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Calculus Homework

Question: Find A_y/A_x for $y(x) = x^3$. Then find dy/dx.

Answer: $A_y/A_x = 3x^2$. $dy/dx = 3x^2$

Question: Find A_y/A_x and dy/dx for $y(x) = 1 - 2x + 3x^2$.

Answer: $A_y/A_x = -2 + 6x$. $dy/dx = -2 + 6x$

Question: When $f(t) = 4/t$, simplify the difference $f(t + \Delta t) - f(t)$, divide by Δt, and set $\Delta t = 0$. The result is $f'(t)$.

Answer: $f'(t) = -4/t^2$

Question: Find the derivative of $1/t^2$ from $Af(t) = 1/(t + \Delta t)^2 - 1/t^2$. Write Af as a fraction with the denominator $t^2(t + \Delta t)^2$.

Answer: $Af = -2\Delta t/(t^4 + \Delta t(2t^3 + \Delta t^2))$. Divide the numerator by Δt to find $Af/\Delta t$. Set $\Delta t = 0$. Answer: $f'(t) = -2/t^3$

Question: Find the second derivative of $y = 3x^2$. What is the third derivative?

Answer: $y'' = 6$. $y''' = 0$

Question: Find numbers A and B so that the straight line $y = x$ fits smoothly with the curve $Y = A + Bx - x^2$ at $x = 1$. Smoothly means that $y = Y$ and $dy/dx = dY/dx$ at $x = 1$.

Answer: $A = 0$, $B = 2$

Question: Find numbers A and B so that the horizontal line $y = 4$ fits smoothly with the curve $y = A + Bx - x^2$ at the point $x = 2$.

Answer: $A = 4$, $B = 4$

Question: For $f(x) = 1/x$, the centered difference $f(x + h) - f(x - h)$ is $1/(x+h) - 1/(x-h)$. Subtract by using the common denominator $(x+h)(x-h)$. Then divide by $2h$ and set $h = 0$. Why divide by $2h$ to obtain the correct derivative?

Answer: The centered difference is an approximation of the derivative, so we divide by $2h$ to account for the average over the interval between $(x - h)$ and $(x + h)$. When taking the limit as h approaches 0, it becomes the true derivative: $f'(x) = -1/x^2$.

Question: The slope of $y = 1/x$ at $x = 1/4$ is $y' = -1/x^2 = -16$. At $h = 1/12$, which of these ratios is closest to -16?

Answer:

$$(y(x+h) - y(x-h))/2h = (-1/(1/4+1/12) + 1/(1/4-1/12))/(2*1/12) = -16$$

Question: Find the average slope of $y = x^2$ between $x = x_1$ and $x = x_2$. What does this average approach as x_2 approaches x_1?

Answer: Average slope $= (x_2^2 - x_1^2)/(x_2 - x_1) = (x_1 + x_2)$. As x_2 approaches x_1, the average slope approaches $2x_1$.