Magic of Log Returns: Concept

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Magic of Log Returns: Concept – Part 1

Ever heard of the term log returns? No, it is not a written record of your fund performance or anything to do with a wooden block. But it is also nothing to fret about if this is the first time you hear of it. While log returns may be unfamiliar, most of us should have come across the mathematical term logarithm during school days. Except that back then, you are probably wondering why you would even need to know anything about this cryptic expression. I did not fully appreciate it until much later as well.

Making mathematics simple can be a rather challenging task. So when I started writing this article, I was thinking whether I can just do away with explaining all the mathematical complexities and dive straight into how you can use log returns. At the end, I decided it is not going to look complete without some academic treatment on the topic. I know some will cringe at the thought of having to relive their school days again, but not to worry, I am more of a practitioner than a theorist, so I will try to make this as painless as I can. Or if you rather not go through this, then feel free to skip it and dive straight into Part 2 to look at the practical applications.

Let's Compound Away – Compound Returns

The logarithm function is inexplicably linked to the compounding function. To revisit the concept of compounding, let's use a typical bank deposit example. If you put $100 in a bank with an annual interest rate of 10% and a yearly compounding period. What you get in a year can be expressed as follows:

$$D_t = D_0(1 + \frac{R}{n})^{nt} = 100(1 + \frac{0.1}{1})^{1\times1} = 110$$

Note that this formula is the same as the sum of a geometric series where

D_0 is the initial deposit of $100

D_t is the value of the deposit at the end of the period

R is the 10% annual interest rate

n is the number of compounding period per year which is 1 since we are doing yearly compounding t is the total time period in years which is 1 year

What if we increase n to 365 i.e. daily compounding? Then we would get

$$D_t = D_0(1 + \frac{R}{n})^{nt} = 100(1 + \frac{0.1}{365})^{365\times 1} = 110.51558$$

With a higher frequency of compounding, you ended up with $110.51558 at the end of the year. $10.51558 is the effective interest you make a year from compounding a 10% annual rate daily. This is more than the $10 in the yearly compounding scenario. Why? Because in a daily compounding scenario, you earn interest everyday. And those interest in turns makes you more interest i.e. interest on interest.

A Special Case of Compounding – Continuous Compounding

So, what happen if we compound infinitely during the period? This is what we called continuous compounding. But, how do we even compute this? First, we have to re-express the equation into the following form:

$$D_t = D_0 \left[\left(1 + \frac{1}{n/R} \right)^{n/R} \right]^{Rt}$$

It might take a while for you to figure out that this is the same equation. All I did was just a little manipulation. Why go through such a seemingly superfluous step? Because we need to get the expression highlighted in red. This red portion has a unique property. When n goes to infinity, the number within this red bracket magically converges to the Euler number called **e** which has a value of 2.7182818284590 .... This is also the number used in your exponential function, and with it, we can further simplify our equation to arrive at a figure for a continuous compounding scenario:

$$D_t = D_0 e^{Rt} = 100e^{0.1\times 1} = 110.51709$$

Under continuous compounding, you will get an effective interest of $10.51709 or in percentage terms 10.51709%. The key takeaway at this stage is to know the relationship between effective interest and continuously compounded interest: (1) Effective interest rate – this is the interest you get as a proportion to the amount you put in and it depends on the frequency of compounding, (2) Continuous compounded interest rate – this is the interest rate that, when compounded continuously, gives you the effective interest rate for this scenario.

Now Let's Log Away – Log Returns

So how do all these relate to log returns? Let us now use stock prices instead of bank deposits to illustrate the concept.

P_0 – Initial price of the stock

P_t – Price of the stock at the end of the period

R – continuously compounded rate over the period

r – simple return of the stock over time t

We can equate P_t as follows:

$$P_t = P_0(1 + r) = P_0 e^R$$

If you compare this against our earlier bank deposit example, you will notice that the simple return r is actually the equivalent of the effective interest, and R is the continuously compounded return needed to produce the simple return r during that period.

