Study Guide: Chapter 5 - Several Useful Discrete Distributions

Overview

Discrete random variables often model real-life applications using specific probability distributions. Three e

1. Binomial Probability Distribution

- **Definition**: A Binomial experiment consists of n identical Bernoulli trials with two outcomes: succes
- **Characteristics**:
  - Fixed number of trials n.
  - Each trial is independent.
  - Probability of success p is constant across trials.
  - Interest is in X, the number of successes in n trials.
- **Probability Formula**:

$$
P(X = x) = C(n, x) p^x q^{(n-x)}
$$

  where $C(n, x) = \frac{n!}{x!(n-x)!}$.

- **Mean and Variance**:
  - Mean: $E(X) = \mu = np$
  - Variance: $Var(X) = npq$
  - Standard deviation: $\sigma = \sqrt{npq}$

- **Examples**:
  - **Coin Tossing**: When tossing a coin 3 times, analyze probabilities for outcomes (e.g., observing 2 he
  - **Multiple Choice Test**: Calculate passing probabilities with random guessing.

2. Hypergeometric Probability Distribution

- **Definition**: This distribution applies to scenarios of sampling without replacement from a finite populati

- **Probability Formula**:

$$
P(X = x) = \frac{C(M, x) C(N-M, n-x)}{C(N, n)}
$$

  where N is the population size, M is the number of successes, and n is the sample size.

- **Mean and Variance**:

  - Mean: $\mu = \frac{nM}{N}$

  - Variance: $\sigma^2 = \frac{nM(N-M)(N-n)}{N^2(N-1)}$

- **Examples**:

  - **Urn Problem**: Determine probabilities of selecting red balls from a mixed set.

  - **Investing in Stocks**: Calculate the likelihood of selecting negatively performing stocks.

3. Binomial Approximation to the Hypergeometric Distribution

- **Approximation**: When N is much larger than n, sampling without replacement may be approxi

4. Poisson Probability Distribution

- **Definition**: A Poisson distribution models the probability of a number of events occurring within a fixed

- **Probability Formula**:

$$
P(X = x) = \frac{e^{-\mu} \mu^x}{x!}
$$

- **Mean and Variance**:

  - Mean \(\mu \)

  - Variance \(\mu \)

- **Examples**:

  - **Ship Collisions**: Calculate probabilities of occurrences over specified periods.

  - **Psychic Hotline Calls**: Use the Poisson model for call frequency.

5. Poisson Approximation to the Binomial Distribution

- **When to Use**: This approximation is best applied when \(n \) is large and \(p \) is small, ensuring \(np

Exercises

- **Binomial Examples**: Coin tossing, multiple choice tests, defective light bulbs.

- **Hypergeometric Examples**: Selecting red balls, choosing stocks with potential failures.

- **Poisson Examples**: Ship collisions, call predictions for hotlines, life insurance claims.

Summary

This chapter outlines significant discrete probability distributions?Binomial, Hypergeometric, and Poisson?e