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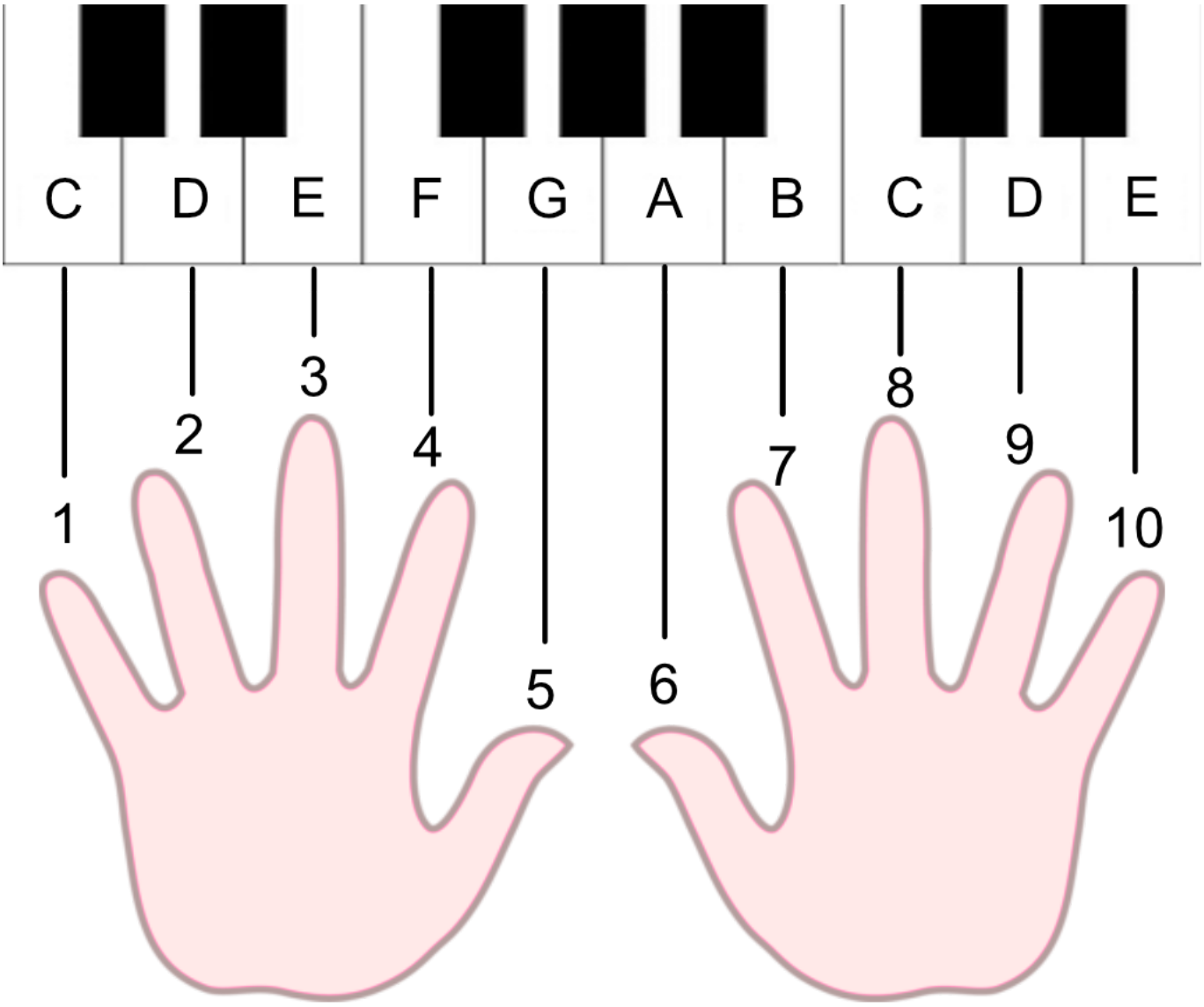


