Calculus Homework

1. Question: Which of the following numbers (as is) gives df/dt at time t?
   Answer: (c) $\lim_{\Delta t \to 0} \frac{f(t-\Delta t) - f(t)}{-\Delta t}$.

2. (a) Question: Compute the ratio for $f(x) = x^2$: $\frac{f(x+h) - f(x)}{h}$. Answer:
   When $h = 0$, it gives $2x$.

   (b) Question: Compute the ratio for $f(x) = x^2$: $\frac{f(x+5h) - f(x)}{5h}$. Answer: When
   $h = 0$, it gives $2x$.

   (c) Question: Compute the ratio for $f(x) = x^2$: $\frac{f(x+h) - f(x-h)}{2h}$. Answer:
   When $h = 0$, it gives $2x$.

   (d) Question: Compute the ratio for $f(x) = x^2$: $\frac{f(x+1) - f(x)}{h}$. Answer: It is
   undefined when $h = 0$.

3. Question: For $f(x) = 3x$ and $g(x) = 1 + 3x$, find $f(4+h)$, $g(4+h)$, $f'(4)$,
   and $g'(4)$. Answer: $f(4+h) = 3(4+h) = 12 + 3h$, $g(4+h) = 1 + 3(4+h) = 13 + 3h$, $f'(4) = 3$, $g'(4) = 3$.

4. Question: Find three functions with the same slope as $f(x) = x^2$. Answer:
   $f_1(x) = x^2 + 10$, $f_2(x) = x^2 - 5x$, $f_3(x) = x^2 + 2x + 4$.

5. Question: For $f(x) = 1/x$, which function has the derivative $-1/x^2$:
   $f(x) + 1$ or $f(x+1)$? Answer: $f(x+1)$.

6. Question: Choose c so that the line $y = x$ is tangent to the parabola
   $y = x^2 + c$. Answer: $c = -\frac{1}{4}$.

7. Question: Sketch the curve $y(x) = 1 - x^2$ and compute its slope at $x = 3$.
   Answer: Slope at $x = 3$ is -6.

8. Question: If $f(t) = 1/t$, what is the average velocity between $t = 1/2$ and
   $t = 2$? Between $t = 1/2$ and $t = 1$? Between $t = 1/2$ and $t = 101/200$?
   Answer: Average velocity between $t = 1/2$ and $t = 2$ is $-\frac{3}{4}$, between
   $t = 1/2$ and $t = 1$ is -1, between $t = 1/2$ and $t = 101/200$ is approximately
   -2.0.

9. Question: Find $\Delta y / \Delta x$ for $y(x) = x + x^2$. Then find dy/dx. Answer:
   $\Delta y / \Delta x = 1 + 2x$, $dy/dx = 1 + 2x$.

10. Question: Find $\Delta y / \Delta x$ and dy/dx for $y(x) = 1 + 2x + 3x^2$. Answer:
    $\Delta y / \Delta x = 2 + 6x$, $dy/dx = 2 + 6x$.

11. Question: When $f(t) = 4/t$, simplify the difference $f(t+\Delta t) - f(t)$, divide
    by Δt, and set $\Delta t = 0$. Answer: $f'(t) = -4/t^2$.

12. Question: Find the derivative of $1/t^2$ from $\Delta f(t) = 1/(t + \Delta t)^2 - 1/t^2$.
    Answer: $f'(t) = -2/t^3$.

13. (a) Question: For $f(t) = 7t$ with $t \leq 1$ and $f(t) = 7 + 9(t - 1)$ with
    $t > 1$, find df/dt at $t = \frac{1}{2}$ and $t = \frac{3}{2}$. Answer: $df/dt|_{t=1/2} = 7$,
    $df/dt|_{t=3/2} = 9$.

(b) Question: Why doesn't $f(t)$ have a derivative at $t = 1$? Answer: Because the slope changes abruptly at $t = 1$.

14. Question: Find the second and third derivatives of $y = 3x^2$. Answer: Second derivative: 6, Third derivative: 0.

15. Question: Find numbers A and B so that $y = x$ fits smoothly with $Y = A + Bx + x^2$ at $x = 1$. Answer: $A = 0$, $B = -1$.

16. Question: Find numbers A and B so that $y = 4$ fits smoothly with the curve $y = A + Bx + x^2$ at the point $x = 2$. Answer: $A = 0$, $B = 8$.

17. (a) Question: True or false: If $f(t) < 0$ then $df/dt < 0$. Answer: False. Example: $f(t) = -t^2$.

(b) Question: True or false: The derivative of $(f(t))^2$ is $2df/dt$. Answer: False. Correct statement: The derivative of $(f(t))^2$ is $2f(t)df/dt$.

(c) Question: True or false: The derivative of $2f(t)$ is $2df/dt$. Answer: True.

(d) Question: True or false: The derivative is the limit of Δf divided by the limit of Δt. Answer: False.

18. Question: Why divide by $2h$ to obtain the correct derivative when finding the derivative for $f(x) = 1/x$? Answer: Because this is for the centered difference approach; dividing by $2h$ accounts for the equal change on both sides of x.

19. Question: Find the conditions for graphs of $y = mx + b$ for $x < 0$ and $y = Mx + B$ for $x \geq 0$ to meet at $x = 0$. Answer: $b = B$ and $m = M$.

20. Question: For $y = 1/x$ at $x = 1/4$, which of these ratios is closest to -16: $\frac{y(x+h)-y(x)}{h}$, $\frac{y(x)-y(x-h)}{h}$, $\frac{y(x+h)-y(x-h)}{2h}$? Answer: $\frac{y(x+h)-y(x)}{h}$ when $h = 1/12$.

21. Question: Find the average slope of $y = x^2$ between $x = x_1$ and $x = x_2$. Answer: Average slope: $\frac{(x_2^2 - x_1^2)}{(x_2 - x_1)}$.

22. Question: Redraw Figure 2.1 when $f(t) = 3 - 2t$ for $t \leqslant 2$ and $f(t) = -1$ for $t \geqslant 2$. What is df/dt? Answer: $df/dt = -2$ for $t \leq 2$ and $df/dt = 0$ for $t > 2$.