Explicit time-evolution in first order:

$$|\Psi(t_{k+1})\rangle = U(t_{k+1}, t_k)\,|\Psi(t_k)\rangle$$

$$U(t_{k+1}, t_k) = \exp\left[-\frac{i}{\hbar}\int_{t_k}^{t_{k+1}} \left(H_S + H_{fB}(t)\right)dt\right]$$

first order

$$U = \exp(M_S + M_B) = \mathbb{1} + M_B + M_S + \frac{1}{2}M_B^2 + \frac{1}{2}(M_S M_B + M_B M_S) + \frac{1}{6}M_B^3 + $$
$$+ \frac{1}{2}M_S^2 + \frac{1}{6}\left(M_B^2 M_S + M_B M_S M_B + M_S M_B^2\right) + \frac{1}{24}M_B^4 + \mathcal{O}(\Delta t^{5/2})$$

$$M_S = -\frac{i}{\hbar}H_S\Delta t \qquad M_B = -\frac{i}{\hbar}\int_{t_k}^{t_{k+1}} H_{fB}(t)\,dt$$

$$H_{fB}(t) = -i\hbar\left\{\left[\sqrt{\gamma_R}\, b(t-\tau)e^{-i\varphi} + \sqrt{\gamma_L}\, b(t)\right]c^+ - \left[\sqrt{\gamma_R}\, b^+(t-\tau)e^{i\varphi} + \sqrt{\gamma_L}\, b^+(t)\right]c\right\}$$

$$H_S = \hbar\Delta_c c^+c + \hbar\Delta_e \sigma_+\sigma_- + \hbar g(c^+\sigma_- + c\sigma_+) + \frac{\hbar\Omega_c}{2}(c + c^+) + \frac{\hbar\Omega_e}{2}(\sigma_- + \sigma_+)$$

$$M_B = -\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\underbrace{\int_{t_k}^{t_{k+1}} b(t-\tau)dt}_{\tau = l\Delta t} + \sqrt{\gamma_L}\underbrace{\int_{t_k}^{t_{k+1}} b(t)dt}_{\Delta B(t_k)}\right]c^+ - \left[\sqrt{\gamma_R}e^{i\varphi}\underbrace{\int_{t_k}^{t_{k+1}} b^+(t-\tau)dt}_{\Delta B^+(t_{k-l})} + \sqrt{\gamma_L}\underbrace{\int_{t_k}^{t_{k+1}} b^+(t)dt}_{\Delta B^+(t_k)}\right]c\right\}$$

$$\int_{t_{k-l}}^{t_{k-l+1}} b(t)dt = \Delta B(t_{k-l}) \qquad \left[\Delta B(t_k), \Delta B^+(t_j)\right] = \int_{t_k}^{t_{k+1}}\int_{t_j}^{t_{j+1}} \underbrace{\left[b(t), b^+(t')\right]}_{\delta(t-t')} dt'dt = $$
$$= \int_{t_k}^{t_{k+1}} dt\,\delta_{jk} = \Delta t\,\delta_{jk}$$

$$M_B^2 = \left[\gamma_R e^{-2i\varphi}\Delta B(t_{k-l})\Delta B(t_{k-l}) + \sqrt{\gamma_R\gamma_L}\left(\Delta B(t_{k-l})\Delta B(t_k) + \Delta B(t_k)\Delta B(t_{k-l})\right)e^{-i\varphi} + \gamma_L\Delta B(t_k)\Delta B(t_k)\right]c^+c^+ -$$
$$-\left[\gamma_R\Delta B(t_{k-l})\Delta B^+(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(\Delta B(t_{k-l})\Delta B^+(t_k)e^{-i\varphi} + \Delta B(t_k)\Delta B^+(t_{k-l})e^{i\varphi}\right) + \gamma_L\Delta B(t_k)\Delta B^+(t_k)\right]c^+c -$$
$$-\left[\gamma_R\Delta B^+(t_{k-l})\Delta B(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(e^{i\varphi}\Delta B^+(t_{k-l})\Delta B(t_k) + e^{-i\varphi}\Delta B^+(t_k)\Delta B(t_{k-l})\right) + \gamma_L\Delta B^+(t_k)\Delta B(t_k)\right]cc^+ +$$
$$+\left[\gamma_R e^{2i\varphi}\Delta B^+(t_{k-l})\Delta B^+(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(\Delta B^+(t_{k-l})\Delta B^+(t_k) + \Delta B^+(t_k)\Delta B^+(t_{k-l})\right)e^{i\varphi} + \gamma_L\Delta B^+(t_k)\Delta B^+(t_k)\right]cc$$

$$|i_p\rangle = \frac{(\Delta B^+(t_p))^{i_p}}{\sqrt{i_p!\,\Delta t^{i_p}}}|vac\rangle \qquad \Delta B(t_k)|i_k\rangle = \frac{\Delta B(t_k)\left[\Delta B^+(t_k)\right]^{i_k}}{\sqrt{i_k!\,\Delta t^{i_k}}}|vac\rangle = \frac{i_k\Delta t\,(\Delta B^+(t_k))^{i_k-1}}{\sqrt{i_k!\,\Delta t^{i_k}}}|vac\rangle = \sqrt{i_k\Delta t}\,|i_k-1\rangle$$

$$\left[\Delta B(t_k), (\Delta B^+(t_k))^{i_k}\right] = \left[\Delta B(t_k), \Delta B^+(t_k)\right](\Delta B^+(t_k))^{i_k-1} + \Delta B^+(t_k)\left[\Delta B(t_k), (\Delta B^+(t_k))^{i_k-1}\right] = \Delta t(\Delta B^+(t_k))^{i_k-1} + \Delta B^+(t_k)\Delta t(\Delta B^+(t_k))^{i_k-2} + \ldots + (\Delta B^+(t_k))^{i_k-1}\Delta t = i_k\Delta t(\Delta B^+(t_k))^{i_k-1}$$

$$\Delta B^+(t_k)|i_k\rangle = \frac{\Delta B^+(t_k)\left[\Delta B^+(t_k)\right]^{i_k}}{\sqrt{i_k!\,\Delta t^{i_k}}}|vac\rangle = \sqrt{(i_k+1)\Delta t}\,|i_k+1\rangle$$

$$\Phi = |i_c, i_e\rangle \bigotimes_{p=-\infty}^{\infty}|i_p\rangle \qquad p \text{ shows the different timebins for the environment}$$

Matrix elements of the evolution operator:

$$\langle j_c, j_e, \{j_p\}|\,U(t_{k+1}, t_k)\,|i_c, i_e, \{i_p\}\rangle = \delta_{j_{-\infty}i_{-\infty}}\cdots\delta_{j_{k-l-1}i_{k-l-1}}\delta_{j_{k-l+1}i_{k-l+1}}\cdots\delta_{j_{k-1}i_{k-1}}\delta_{j_{k+1}i_{k+1}}\cdots\delta_{j_\infty i_\infty}\cdot$$
$$\cdot\Big\{\delta_{j_ci_c}\delta_{j_ei_e}\delta_{j_ki_k}\delta_{j_{k-l}i_{k-l}} +$$
$$+(-\sqrt{\Delta t})\,\delta_{j_ei_e}\Big[\delta_{j_ki_k}\sqrt{\gamma_R}e^{-i\varphi}\delta_{j_{k-l}i_{k-l}-1}\delta_{j_ci_c+1}\sqrt{i_{k-l}(i_c+1)} + \delta_{j_{k-l}i_{k-l}}\sqrt{\gamma_L}\delta_{j_ki_k-1}\delta_{j_ci_c+1}\sqrt{i_k(i_c+1)} -$$
$$-\delta_{j_ki_k}\sqrt{\gamma_R}e^{i\varphi}\delta_{j_{k-l}i_{k-l}+1}\delta_{j_ci_c-1}\sqrt{(i_{k-l}+1)\,i_c} + \delta_{j_{k-l}i_{k-l}}\sqrt{\gamma_L}\delta_{j_ki_k+1}\delta_{j_ci_c-1}\sqrt{(i_k+1)\,i_c}\Big] +$$
$$-i\,\delta_{j_ki_k}\delta_{j_{k-l}i_{k-l}}\Big[\Delta_c i_c\delta_{j_ci_c}\delta_{j_ei_e} + \Delta_e\delta_{j_ci_c}\delta_{j_ee}\delta_{i_ee} + g\left(\sqrt{i_c+1}\,\delta_{j_ci_c+1}\delta_{j_eg}\delta_{i_ee} + \sqrt{i_c}\,\delta_{j_ci_c-1}\delta_{j_ee}\delta_{i_eg}\right) + \frac{\Omega_c}{2}\left(\sqrt{i_c}\,\delta_{j_ci_c-1} + \sqrt{i_c+1}\,\delta_{j_ci_c+1}\right)\delta_{j_ei_e} +$$
$$+\frac{\Omega_e}{2}\left(\delta_{j_eg}\delta_{i_ee} + \delta_{j_ee}\delta_{i_eg}\right)\delta_{j_ci_c}\Big]\Delta t +$$
$$+\delta_{j_ei_e}\frac{\Delta t}{2}\Big[\Big(\gamma_R e^{-2i\varphi}\delta_{j_{k-l}i_{k-l}-2}\sqrt{i_{k-l}(i_{k-l}-1)}\,\delta_{j_ki_k} + \sqrt{\gamma_L\gamma_R}e^{-i\varphi}2\,\delta_{j_{k-l}i_{k-l}-1}\delta_{j_ki_k-1}\sqrt{i_ki_{k-l}} + \gamma_L\delta_{j_ki_k-2}\sqrt{i_k(i_k-1)}\Big)\delta_{j_ci_c+2}\sqrt{(i_c+1)(i_c+2)} -$$
$$-\Big(\gamma_R\delta_{j_{k-l}i_{k-l}}\delta_{j_ki_k}(i_{k-l}+1) + \sqrt{\gamma_L\gamma_R}\left\{\delta_{j_{k-l}i_{k-l}-1}\delta_{j_ki_k+1}\sqrt{i_{k-l}(i_k+1)}\,e^{-i\varphi} + \delta_{j_ki_k-1}\delta_{j_{k-l}i_{k-l}+1}\sqrt{i_k(i_{k-l}+1)}\,e^{i\varphi}\right\} + \gamma_L\delta_{j_{k-l}i_{k-l}}\delta_{j_ki_k}(i_k+1)\Big)\delta_{j_ci_c}\,i_c -$$
$$-\Big(\gamma_R\delta_{j_{k-l}i_{k-l}}\delta_{j_ki_k}\,i_{k-l} + \sqrt{\gamma_L\gamma_R}\left\{e^{i\varphi}\delta_{j_{k-l}i_{k-l}+1}\delta_{j_ki_k-1}\sqrt{i_k(i_{k-l}+1)} + e^{-i\varphi}\delta_{j_ki_k+1}\delta_{j_{k-l}i_{k-l}-1}\sqrt{i_{k-l}(i_k+1)}\right\} + \gamma_L\delta_{j_{k-l}i_{k-l}}\delta_{j_ki_k}\,i_k\Big)\delta_{j_ci_c}(i_c+1) +$$
$$+\Big(\gamma_R e^{2i\varphi}\delta_{j_{k-l}i_{k-l}+2}\delta_{j_ki_k}\sqrt{(i_{k-l}+1)(i_{k-l}+2)} + \sqrt{\gamma_L\gamma_R}\,2e^{i\varphi}\delta_{j_{k-l}i_{k-l}+1}\delta_{j_ki_k+1}\sqrt{(i_k+1)(i_{k-l}+1)} + \gamma_L\delta_{j_ki_k+2}\sqrt{(i_k+1)(i_k+2)}\Big)\delta_{j_ci_c-2}\sqrt{i_c(i_c-1)}\Big]\Big\}$$

