

Problem 2709: Greatest Common Divisor Traversal

Problem Information

Difficulty: Hard

Acceptance Rate: 41.75%

Paid Only: No

Tags: Array, Math, Union Find, Number Theory

Problem Description

You are given a **0-indexed** integer array `nums`, and you are allowed to **traverse** between its indices. You can traverse between index `i` and index `j`, `i != j`, if and only if $\text{gcd}(\text{nums}[i], \text{nums}[j]) > 1$, where `gcd` is the **greatest common divisor**.

Your task is to determine if for **every pair** of indices `i` and `j` in `nums`, where `i < j`, there exists a **sequence of traversals** that can take us from `i` to `j`.

Return `true` if it is possible to traverse between all such pairs of indices, `false` otherwise.

Example 1:

Input: `nums = [2,3,6]` **Output:** `true` **Explanation:** In this example, there are 3 possible pairs of indices: (0, 1), (0, 2), and (1, 2). To go from index 0 to index 1, we can use the sequence of traversals 0 -> 2 -> 1, where we move from index 0 to index 2 because $\text{gcd}(\text{nums}[0], \text{nums}[2]) = \text{gcd}(2, 6) = 2 > 1$, and then move from index 2 to index 1 because $\text{gcd}(\text{nums}[2], \text{nums}[1]) = \text{gcd}(6, 3) = 3 > 1$. To go from index 0 to index 2, we can just go directly because $\text{gcd}(\text{nums}[0], \text{nums}[2]) = \text{gcd}(2, 6) = 2 > 1$. Likewise, to go from index 1 to index 2, we can just go directly because $\text{gcd}(\text{nums}[1], \text{nums}[2]) = \text{gcd}(3, 6) = 3 > 1$.

Example 2:

Input: `nums = [3,9,5]` **Output:** `false` **Explanation:** No sequence of traversals can take us from index 0 to index 2 in this example. So, we return false.

****Example 3:****

****Input:**** nums = [4,3,12,8] ****Output:**** true ****Explanation:**** There are 6 possible pairs of indices to traverse between: (0, 1), (0, 2), (0, 3), (1, 2), (1, 3), and (2, 3). A valid sequence of traversals exists for each pair, so we return true.

****Constraints:****

*`1 <= nums.length <= 105` *`1 <= nums[i] <= 105`

Code Snippets

C++:

```
class Solution {
public:
    bool canTraverseAllPairs(vector<int>& nums) {

    }
};
```

Java:

```
class Solution {
    public boolean canTraverseAllPairs(int[] nums) {

    }
}
```

Python3:

```
class Solution:
    def canTraverseAllPairs(self, nums: List[int]) -> bool:
```