



# 15.415x Foundations of Modern Finance

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## Lecture 10: Capital Budgeting I

## Key concepts

- NPV rule
- Cash flows from capital investments
- Discount rates and project interaction
- Alternative capital budgeting rules
- Additional considerations

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## NPV rule

A firm's business involves capital investments (capital budgeting), e.g., the acquisition of real assets. The objective is to increase the firm's current market value. Decision reduces to valuing real assets, i.e., their cash flows.

Let the expected cash flow of an investment (a project) be:

$$\{CF_0, CF_1, \dots, CF_T\}$$

Its current market value is:

$$NPV = CF_0 + \frac{CF_1}{(1+r_1)} + \frac{CF_2}{(1+r_2)^2} + \dots + \frac{CF_T}{(1+r_T)^T}$$

This is the increase in the firm's current market value by the project.

## NPV rule

Investment Criteria:

- For a single project, take it if and only if its NPV is positive.
- For many independent projects, take all those with positive NPV.
- For **mutually exclusive** projects, take the one with positive and highest NPV.

In order to compute the NPV of a project, we need to analyze

1. Cash flows,     **expected cash flow**.
2. Discount rates,
3. Strategic options.

We will focus mostly on 1 here and return to 2 and 3 later in Part II.

# Cash flow calculations

## Important Points:

1. Use cash flows, not accounting earnings.  
Earnings reported in accounting statements follow certain accounting rules which may not adhere to the actual timing of the cash flow.
2. Use after-tax cash flows.
3. Use cash flows attributable to the project (compare firm value with and without the project):
  - Use incremental cash flows.
  - Forget sunk costs: bygones are bygones.  
sunk costs are cash flows associated with previous decisions, not the current decision, which is about the current and future cash flows.
  - Include investment in working capital ~~as capital expenditure.~~  
Working Capital WC = Inventory + A/R - A/P
  - Include opportunity costs of using existing facilities.

## Cash flow calculations

In what follows, all cash flows are attributable to the project.

We can write a project's CF as follows:

$$CF = [\text{Project Cash Inflows}] - [\text{Project Cash Outflows}]$$

$$\begin{aligned} &= [\text{Operating Revenues}] - [\text{Operating Expenses without depreciation}] \\ &\quad - [\text{Taxes}] \\ &\quad - [\text{Capital Expenditures}] \end{aligned}$$

not complete! missing working capital

## Cash flow calculations

Defining operating profit by:

Operating Profit = Operating Revenues

– Operating Expenses w/o Depreciation

Accounting Profit = Operating Profit - Accounting Depreciation

Let  $\tau$  be the “effective” tax rate. The income taxes are:

Taxes = tax\_rate \* Accounting Profit

[Taxes] =  $(\tau)[\text{Operating Profit}] - (\tau)[\text{Depreciation}]$

comes from the tax credit the firm can claim from its capital investments.

Accounting depreciation affects CF because it reduces firm's tax bill.

CF =  $(1 - \tau)[\text{Operating Profit}] - [\text{Capital Expenditures}]$

+  $(\tau)[\text{Depreciation}]$

accounting depreciation itself is not a cash flow, but it does affect the project's after tax cash flow because it can be used to reduce the firm's tax bill

The term depreciation refers to an accounting method used to allocate the cost of a tangible or physical asset over its useful life or life expectancy.

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## Cash flow calculations

Capital expenditures (CAPEX) are major purchases a company makes that are designed to be used over the long term.  
Operating expenses (OPEX) are the day-to-day expenses a company incurs to keep its business operational.

**Example. Accounting earnings vs. cash flows.** A machine purchased for \$1,000,000 with a life of 10 years generates annual revenue of \$300,000 and operating expense of \$100,000. Assume that the machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%.

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	-1,000,000 CapEx
1	300,000-100,000-100,000 =100,000	(1-0.4)(100,000) =60,000	(1-0.4)(300,000-100,000)+ 40,000=160,000
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

the accounting earnings did not accurately reflect the timing of the project's cash flow.