Dividing all by P_0 and applying a special logarithm called the natural logarithm (ln) which uses the Euler number e as its base, it gives us

$$ln\left(\frac{P_t}{P_0}\right) = \ln(1 + r) = lne^R$$

$$ln\left(\frac{P_t}{P_0}\right) = \ln(1 + r) = Rlne$$

The logarithm of a number that is equal to its base gives you a value of 1. So,

$$ln\left(\frac{P_t}{P_0}\right) = \ln(1 + r) = R$$

ln(1+r) is what we called the log returns. It is the same as R which is the continuously compounded rate of return that will grow the price of the stock from P_0 to P_t.

Cool Stuffs About Log Returns

There a couple of interesting things about log return.

1. **Log returns can be added across time periods**. But adding simple returns, on the other hand, can lead to misleading outcomes. What do I mean by that? For example, let's say you have a stock worth $100 that rose to $120 in the first time period and then goes back to $100 in the second time period. Going by simple returns, you will get a 20% increase in the first time period and -16.7% decrease in the second time period. If you just add them up or even take an average, you will get a total return of 3.3% and an average return of 1.7% even though you did not make any money at all. Log returns, however, being the continuously compounded return, can be added across time. Adding up the log returns over the period gives you a total and average return of 0% in this example.

Time	Price	r	ln(1+r)
0	100		
1	120	20.0%	18.2%
2	100	-16.7%	-18.2%
	Total Ret	3.3%	0.0%
	Avg Ret	1.7%	0.0%

If you want a little mathematical rigour:

$$ln\left(\frac{P_2}{P_0}\right) = ln\left(\frac{P_2}{P_1}\frac{P_1}{P_0}\right) = ln\left(\frac{P_2}{P_1}\right) + ln\left(\frac{P_1}{P_0}\right) = ln(1 + r_2) + ln(1 + r_1)$$

The convenience of being able to use addition instead of multiplication makes manipulation in Excel a lot easier.

2. **Log returns can be easily converted back into simple returns**. To get simple returns out from the log returns, you can easily do it by applying the exponential function.

$$r = e^{\ln(1+r)} - 1$$

3. **Log returns follows normal distribution.** In the realm of asset pricing, stock prices are assume to follow a lognormal distribution. Note that this is an approximation and not necessarily reflective of the actual distribution. But a lognormal distribution gives the desirable property to ensure that stock prices never go below zero. Stock price also seem to move in an exponential way over time. In addition, if the prices are lognormal distributed, then it follows that the log of its prices will have a normal distribution. And we know that log returns can be expressed in terms of the stock prices (see the equation below). So log returns are normally distributed.

$$ln\left(\frac{P_t}{P_0}\right) = \ln(P_t) - \ln(P_0) = ln(1 + r_t)$$

Not So Cool Stuffs About Log Returns

1. **Log returns is not intuitive**. While log returns is more manageable, simple returns has the advantage of being more intuitive. When people ask you how much you make this year, your reply is going to be something along the lines like “Oh, I make 20%.” And that 20% refers to the simple return. You are almost never going to say things like “Oh, I make a continuously compounded 18.23%.” I mean who speaks like that? If the person who hears it flips out a calculator and start keying the numbers, do not be surprised. Because that is actually a good sign. It means he understands what you are talking about.

2. **Log returns cannot be added across securities of a portfolio in the same time period.** If you managed a portfolio, you are almost going to have at least a few securities. The simple return of your portfolio over any time period is the weighted sum of all the simple returns from each of the security. However, you cannot pull the same stunt with log returns. It will give you an erroneous number. Remember that log returns is a continuous compounded rate over time. When you add log returns, you compound. Across different stocks within the same time period, there are no compounding element here. So for computing portfolio return across contribution from securities within the same time period, use simple returns instead.