Evolving the wave function in MPS representation:

$$U^{i_c, i_e, \{i_p\}}_{j_c, j_e, \{j_p\}}\,\Psi_{i_c, i_e, \{i_p\}}(0) = \Psi_{\underbrace{j_c, j_e}_{A_c \otimes A_e}, \{j_p\}}$$

diagonalization → diagonalized Hamiltonian gives only 1 index for the system

$$\Psi(0) = A_\infty \otimes \ldots \otimes A_1 \otimes A_0 \otimes A_S \otimes A_{-1} \otimes \ldots \otimes A_{-l+1} \otimes \underset{\tau}{A_{-l}} \otimes A_{-l-1} \otimes \ldots \otimes A_{-\infty}$$

tensor product of vectors

Basis for the system Hamiltonian:

$$\hbar\Delta_c c^+c + \hbar\Delta_e \sigma_+\sigma_- + \hbar g\,(c^+\sigma_- + c\sigma_+) + \frac{\hbar\Omega_c}{2}(c+c^+) + \frac{\hbar\Omega_e}{2}(\sigma_- + \sigma_+)$$

i_s	i_c i_e	
0	$\lvert 0\rangle\lvert g\rangle$	$\rightarrow \frac{\hbar\Omega_c}{2}\lvert 1\rangle\lvert g\rangle + \frac{\hbar\Omega_e}{2}\lvert 0\rangle\lvert e\rangle$
1	$\lvert 0\rangle\lvert e\rangle$	$\rightarrow \hbar\Delta_e\lvert 0\rangle\lvert e\rangle + \hbar g\lvert 1\rangle\lvert g\rangle + \frac{\hbar\Omega_c}{2}\lvert 1\rangle\lvert e\rangle + \frac{\hbar\Omega_e}{2}\lvert 0\rangle\lvert g\rangle$
2	$\lvert 1\rangle\lvert g\rangle$	$\rightarrow \hbar\Delta_c\lvert 1\rangle\lvert g\rangle + \hbar g\lvert 0\rangle\lvert e\rangle + \frac{\hbar\Omega_c}{2}(\lvert 2\rangle\lvert g\rangle + \lvert 0\rangle\lvert g\rangle) + \frac{\hbar\Omega_e}{2}\lvert 1\rangle\lvert e\rangle$
3	$\lvert 1\rangle\lvert e\rangle$	$\rightarrow \hbar(\Delta_c+\Delta_e)\lvert 1\rangle\lvert e\rangle + \hbar g\lvert 2\rangle\lvert g\rangle + \frac{\hbar\Omega_c}{2}(\lvert 2\rangle\lvert e\rangle + \lvert 0\rangle\lvert e\rangle) + \frac{\hbar\Omega_e}{2}\lvert 1\rangle\lvert g\rangle$
4	$\lvert 2\rangle\lvert g\rangle$	$\rightarrow \hbar\Delta_c\cdot 2\lvert 2\rangle\lvert g\rangle + \hbar g\sqrt{2}\lvert 1\rangle\lvert e\rangle + \frac{\hbar\Omega_c}{2}(\sqrt{2}\lvert 1\rangle\lvert g\rangle + \sqrt{3}\lvert 3\rangle\lvert g\rangle) + \frac{\hbar\Omega_e}{2}\lvert 2\rangle\lvert e\rangle$
5	$\lvert 2\rangle\lvert e\rangle$	$\rightarrow \hbar(2\Delta_c+\Delta_e)\lvert 2\rangle\lvert e\rangle + \hbar g\sqrt{3}\lvert 3\rangle\lvert g\rangle + \frac{\hbar\Omega_c}{2}(\sqrt{2}\lvert 1\rangle\lvert e\rangle + \sqrt{3}\lvert 3\rangle\lvert e\rangle) + \frac{\hbar\Omega_e}{2}\lvert 2\rangle\lvert g\rangle$
6	$\lvert 3\rangle\lvert g\rangle$	$\rightarrow \hbar 3\Delta_c\lvert 3\rangle\lvert g\rangle + \hbar g\sqrt{3}\lvert 2\rangle\lvert e\rangle + \frac{\hbar\Omega_c}{2}(\sqrt{3}\lvert 2\rangle\lvert g\rangle + \sqrt{4}\lvert 4\rangle\lvert g\rangle) + \frac{\hbar\Omega_e}{2}\lvert 3\rangle\lvert e\rangle$
7	$\lvert 3\rangle\lvert e\rangle$	$\rightarrow \hbar(3\Delta_c+\Delta_e)\lvert 3\rangle\lvert e\rangle + \hbar g\sqrt{4}\lvert 4\rangle\lvert g\rangle + \frac{\hbar\Omega_c}{2}(\sqrt{3}\lvert 2\rangle\lvert e\rangle + \sqrt{4}\lvert 4\rangle\lvert e\rangle) + \frac{\hbar\Omega_e}{2}\lvert 3\rangle\lvert g\rangle$
8	$\lvert 4\rangle\lvert g\rangle$	$\rightarrow \hbar 4\Delta_c\lvert 4\rangle\lvert g\rangle + \hbar g\sqrt{4}\lvert 3\rangle\lvert e\rangle + \frac{\hbar\Omega_c}{2}(\sqrt{4}\lvert 3\rangle\lvert g\rangle + \sqrt{5}\lvert 5\rangle\lvert g\rangle) + \frac{\hbar\Omega_e}{2}\lvert 4\rangle\lvert e\rangle$

$i_c = \mathrm{int}\left(\frac{i_s}{2}\right)$ $\quad i_e = \mathrm{mod}_2(i_s)$

$i_c = \mathrm{int}\left(\frac{i_s}{2}\right)$ $\quad i_e = \mathrm{mod}_2(i_s)$

$$\langle j_s, \{j_p\} \,|\, U(t_{k+1}, t_k) \,|\, i_s, \{i_p\}\rangle = \delta_{j_{-\infty} i_{-\infty}} \cdots \delta_{j_{k-e-1} i_{k-e-1}} \delta_{j_{k-e+1} i_{k-e+1}} \cdots \delta_{j_{k-1} i_{k-1}} \delta_{j_{k+1} i_{k+1}} \cdots \delta_{j_\infty i_\infty} \cdot$$

$$\cdot\Big\{ \delta_{j_s i_s}\delta_{j_k i_k}\delta_{j_{k-e} i_{k-e}} +$$

$$+(-\sqrt{\Delta t})\Big[\delta_{j_k i_k}\sqrt{\gamma_R}e^{-i\varphi}\delta_{j_{k-e} i_{k-e}-1}\,\delta_{j_s i_s+2}\sqrt{i_{k-e}(i_c+1)} + \delta_{j_{k-e} i_{k-e}}\sqrt{\gamma_L}\,\delta_{j_k i_k-1}\,\delta_{j_s i_s+2}\sqrt{i_k(i_c+1)} -$$

$$-\delta_{j_k i_k}\sqrt{\gamma_R}e^{i\varphi}\delta_{j_{k-e} i_{k-e}+1}\,\delta_{j_s i_s-2}\sqrt{(i_{k-e}+1)\,i_c} + \delta_{j_{k-e} i_{k-e}}\sqrt{\gamma_L}\,\delta_{j_k i_k+1}\,\delta_{j_s, i_s-2}\sqrt{(i_k+1)\,i_c}\Big] +$$

