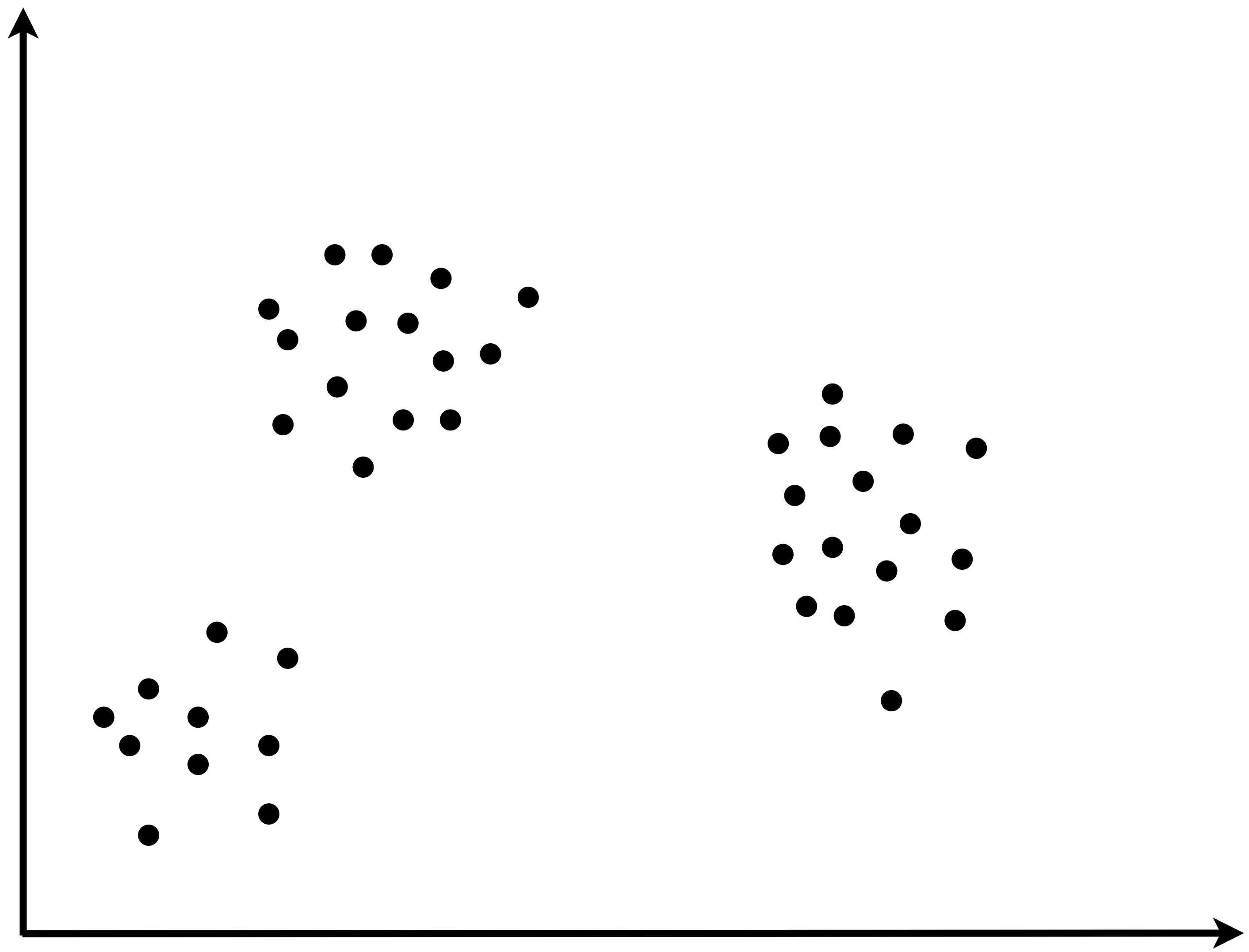
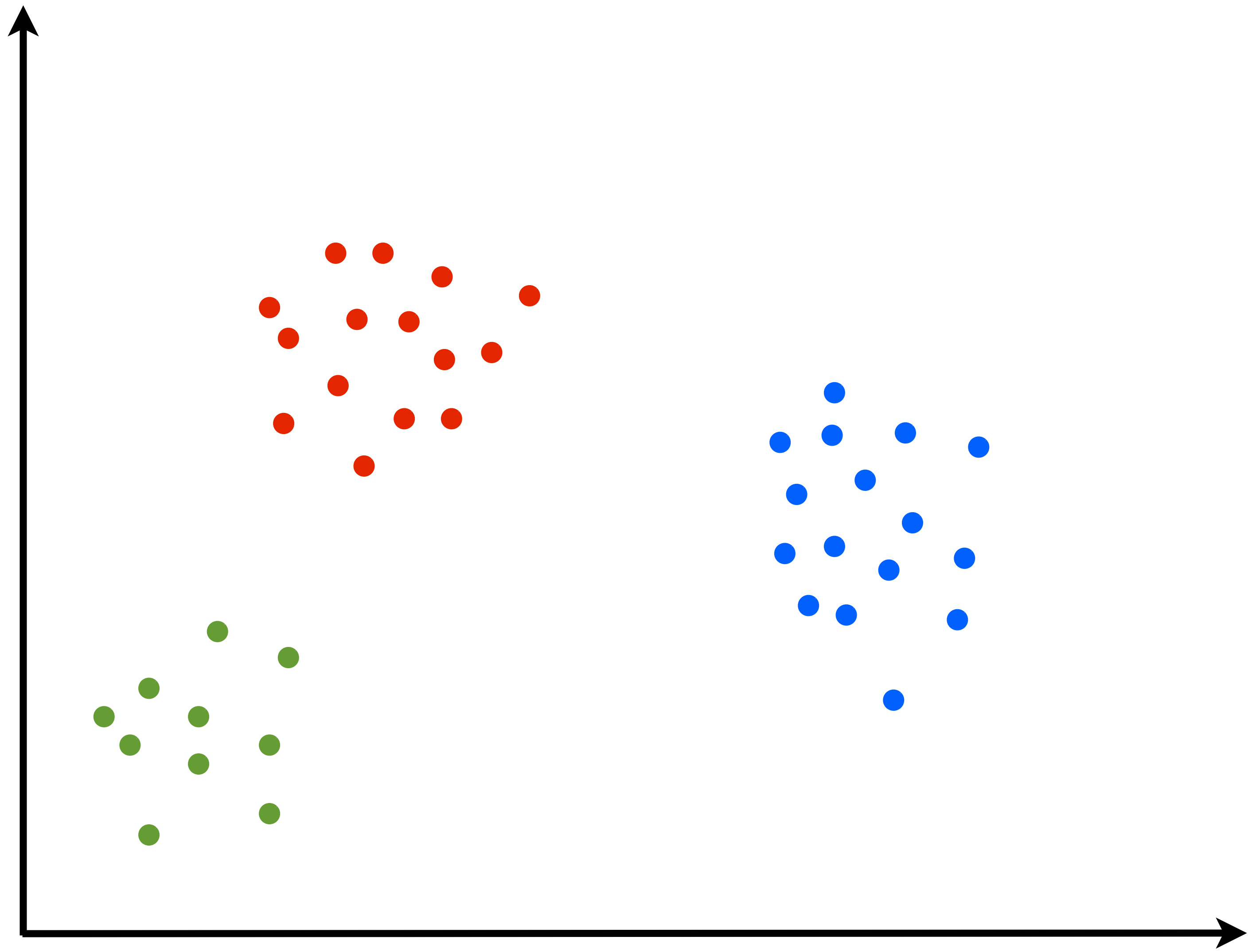
Feature allocations, probability functions, and paintboxes

Tamara Broderick, Jim Pitman, Michael I. Jordan
UC Berkeley

Clustering/Partition

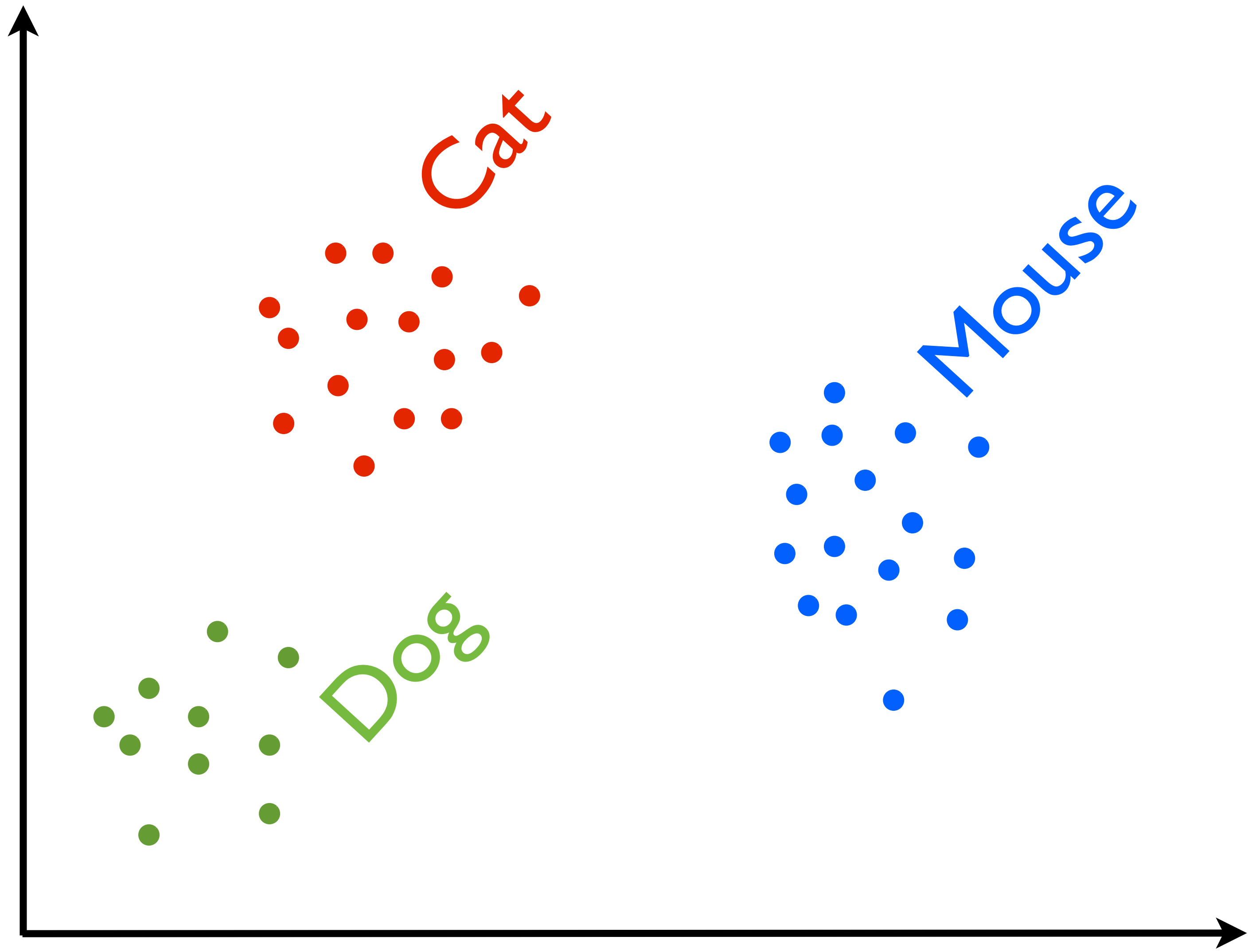


Clustering/Partition



"clusters",
"classes",
"blocks (of a partition)"

Clustering/Partition



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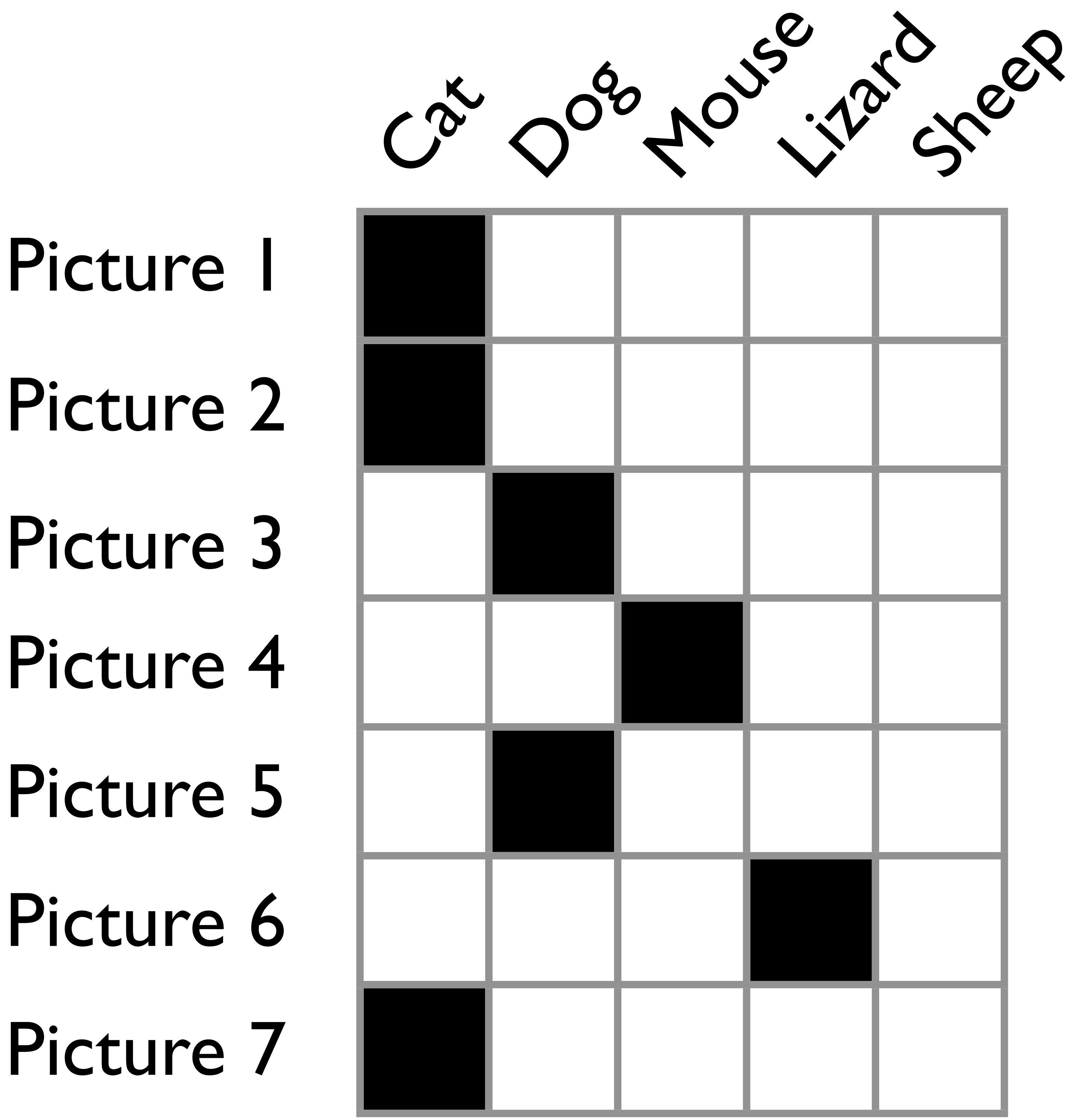
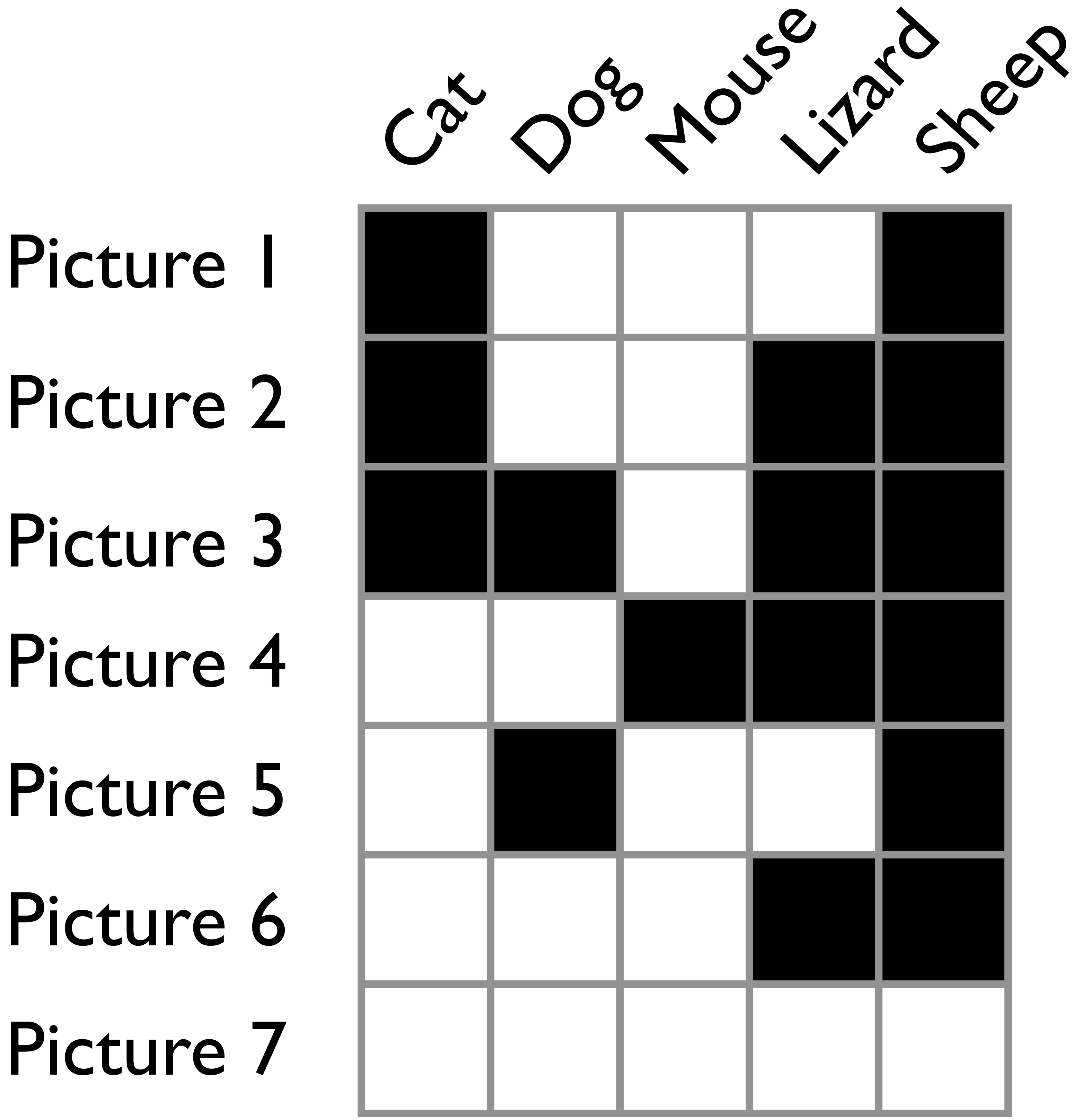


Latent feature allocation



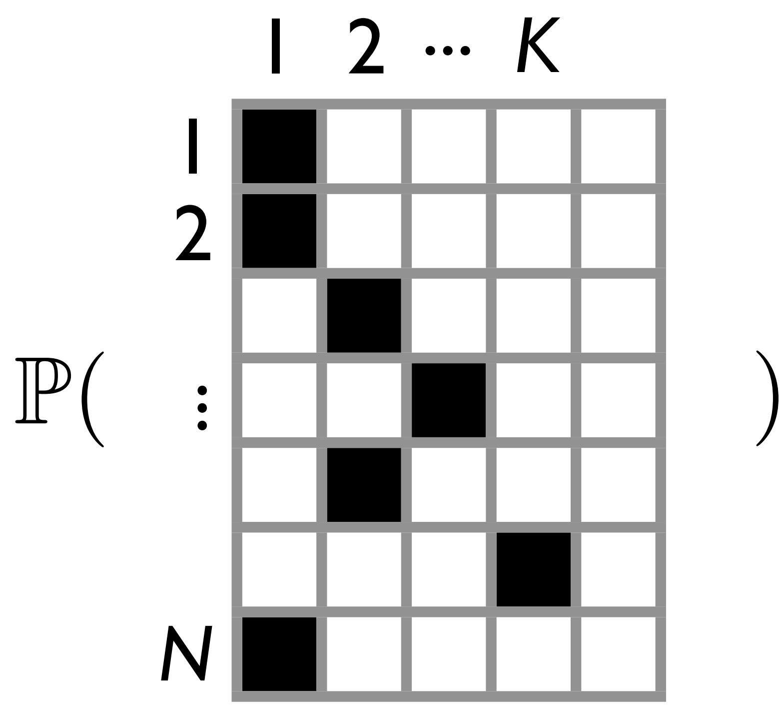
“features”,
“topics”

- Exchangeable
- Finite # of features per data point

Characterizations

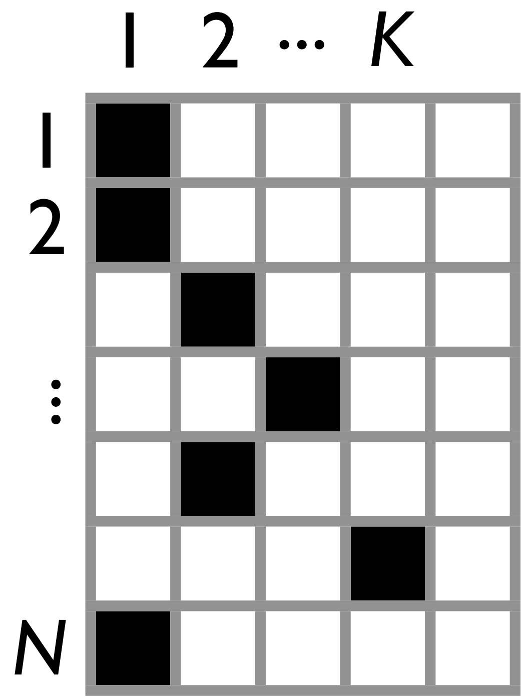
- Exchangeable cluster distributions are characterized
- What about exchangeable feature distributions?

Exchangeable probability functions



Exchangeable probability functions

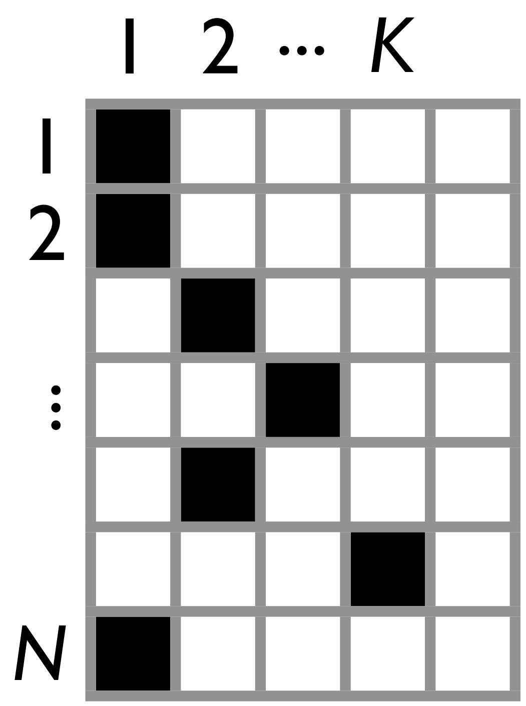
$\mathbb{P}($



$) = p\left(S_{N,1}, \ldots, S_{N,K}\right)$

Exchangeable probability functions

Size of *K*th cluster

$\mathbb{P}($ 

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Exchangeable probability functions

Exchangeable partition probability function (EPPF)

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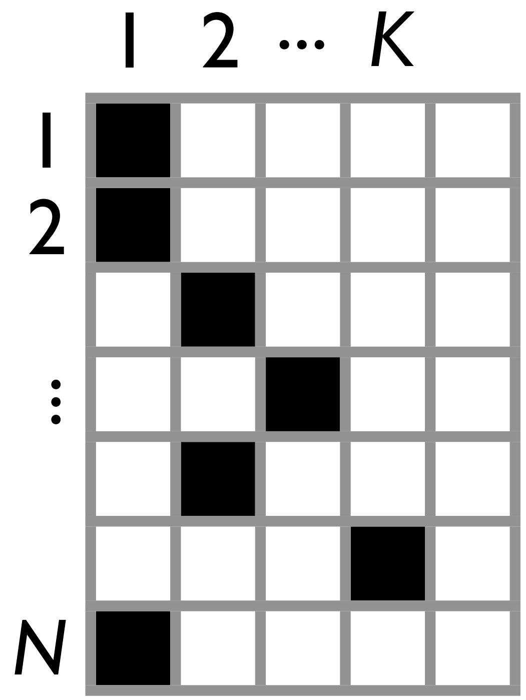

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[Pitman 1995]

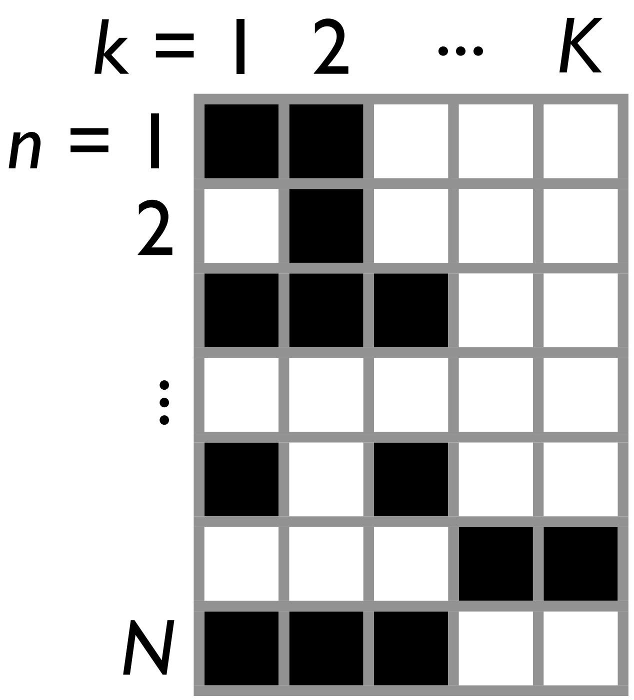
Exchangeable probability functions

"Exchangeable feature probability function" (EFPF)?

Example: Indian buffet process

[Griffiths, Ghahramani 2006]

Example: Indian buffet process



[Griffiths, Ghahramani 2006]

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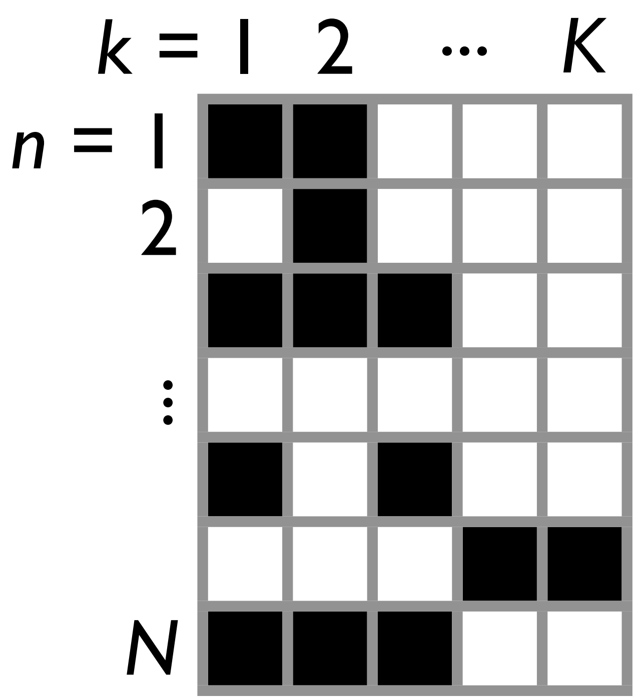

For n = 1, 2, ..., N

[Griffiths, Ghahramani 2006]

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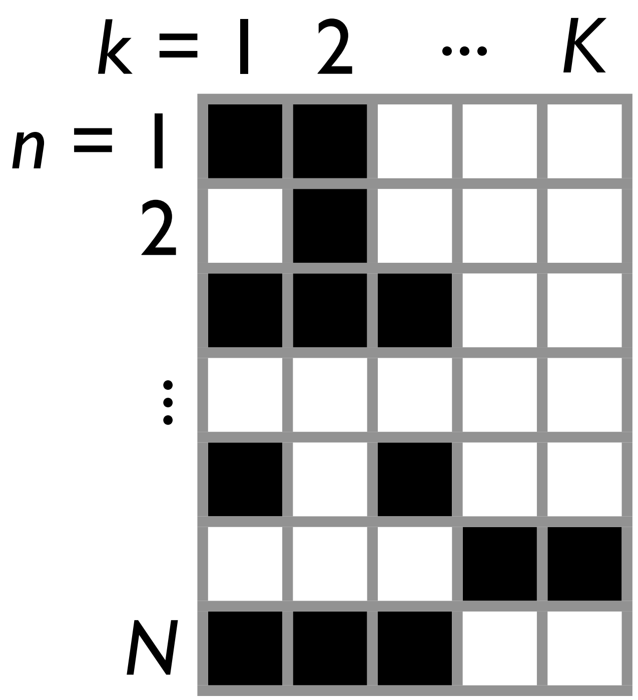
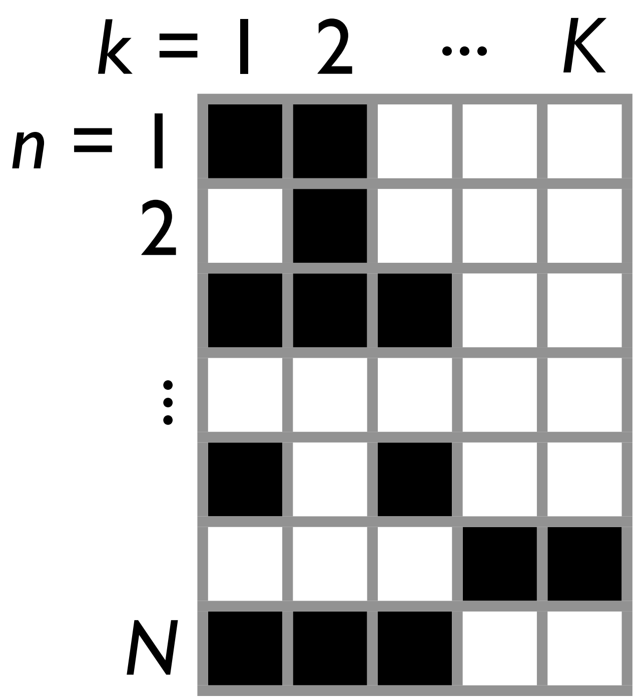


For n = 1, 2, ..., N

1. Data point n has an existing feature k that has already occurred $S_{n-1,k}$ times with probability $\frac{S_{n-1,k}}{\theta + n - 1}$

[Griffiths, Ghahramani 2006]

Example: Indian buffet process



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2. Number of new features for data point n: $K_n^+ = \mathrm{Poisson}\left(\gamma \dfrac{\theta}{\theta + n - 1}\right)$

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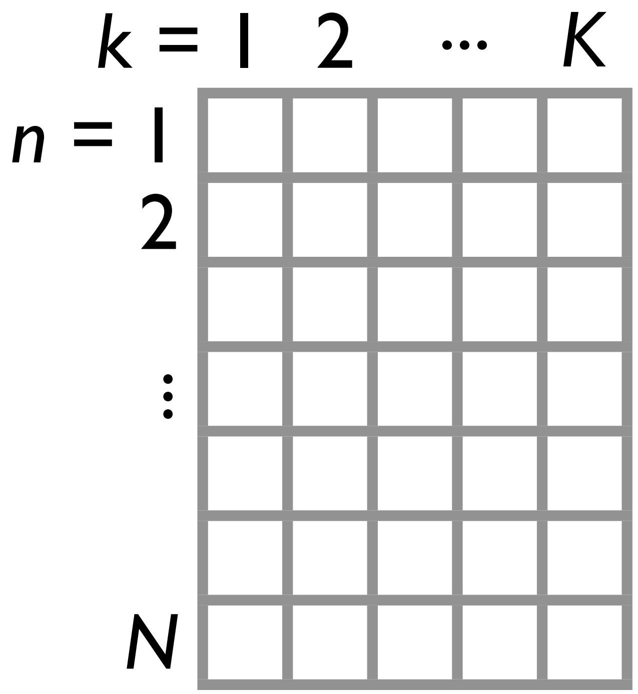

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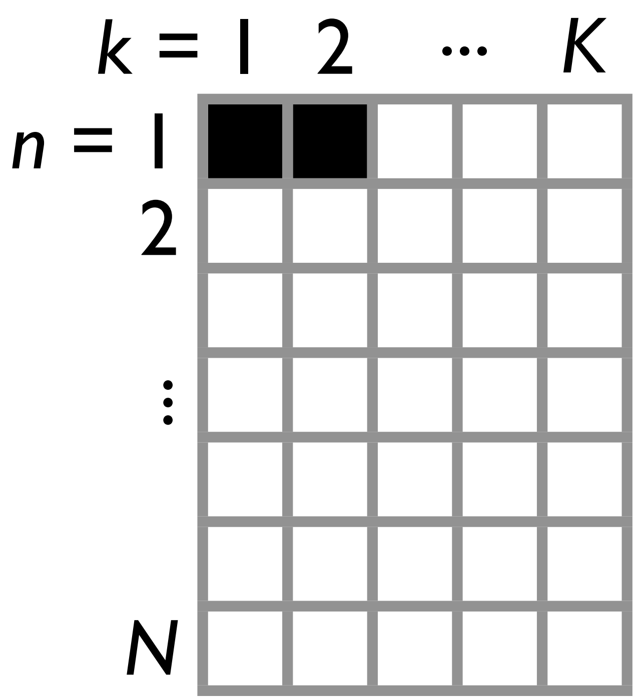

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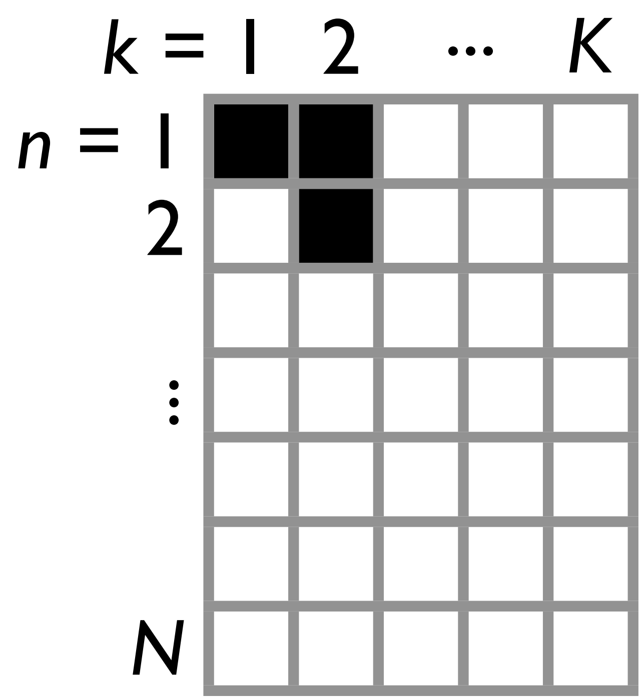

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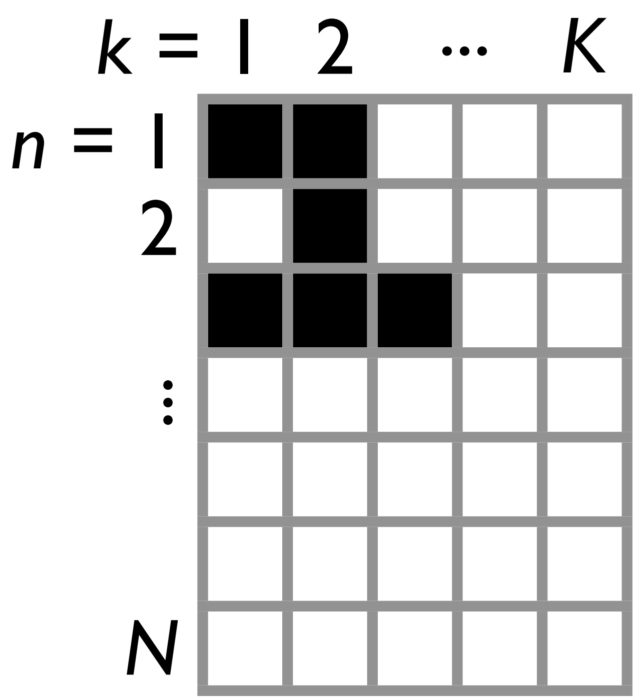

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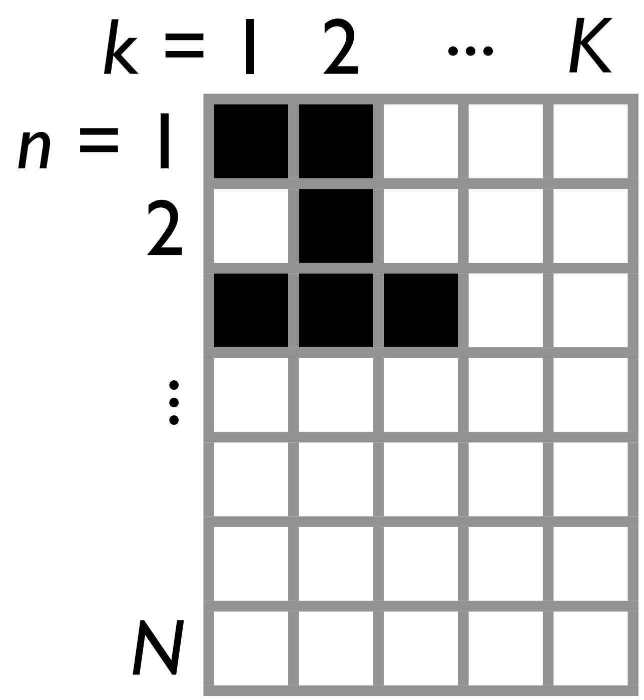

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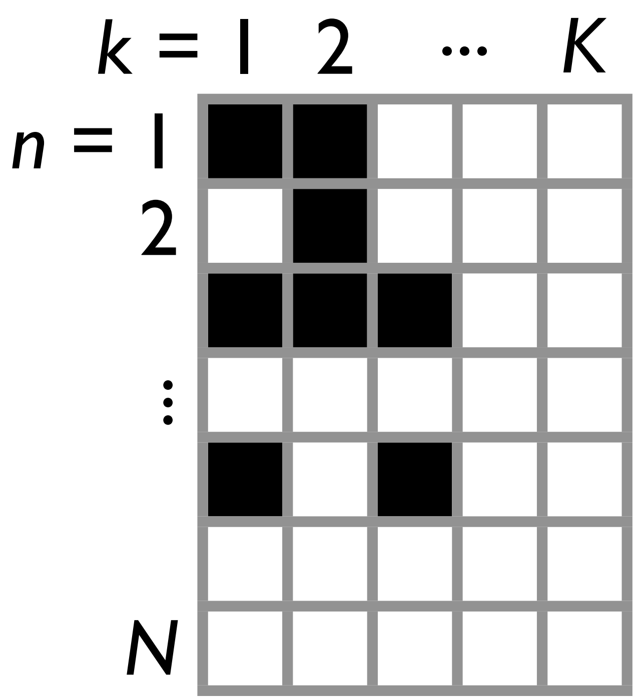

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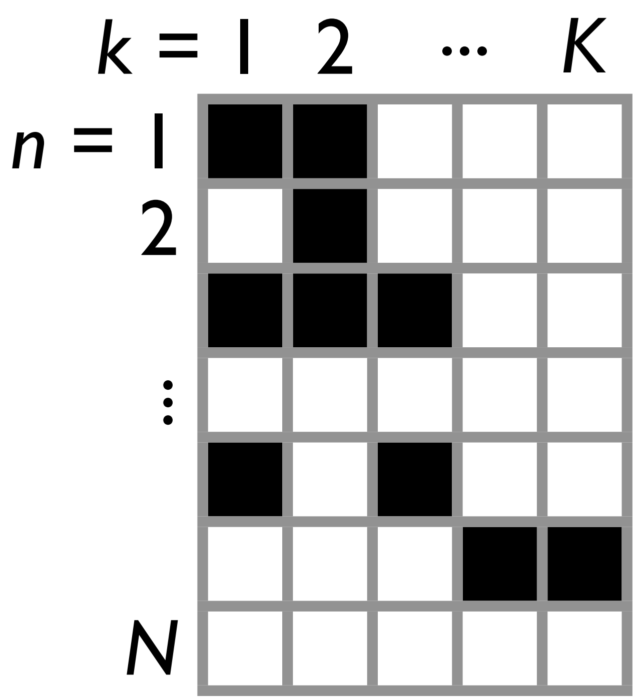

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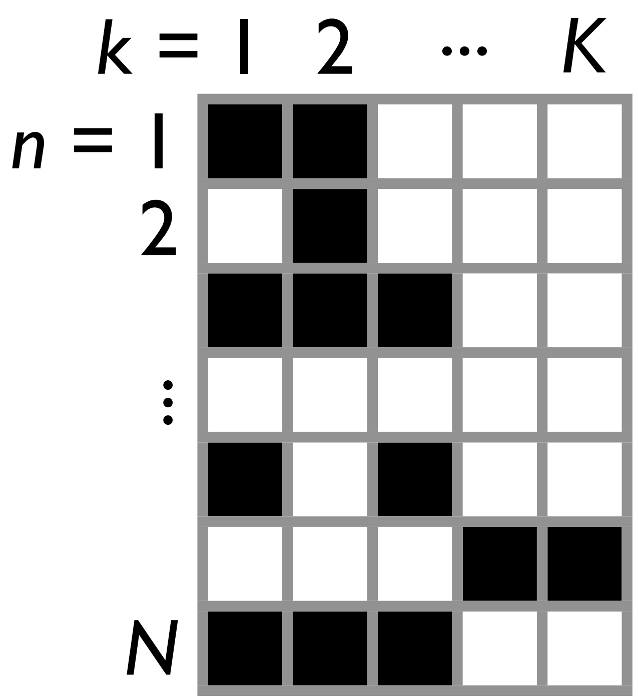

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Exchangeable probability functions

"Exchangeable feature probability function" (EFPF)?

