Exam: (SAVE to UPDATE excel #!!!) - Ans2 Cal6- Unit – Delete EXCEL Perm

Zero Coupon Bond **(ZCB)** price: $\boldsymbol{P_0 = FV * e^{-r_T T}}$;

Forward Price: $F_0^{(T_1,T_2)} = FV * e^{(r_1T_1 - r_2T_2)}$; *(NPV=0, ↲@no cost)*

Implied fwd yield *[T1, T2]*: $\boldsymbol{R_{fwd}^{(T_1,T_2)}} = \frac{T_2r_2 - T_1r_1}{T_2 - T_1}; r_{1,2}: zero\ rate\ for\ T_{1,2}$

$General\ formula: 1 + f_n = (1 + r_n)^n/(1 + r_{n-1})^{n-1}; Def: f_0 = r_0$

Long fwd pos = (short) Bond + (long) Assets *(Short position: reverse)*

Fwd. (Stock): $F_0 = (P_{S,0} - D_t * e^{-rt}) * e^{rT}; D_t: div\ @\ t < T$

$\boldsymbol{F_0 = P_{S,0} * e^{(r-q)}T}; q = div\ yield, r = risk\ free\ rate, P_{S,0}: price\ @\ 0$

Fwd. (Bond): $F_0 = (P_{B,0} - C_t * e^{-rt}) * e^{rT}$; Ct=Coupon @ t

Fwd. (Currency): $F_{0,T}^{\$/€} = S_0^{\$/€} * e^{(r_\$ - r_€)T}\ ; r_\$, r_€ \sim r_f; X \upharpoonleft\downharpoonright principal$

Fwd. (Com): $F_0 = (S_0 + PV_U) * e^rT$; *U = lump sum storage cost*

$F_0 = S_0 * e^{(r+u-y)}T$; *u=proportion of storage cost, y = convenience yield (%spot)*

Value (price) of Fwd at any time t: $f_t = (F_0 - F_t) * e^{-r(T-t)}$

Basis = Spot price – Forward price; *(Basic = 0 @ expiry, **100 bps=1%**)*

Hedge ratio *(⥮ Δy)*: $HR(N\#) = D_d = \frac{PD_m}{FD_F}(\Delta) = \frac{Port_{Value}^{Hedged}}{Future_{Unit}^{size}}$

Swap Interest Rate IRS (Plain Vanilla): NPV = 0 (No principal X-change)

(1) Float leg always price @Par @ reset: $P_{(N-1)} = F = P_{(N-2)} = \cdots$

(2) Fix rate: $c = \left(1 - \frac{1}{(1+Y_N)^N}\right)/(\Sigma 1/(1 + Y_i)^i)$; *$Y_i$: spot yield curve (APR)* (3)

Swap value = Interest paid (C)+ Principal (F+C): *discounted to PV*

Fix rate receiver *(short swap, "sold the swap")* = *long fixed bond + short floating.*

Duration (*Macaulay≅**promised CF***): $D = \left(-\frac{dP}{dy} * \frac{1+y_{APR}/k}{P}\right) = -\left(\frac{dP}{P}\right)/\left(\frac{dy}{1+\frac{y}{k}}\right)$

$$D = \Sigma\left(\frac{t}{k}\right) * \frac{PV(CF_t)}{PVTotal(\Sigma CF_i) = P_B} = \Sigma\left(\frac{t}{k}\right) * \left(\frac{1}{P_B}\right) * CF_t/\left(1 + \frac{y_{APR}}{k}\right)^t$$

(1) **D(option free)** ≤ Time 2 Maturity ∘∘∘ **D (Zero Coupon Bond)** = T to maturity

(2) Coupon rate C ↑↑ Y ↑↑ → D ↓↓ ***(Yield, y, annual % rate, APR)***

(3) Higher D → Higher price sensitivity to interest → more price Volatility

Modified Duration: $D_m = \frac{D}{1+\frac{y_{APR}}{k}} \to \frac{dP_B}{P_B} = -\boldsymbol{D_m} * \boldsymbol{dy}; dP_B = -\boldsymbol{PD_m}dy$

Dollar Duration: $D_d = D_m * P_{B(PresentV)}^{prepaid} \to dP_B = -D_d * dy$;

Port Duration: $D_M(P) = \Sigma \boldsymbol{w_i} D_{Mi}\ (w_i = P_i/P); D_d(P) = \Sigma D_{di}$

Convexity: $C_0 = \frac{d^2P}{Pdy^2} = \frac{1}{P_B} * \Sigma \frac{t(t+1)CF_t}{\left(1+\frac{y}{k}\right)^{t+2}k^2}; C_{Dollar} = C_0 * P_B$.