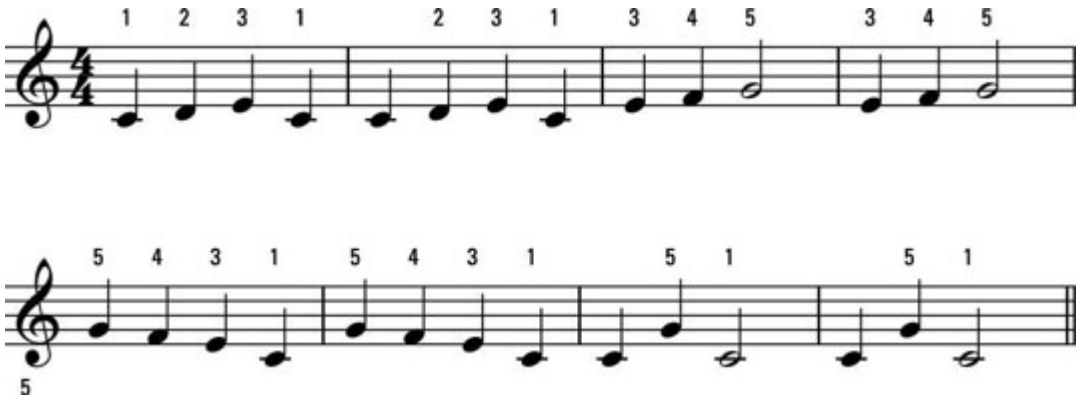
Mapping fingers to numbers and musical notes

How Playing The Piano Taught Me Math

Xiaoyun Yang Mar 28, 2016 · 7 min read

I was first introduced to numbers when I learned to play the piano at age 4. In particular, I was introduced to an array of symbols used in the encoding of music.

In the Frère Jacques example below, the numbers you see above each note in a piano sheet music corresponds to a finger on your right hand.



Frère Jacques with piano fingering notation in numerals

Piano Fingering

Adding numbers above the musical notes helps guide placement of fingers on the piano keyboard. The numbers act as some kind of training wheels to guide finger placement and hand movement across a piano keyboard in a way that's both comfortable and intuitive for the player to play the corresponding notes without missing a beat. As part of what's commonly referred to as Piano Fingering, the accepted mapping of numbers to fingers is as follows:

