

# Combining latent topics with document attributes in text analysis

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## Abstract

This paper introduces a variant to existing models of multinomial regression for text analysis. Using the base model introduced by Taddy (2013), we extend the data-generating model to incorporate topics not explained by metadata. In doing so, we seek to increase the prediction accuracy over existing techniques, bridge the gap between multinomial regression and standard topic models, and investigate methods for discovering new topics in a corpus. We explore computational aspects of our approach, provide software for parallelization of the algorithm, and conclude by proposing areas of future research.

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# Introduction

## Text Data

A common technique for modeling text data is the use of multinomial models on word stems derived from the original document. Typically, text information is naturally grouped by documents, and each document is represented by counts of words, or "tokens". A document might be a single written text (e.g. an academic article), or a collection of works by the same author (e.g. all of the lyrics of an album by the rolling stones) A token is often a single word (called unigram) but may also be a sequence of two or more words (e.g. bi-grams, like 'good swimmer' is a bigram, or tri-grams, like 'I eat cheese'). The word components of tokens are often reduced to a root form by removing suffixes (e.g. 'illuminated', 'illumination' and 'illuminating' all become 'illuminate').

These tokens are then aggregated by document: For  $i$  in  $i = 1, \dots, N$ , the count vector  $x_i = [x_{i1}, x_{i2}, \dots, x_{ip}]$  contains the number of occurrences of first, second,  $\dots$   $p$ th token in the  $i$ th document, where  $p$  is the total number of unique tokens in all documents. This forms the complete count matrix  $X$ , where each  $x_{ij}$  is the number of occurrences of word  $j$  in document  $i$ .

Since the number of unique words that appear in a large number of documents can be extensive, we often restrict the number of tracked tokens,  $p$  to words that occur in at least two documents. We may also remove common tokens that add little meaning and are found in all documents (i.e. 'the' or 'of').

A trivial example of such content might be student's answers to the question "What did homework assignments involve?", with the following four responses:

Table 1: Example of text data from course reviews

Document	Content
1	Some computation and formula proving, a lot of R code
2	Problems, computation using R
3	Some computations and writing R code
4	Proofs, problems, and programming work

After removing common words and stemming the remaining words, we might produce the count matrix in Table 2.

## Multinomial Model

We then model each document  $x_i$  as the realization of a multinomial distribution. That is,

$$x_i \sim MN(q_i, m_i)$$

Table 2: Creating a word-count matrix from text

Document	Some	comp	formula	prov	R	code	use	problem	writ	program	work
1	1	1	1	1	1	1	0	0	0	0	0
2	0	1	0	0	1	0	1	1	0	0	0
3	1	1	0	0	1	0	0	0	1	0	0
4	0	0	0	1	0	0	0	1	0	1	1

Where  $q_i$  is the vector  $[q_{i1}, \dots, q_{ip}]$  of token probabilities for document  $x_i$  and  $m_i$  is  $\sum_{j=1}^p x_{ij}$ , or the total number of tokens in document  $i$

It is trivial to show that the maximum likelihood estimator of  $q_i$  is  $f_i = x_i/m_i$ , but by imposing structure on  $q_i$ , we can model features of the data. The two most common techniques for creating structure are *topic models* and *metadata*

## Topic Models

A topic model structure assumes that each document is created from a linear combination of  $K$  topics. Each topic  $l = 1, \dots, K$  represents a distribution, or vector of probability weights  $\omega_l = [\omega_{l1}, \dots, \omega_{lp}]$ , over words. As a simple example, we can imagine a fitness store that primarily sells books on biking, running, and swimming. We can see that a probability distribution of these topics would have high probability weights on the terms (“pedal”, “helment”) for biking, (“stride”) for running, and (“breath”, “stroke”, “water”) for swimming. By denoting the proportion of topic  $l$  as  $\theta_l$ , we can imagine each document as being generated by a linear combination of topics  $\omega_1\theta_1 + \omega_2\theta_2 + \omega_3\theta_3$ , described as the following data-generating process:

1. Choose  $\theta = \theta_1, \dots, \theta_K$  the proportion of topics. (i.e., a book completely about swimming would have  $\theta = (1, 0, 0)$ , a book about triathalons might have  $\theta = (1/3, 1/3, 1/3)$ ).
2. Choose  $m_i$ , the number of words in the document
3. For each word  $j \in 1, 2, \dots, m_i$ , choose topic  $l$  with probability  $\theta_l$ . With the corresponding weighting vector  $\omega_l$ , choose a word  $x_{ij}$

Traditionally, the topic model proportions are given a dirchelet prior, and the model is also known as Latend Dirichlet Allocation, or LDA. For a thorough introduction to topic models, we refer the reader to Blei, Ng, and Jordan (2003).