(C>0 ↪: underestimate y↓ (higher: Pa>Pe) & overestimate y↑ (lower Pa<Pe)

Effective Duration & Convexity (for Bond with embedded option)

$$\boldsymbol{D_{eff}} = \frac{1}{P_{init}} * \frac{P_{rate}^{fall} - P_{rate}^{rise}}{2S}, \boldsymbol{S} = amt\ of\ interest \upharpoonleft\downharpoonright (+S, -S)$$

$$\boldsymbol{C_{eff}} = \frac{1}{P_{init}} * \frac{(P_{rate}^{fall} - P_{init}) - (P_{init} - P_{rate}^{rise})}{S^2};$$

Price of bond w option: $dP_e \approx -D_{eff}Pdy + \frac{1}{2}C_{eff}P(dy)^2$

Hedging *(offset, protect Port PresentV) (δ, delta ↔**Duration**, γ gamma ↔ **Convexity**)*:

$\boldsymbol{\frac{dP}{P} = -D_M dy + \frac{1}{2}C_0(dy)^2}$; **Neutral** hedge: $\updownarrow Assets = \updownarrow Liability$

$\leftrightarrow \Sigma P_i D_m^i = 0 \leftrightarrow P_{assets} * D_m^{assets} = P_{liab} * D_m^{liab}\ (same\ for\ \gamma)$

Dollar Duration of Fwd: $D_d^f = D_{fwd} * F_0; F_0 = PV(forward_{price})$

BEB = semi-annual compounding: $y_{EAR} + 1 = \left(1 + \frac{y_{beb}}{2}\right)^2, P_{bond} = FV/\left(1 + \frac{BEB}{2}\right)^2; \boldsymbol{y_{period}^{1y} = y_{EAR}} = \left(1 + \frac{y_{APR}}{k}\right)^k - 1$;

To immunize = match both the **modified** duration & **present values** of our assets and liabilities: $(1) D_m^p = \Sigma w_i D_m^i; (2)\ PV_p = \Sigma PV_i$

Interest Rate Swap (IRS): *care on y_beb vs T period for float*

(1) D (Floating, freq=**1**) = $\frac{reset\ period}{1+y_{period}}; D_{1y}^{fl} = \frac{1}{\left(1+\frac{y_{BEB}}{2}\right)^2}; D_{0.5}^{fl} = \frac{0.5}{1+\frac{y_{beb}}{2}}$

(Bond price @ par → coupon = yield but y_beb → convert C → APR)

(2) D (Fixed) = normal = $D_M\ (\to File\ Duration\ Cal)$

Swap Dollar Duration: *<u>fixed receiver</u> (long+) float payer reset@Par (short-)*

$$\boldsymbol{D_D^S} = -\frac{dP}{dy} = D_m P = +P_{fix} * D_m^{fix} - P_{fl} * D_{eff}^{fl}; \boldsymbol{D_m^d}\left(\equiv \boldsymbol{P_{fl}^f}\right) = D_{eff}^{fl} - D_m^f$$

Delta Δ hedging: $D_M^{fund} * P^{fund} + D_M^{swap} * P^{swap} = 0$

Put Call Parity (EO): $P + S_0^{spot}e^{-pT} = C + Ke^{-rT}; p = \delta\ or\ r_{foreign}$

Replicating Portfolio (Call Option):

$$\boldsymbol{\Delta_0} = \frac{C_{1u} - C_{1d}}{S_{1u} - S_{1d}}; \boldsymbol{B_0} = e^{-rT} * (C_{1u} - \Delta_0 * S_{1u}); \boldsymbol{C_0}(V_0) = \Delta_0 S_0 + B_0$$

<u>BINOMIAL TREE</u> - Risk Neutral Pricing=***asset @Rf*** (q* ≠ q: physical risk)

$$\boldsymbol{q^*} = \frac{S_0 * e^{(r-p)h} - S_{1d}}{S_{1u} - S_{1d}} = \frac{e^{(r-p)h} - d}{u - d}; Price = E^*[e^{-r_f T} * Payoff]$$