	Weight	r	ln (1+r)
Stock A	0.20	45.00%	37.16%
Stock B	0.20	12.00%	11.33%
Stock C	0.20	15.00%	13.98%
Stock D	0.20	16.00%	14.84%
Stock E	0.20	-30.00%	-35.67%
Porfolio	1.00	11.60%	8.33%

wrong portfolio return is implied by log return--> 8.68%

Concluding Part I

Here is the end of Part I or the more theoretical stuffs. I have included an excel file: **Log-Returns-Part-1.xlsx** to illustrate some of the things I talked about here. It is very straightforward, so there is not much to explain. In Part 2 Magic of Log Returns – Practice, I will show more examples in Excel on how log returns can be used including one where I construct a performance table making use of log returns and a few other functions in Excel. It will give you a better appreciation, in particular as to why I say log returns makes things easier.

See you soon!

Magic of Log Returns: Practical – Part 2

This is Part 2 and the final part of the series on Log Returns. I will be focusing solely on its application here using Microsoft Excel. I am using the 2013 version. If you are using a different version, it is fine as well. While you might find some changes in the layout, the Excel built-in formulas I am using should be the same. For those who have no idea what log returns is about, you can take a look at my previous post on Part 1 Magic of Log Returns – Concept.

For this exercise, I am using the historical data for SPDR S&P 500 ETF – SPY from yahoo finance. Yahoo finance is a very useful site for free End-of-Day (EOD) stock data. They use to support APIs that allow us to pull data directly into Excel. But unfortunately, they seemed to have terminated this service permanently after Verizon acquired Yahoo. Nonetheless, you can still download the data manually from the website.

This is the excel file to go with with this post: **Log-Returns-Part-2.** I have retained only the 'Date' and 'Adj Close' of SPY column here. The rest of the columns are not required. 'Adj Close', or in full Adjusted Close, are prices that accounts for dividends and share splits. Unless you have good reasons to exclude dividends and splits from your calculations, it is a good practice to make use of it.

I will show 4 simple applications of log returns here:

(1) Calculating daily simple and daily log returns.

(2) Constructing the NAV that starts at 1.

(3) Calculating annualized returns using both simple and log returns.

(4) Creating a performance table using log returns.

1. Calculating and Comparing Simple and Log Daily Returns

This is the most straightforward part. Open up the Excel file and go to sheet 'Log & Simple Returns'. Then refer to column E and F for the formula keyed into the cells. For simple returns, you can get them by dividing today's price with yesterdays' and then subtracting 1 from the result.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	11993	1993	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01
5	21993	1993	2/2/1993	27.48679	=D5/D4-1	0.21%	1.01	1.01
6	21993	1993	3/2/1993	27.777365	1.06%	1.05%	1.02	1.02

Then using the simple returns in Column E, you can compute your log returns. This is done by applying the Excel built-in formula for natural logarithm, ***ln()*** on the simple return. Nice and simple.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	11993	1993	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01
5	21993	1993	2/2/1993	27.48679	0.21%	=LN(1+E5)	1.01	1.01
6	21993	1993	3/2/1993	27.777365	1.06%	1.05%	1.02	1.02

You might observed that the daily log returns and simple returns are very close in value to each other. But that is not always the case. If the simple daily returns are very large, the difference can be significant. I show an example on this later.

2. Calculating NAV

NAV is really the same series as Adjusted Close, except that I have rebased the Adjusted Close to start at 1. It is more apparent from the NAV how much a security or portfolio has gained since inception. For example, if the NAV today is 10, that means the value of portfolio has grown from 1 to 10 since inception, or a 900% increase. If we talk in terms of adjusted close, then it is harder to see the performance without doing further calculations.

There are 2 ways you can go about building the NAV column.