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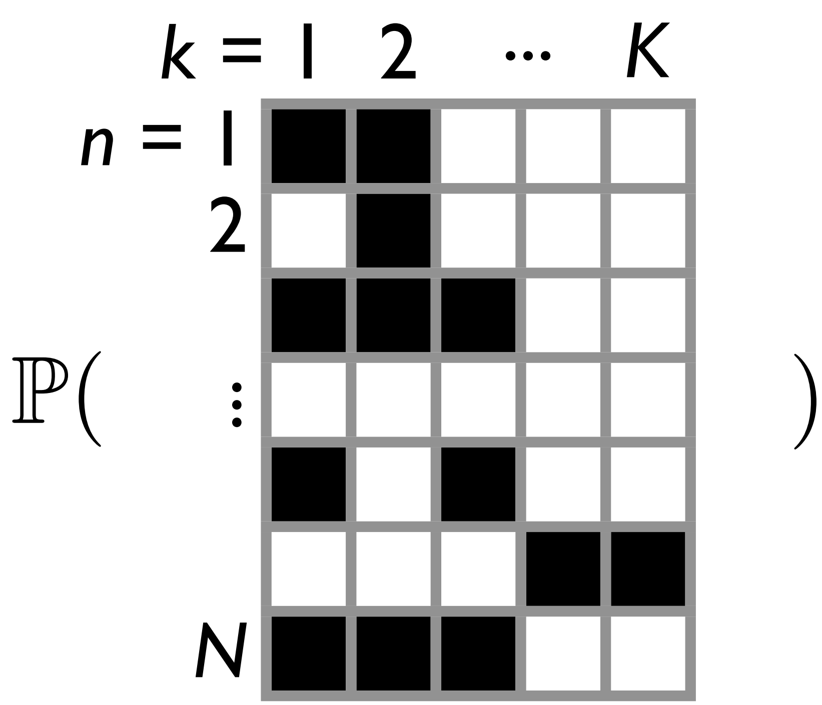
Example: Indian buffet process (IBP)



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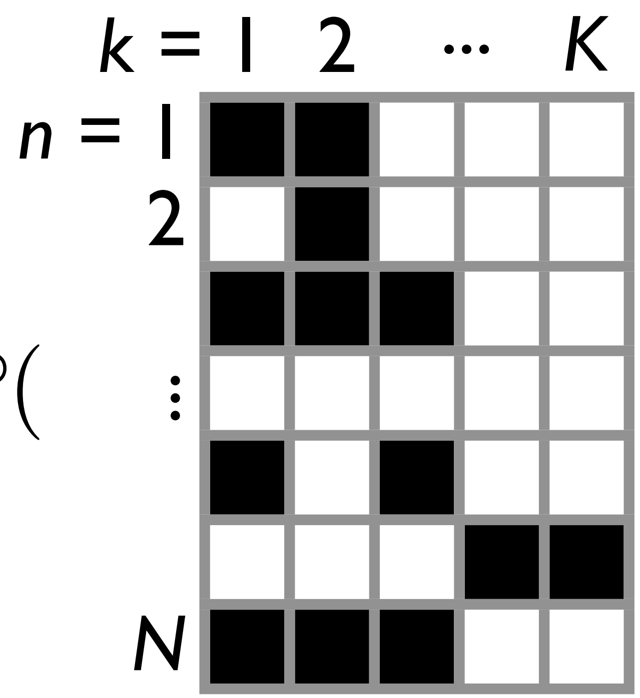
$\mathbb{P}($


$)$

$$= \frac{1}{K_N!}(\theta\gamma)^{K_N} \exp\left(-\theta\gamma \sum_{n=1}^{N} (\theta + n - 1)^{-1}\right) \prod_{k=1}^{K_N} \frac{\Gamma(S_{N,k})\Gamma(N - S_{N,k} + \theta)}{\Gamma(N + \theta)}$$

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$$\mathbb{P}\left(\begin{array}{r|ccccc} & k=1 & 2 & & \cdots & K \\ \hline n=1 & \blacksquare & \blacksquare & \square & \square & \square \\ 2 & \square & \blacksquare & \square & \square & \square \\ & \blacksquare & \blacksquare & \blacksquare & \square & \square \\ \vdots & \square & \square & \square & \square & \square \\ & \blacksquare & \square & \blacksquare & \square & \square \\ & \square & \square & \square & \blacksquare & \blacksquare \\ N & \blacksquare & \blacksquare & \blacksquare & \square & \square \end{array}\right)$$

Size of *k*th feature

$$= \frac{1}{K_N!}(\theta\gamma)^{K_N} \exp\left(-\theta\gamma \sum_{n=1}^{N} (\theta + n - 1)^{-1}\right) \prod_{k=1}^{K_N} \frac{\Gamma(S_{N,k})\Gamma(N - S_{N,k} + \theta)}{\Gamma(N + \theta)}$$

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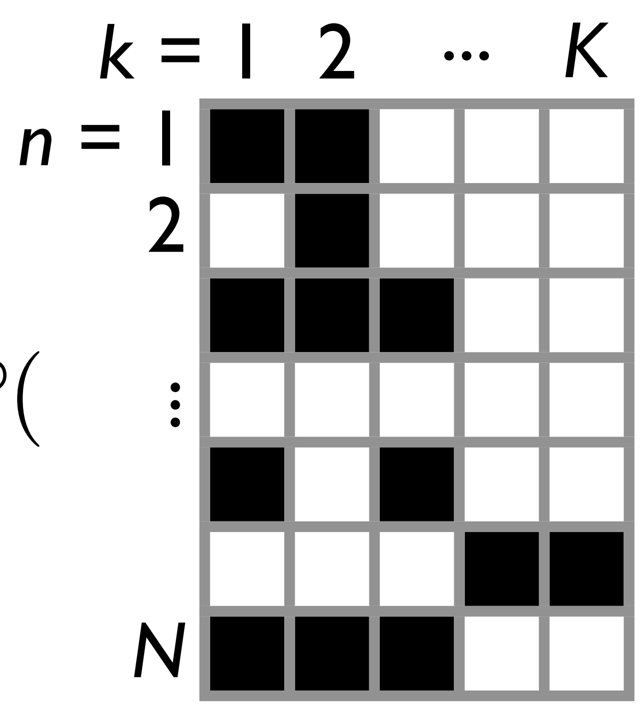
$\mathbb{P}($  $)$

Size of *k*th feature

Number of features

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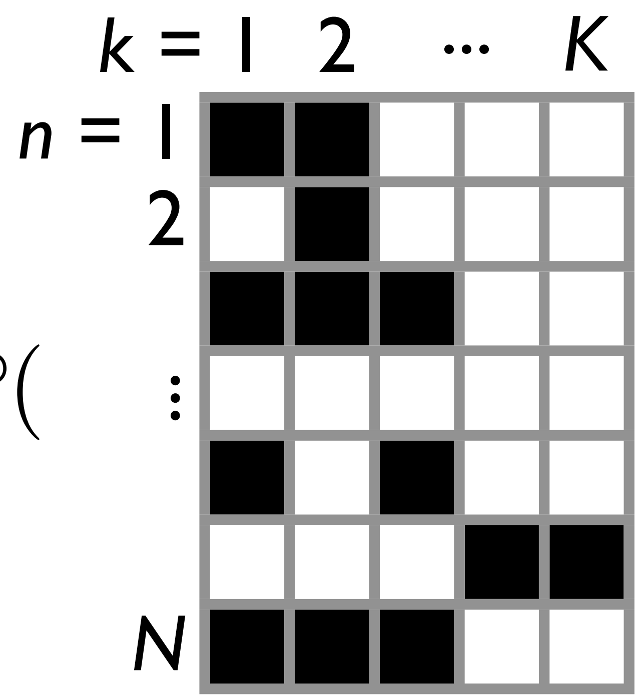
$\mathbb{P}($  $)$

Number of data points

Size of *k*th feature

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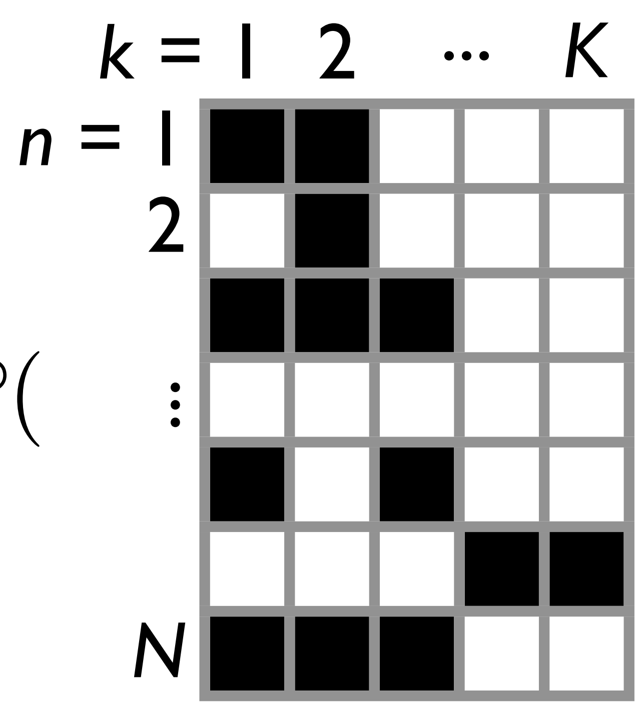
$\mathbb{P}($  $)$

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$$= p(N; S_{N,1}, S_{N,2}, \ldots, S_{N,K})$$

[Broderick, Jordan, Pitman 2012]

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$$
\mathbb{P}\left(
\begin{array}{c|ccccc}
 & k=1 & 2 & & \cdots & K \\
\hline
n=1 & 1 & 1 & 0 & 0 & 0 \\
2 & 0 & 1 & 0 & 0 & 0 \\
 & 1 & 1 & 1 & 0 & 0 \\
\vdots & 0 & 0 & 0 & 0 & 0 \\
 & 1 & 0 & 1 & 0 & 0 \\
 & 0 & 0 & 0 & 1 & 1 \\
N & 1 & 1 & 1 & 0 & 0
\end{array}
\right)
$$

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"EFPF"

[Broderick, Jordan, Pitman 2012]

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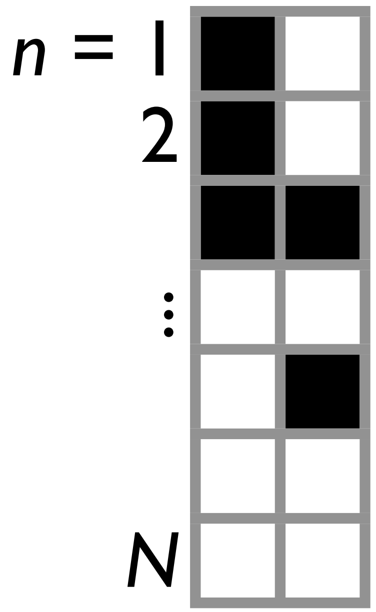
Counterexample



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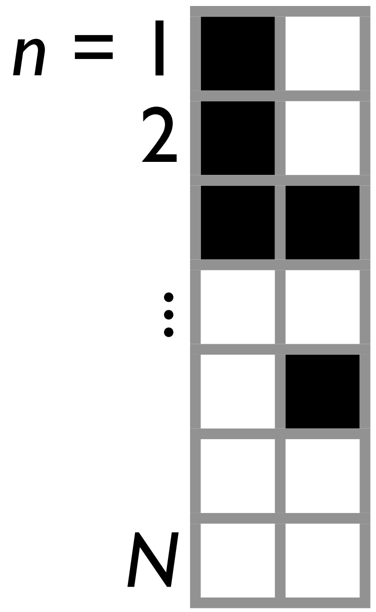
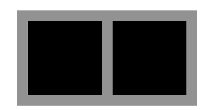
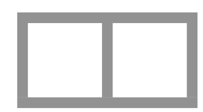


$\mathbb{P}(\text{row} =$  $) = p_1$

$\mathbb{P}(\text{row} =$  $) = p_2$

$\mathbb{P}(\text{row} =$  $) = p_3$

$\mathbb{P}(\text{row} =$  $) = p_4$

[Broderick, Jordan, Pitman 2012]

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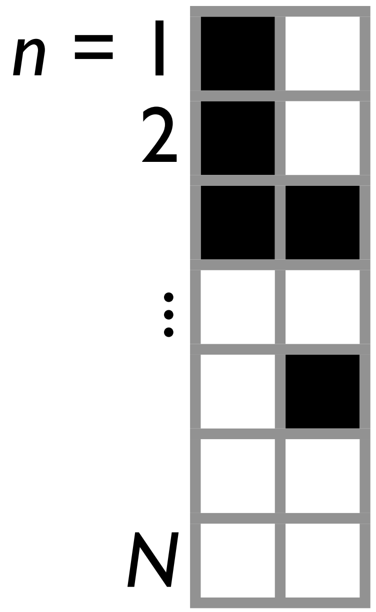

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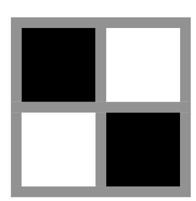

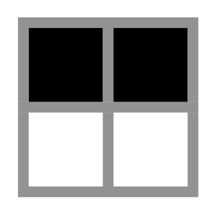

[Broderick, Jordan, Pitman 2012]

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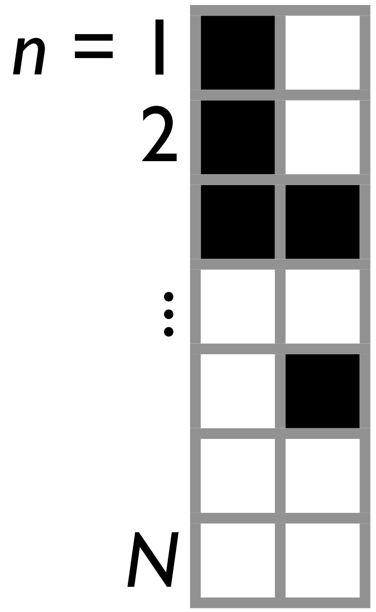

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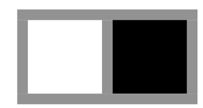
$\mathbb{P}(\mathrm{row} =$  $) = p_2$

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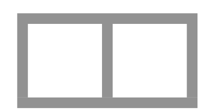
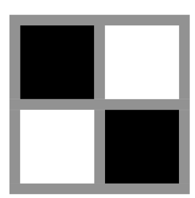
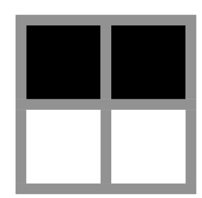
$\mathbb{P}(\mathrm{row} =$  $) = p_4$

$\mathbb{P}($  $)$ $\mathbb{P}($  $)$

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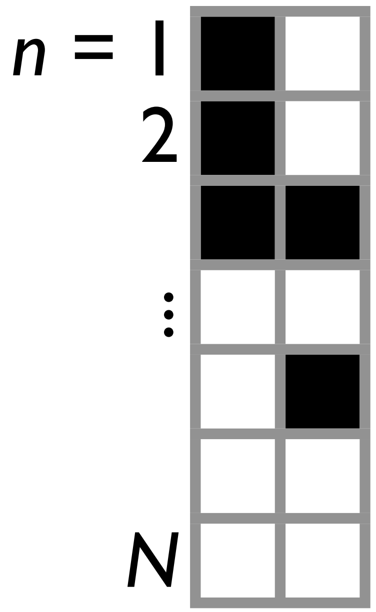

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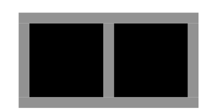
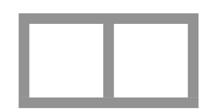
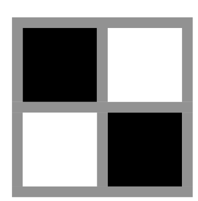
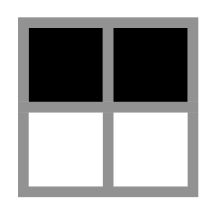
$\mathbb{P}(\text{row} =$  $) = p_3$

$\mathbb{P}(\text{row} =$  $) = p_4$

$\mathbb{P}($  $)$ $\mathbb{P}($  $)$

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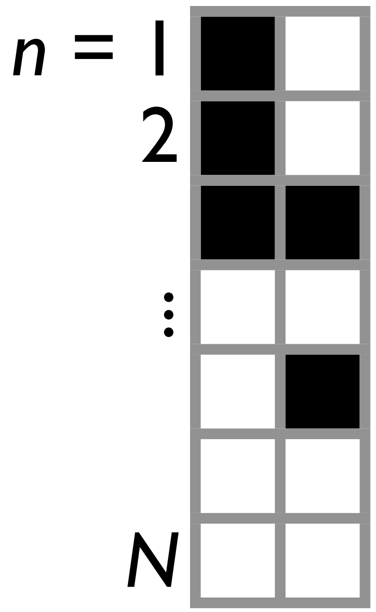
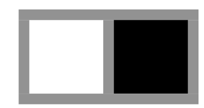
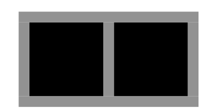
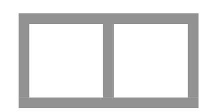
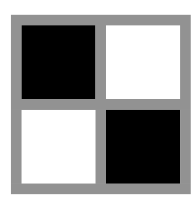
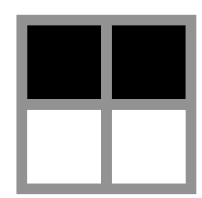


ℙ(row = ) = p_1

ℙ(row = ) = p_2

ℙ(row = ) = p_3

ℙ(row = ) = p_4

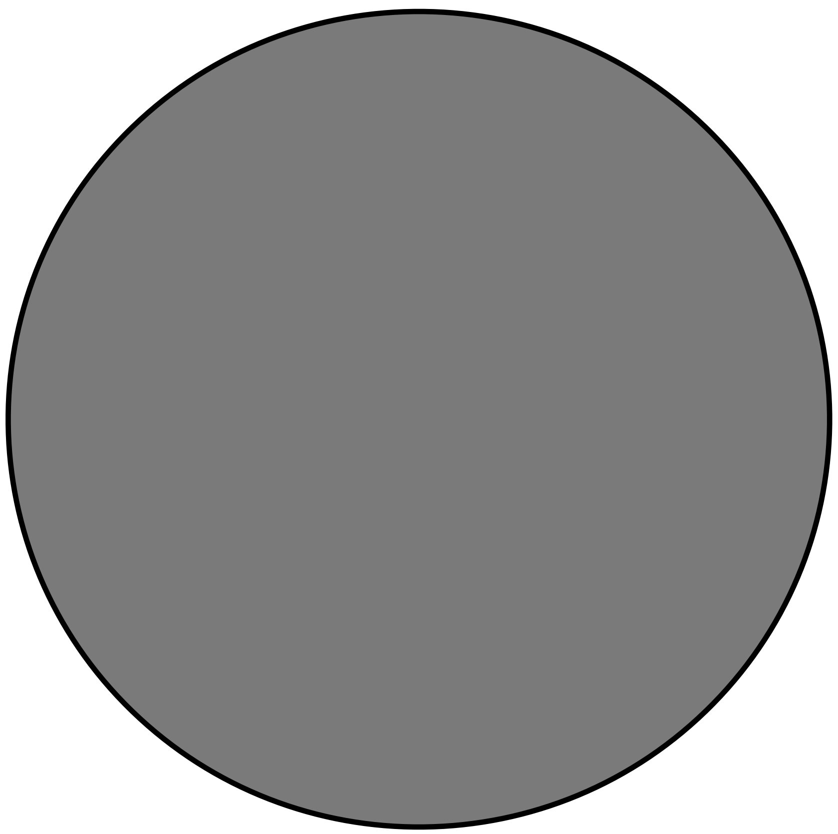
ℙ() ≠ ℙ()

$$p_1 p_2 \neq p_3 p_4$$

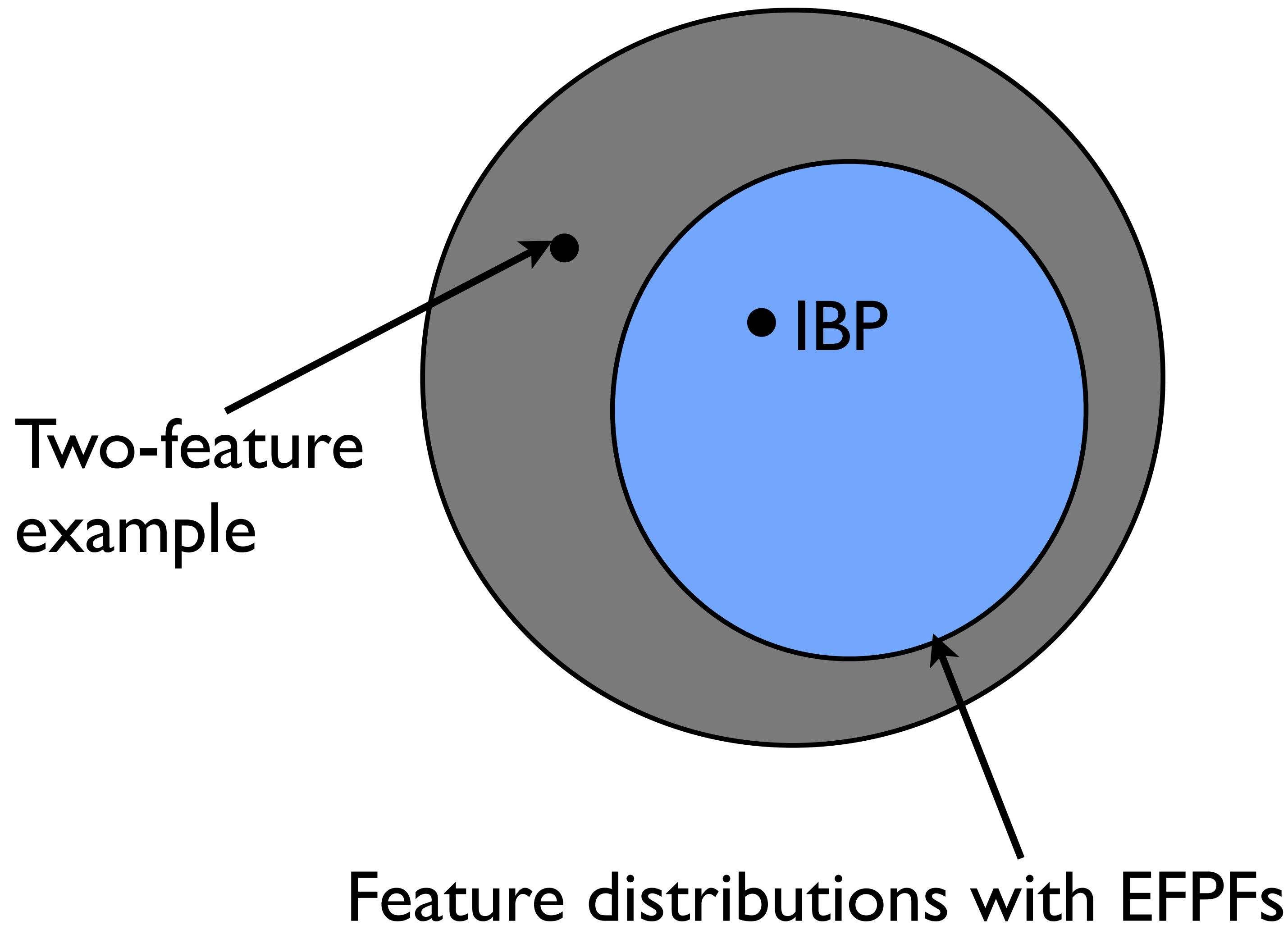
[Broderick, Jordan, Pitman 2012]

Exchangeable probability functions

Exchangeable cluster distributions
= Cluster distributions with EPPFs



Exchangeable feature distributions



[Broderick, Jordan, Pitman 2012]

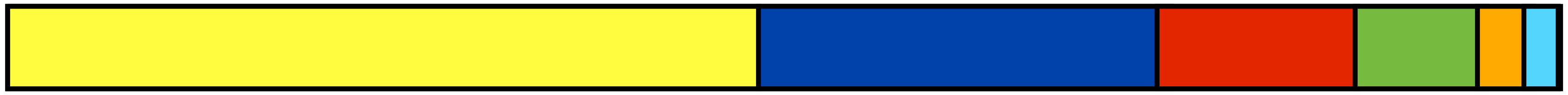
Paintboxes

Exchangeable partition: Kingman paintbox

[Kingman 1978]

Paintboxes

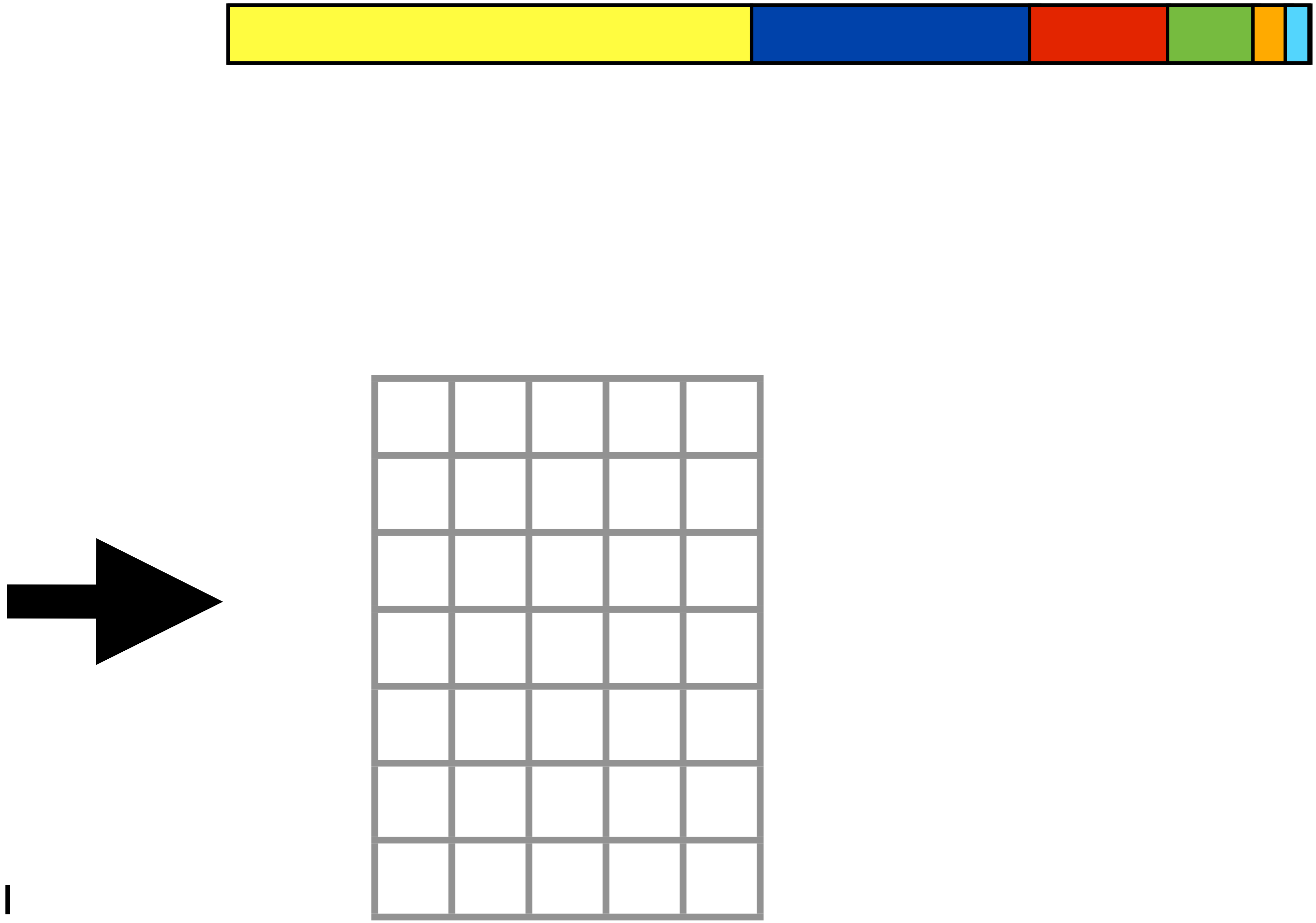
Exchangeable partition: Kingman paintbox



[Kingman 1978]

Paintboxes

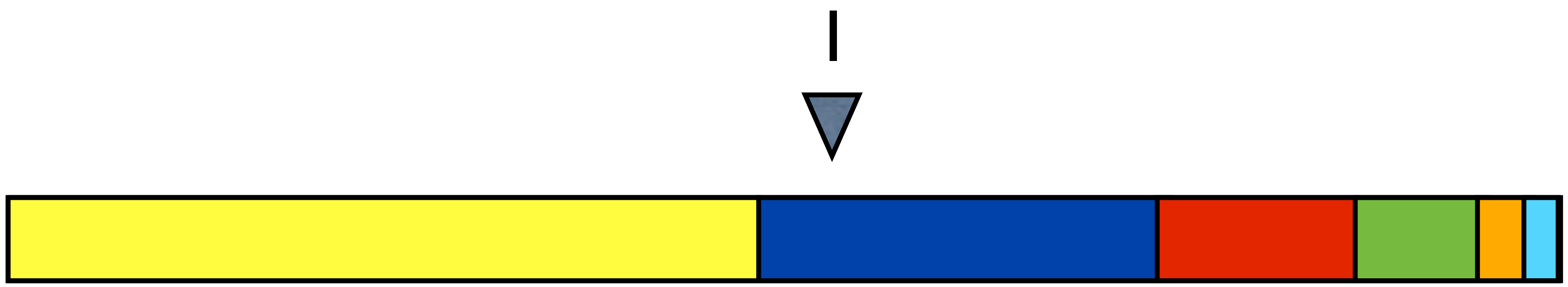
Exchangeable partition: Kingman paintbox

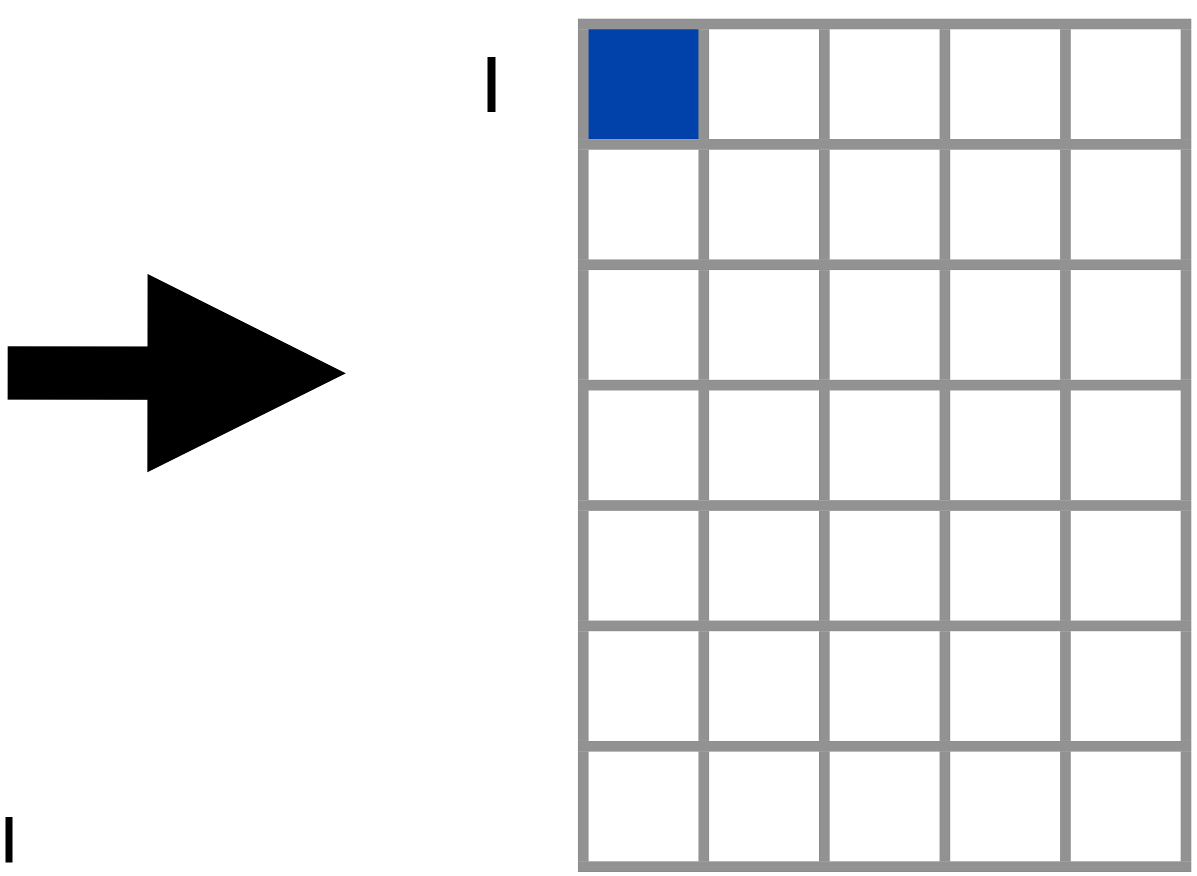


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox





[Kingman 1978]

