

# Problem 3671: Sum of Beautiful Subsequences

## Problem Information

**Difficulty:** Hard

**Acceptance Rate:** 21.57%

**Paid Only:** No

**Tags:** Array, Math, Tree, Number Theory

## Problem Description

You are given an integer array `nums` of length `n`.

For every **positive** integer `g`, we define the **beauty** of `g` as the **product** of `g` and the number of **strictly increasing** **subsequences** of `nums` whose greatest common divisor (GCD) is exactly `g`.

Return the **sum** of **beauty** values for all positive integers `g`.

Since the answer could be very large, return it modulo  $10^9 + 7$ .

**Example 1:**

**Input:** `nums = [1,2,3]`

**Output:** 10

**Explanation:**

All strictly increasing subsequences and their GCDs are:

Subsequence | GCD ---|--- [1] | 1 [2] | 2 [3] | 3 [1,2] | 1 [1,3] | 1 [2,3] | 1 [1,2,3] | 1 Calculating beauty for each GCD:

GCD | Count of subsequences | Beauty (GCD × Count) ---|---|--- 1 | 5 |  $1 \times 5 = 5$  2 | 1 |  $2 \times 1 = 2$  3 | 1 |  $3 \times 1 = 3$  Total beauty is  $5 + 2 + 3 = 10$ .

**\*\*Example 2:\*\***

**\*\*Input:\*\*** nums = [4,6]

**\*\*Output:\*\*** 12

**\*\*Explanation:\*\***

All strictly increasing subsequences and their GCDs are:

Subsequence | GCD ---|--- [4] | 4 [6] | 6 [4,6] | 2 Calculating beauty for each GCD:

GCD | Count of subsequences | Beauty (GCD × Count) ---|---|--- 2 | 1 |  $2 \times 1 = 2$  4 | 1 |  $4 \times 1 = 4$  6 | 1 |  $6 \times 1 = 6$  Total beauty is  $2 + 4 + 6 = 12$ .

**\*\*Constraints:\*\***

$1 \leq n == \text{nums.length} \leq 10^4$   $1 \leq \text{nums}[i] \leq 7 \times 10^4$

## Code Snippets

**C++:**

```
class Solution {
public:
    int totalBeauty(vector<int>& nums) {

    }
};
```

**Java:**

```
class Solution {
    public int totalBeauty(int[] nums) {

    }
}
```

**Python3:**

```
class Solution:
    def totalBeauty(self, nums: List[int]) -> int:
```