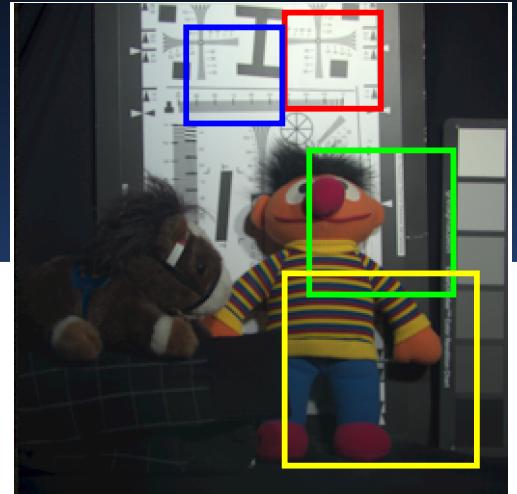


Comparison: SAM

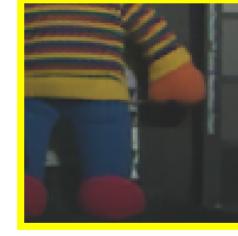
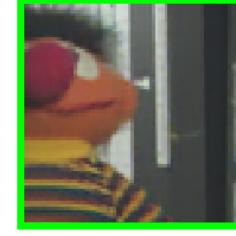
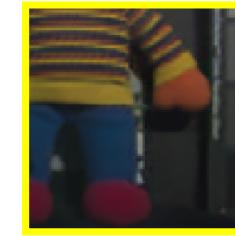
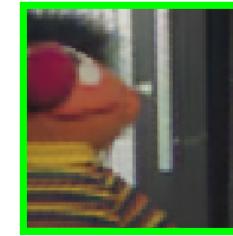
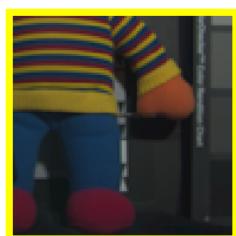
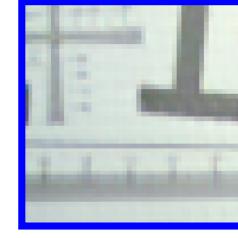
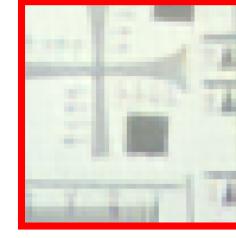
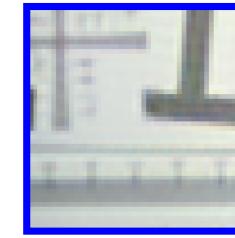
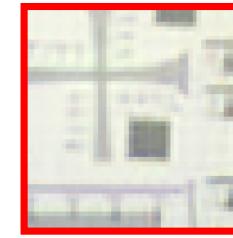
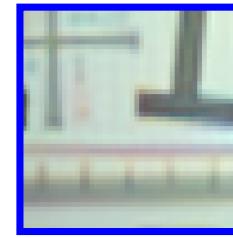
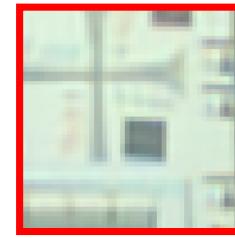
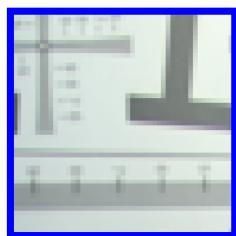
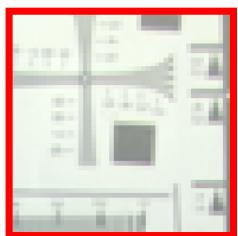