- Using simple returns – Multiply the previous day's NAV with today's 1 + simple returns.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	11993	1993	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01
5	21993	1993	2/2/1993	27.48679	0.21%	0.21%	=(1+E5)*G4	1.01
6	21993	1993	3/2/1993	27.777365	1.06%	1.05%	1.02	1.02

- Using log returns – Sum up all prior log returns including today's and then apply the exponential function on it to get the NAV.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	11993	1993	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01
5	21993	1993	2/2/1993	27.48679	0.21%	0.21%	1.01	=EXP(SUM(F3:F5))
6	21993	1993	3/2/1993	27.777365	1.06%	1.05%	1.02	1.02

3. Annualized Returns

We always like to talk in terms of annual performance as people like to know how much they can expect to make a year in percentage terms. That is why in most of the fund reports, you will find a standard metric called annualized returns. It is also known as the Compound Annual Growth Rate (CAGR) or the Geometric Annual Return. And how do we calculate this in Excel?

- Using Simple Returns – We can make use of the NAV we have computed earlier. You take the final NAV and divide it by the inception NAV (which by the way is a value of 1). The value is then raise to the power of 252/T where T is the number of trading days over the entire period considered. 252 is the number of US trading days in a year. The result is then subtracted by 1 to get the annualized return. To generalize it further, if we are using weekly data instead, then we will be raising it to the power of 52/T. There are 52 weeks in a year and T is now in terms of weeks. I believe you get the idea.

$$AnnRet = \left(\frac{NAV_T}{NAV_0}\right)^{252/T} - 1$$

C	D	E	F	G	H	I	J	K	L
		Daily Return		NAV				Annualized Return	
Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)			Using r	Using ln(1+r)
29/1/1993	27.234995			1.00	1.00		Geometric Ann Ret	=(G6452/G3)^(252/COUNT(G4:G6452))-1	9.65%
1/2/1993	27.428684	0.71%	0.71%	1.01	1.01		Arithmetic Ann Ret	10.9%	
2/2/1993	27.48679	0.21%	0.21%	1.01	1.01				

- Using Log Returns – We multiply the average of the daily log returns over the period by 252 and then apply the exponential function on it. Then we subtract 1 from the result to get the annualized return. If we are working with weekly returns, then we multiply the average by 52, or if monthly, then by 12.

$$AnnRet = e^{\frac{252}{T}\sum_{t=1}^{T}\ln(1+r_t)} - 1$$

C	D	E	F	G	H	I	J	K	L
		Daily Return		NAV					Annualized Return
Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)			Using r	Using ln(1+r)
29/1/1993	27.234995			1.00	1.00		Geometric Ann Ret	9.65%	=EXP(AVERAGE(F3:F6452)*252)-1
1/2/1993	27.428684	0.71%	0.71%	1.01	1.01		Arithmetic Ann Ret	10.9%	
2/2/1993	27.48679	0.21%	0.21%	1.01	1.01				

As a word of caution, some people find it more convenient to just use arithmetic average as a proxy for annualized returns. And what do I mean by that? Using our exercise as an example, they will use the average of the daily simple return series and multiply it by 252. But for reasons that I have already mentioned in **Part 1**, you should use the geometric return instead. The arithmetic approach can differ significantly if the return series is volatile. You can refer to the worksheet "Inflated Returns". In this sheet, I have inflated the daily returns of SPY by 5 times. The CAGR or geometric annualized returns is 12.73%. But if you used an arithmetic average on the simple returns instead, you would end up with a whopping 54.4%.

4. Constructing A Performance Table

Performance table is a common sight in monthly reports. You may want to create one, but find it too much of a pain to manually locate and find all the corresponding NAVs to compute the monthly returns. Actually, it does not have to be that difficult. All you need is to make use of log returns and a few functions in Excel.

	1	2	3	4	5	6	7	8	9	10	11	12	YTD
1993		1.07%	2.24%	-2.56%	2.70%	0.36%	-0.49%	3.83%	-0.73%	1.97%	-1.07%	1.23%	8.71%
1994	3.49%	-2.92%	-4.19%	1.12%	1.59%	-2.29%	3.23%	3.81%	-2.53%	2.84%	-3.98%	0.73%	0.40%
1995	3.36%	4.08%	2.78%	2.96%	3.97%	2.02%	3.22%	0.45%	4.24%	-0.29%	4.45%	1.57%	38.05%
1996	3.56%	0.32%	1.72%	1.09%	2.27%	0.88%	-4.49%	1.93%	5.59%	3.23%	7.30%	-2.38%	22.50%