$Stock: p = \delta_{yield}^{div}\ (jump\ u, d \to paid\ div \to ex. div\ jump\ again)\ ; Futures\ p = r_f^{domestic}, Currency\ p = r_{free}^{foreign}$

$$S_0 = E^*(e^{-rT} * S1) = \frac{q^* S_{1u}}{e^{rT}} + \frac{(1 - q^*)S_{1d}}{e^{rT}}; C_0 = \Sigma e^{-rT} * C_i * q_i^*$$

$$u = e^{\sigma\sqrt{h}}; d = \frac{1}{u}; \boldsymbol{q}\ (real) = \frac{e^{\mu h} - d}{u - d}; \mu = const\ exp\ return$$

<u>BSM</u> (EU): $d_1 \in \ln\left(\frac{S_t}{K}\right); E^* = e^{\mu h}, Var(E) = \sigma^2 h$

$\boldsymbol{C_t = \Delta_C S - B} = S_t^* N(d_1) - Ke^{-r(T-t)}N(d_2);\ P_t = -S_t^* N(-d_1) + Ke^{-r(T-t)}N(-d_2)$; **Put Hedge**: *-#N(-d1)*S + BOND = PUT; $r = r_f$;*

$\boldsymbol{S^*} = S_0 - PV_0(Div); \boldsymbol{S^*} = S_0 * e^{-\delta T}(yield); \boldsymbol{S^*} = S_0 * e^{-r_{foreign}T}(currency)$;

$S^* = F_0 * e^{-(r-r_f)T}\ (fut\ or\ fwd: r_f = foreign, r = domestic)$

$$:: d_1 = \frac{\ln\left(\frac{S_0}{K}\right) + \left(r - p + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}};\ d_2 = d_1 - \sigma\sqrt{T}; p = (\delta, r_f, r); \boldsymbol{F_0 = S_0 e^{rT}}$$

BSM: $\boldsymbol{\sigma = const, lognormall, r_f = const, future\ price} \nearrow$

BSM Delta-Hedge CPN: $\Delta_C^{Stock}(Call) = \frac{\delta C}{\delta S} = N(d_1); \Delta_P(Put) = \frac{\delta P}{\delta S} = -N(d_1); \boldsymbol{B_{position}^{Bond}} = C - \Delta_c * S = \boldsymbol{-Ke^{-r(T-t)}N(d_2)}$

The Greeks: Δ(slope) $= \frac{dP}{dS} = \begin{cases} N(d_1): for\ Call \\ -N(-d_1): for\ Put \end{cases};$

Γ(curve) $= \frac{d\Delta}{dS} = \frac{N'(d_1)}{S\sigma\sqrt{T}}, N'(x) = e^{-\frac{x^2}{2}}/\sqrt{2\pi}$); **Θ** $= \frac{dP}{dt}: \theta_{Put}^{low\ S} > 0, \theta_{Put}^{high\ S} < 0;\ \theta_{Call}^{div=0} < 0,\ \theta_{Call}^{div\uparrow} > 0$ (Hold T const, $t = time\ to\ T$)

$\rho^{Rho} = \frac{dP}{dr} = \begin{cases} KTe^{-rT}N(d_2) > 0: for\ Call \\ -KTe^{-rT}N(-d_2) < 0: for\ Put \end{cases}; K \downarrow with\ r \uparrow \to \downarrow Call$

$\nu^{Vega} = \frac{dP}{d\sigma} = S\sqrt{T}N'(d_1) > 0$; *Γ: est, 2 sides S_0 (5% = +2.5% & -2.5%)*

Delta-Gamma Hedge for CPN (fixed income)

Port pay-off: $i = -Call(S,T) + N * S + N^C * Call(S,T_1)$

$N^C = \frac{\Gamma(S,T)}{\Gamma(S,T_1)}; N = \Delta(S,T) - N^C * \Delta(S,T_1); \Gamma = \frac{d\Delta}{dS}$; *Bond position = - i*

Black's Model for Futures /Option EU: lognormal + cont trading

$C_0 = e^{-rT}[F_0N(d_1) - KN(d_2)]; P_0 = e^{-rT}[-F_0N(-d_1) + KN(-d_2)];$

$$d1 = \frac{\ln\left(\frac{F_0}{K}\right) + \left(\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}; d_2 = d_1 - \sigma\sqrt{T}; r = R_f$$