## Metadata

Text data is frequently accompanied by information, or metadata, about the text itself. For example, in academic journals, metadata on an article could include the number of times the

article has been cited, and the journal in which the article has been published. When this metadata is believed to be relevant to the composition of the document, we use the generic term *sentiment*. For example, given a database of written movie reviews and final rating out of five  $y \in (1, 2, 3, 4, 5)$ , we might want to model the relationship between the words used in the document and the final rating.

## Metadata and Unigram Models

If the support  $Y$  of metadata  $y$  takes discrete values  $y^{(1)}, y^{(2)}, \dots, y^{(m)}$  with few unique observations ( $m$  small), large computational gains can be had by collapsing the token counts over levels of metadata. That is, each dataset of  $n$  ordered (text, metadata) pairs

$$\left[ (x_1, y_1), (x_2, y_2), \dots, (x_n, y_n) \right] \quad (1)$$

can be expressed as  $m$  collapsed observations:

$$\left[ \left( \sum_{x_i: y_i = y^{(1)}} x_i \right), \left( \sum_{x_i: y_i = y^{(2)}} x_i \right), \dots, \left( \sum_{x_i: y_i = y^{(m)}} x_i \right) \right] \quad (2)$$

Then, a simple log-link model allows us to express text  $x$ , with given metadata rating  $y$ , denoted  $x_y$ , as  $x_y \sim MN(q_y, m_y)$ , with

$$q_{yj} \sim \frac{\exp[\alpha_j + y\phi_j]}{\sum_{l=1}^p \exp[\alpha_l + y\phi_l]} \quad (3)$$

## Theory and Approach

### Mixture models and cluster membership

We now turn our attention to the main purpose of this paper, which is to incorporate latent topics across documents while maintaining the computational simplicity of a collapseable multinomial model. To do so we will restrict our model by assuming that every document is a member of one and only one topic. In order not to confused our approach with a traditional topic models, where each topic can take a weight  $\theta \in (0, 1)$ , we refer to the model in which a document can only belong to one topic as a "cluster membership". This also emphasizes the theoretical relationship between our model and finite mixture models.

As a simple motivating example, we might imagine a corpus of movie reviews written by several bloggers. After accounting for text information explained by the rating (e.g. relating a 5-star rating to 'good plot'), the remaining heterogeneity in the movie review content could be related traits of the blogger (e.g. gender, or home city) We may be interested in using predicting traits about bloggers given their movie reviews, and also in determining how movie review content changes across these traits.

## Model Specification

Denoting the word count of a document as the vector  $x_i$ , we propose that words in a document are distributed as a multinomial with a log-link to related sentiment and cluster membership. That is:

$$x_i \sim MN(q_{ij}, m_{ij}); \quad q_{ij} = \frac{\exp(\alpha_j + y_i \phi_j + u_i \Gamma_{kj})}{\sum_{l=1}^p \exp(\alpha_l + y_i \phi_l + u_i \Gamma_{kl})} \quad (4)$$

where  $y_i$ ,  $u_i$  are the metadata and cluster membership associated with document  $i$ , and  $\phi_j$  and  $\Gamma_{kj}$  are the distortion coefficients for metadata and cluster membership, respectively. We use the subscript  $k$  to denote that each document  $x_i$  is considered a member of  $k = 1, \dots, K$  clusters, with their own distortion vectors  $\Gamma_1, \dots, \Gamma_K$

