

Problem 3621: Number of Integers With Popcount-Depth Equal to K

Problem Information

Difficulty: **Hard**

Acceptance Rate: 21.31%

Paid Only: No

Tags: Math, Dynamic Programming, Bit Manipulation, Combinatorics

Problem Description

You are given two integers n and k .

For any positive integer x , define the following sequence:

$p_0 = x$ $p_{i+1} = \text{popcount}(p_i)$ for all $i \geq 0$, where $\text{popcount}(y)$ is the number of set bits (1's) in the binary representation of y .

This sequence will eventually reach the value 1.

The **popcount-depth** of x is defined as the **smallest** integer $d \geq 0$ such that $p_d = 1$.

For example, if $x = 7$ (binary representation "111"). Then, the sequence is: $7 \rightarrow 3 \rightarrow 2 \rightarrow 1$, so the popcount-depth of 7 is 3.

Your task is to determine the number of integers in the range $[1, n]$ whose popcount-depth is **exactly** equal to k .

Return the number of such integers.

Example 1:

Input: $n = 4, k = 1$

Output: 2

****Explanation:****

The following integers in the range $[1, 4]$ have popcount-depth exactly equal to 1:

x | Binary | Sequence ---|---|--- 2 | "10" | 2 -> 1 4 | "100" | 4 -> 1 Thus, the answer is 2.

****Example 2:****

****Input:**** n = 7, k = 2

****Output:**** 3

****Explanation:****

The following integers in the range $[1, 7]$ have popcount-depth exactly equal to 2:

x | Binary | Sequence ---|---|--- 3 | "11" | 3 -> 2 -> 1 5 | "101" | 5 -> 2 -> 1 6 | "110" | 6 -> 2 -> 1 Thus, the answer is 3.

****Constraints:****

$1 \leq n \leq 10^{15}$ $0 \leq k \leq 5$

Code Snippets

C++:

```
class Solution {
public:
    long long popcountDepth(long long n, int k) {

    }
};
```

Java:

```
class Solution {
    public long popcountDepth(long n, int k) {
```

```
}  
}
```

Python3:

```
class Solution:  
    def popcountDepth(self, n: int, k: int) -> int:
```