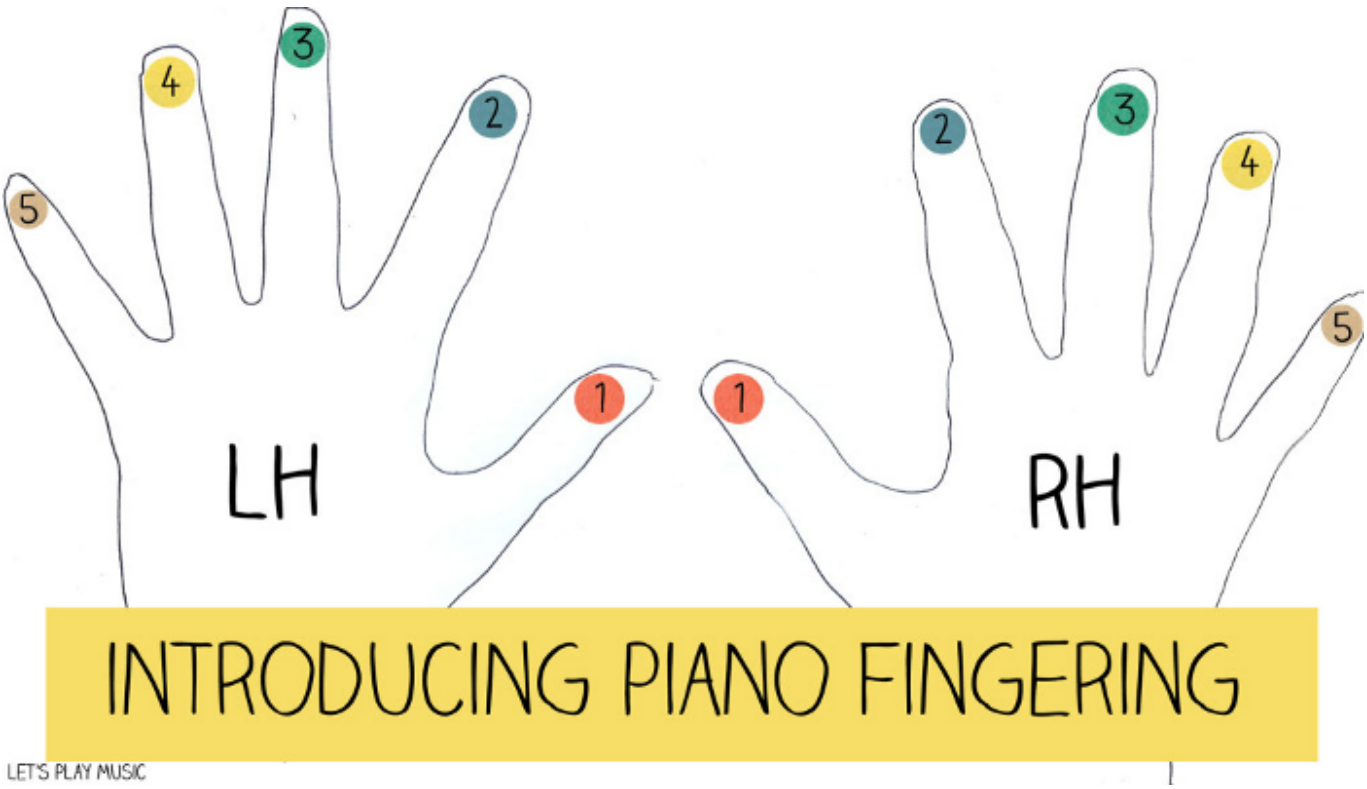
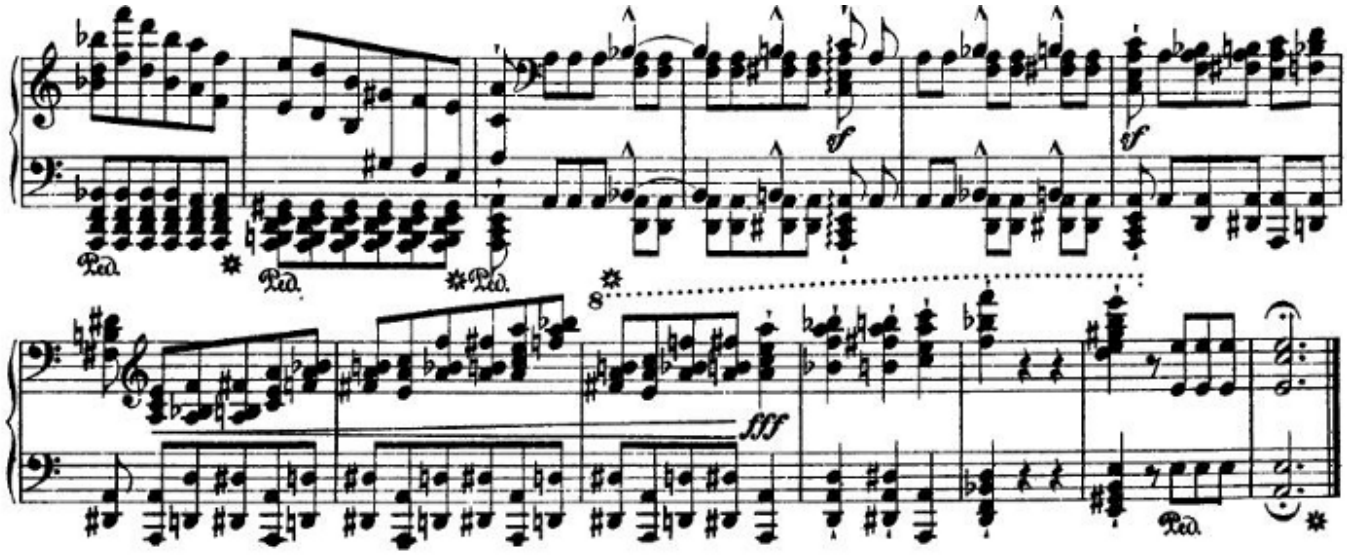


