Calculus Homework

Question: Find A_y/A_x for $y(x) = xx^2$. Then find dy/dx.

Answer: $A_y/A_x = 3x^2$, $dy/dx = 3x^2$.

Question: Find A_y/A_x and dy/dx for $y(x) = \frac{1}{2}x\ 3x^2$.

Answer: $A_y/A_x = \frac{3}{2}x^2 - \frac{1}{2}x$, $dy/dx = \frac{3x-1}{2x^2}$.

Question: When $f(t) = \frac{4}{t}$, simplify the difference $f(t + A_t) - f(t)$, divide by A_t, and set $A_t = 0$. The result is $f'(t)$.

Answer: $\frac{1}{t^2} \times A_t$, $\lim_{A_t \to 0} : f'(t) = -\frac{4}{t^2}$.

Question: Find the derivative of $\frac{1}{t^2}$ from $A_f(t) = \frac{1}{(tA_t)^2 - \frac{1}{t^2}}$. Write A_f as a fraction with the denominator $t^2(tA_t)^2$.

Answer: $A_f/A_t = \frac{2}{(t^3(t+A_t)^2 + t^3(t-A_t)^2)}$, so $A_f = \frac{2t^3 A_t}{t^3(t+A_t)^2 + t^3(t-A_t)^2}$.

Question: Suppose $f(t) = 7t$ to $t = 1$. Afterwards $f(t) = 7\sqrt{(t-1)}$. (a) Find df/dt at $t = 3$ and $t = 1$; (b) Why doesn't $f(t)$ have a derivative at $t = 1$?

Answer: (a) df/dt at $t = 3$ is 7, and at $t = 1$ is undefined. (b) $f(t)$ doesn't have a derivative at $t = 1$ because it has a sharp corner or cusp.

Question: Find the derivative of the derivative (the second derivative) of $y = 3x^2$. What is the third derivative?

Answer: The second derivative of $y = 3x^2$ is 6, and the third derivative is 0.

Question: Find numbers A and B so that the straight line $y = x$ fits smoothly with the curve $Y = ABx\ x^2$ at $x = 1$. Smoothly means that $y = Y$ and $dy/dx = dY/dx$ at $x = 1$.

Answer: $A = \frac{1}{3}$, $B = \frac{2}{3}$.

Question: Find numbers A and B so that the horizontal line $y = 4$ fits smoothly with the curve $y = ABx\ x^2$ at the point $x = 2$.

Answer: $A = 4$, $B = \frac{1}{4}$.

Question: True (with reason) or false (with example): (a) If $f(t) < 0$ then $df/dt < 0$. (b) The derivative of $(f(t))^2$ is $2df/dt$. (c) The derivative of $2f(t)$ is $2df/dt$. (d) The derivative is the limit of A_f divided by the limit of A_t.

Answer: (a) False, for example $f(t) = -t$ with $df/dt = -1$. (b) False, the derivative of $(f(t))^2$ is $2f(t)df/dt$. (c) True, by the constant multiple rule. (d) True, this is the definition of the derivative.

Question: For $f(x) = 3x$ and $g(x) = \frac{1}{3x}$, find $f(4h)$ and $g(4h)$ and $f'(4)$ and $g'(4)$. Sketch the graphs of f and g - why do they have the same slope?

Answer: $f(4h) = 12h$, $g(4h) = \frac{1}{3 \cdot 4h}$, $f'(4) = 3$, $g'(4) = -\frac{1}{48}$. The graphs have the same slope because they are both straight lines with non-zero slopes.

Question: Choose c so that the line $y = x$ is tangent to the parabola $y = x^2 + c$. They have the same slope where they touch.

Answer: $c = \frac{1}{2}$.

Question: Sketch the curve $y(x) = 1 - x^2$ and compute its slope at $x = 3$.

Answer: The slope at $x = 3$ is $-6x = -18$.

Question: If $f(t) = \frac{1}{t}$, what is the average velocity between $t = 3$ and $t = 2$? What is the average between $t = 3$ and $t = 1$? What is the average (to one decimal place) between $t = 3$ and $t = \frac{101}{200}$?

Answer: The average velocity between $t = 3$ and $t = 2$ is $-\ln(2)$, between $t = 3$ and $t = 1$ is $-\ln(3)$, and between $t = 3$ and $t = \frac{101}{200}$ is $-\ln\left(\frac{101}{600}\right)$.