Paintboxes

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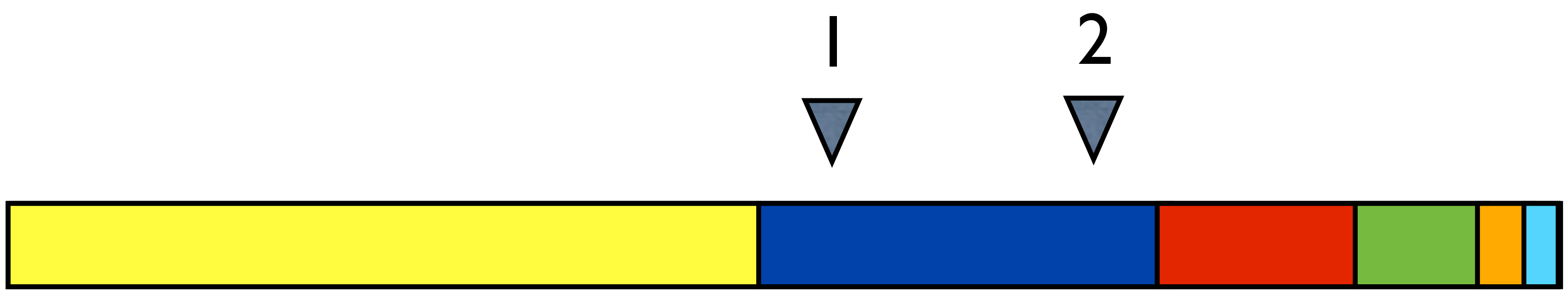
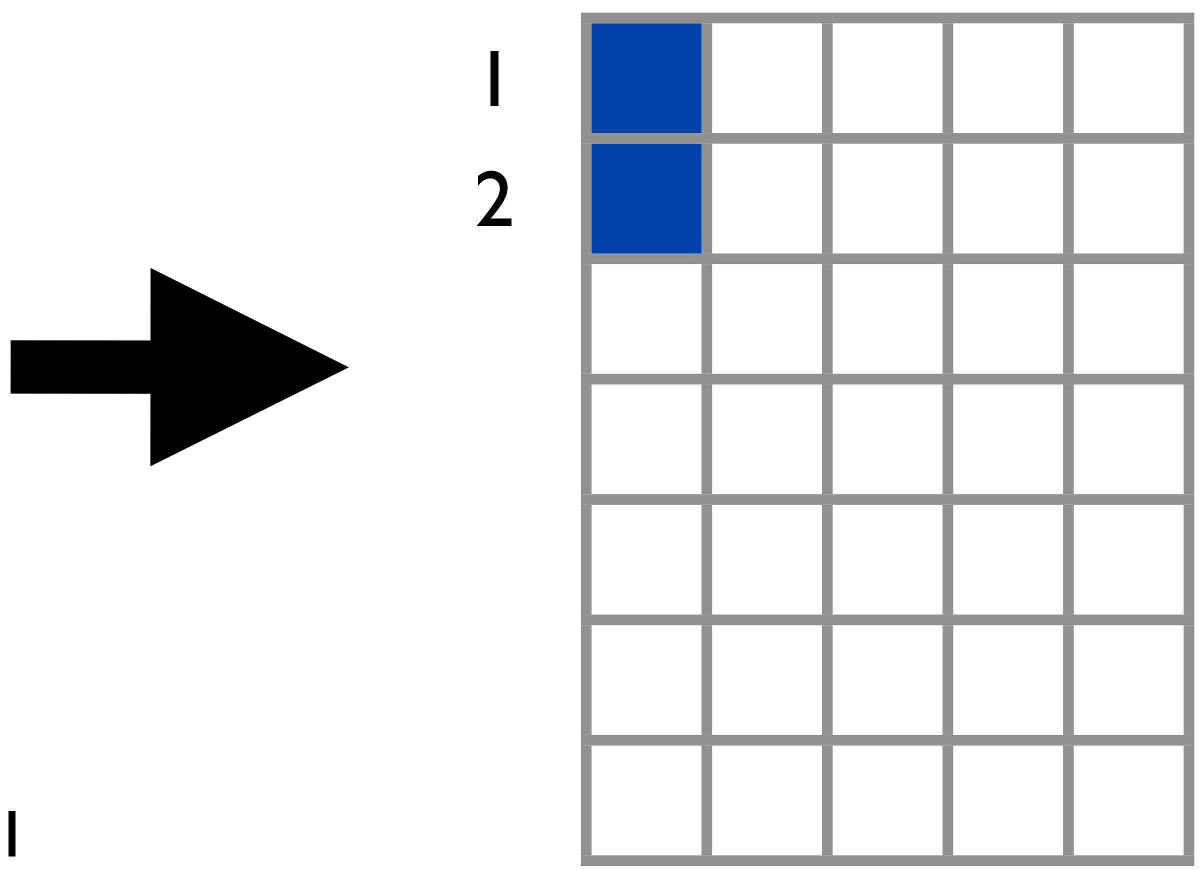
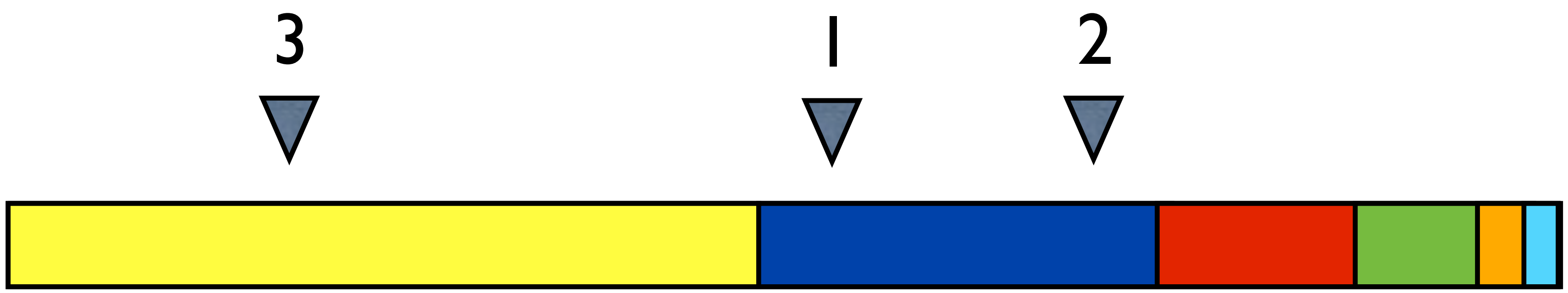


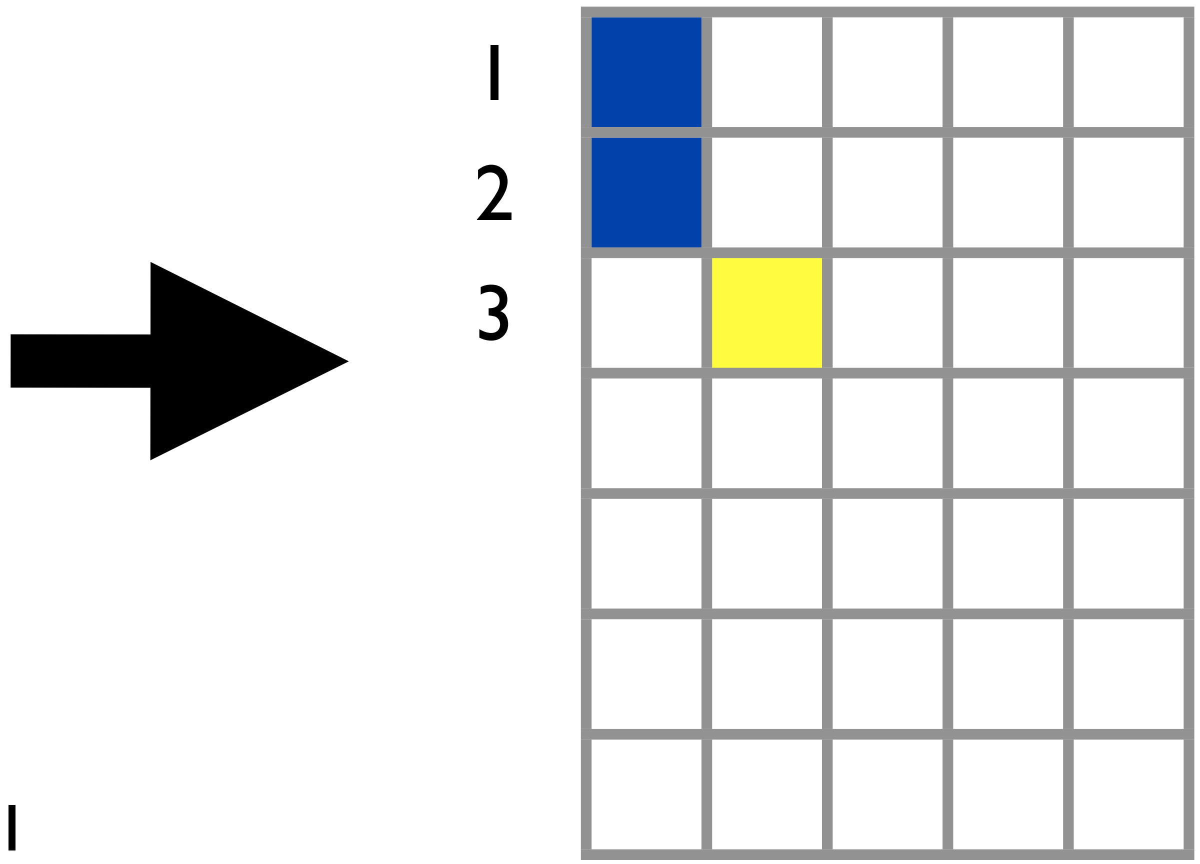


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

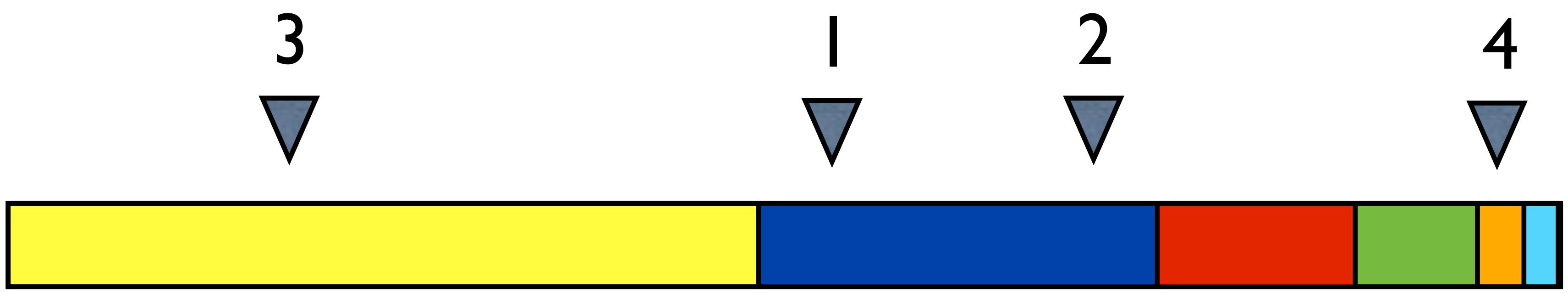


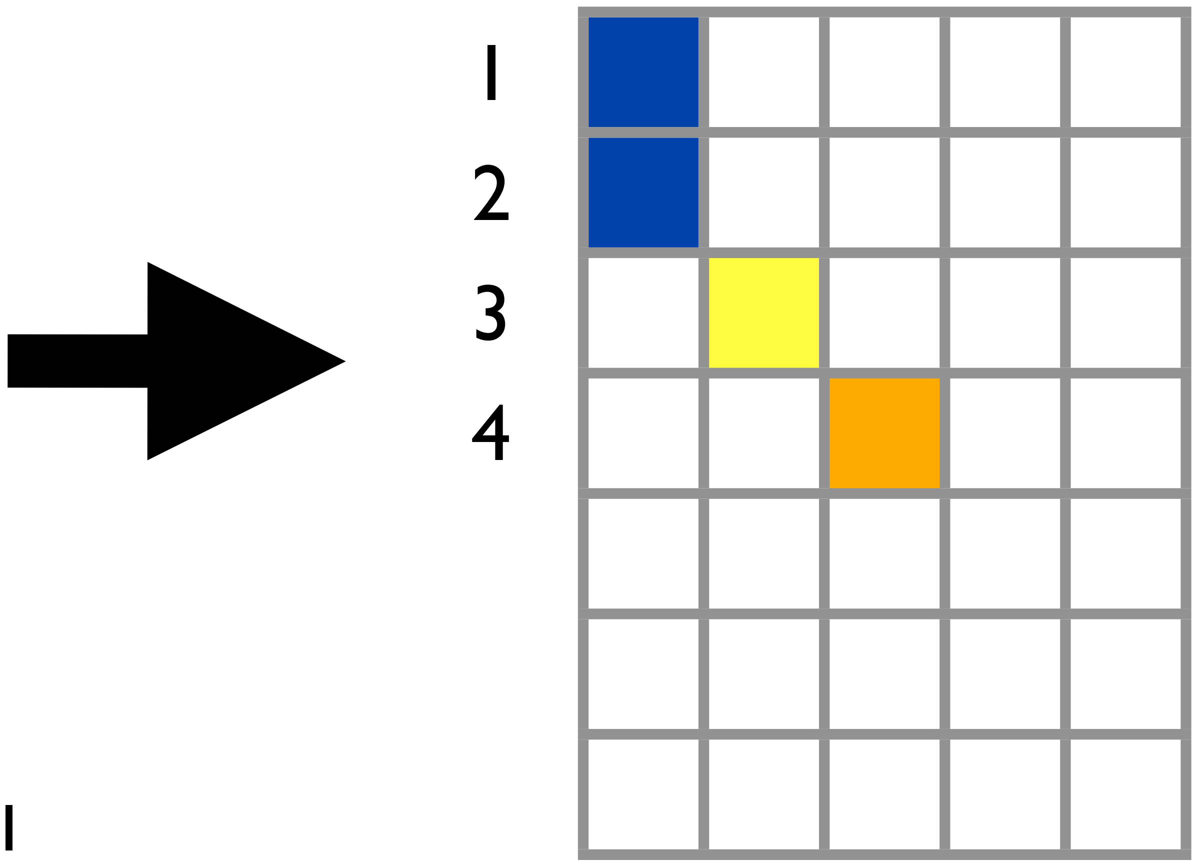


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

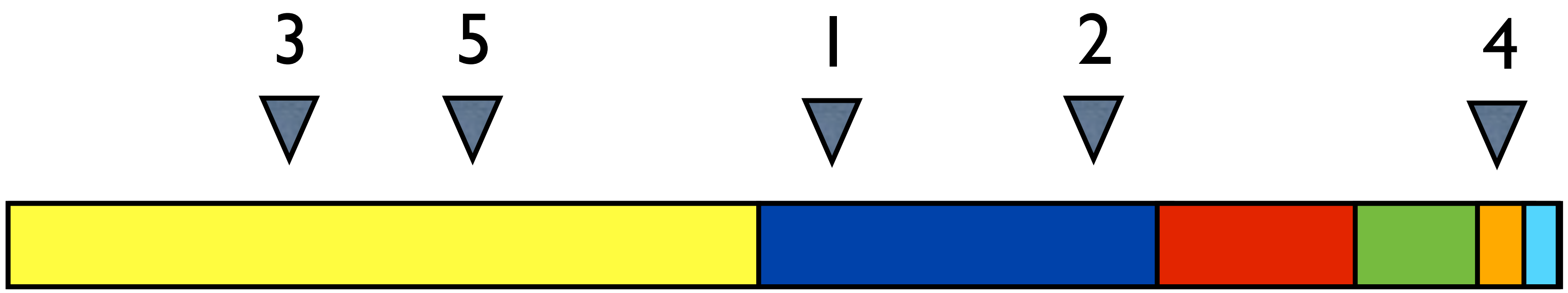


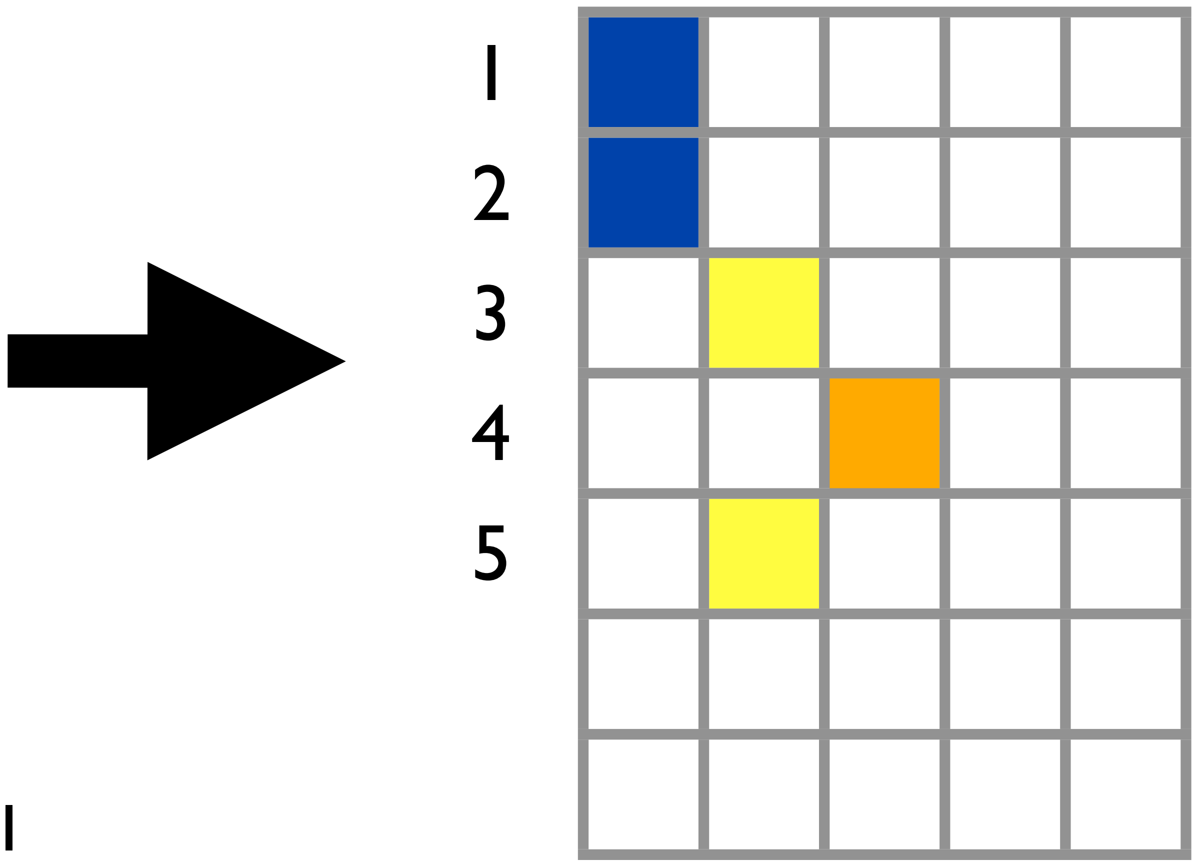


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

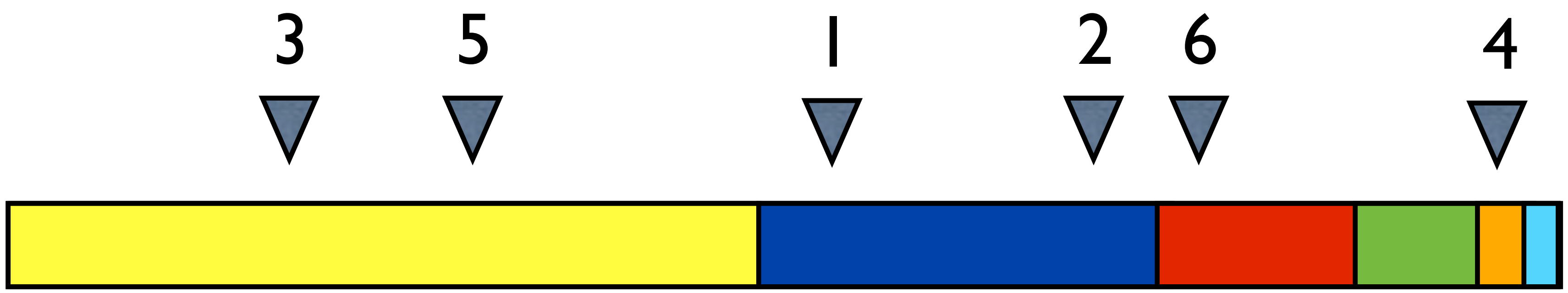


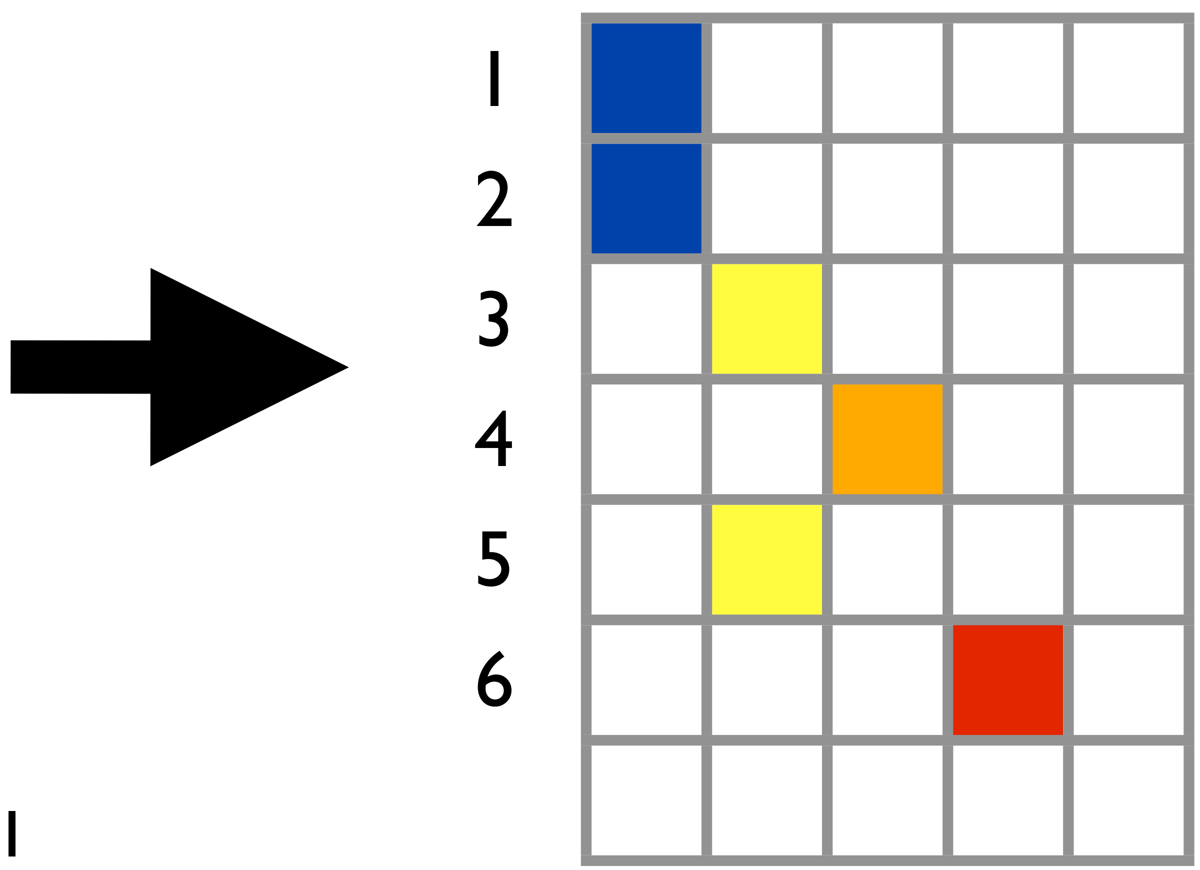


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

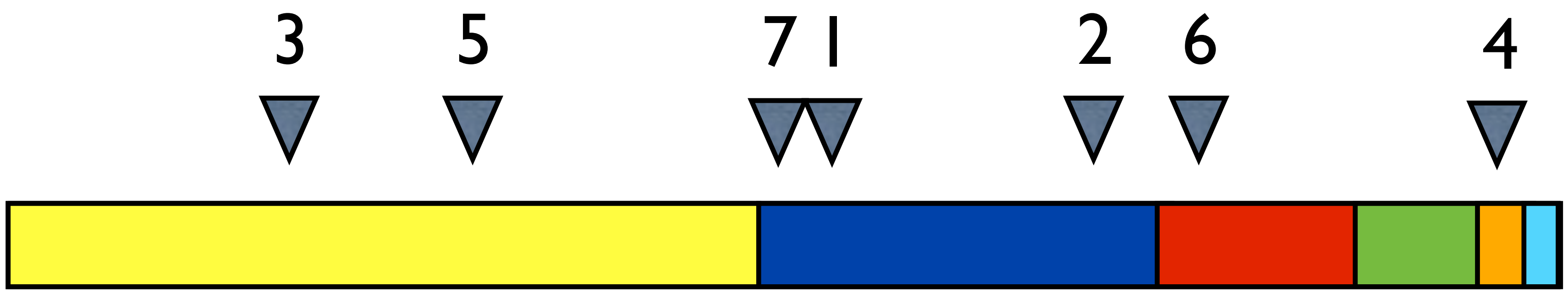


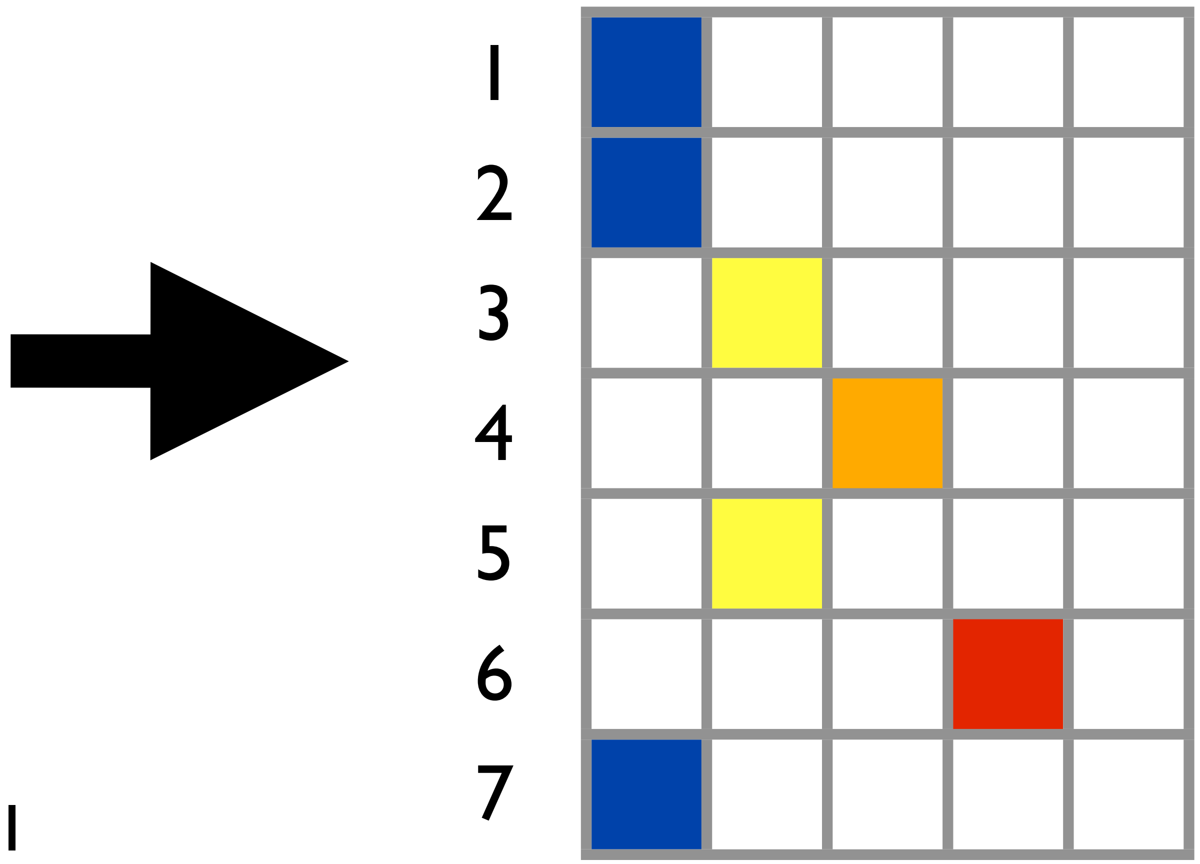


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

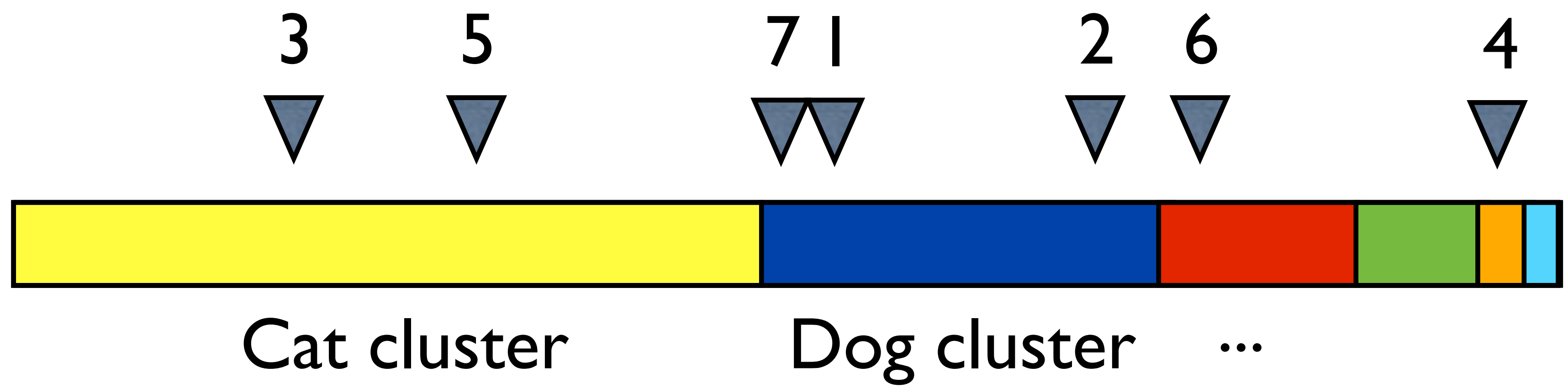


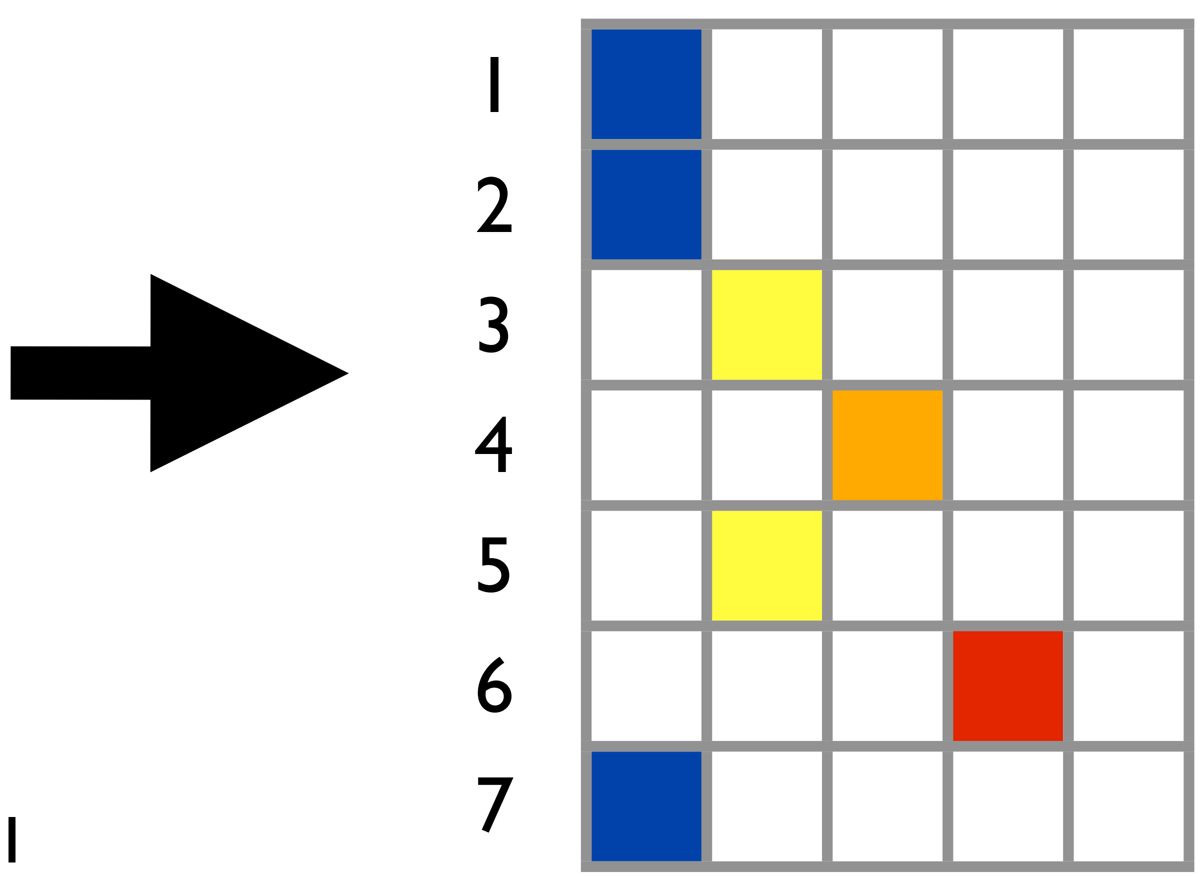


[Kingman 1978]

Paintboxes

Exchangeable partition: Kingman paintbox

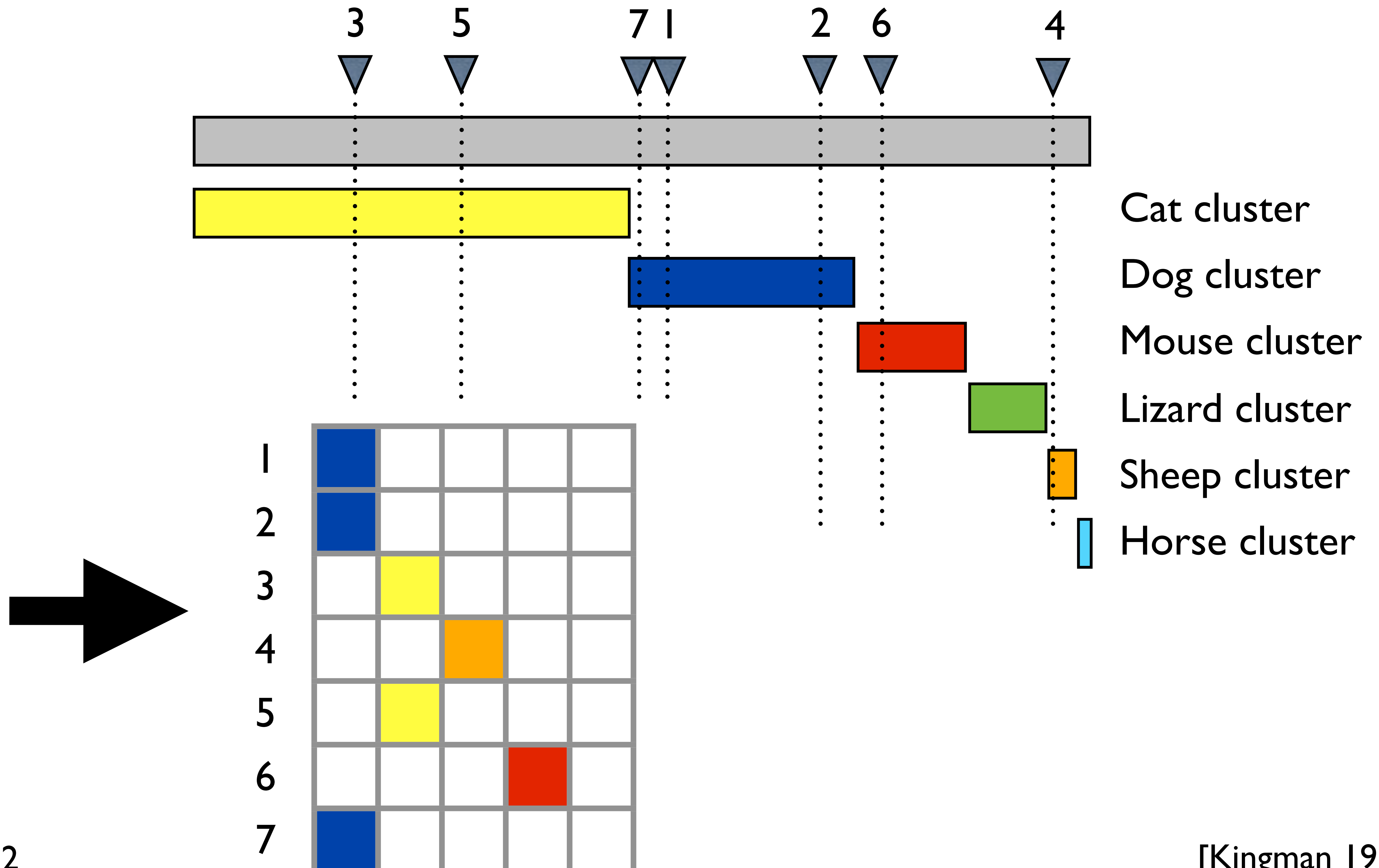




[Kingman 1978]