EXOTIC OPTIONS: *(d1 & d2 follow BSM)*

1. **Binary** Option *(Cash/Assets or Nothing)* Pay $1 or Stock (Asset)

$Cash\boldsymbol{Call} = e^{-r(T-t)}N(d_2); Cash\boldsymbol{Put} = e^{-r(T-t)}N(-d_2);$

$Asset\boldsymbol{Call} = Se^{-\delta(T-t)}N(d_1); Asset\boldsymbol{Put} = e^{-\delta(T-t)}N(-d_1);$

[Cash: Pay $1]/ [Asset: Pay stock price S] if S > (<) K & 0 otherwise

2. **Asian** Option *(Average price over some period*

$$Arith(Avg): A(T) = \left(\frac{1}{N}\right)\Sigma S_{ih}; Geo(Avg): G(T) = (S_h * \ldots * S_{Nh})^{\frac{1}{N}}$$

3. **Barrier** Option *(under or over, increase or decrease over time)*

Knock-out (of exist): Down & Out (↓<B), Up & Out (↑>B)

Knock-in (of exist): Down & In (↓<B), Up & In (↑>B)

Rebate: fixed payment if Down Rebate (↓<B) & Up Rebate (↑>B)

4. **Lookback** Option *(expensive, cont' looks & lognormal)*

$Floating: Call = S_T - S_{min}; Put = S_{max} - S_T$

$Fixed: Call = \max(S_{max} - K, 0); Put = \max(K - S_{max}, 0);$

5. **Exchange** Option: $Payoff = \max(0, S_T - N_T);$

$\boldsymbol{C_0 = S_0e^{-\delta_S T}N(d_1) - N_0e^{-\delta_N T}N(d_2)}: d_1 = (\ln\left(\frac{S_0e^{-\delta_S T}}{N_0e^{-\delta_N T}}\right) + \frac{1}{2}\sigma^2 T) * \frac{1}{\sigma\sqrt{T}}; d_2 = d_1 - \sigma\sqrt{T}; \sigma = \sqrt{(\sigma_S^2 + \sigma_N^2 - 2\rho\sigma_S\sigma_N)}$

6. **GAP** Option *(pays S-K1 when S>K2)*: $d_2 = d_1 - \sigma\sqrt{T};$

$$C = S_0e^{-\delta T}N(d_1) - K_1e^{-rT}N(d_2); d_1 = \frac{\ln\left(\frac{S_0}{K_2}\right) + \left(r - \delta + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

Monte Carlo *(or EO only)* on Risk-Neutral Tree: RAND() = [0,1] (RAND()**>q*** -> probability **1-q***): $\widehat{V_0} = avgPV(V_i) = \frac{1}{N}\Sigma e^{-rT}V(S_1^i)$

MC with lognormality *(Good for path dependent security)*

$S_{t+h} = S_te^{\left(r-\frac{\sigma^2}{2}\right)h + \sigma * \epsilon_t * \sqrt{h}}$; $\epsilon_t \sim N(0,1)$: = NORMINV(RAND(),0,1)

Implied: $E^*\left(\frac{S_{t+h}}{S_t}\right) = e^{rh}, and\ \sigma^2 converge\ to\ annu\ Var(\log\left(\frac{S_{t+h}}{S_h}\right)$

MC with multi-factor: $Payoff_T = \max\left(\frac{S_T}{S_0}; \frac{N_T}{N_0}\right)$, stock *(S, N) same* ↑

$\widehat{\epsilon_t}: std\ norm, uncorrelated\ \epsilon_{1,t}\ then: \epsilon_{2,t} = \rho\epsilon_{1t} + \widehat{\epsilon_t}\sqrt{1-\rho^2};\ (\bar{0}, 1)$

$\widehat{V_0} = avgPV(V_i) = \frac{1}{N}\Sigma V^i; V^i = e^{-rT}Max\left(\frac{S_T^i}{S_0}\right), N_T^i/N_0);$

Interest rate *(eg: 2 year)*: $V_0 = p_1 * \frac{V_1}{(1+r_0)(1+r_1)} + p_2 * \ldots +$

CIR Model for continuous interest rate: $dz \sim (N(0,1) \approx \epsilon_t)\sqrt{\Delta t}$