$$-i\,\delta_{j_k i_k}\delta_{j_{k-e} i_{k-e}}\Big[\mathrm{int}\left(\tfrac{i_s}{2}\right)\Delta_c + \mathrm{mod}_2(i_s)\;\Delta_e\Big]\delta_{j_s, i_s} + g\Big[\mathrm{mod}_2(i_s)\sqrt{\mathrm{int}\left(\tfrac{i_s}{2}\right)+1}\,\delta_{j_s, i_s+1} + \mathrm{mod}_2(i_s+1)\sqrt{\mathrm{int}\left(\tfrac{i_s}{2}\right)}\,\delta_{j_s, i_s-1}\Big] +$$

$$+\frac{\Omega_c}{2}\Big[\sqrt{\mathrm{int}\left(\tfrac{i_s}{2}\right)}\,\delta_{j_s, i_s-2} + \sqrt{\mathrm{int}\left(\tfrac{i_s}{2}\right)+1}\,\delta_{j_s, i_s+2}\Big] + \frac{\Omega_e}{2}\Big[\mathrm{mod}_2(i_s)\,\delta_{j_s, i_s-1} + \mathrm{mod}_2(i_s+1)\,\delta_{j_s, i_s+1}\Big]\Delta t +$$

$$+\tfrac{1}{2}\Delta t\Big[\Big(\gamma_R e^{-2i\varphi}\delta_{j_{k-e} i_{k-e}-2}\sqrt{i_{k-e}(i_{k-e}-1)}\,\delta_{j_k i_k} + \sqrt{\gamma_L\gamma_R}\,e^{-i\varphi}2\,\delta_{j_{k-e} i_{k-e}-1}\delta_{j_k i_k-1}\sqrt{i_k i_{k-e}} + \gamma_L\,\delta_{j_k i_k-2}\sqrt{i_k(i_k-1)}\Big)\delta_{j_s i_s+4}\sqrt{(i_c+1)(i_c+2)} -$$

$$-\Big(\gamma_R\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}(i_{k-e}+1) + \sqrt{\gamma_L\gamma_R}\Big\{\delta_{j_{k-e} i_{k-e}-1}\delta_{j_k i_k+1}\sqrt{i_{k-e}(i_k+1)}\,e^{-i\varphi} + \delta_{j_k i_k-1}\delta_{j_{k-e} i_{k-e}+1}\sqrt{i_k(i_{k-e}+1)}\,e^{i\varphi}\Big\} + \gamma_L\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}(i_k+1)\Big)\delta_{j_s i_s}\,i_c -$$

$$-\Big(\gamma_R\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}\,i_{k-e} + \sqrt{\gamma_L\gamma_R}\Big\{e^{i\varphi}\delta_{j_{k-e} i_{k-e}+1}\delta_{j_k i_k-1}\sqrt{i_k(i_{k-e}+1)} + e^{-i\varphi}\delta_{j_k i_k+1}\delta_{j_{k-e} i_{k-e}-1}\sqrt{i_{k-e}(i_k+1)}\Big\} + \gamma_L\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}\,i_k\Big)\delta_{j_s i_s}(i_c+1) +$$

$$+\Big(\gamma_R e^{2i\varphi}\delta_{j_{k-e} i_{k-e}+2}\delta_{j_k i_k}\sqrt{(i_{k-e}+1)(i_{k-e}+2)} + \sqrt{\gamma_L\gamma_R}\,2e^{i\varphi}\delta_{j_{k-e} i_{k-e}+1}\,\delta_{j_k i_k+1}\sqrt{(i_k+1)(i_{k-e}+1)} + \gamma_L\,\delta_{j_k i_k+2}\sqrt{(i_k+1)(i_k+2)}\Big)\delta_{j_s i_s-4}\sqrt{i_c(i_c-1)}\Big]\Big\}$$

SWAP gate

$\lvert 0\rangle\lvert 0\rangle$, $\lvert 0\rangle\lvert 1\rangle$, $\lvert 0\rangle\lvert 2\rangle$, $\lvert 0\rangle\lvert 3\rangle$, $\lvert 0\rangle\lvert 4\rangle$

$\lvert 1\rangle\lvert 0\rangle$, $\lvert 1\rangle\lvert 1\rangle$, $\lvert 1\rangle\lvert 2\rangle$, $\lvert 1\rangle\lvert 3\rangle$, $\lvert 1\rangle\lvert 4\rangle$

$\lvert 2\rangle\lvert 0\rangle$, $\lvert 2\rangle\lvert 1\rangle$, $\lvert 2\rangle\lvert 2\rangle$, $\lvert 2\rangle\lvert 3\rangle$, $\lvert 2\rangle\lvert 4\rangle$

$\lvert 3\rangle\lvert 0\rangle$, $\lvert 3\rangle\lvert 1\rangle$, $\lvert 3\rangle\lvert 2\rangle$, $\lvert 3\rangle\lvert 3\rangle$, $\lvert 3\rangle\lvert 4\rangle$

$\lvert 4\rangle\lvert 0\rangle$, $\lvert 4\rangle\lvert 1\rangle$, $\lvert 4\rangle\lvert 2\rangle$, $\lvert 4\rangle\lvert 3\rangle$, $\lvert 4\rangle\lvert 4\rangle$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ & & & & \\ & & & & \end{pmatrix}$$

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ & \lvert 0\rangle\lvert 0\rangle & \lvert 0\rangle\lvert 1\rangle & \lvert 1\rangle\lvert 0\rangle & \lvert 1\rangle\lvert 1\rangle \\ \hline \lvert 0\rangle\lvert 0\rangle & 1 & 0 & 0 & 0 \\ \lvert 0\rangle\lvert 1\rangle & 0 & 0 & 1 & 0 \\ \lvert 1\rangle\lvert 0\rangle & 0 & 1 & 0 & 0 \\ \lvert 1\rangle\lvert 1\rangle & 0 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccccc} \lvert 0\rangle\lvert 0\rangle & \lvert 0\rangle\lvert 1\rangle & \lvert 1\rangle\lvert 0\rangle & \lvert 1\rangle\lvert 1\rangle & \lvert 1\rangle\lvert 2\rangle & \lvert 2\rangle\lvert 1\rangle & \lvert 2\rangle\lvert 2\rangle & \lvert 0\rangle\lvert 2\rangle & \lvert 2\rangle\lvert 0\rangle \end{array}$$
$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$\lvert 4\rangle\lvert 4\rangle$
5 diag & $\overset{10}{\frac{5\cdot 4}{2}}$ pair

3 diag & $\frac{3\cdot 2}{2} = 3$ pair

The code

1) Creation of the wave function

• The wave function is initially a tensor product of the states in the different time bins. The objective of the MPS representation is to keep these time bins separate, recording the connections between them. In this way the created tensors can be stored separately on the hard drive.

• Depending on the order of precision, the photon limit in each environment bin is given. E.g. for $\mathcal{O}(\Delta t^{3/2})$ it is 4. $\Rightarrow$ basis size: 5

• Simplest system: 1 TLS with feedback. System basis size: 2

$$H_{SYS,TLS} = \hbar\omega_{TLS}\sigma_+\sigma_- + \hbar\Omega_{TLS}(e^{-i\omega_L t}\sigma_+ + e^{i\omega_L t}\sigma_-)$$

• Number of time bin: "$\gamma\tau >> 1$" + "$\tau = \Delta t \cdot e$"

2.) SWAP: Actually it might just be easier to swap the indices in numpy.

• The time bins to be SWAP-ed have to be merged into a common state vector.

• SWAP matrix should be built as n = max num of photons in a bin:

$n+1$ diagonal & $\frac{(n+1)n}{2}$ pairs $\quad \left(n+1+2\frac{(n+1)n}{2} = n^2+2n+1 = (n+1)^2\right)$

• Act on the common state vector with the SWAP

• Decompose state vectors

e.g. $\begin{matrix}|0\rangle\\|1\rangle\\|2\rangle\end{matrix}\Big(\;\Big) \otimes \begin{matrix}|0\rangle\\|1\rangle\\|2\rangle\end{matrix}\Big(\;\Big) = \begin{matrix}|00\rangle & |01\rangle & |02\rangle\\|10\rangle & |11\rangle & |12\rangle\\|20\rangle & |21\rangle & |22\rangle\end{matrix}\Big(\qquad\Big) \Rightarrow \begin{matrix}|00\rangle\\|01\rangle\\|02\rangle\\|10\rangle\\|11\rangle\\|12\rangle\\|20\rangle\\|21\rangle\\|22\rangle\end{matrix}\Big\|\;\Big\|$

$$\text{SWAP} \quad \begin{array}{c} \\ |00\rangle\\|01\rangle\\|02\rangle\\|10\rangle\\|11\rangle\\|12\rangle\\|20\rangle\\|21\rangle\\|22\rangle \end{array} \begin{array}{c} \begin{matrix} |00\rangle & |01\rangle & |02\rangle & |10\rangle & |11\rangle & |12\rangle & |20\rangle & |21\rangle & |22\rangle \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

$$\delta_{00} + \delta_{13} + \delta_{26} + \delta_{31} + \delta_{44} + \delta_{57} + \delta_{62} + \delta_{75} + \delta_{88}$$

From this, the general vector state is:

MERGED WAVE FUNCTION

$|00\rangle, |01\rangle, \ldots, |0n\rangle, |10\rangle, \ldots |1,n\rangle, |2,0\rangle \ldots \quad \ldots |n-1,n\rangle, |n,0\rangle \ldots |n,n\rangle$

$$\sum_{i=0}^{n}\sum_{j=0}^{n} \delta_{(n+1)i+j,\,(n+1)j+i}$$

SWAP OPERATOR

$n=2 \Rightarrow \delta_{00} + \delta_{13} + \delta_{26} + \delta_{31} + \delta_{44} + \delta_{57} + \delta_{62} + \delta_{75} + \delta_{88}$ ✓

Check: $n=3$

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$\|00\rangle$	$\|01\rangle$	$\|02\rangle$	$\|03\rangle$	$\|10\rangle$	$\|11\rangle$	$\|12\rangle$	$\|13\rangle$	$\|20\rangle$	$\|21\rangle$	$\|22\rangle$	$\|23\rangle$	$\|30\rangle$	$\|31\rangle$	$\|32\rangle$	$\|33\rangle$

$$\delta_{00} + \delta_{14} + \delta_{28} + \delta_{3,12} + \delta_{41} + \delta_{55} + \delta_{69} + \delta_{7,13} + \delta_{8,2} + \delta_{96} + \delta_{10,10} + \delta_{11,14} + \delta_{12,3} + \delta_{13,7} + \delta_{14,11} + \delta_{15,15}$$

$$\delta_{00} + \delta_{41} + \delta_{82} + \delta_{12,3} + \delta_{1,4} + \delta_{55} + \delta_{96} + \delta_{13,7} + \delta_{2,8} + \delta_{6,9} + \delta_{10,10} + \delta_{14,11} + \delta_{3,12} + \delta_{7,13} + \delta_{11,14} + \delta_{15,15}$$

Decomposing: convert back to $(n+1)\otimes(n+1) \rightarrow$ SVD

$i_1 \cdot i_2 \cdot i_3$ $\quad \begin{pmatrix} \overset{000}{0} & \overset{001}{0} & \overset{002}{0} \\ \overset{010}{0} & \overset{011}{0} & \overset{012}{0} \end{pmatrix}, \begin{pmatrix} \overset{100}{0} & \overset{101}{0} & \overset{102}{0} \\ \overset{110}{0} & \overset{111}{1} & \overset{112}{2} \end{pmatrix}, \begin{pmatrix} \overset{200}{0} & \overset{201}{0} & \overset{202}{0} \\ \overset{210}{0} & \overset{211}{2} & \overset{212}{4} \end{pmatrix}$

$\overset{i_2=0}{0},0,0,\overset{i_2=1}{0},0,0,\overset{i_2=0}{0},0,0,\overset{i_2=1}{0},1,2,\overset{i_2=0}{0},0,0,\overset{i_2=1}{0},2,4$

$$H'_{SYS,TLS} = \hbar\omega_{TLS}\sigma_+\sigma_- + \hbar\Omega_{TLS}(e^{-i\omega_L t}\sigma_+ + e^{i\omega_L t}\sigma_-)$$

$$H'_{fb} = \int_B d\omega\, \hbar\omega\, b^+(\omega)b(\omega) + i\hbar\int_B d\omega \left[(\kappa_R(\omega)e^{i\omega L/c_0} - \kappa_L(\omega)e^{-i\omega L/c_0})\, b^+(\omega)\sigma_- - (\kappa_R^*(\omega)e^{-i\omega L/c_0} - \kappa_L^*(\omega)e^{i\omega L/c_0}) b(\omega)\sigma_+\right]$$

$$H_{SYS,TLS} = \hbar\Delta_T\sigma_+\sigma_- + \hbar\Omega_{TLS}(\sigma_+ + \sigma_-)$$

$$H_{fb} = -i\hbar\left\{\left[\sqrt{\gamma_R}\, b(t-\tau)e^{-i\varphi} + \sqrt{\gamma_L}\, b(t)\right]\sigma_+ - \left[\sqrt{\gamma_R}\, b^+(t-\tau)e^{i\varphi} + \sqrt{\gamma_L}\, b^+(t)\right]\sigma_-\right\}$$

$$\langle j_{TLS}, \{j\} | U(t_{k+1}, t_k) | i_{TLS}, \{i_p\}\rangle = \delta_{j_{-\infty} i_{-\infty}} \cdots \delta_{j_{k-e-1} i_{k-e-1}} \delta_{j_{k-e+1} i_{k-e+1}} \cdots \delta_{j_{k-1} i_{k-1}} \delta_{j_{k+1} i_{k+1}} \cdots \delta_{j_\infty i_\infty} \cdot$$

$$\cdot \Big\{ \delta_{j_T i_T} \delta_{j_k i_k} \delta_{j_{k-e} i_{k-e}} +$$

$$+(-\sqrt{\Delta t})\Big[\delta_{j_k i_k}\sqrt{\gamma_R}\, e^{-i\varphi}\delta_{j_{k-e} i_{k-e}-1}\sqrt{i_{k-e}}\,\delta_{i_T g}\delta_{j_T e} + \delta_{j_{k-e} i_{k-e}}\sqrt{\gamma_L}\,\delta_{j_k i_k-1}\sqrt{i_k}\,\delta_{i_T g}\delta_{j_T e} -$$

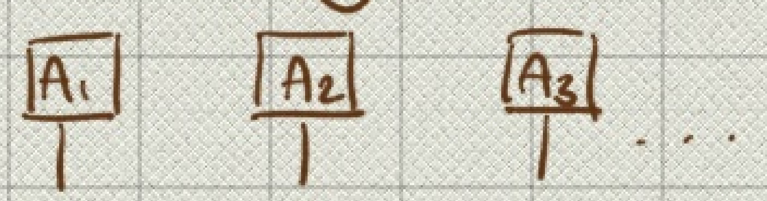
$$-\delta_{j_k i_k}\sqrt{\gamma_R}\, e^{i\varphi}\delta_{j_{k-e} i_{k-e}+1}\sqrt{i_{k-e}+1}\,\delta_{i_T e}\delta_{j_T g} - \delta_{j_{k-e} i_{k-e}}\sqrt{\gamma_L}\,\delta_{j_k i_k+1}\sqrt{i_k+1}\,\delta_{i_T e}\delta_{j_T g}\Big] +$$

$$-i\delta_{j_k i_k}\delta_{j_{k-e} i_{k-e}}\left[\Delta_T\delta_{j_T e}\delta_{i_T e} + \Omega_{TLS}(\delta_{j_T e}\delta_{i_T g} + \delta_{j_T g}\delta_{i_T e})\right]\Delta t +$$

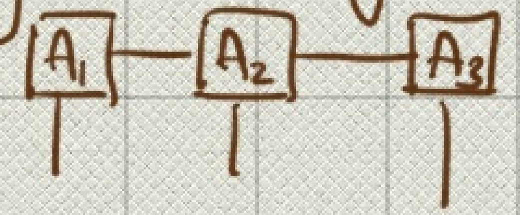
$$-\frac{1}{2}\Big[\Big(\gamma_R\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}(i_{k-e}+1) + \sqrt{\gamma_L\gamma_R}\{\delta_{j_{k-e} i_{k-e}-1}\delta_{j_k i_k+1}\sqrt{i_{k-e}(i_k+1)}\,e^{-i\varphi} + \delta_{j_k i_k-1}\delta_{j_{k-e} i_{k-e}+1}\sqrt{i_k(i_{k-e}+1)}\,e^{i\varphi}\} + \gamma_L\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}(i_k+1)\Big)\delta_{i_T e}\delta_{j_T e} +$$

$$+\Big(\gamma_R\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}\, i_{k-e} + \sqrt{\gamma_L\gamma_R}\{e^{i\varphi}\delta_{j_{k-e} i_{k-e}+1}\delta_{j_k i_k-1}\sqrt{i_k(i_{k-e}+1)} + e^{-i\varphi}\delta_{j_k i_k+1}\delta_{j_{k-e} i_{k-e}-1}\sqrt{i_{k-e}(i_k+1)}\} + \gamma_L\,\delta_{j_{k-e} i_{k-e}}\delta_{j_k i_k}\, i_k\Big)\delta_{i_T g}\delta_{j_T g}\Big]\Delta t\Big\}$$

Initially



After the first time step



$U = \sum_i U_i$

$\sum_i U'_{i1} \otimes U'_{i2} \otimes U'_{i3}$

quantity with 3 indices

formulate as a matrix with S up front

$$M_S = -\frac{i}{\hbar} H_S \Delta t = -i\Delta t\left\{\Delta_e \sigma_+\sigma_- + \Omega_e(\sigma_- + \sigma_+)\right\} \qquad M_B = -\frac{i}{\hbar}\int_{t_k}^{t_{k+1}} H_{fB}(t)\,dt$$

$$H_{fB}(t) = -i\hbar\left\{\left[\sqrt{\gamma_R}\, b(t-\tau)e^{-i\varphi} + \sqrt{\gamma_L}\, b(t)\right]\sigma_+ - \left[\sqrt{\gamma_R}\, b^+(t-\tau)e^{i\varphi} + \sqrt{\gamma_L}\, b^+(t)\right]\sigma_-\right\}$$

$$H_S = \hbar\Delta_e\sigma_+\sigma_- + \hbar\Omega_e(\sigma_- + \sigma_+)$$

$$M_B = -\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\underbrace{\int_{t_k}^{t_{k+1}} b(t-\tau)dt}_{\tau = l\Delta t} + \sqrt{\gamma_L}\underbrace{\int_{t_k}^{t_{k+1}} b(t)dt}_{\Delta B(t_k)}\right]\sigma_+ - \left[\sqrt{\gamma_R}e^{i\varphi}\underbrace{\int_{t_k}^{t_{k+1}} b^+(t-\tau)dt}_{\Delta B^+(t_{k-l})} + \sqrt{\gamma_L}\underbrace{\int_{t_k}^{t_{k+1}} b^+(t)dt}_{\Delta B^+(t_k)}\right]\sigma_-\right\}$$