The performance table we want to create is segregated by month and year. So to get those figures, you will have to identify the returns by their corresponding month and year. We can do this by creating identifier columns that extract the month and year of each date using the MONTH() and YEAR() function. In column A, we concatenate MONTH() & YEAR() to get a unique MTHYR ID that corresponds each date to a specific month and year. This will assist us later in getting the monthly returns.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	=MONTH(C3)&YEAR(C3)	1993	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01

Column B just gives us the Year. This is to help us in computing the yearly returns.

	A	B	C	D	E	F	G	H
1					Daily Return		NAV	
2	MTHYR ID	YR ID	Date	Adj Close	Simple Ret r	Log Ret ln(1+r)	Using r	Using ln(1+r)
3	11993	=YEAR(C3)	29/1/1993	27.234995			1.00	1.00
4	21993	1993	1/2/1993	27.428684	0.71%	0.71%	1.01	1.01

Now, to get the returns for each specific month and year, we just need to sum up the log returns for the days where their IDs matches that particular month and year. This can be done using the SUMIF function. You then apply the exponential function on the result and subtract 1 to back out the simple returns for the month. You can lookup the SUMIF function in Excel Help. They have a writeup and examples on how it can be used.

	A	B	E	F	I	J	K	L	M	N
1			Daily Return					Annualized Return		
2	MTHYR ID	YR ID	Simple Ret r	Log Ret ln(1+r)			Using r	Using ln(1+r)		
3	11993	1993				Geometric Ann Ret	9.65%	9.65%		
4	21993	1993	0.71%	0.71%		Arithmetic Ann Ret	10.9%		<----------	some people
5	21993	1993	0.21%	0.21%						
6	21993	1993	1.06%	1.05%			1	2	3	4
7	21993	1993	0.42%	0.42%		1993		=EXP(SUMIF($A:$A,L$6&$J7,$F:$F))-1	2.24%	-2.56%
8	21993	1993	-0.07%	-0.07%		1994	3.49%	-2.92%	-4.19%	1.12%
9	21993	1993	0.00%	0.00%		1995	3.36%	4.08%	2.78%	2.96%
10	21993	1993	-0.69%	-0.70%		1996	3.56%	0.32%	1.72%	1.09%
11	21993	1993	0.14%	0.14%		1997	6.18%	0.96%	-4.41%	6.26%
12	21993	1993	0.49%	0.49%		1998	1.29%	6.93%	4.88%	1.28%

The yearly returns is done in a similar way. You just need to add up the log returns of the days that has an ID that matches the specific year you are looking for using SUMIF.

	A	B	E	F	I	J	S	T	U	V	W
1			Daily Return								
2	MTHYR ID	YR ID	Simple Ret r	Log Ret ln(1+r)							
3	11993	1993				Geometric Ann Ret					
4	21993	1993	0.71%	0.71%		Arithmetic Ann Ret					
5	21993	1993	0.21%	0.21%							
6	21993	1993	1.06%	1.05%			9	10	11	12	YTD
7	21993	1993	0.42%	0.42%		1993	-0.73%	1.97%	-1.07%	1.23%	=EXP(SUMIF($B:$B,$J7,$F:$F))-1
8	21993	1993	-0.07%	-0.07%		1994	-2.53%	2.84%	-3.98%	0.73%	0.40%
9	21993	1993	0.00%	0.00%		1995	4.24%	-0.29%	4.45%	1.57%	38.05%
10	21993	1993	-0.69%	-0.70%		1996	5.59%	3.23%	7.30%	-2.38%	22.50%

End Of The Magic

That is all I have regarding log returns for now. There are other ways you can implement the things I have done here. For example, you can also write a VBA script to churn out a performance table. Feel free to explore other means of implementation. Meanwhile, hope you gained something out of this and have a good day.

Feel free to share the post if you find it useful.