$dr = \alpha(b - r)dt + \sigma\sqrt{r}dz; r_{(t+\Delta t)} = r_t + a(r,t)\Delta t + \sigma_{r,t}\epsilon_t\sqrt{\Delta t}$

Issuer: $\boldsymbol{V_{Callable}^{Bond}} = V_{NonCall} - V_{CallOp}; \boldsymbol{V_{Putable}^{Bond}} = V_{Bond} + \boldsymbol{V_{PutO}}$

Puttable Callable: $V^i = V_{Put}^i + V_{no-option}^i - V_{Call}^i + Coupon^i$

Valuing Caps as Call option on rates: $Payoff = Principle * Period * \max(R_t - R_x, 0): where\ R_t = rate\ @t,\ R_x = cap\ rate,$

Credit (Yield) spread ≈ fair CDS$_{prem}$: $CS = Y_{Risky}^T - Y_{Rf}^T = \frac{1}{T}\ln\left[1 - e^{r_fT} * Put\left(\frac{V_0}{F}, 1, r_f, T, \sigma\right)\right]$ *(Merton Model), YTM # E, ref: P(V0,F,r,T,σ)*

CDO *(paid over T = **swap**, up-front =**option**)*: $CDO_{premium}^{>0} = P_{R_f} - P_{Risky}$

Cash settlement = Face Value – Market Value @ Trigger

Risky Bond Price: $P = \frac{(1-d)^T}{(1+r)^T} + \Sigma\left\{(1-d)^{i-1} * \frac{[dg(1+c)+(1-d)c]}{(1+r)^i}\right\}; d = default\ rate, g = recovery, c = coupon, r = expected\ return\ @\frac{T}{k}$

YTM: $P = \frac{1}{(1+y)^T} + \frac{\Sigma c}{(1+y)^i}$;For **i=1**: $P = \frac{R*d+FV(1-d)}{1+r}; R = g(1+c)$

Merton Model for Corp Debt ***(FV=F**, T)*: V[A]=E+D, log-normally, all ZCB

$V_T = V_0e^{\left(\mu-\frac{\sigma^2}{2}\right)T + \epsilon\sigma\sqrt{T}}$; $\boldsymbol{E_0} = V_0N(d_1) - Fe^{-rT}N(d_2) = BSM\boldsymbol{C}(V_0, F)$

$$d1 = \frac{\ln\left(\frac{V_0}{F}\right) + \left(r_f + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}; d_2 = d_1 - \sigma_A\sqrt{T}; \sigma_E * E_0 = \sigma_A * V_0N(d_1)$$

$\boldsymbol{D_0 = V_0 - E_0} = Fe^{-r_fT} - Put(V_0, F): with\ Put\ Call\ Parity$

$D_0^{Sen} = V_0 - BSM\boldsymbol{C}(V_0, F_S); D_0^{Jun} = BSM\boldsymbol{C}(V_0, F_S) - BSM\boldsymbol{C}(V_0, F_S + F_J)$

MBS-↕ in Equity: $dE = d(STM) + d(LTM) - d(Loans) + d(Swap)$

$\overline{A \cup B}(neither\ A\ or\ B) = 100 - (A + B + A \cap B)$

∴·∵· **Moneyness** = K/S (Strike/Stock); Implied Volatility = **BSM @ Solver**.EUO: **call = put** with any **Strike = Forward** Price; **RATE** (#CF period, coupon/period, -PV, FV); **Tree Bino: 2σ** ≅ e^2σ -1; **Hedge Ratio for IRS** = Dollar Duration; **// shift** upward: Δy = -1% (note); **fixed-for-floating** = floating payor; $D_{IO} < D_{PO}, D_{PSA}^{300\%} < D_{PSA}^{200\%}$; N(d1>0)>0.5; $N(-d) + N(d) = 1;$ $N() = \boldsymbol{normSdist;}$

Option type *(long=+, short=-)*: **Protective Put** = St + Put (St,K)
Covered Call = St - Call (St,K) **Bear Spread**=Put(K2>K1)-Put(K1)
Bull Spread=Call(K1)-Call (K2>K1) **Butterfly Spread** = Call(K0) -2Call(K1) + Call (K2), where: K1 = (K0+K2)/2
Straddle = Call(K) + Put(K) (bet on high σ)
Strangle = Call (K1>K0) + Put (K0) (bet on large movement)