$$\int_{t_{k-l}}^{t_{k-l+1}} b(t)dt = \Delta B(t_{k-l}) \qquad \left[\Delta B(t_k), \Delta B^+(t_j)\right] = \int_{t_k}^{t_{k+1}}\int_{t_j}^{t_{j+1}} \left[b(t), b^+(t')\right]^{\delta(t-t')} dt'dt = \int_{t_k}^{t_{k+1}} dt\, \delta_{jk} = \Delta t\,\delta_{jk}$$

$$M_B = -\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\Delta B(t_{k-l}) + \sqrt{\gamma_L}\Delta B(t_k)\right]\sigma_+ - \left[\sqrt{\gamma_R}\Delta B^+(t_{k-l})e^{i\varphi} + \sqrt{\gamma_L}\Delta B^+(t_k)\right]\sigma_-\right\}$$

$$M_B^2 = \left[\gamma_R e^{-2i\varphi}\Delta B(t_{k-l})\Delta B(t_{k-l}) + \sqrt{\gamma_R\gamma_L}\left(\Delta B(t_{k-l})\Delta B(t_k) + \Delta B(t_k)\Delta B(t_{k-l})\right)e^{-i\varphi} + \gamma_L \Delta B(t_k)\Delta B(t_k)\right]\underbrace{\sigma_+\sigma_+}_{0} -$$
$$-\left[\gamma_R \Delta B(t_{k-l})\Delta B^+(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(\Delta B(t_{k-l})\Delta B^+(t_k)e^{-i\varphi} + \Delta B(t_k)\Delta B^+(t_{k-l})e^{i\varphi}\right) + \gamma_L \Delta B(t_k)\Delta B^+(t_k)\right]\sigma_+\sigma_- -$$
$$-\left[\gamma_R \Delta B^+(t_{k-l})\Delta B(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(e^{i\varphi}\Delta B^+(t_{k-l})\Delta B(t_k) + e^{-i\varphi}\Delta B^+(t_k)\Delta B(t_{k-l})\right) + \gamma_L \Delta B^+(t_k)\Delta B(t_k)\right]\sigma_-\sigma_+ +$$
$$+\left[\gamma_R e^{2i\varphi}\Delta B^+(t_{k-l})\Delta B^+(t_{k-l}) + \sqrt{\gamma_L\gamma_R}\left(\Delta B^+(t_{k-l})\Delta B^+(t_k) + \Delta B^+(t_k)\Delta B^+(t_{k-l})\right)e^{i\varphi} + \gamma_L \Delta B^+(t_k)\Delta B^+(t_k)\right]\underbrace{\sigma_-^2}_{0}$$

$$\langle\sigma_+\sigma_+\sigma_+\rangle = \langle\sigma_+\sigma_+\sigma_-\rangle = \langle\sigma_+\sigma_-\sigma_-\rangle = \langle\sigma_-\sigma_+\sigma_+\rangle = \langle\sigma_-\sigma_-\sigma_+\rangle = \langle\sigma_-\sigma_-\sigma_-\rangle = 0$$

$2^3 = 8$ 2 left : $\langle\sigma_+\sigma_-\sigma_+\rangle$ & $\langle\sigma_-\sigma_+\sigma_-\rangle$

$\delta_{g i_T}\delta_{e j_T}$ $\delta_{g j_T}\delta_{e i_T}$

$$\Delta B(t_k)|i_k\rangle = \frac{\Delta B(t_k)\left[\Delta B^+(t_k)\right]^{i_k}}{\sqrt{i_k!\,\Delta t^{i_k}}}|vac\rangle = \frac{i_k\Delta t\,(\Delta B^+(t_k))^{i_k-1}}{\sqrt{i_k!\,\Delta t^{i_k}}}|vac\rangle = \sqrt{i_k\Delta t}\,|i_k-1\rangle$$

$$M_B^3 = \Big\{\gamma_R^{3/2}e^{-i\varphi}\Delta B(t_{k-l})\Delta B^+(t_{k-l})\Delta B(t_{k-l}) + \gamma_L^{3/2}\Delta B(t_k)\Delta B^+(t_k)\Delta B(t_k) +$$
$$+\gamma_R\sqrt{\gamma_L}\left[\Delta B(t_{k-l})\Delta B^+(t_{k-l})\Delta B(t_k) + \Delta B(t_k)\Delta B^+(t_{k-l})\Delta B(t_{k-l}) + \Delta B(t_{k-l})\Delta B^+(t_k)\Delta B(t_{k-l})\cdot e^{-2i\varphi}\right] +$$
$$+\gamma_L\sqrt{\gamma_R}\left[\Delta B(t_k)\Delta B^+(t_k)\Delta B(t_{k-l})e^{-i\varphi} + \Delta B(t_{k-l})\Delta B^+(t_k)\Delta B(t_k)e^{-i\varphi} + \Delta B(t_k)\Delta B^+(t_{k-l})\Delta B(t_k)e^{i\varphi}\right]\Big\}\sigma_+ +$$
$$-\Big\{\gamma_R^{3/2}e^{i\varphi}\Delta B^+(t_{k-l})\Delta B(t_{k-l})\Delta B^+(t_{k-l}) + \gamma_L^{3/2}\Delta B^+(t_k)\Delta B(t_k)\Delta B^+(t_k) +$$
$$+\gamma_R\sqrt{\gamma_L}\left[\Delta B^+(t_{k-l})\Delta B(t_{k-l})\Delta B^+(t_k) + \Delta B^+(t_k)\Delta B(t_{k-l})\Delta B^+(t_{k-l}) + \Delta B^+(t_{k-l})\Delta B(t_k)\Delta B^+(t_{k-l})e^{2i\varphi}\right] +$$
$$+\gamma_L\sqrt{\gamma_R}\left[\Delta B^+(t_k)\Delta B(t_k)\Delta B^+(t_{k-l})e^{i\varphi} + \Delta B^+(t_{k-l})\Delta B(t_k)\Delta B^+(t_k)e^{i\varphi} + \Delta B^+(t_k)\Delta B(t_{k-l})\Delta B^+(t_k)e^{-i\varphi}\right]\Big\}\sigma_-$$

$$M_S M_B = i\left(\Delta_e\sigma_+\sigma_- + \Omega_e(\sigma_- + \sigma_+)\right)\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\Delta B(t_{k-l}) + \sqrt{\gamma_L}\Delta B(t_k)\right]\sigma_+ - \left[\sqrt{\gamma_R}e^{i\varphi}\Delta B^+(t_{k-l}) + \sqrt{\gamma_L}\Delta B^+(t_k)\right]\sigma_-\right\} =$$
$$= i\left\{\left[\Delta_e\sigma_+ + \Omega_e\sigma_-\sigma_+\right]\left[\sqrt{\gamma_R}e^{-i\varphi}\Delta B(t_{k-l}) + \sqrt{\gamma_L}\Delta B(t_k)\right] - \Omega_e\sigma_+\sigma_-\left[\sqrt{\gamma_R}e^{i\varphi}\Delta B^+(t_{k-l}) + \sqrt{\gamma_L}\Delta B^+(t_k)\right]\right\}$$

$$M_B M_S = i\Delta t\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\Delta B(t_{k-l}) + \sqrt{\gamma_L}\Delta B(t_k)\right]\sigma_+ - \left[\sqrt{\gamma_R}e^{i\varphi}\Delta B^+(t_{k-l}) + \sqrt{\gamma_L}\Delta B^+(t_k)\right]\sigma_-\right\}\left\{\Delta_e\sigma_+\sigma_- + \Omega_e(\sigma_- + \sigma_+)\right\} =$$
$$= i\Delta t\left\{\Omega_e\left[\sqrt{\gamma_R}e^{-i\varphi}\Delta B(t_{k-l}) + \sqrt{\gamma_L}\Delta B(t_k)\right]\sigma_+\sigma_- - \left[\sqrt{\gamma_R}e^{i\varphi}\Delta B^+(t_{k-l}) + \sqrt{\gamma_L}\Delta B^+(t_k)\right]\left(\Delta_e\sigma_- + \Omega_e\sigma_-\sigma_+\right)\right\}$$