For evaluation purposes, we cannot arbitrarily shift cash flows over time (depreciation) without proper discounting.

**Accounting earnings may not accurately reflect the actual CF timing.**

no matter what discount rate we use, the discount value for all accounting earnings will always be positive for actual cash flow, high enough of a cost of capital will make the NPV of the project negative

# Cash flow calculations

**Example.** Use after-tax cash flows. Consider the following project (the cash flow is in thousands of dollars and tax rate is 50%):

$$100 = 200 \text{ (accounting income)} * 0.5$$

$$100 = \text{tax} - \text{tax credit} = 300 \text{ (profit)} * 0.5 - 100 \text{ (depreciation)} * 0.5$$

Year	0	1	2	3	4	5
Investment	500					
Operating CF	operating profit	0	100	300	300	300
Depreciation		100	100	100	100	100
Income		-100	0	200	200	200
accounting income = profit - depreciation		-50	0	100	100	100
Tax		50	100	200	200	200
After-tax CF	discount at 10%	-500	50	100	200	200
PV at 10%		-500	45.45	82.64	150.26	136.60
						124.18

cost of capital for this project is 10%,

this tax credit can be used to reduce the tax bill on other incomes, \$1 saved is \$1 earned

NPV = +39.13.

accounting income is not actual income because depreciation is not itself a cash flow.

depreciation takes effect in cash flow through tax credit

$$200 = 300 \text{ (real profit)} - 100 \text{ (real tax)}$$

$$200 = 300 * (1 - 0.5) + 0.5 * 100 \text{ (depreciation)}$$

# Cash flow calculations

**Example. Inventory.** You run a chain of shoe stores.

- This quarter, you buy 1,000,000 pairs of shoes at a price of \$30.00 each.
- For the next two quarters, you sell 500,000 pairs each quarter for \$60.00 each.  $\text{accounting after tax profit} + (-1) * (\Delta \text{Inventory}) = \text{cash flow}$
- The effective corporate tax rate is 40%.

In million dollars, your cash flows are: a change in inventory implies an opposite cash flow of the same amount.

accounting	after tax
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Date	After Tax Profit	Inventory	Cash Flow
0	0	(1)(30)=30 -30	-30
1	$(0.5)(60-30)(1-0.4)=9$	$(0.5)(30)=15$ -15	$(0.5)(60)-(0.5)(60-30)(0.4)=24$ <small>operating profit * tax rate</small>
2	$(0.5)(60-30)(1-0.4)=9$	0	$(0.5)(60)-(0.5)(60-30)(0.4)=24$ <small><math>24=9+15</math></small>

After Tax Profit is not the actual after tax cash flow, the cost of shoes

$24=9+15$

Notice: are paid in quarter 0 in the form of inventory, not in quarter 1 and 2.

$$\text{Cash Flow} = \text{Profit (after tax)} - \text{Change in Inventory}$$

## Cash flow calculations

inventory in previous slide is a special case of working capital

Typically, there are timing differences between the accounting measure of earnings (Sales - Cost of Goods Sold) and cash flows.

$$\text{Working Capital (WC)} = \text{Inventory} + \frac{\text{Accounts Receivable}}{\text{Accounts Payable}}$$

**Changes in Working Capital** changes in working capital implied actual cash flows.

- Inventory: Cost of goods sold includes only the cost of items sold. When inventory is rising, the cost of goods sold understates cash outflows. When inventory is falling, cost of goods sold overstates cash outflows.
- Accounts Receivable (A/R): Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.  
which goes into accounts receivable.  
which corresponds to decreases in accounts receivable.
- Accounts Payable (A/P) -- conceptually the reverse of A/R.

$$\begin{aligned} \text{CF} = & (1-\tau)[\text{Operating Profits}] + (\tau)[\text{Depreciation}] \\ & - [\text{Capital Expenditures}] - \Delta[\text{Working Capital}] \end{aligned}$$

## Putting all together

**Example.** MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

Salvage value is the book value of an asset after all depreciation has been fully expensed.