Now, the unigram collapsing can be expressed as  $x_{yk}$ , with

$$x_{yk} \sim MN(q_{yk}, m_{yk}), \quad q_{yk} = \sum_{i: y_i=y, u_i=k} \left[ \sum_{j=1}^p x_{ij} \right] \quad (5)$$

and

$$q_{yk} \sim \frac{\exp[\alpha_j + y \phi_j + u_k \Gamma_{kj}]}{\sum_{l=1}^p \exp[\alpha_l + y \phi_l + u_k \Gamma_{kl}]} \quad (6)$$

## Initializing Cluster Membership

Our focus is on predicting cluster membership  $u_i$  and the corresponding probability distortion  $\Gamma_i$ . Because each document can only be a member of one topic (unlike a traditional topic model), we want to investigate how important the cluster member initialization is to the final coefficients. For this paper, we initialize cluster membership using one of the three following methods:

1. Random Initialization
2. K-means on the word count data  $X$
3. K-means on the residual of the word count data after incorporating metadata  $y$  (That is, given predicted word count  $\hat{X}$ , clustering on  $X - \hat{X}$ )

## Estimation of Parameters via Maximum a Posteriori

The negative log likelihood of a multinomial distribution can be written as

$$\ell(\alpha, \phi, \Gamma, u) = \sum_{i=1}^N x_i^\top (\alpha + \phi v_i + u_i \Gamma_{kj}) - m_i \log \left( \sum_{j=1}^p \exp[\alpha + \phi v_i + u_i \Gamma_{kj}] \right) \quad (7)$$

Following previous literature on text regression (Taddy, 2013), We specify laplace priors and gamma hyperprior on coefficients, as well as a gamma lasso penalty on coefficients  $c(\Phi, \Gamma)$ . This procedure leads us to minimize

$$\ell(\alpha, \Phi, \Gamma, u) + \sum_{j=1}^p (\alpha_j / \sigma_\alpha)^2 + c(\Phi, \Gamma) \quad (8)$$

The basic algorithm we use to fit coefficients  $\alpha$ ,  $\phi$ ,  $\Gamma$  and cluster memberships  $u_i$  is two main steps iterated until convergence:

#### Algorithm for Cluster Membership Model with Gamma Lasso Penalty

1. Initialize  $u_i$  for  $i = 1, \dots, n$
2. Determine parameters  $\alpha, \phi, \Gamma$  by fitting a multinomial regression on  $y_i | x_i, u_i$  with a gamma lasso penalty
3. For each document  $i$ , determine new cluster  $u_i$  membership as  $\argmax_{k=1, \dots, K} [\ell(u_i | \alpha, \phi, \Gamma)]$
4. Check if current cluster assignment is different from previous cluster assignment, ( $\mathbf{u}^{(t)} = \mathbf{u}^{(t-1)}$ ). If so, return to step 2. If not, end algorithm.

By alternating between the first two steps, we aim to converge to optimal parameter estimates  $\alpha, \phi, \Gamma$  as well as optimal cluster membership  $u$ .

## Computation

As noted previously, multinomial regression enjoys the ability of being able to collapse observations across levels of metadata. This attractive property is preserved in the cluster membership model.

In step two, we can increase the speed of step two by only evaluating portions of the likelihood function relevant to  $u_i$  and  $\Gamma$  by eliminating first two terms from equation one:

$$L(u_i | \alpha, \phi, \Gamma) = \sum_{i=1}^N x_i^\top (u_i \Gamma_{kj}) - m_i \log \left( \sum_{j=1}^p \exp[\alpha + \phi v_i + u_i \Gamma_{kj}] \right) \quad (9)$$

In addition, the right hand side does not depend on  $x_i$  and can be precalculated for each cluster  $u_i$ . This will lead to an order-of-magnitude speed-up as long as the number of clusters is relatively small compared to the number of documents.