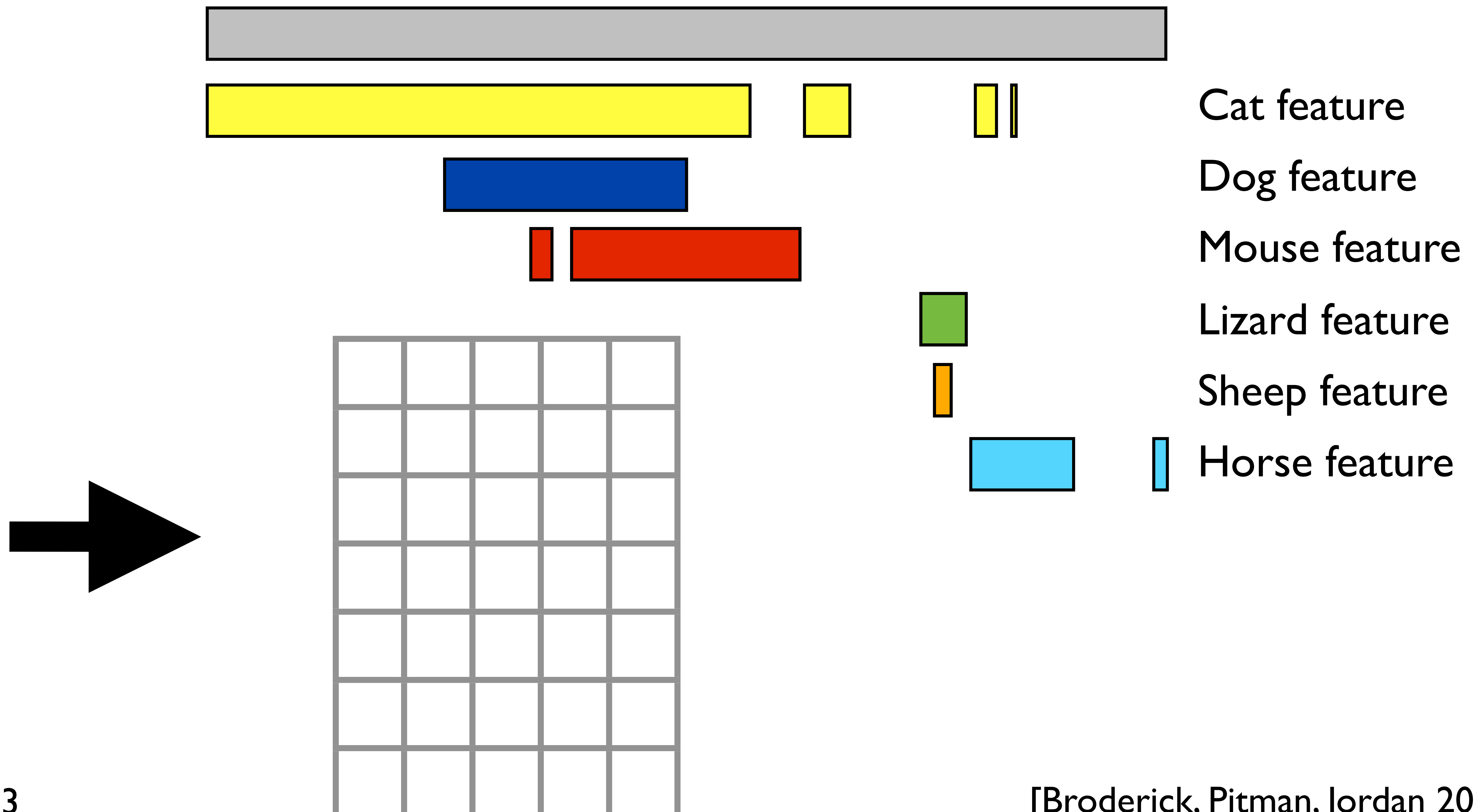
Paintboxes

Exchangeable partition: Kingman paintbox



[Kingman 1978]

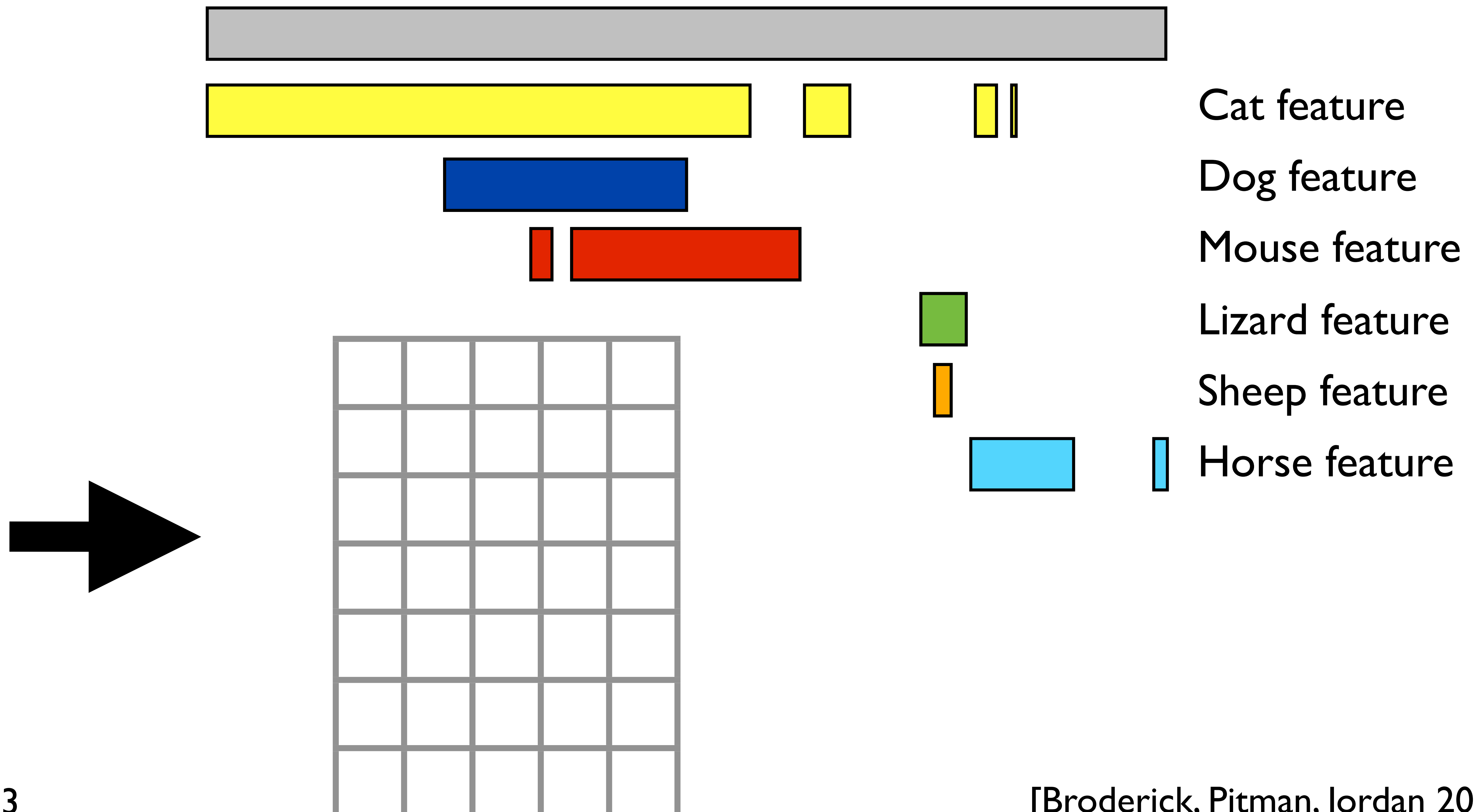
Paintboxes



[Broderick, Pitman, Jordan 2013]

Paintboxes

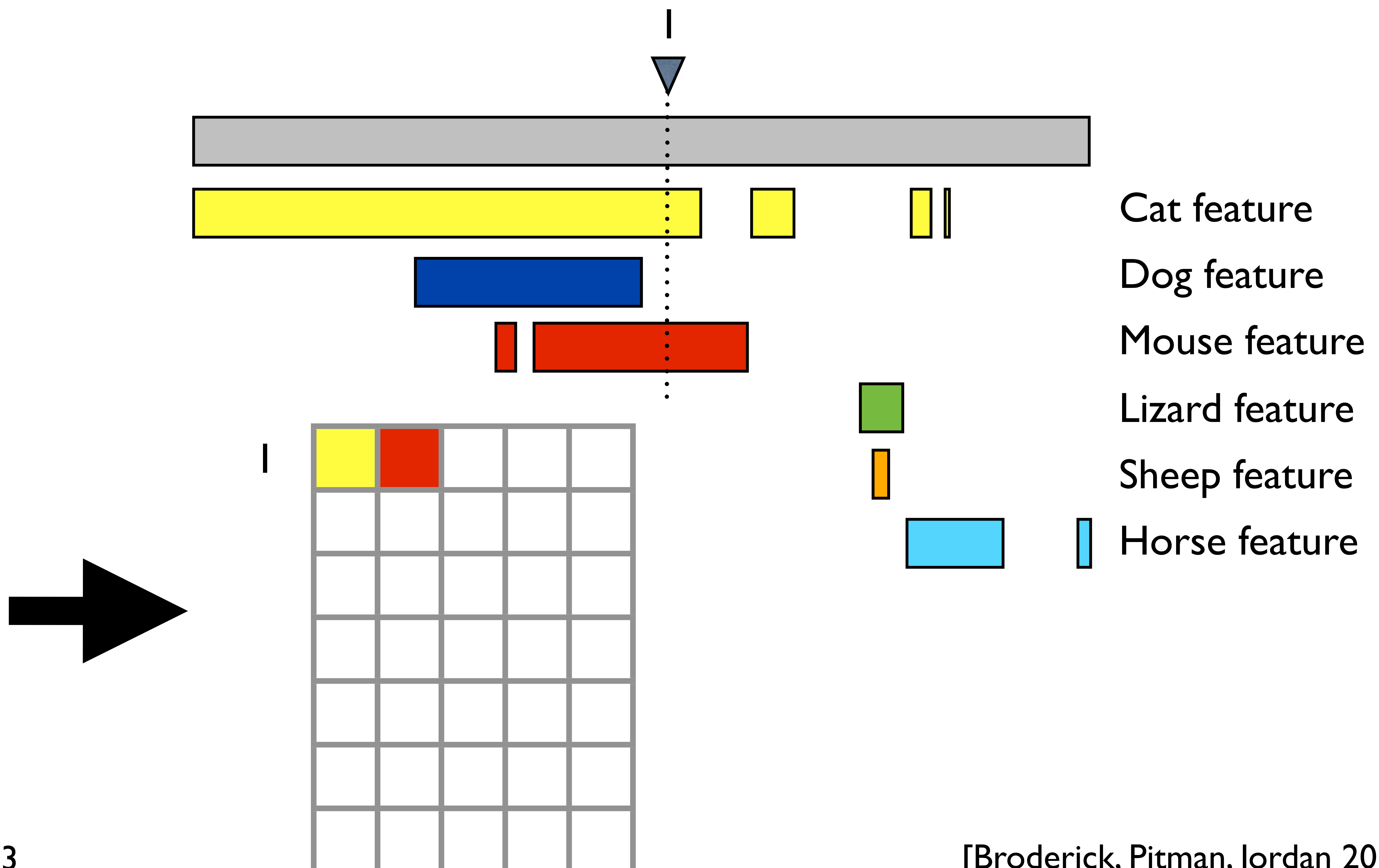
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

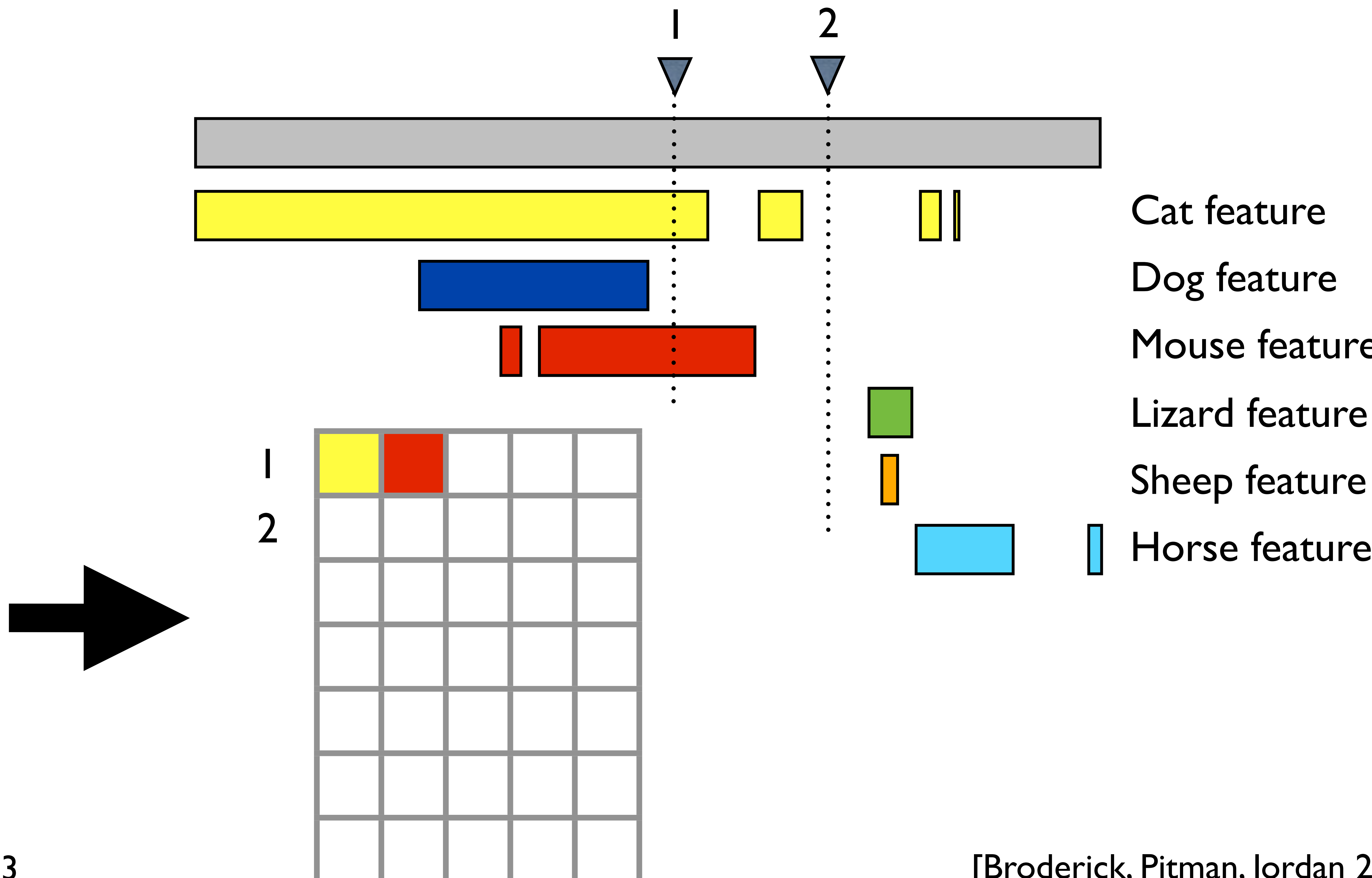
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

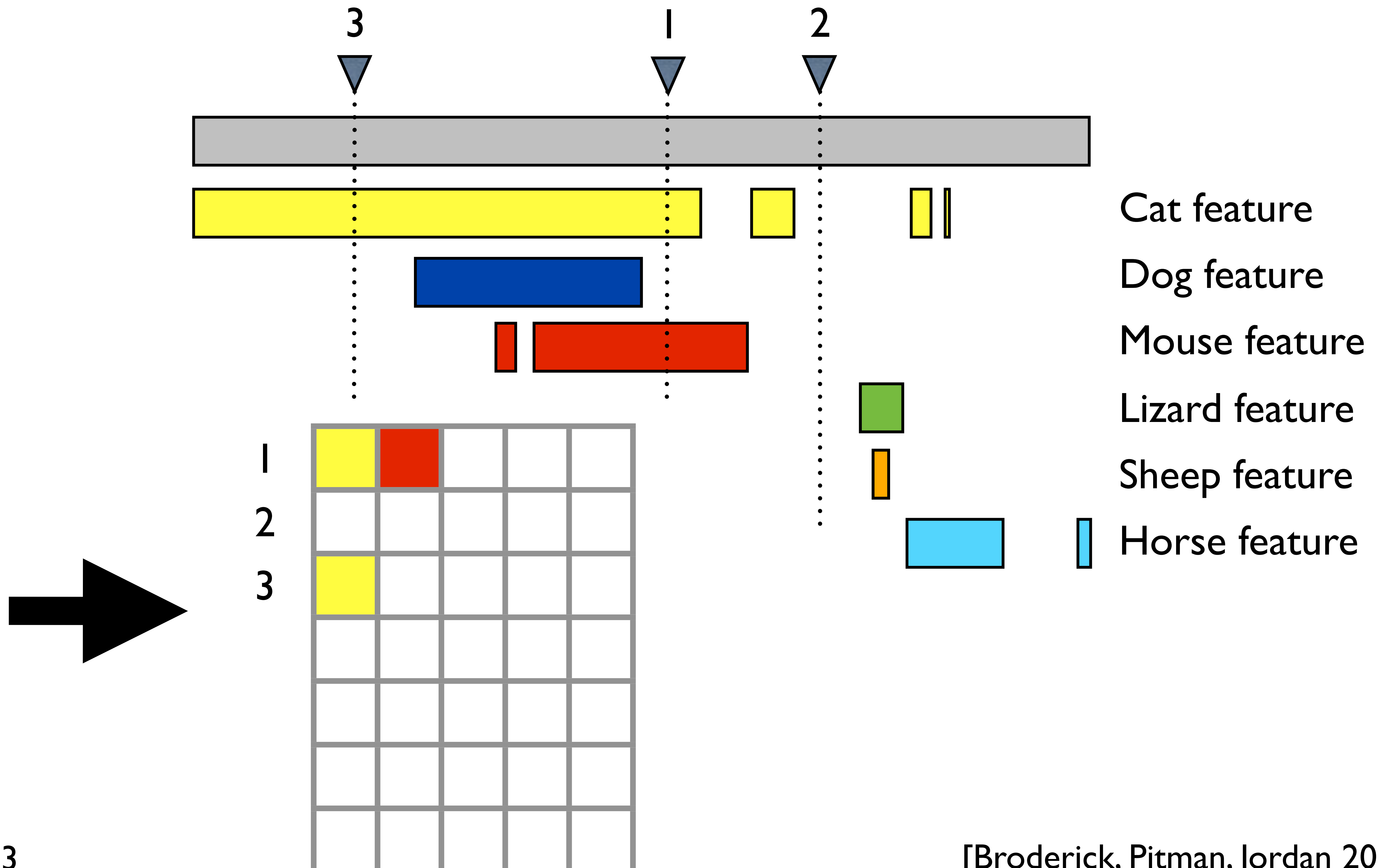
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

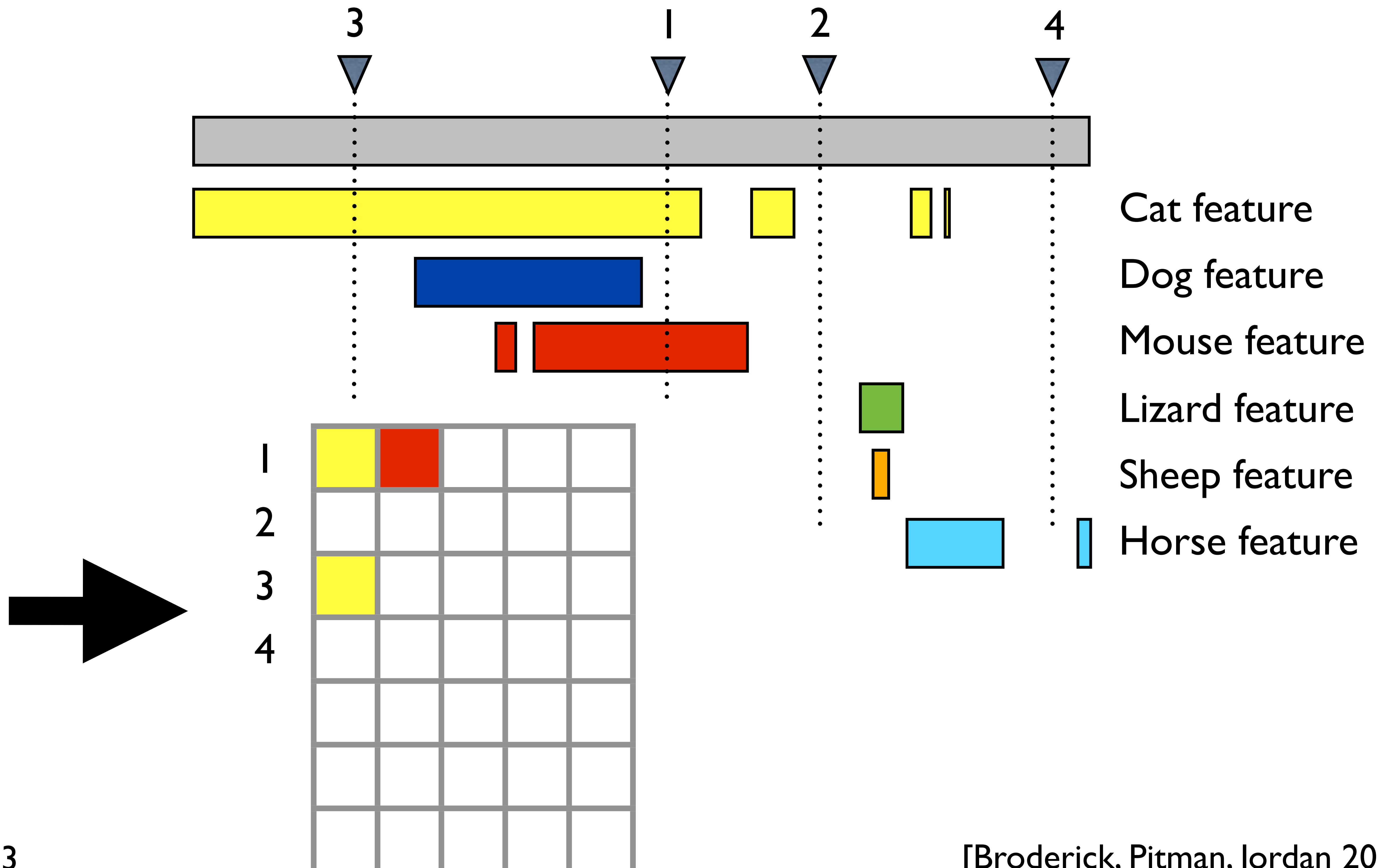
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

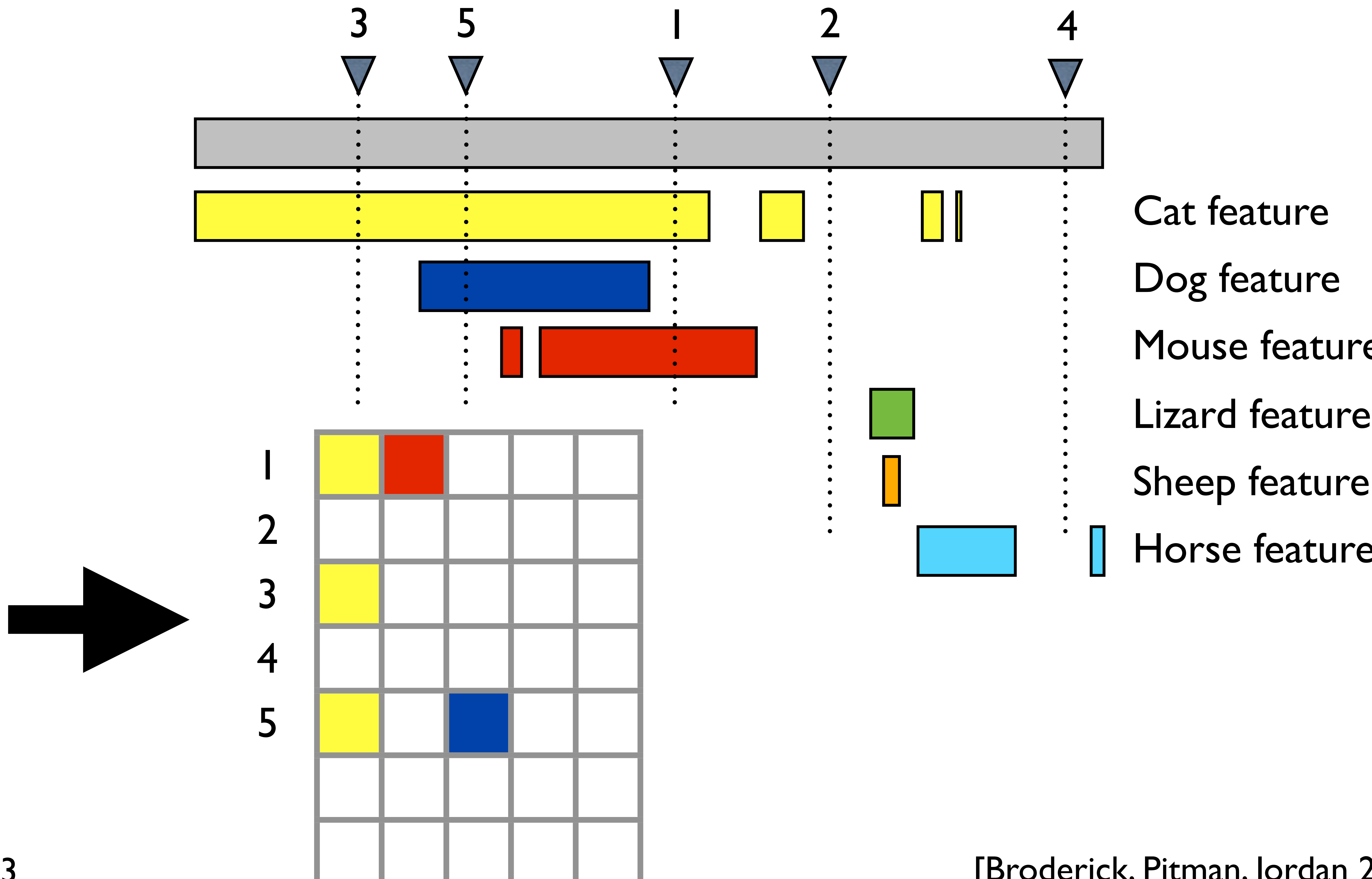
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

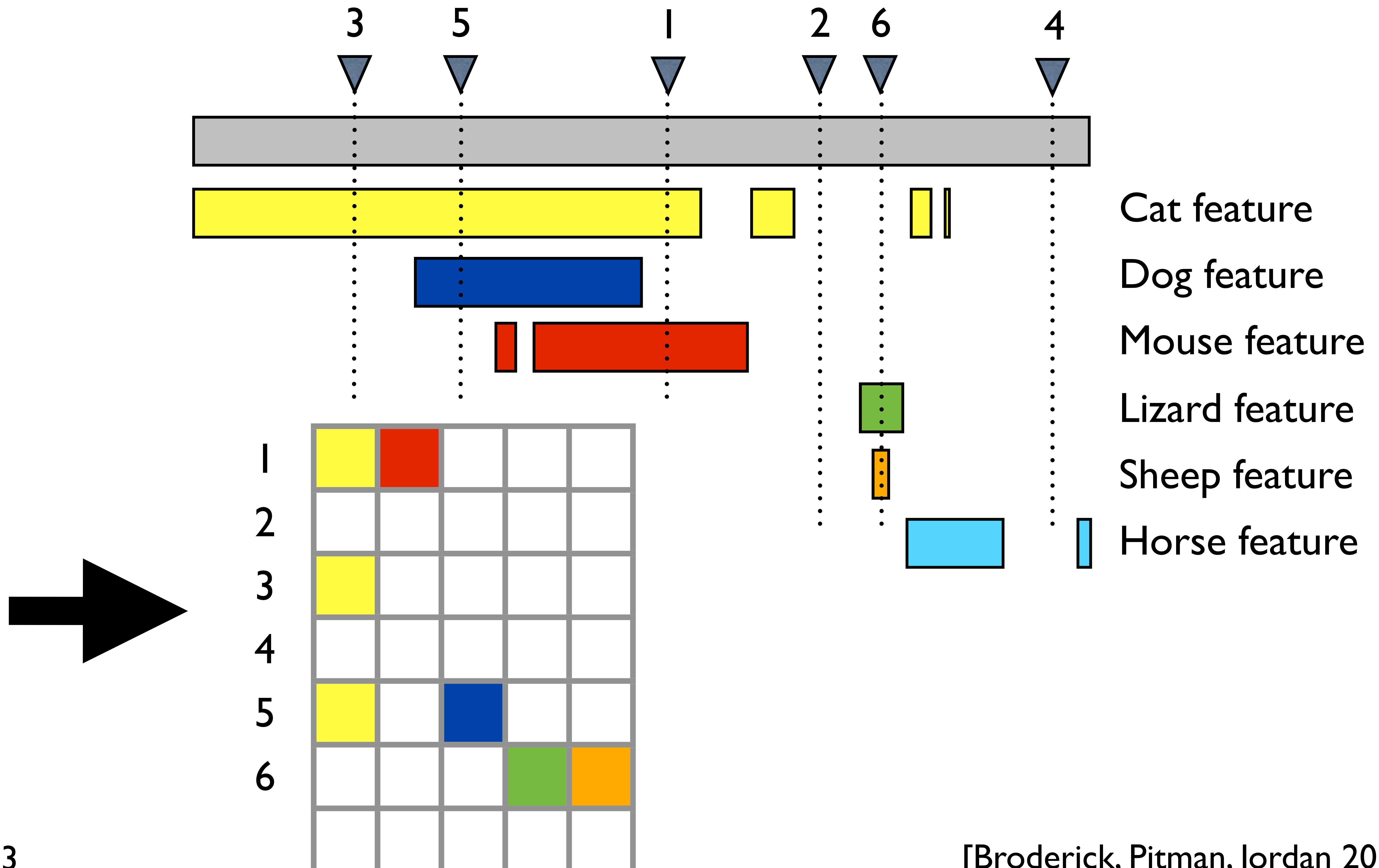
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

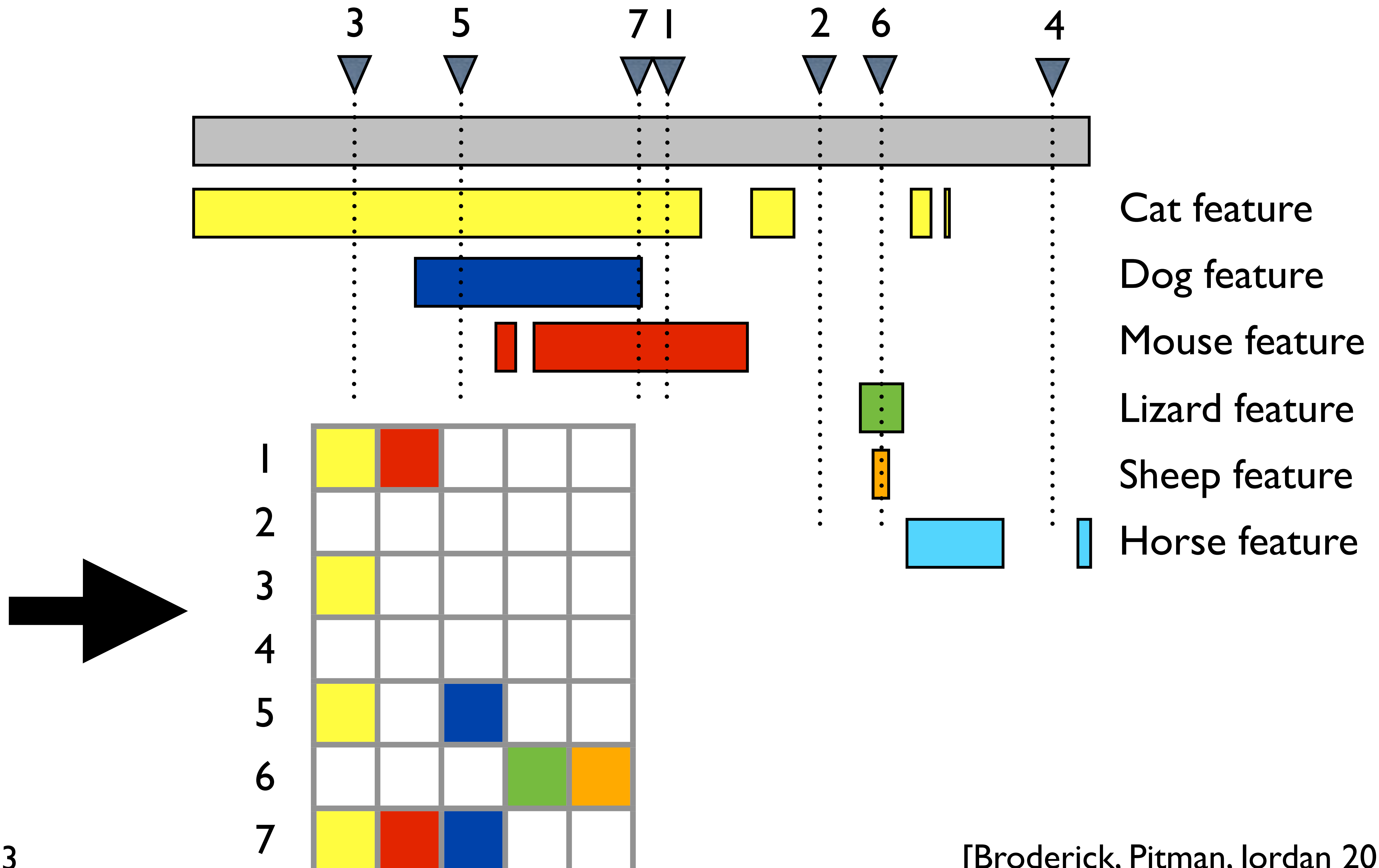
Exchangeable feature allocation: feature paintbox



[Broderick, Pitman, Jordan 2013]

Paintboxes

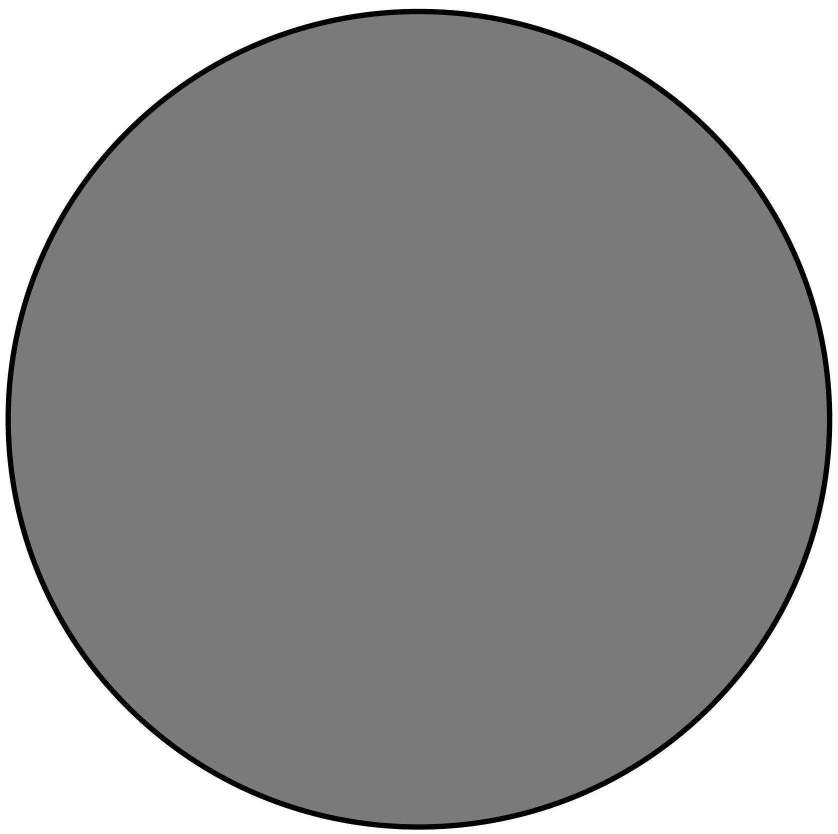
Exchangeable feature allocation: feature paintbox



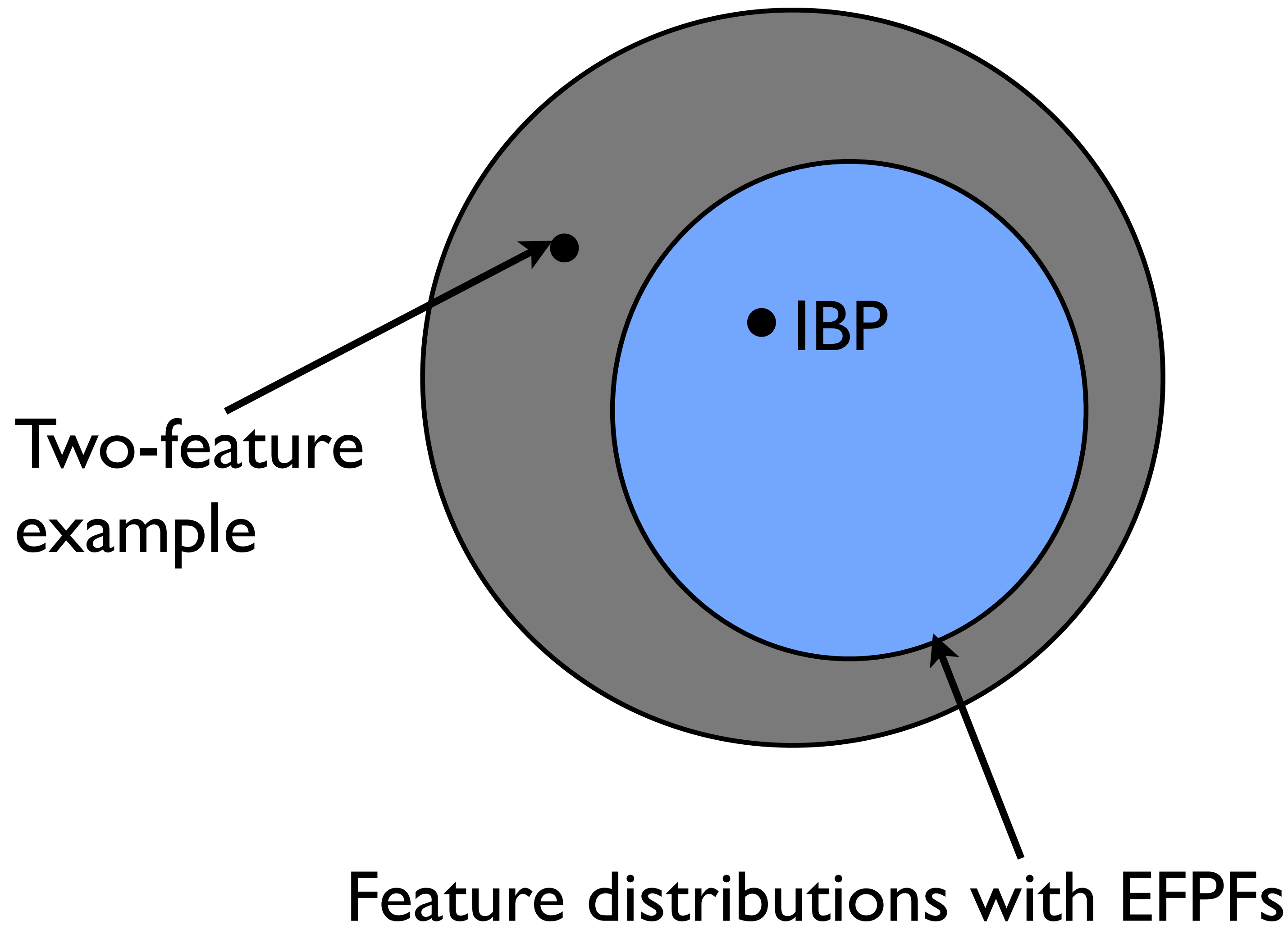
[Broderick, Pitman, Jordan 2013]