$$\langle j_T, j_{\{k\}}|U(t_k, t_{k+1})|i_T, i_{\{k\}}\rangle = \delta_{j_T i_T}\delta_{j_k i_k} -$$
$$-\left\{\left[\sqrt{\gamma_R}e^{-i\varphi}\sqrt{i_{k-l}}\,\delta_{j_{k-l} i_{k-l}-1} + \sqrt{\gamma_L}\sqrt{i_k}\,\delta_{j_k i_k-1}\right]\delta_{j_T e}\delta_{i_T g} - \left[\sqrt{\gamma_R}e^{i\varphi}\sqrt{i_{k-l}+1}\,\delta_{j_{k-l} i_{k-l}+1} + \sqrt{\gamma_L}\sqrt{i_k+1}\,\delta_{j_k i_k+1}\right]\delta_{j_T g}\delta_{i_T e}\right\}\sqrt{\Delta t} -$$
$$-i\left\{\Delta_e\delta_{j_T e}\delta_{i_T e} + \Omega_e\left(\delta_{j_T e}\delta_{i_T g} + \delta_{i_T e}\delta_{j_T g}\right)\right\}\Delta t - \Big[\gamma_R(i_{k-l}+1)\delta_{i_{k-l} j_{k-l}} + \sqrt{\gamma_R\gamma_L}\Big(\sqrt{(i_k+1)i_{k-l}}\,e^{-i\varphi}\delta_{j_{k-l} i_{k-l}-1}\delta_{j_k i_k+1} + \sqrt{(i_{k-l}+1)i_k}\,e^{i\varphi}\delta_{j_{k-l} i_{k-l}+1}\delta_{j_k i_k-1}\Big) +$$
$$+\gamma_L(i_k+1)\delta_{j_k i_k}\Big]\frac{1}{2}\delta_{i_T e}\delta_{j_T e}\Delta t - \frac{1}{2}\Big[\gamma_R i_{k-l}\delta_{j_{k-l} i_{k-l}} + \sqrt{\gamma_L\gamma_R}\Big(e^{i\varphi}\sqrt{(i_{k-l}+1)i_k}\,\delta_{j_{k-l} i_{k-l}+1}\delta_{j_k i_k-1} + e^{-i\varphi}\sqrt{(i_k+1)i_{k-l}}\,\delta_{j_{k-l} i_{k-l}-1}\delta_{j_k i_k+1}\Big) + \gamma_L i_k\Big]\delta_{j_T g}\delta_{i_T g}\Delta t +$$
$$+\frac{1}{6}\Big\{\gamma_R^{3/2}e^{-i\varphi}i_{k-l}^{3/2}\delta_{j_{k-l} i_{k-l}-1} + \gamma_L^{3/2}i_k^{3/2}\delta_{j_k i_k-1} + \gamma_R\sqrt{\gamma_L}\Big[\sqrt{i_k}(i_{k-l}+1)\delta_{j_k i_k-1}\delta_{j_{k-l} i_{k-l}} + \sqrt{i_k}\,i_{k-l}\delta_{j_k i_k-1}\delta_{j_{k-l} i_{k-l}} + \sqrt{(i_k+1)i_{k-l}(i_{k-l}-1)}\,e^{-2i\varphi}\delta_{j_{k-l} i_{k-l}-2}\delta_{j_k i_k+1}\Big] +$$
$$+\gamma_L\sqrt{\gamma_R}\Big[(i_k+1)\sqrt{i_{k-l}}\,\delta_{j_k i_k}\delta_{j_{k-l} i_{k-l}-1}e^{-i\varphi} + i_k\sqrt{i_{k-l}}\,\delta_{j_k i_k}\delta_{j_{k-l} i_{k-l}-1}e^{-i\varphi} + \sqrt{(i_{k-l}+1)i_k(i_k-1)}\,\delta_{j_k i_k-2}\delta_{j_{k-l} i_{k-l}+1}e^{i\varphi}\Big]\Big\}\delta_{j_T e}\delta_{i_T g}\Delta t^{3/2} -$$
$$-\frac{1}{6}\Big\{\gamma_R^{3/2}e^{i\varphi}(i_{k-l}+1)^{3/2}\delta_{j_{k-l} i_{k-l}+1} + \gamma_L^{3/2}(i_k+1)^{3/2}\delta_{j_k i_k+1} + \gamma_R\sqrt{\gamma_L}\Big[i_{k-l}\sqrt{i_k+1}\,\delta_{j_{k-l} i_{k-l}}\delta_{j_k i_k+1} + (i_{k-l}+1)\sqrt{i_k+1}\,\delta_{j_{k-l} i_{k-l}}\delta_{j_k i_k+1} + \sqrt{(i_{k-l}+1)(i_{k-l}+2)i_k}\,\delta_{j_{k-l} i_{k-l}+2}\delta_{j_k i_k-1}e^{2i\varphi}\Big] +$$
$$+\gamma_L\sqrt{\gamma_R}\Big[\sqrt{i_{k-l}+1}\,i_k\delta_{j_k i_k}\delta_{j_{k-l} i_{k-l}+1}e^{i\varphi} + \sqrt{i_{k-l}+1}(i_k+1)\delta_{j_{k-l} i_{k-l}+1}\delta_{j_k i_k}e^{i\varphi} + \sqrt{(i_k+1)(i_k+2)i_{k-l}}\,\delta_{j_k i_k+2}\delta_{j_{k-l} i_{k-l}-1}e^{-i\varphi}\Big]\Big\}\delta_{j_T g}\delta_{i_T e}\Delta t^{3/2} +$$
$$+\frac{1}{2}i\Big\{\left[\Delta_e\delta_{j_T e}\delta_{i_T g} + \Omega_e\delta_{j_T g}\delta_{i_T g}\right]\left[\sqrt{\gamma_R}e^{-i\varphi}\sqrt{i_{k-l}}\,\delta_{j_{k-l} i_{k-l}-1} + \sqrt{\gamma_L}\sqrt{i_k}\,\delta_{j_k i_k-1}\right] - \Omega_e\delta_{j_T e}\delta_{i_T e}\left[\sqrt{\gamma_R}e^{i\varphi}\sqrt{i_{k-l}+1}\,\delta_{j_{k-l} i_{k-l}+1} + \sqrt{\gamma_L}\sqrt{i_k+1}\,\delta_{j_k i_k+1}\right]\Big\}\Delta t^{3/2} +$$
$$+\frac{i}{2}\Big\{\Omega_e\left[\sqrt{\gamma_R}e^{-i\varphi}\delta_{j_{k-l}}\delta_{i_{k-l}-1}\sqrt{i_{k-l}} + \sqrt{\gamma_L}\,\delta_{j_k i_k-1}\sqrt{i_k}\right]\delta_{j_T e}\delta_{i_T e} - \left[\sqrt{\gamma_R}e^{i\varphi}\delta_{j_{k-l} i_{k-l}+1}\sqrt{i_{k-l}+1} + \sqrt{\gamma_L}\,\delta_{j_k i_k+1}\sqrt{i_k+1}\right]\left(\Delta_e\delta_{j_T g}\delta_{i_T e} + \Omega_e\delta_{j_T g}\delta_{i_T g}\right)\Big\}\Delta t^{3/2}$$

$$M_S = -\frac{i}{\hbar} H_S \Delta t = -i\Delta t \left\{ \Delta_e \hat{\sigma}_+ \hat{\sigma}_- + \Omega_e (\hat{\sigma}_- + \hat{\sigma}_+) \right\}$$

$$M_B = -\left\{ \left[\sqrt{\gamma_R}\, e^{-i\varphi} \underbrace{\int_{t_\ell}^{t_{\ell+1}} b(t-\tau)\,dt}_{\substack{\tau = e\Delta t \\ \Delta B(t_{\ell-e})}} + \sqrt{\gamma_L} \underbrace{\int_{t_\ell}^{t_{\ell+1}} b(t)\,dt}_{\Delta B(t_\ell)} \right] \hat{\sigma}_+ - \left[\sqrt{\gamma_R}\, e^{i\varphi} \underbrace{\int_{t_\ell}^{t_{\ell+1}} b^+(t-\tau)\,dt}_{\Delta B^+(t_{\ell-e})} + \sqrt{\gamma_L} \underbrace{\int_{t_\ell}^{t_{\ell+1}} b^+(t)\,dt}_{\Delta B^+(t_\ell)} \right] \hat{\sigma}_- \right\}$$

$$M_S^2 = -\Delta t^2 \left\{ \Delta_e^2 \hat{\sigma}_+ \hat{\sigma}_- + \Omega_e \Delta_e (\hat{\sigma}_+ + \hat{\sigma}_-) + \Omega_e^2 (\hat{\sigma}_- \hat{\sigma}_+ + \hat{\sigma}_+ \hat{\sigma}_-) \right\}$$

$$\langle \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_- \rangle = \langle \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_- \rangle = \langle \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_- \rangle = \langle \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_- \rangle = \langle \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_- \rangle =$$

$$= \langle \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_- \rangle = \langle \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_+ \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_+ \rangle = \langle \hat{\sigma}_- \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_- \rangle = 0$$

$$2^4 - 14 = 2 \qquad \langle \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_- \rangle \;\&\; \langle \hat{\sigma}_- \hat{\sigma}_+ \hat{\sigma}_- \hat{\sigma}_+ \rangle$$

$$M_B^3 = \Big\{ \gamma_R^{3/2} e^{-i\varphi} \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) + \gamma_L^{3/2} \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_\ell) +$$

$$+ \gamma_R \sqrt{\gamma_L} \left[\Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) + \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) + \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \cdot e^{-2i\varphi} \right] +$$

$$+ \gamma_L \sqrt{\gamma_R} \left[\Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) e^{-i\varphi} + \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_\ell) e^{-i\varphi} + \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) e^{i\varphi} \right] \Big\} \hat{\sigma}_+ +$$

$$- \Big\{ \gamma_R^{3/2} e^{i\varphi} \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) + \gamma_L^{3/2} \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_\ell) +$$

$$+ \gamma_R \sqrt{\gamma_L} \left[\Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) + \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) + \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_{\ell-c}) e^{2i\varphi} \right] +$$

$$+ \gamma_L \sqrt{\gamma_R} \left[\Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) e^{i\varphi} + \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_\ell) e^{i\varphi} + \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) e^{-i\varphi} \right] \Big\} \hat{\sigma}_-$$

$$M_B^4 = \Big\{ \gamma_R^2 \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) + \gamma_L \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_\ell) +$$