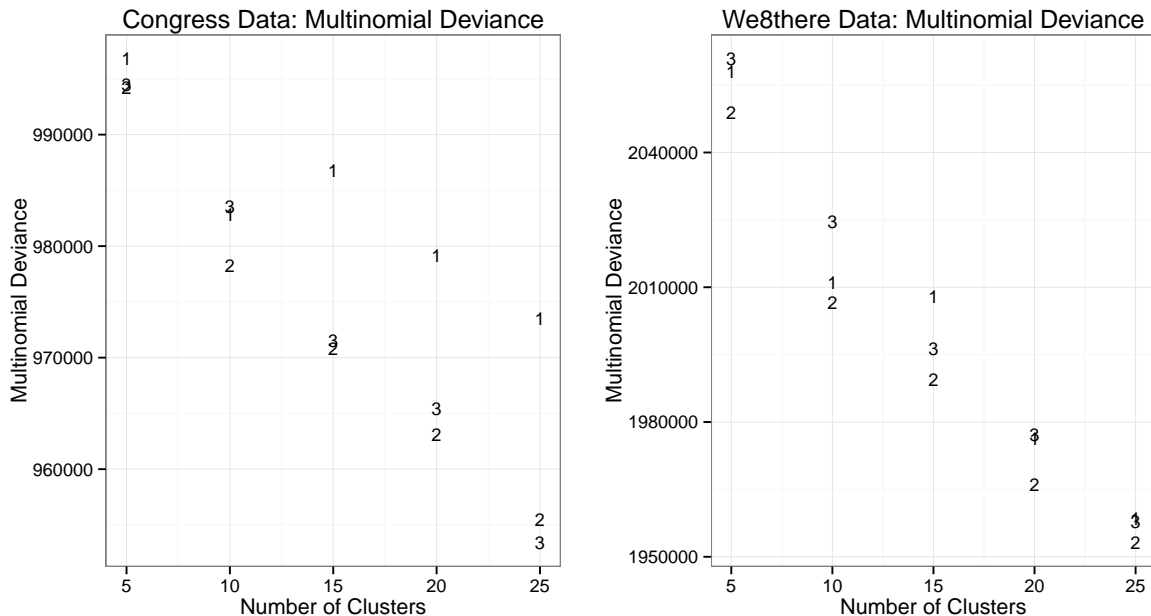
# Application and Evaluation of Algorithm

We applied the Cluster Membership model to two datasets; Congressional Speech records, most famously used to investigate media slant (Moskowitz and Shapiro, 2010) and a corpus of restaurant reviews called we8there.

## Congressional Speech

We first investigate the performance of the algorithm on the congressional speech data of. The data consists of text from 579 speeches of the members of the 109th Congress. For the analysis, party membership was regressed onto speech data. The algorithm was run for 5, 10, 15, 20, and 25 clusters, each over the 3 different initialization methods. We then report the multinomial deviance, or two times the negative log likelihood, in figure 1.

Figure 1: Multinomial Deviance from fitted model cluster membership model



1 : random cluster initialization, 2 : K Means, 3 : K means on residuals

We note that, for any given number of clusters, a better-fitting model is almost always obtained by initializing the cluster memberships on the text data, compared to assigning each document a random cluster. We also note that initializing the cluster memberships to the residuals of the text data regressed on the outcome variable (in this case, GOP party membership) usually produces a worse-fitting model than simply initializing membership on the original text data.

Previous research (Taddy, 2012) has shown that the optimal number of topics for a topic model on this dataset is around 10. This fact is not shown in our data for a couple of

reasons. First, we are possibly overfitting the data, since these models are not run under cross-validation. Second, due to its unigram design, the cluster membership model is much less flexible than a standard topic model, and would likely need many more clusters to model the same lexical variation.

## Interpretation of Topics

An essential aspect of topic modeling is determining the overall theme from the loadings vector. Previous research (Blei and Lafferty, 2009) shows that correctly-specified topic models allow for rich interpretation of themes that can change over time, as well as model relationships between themes.

Of particular importance in our model is that our extreme simplification of topic weights does not result in "meaningless" topics. If the topics produced by our method bear no relation to the topics produced by more complex topic modeling approaches, then there is little benefit of our model, regardless of computational improvements and model simplicity.

To test topic fit, we compare topics produced from our method with the topics produced by fitting a topic-only model, via MAP, with 12 topics on Gentzkow and Shapiro's Congressional Data. Fortunately, we find that topics from our model are, in many cases, similar to topics obtained by traditional methods. For example, the following table shows a "stem cell" topic found by our method, compared to a similar topic found using the topic-only model mentioned above.