- TW were developed at an R&D cost of \$1M over past 3 years.
- New machine to produce TW would cost \$2M.
- New machine lasts for 15 years, with salvage value of \$50,000.
- New machine can be depreciated linearly to \$0 over 10 years.
- TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of \$30,000 (regardless of how much it is used). no additional cost
- Operating cost: \$40,000 per year.
- Sales: \$400,000, but cannibalization would lead existing sales of regular widgets to decrease by \$20,000.
- Working Capital (WC): \$250,000 needed over the life of the project.
- Tax rate: 34%.
- Opportunity cost of capital: 10%.

Should MSW go ahead to produce TW?

## Putting all together

- R&D expense is sunk cost. Bygones are bygones. Unless the R&D cost are part of the CF after year 0.
- Initial investment includes capital expenditure.
- Depreciation is  $\$2M/10 = \$0.2M$  for first 10 years. for tax purposes.  
salvage value is an income,
- Salvage value is fully taxable since the book value at the end of year 10 is \$0 (the machine cost has been fully depreciated).
- Project should not be charged for painting-machine time.
- Project should be charged for cannibalization of regular widget sales.
- Working capital is an initial investment, but recovered at the end.

The cash flows (in thousand dollars) are calculated as follows:

Year	$\Delta[\text{WorkingCapital}]$			Cash Flow
0	<small>sales expenses cannibalization</small>	<small>CapEx</small>	<small>working capital</small>	$-(2,000 + 250) = -2,250.0$
1-10	$(400 - 40 - 20)(1 - 0.34) + (200)(0.34)$	<small>tax credit</small>		= 292.4
11-14		$(400 - 40 - 20)(1 - 0.34)$		= 224.4
15		<small>salvage value</small>		$224.4 + (50)(1 - 0.34) + 250 = 507.4$

$\Rightarrow \text{NPV} = -\$57,617$

$\text{CF} = (1 - \tau)[\text{Operating Profits}] + (\tau)[\text{Depreciation}]$

$- [\text{Capital Expenditures}] - \Delta[\text{Working Capital}]$

working capital recovered

Even though working capital will net to zero by the end of the project, their different timing implies that their Net Present Value (NPV) will in general not be zero.

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## Discount rates

**So far, we have shown that:**

- A project's discount rate (required rate of return or cost of capital) is the expected rate of return demanded by investors **for the project**.  
and for similar traded assets
- Discount rate(s) in general depend on the timing and risk of the cash flow(s).
- Discount rate is usually different for different projects.
- It is in general incorrect to use a company-wide “cost of capital” to discount cash flows of all projects.

**What is the required rate of return on a project?**

- Simple case: a single discount rate used for all cash flows of a project.
  - General case: different discount rates for different cash flows.
    - Different timing and risk. **safe cash flows \$1 in different years should be discounted at different interest rates if the term structure of interest rates is not flat**
- We will return to the determination of discount rates after developing more asset pricing tools. **in this lecture, we will use a single discount rate**

# Project interaction

Often we have to decide on more than one project.

- For mutually independent projects, apply NPV rule to each project.
- For projects dependent of each other (e.g., **mutually exclusive**), we have to compare their NPVs.

**OR sequential: project B is contingent on project A**

The decision is when to bring the new product to the market. If we view each choice as the project itself, then the timing decision is between a set of mutually exclusive projects.

**Example.** Potential demand for your product is projected to increase over time.

If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity **cost of capital is 10%**. Denoting by FPV the project's NPV at the time of introduction, we have:

Year to start	Future PV	% Change in FPV	NPV
1	100		91
2	120	20	99
3	138	15	104
4	149	8	102

waiting is  
desirable only if  
it yields a  
return higher  
than the cost of  
capital.  
 $20, 15 > 10$   
 $8 < 10$

FPV: future present value, NPV for each launching date at the time of launch

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## Alternatives to NPV

In practice, investment rules other than NPV are also used:

- Payback Period,
- Internal Rate of Return (IRR),
- Profitability Index (PI) ...

Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.

These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.

The bottom line:

The NPV rule dominates the alternative rules.

## Payback period

**Payback period** is the minimum length of time  $s$  such that the sum of net cash flows from a project becomes positive:

$$CF_1 + CF_2 + \cdots + CF_s \geq -CF_0 = I_0$$

### Decision Criterion Using Payback Period

- For independent projects: Accept if  $s$  is less than or equal to some fixed threshold  $t^*$ :  $s \leq t^*$ .
- For mutually exclusive projects: Among all the projects having  $s \leq t^*$ , accept the one that has the minimum payback period.

## Payback period

**Example.** Let  $t^* = 3$ . Consider the two independent projects, 1 and 2, with the following cash flows (in thousand dollars):

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$	$s$
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	10	80	5	10	10	3

Decision: Accept Project 2.

## Payback period

- Payback period rule ignores cash flows after the payback period;
- It ignores discounting.

**Example (cont'd).** Suppose that the appropriate discount rate is a constant 10% per period. Then,

$$NPV_1 = 39,315 \quad \text{and} \quad NPV_2 = -7,270$$

But we accepted project 2 and not project 1!

Taking into account appropriate discounting, we have the **discounted payback period**, which is the minimum  $s$  so that:

$$\frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_s}{(1+r)^s} \geq -CF_0$$

where  $r$  is the discount rate (cost of capital). (It still ignores the cash flows after the discounted payback period.)

## Internal rate of return (IRR)

A project's **internal rate of return (IRR)** is the number that satisfies:

$$0 = CF_0 + \frac{CF_1}{(1 + IRR)} + \frac{CF_2}{(1 + IRR)^2} + \cdots + \frac{CF_t}{(1 + IRR)^t}$$

### Decision Criteria Using IRR

- For independent projects: Accept a project if its IRR is greater than some fixed  $IRR^*$ , the threshold rate/hurdle rate.
- For mutually exclusive projects: Among the projects having IRR's greater than  $IRR^*$ , accept one with the highest IRR.

# Internal rate of return (IRR)

**Example (IRR).** Consider the following mutually exclusive projects:

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$
Project 1	-100	20	40	30	10	40	60
Project 2	-100	10	10	80	5	10	10

Then,  $IRR_1 = 21\%$  and  $IRR_2 = 7\%$ .

IRR rule leads to the same decisions as NPV if:

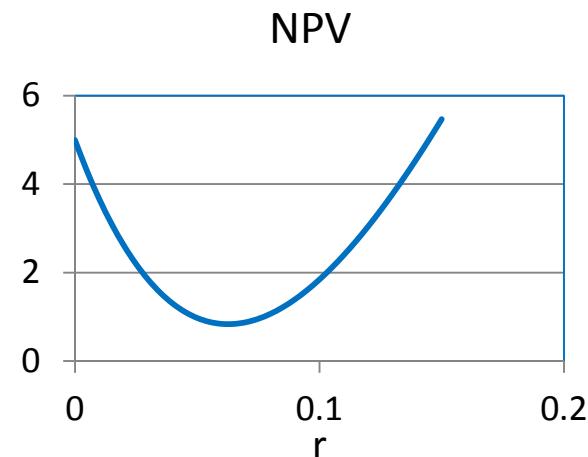
- Cash outflow occurs only at time 0,
- Only one project is under consideration, reason in page 27
  - From the definition of IRR,
- Opportunity cost of capital is the same for all periods, It is used to discount cash flows for all different periods
- Threshold rate is set equal to opportunity cost of capital.

## Internal rate of return (IRR)

An IRR is given by the point where the cash flow's discounted value curve crosses the x-axis.