Comparative with TRevSCI

- Aliasing
- Zipper effect
- Color artifacts



Groundtruth



Groundtruth

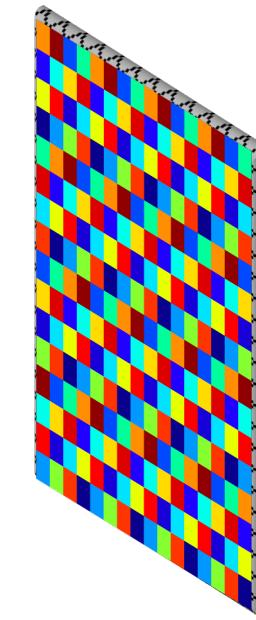
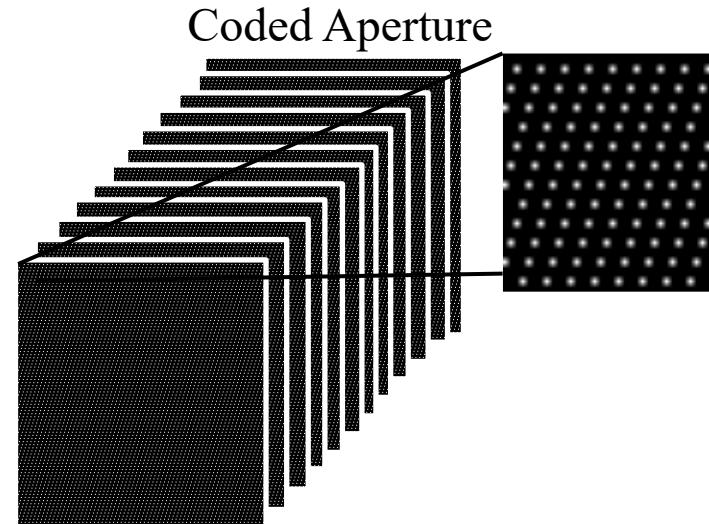
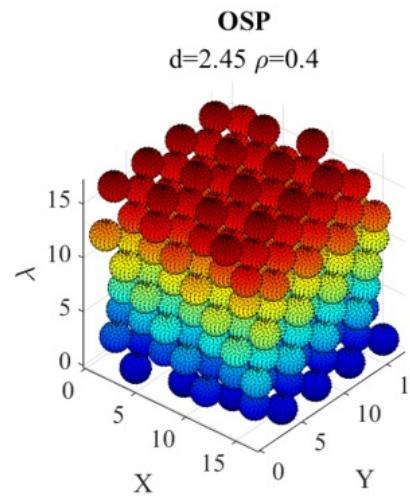
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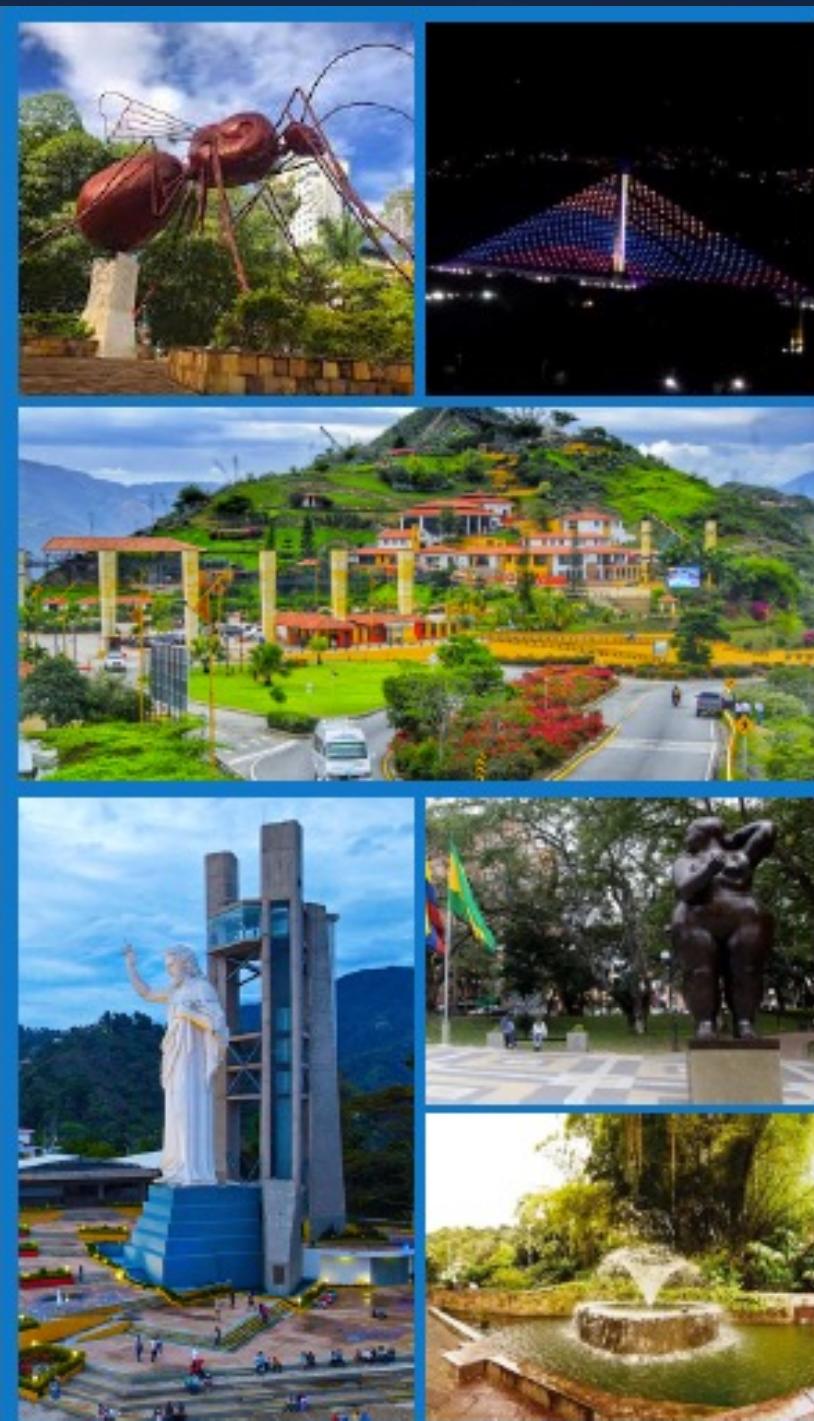
BTES16

OSP16

Conclusion

- We present a Multispectral Filter Array by Optimal Sphere Packing (MSFA-OSP). This approach extends the idea of CFA (RGB) to multispectral imaging.
- Our MSFA-OSP provides 2 [dB] extra of PSNR compared to the best of other SOTA MSFA.
- The advantages of the optimal filter distribution include reducing artifacts such as false colors and the zipper effect of demosaicking algorithms.
- Future works will extend the sphere packing framework to higher dimensions of the plenoptic function, such as compressive spectral-video.





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