Paintboxes

Exchangeable cluster distributions
= Cluster distributions with EPPFs



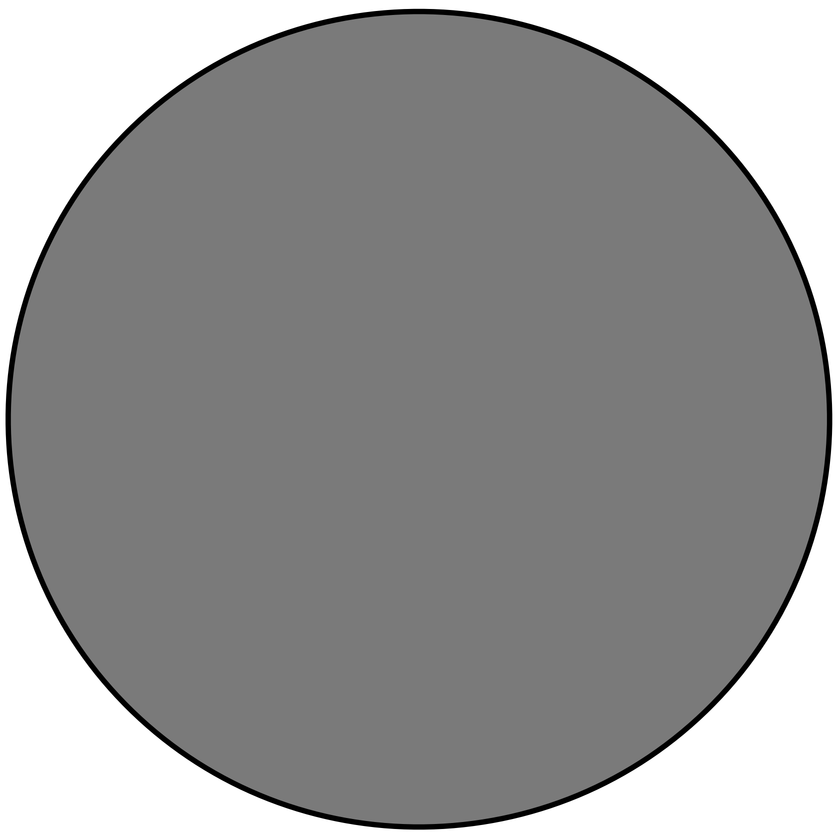
Exchangeable feature distributions



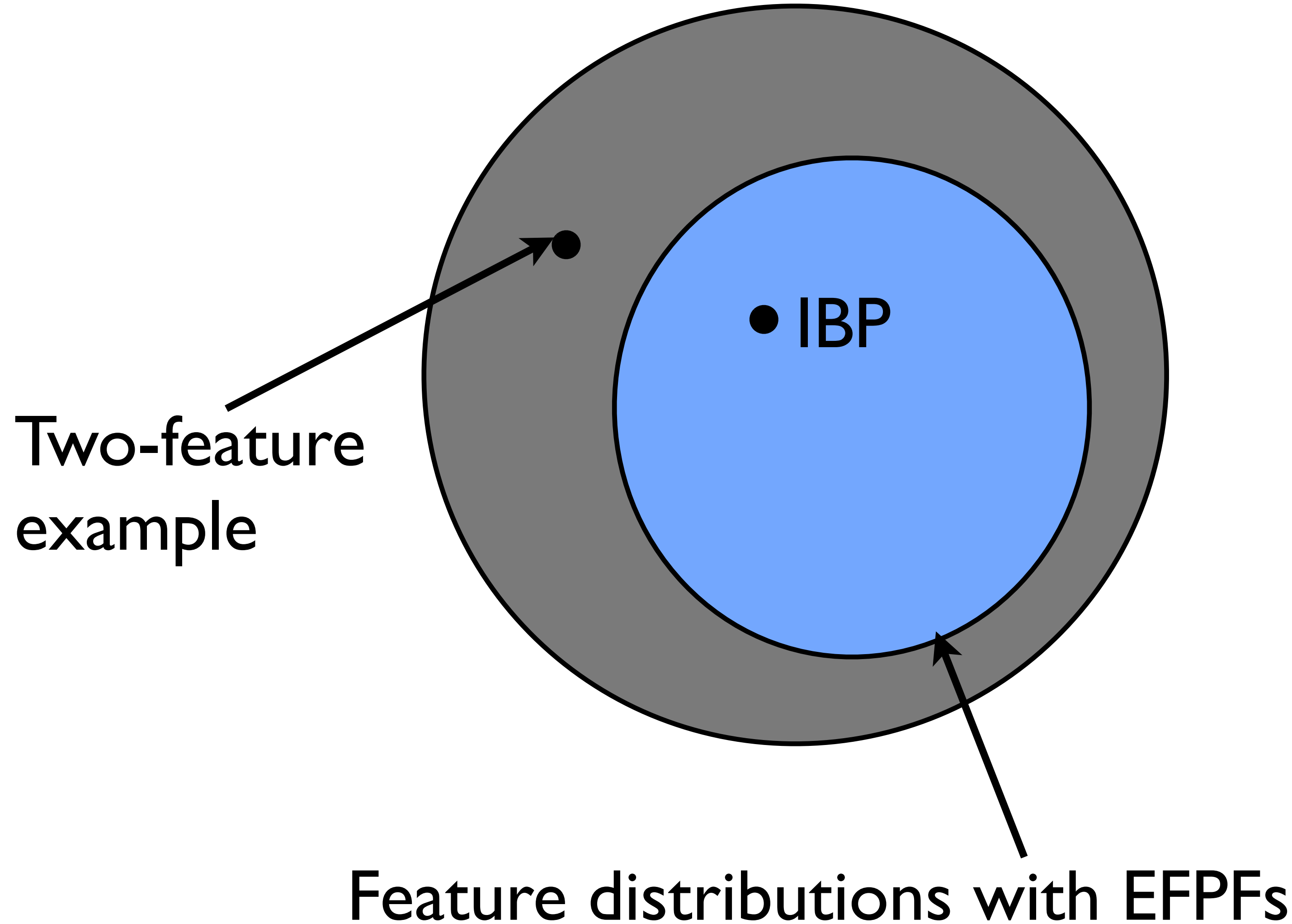
[Broderick, Pitman, Jordan 2013]

Paintboxes

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



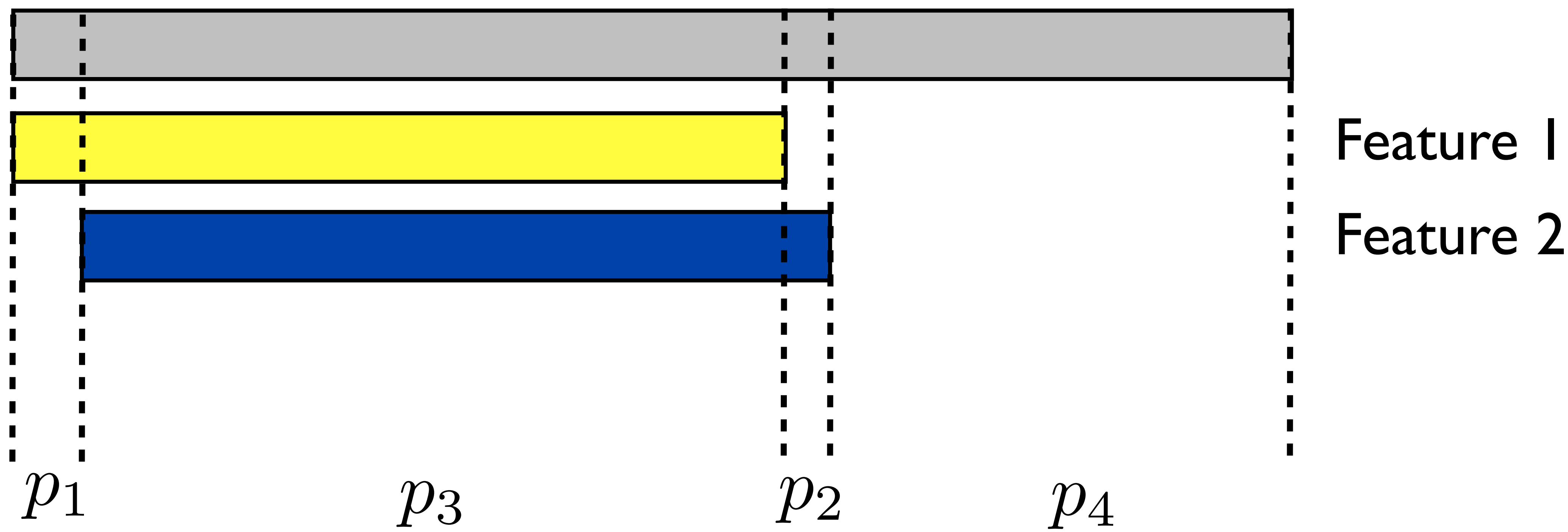
Exchangeable feature distributions
= Feature paintbox allocations



[Broderick, Pitman, Jordan 2013]

Paintboxes

Two feature example



$$\mathbb{P}(\text{row} = \blacksquare\square) = p_1$$

$$\mathbb{P}(\text{row} = \square\blacksquare) = p_2$$

$$\mathbb{P}(\text{row} = \blacksquare\blacksquare) = p_3$$

$$\mathbb{P}(\text{row} = \square\square) = p_4$$

Paintboxes

Indian buffet process: beta feature frequencies

[Thibaux, Jordan 2007]

Paintboxes

Indian buffet process: beta feature frequencies

For m = 1, 2, ...
1. Draw $K_m^+ = \mathrm{Poisson}\left(\gamma \frac{\theta}{\theta + m - 1}\right)$

[Thibaux, Jordan 2007]

Paintboxes

Indian buffet process: beta feature frequencies

For *m* = 1, 2, ...

1. Draw $K_m^+ = \mathrm{Poisson}\left(\gamma \frac{\theta}{\theta + m - 1}\right)$

   Set $K_m = \sum_{j=1}^{m} K_j^+$

2. For k = $K_{m-1}, \ldots, K_m$

   Draw a frequency of size

$$q_k \sim \mathrm{Beta}(1, \theta + m - 1)$$

[Thibaux, Jordan 2007]

Paintboxes

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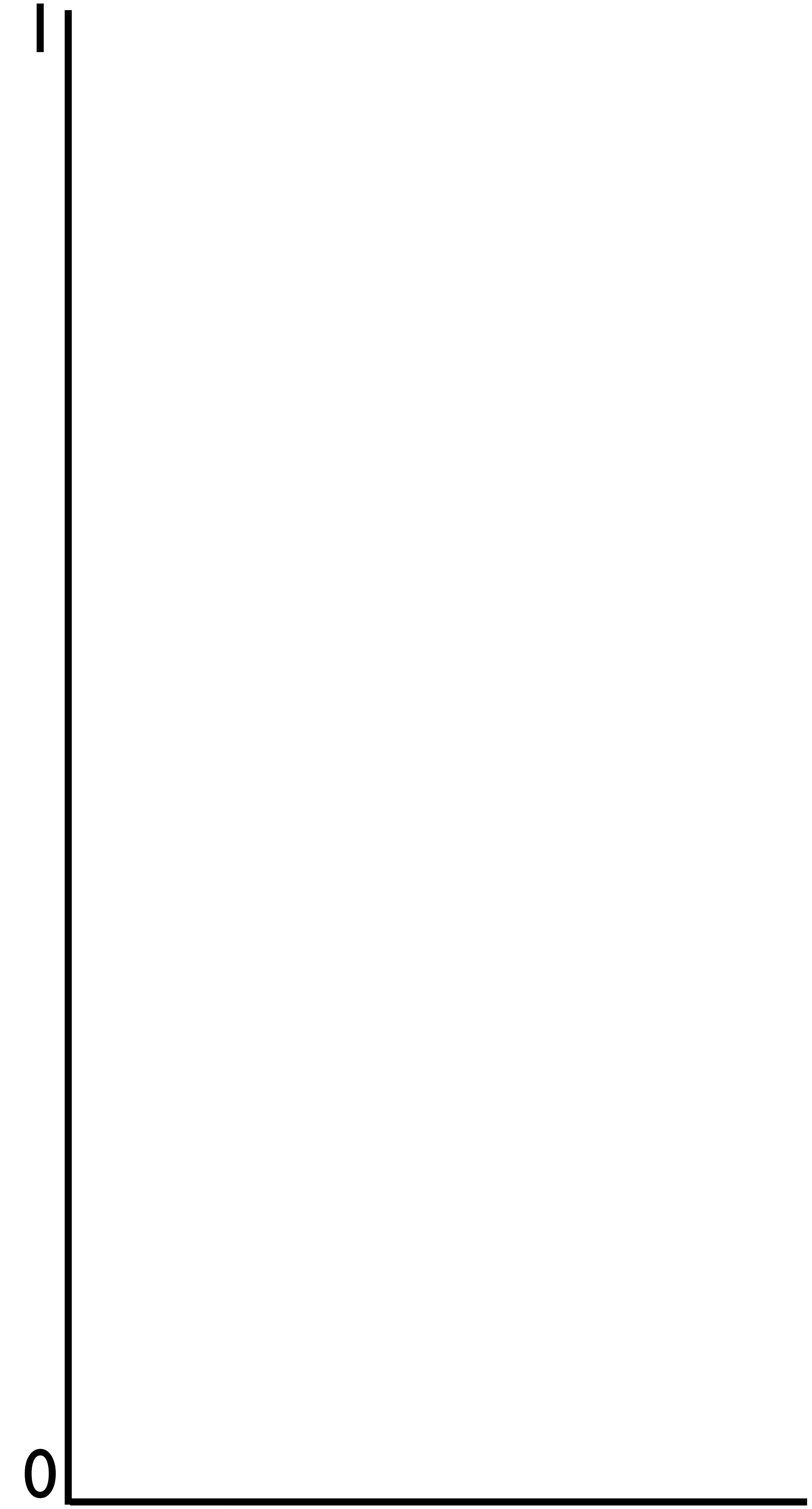

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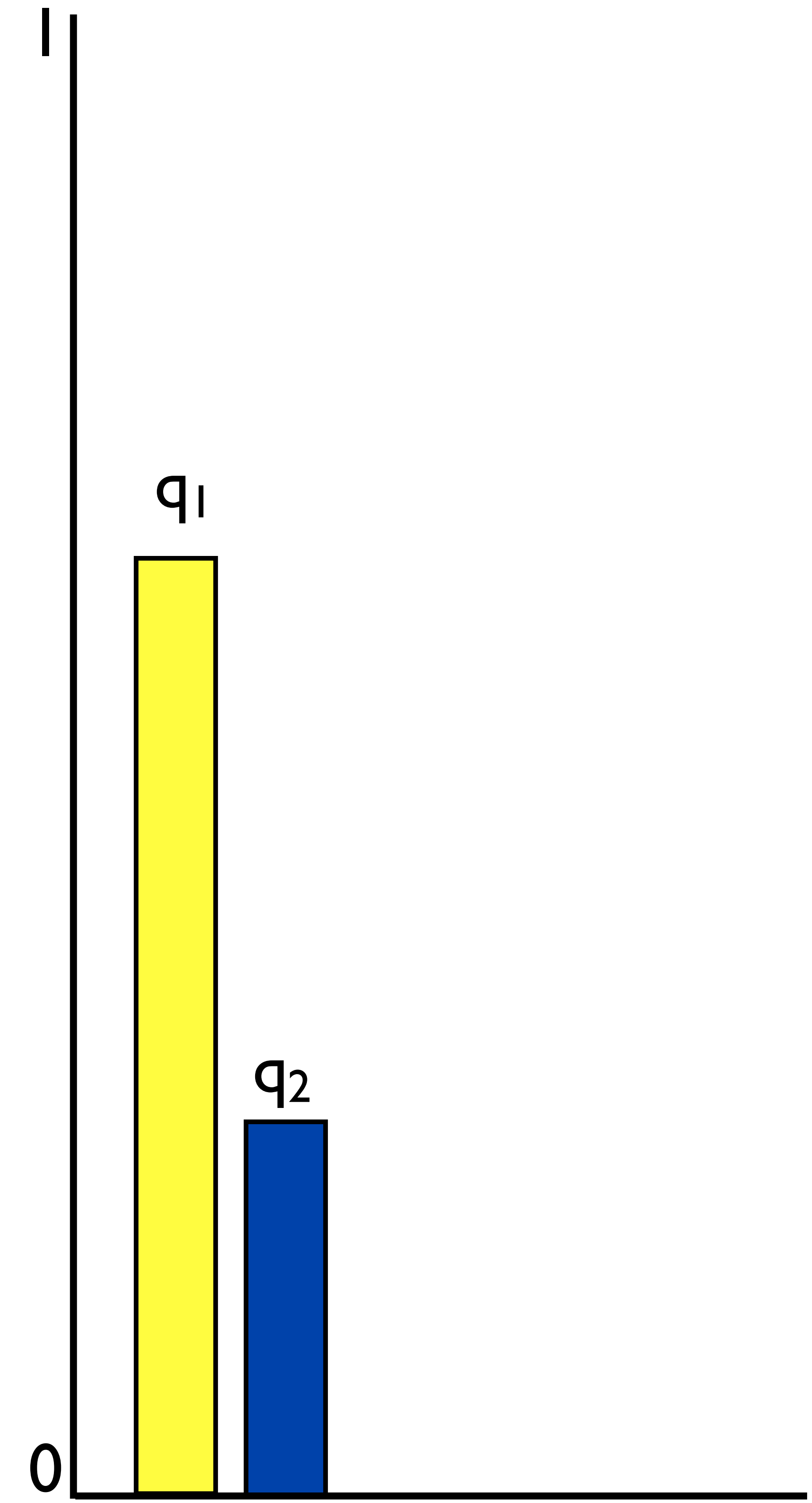

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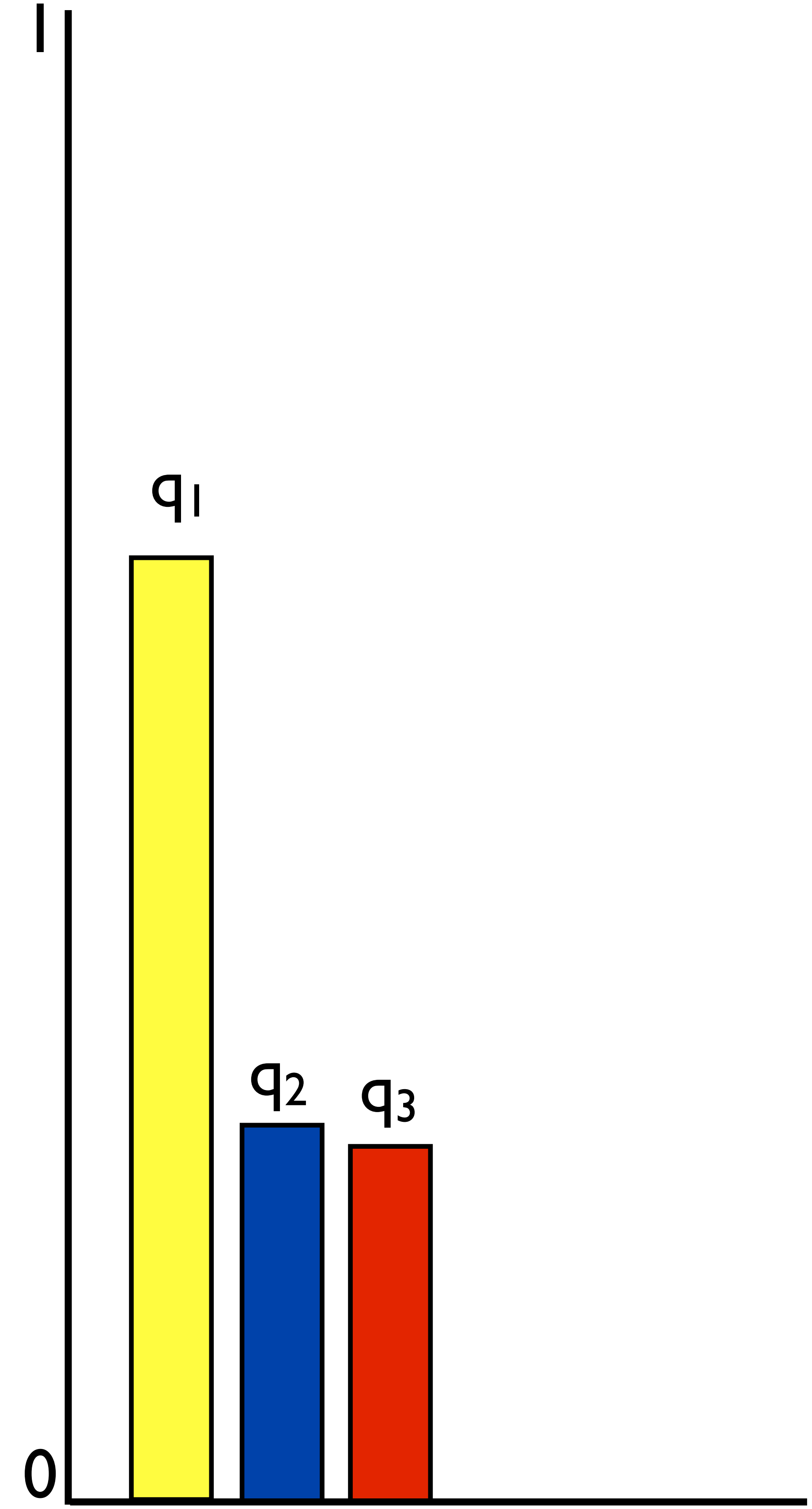

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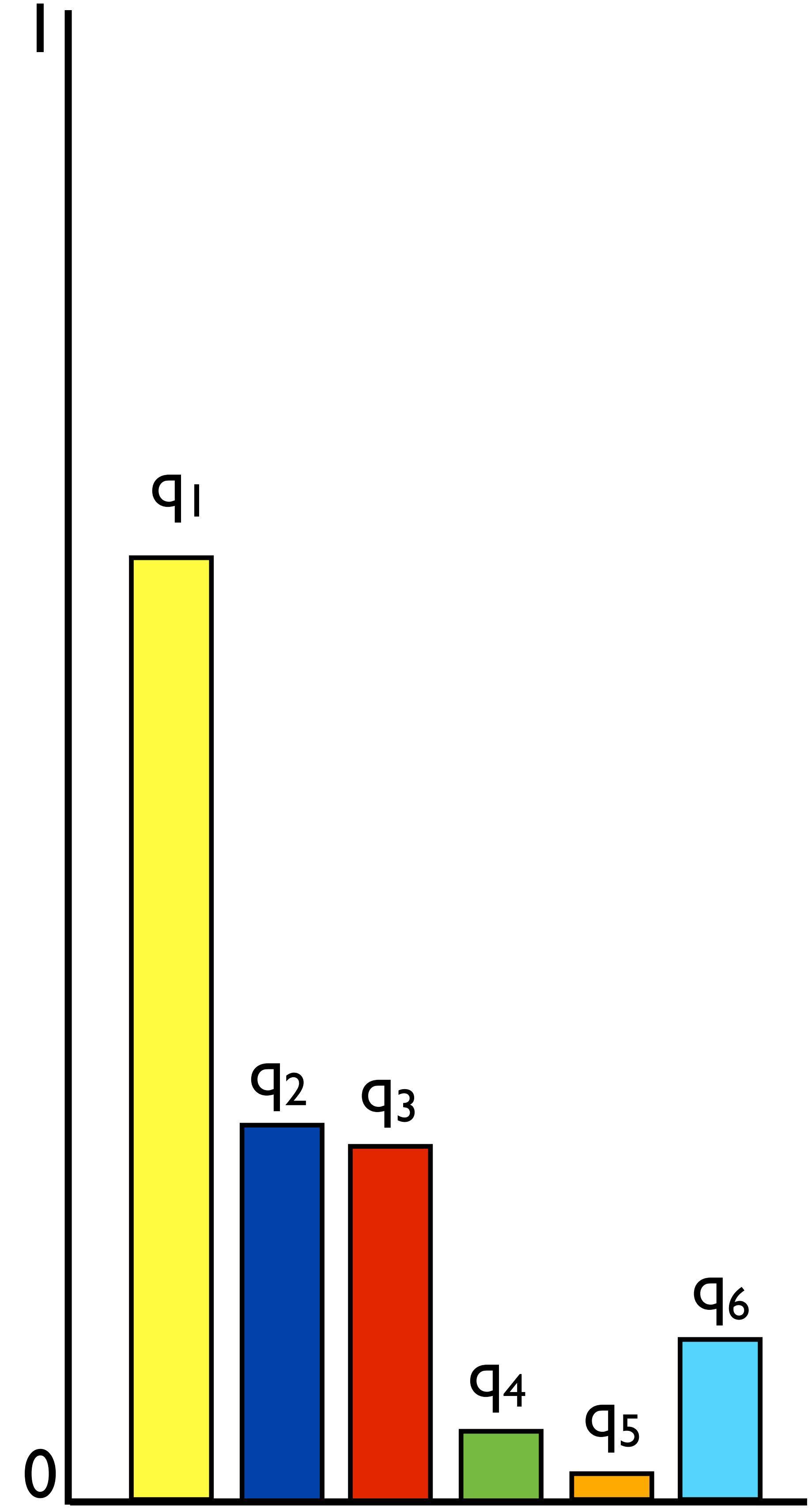

[Thibaux, Jordan 2007]

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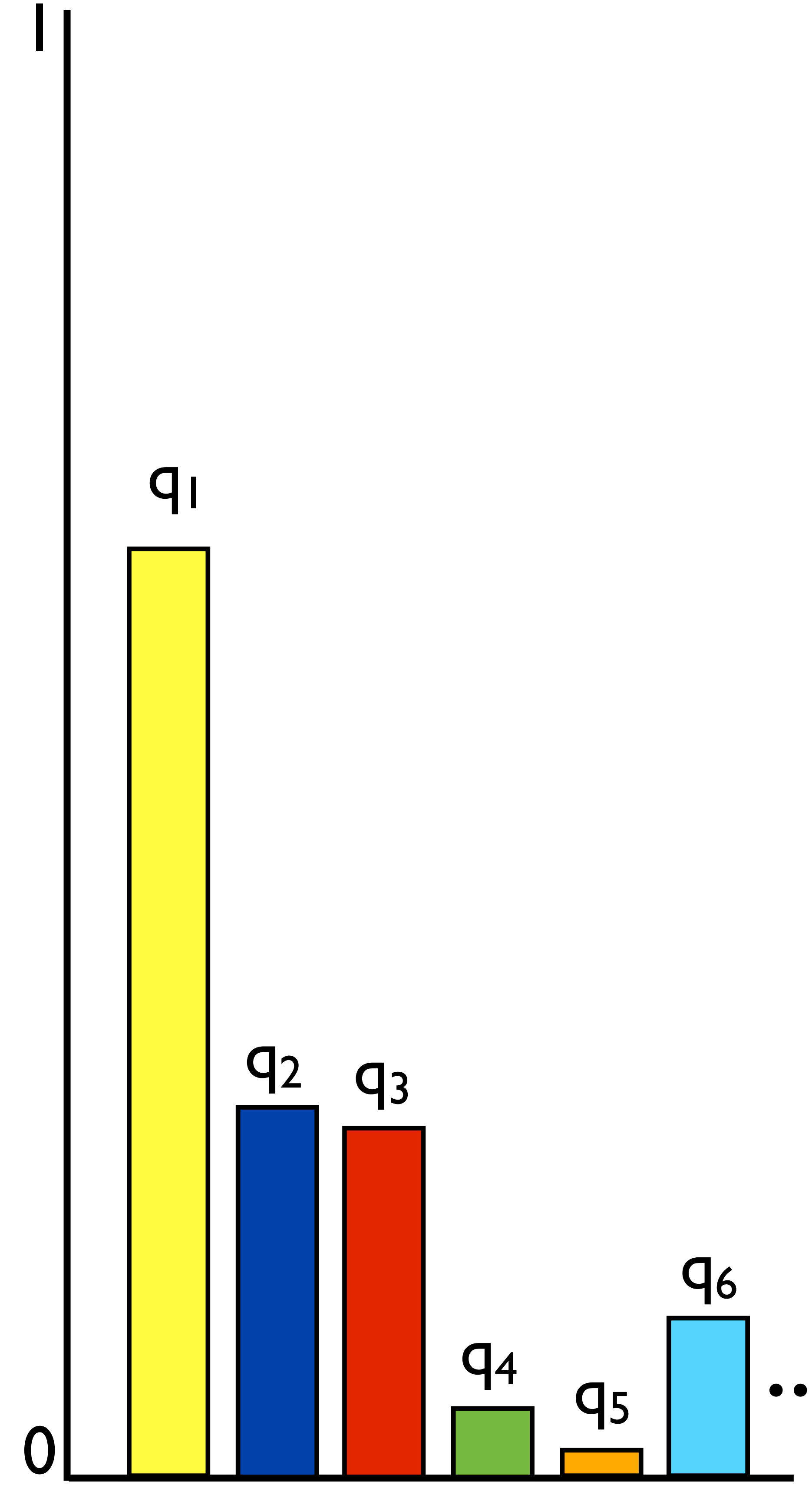

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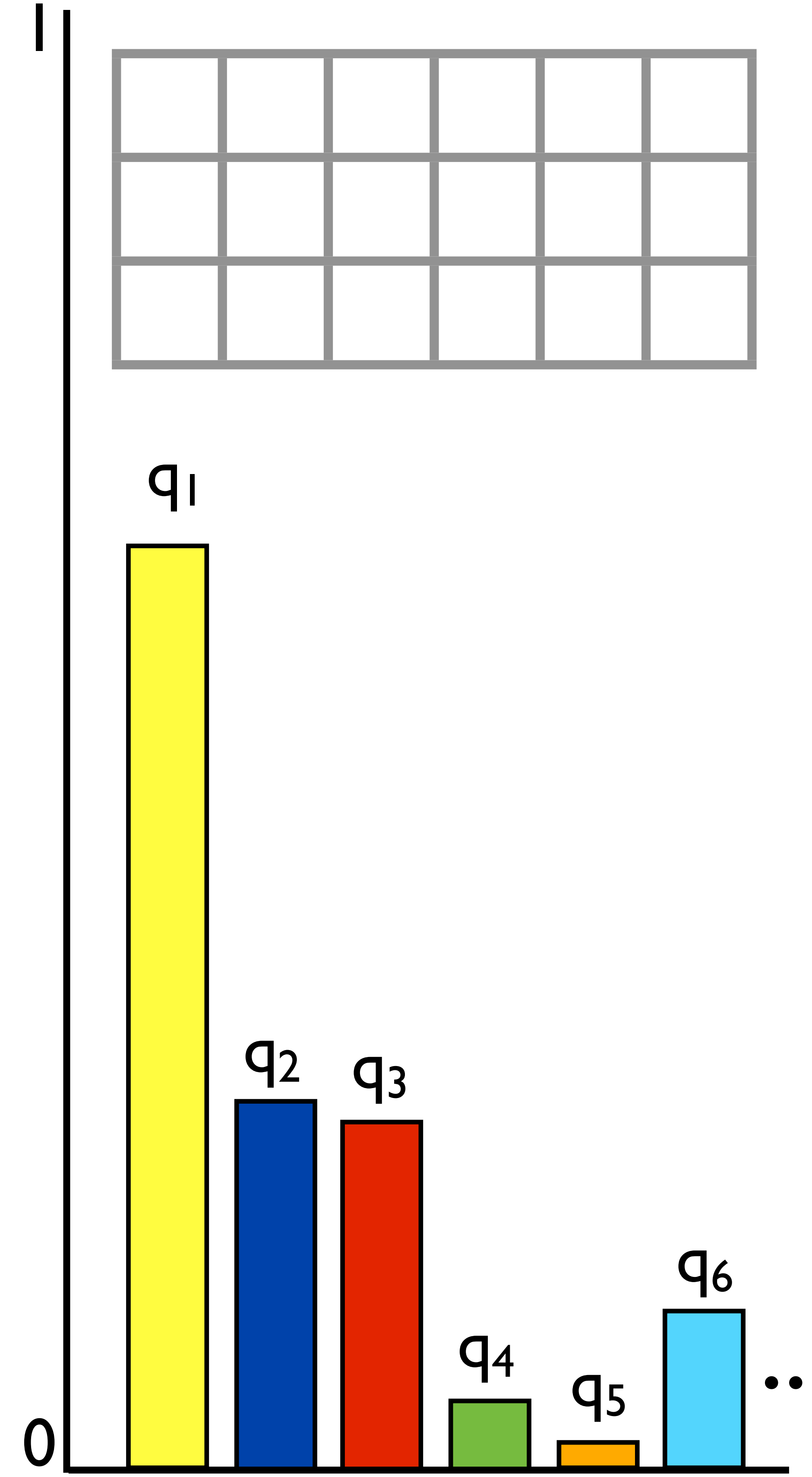

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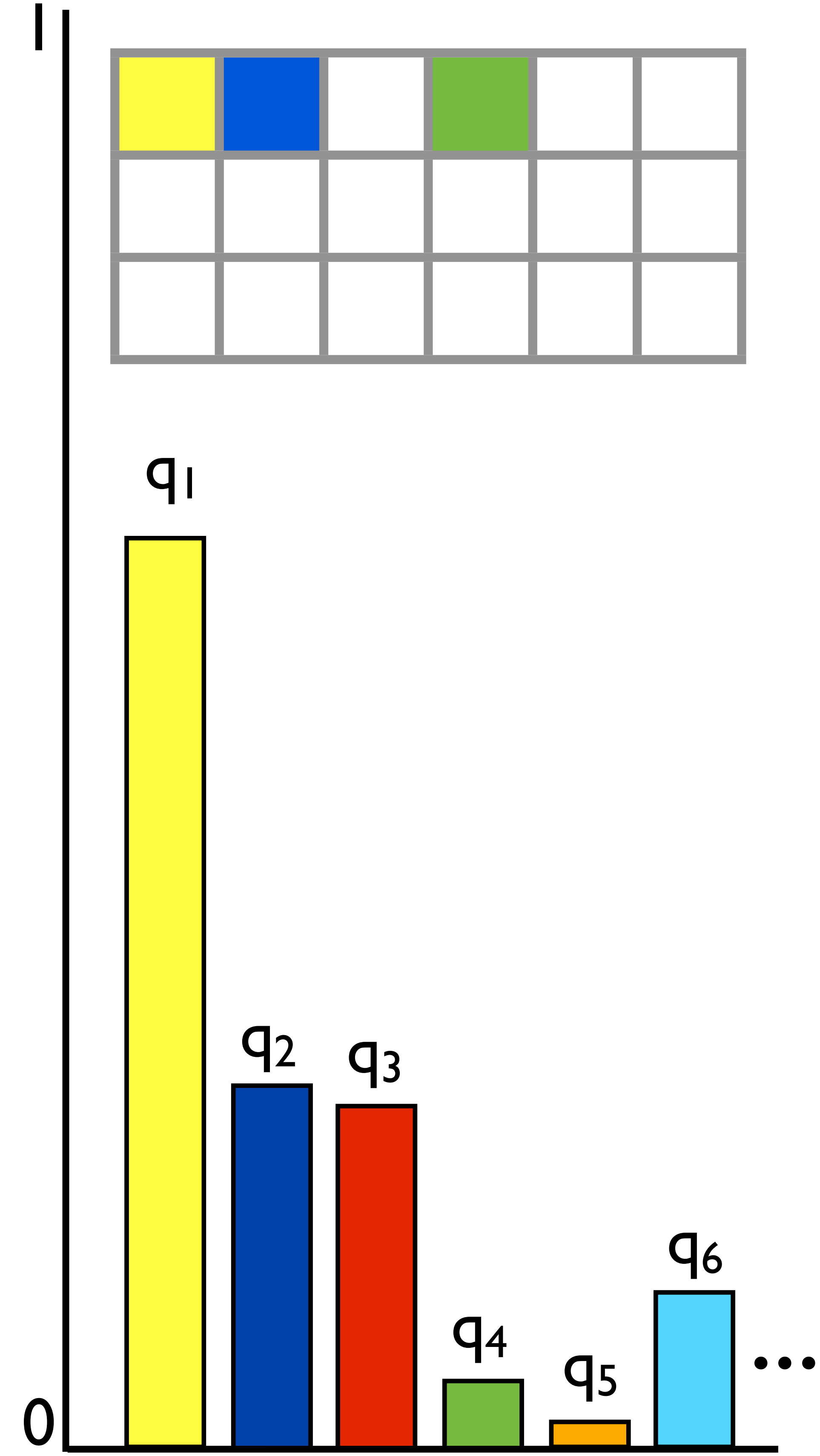

