

Calculus Homework

1. Question: Which of the following numbers (as is) gives df/dt at time t ?
If in doubt test on $f(t) = t^2$. Answer: (c) $\lim_{\Delta t \rightarrow 0} \frac{f(t-\Delta t)-f(t)}{-\Delta t}$.
2. Question: Suppose $f(x) = x^2$. Compute each ratio and set $h = 0$. Answer:
(a) $\frac{2xh+h^2}{h}$, (b) $\frac{5(2xh+h^2)}{5h}$, (c) $\frac{2h^2}{2h}$, (d) $2x + 1$.
3. Question: For $f(x) = 3x$ and $g(x) = 1 + 3x$, find $f(4+h)$ and $g(4+h)$ and $f'(4)$ and $g'(4)$. Sketch the graphs of f and g -why do they have the same slope? Answer: $f(4+h) = 3(4+h)$, $g(4+h) = 1 + 3(4+h)$, $f'(4) = 3$, $g'(4) = 3$. The graphs have the same slope because they both have a linear relationship with the same rate of change; they are parallel.
4. Question: Find three functions with the same slope as $f(x) = x^2$. Answer: $f(x) = x^3 + x^2$, $f(x) = 2x^2$, $f(x) = x^2 + 3$.
5. Question: For $f(x) = 1/x$, sketch the graphs of $f(x) + 1$ and $f(x + 1)$. Which one has the derivative $-1/x^2$? Answer: The graph of $f(x + 1)$ has the derivative $-1/x^2$.
6. Question: Choose c so that the line $y = x$ is tangent to the parabola $y = x^2 + c$. They have the same slope where they touch. Answer: $c = -1/4$.
7. Question: Sketch the curve $y(x) = 1 - x^2$ and compute its slope at $x = 3$. Answer: Slope $= -2x = -6$.
8. Question: If $f(t) = 1/t$, what is the average velocity between $t = \frac{1}{2}$ and $t = 2$? What is the average between $t = \frac{1}{2}$ and $t = 1$? What is the average (to one decimal place) between $t = \frac{1}{2}$ and $t = 101/200$? Answer: Avgs are $-1/4$, -1 , and -1.9 , respectively.
9. Question: Find $\Delta y/\Delta x$ for $y(x) = x + x^2$. Then find dy/dx . Answer: $\Delta y/\Delta x = 1 + 2x$, $dy/dx = 1 + 2x$.
10. Question: Find $\Delta y/\Delta x$ and dy/dx for $y(x) = 1 + 2x + 3x^2$. Answer: $\Delta y/\Delta x = 2 + 6x$, $dy/dx = 2 + 6x$.
11. Question: When $f(t) = 4/t$, simplify the difference $f(t + \Delta t) - f(t)$, divide by Δt , and set $\Delta t = 0$. The result is $f'(t)$. Answer: $f'(t) = -4/t^2$.
12. Question: Find the derivative of $1/t^2$ from $\Delta f(t) = 1/(t + \Delta t)^2 - 1/t^2$; Write Δf as a fraction with the denominator $t^2(t + \Delta t)^2$. Divide the numerator by Δt to find $\Delta f/\Delta t$. Set $\Delta t = 0$. Answer: $d(1/t^2)/dt = -2/t^3$.
- 13a. Question: Find df/dt at $t = \frac{1}{2}$ and $t = \frac{3}{2}$. Answer: at $t = \frac{1}{2}$, $df/dt = 7$; at $t = \frac{3}{2}$, $df/dt = 9$.
- 13b. Question: Why doesn't $f(t)$ have a derivative at $t = 1$?. Answer: $f(t)$ has a kink, or a sudden change in slopes, at $t = 1$; the function is not smooth at this point.

14. Question: Find the derivative of the derivative (the second derivative) of $y = 3x^2$. What is the third derivative? Answer: Second derivative: 6. Third derivative: 0.
15. Question: Find numbers A and B so that the straight line $y = x$ fits smoothly with the curve $Y = A + Bx + x^2$ at $x = 1$. Smoothly means that $y = Y$ and $dy/dx = dY/dx$ at $x = 1$. Answer: $A = -1$, $B = 1$.
16. Question: Find numbers A and B so that the horizontal line $y = 4$ fits smoothly with the curve $y = A + Bx + x^2$ at the point $x = 2$. Answer: $A = 0$, $B = 4$.
- 17a. Question: True (with reason) or false (with example): If $f(t) < 0$ then $df/dt < 0$. Answer: False, e.g. $f(t) = -t^2$.
- 17b. Question: The derivative of $(f(t))^2$ is $2df/dt$. Answer: False.
- 17c. Question: The derivative of $2f(t)$ is $2df/dt$. Answer: True.
- 17d. Question: The derivative is the limit of Δf divided by the limit of Δt . Answer: False; the derivative is the limit of $\Delta f/\Delta t$ as Δt approaches 0.
18. Question: Why divide by $2h$ to obtain the correct derivative? Answer: You used the centered difference method, where the changes in x were symmetric ($x + h$ and $x - h$), so dividing by $2h$ gives the correct average rate of change.
- 19a. Question: The graphs meet if: Answer: $mx + b = Mx + B$.
- 19b. Question: The two slopes are: Answer: m and M .
- 19c. Question: What is the slope at $x = 0$ (what is possible)? Answer: Any value is possible.
20. Question: The slope of $y = 1/x$ at $x = 1/4$ is $y' = -1/x^2 = -16$. At $h = 1/12$, which of these ratios is closest to -16 ? Answer: $\frac{y(x+h)-y(x)}{h}$.
21. Question: Find the average slope of $y = x^2$ between $x = x_1$ and $x = x_2$. What does this average approach as x_2 approaches x_1 ? Answer: Average slope: $(2x_1 + x_2 - x_1)/(x_2 - x_1)$. As x_2 approaches x_1 , the average slope approaches $2x_1$.
22. Question: Redraw Figure 2.1 when $f(t) = 3 - 2t$ for $t \leq 2$ and $f(t) = -1$ for $t \geq 2$. Include df/dt . Answer: Redrawn figure would show a piecewise-defined function with slope -2 for $t \leq 2$ and slope 0 (horizontal) for $t \geq 2$.