

writing dy/dx —which caught on. It is the perfect way to suggest the limit of $\Delta y/\Delta x$. Newton was one of the great scientists of all time, and calculus was one of the great inventions of all time—but the notation must help. You now can write and speak about the derivative. What is needed is a longer list of functions and derivatives.

2.1 EXERCISES

Read-through questions

The derivative is the a of $\Delta f/\Delta t$ as Δt approaches b. Here Δf equals c. The step Δt can be positive or d. The derivative is written v or e or f. If $f(x) = 2x + 3$ and $\Delta x = 4$ then $\Delta f =$ g. If $\Delta x = -1$ then $\Delta f =$ h. If $\Delta x = 0$ then $\Delta f =$ i. The slope is not $0/0$ but $df/dx =$ j.

The derivative does not exist where $f(t)$ has a k and $v(t)$ has a l. For $f(t) = 1/t$ the derivative is m. The slope of $y = 4/x$ is $dy/dx =$ n. A decreasing function has a o derivative. The p variable is t or x and the q variable is f or y . The slope of y^2 (is) (is not) $(dy/dx)^2$. The slope of $(u(x))^2$ is r by the square rule. The slope of $(2x + 3)^2$ is s.

1 Which of the following numbers (as is) gives df/dt at time t ? If in doubt test on $f(t) = t^2$.

- (a) $\frac{f(t + \Delta t) - f(t)}{\Delta t}$ (b) $\lim_{h \rightarrow 0} \frac{f(t + 2h) - f(t)}{2h}$
 (c) $\lim_{\Delta t \rightarrow 0} \frac{f(t - \Delta t) - f(t)}{-\Delta t}$ (d) $\lim_{t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$

2 Suppose $f(x) = x^2$. Compute each ratio and set $h = 0$:

- (a) $\frac{f(x + h) - f(x)}{h}$ (b) $\frac{f(x + 5h) - f(x)}{5h}$
 (c) $\frac{f(x + h) - f(x - h)}{2h}$ (d) $\frac{f(x + 1) - f(x)}{h}$

3 For $f(x) = 3x$ and $g(x) = 1 + 3x$, find $f(4 + h)$ and $g(4 + h)$ and $f'(4)$ and $g'(4)$. Sketch the graphs of f and g —why do they have the same slope?

4 Find three functions with the same slope as $f(x) = x^2$.

5 For $f(x) = 1/x$, sketch the graphs of $f(x) + 1$ and $f(x + 1)$. Which one has the derivative $-1/x^2$?

6 Choose c so that the line $y = x$ is tangent to the parabola $y = x^2 + c$. They have the same slope where they touch.

7 Sketch the curve $y(x) = 1 - x^2$ and compute its slope at $x = 3$.

8 If $f(t) = 1/t$, what is the average velocity between $t = \frac{1}{2}$ and $t = 2$? What is the average between $t = \frac{1}{2}$ and $t = 1$? What is the average (to one decimal place) between $t = \frac{1}{2}$ and $t = 101/200$?

9 Find $\Delta y/\Delta x$ for $y(x) = x + x^2$. Then find dy/dx .

10 Find $\Delta y/\Delta x$ and dy/dx for $y(x) = 1 + 2x + 3x^2$.

11 When $f(t) = 4/t$, simplify the difference $f(t + \Delta t) - f(t)$, divide by Δt , and set $\Delta t = 0$. The result is $f'(t)$.

12 Find the derivative of $1/t^2$ from $\Delta f(t) = 1/(t + \Delta t)^2 - 1/t^2$. Write Δf as a fraction with the denominator $t^2(t + \Delta t)^2$. Divide the numerator by Δt to find $\Delta f/\Delta t$. Set $\Delta t = 0$.

13 Suppose $f(t) = 7t$ to $t = 1$. Afterwards $f(t) = 7 + 9(t - 1)$.

- (a) Find df/dt at $t = \frac{1}{2}$ and $t = \frac{3}{2}$.
 (b) Why doesn't $f(t)$ have a derivative at $t = 1$?

14 Find the derivative of the derivative (the *second derivative*) of $y = 3x^2$. What is the third derivative?

15 Find numbers A and B so that the straight line $y = x$ fits smoothly with the curve $Y = A + Bx + x^2$ at $x = 1$. Smoothly means that $y = Y$ and $dy/dx = dY/dx$ at $x = 1$.

16 Find numbers A and B so that the horizontal line $y = 4$ fits smoothly with the curve $y = A + Bx + x^2$ at the point $x = 2$.

17 True (with reason) or false (with example):

- (a) If $f(t) < 0$ then $df/dt < 0$.
 (b) The derivative of $(f(t))^2$ is $2 df/dt$.
 (c) The derivative of $2f(t)$ is $2 df/dt$.
 (d) The derivative is the limit of Δf divided by the limit of Δt .

18 For $f(x) = 1/x$ the *centered difference* $f(x + h) - f(x - h)$ is $1/(x + h) - 1/(x - h)$. Subtract by using the common denominator $(x + h)(x - h)$. Then divide by $2h$ and set $h = 0$. Why divide by $2h$ to obtain the correct derivative?

19 Suppose $y = mx + b$ for negative x and $y = Mx + B$ for $x \geq 0$. The graphs meet if _____. The two slopes are _____. The slope at $x = 0$ is _____ (what is possible?).

20 The slope of $y = 1/x$ at $x = 1/4$ is $y' = -1/x^2 = -16$. At $h = 1/12$, which of these ratios is closest to -16 ?

$$\frac{y(x + h) - y(x)}{h} \quad \frac{y(x) - y(x - h)}{h} \quad \frac{y(x + h) - y(x - h)}{2h}$$

21 Find the average slope of $y = x^2$ between $x = x_1$ and $x = x_2$. What does this average approach as x_2 approaches x_1 ?

22 Redraw Figure 2.1 when $f(t) = 3 - 2t$ for $t \leq 2$ and $f(t) = -1$ for $t \geq 2$. Include df/dt .