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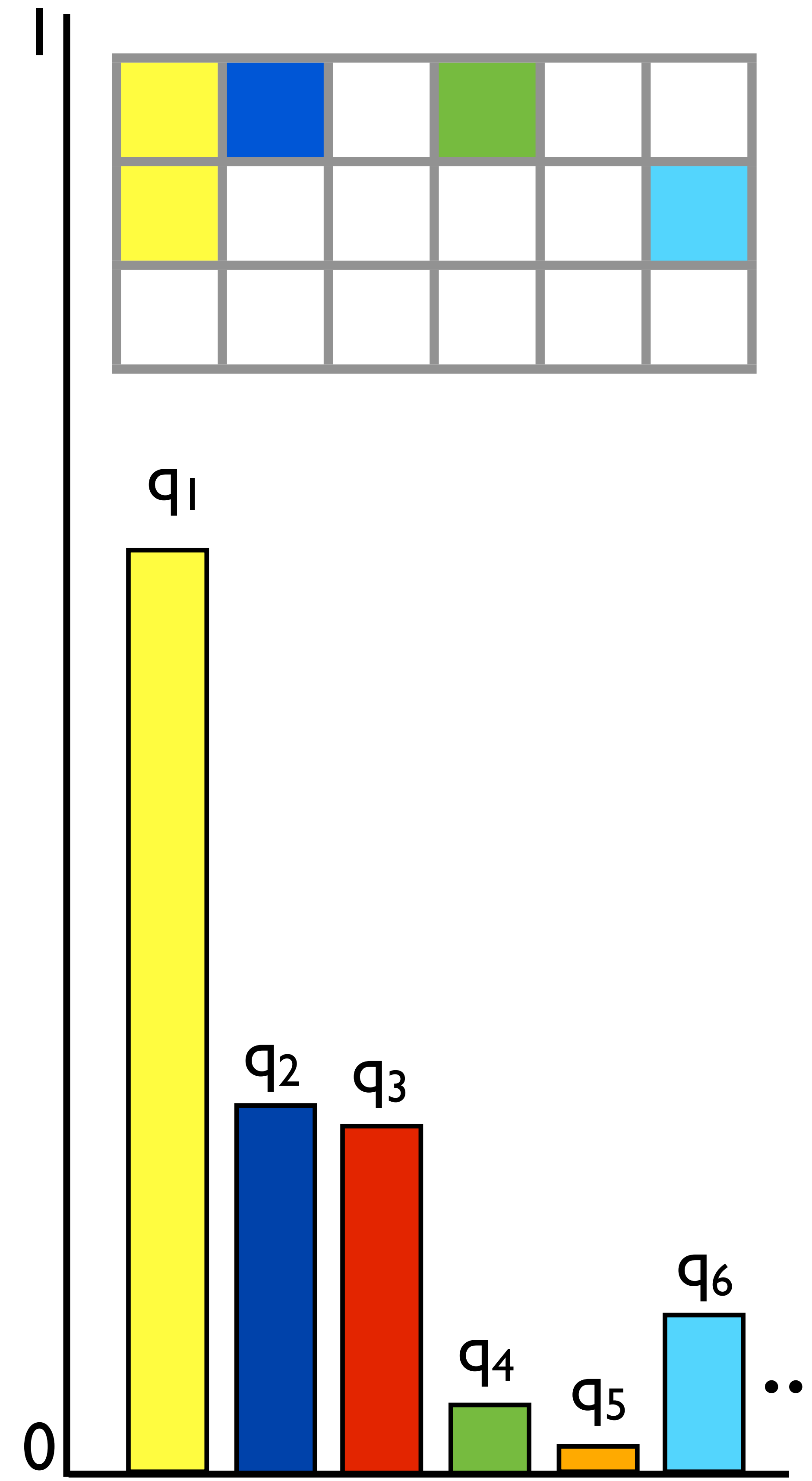
Paintboxes

Indian buffet process: beta feature frequencies



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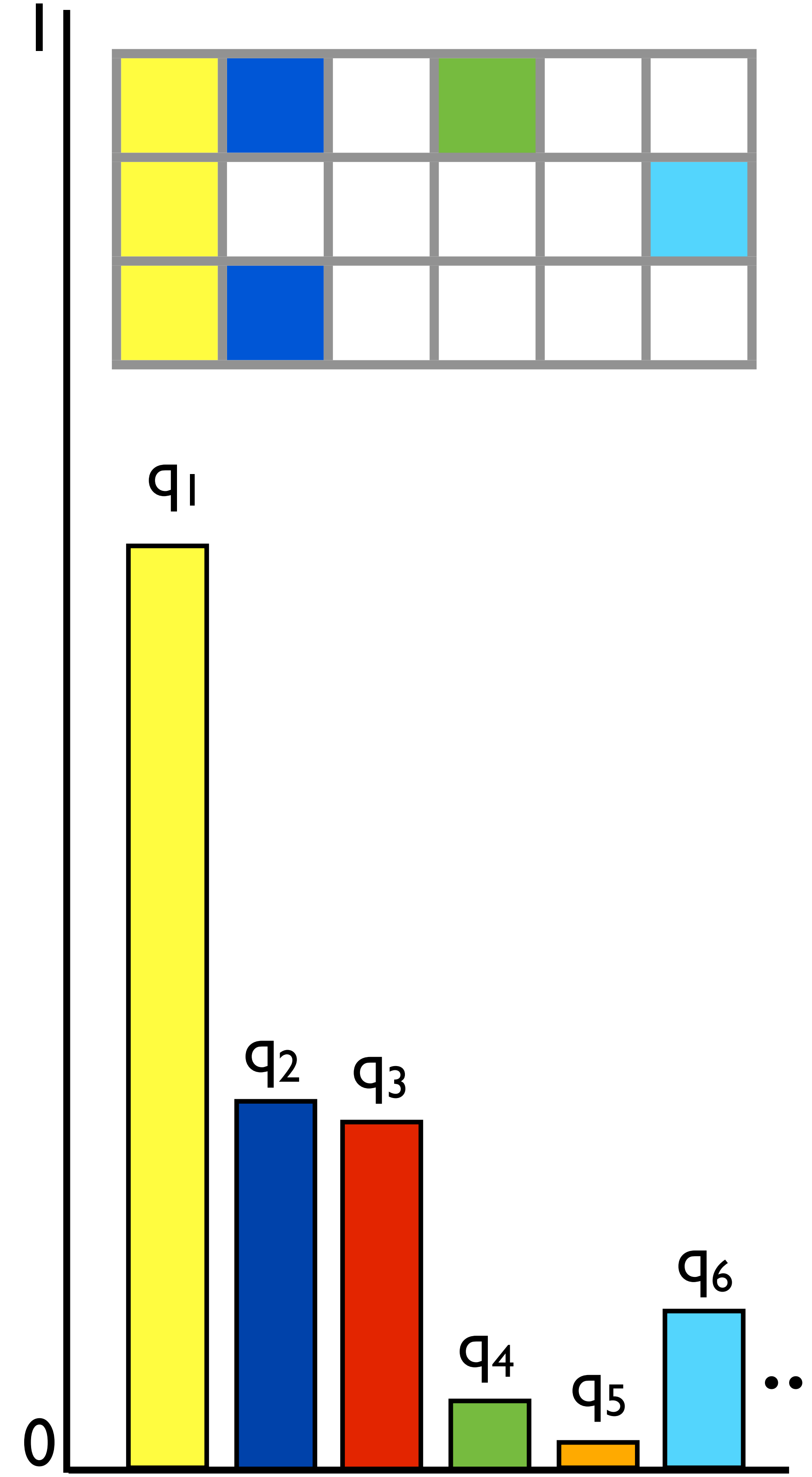
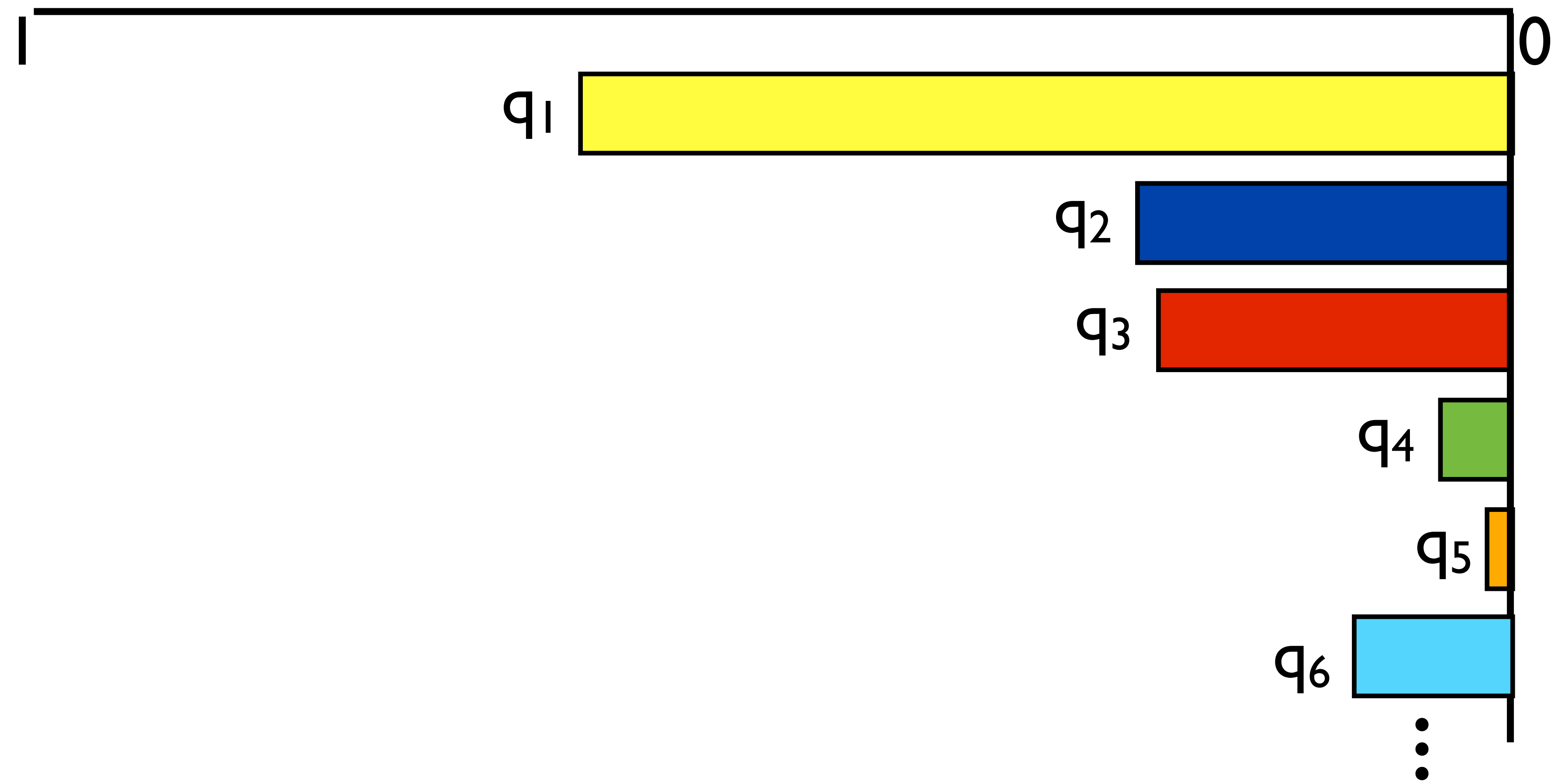


[Thibaux, Jordan 2007]

Paintboxes

Indian buffet process: beta feature frequencies

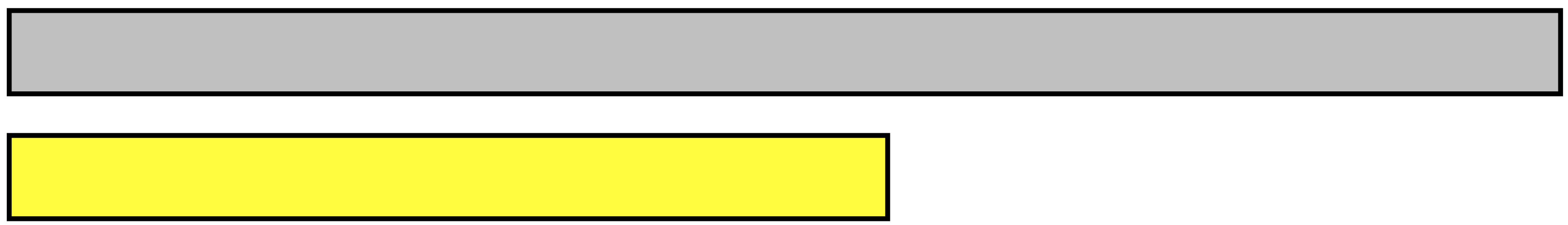


Paintboxes

Indian buffet process: beta feature frequencies

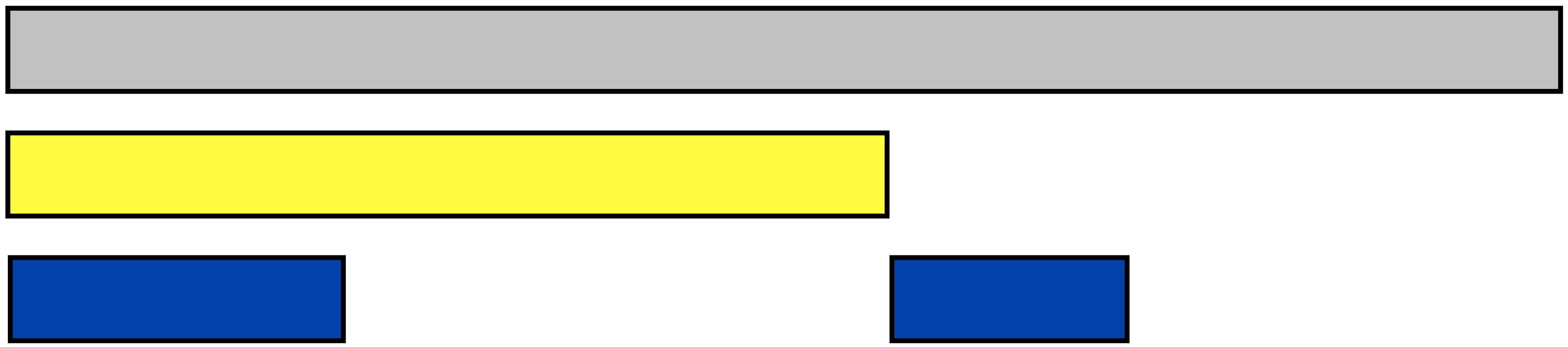
Paintboxes

Indian buffet process: beta feature frequencies



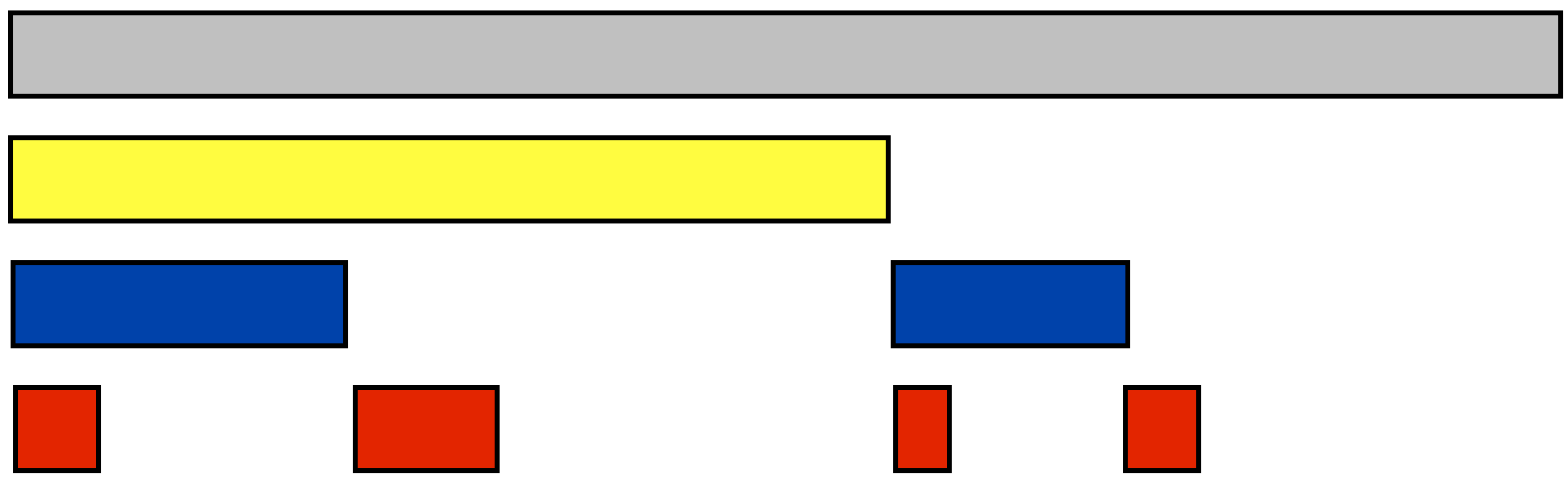
Paintboxes

Indian buffet process: beta feature frequencies



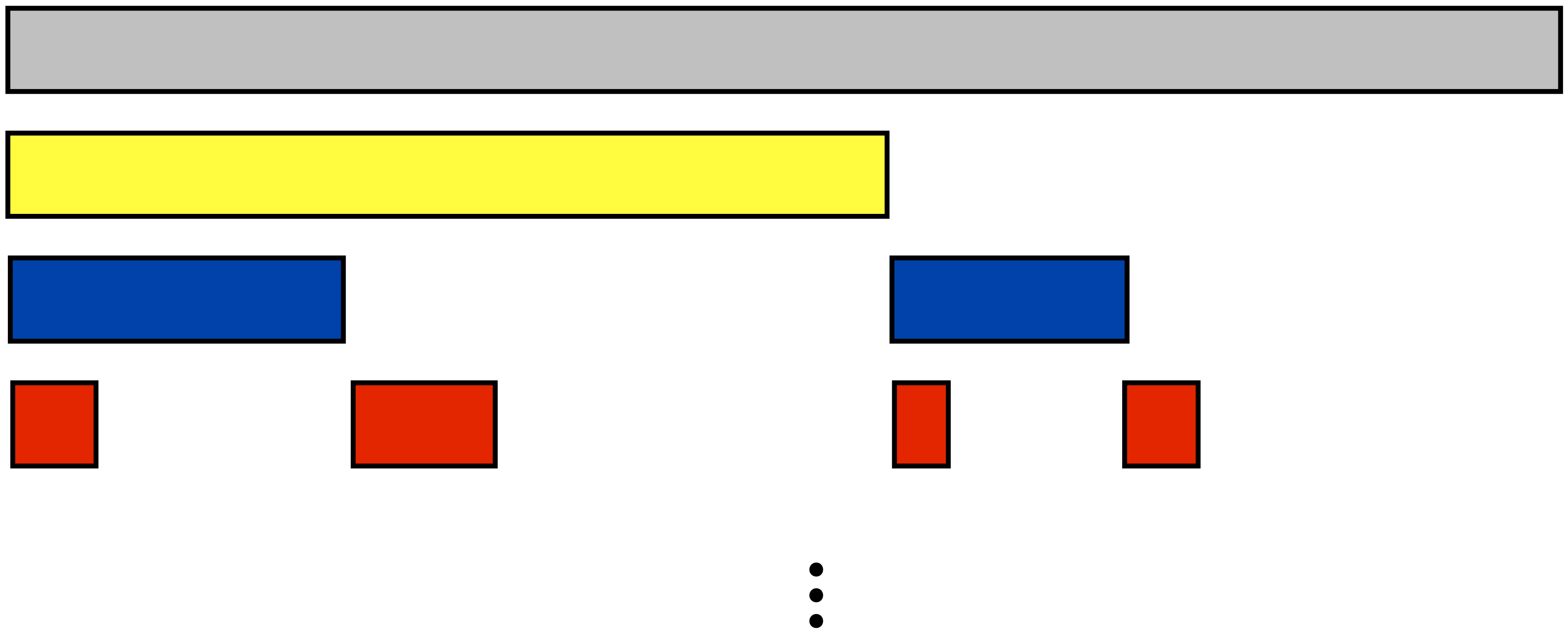
Paintboxes

Indian buffet process: beta feature frequencies

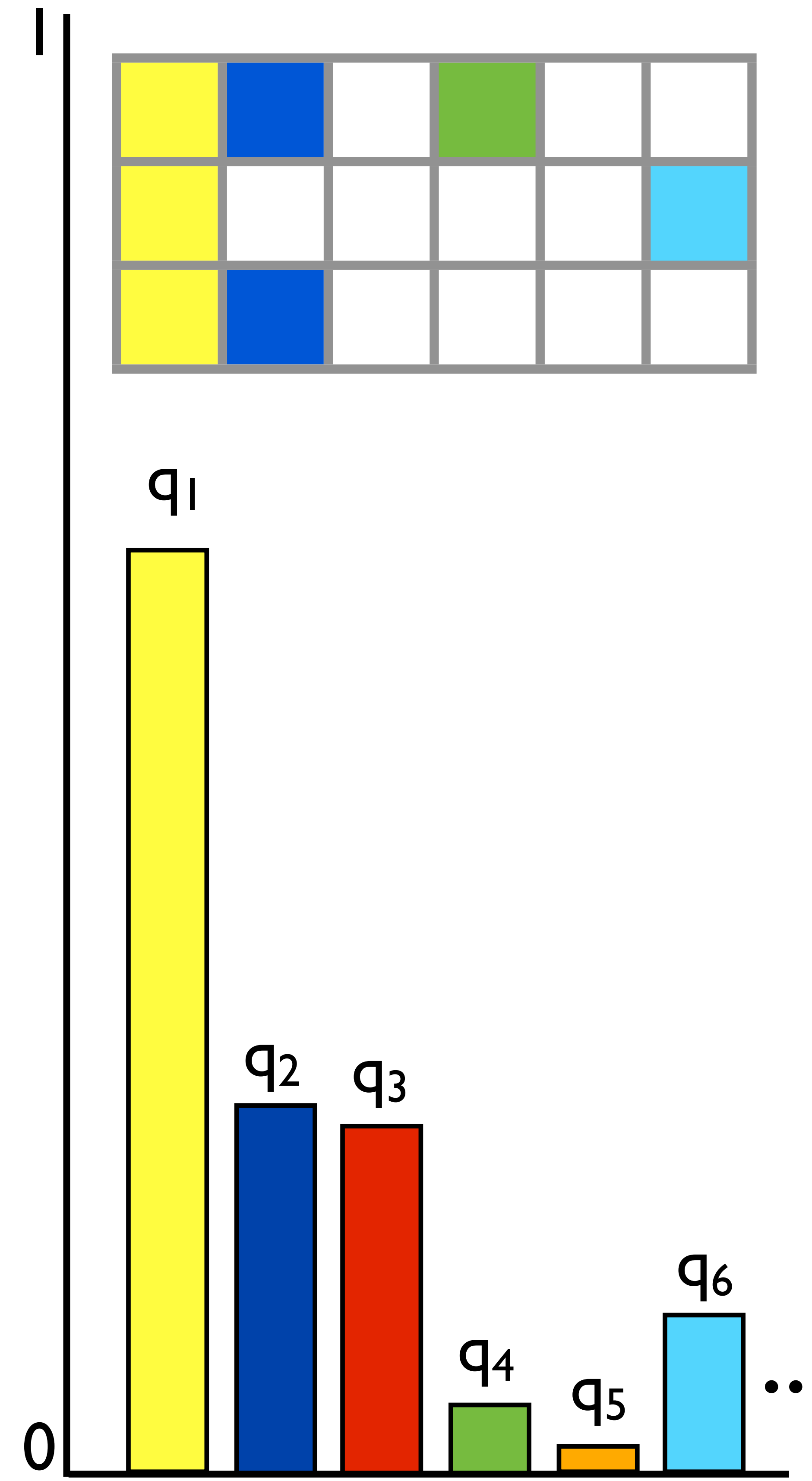


Paintboxes

Indian buffet process: beta feature frequencies

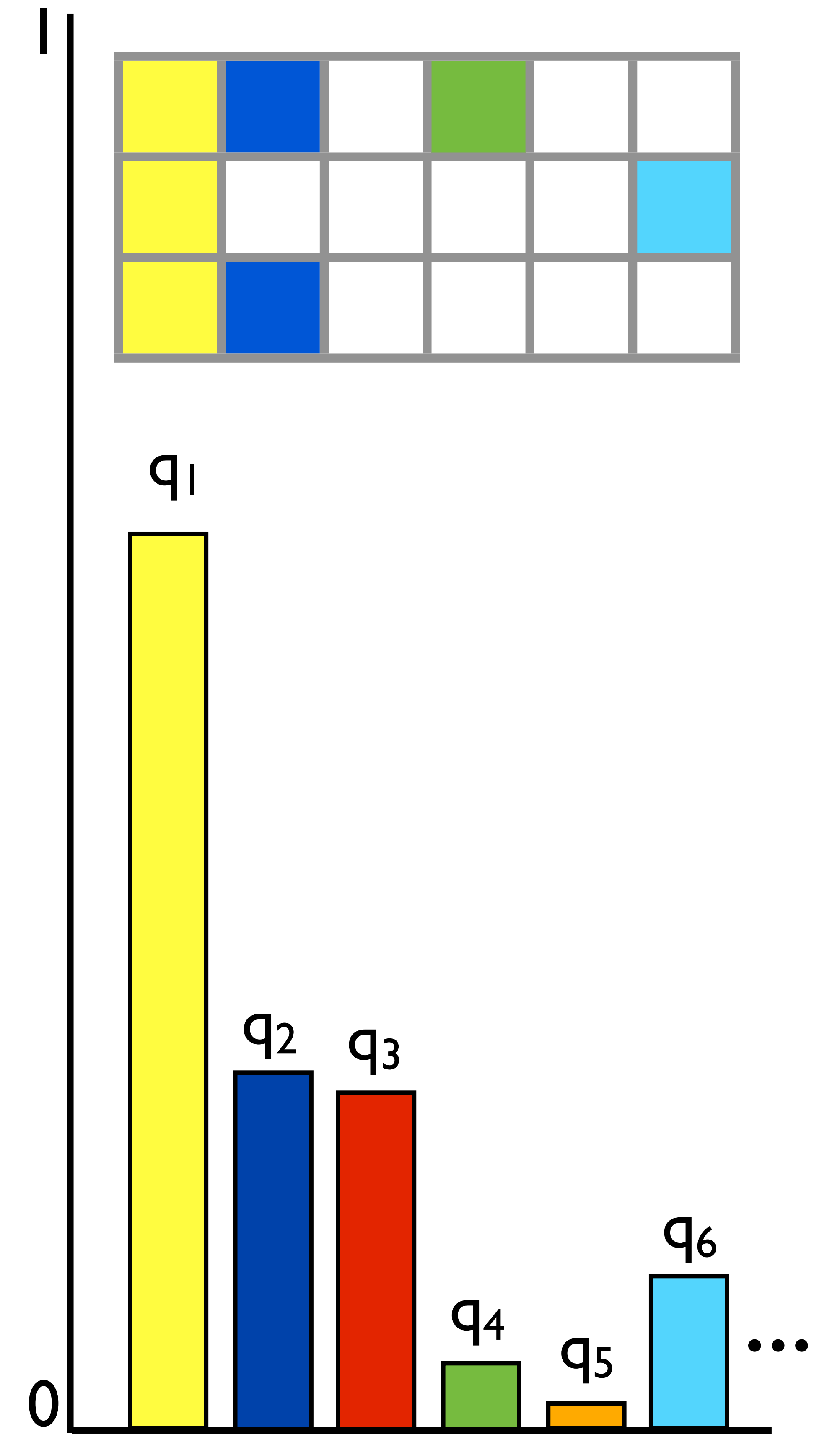


Paintboxes



Paintboxes

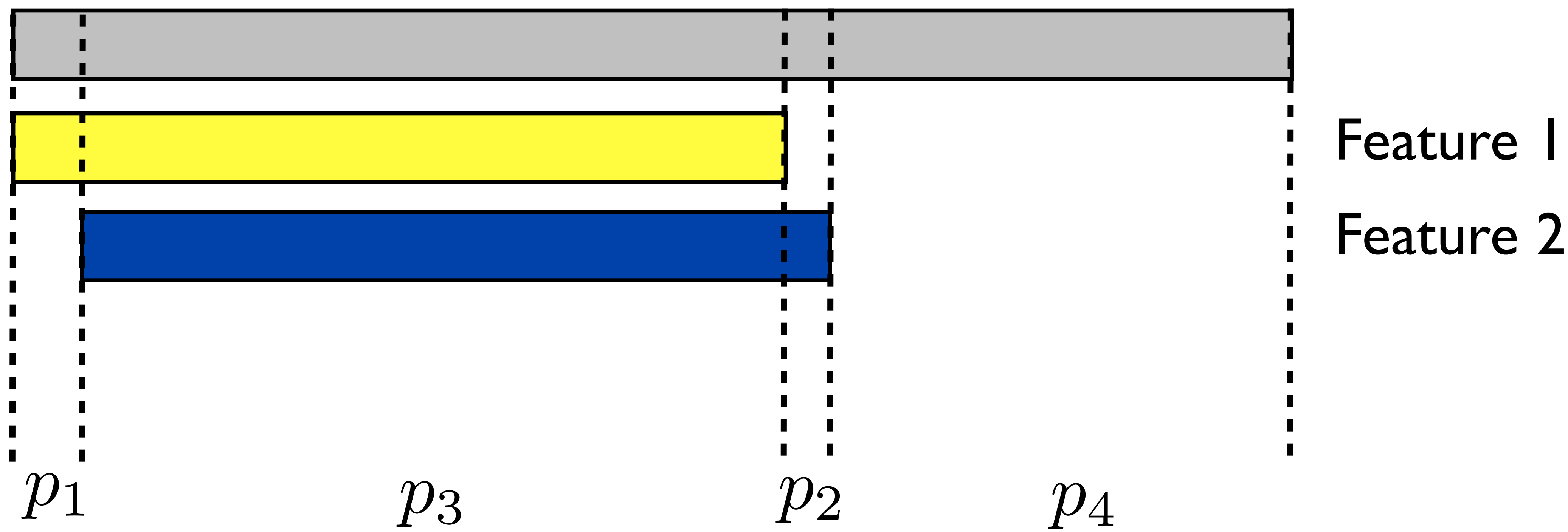
“Feature frequency models”



[Broderick, Pitman, Jordan 2013]

Paintboxes

Two feature example



$$\mathbb{P}(\text{row} = \blacksquare\square) = p_1$$

$$\mathbb{P}(\text{row} = \square\blacksquare) = p_2$$

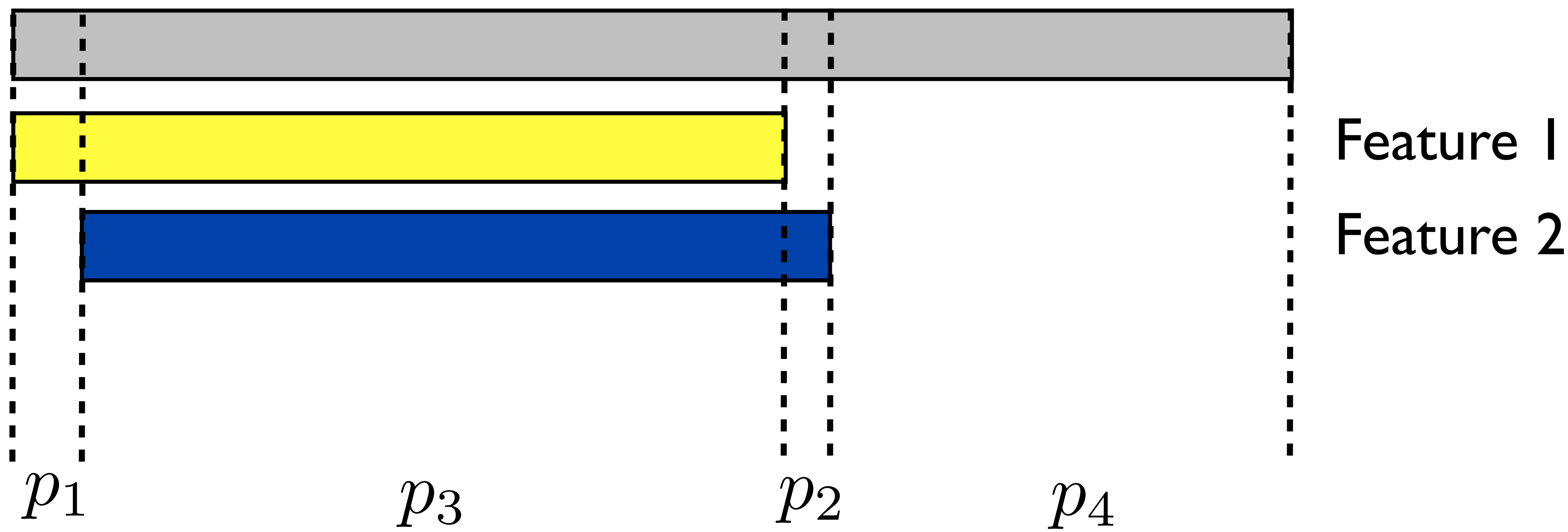
$$\mathbb{P}(\text{row} = \blacksquare\blacksquare) = p_3$$

$$\mathbb{P}(\text{row} = \square\square) = p_4$$

Paintboxes

Two feature example

Not a feature frequency model



$$\mathbb{P}(\text{row} = \blacksquare\square) = p_1$$

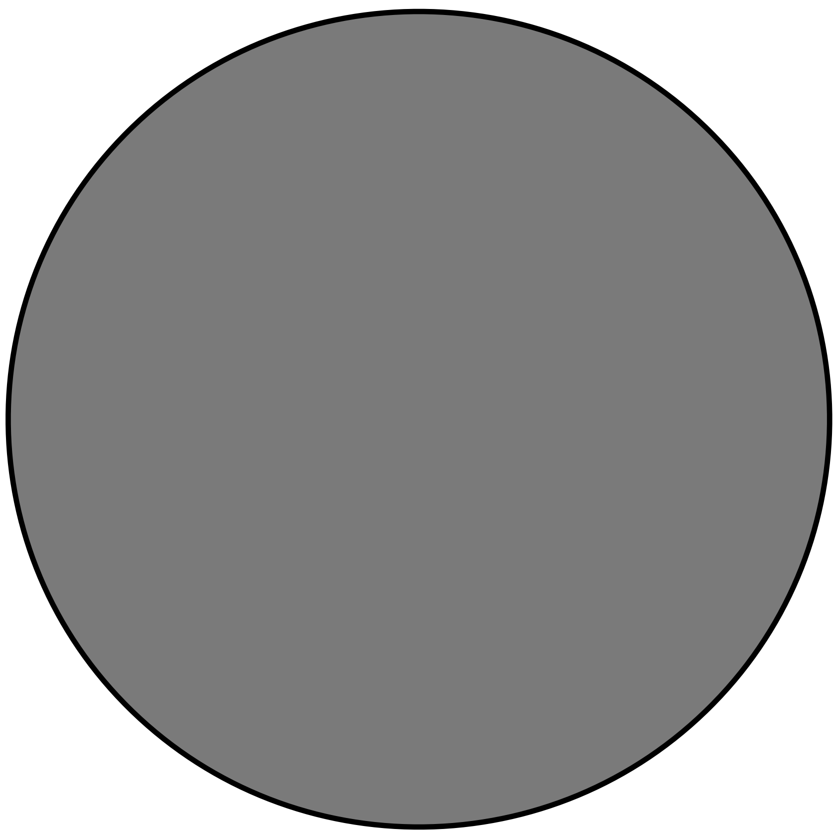
$$\mathbb{P}(\text{row} = \square\blacksquare) = p_2$$

$$\mathbb{P}(\text{row} = \blacksquare\blacksquare) = p_3$$

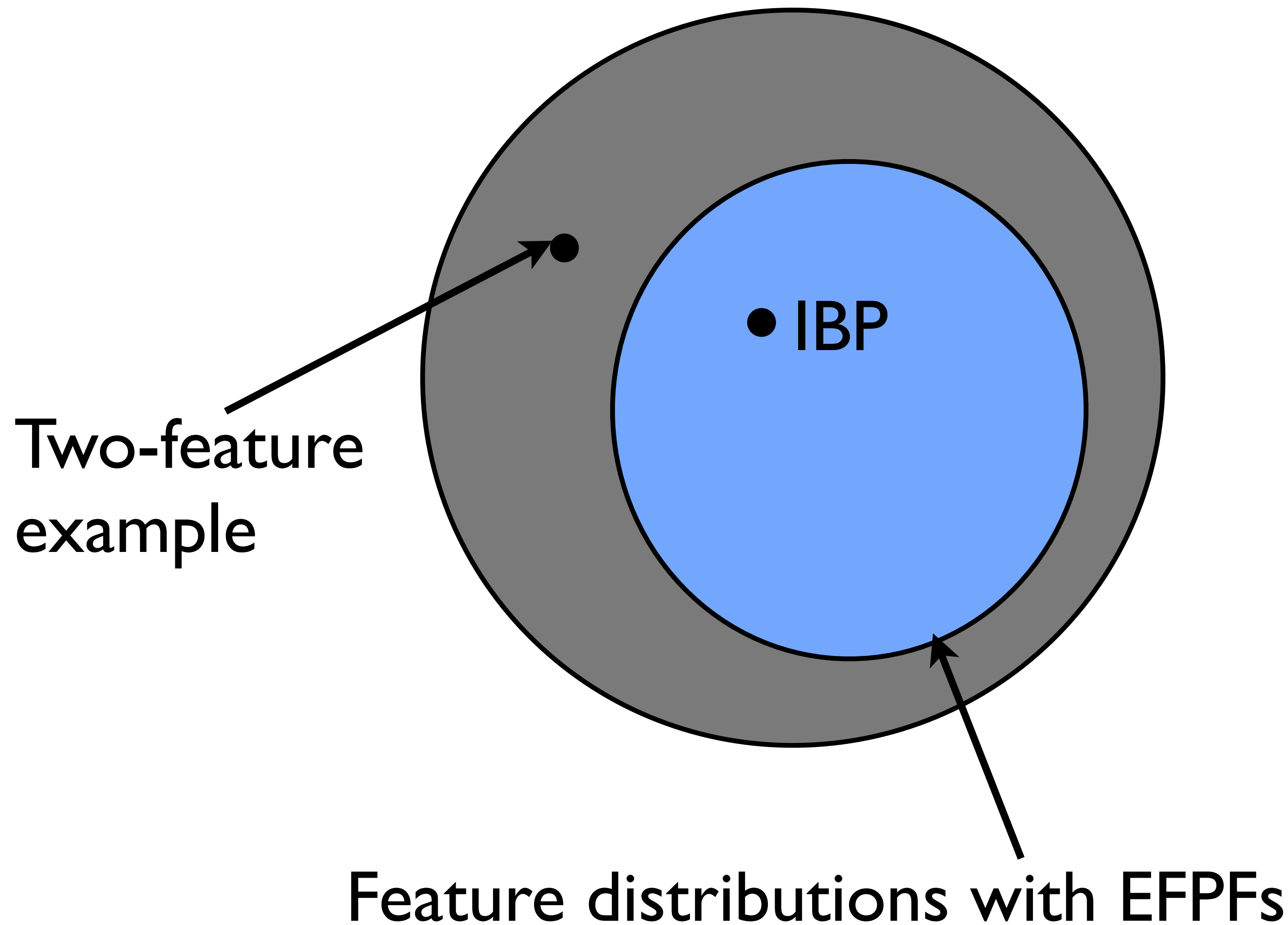
$$\mathbb{P}(\text{row} = \square\square) = p_4$$

