1 Question

$$x \in R^n$$

$$\Delta\tau = \sum_{i=1}^{n} \frac{\partial^2 \tau}{\partial x_i^2}$$

$$\boldsymbol{\nabla}\tau = \begin{bmatrix} \frac{\partial\tau}{\partial x_1} \\ \frac{\partial\tau}{\partial x_2} \\ \dots \\ \frac{\partial\tau}{\partial x_n} \end{bmatrix}$$

We know that

$$\frac{\partial\tau}{\partial t} + \frac{r^2}{2}\Delta\tau + b\boldsymbol{\nabla}\tau = 0$$

and

$$v = -\ln\tau$$

2 Solution

We get the partial derivatives based on v in regards of different variables

$$\frac{\partial v}{\partial t} = -\frac{1}{\tau}\frac{\partial\tau}{\partial t}$$

$$\frac{\partial v}{\partial x} = -\frac{1}{\tau}\frac{\partial\tau}{\partial x}$$

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{\tau^2}(\frac{\partial\tau}{\partial x})^2 - \frac{1}{\tau}\frac{\partial^2\tau}{\partial x^2}$$

Therefore, we get

$$\sum_{i=1}^{n}\frac{\partial^2 v}{\partial x_i^2} = \sum_{i=1}^{n}\frac{1}{\tau^2}(\frac{\partial\tau}{\partial x_i})^2 - \sum_{i=1}^{n}\frac{1}{\tau}\frac{\partial^2\tau}{\partial x_i^2}$$

By the equation

$$\frac{\partial\tau}{\partial t} + \frac{r^2}{2}\Delta\tau + b\boldsymbol{\nabla}\tau = 0$$

We get

$$-\tau\frac{\partial v}{\partial t} - \frac{r^2}{2}\tau\sum_{i=1}^{n}(\frac{\partial v}{\partial x_i})^2 + \frac{r^2}{2}\tau\sum_{i=1}^{n}\frac{\partial^2 v}{\partial x_i^2} - \tau\sum_{i=1}^{n} b_i\frac{\partial v}{\partial x_i} = 0$$

By eliminating τ, we get the final equation based on the partial differential of v.

$$\frac{\partial v}{\partial t} + \frac{r^2}{2}\sum_{i=1}^{n}(\frac{\partial v}{\partial x_i})^2 - \frac{r^2}{2}\sum_{i=1}^{n}\frac{\partial^2 v}{\partial x_i^2} + \sum_{i=1}^{n} b_i\frac{\partial v}{\partial x_i} = 0$$