Image from LetsPlayMusic First Piano Lessons

While we only have 10 fingers, there are many more keys on the piano on which the fingers can be place. As one would expect, the mapping of fingers to notes varies. For example, you can use your right thumb to play the C note, or the D note, depending on

what's more convenient. In other words, there is no one-to-oneness between the fingers and the keys on the keyboard.

The numerals encoding this fingers-to-note mapping are often not printed on piano sheet music for more complicated musical compositions, as they tend to be distracting and "get in the way" of other symbols that need to go on the sheet music given the limited real estate available.



Transcendental Étude by Franz Liszt. Étude (means study in French) is a musical composition designed to exercise a pianist's fingers to perfect a particular musical skill.

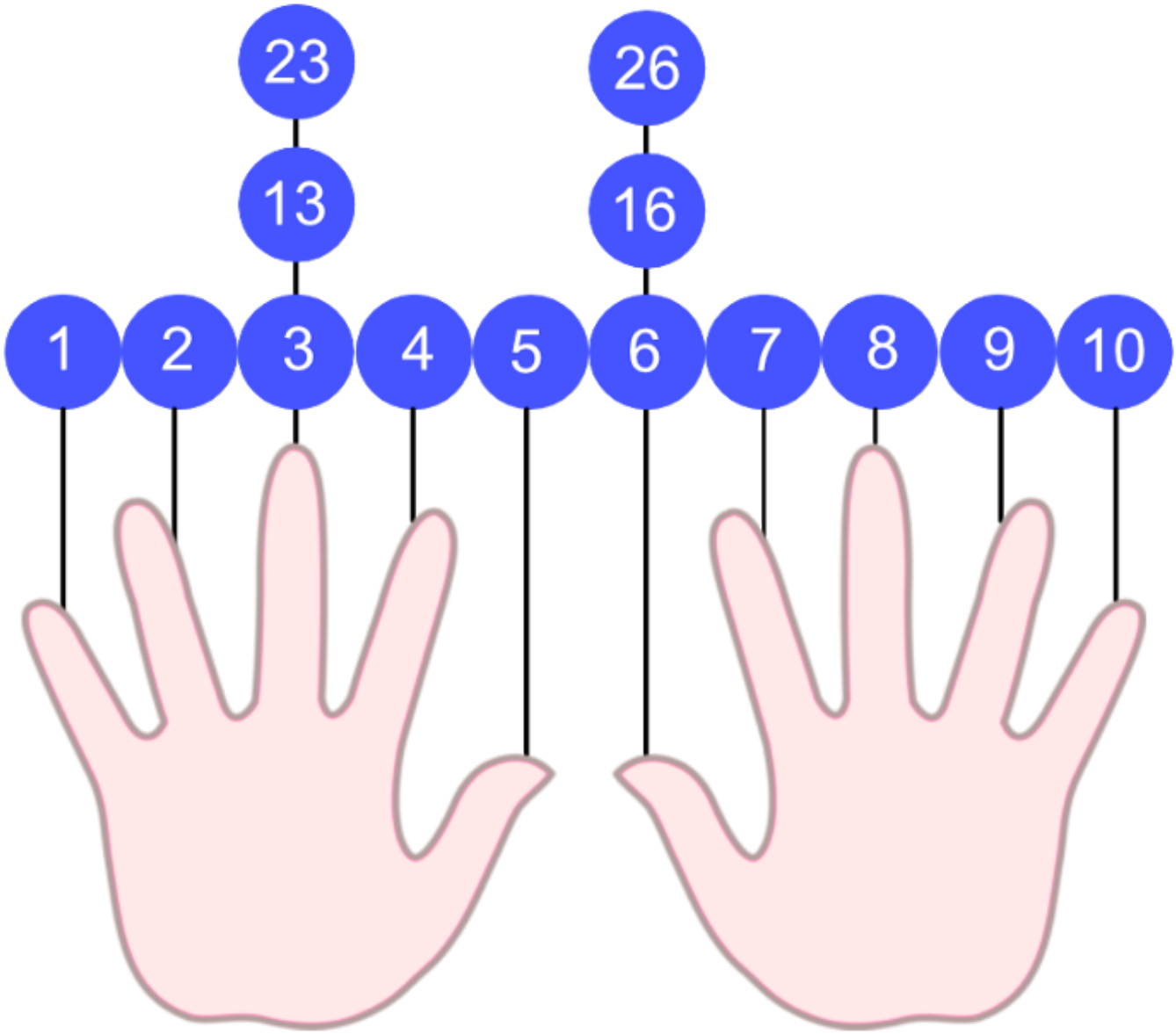
When I began my math education at age 6, I used my fingers to count, add and subtract. The notion of fingers representing things in other domains carried over from my music education. **Surely, if fingers can be mapped to various musical notations, then they can be mapped to various numbers.** Fiddling with my fingers to add and subtract numbers, I discovered a system of "number fingering" such that I can consistently use my fingers to derive results of addition and subtraction.