Paintboxes

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



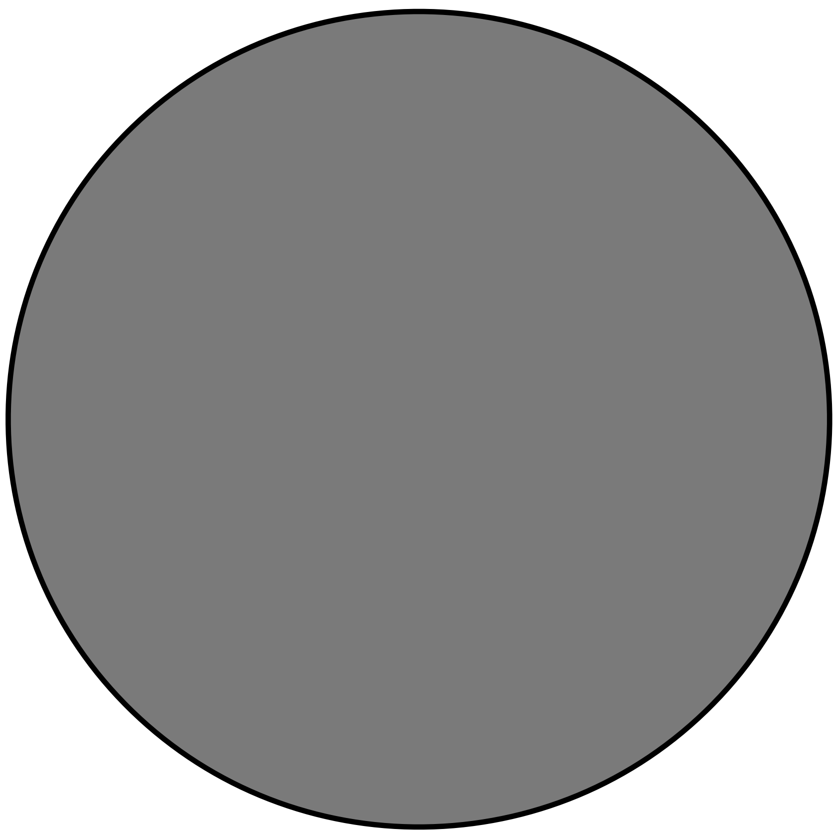
Exchangeable feature distributions
= Feature paintbox allocations



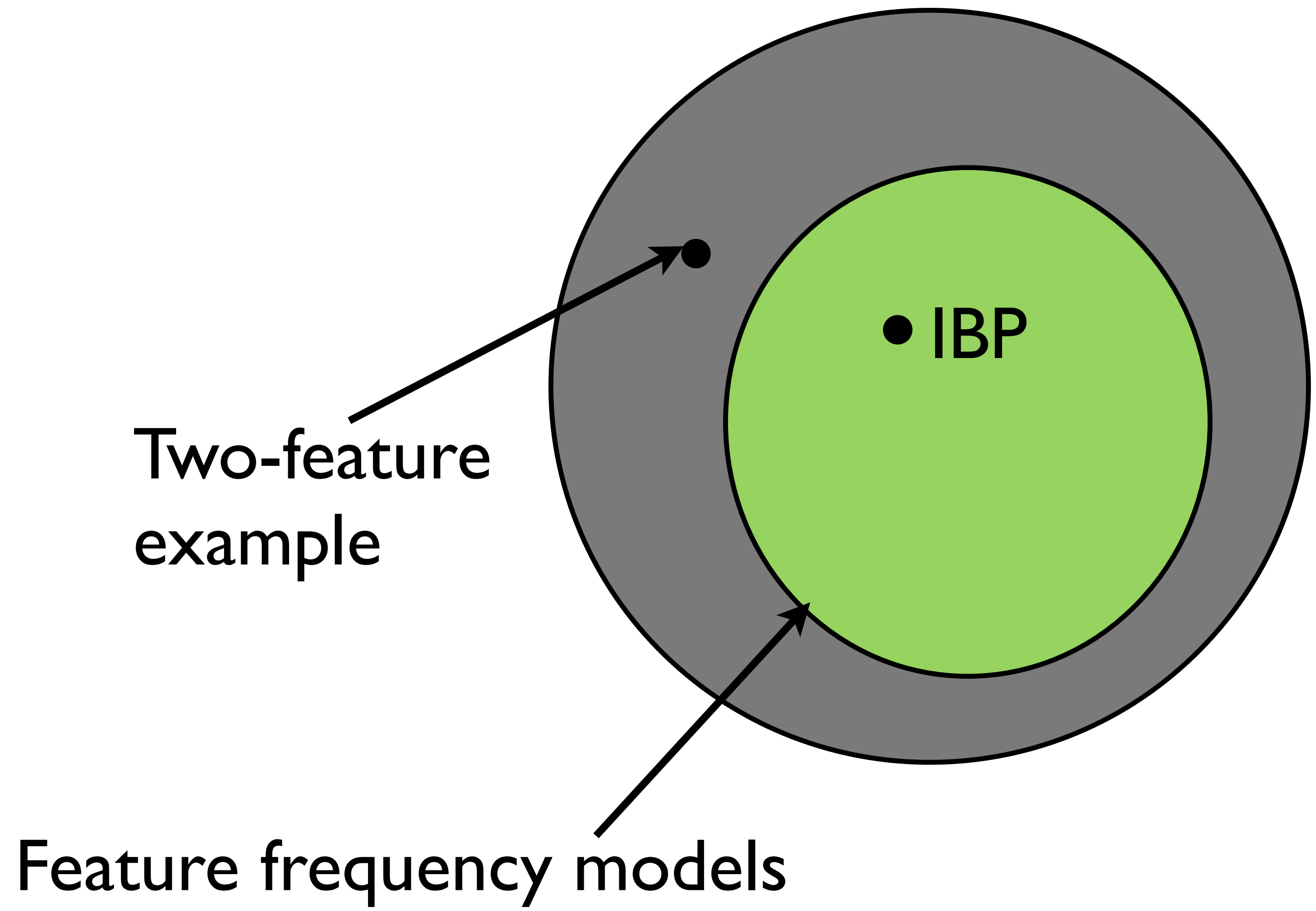
[Broderick, Pitman, Jordan 2013]

Feature frequency models: EFPFs?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



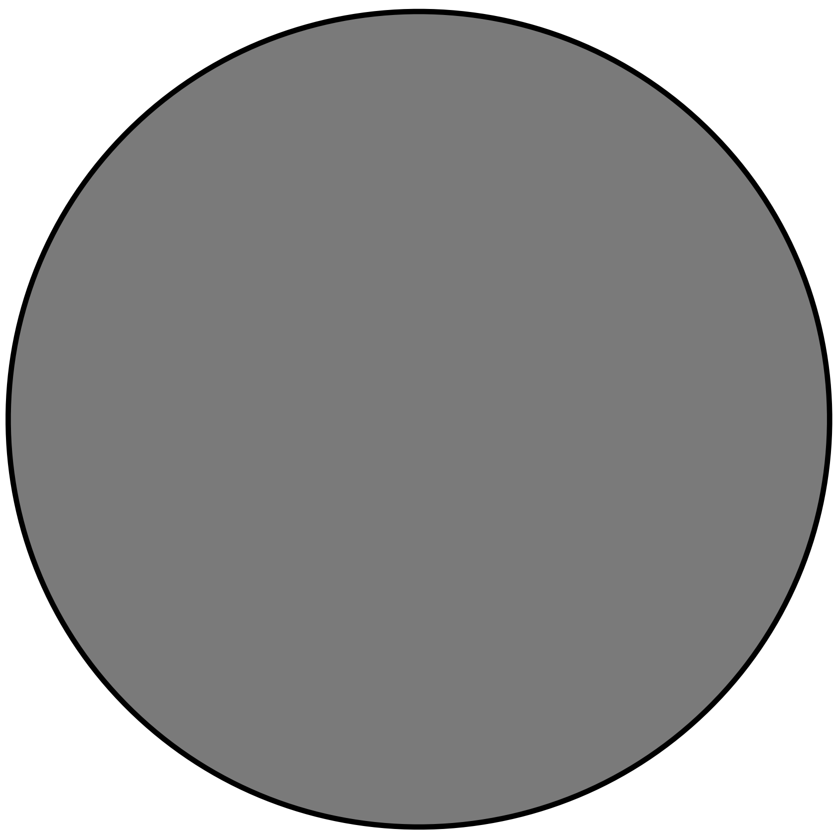
Exchangeable feature distributions
= Feature paintbox allocations



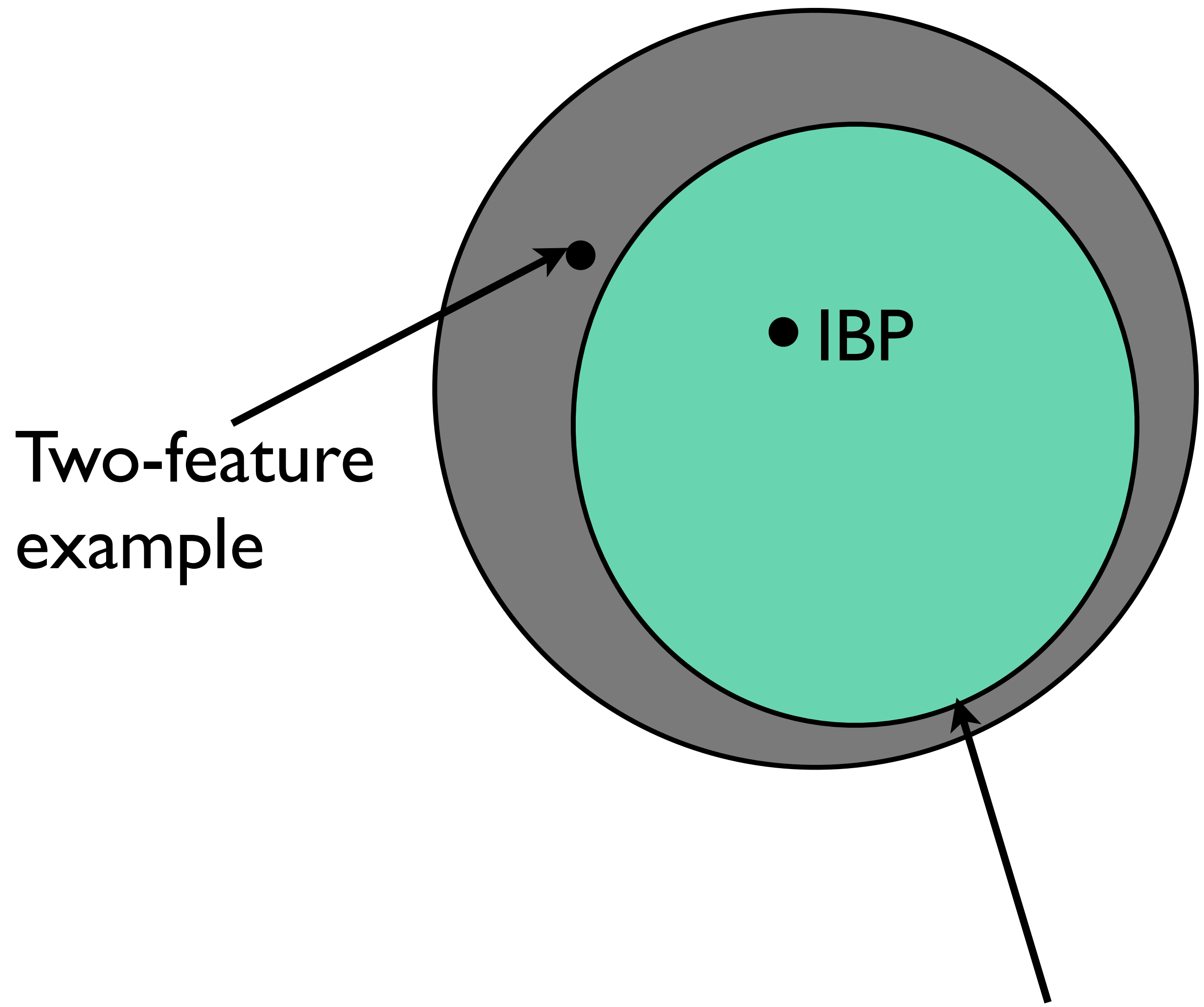
[Broderick, Pitman, Jordan 2013]

Distributions with EFPFs: frequencies?

Exchangeable cluster distributions
= Cluster distributions with EPPFs
= Kingman paintbox partitions



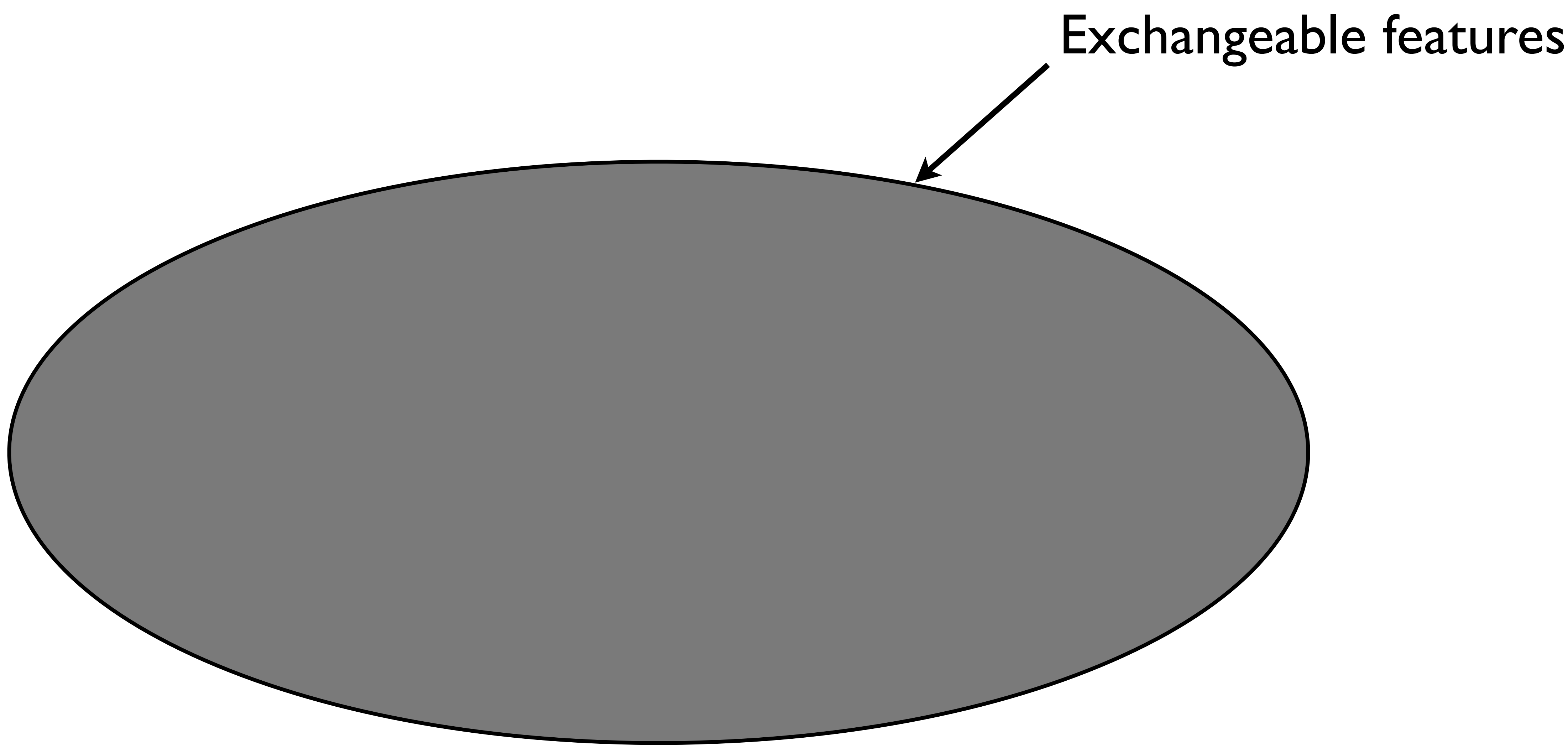
Exchangeable feature distributions
= Feature paintbox allocations



Feature distributions with EFPFs
= Feature frequency models

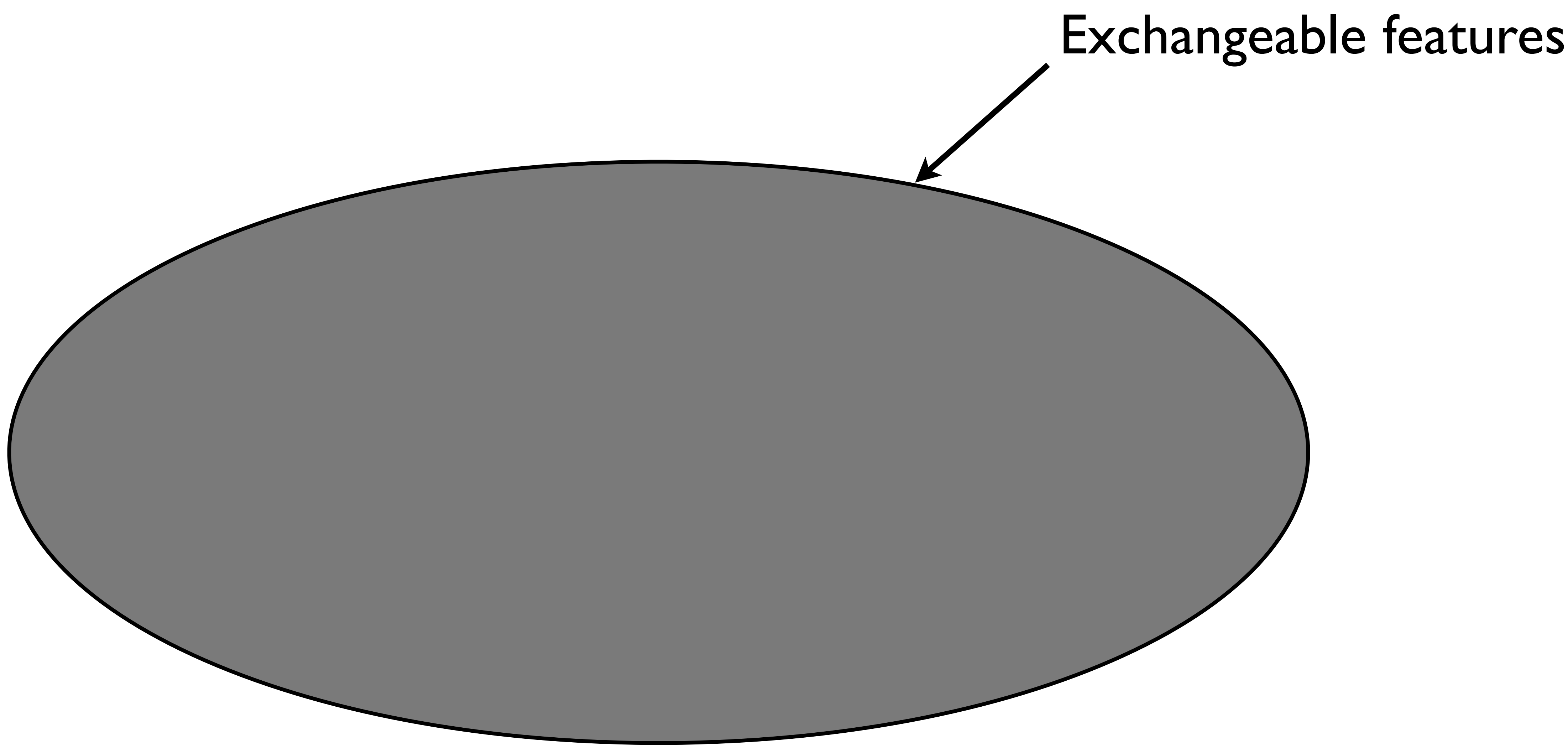
[Broderick, Pitman, Jordan 2013]

Conclusions



Conclusions

- Feature paintbox: characterization of exchangeable feature models



Conclusions

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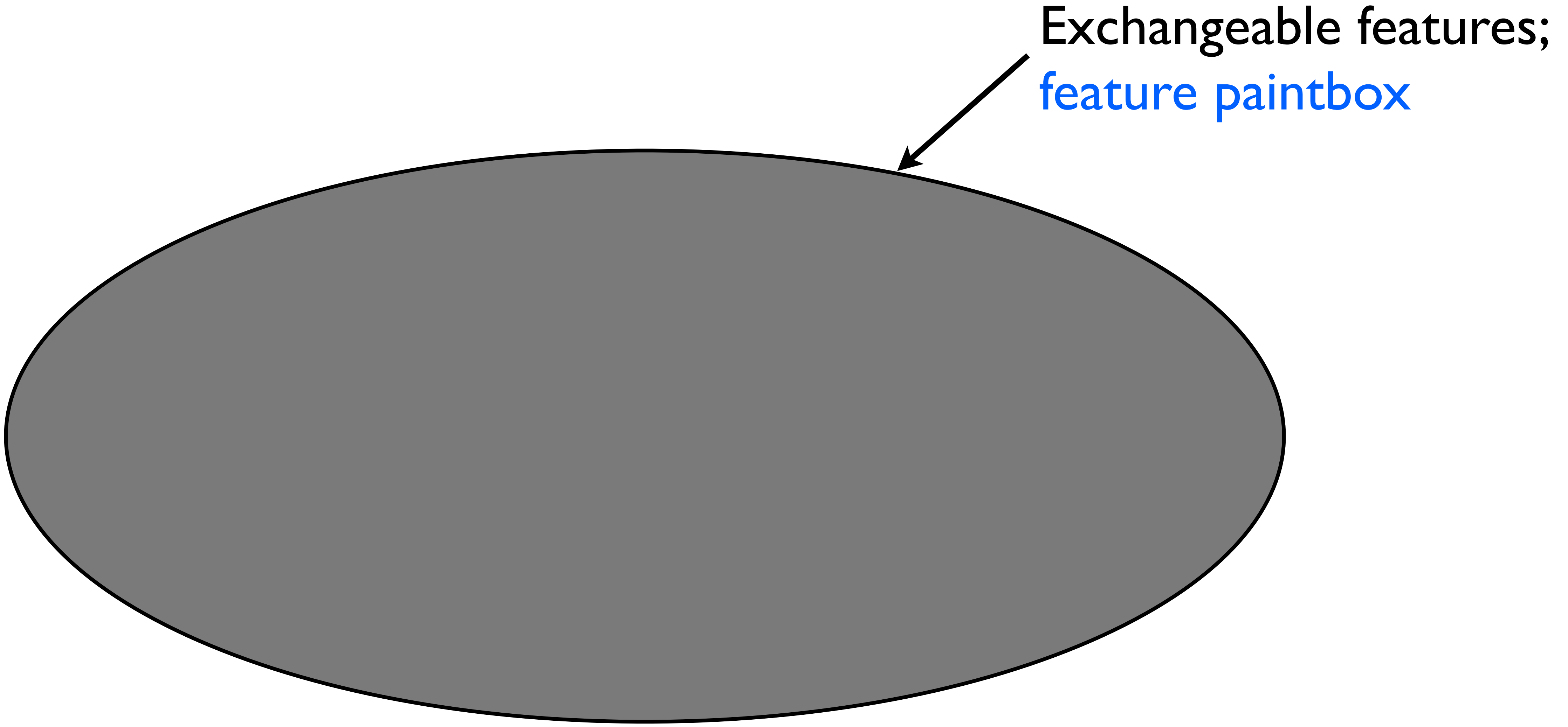
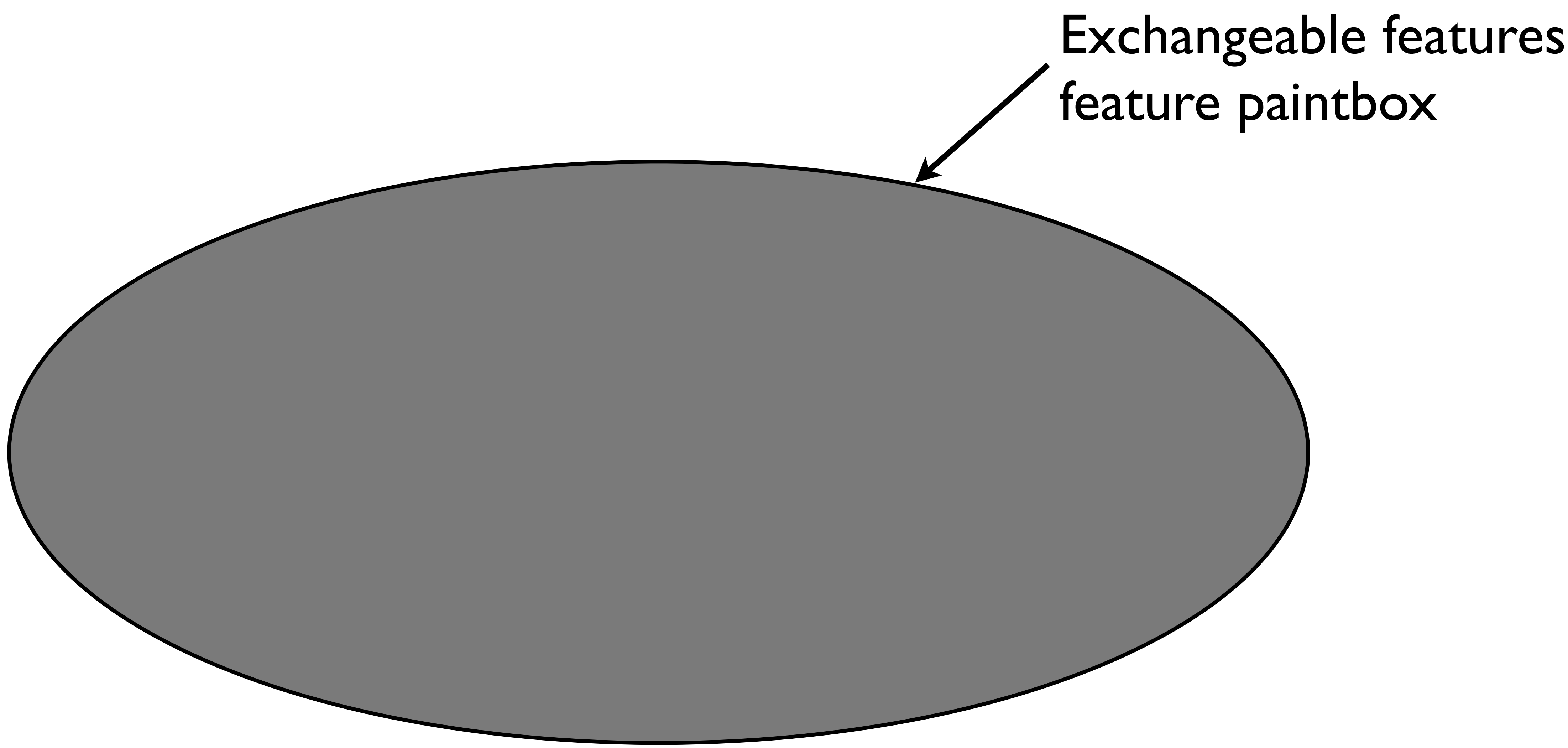


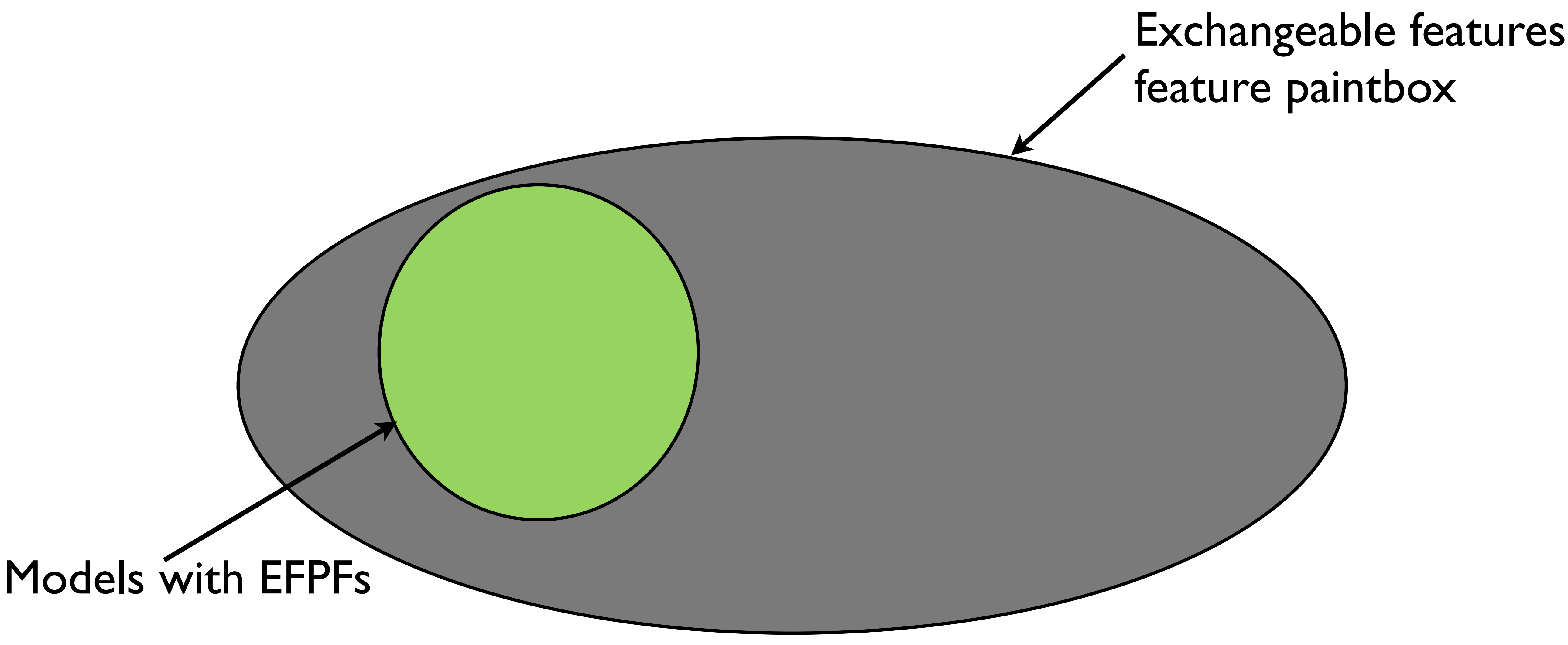
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



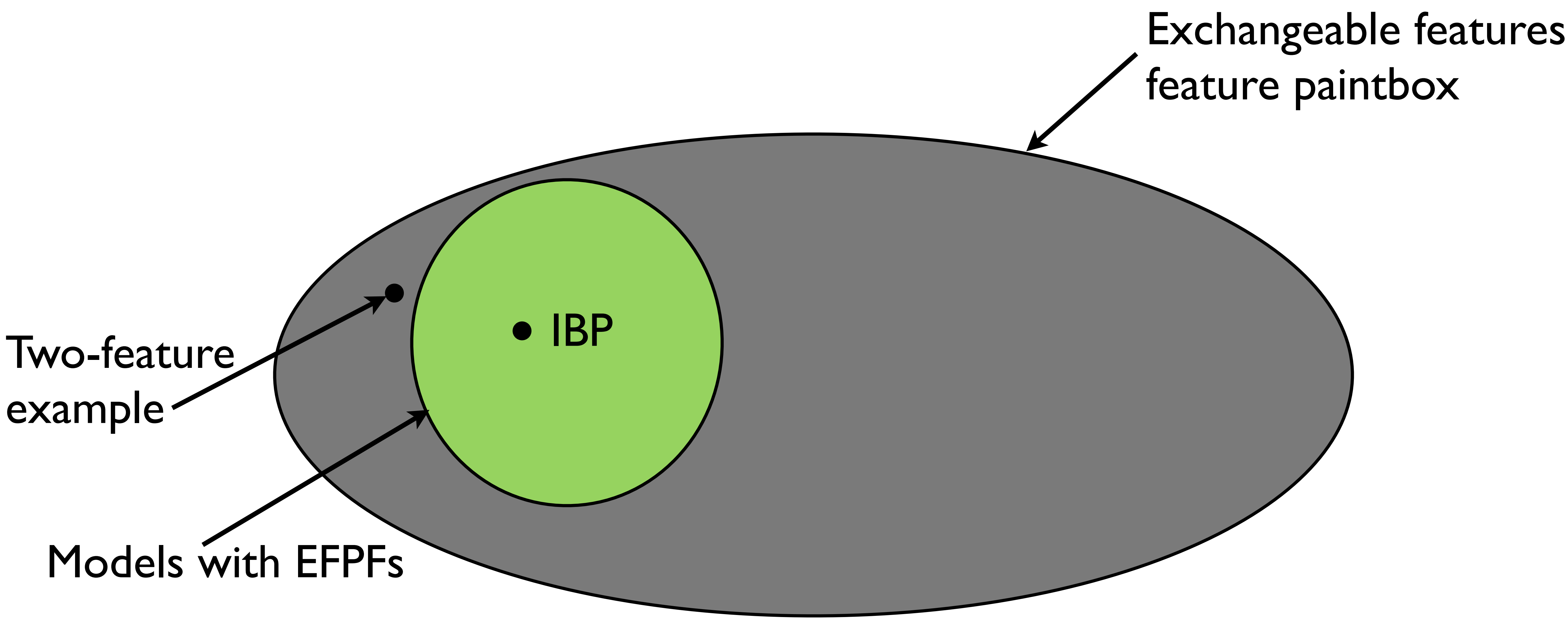
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



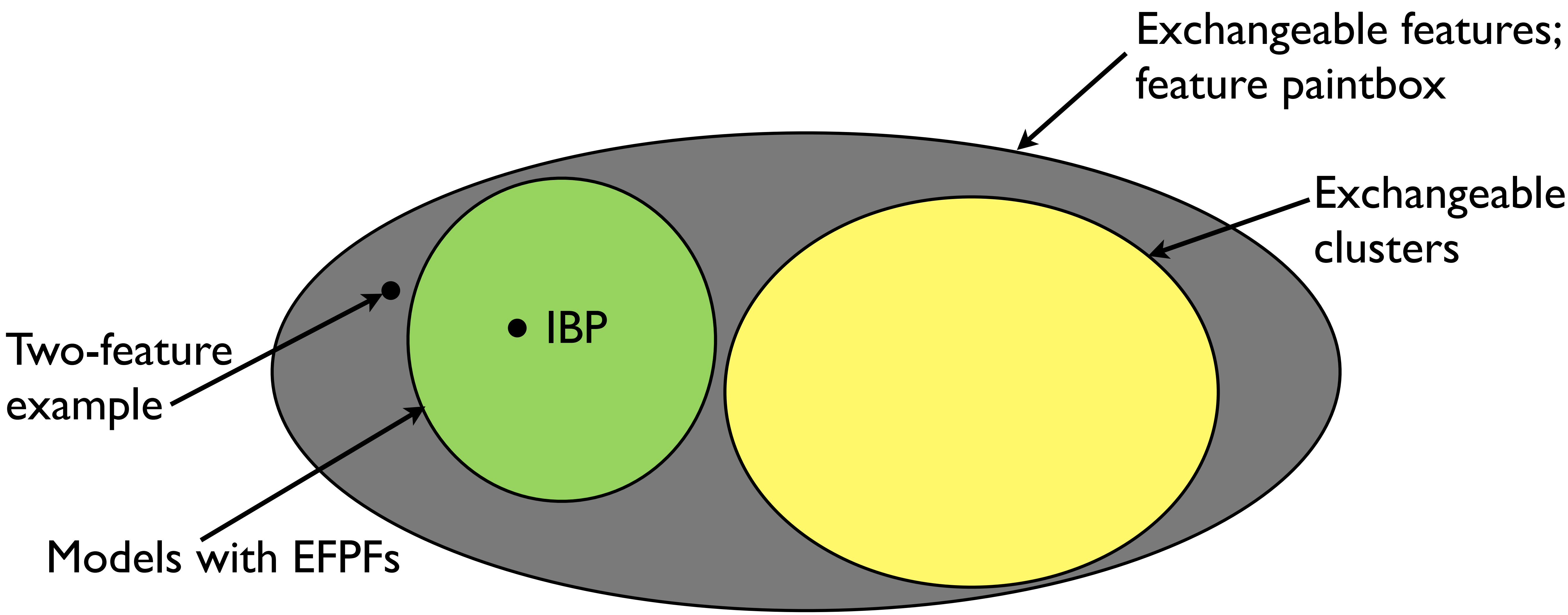
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



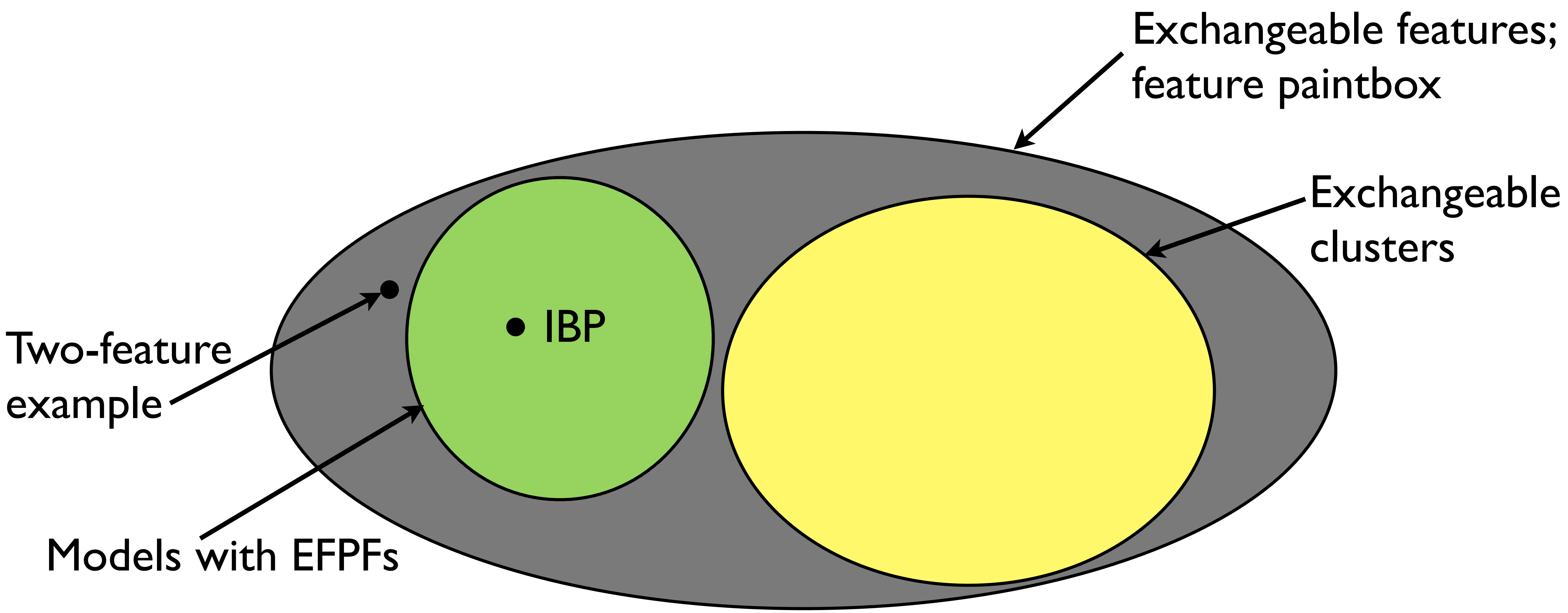
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Limits of clustering characterizations in feature case?