$$+ \gamma_L^{3/2} \sqrt{\gamma_R} \left[\Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) e^{i\varphi} + \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) e^{-i\varphi} + \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_\ell) e^{-i\varphi} + \right.$$

$$\left. + \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_\ell) e^{i\varphi} \right] + \gamma_L \gamma_R \Big[\Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_\ell) + \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) +$$

$$+ \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) e^{-2i\varphi} + \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) + \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) +$$

$$+ \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) e^{2i\varphi} \Big] + \sqrt{\gamma_L} \gamma_R^{3/2} \Big[e^{-i\varphi} \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) +$$

$$+ \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) e^{+i\varphi} + \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) e^{i\varphi} + \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) e^{-i\varphi} \Big] \Big\} \hat{\sigma}_+ \hat{\sigma}_- +$$

$$+ \Big\{ \gamma_R^2 \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) + \gamma_L \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_\ell) +$$

$$+ \gamma_L^{3/2} \sqrt{\gamma_R} \left[\Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) e^{-i\varphi} + \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) e^{+i\varphi} + \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_\ell) e^{+i\varphi} + \right.$$

$$\left. + \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_\ell) e^{-i\varphi} \right] + \gamma_L \gamma_R \Big[\Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_\ell) + \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) +$$

$$+ \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) e^{+2i\varphi} + \Delta B^+(t_\ell) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) + \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) +$$

$$+ \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) e^{-2i\varphi} \Big] + \sqrt{\gamma_L} \gamma_R^{3/2} \Big[e^{+i\varphi} \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) +$$

$$+ \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) e^{-i\varphi} + \Delta B^+(t_\ell) \Delta B(t_{\ell-e}) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) e^{-i\varphi} + \Delta B^+(t_{\ell-e}) \Delta B(t_\ell) \Delta B^+(t_{\ell-e}) \Delta B(t_{\ell-e}) e^{+i\varphi} \Big] \Big\} \hat{\sigma}_- \hat{\sigma}_+$$

$$M_S M_B = i \left(\Delta_e \hat{\sigma}_+ \hat{\sigma}_- + \Omega_e (\hat{\sigma}_- + \hat{\sigma}_+) \right) \left\{ \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] \hat{\sigma}_+ - \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \hat{\sigma}_- \right\} =$$

$$= i \left\{ \left[\Delta_e \hat{\sigma}_+ + \Omega_e \hat{\sigma}_- \hat{\sigma}_+ \right] \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] - \Omega_e \hat{\sigma}_+ \hat{\sigma}_- \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \right\}$$

$$M_S M_B^2 = i \left\{ \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \left[\Delta_e \hat{\sigma}_+ \hat{\sigma}_- + \Omega_e \hat{\sigma}_- \right] \cancel{\hat{\sigma}_+ \hat{\sigma}_-} + \right.$$

$$\left. + \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] \Omega_e \hat{\sigma}_+ \cancel{\hat{\sigma}_- \hat{\sigma}_+} \right.$$

$$M_B M_S M_B = i \Big\{ \Delta_e \hat{\sigma}_- \hat{\sigma}_+ \left[\sqrt{\gamma_R} e^{+i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] -$$

$$- \Omega_e \hat{\sigma}_+ \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] \left[\sqrt{\gamma_R} e^{-i\varphi} \Delta B(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B(t_\ell) \right] -$$

$$- \Omega_e \hat{\sigma}_- \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \left[\sqrt{\gamma_R} e^{i\varphi} \Delta B^+(t_{\ell-e}) + \sqrt{\gamma_L} \Delta B^+(t_\ell) \right] \Big\} \Delta t$$

$$M_3 M_S = i\Delta t\left\{\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_{2-e}) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\sigma_+ - \left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\sigma_-\right\}\left\{\Delta_e\sigma_+\sigma_- + \Omega_e(\sigma_- + \sigma_+)\right\} =$$

$$= i\Delta t\left\{\Omega_e\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\sigma_+\sigma_- - \left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\left(\Delta_e\sigma_- + \Omega_e\sigma_-\sigma_+\right)\right\}$$

$$M_3 = -\left\{\left[\sqrt{\gamma_R}\, e^{-i\varphi}\underbrace{\int_{t_2}^{t_2+1} b(t-\tau)\,dt}_{\substack{\tau = e\Delta t \\ \Delta B(t_{2-e})}} + \sqrt{\gamma_L}\underbrace{\int_{t_2}^{t_2+1} b(t)\,dt}_{\Delta B(t_2)}\right]\sigma_+ - \left[\sqrt{\gamma_R}\, e^{i\varphi}\underbrace{\int_{t_2}^{t_2+1} b^+(t-\tau)\,dt}_{\Delta B^+(t_2-e)} + \sqrt{\gamma_L}\underbrace{\int_{t_2}^{t_2+1} b^+(t)\,dt}_{\Delta B^+(t_2)}\right]\sigma_-\right\}$$

$$M_3^2 M_S = i\left\{\Omega_e\left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\sigma_- + \right.$$
$$\left. + \left(\Delta_e\sigma_+\sigma_- + \Omega_e\sigma_+\right)\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\right\}$$

$j_T e\ i_T e$ $\quad$ $j_T g\ i_T g$ $\quad$ $j_T e\ i_T g$ $\quad$ $j_T g\ i_T e$

$\delta_{j_2}\ i_2\ \delta_{j_T i_T}$

$$M_3 = \left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\sigma_- - \left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\sigma_+$$

$$M_S = -i\Delta t\left\{\Delta_e\sigma_+\sigma_- + \Omega_e(\sigma_+ + \sigma_-)\right\}$$

$$\frac{1}{2}M_3^2 = -\frac{1}{2}\left\{\left[\gamma_R\Delta B^+(t_2-e)\Delta B(t_2-e) + \sqrt{\gamma_R\gamma_L}\left(e^{i\varphi}\Delta B^+(t_2-e)\Delta B(t_2) + e^{-i\varphi}\Delta B^+(t_2)\Delta B(t_2-e)\right) + \gamma_L\Delta B^+(t_2)\Delta B(t_2)\right]\sigma_-\sigma_+ + \right.$$
$$\left. + \left[\gamma_R\Delta B(t_2-e)\Delta B^+(t_2-e) + \sqrt{\gamma_R\gamma_L}\left(e^{-i\varphi}\Delta B(t_2-e)\Delta B^+(t_2) + e^{i\varphi}\Delta B(t_2)\Delta B^+(t_2-e)\right) + \gamma_L\Delta B(t_2)\Delta B^+(t_2)\right]\sigma_+\sigma_-\right\}$$

$$\frac{1}{2}M_3 M_S = \frac{i}{2}\Delta t\left\{\Omega_e\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right]\sigma_+\sigma_- - \left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\left(\Delta_e\sigma_- + \Omega_e\sigma_-\sigma_+\right)\right\}$$

$$\frac{1}{2}M_S M_3 = \frac{i}{2}\Delta t\left\{\left[\Delta_e\sigma_+ + \Omega_e\sigma_-\sigma_+\right]\left[\sqrt{\gamma_R}\, e^{-i\varphi}\Delta B(t_2-e) + \sqrt{\gamma_L}\,\Delta B(t_2)\right] - \Omega_e\sigma_+\sigma_-\left[\sqrt{\gamma_R}\, e^{i\varphi}\Delta B^+(t_2-e) + \sqrt{\gamma_L}\,\Delta B^+(t_2)\right]\right\}$$

$$\frac{1}{6}M_3^3 = \frac{1}{6}\left\{\gamma_R^{3/2} e^{-i\varphi}\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2-e) + \gamma_L^{3/2}\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2) + \right.$$
$$+ \gamma_R\sqrt{\gamma_L}\left[\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2) + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2-e) + \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2-e)\cdot e^{-2i\varphi}\right] +$$
$$\left. + \gamma_L\sqrt{\gamma_R}\left[\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2-e)e^{-i\varphi} + \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2)e^{-i\varphi} + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2)e^{i\varphi}\right]\right\}\sigma_+ +$$
$$- \left\{\gamma_R^{3/2} e^{i\varphi}\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2-e) + \gamma_L^{3/2}\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2) + \right.$$
$$+ \gamma_R\sqrt{\gamma_L}\left[\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2) + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2-e) + \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2-e)e^{2i\varphi}\right] +$$
$$\left. + \gamma_L\sqrt{\gamma_R}\left[\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2-e)e^{i\varphi} + \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2)e^{i\varphi} + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2)e^{-i\varphi}\right]\right\}\sigma_-$$

$$\frac{1}{2}M^2 = -\frac{1}{2}\Delta t^2\left\{\Delta_e^2\sigma_+\sigma_- + \Omega_e\Delta_e(\sigma_+ + \sigma_-) + \Omega_e^2(\sigma_-\sigma_+ + \sigma_+\sigma_-)\right\}$$

$$\frac{1}{6}M_3^2 M_S = \frac{i\Delta t}{6}\left\{\Omega_e\left[\gamma_R\Delta B^+(t_2-e)\Delta B(t_2-e) + \sqrt{\gamma_R\gamma_L}\left(e^{i\varphi}\Delta B^+(t_2-e)\Delta B(t_2) + e^{-i\varphi}\Delta B^+(t_2)\Delta B(t_2-e)\right) + \gamma_L\Delta B^+(t_2)\Delta B(t_2)\right]\sigma_- + \right.$$
$$\left. + \left[\gamma_R\Delta B(t_2-e)\Delta B^+(t_2-e) + \sqrt{\gamma_R\gamma_L}\left(e^{-i\varphi}\Delta B(t_2-e)\Delta B^+(t_2) + e^{i\varphi}\Delta B(t_2)\Delta B^+(t_2-e)\right) + \gamma_L\Delta B(t_2)\Delta B^+(t_2)\right]\left(\Delta_e\sigma_+\sigma_- + \Omega_e\sigma_+\right)\right\}$$