Number Fingering

The mapping of fingers to numbers (natural numbers) are as follows:

- The left pinky maps to 1, 11, 21, …
- The left ring finger maps to 2, 12, 22, …
- …
- The right ring finger maps to 9, 19, 29, …
- The right pinky maps to 10, 20, 30, …

To begin counting and doing arithmetic with this finger counting system, you lay your hands out in front of you, allowing them to levitate above the table like this:



In this Number Fingering system, each finger is mapped to a group of numbers

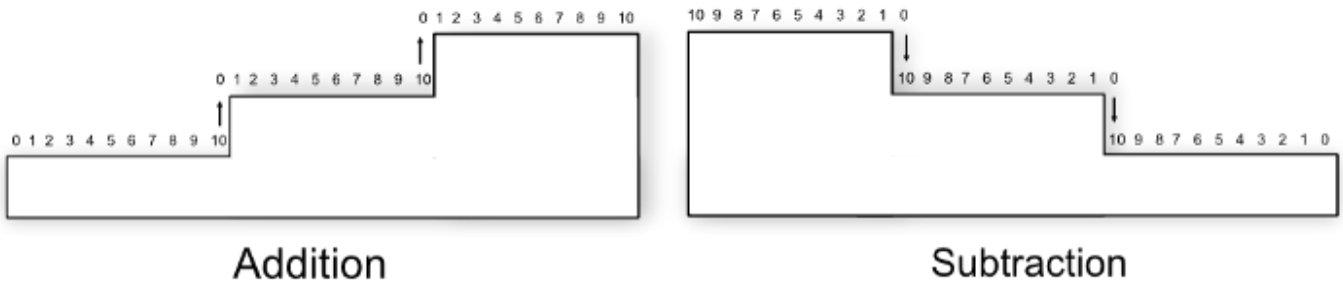
Adding is moving from left to right. To find “4 + 2”, you would start the left index finger and all fingers to the left pressed down on the table, then tap down the next two fingers to the right (left thumb, then right thumb). The right thumb represents “6”, so the answer is 6. More intuitively, you will end up with 6 fingers pressed down on the table after doing this fingering for “4 +2” so it’s obvious the final answer is 6. I liked to actually tap my fingers on the table to simulate “playing the note” on the “number keyboard”. The same sequence of finger placement is used for “14 + 2”, “24 + 2”, “34 + 2”, etc. Deriving the commutative property of addition using this finger counting system is trivial. It’s clear that if you finger “2 + 4” (start with left pinky and ring finger pressed down then tapping fingers to the right four times), you will end up at the right thumb and six fingers pressed down on the table, which is precisely where you end up if you finger “4 + 2”.

