1 Mission

We know that

$$\tau(x,t,y,s) \triangleq \frac{1}{\sqrt{2\pi(s-t)}} \exp\left(-\frac{1}{2}\frac{(y-x)^2}{s-t}\right)$$

we get the conclusion with initial or terminal conditions in terms of Dirac Delta function that

$$\frac{\partial \tau}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 \tau}{\partial x^2} = 0$$

$$\frac{\partial \tau}{\partial s} - \frac{\sigma^2}{2}\frac{\partial^2 \tau}{\partial y^2} = 0$$

2 Mission

After we get the two backward and forward equations based on the Dirac Delta Function, we should continue represents σ based on (x,t) and (y,s).

First. we need to define $\frac{\partial \tau}{\partial t}, \frac{\partial^2 \tau}{\partial x^2}, \frac{\partial \tau}{\partial s}, \frac{\partial^2 \tau}{\partial y^2}$

$$\begin{aligned}
\frac{\partial \tau}{\partial t} &= \frac{1}{2}\frac{1}{\sqrt{2\pi(s-t)^3}} \exp\left(-\frac{1}{2}\frac{(y-x)^2)}{s-t}\right) + \frac{1}{\sqrt{2\pi(s-t)}} \exp\left(-\frac{1}{2}\frac{(y-x)^2}{s-t}\right) * -\frac{1}{2}\frac{(y-x)^2}{(s-t)^2} \\
&= \frac{1}{2}\frac{1}{\sqrt{2\pi(s-t)}} \exp\left(-\frac{1}{2}\frac{(y-x)^2}{s-t}\right)(\frac{1}{(s-t)} - \frac{(y-x)^2}{(s-t)^2}) \\
\frac{\partial \tau}{\partial x} &= \frac{1}{2\pi(s-t)} exp(-\frac{1}{2}\frac{(y-x)^2}{s-t}(\frac{y-x}{s-t}) \\
\frac{\partial^2 \tau}{\partial x^2} &= \frac{1}{\sqrt{2\pi(s-t)}} \exp\left(-\frac{1}{2}\frac{(y-x)^2}{s-t}\right)(-\frac{1}{s-t} + (\frac{(y-x)}{s-t})^2)
\end{aligned}$$

Therefore, $\sigma = 1$.