### ■ Non-existence of IRR

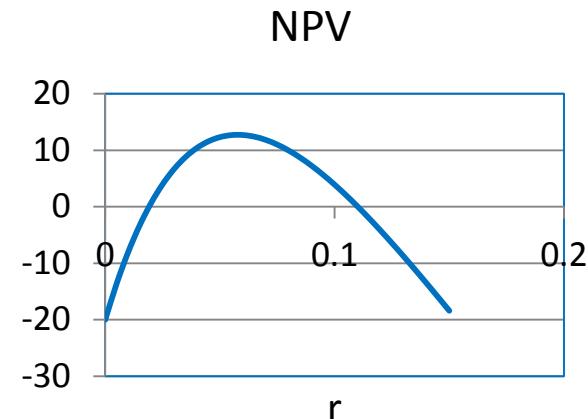
Period	0	3	6
CF	105	-250	150



### ■ Multiple IRR's

Period	0	3	6	9	12
CF	-200	100	300	280	-500

project's cash flow changes side twice, violating one of the conditions needed for IRR to work.



## Internal rate of return (IRR)

Project ranking using IRR for mutually exclusive projects:

- Projects of different scales hurdle rate is 10% (the cost of capital for these projects)

$CF_t$	$CF_0$	$CF_1$	$IRR$	$NPV \text{ at } 10\%$
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27

- Project 1 has higher IRR.
- Project 2 has higher NPV.
- Project 2 has a larger scale than Project 1. Even though it has a smaller IRR, its larger scale leads to a larger NPV.

IRR, being a rate itself, does not capture the scale of the project.

## Profitability index (PI)

Profitability index (PI) is the ratio of the present value of future cash flows and the initial cost of a project:

$$PI = \frac{PV}{-CF_0} = \frac{PV}{I_0}$$

### Decision criterion using PI

- For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule).
- For mutually exclusive projects: Among the projects with PI greater than one, accept the one with the highest PI.