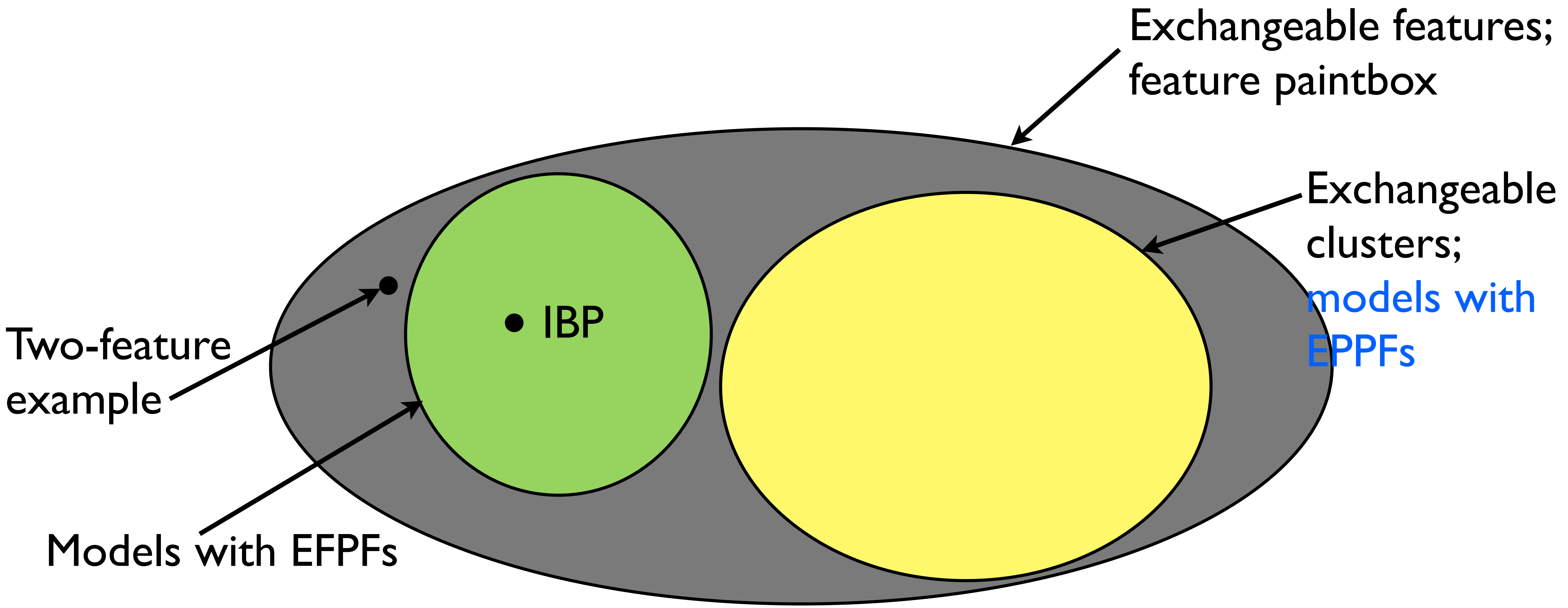
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



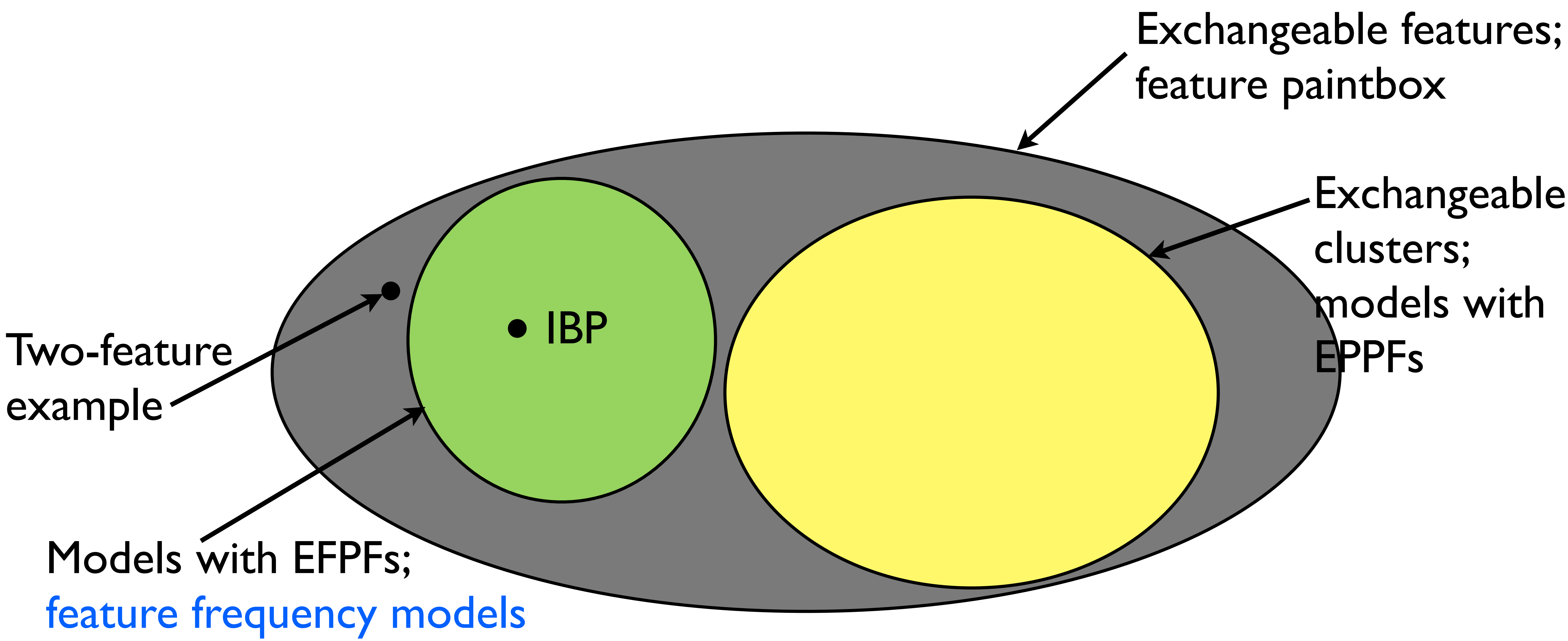
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



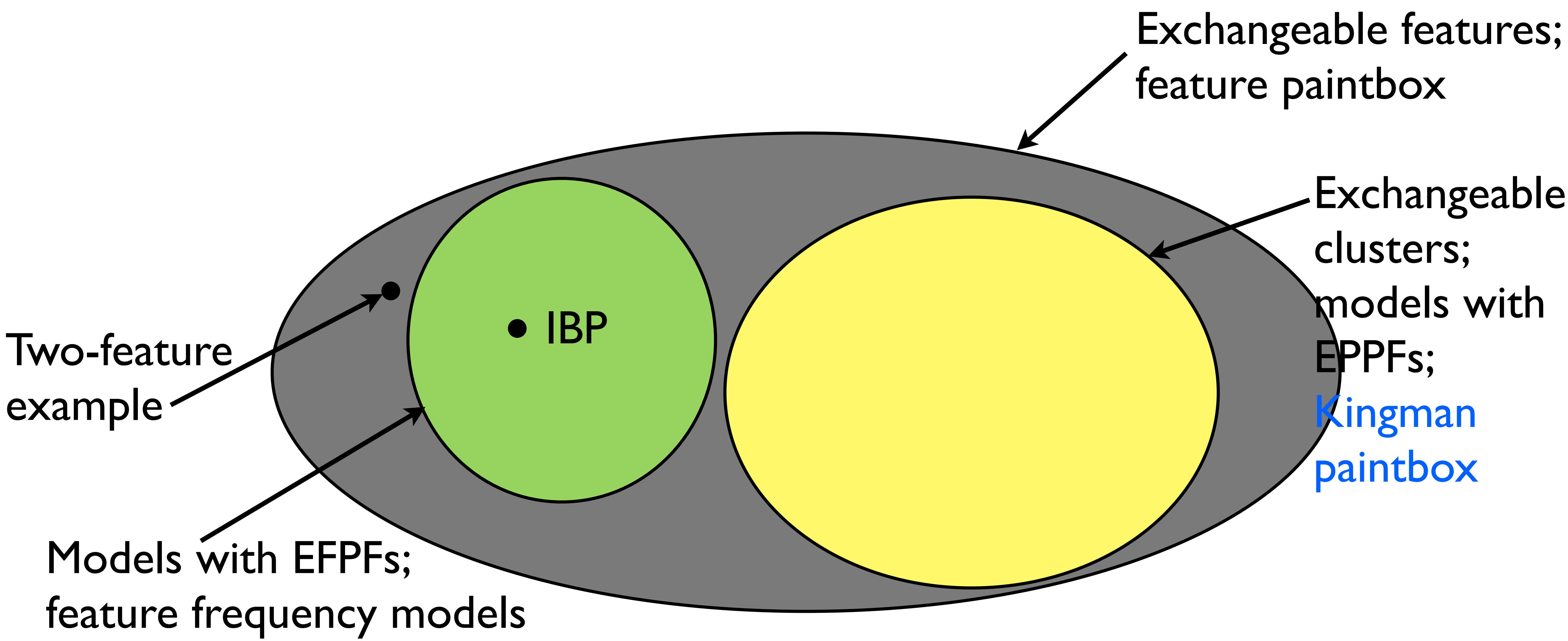
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



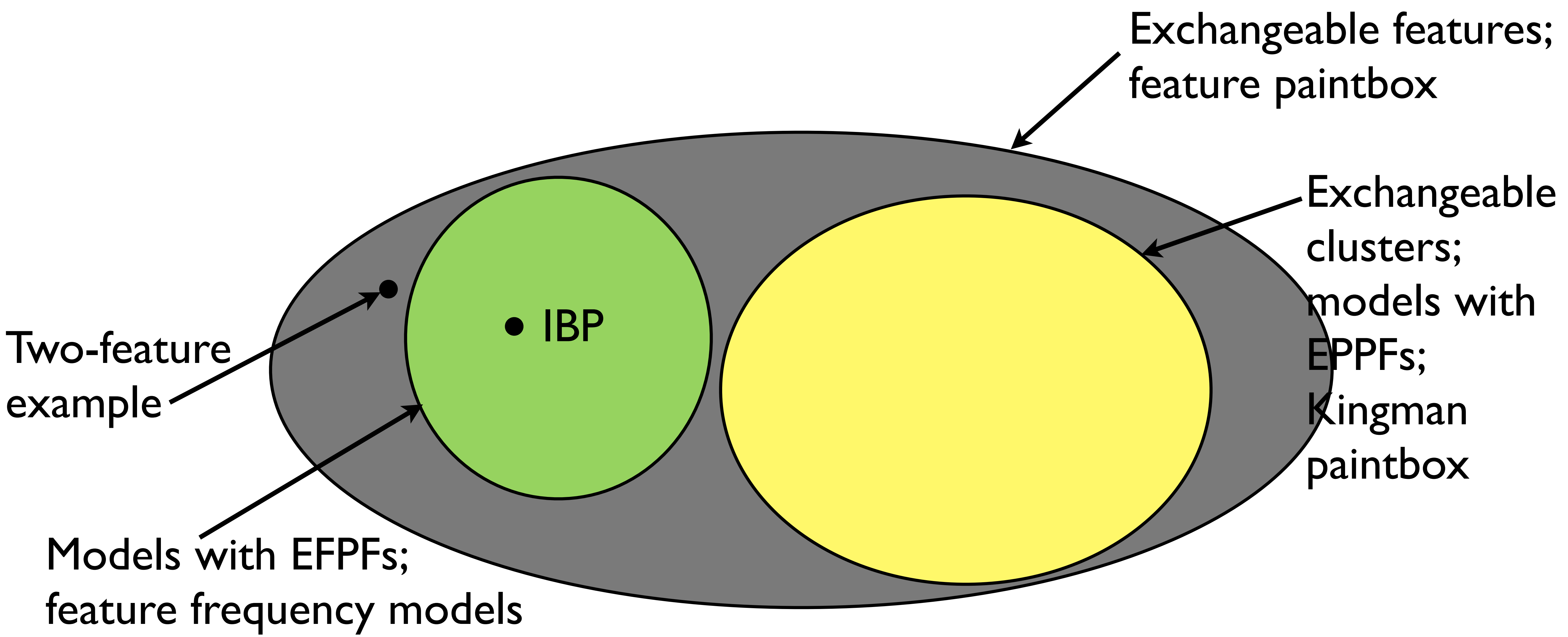
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure



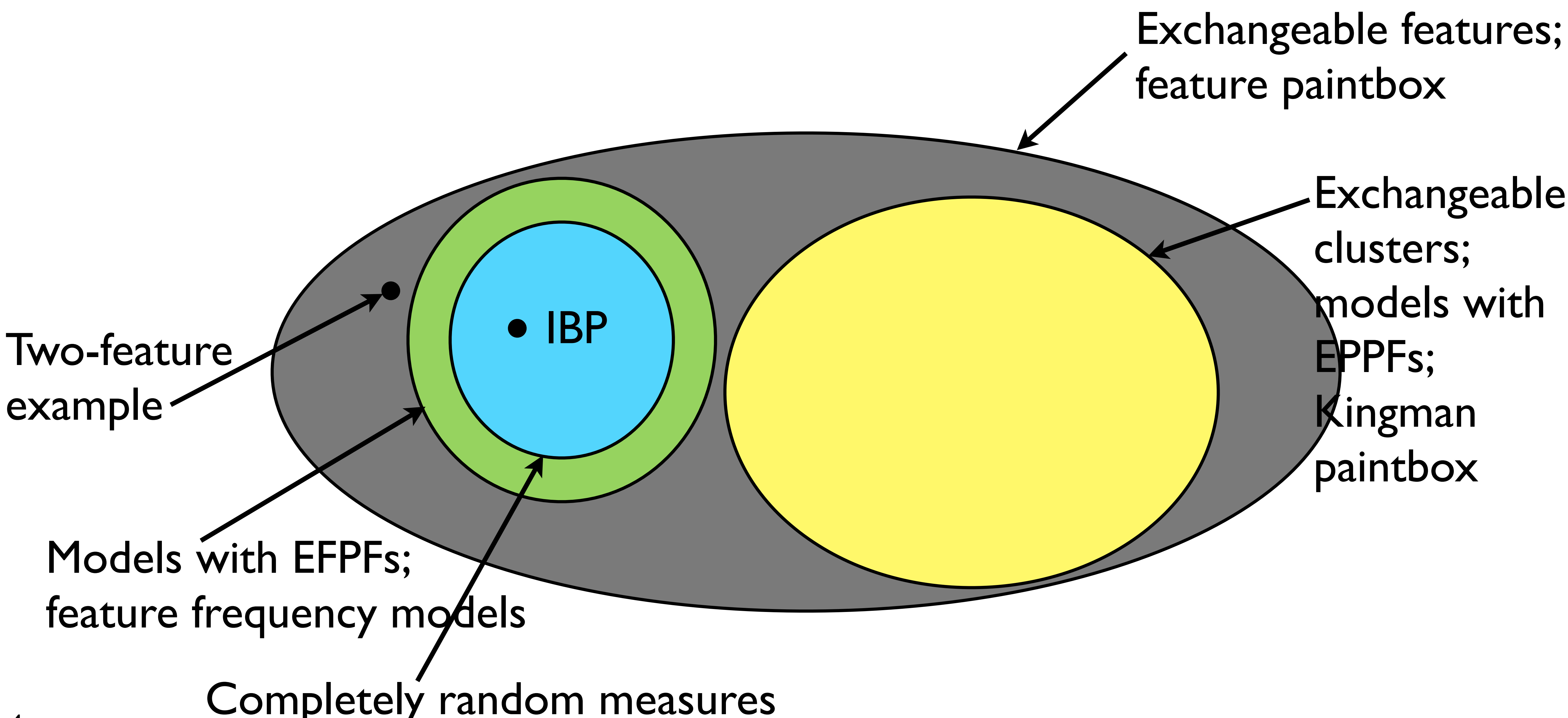
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections



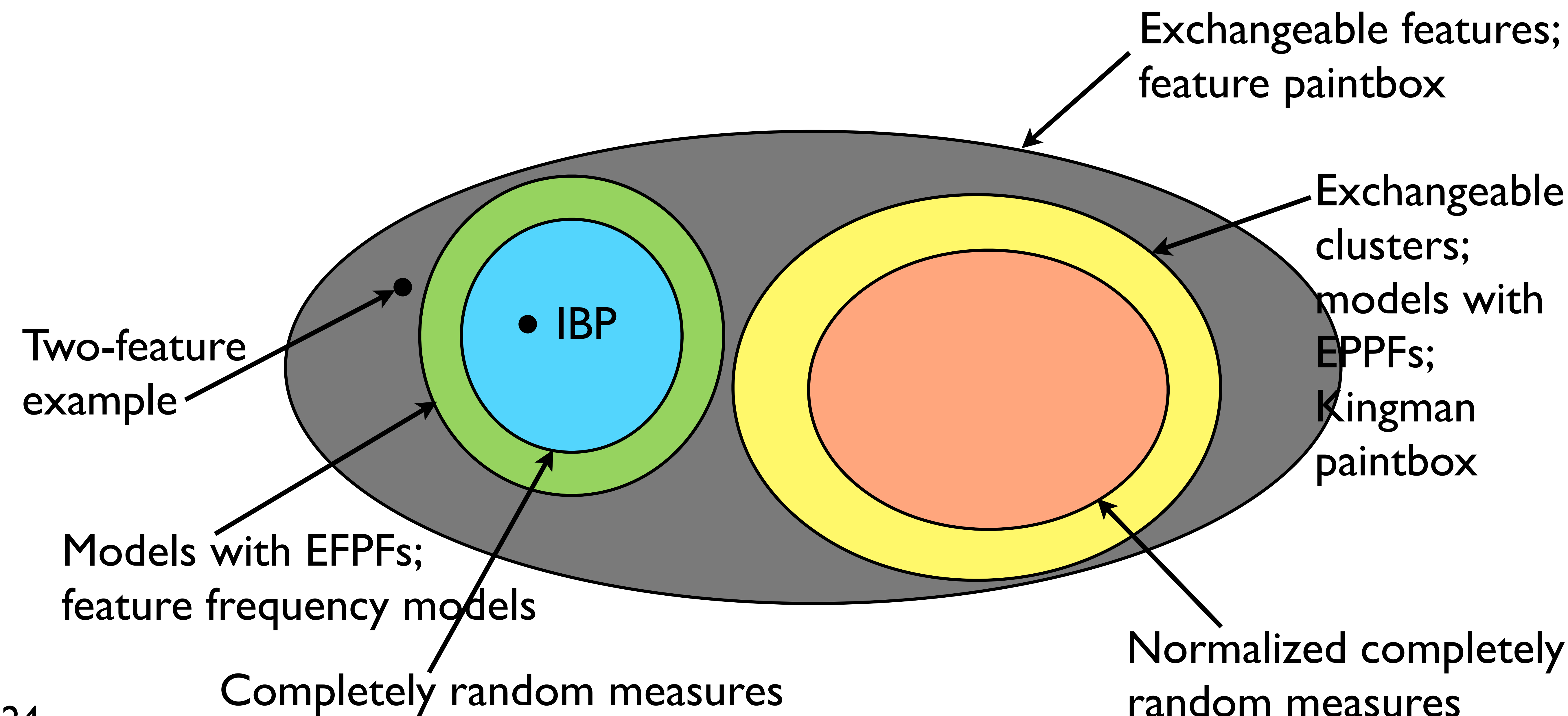
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



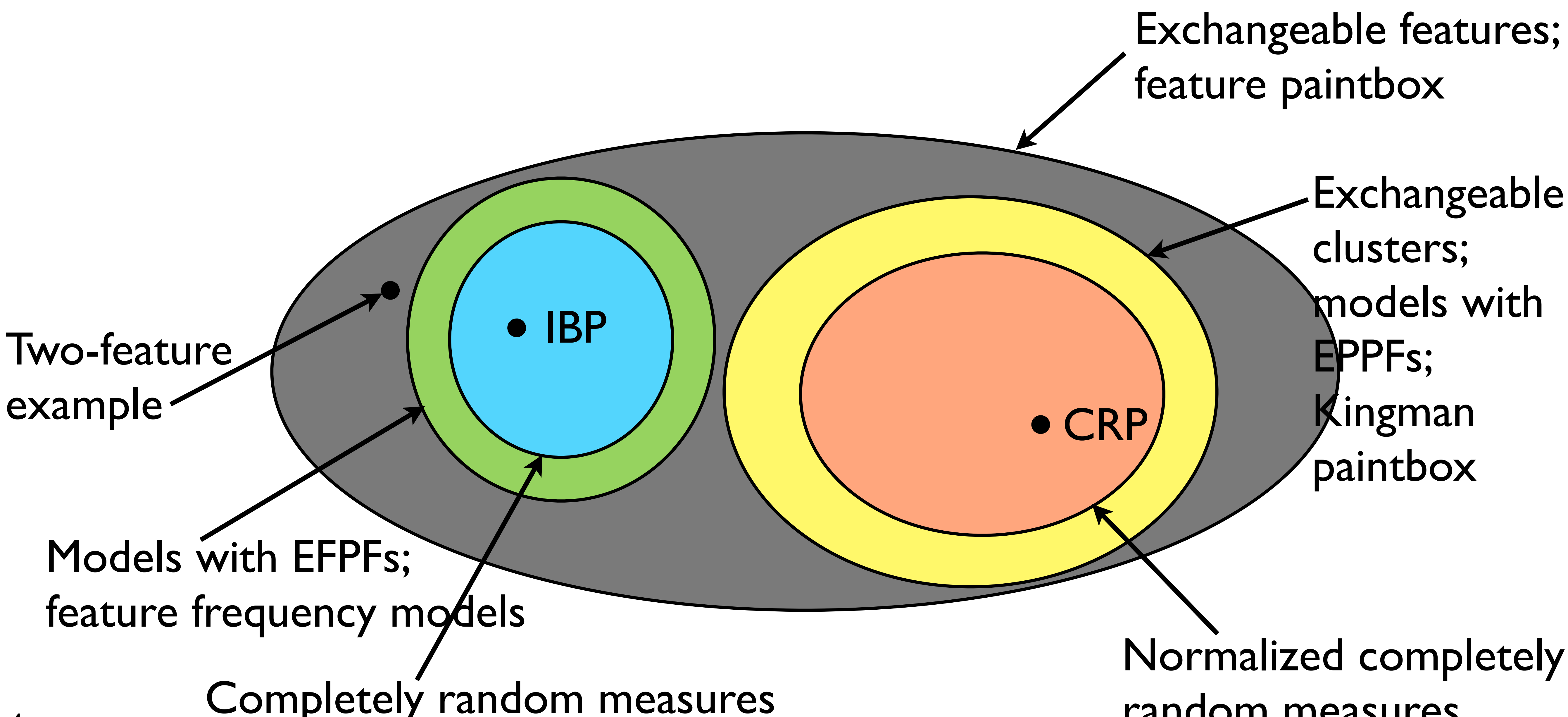
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



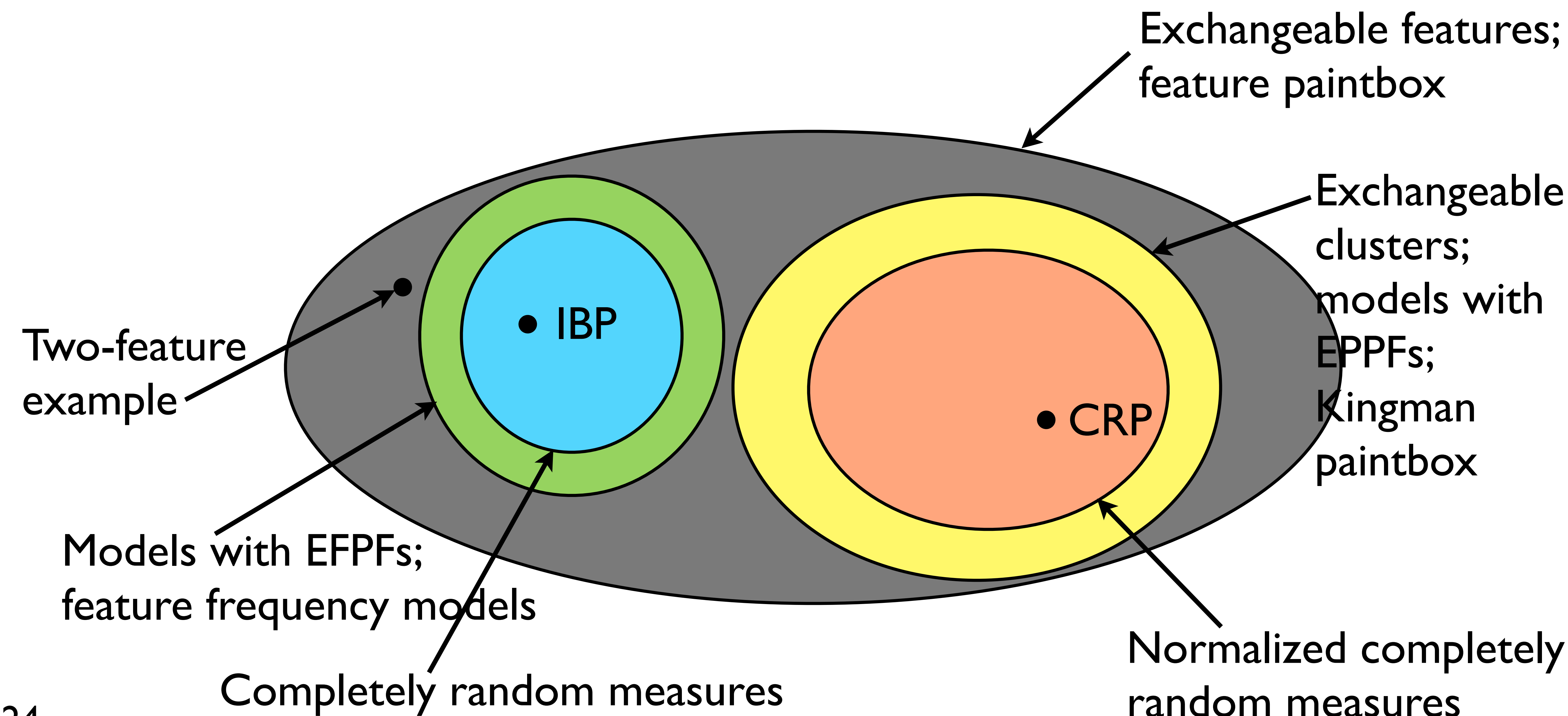
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs)



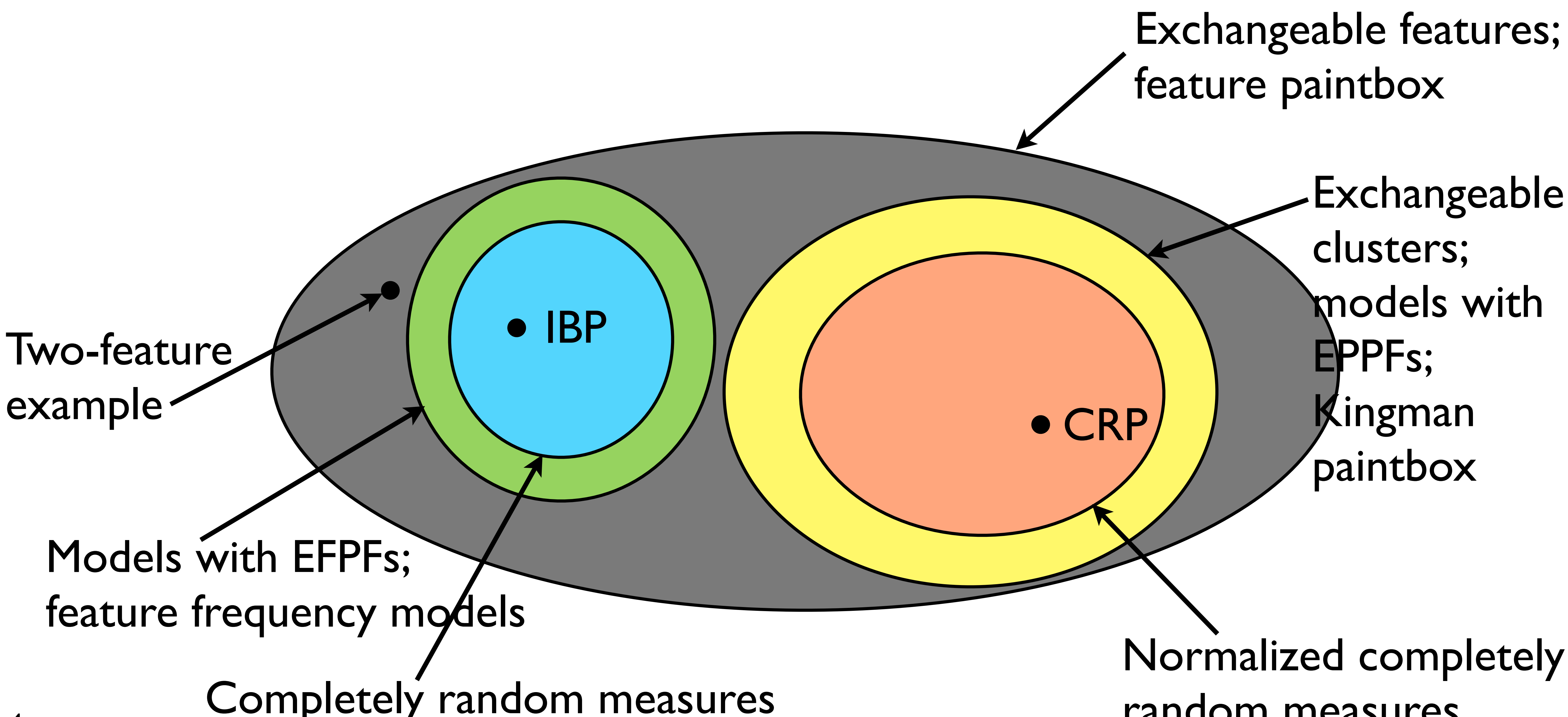
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust)



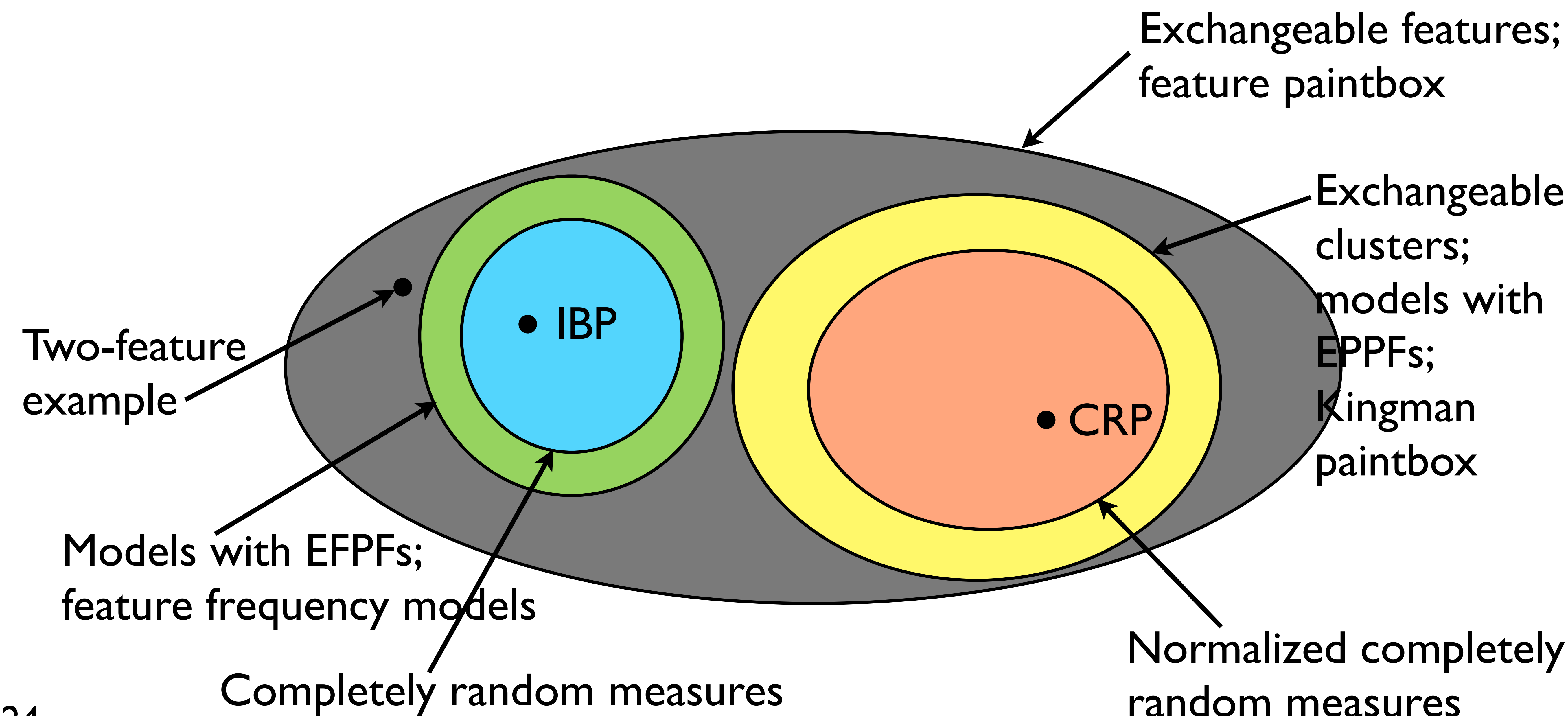
Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)



Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)
- Other combinatorial structures



Conclusions

- Feature paintbox: characterization of exchangeable feature models
- Characterization of alternative correlation structure
- Remaining connections (CRMs, dust, etc)
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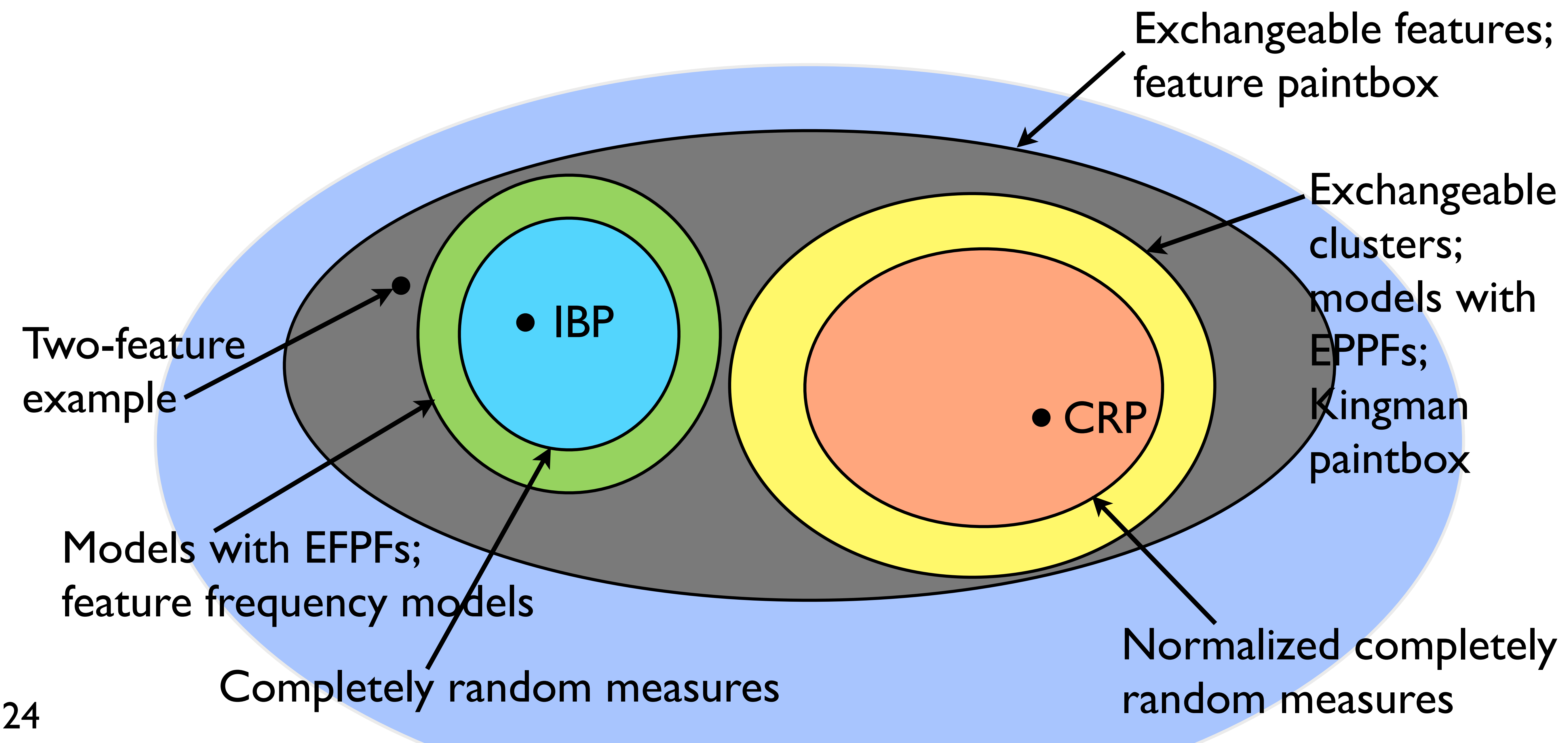

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