

Logarithms 對數函數

Introduction

- If $x = a^y$, then $y = \log_a x$
- If $y = \log_a x$, then $x = a^y$

Example

Exponential form 指數形式	Logarithmic form 對數形式
$1 = 3^0$	$0 = \log_3 1$
$10000 = 10^4$	$4 = \log_{10} 10000$
$\frac{1}{16} = \left(\frac{1}{4}\right)^2$	$2 = \log_{\frac{1}{4}} \frac{1}{16}$
$\frac{1}{32} = 2^{-5}$	$-5 = \log_2 \frac{1}{32}$

Practice Round

Find the following equation in Logarithmic form.

1. $10000 = 10^4$
2. $0.00001 = 10^{-5}$

Find the value of the unknown in the following questions.

1. $7 = 10^b$
2. $\log d = -0.3$

Property

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $a^{\log_a N} = N$
4. $\log_a(MN) = \log_a M + \log_a N$
5. $\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$
6. $\log_a M^n = n \log_a M$
7. $\log_a x = \frac{\log_b x}{\log_b a}$

Quick Practice

Find the values of the following logarithms correct to 3 significant figures.

(a) $\log_5 30$

(b) $\log_3 0.75$

Find the values of the following expressions without using a calculator.

(a) $\log_2 48 - \log_2 3$

(b) $\log_9 27$

In the t -th year after 2012, the population P of a town is given by $P = 50000 \times 1.07^t$.
某城市在2012年後第 t 年的人口是以 $P = 50000 \times 1.07^t$ 估算。

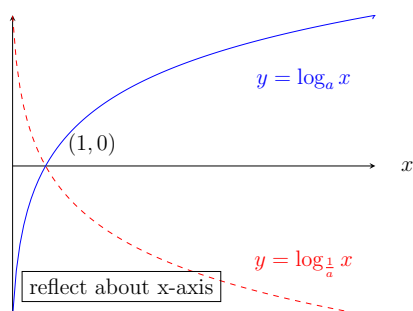
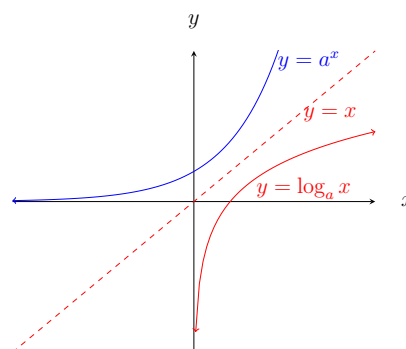
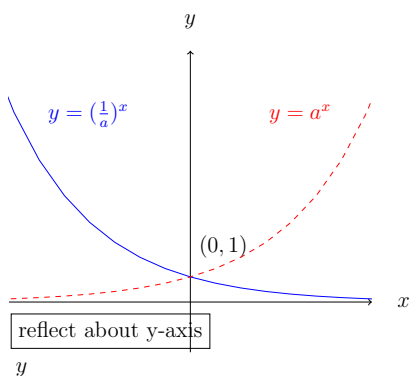
(a) Find the population of the town 3 years after 2012.

求城市A在2012年後第3年的人口。

(b) In which year will the population of the town first exceed 80000?

城市A的人口將於那年超過80000?

Graphs of Exponential and Logarithmic Function



Answers

Practice Round

1.

$$\log_{10} 10000 = 4$$

2.

$$\log_{10} 0.00001 = -5$$

3.

$$7 = 10^b$$

$$\log_{10} 7 = \log_{10} 10^b$$

$$\log_{10} 7 = b \log_{10} 10$$

$$\log_{10} 7 = b$$

$$b = \log_{10} 7$$

4.

$$\log d = -0.3$$

$$\log d = -0.3 \log 10$$

$$\log d = \log 10^{-0.3}$$

$$\log d = \log \frac{1}{10^{0.3}}$$

$$d = \frac{1}{10^{0.3}}$$

$$d = 10^{-0.3}$$

Quick Practice

1.

$$\begin{aligned}\log_5 30 &= \frac{\log_{10} 30}{\log_{10} 5} \\ &= 2.11\end{aligned}$$

2.

$$\begin{aligned}\log_3 0.75 &= \frac{\log_{10} 0.75}{\log_{10} 3} \\ &= -0.263\end{aligned}$$

3. (a)

$$\begin{aligned}P &= 50000(1.07)^3 \\ &= 61252\end{aligned}$$

The population 3 years after 2020 is 61,252.

(b)

$$P > 80000$$

$$50000 \times 1.07^t > 80000$$

$$1.07^t > \frac{80000}{50000}$$

$$\log 1.07^t > \log \frac{80000}{50000}$$

$$t \log 1.07 > \log \frac{80000}{50000}$$

$$t > \frac{\log \frac{80000}{50000}}{\log 1.07}$$

$$t > 6.95$$

The population will exceed 80000 in 2019.