XII. Conclusion

We introduced a groundbreaking approach to the SAMPLING INVERSION with important implications to INFORMATION THEORY which underlies the foundation of ALL applications that rely on SAMPLING AND MEASUREMENTS of RANDOM PROCESSES. As such, its consequences are vast.

The approach (i.e., the **Harmonic Process State** (**HPS**) **transform**) allows to convert an arbitrary input signal (e.g., representing sampled observations from any random process) into two equivalent signals (i.e., the **HPS fast signal** and the **HPS slow signal**), which in turn are used to span a decision-making space (i.e., the $\hat{H}_{CLT}$ DMS) of very robust properties. A decision signal (i.e., the **HPS decision signal**) is generated from this decision-making space, which in turn is used to generate a forecast signal (i.e., the **HPS monitor signal**). This forecast signal approximates the original signals (i.e., **HPS fast signal**, **HPS slow signal**, and input signal) through a time series containing (zero or more) segments (i.e., **ATS segments**), each exhibiting what is referred to be as "**approximate temporal stability**".

The **HPS transform** has truly remarkable properties. First, the **HPS transform** approximates the input signal as a sequence of segments of constant values but of unknown and random duration, where any two consecutive segments are interconnected through random walks. Second, the computation of this transform is optimal in computational time under modest memory requirements while requiring the specification of few and simple parameters that operate under a stable optimality region. Third, the transformed series is an unbiased and consistent estimator of both the **HPS fast signal** and the **HPS slow signal**, which in turn are based on ***MLE*** estimation of the input signal. Fourth, the **HPS approximation** is throughout error-constrained at known and consistent confidence levels. Fifith, the **HPS approximation** describes a stable "trajectory" that can be highly compressible – depending on the presence of "approximate temporal stability" on the input signal.

Effectively, the **HPS transform** allows taking any input signal consisting on N tokens and transforming it into an equivalent stream of $\langle n \rangle$ tokens, where $\langle n \rangle$ (i.e., fractality) can be determined *w.r.t.* error and confidence levels. Moreover, we showed that for ANY random signal, we could determine within ONE sample – with known BOUNDED error at a CONSISTENT confidence – if the current observation is part of a stable known state or more importantly, if the current observation represents a departure from such known process state.[1] The approach (i.e., the **HPS transform**) generates an intermediary representation (i.e., the **HPS approximation**), which relates the presence (or lack) of "approximate temporal stability" conditions (if any) on an input signal. This is a result of vast implications. More importantly, because approximate temporal stability" is a desirable condition (i.e., stable conditions buried under noise sources) in most random systems, it is bound to be present often enough. Therefore, the intelligent identification, mining, and exploitation of this resource have very significant consequences to the handling of adaptive process control systems. Most applications that seek to build an indirect indicator of process state stand to derive very significant benefit from the implementation of the **HPS transform** ideas found in this paper. Applications that stand to benefit the most are those for which tolerance to lag-delayed knowledge acquisition about process state changes exceeds the CLT-stabilization requirements of the applicable **online HPS monitors**. Moreover, for other applications, the lag-delay could be reduced by trading off a loss of discrimination power *w.r.t.* operation within the $\hat{H}_{CLT}$ DMS.

On its most simple interpretation, this paper advocates very simple changes (i.e., $\hat{H}_{CLT}$ or approximate $\hat{H}_{CLT}$ DMS operation)[2] applicable to MOST – if not all – sampling and measurements applications. Once implemented, these changes will result in a robust, stable, and highly compressible indicator of "approximate temporal stability" that lag-tracks the process state of the random process being observed. However, this indicator is such that it provides with a trajectory that is unbiased and consistent (*w.r.t.* the process' running sampling process mean), as well as optimal *w.r.t.* known bounded (quantization) error at associated confidence levels. More generally, this paper pioneers a rigorous theoretical basis (i.e., the **HPS transform** and $\hat{H}_{CLT}$ DMS) for robust coupling of distributed adaptive process control for the class of complex random processes for which today, only ad-hoc sampling and measurements methods are currently in place.[3]

The class of applications of interest is those on which the **HPS fundamental frequencies** of a signal are the subject of interest, but that because the nature of the application domain, said frequencies are typically either concealed or unrecoverable because of the effect of inherent, natural, or introduced noise and/or variation.[4] To this end, in **Appendix B**, we illustrate next the application of the **HPS transform** to various problems.

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[1] Of course, provided that some small amount of process history is maintained, which is shown here to be of relatively negligible computational overhead but at the expense of some customizable delay.

[2] Given an input signal, $\hat{H}_{CLT}$ DMS operation entails the super-heterodyned generation (from the input signal) of fast and time-delayed slow signals over which HPS hypothesis testing is applied to determine the presence (or lack) of approximate temporal stability (if any on the input signal) and consequently, the generation of a process state forecast for the underlying input signal. The decision of whether or not to rely on CLT-stabilization for the generation of this signal set leads to the term approximate $\hat{H}_{CLT}$ DMS operation.

[3] Examples of such relevant areas are multimedia coding, internet bandwidth, router stability, internet resource management, computer performance estimation, resource scheduling, noise reduction, feature extraction, etc.

[4] For example, traveling objects due to dynamics traverse along long-term stable paths, yet are subject to minutiae of small-scale perturbations.