Subtracting is moving your fingers from right to left. To find “8 – 3”. Start with the right middle finger and all the fingers to the left of it pressed down on the table. After lifting 3 fingers one at a time from the right to left (starting with the right middle

finger), the numerical decoding of the next finger to the left is “5”. You will also end up with 5 fingers pressed down on the table after performing the fingering for “8 – 3”. Thus the answer is 5. You probably noticed that the fingering operation for subtraction is exactly the opposite of the fingering operation for addition. Specifically, fingers are lifted off the table from right to left for subtraction whereas fingers are pressed down from left to right for addition. The fingering for subtraction being the opposite of the fingering for addition demonstrates that subtraction is the inverse operation of addition.

Running out of fingers?

You probably have been wondering this entire time what happens if you add to a sum greater than 10. What would the fingering look like for “9 + 2”? There are no fingers to the right of the right hand pinky, but we know that 11 comes after 10 and in this finger counting system, the left hand pinky represents “11”. So you start with the right hand ring finger (and all fingers to the left of it) pressed down on the table, tap the right hand pinky, then wrap around to tap the left hand pinky. Wrapping around requires lifting all 10 fingers off the table. In effect, wrapping around from right to left hand signifies that we have rolled over to the beginning and the pattern continues just as before. The act of wrapping around from right to left hand signifies transcending to a “higher baseline” from which we count. Wrapping around to a “higher baseline level” is thus mappable to operation of adding 10 to the initial baseline. Similarly, using this fingering system to find 13–5 requires wrapping around from left to right to a “lower baseline” from which we count down, which corresponds to subtracting 10 from where the initial baseline. In electrical engineering language, this is like adding and subtracting a certain DC offset.



The inverse relationship between addition and subtraction and the DC offset applied when you “wrap around”

Getting caught counting with my fingers

Naturally, numbers are mappable to fingers. Our ancestors must have counted with their fingers because it cannot be just a coincidence that we have 10 fingers and we use the decimal system. When all my first-grade classmates were memorizing math facts like “2 + 3 = 5”, I was using this number fingering system I invented to derive the answers and in the mean time, discovering intuition and understanding about the commutative property of addition, the nature of counting, modular arithmetics and cycles, inverse

operations, and numerical encoding. I was pretty happy with my system until I was called out during class when my teacher saw me fiddling with my fingers during a classroom exercise. Despite the initial embarrassment, I never abandoned my system and relied on it to derive answers for adding, subtracting natural numbers, and eventually generalized it for negative numbers and hexadecimal system.

Many educators discourage children from using fingers for basic arithmetic operations because they think memorization of math facts frees the brain to focus on other processes. But from a learning and understanding standpoint, finger counting is a wonderful tool for introducing math concepts and acquiring intuitions about patterns in counting. Using my fingers for adding and subtracting never inhibited my math abilities (I have a masters in electrical and computer engineering). As I got more practice with adding and subtracting with fingers, muscle memory formed such that I can derive sums and differences without even thinking. Eventually, I completely stopped using my fingers as training wheels, although I still visualize fingers moving in my mind when I do addition and subtraction. Relating numbers to fingers endows them with physical meaning — I strongly believe this kind of intuition about numbers wouldn't have developed for me from pure memorization.

Many thanks to Andrew Fenner for creating the Number Fingering illustrations for this article. Check out our joint project NumberShapes, a playground for kids to discover math. Check out my other projects at http://xiaoyunyang.github.io/

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Math Music Learning