Table 3: Comparison of top word loadings on a stem-cell topic

Cluster Membership	Topic Model (LDA)*
umbilic.cord.blood	pluripotent.stem.cel
cord.blood.stem	national.ad.campaign
blood.stem.cel	cel.stem.cel
adult.stem.cel	stem.cel.line

\*Results reported in Taddy (2012)

## Interpretation of results

We first evaluate our model by comparing the coefficients predicting GOP to a gamma lasso regression without any topic models. The words with the highest loading for determining party affiliation are illustrated in Table 4.

The theory behind our mixed topic model regression predicts that the topics should be able to incorporate a specific theme important to a group of individuals, leaving behind the more general predictors of party-affiliation to the regression coefficient. We can test this prediction by examining terms that are significant for a Multinomial Regression without topics, but decrease in importance once topic models are added. For a simple illustration, we choose the

Table 4: Words with highest loadings for predicting Republican-party affiliation

Cluster Membership			Multinomial Regression	
	term	loading	term	loading
1	ready.mixed.concrete	9.25	un.official	5.47
2	driver.education	7.34	people.middle.east	5.47
3	speaker.table	7.2	speaker.table	5.47
4	medic.liability.reform	6.85	term.care.insurance	5.47
5	near.retirement.age	6.42	weapon.grade.plutonium	5.46
6	weapon.grade.plutonium	6.23	national.homeownership.month	5.46
7	death.tax.repeal	5.98	nation.oil.food	5.45
8	commonly.prescribed.drug	5.72	united.nation.oil	5.45
9	national.ad.campaign	5.69	national.heritage.corridor	5.44
10	national.homeownership.month	5.37	feder.air.marshal	5.42

\*Mixed model fit with 15 topics, each topic initialized with K-means on the word count matrix

phrase "nation oil food", which predicts affiliation with the republican party. The term is strongly associated with a topic we might call "domestic issues" topics, with the following high word loading terms:

	term	loading
1	nation.oil.food	20.09
2	united.nation.oil	12.09
3	liberty.pursuit.happiness	8.11
4	life.liberty.pursuit	8.11
5	minority.women.owned	6.73
6	universal.health	6.67
7	white.care.act	6.64
8	ryan.white.care	6.6
9	universal.health.care	5.99
10	growth.job.creation	5.39
11	drilling.arctic.national	5.3
12	tax.relief.package	5.29
13	judge.john.robert	5.26
14	fre.enterprise	5.07
15	arctic.refuge	4.93

One cluster from a model fit with 20-clusters  
, each having been initialized with K-means  
on the residuals of metadata regression

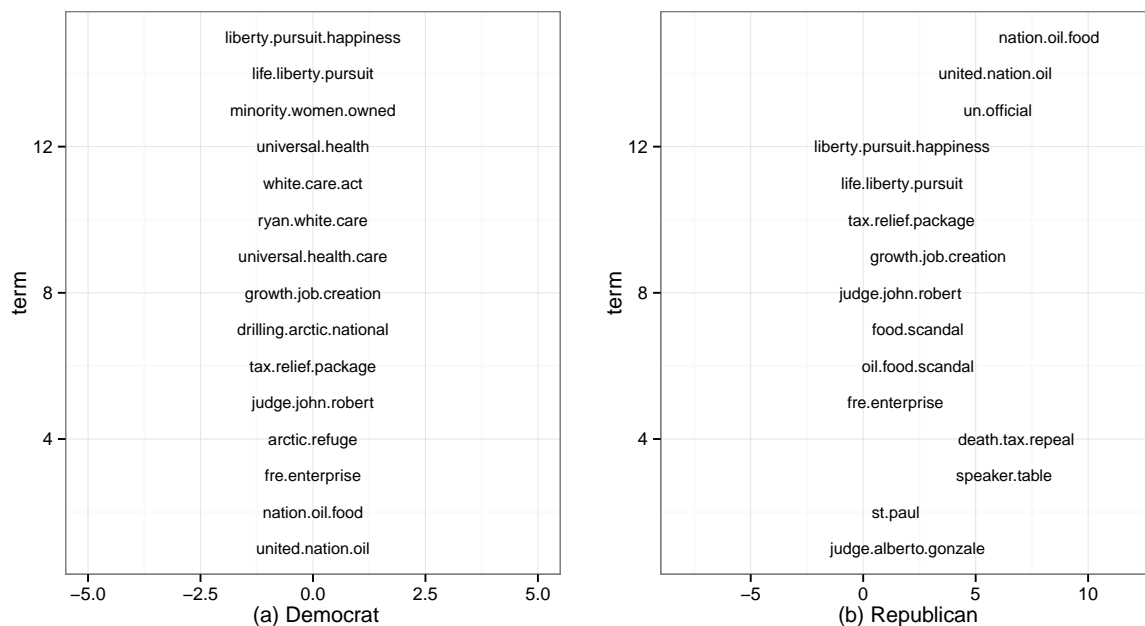
We also notice that the members of this cluster (by our simplification, each observation can only be a member of one topic) are 3 republicans and 8 democrats. The ability to group observations that may have different metadata (in this case, political party) is a benefit of our mixed regression-topic model approach. Under standard multinomial regression,