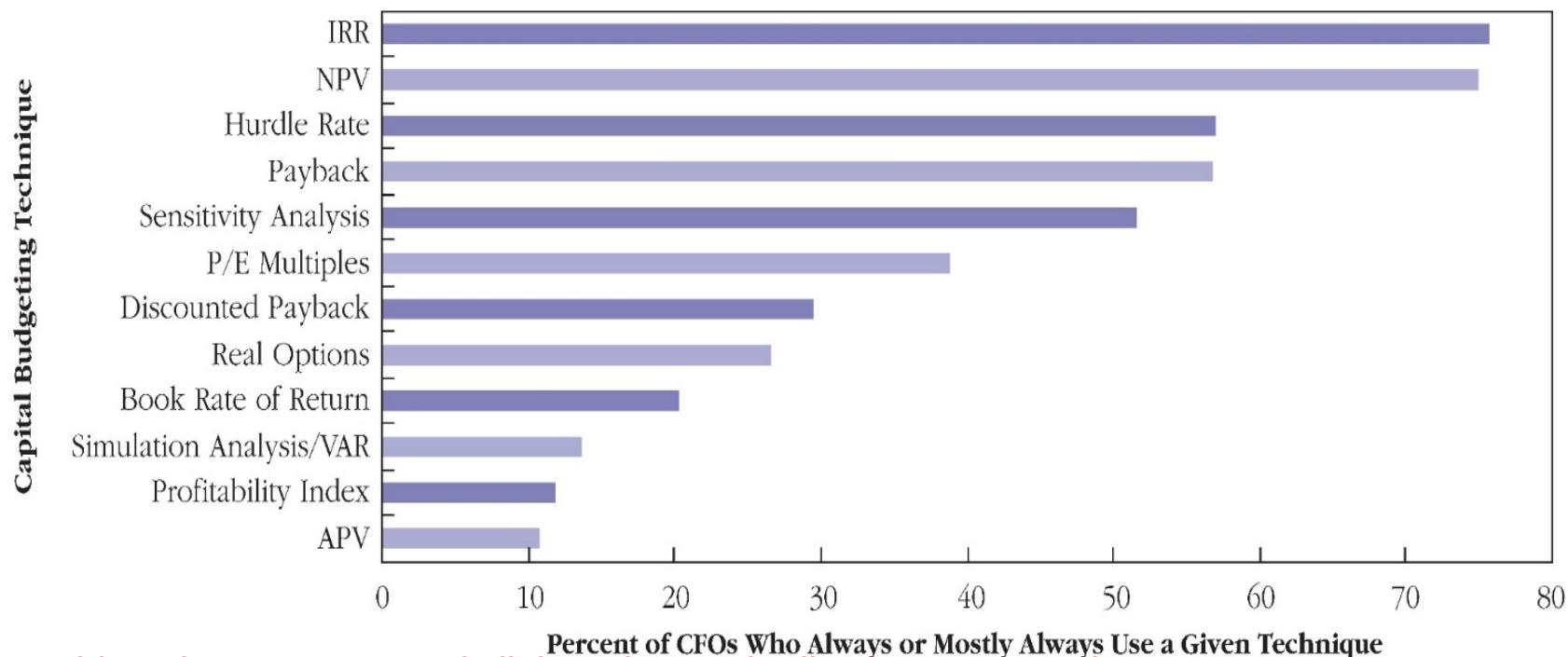
## Problems with PI

- PI gives the same answer as NPV when
  - There is only one cash outflow, which is at time 0,
  - Only one project is under consideration.
- PI suffers the same drawback as IRR in comparing projects with different scales.
- PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.

	$CF_0$	$CF_1$	$IRR$	$NPV \text{ at } 10\%$	$PI \text{ at } 10\%$
Project 1	-1,000	2,000	100%	818.18	1.82
Project 2	-2,000	3,600	80%	1,272.73	1.64

# Capital budgeting in practice

**FIGURE 1 ■ SURVEY EVIDENCE ON THE POPULARITY OF DIFFERENT CAPITAL BUDGETING METHODS\***



if we add up the percentage of all the rules on the list, it way exceeds 100%. This means that multiple rules are often used at the same time.

Source: John Graham and Campbell Harvey, 2002, "How do CFOs make capital budgeting and capital structure decisions?" *Journal of Applied Corporate Finance*.

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## Additional considerations

1. Competitive Response: a more profitable project will more likely attract imitators.

- CF forecasts should consider responses of competitors.  
a firm should take on all the independent projects with positive

2. Capital Rationing. NPVs. One possible constraint is the amount of capital available. But good projects are more scarce than capital.

3. Sources of Positive-NPV Projects:

- Short-run competitive advantage (right place at right time)

- Long-run competitive advantage

brand

Just having a positive number for NPV without knowing what is behind it is far from being sufficient.

❖ Patent

- Sources of Positive-NPV Projects

❖ Technology

- are our assumptions reasonable?

❖ economies of scale, etc.

- how robust is our answer?

- Noise.

pure luck

Capital Rationing: A division inside a firm does not have direct access to the financial market. Its capital is often provided by the firm's headquarters. At a given time, the division may only have a fixed amount of capital at its disposal.

## Additional considerations

### Capital rationing

- $N$  independent projects,
- All with positive NPVs. For  $i = 1, 2, \dots, N$ :
  - Initial investment  $I_i$
  - Net present value  $NPV_i > 0$ ,
- Total capital available  $I$ .

We should choose the set of projects which maximizes the total NPV, subject to the capital constraint:

$$\begin{aligned} \text{Maximize } NPV &= \sum_i NPV_i \\ \text{subject to: } \sum I_i &\leq I \end{aligned}$$

## Summary

- NPV rule
- Cash flows from capital investments
- Discount rates/project interaction
- Alternative capital budgeting rules
- Additional considerations