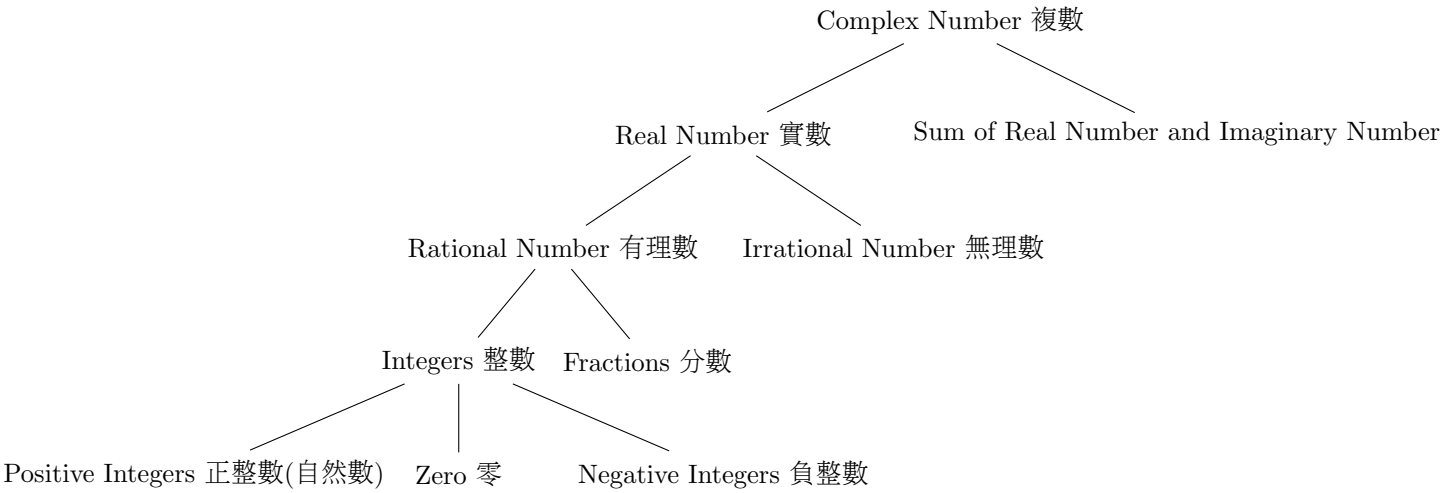
1 Complex Numbers 複數



2 Law of Indices 指數定律

1. $a^0 = 1$ where $a \neq 0$
2. $a^{-n} = \dfrac{1}{a^n}$ where $a \neq 0$
3. $a^m \times a^n = a^{m+n}$
4. $a^m \div a^n = a^{m-n}$
5. $(a^m)^n = a^{mn}$
6. $(ab)^n = a^n b^n$
7. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ where $b \neq 0$
8. $a^{\frac{1}{n}} = \sqrt[n]{a}$ where $n > 0$
9. $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ where $a > 0, n > 0$

3 Surds 無理數

1. $\sqrt{a} \times \sqrt{a} = a$
2. $\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$
3. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$
4. $\dfrac{1}{\sqrt{a}} = \dfrac{1}{\sqrt{a}} \times \dfrac{\sqrt{a}}{\sqrt{a}} = \dfrac{\sqrt{a}}{a}$

4 Questions

1. $(\dfrac{1}{9^{555}})3^{444}$
2. $8^{222} \cdot 5^{666}$
3. $\dfrac{(3y^6)^4}{3y^2}$

4. Simplify $\frac{(mn^{-2})^5}{m^{-4}}$ and express your answer with positive indices.

5. Simplify $\frac{xy^7}{(x^{-2}y^3)^4}$ and express your answer with positive indices.

6. $\sqrt{108}$

7. $\frac{6}{\sqrt{12}}$

8. $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}}$

5 Answers

1.

$$\begin{aligned}(\frac{1}{9^{555}})3^{444} &= (\frac{1}{(3^2)^{555}})3^{444} \\ &= (\frac{1}{3^{1110}})3^{444} \\ &= (\frac{3^{444}}{3^{1110}}) \\ &= (\frac{1}{3^{666}}) \\ &= 3^{-666}\end{aligned}$$

2.

$$\begin{aligned}8^{222} \cdot 5^{666} &= (2^3)^{222} \cdot 5^{666} \\ &= (2)^{666} \cdot 5^{666} \\ &= 10^{666}\end{aligned}$$

3.

$$\begin{aligned}\frac{(3y^6)^4}{3y^2} &= \frac{(3^4y^{24})}{3y^2} \\ &= 3^{4-1}y^{24-2} \\ &= 3^3y^{22} \\ &= 27y^{22}\end{aligned}$$

4.

$$\begin{aligned}\frac{(mn^{-2})^5}{m^{-4}} &= \frac{m^5n^{-10}}{m^{-4}} \\ &= m^{5-(-4)}n^{-10} \\ &= m^{5+4}n^{-10} \\ &= \frac{m^9}{n^{10}}\end{aligned}$$

5.

$$\begin{aligned}\frac{xy^7}{(x^{-2}y^3)^4} &= \frac{xy^7}{(x^{-8}y^{12})} \\ &= \frac{xy^7x^8}{(y^{12})} \\ &= \frac{y^7x^9}{(y^{12})} \\ &= \frac{x^9}{y^{(12-7)}} \\ &= \frac{x^9}{y^5}\end{aligned}$$

6.

$$\begin{aligned}\sqrt{108} &= \sqrt{4 \times 27} \\ &= \sqrt{2^2 \times 3^3} \\ &= \sqrt{2^2} \times \sqrt{3^3} \\ &= 2 \times 3\sqrt{3} \\ &= 6\sqrt{3}\end{aligned}$$

7.

$$\begin{aligned}
\frac{6}{\sqrt{12}} &= \frac{6}{\sqrt{4\times 3}} \\
&= \frac{6}{2\sqrt{3}} \\
&= \frac{3}{\sqrt{3}} \\
&= \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
&= \frac{3\sqrt{3}}{(\sqrt{3})^2} \\
&= \frac{3\sqrt{3}}{3} \\
&= \sqrt{3}
\end{aligned}$$

8.

$$\begin{aligned}
\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} &= \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}+\sqrt{2}} \times \frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}} \\
&= \frac{(\sqrt{6}-\sqrt{2})^2}{(\sqrt{6})^2-(\sqrt{2})^2} \\
&= \frac{(\sqrt{6}-\sqrt{2})^2}{6-2} \\
&= \frac{(\sqrt{6})^2-2\sqrt{6}\sqrt{2}+(\sqrt{2})^2}{6-2} \\
&= \frac{6-2\sqrt{6}\sqrt{2}+2}{6-2} \\
&= \frac{8-2\sqrt{12}}{4} \\
&= \frac{8-4\sqrt{3}}{4} \\
&= 2-\sqrt{3}
\end{aligned}$$