observations cannot be grouped by topics, whereas topic modeling does not immediately offer a computationally simple way to include influence on outside metadata.

## Comparison to Blei's Inverse Regression Topic Model

As mentioned in Blei (IRTM) the addition of latent topics to a model of text data with attributes is the ability to gain an intuitive concept to how metadata affects the distribution of a given topic. To demonstrate this effect, we use the graphic model presented in (Blei) to show the Democrat/GOP distortion to the "domestic issues" topic shown earlier.

Figure 2: Congress 109: Cluster word loadings with covariate distortion



A cluster from the Congressional data. On the left are, from highest to lowest, terms with the top Democratic word loadings. On the right are the words with the highest Republican word loadings. The horizontal position indicates the value of  $\phi$ , or word distortion for Republicans

## We8there Data

We briefly illustrate the results of the cluster membership model on the we8there corpus of restaurant reviews with a table of word loadings for a selected cluster, as well as a distortion graph of that topic. As with the Congress data, we note that despite the simplicity of our model, we obtain meaningful clusters. The following table illustrates a cluster we might label as "pizza diner".

As with the Congressional data, we also show the affect of metadata regression on the topic weights. However, the analysis for the food reviews data is different from the congressional

	term	loading
1	deep dish	7.76
2	italian beef	7.07
3	pizza like	6.85
4	style food	6.69
5	au jus	6.33
6	cut fri	6.16
7	just ok	6.01
8	great pizza	5.96
9	south side	5.94
10	pizza great	5.82
11	just over	5.75
12	took seat	5.72
13	golden brown	5.61
14	behind counter	5.58
15	got littl	5.52

One cluster from a model fit with 15 clusters, each having been initialized with K-means on the original text. This particular cluster has 152 reviews as "members".

data. Instead of a binary response (dummy variable for party affiliation), the covariate is the overall food review score, from 1 to 5.

