Problem 3426: Manhattan Distances of All Arrangements of Pieces

Problem Information

Difficulty: Hard
Acceptance Rate: 34.40%
Paid Only: No
Tags: Math, Combinatorics

Problem Description

You are given three integers `m`, `n`, and `k`.

There is a rectangular grid of size `m × n` containing `k` identical pieces. Return the sum of Manhattan distances between every pair of pieces over all **valid arrangements** of pieces.

A **valid arrangement** is a placement of all `k` pieces on the grid with **at most** one piece per cell.

Since the answer may be very large, return it **modulo** `109 + 7`.

The Manhattan Distance between two cells `(xi, yi)` and `(xj, yj)` is `|xi - xj| + |yi - yj|`.

Example 1:

Input: m = 2, n = 2, k = 2

Output: 8

Explanation:

The valid arrangements of pieces on the board are:

![](https://assets.leetcode.com/uploads/2024/12/25/4040example1.drawio)![](https://assets.leetcode.com/uploads/2024/12/25/untitled- diagramdrawio.png)

* In the first 4 arrangements, the Manhattan distance between the two pieces is 1. * In the last 2 arrangements, the Manhattan distance between the two pieces is 2.

Thus, the total Manhattan distance across all valid arrangements is `1 + 1 + 1 + 1 + 2 + 2 = 8`.

Example 2:

Input: m = 1, n = 4, k = 3

Output: 20

Explanation:

The valid arrangements of pieces on the board are:

![](https://assets.leetcode.com/uploads/2024/12/25/4040example2drawio.png)

* The first and last arrangements have a total Manhattan distance of `1 + 1 + 2 = 4`. * The middle two arrangements have a total Manhattan distance of `1 + 2 + 3 = 6`.

The total Manhattan distance between all pairs of pieces across all arrangements is `4 + 6 + 6 + 4 = 20`.

Constraints:

* `1 <= m, n <= 105` * `2 <= m * n <= 105` * `2 <= k <= m * n`

Code Snippets

C++:

```
class Solution {
public:
int distanceSum(int m, int n, int k) {

}
};
```

Java:

```
class Solution {
public int distanceSum(int m, int n, int k) {

}
}
```

Python3:

```
class Solution:
def distanceSum(self, m: int, n: int, k: int) -> int:
```