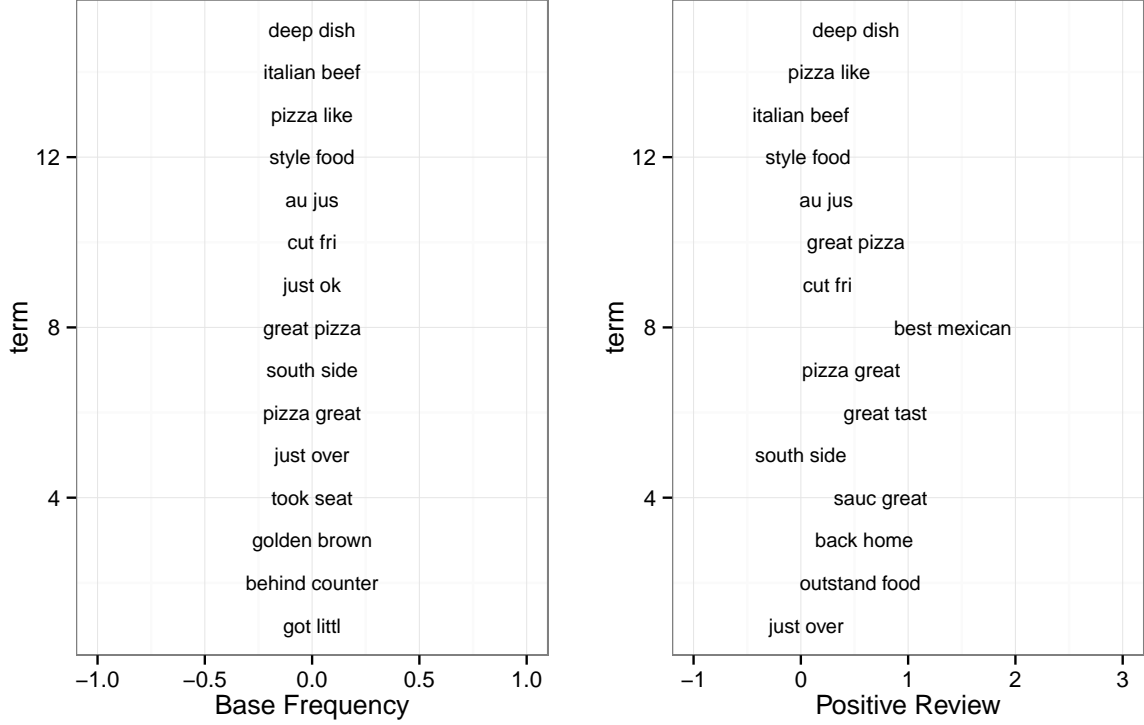
The presence of the term "best mexican" in the "pizza diner" cluster distorted by a positive review illustrates one fundamental drawback of our model. Because we do not model relationship between cluster word loadings and covariate distortion word loadings, combining the two will predict collections of terms that may never appear together in the actual data. The shortcoming is also mentioned in other work on text analysis and metadata (Rabinovich and Blei, 2014), and possible solutions will be mentioned in the next section.

## Extension

### Feature Allocations

A promising extension of the cluster membership model is the generalized "feature allocation" (Broderick 2014), where each observation can be attributed multiple features. This setup is an intermediary between our cluster membership model and a traditional topic model. The algorithm provided in this paper could be extended to incorporate a feature allocation, although the order of step 3 of the algorithm increases from linear  $\mathcal{O}_{cluster} = n_k$  to  $\mathcal{O}_{feature} = 2^{n_k}$ , where  $n_k$  refers to the number of clusters/features in the model. This increase in complexity may be reduced through changes to the algorithm, and the model would still

Figure 3: we8there: Cluster word loadings with covariate distortion



A cluster from the we8there restaurant reviews dataset. On the left are, from highest to lowest, terms with the top word loadings in this given cluster. On the right are the word loadings of the cluster altered by a positive review. The horizontal position indicates the value of  $\phi$ , or word distortion for a positive review

enjoy the ability to collapse observations across unique features.

## Cross Validation and Prediction

One drawback of our is that, in order to train cluster membership, the values of the metadata are required (see algorithm pseudocode in previous section). This hinder the ability to use the method for prediction (where, presumably, we hope to predict missing metadata from the text), and impeded our ability to perform cross validation. One possible solution for this is, after fitting a model with training data, use k-nearest neighbors or other appropriate algorithm to assign each document of the test data to a cluster. However, this approach would be difficult to combine with the feature allocation extension proposed above, since the  $n$  observations would be seperated into  $2^{n_k}$  partitions, instead of  $n_k$ .

## Modeling relationships between clusters and metadata

As noted in a previous section, one drawback of our model is the lack of relationship between clusters and metadata distortion vectors. Previous work on topic models allowed

for correlation between topics (Blei & Lafferty 2006) and has successfully applied these models to complicated datasets (Blei & Lafferty 2007). Other authors have noted this problem (Rabinovich & Blei), and possible solutions include allowing a range of metadata distortion vectors with varying relationships to each cluster or topic.

## Conclusion

In this paper, we have reviewed the theory behind topic modeling and regression on metadata in text data. We introduce an algorithm that combines the metadata regression techniques developed by Taddy (2013a, 2013b) with a simple adaptation of the classic topic model. We then applied our algorithm to two large, sparse text datasets and report the results, noting that the simple cluster membership is able to identify similar topics found in more complex LDA models. We conclude by suggesting possible extensions to our approach to assist with generalizing and comparing the cluster membership model to alternative models for topic and metadata analysis.

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