$$\frac{1}{6}M_1 M_S M_3 = \frac{i\Delta t}{6}\left\{\Delta_e\sigma_-\sigma_+\left[\gamma_R\Delta B^+(t_2-e)\Delta B(t_2-e) + \overset{\sqrt{\gamma_L\gamma_R}\left(\Delta B^+(t_2-e)\Delta B(t_2) + \Delta B^+(t_2)\Delta B(t_2-e)\right)}{+} \gamma_L\Delta B^+(t_2)\Delta B(t_2)\right] - \Omega_e\sigma_+\left[\gamma_R e^{-2i\varphi}\Delta B(t_2-e)^2 + e^{-i\varphi}\sqrt{\gamma_R\gamma_L}\left(\Delta B(t_2-e)\Delta B(t_2) + \Delta B(t_2)\Delta B(t_2-e)\right) + \right.\right.$$
$$\left.\left. + \gamma_L\Delta B(t_2)^2\right] - \Omega_e\sigma_-\left[\gamma_R e^{2i\varphi}\Delta B^{+2}(t_2-e) + \sqrt{\gamma_L\gamma_R}\left(\Delta B^+(t_2)\Delta B^+(t_2-e) + \Delta B^+(t_2-e)\Delta B^+(t_2)\right)e^{i\varphi} + \gamma_L\Delta B^{+2}(t_2)\right]\right\}$$

$$\frac{1}{6}M_1 M_3^2 = \frac{i}{6}\left\{\left[\gamma_R\Delta B(t_2-e)\Delta B^+(t_2-e) + \sqrt{\gamma_L\gamma_R}\left(\Delta B(t_2)\Delta B^+(t_2-e)e^{i\varphi} + \Delta B(t_2-e)\Delta B^+(t_2)e^{-i\varphi}\right) + \gamma_L\Delta B(t_2)\Delta B^+(t_2)\right]\left[\Delta_e\sigma_+\sigma_- + \Omega_e\sigma_-\right] + \right.$$
$$\left. + \left[\gamma_R\Delta B^+(t_2-e)\Delta B(t_2-e) + \sqrt{\gamma_L\gamma_R}\left(\Delta B^+(t_2)\Delta B(t_2-e)e^{-i\varphi} + \Delta B^+(t_2-e)\Delta B(t_2)e^{i\varphi}\right) + \gamma_L\Delta B^+(t_2)\Delta B(t_2)\right]\ \Omega_e\sigma_+\right\}\Delta t$$

$$\frac{1}{24}M_3^4 = \frac{1}{24}\left\{\gamma_R^2\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2-e) + \gamma_L\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2) + \right.$$
$$+ \gamma_L^{3/2}\sqrt{\gamma_R}\left[\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2-e)e^{i\varphi} + \Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2)e^{-i\varphi} + \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2)e^{-i\varphi} + \right.$$
$$\left. + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2)e^{i\varphi}\right] + \gamma_L\gamma_R\left[\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2) + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2) + \right.$$
$$+ \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2)e^{-2i\varphi} + \Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2-e) + \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2-e) +$$
$$\left. + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2-e)e^{2i\varphi}\right] + \sqrt{\gamma_L}\gamma_R^{3/2}\left[e^{-i\varphi}\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2) + \right.$$
$$\left.\left. + \Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2-e)e^{+i\varphi} + \Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2-e)e^{i\varphi} + \Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2-e)e^{-i\varphi}\right]\right\}\sigma_+\sigma_- +$$

$$+ \frac{1}{24}\left\{\gamma_R^2\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2-e) + \gamma_L\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2) + \right.$$
$$+ \gamma_L^{3/2}\sqrt{\gamma_R}\left[\Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2-e)e^{-i\varphi} + \Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2)e^{+i\varphi} + \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2)e^{+i\varphi} + \right.$$
$$\left. + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2)e^{-i\varphi}\right] + \gamma_L\gamma_R\left[\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2) + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2) + \right.$$
$$+ \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2)e^{+2i\varphi} + \Delta B^+(t_2)\Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2-e) + \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2)\Delta B(t_2-e) +$$
$$\left. + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2-e)e^{-2i\varphi}\right] + \sqrt{\gamma_L}\gamma_R^{3/2}\left[e^{+i\varphi}\Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2) + \right.$$
$$\left.\left. + \Delta B^+(t_2-e)\Delta B(t_2-e)\Delta B^+(t_2)\Delta B(t_2-e)e^{-i\varphi} + \Delta B^+(t_2)\Delta B(t_2-e)\Delta B^+(t_2-e)\Delta B(t_2-e)e^{-i\varphi} + \Delta B^+(t_2-e)\Delta B(t_2)\Delta B^+(t_2-e)\Delta B(t_2-e)e^{+i\varphi}\right]\right\}\sigma_-\sigma_+$$

$$\mathcal{U}_m^{\ell} = \sqrt{\gamma_L}\,\Delta B(t_\ell) \qquad \mathcal{U}_p^{\ell} = \sqrt{\gamma_L}\,\Delta B^{+}(t_\ell) \qquad \mathcal{U}_m^{e} = \sqrt{\gamma_R}\,e^{-i\varphi}\Delta B(t_{\ell-e}) \qquad \mathcal{U}_p^{e} = \sqrt{\gamma_R}\,e^{i\varphi}\Delta B^{+}(t_{\ell-e})$$

$$\mathcal{U}_{2np_1}^{\ell} = (2n_\ell+1)\gamma_L\Delta t \qquad \mathcal{U}_{2np_1}^{\ell} = (2n_{\ell-e}+1)\gamma_R\Delta t \qquad \mathcal{U}_n^{\ell} = n_\ell\gamma_L\Delta t \qquad \mathcal{U}_n^{e} = n_e\gamma_R\Delta t$$

$\delta_{j_\ell i_\ell}\,\delta_{j_r i_r}$ $\qquad \delta_{je}\delta_{ig} \qquad \delta_{jg}\delta_{ie} \qquad \delta_{je}\delta_{ie} \qquad \delta_{jg}\delta_{ig}$

$$M_B = \left[\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\right]\hat\sigma_- - \left[\mathcal{U}_m^{e} + \mathcal{U}_m^{\ell}\right]\hat\sigma_+$$

$$M_S = -i\Delta t\left\{\Delta_e\hat\sigma_+\hat\sigma_- + \Omega_e\left(\hat\sigma_+ + \hat\sigma_-\right)\right\}$$

$$\frac{1}{2}M_B^2 = -\frac{1}{2}\left\{\left[\mathcal{U}_n^{e}\mathbb{1} + \mathcal{U}_p^{e}\mathcal{U}_m^{\ell} + \mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_n^{\ell}\mathbb{1}\right]\hat\sigma_-\hat\sigma_+ + \left[(\mathcal{U}_n^{e}+1)\mathbb{1} + \mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + (\mathcal{U}_n^{\ell}+1)\mathbb{1}\right]\hat\sigma_+\hat\sigma_-\right\}$$

$$\frac{1}{2}M_B M_S = \frac{i}{2}\Delta t\left\{\Omega_e\left[\mathcal{U}_m^{e} + \mathcal{U}_m^{\ell}\right]\hat\sigma_+\hat\sigma_- - \left[\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\right]\left(\Delta_e\hat\sigma_- + \Omega_e\hat\sigma_-\hat\sigma_+\right)\right\}$$

$$\frac{1}{2}M_S M_B = \frac{i}{2}\Delta t\left\{-\Omega_e\left[\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\right]\hat\sigma_+\hat\sigma_- + \left[\mathcal{U}_m^{e} + \mathcal{U}_m^{\ell}\right]\left(\Delta_e\hat\sigma_+ + \Omega_e\hat\sigma_-\hat\sigma_+\right)\right\}$$

$$\frac{1}{6}M_B^3 = \frac{1}{6}\Big\{\left[\mathcal{U}_n^{e}\mathcal{U}_m^{\ell} + \mathcal{U}_n^{\ell}\mathcal{U}_m^{\ell} + (\mathcal{U}_n^{e}+1)\mathcal{U}_m^{\ell} + \mathcal{U}_n^{e}\mathcal{U}_m^{\ell} + \mathcal{U}_m^{e2}\mathcal{U}_p^{\ell} + (\mathcal{U}_n^{\ell}+1)\mathcal{U}_m^{e} + \mathcal{U}_n^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_m^{\ell 2}\mathcal{U}_p^{e}\right]\hat\sigma_+ -$$
$$-\left[(\mathcal{U}_n^{e}+1)\mathcal{U}_p^{e} + (\mathcal{U}_n^{\ell}+1)\mathcal{U}_p^{\ell} + (2\mathcal{U}_n^{e}+1)\mathcal{U}_p^{\ell} + \mathcal{U}_p^{e2}\mathcal{U}_m^{\ell} + (2\mathcal{U}_n^{\ell}+1)\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell 2}\mathcal{U}_m^{e}\right]\hat\sigma_-\Big\}$$

$$\frac{1}{2}M_S^2 = \frac{1}{2}\Delta t^2\left\{\Delta_e^2\hat\sigma_+\hat\sigma_- + \Omega_e\Delta_e\left(\hat\sigma_+ + \hat\sigma_-\right) + \Omega_e^2\left(\hat\sigma_-\hat\sigma_+ + \hat\sigma_+\hat\sigma_-\right)\right\}$$

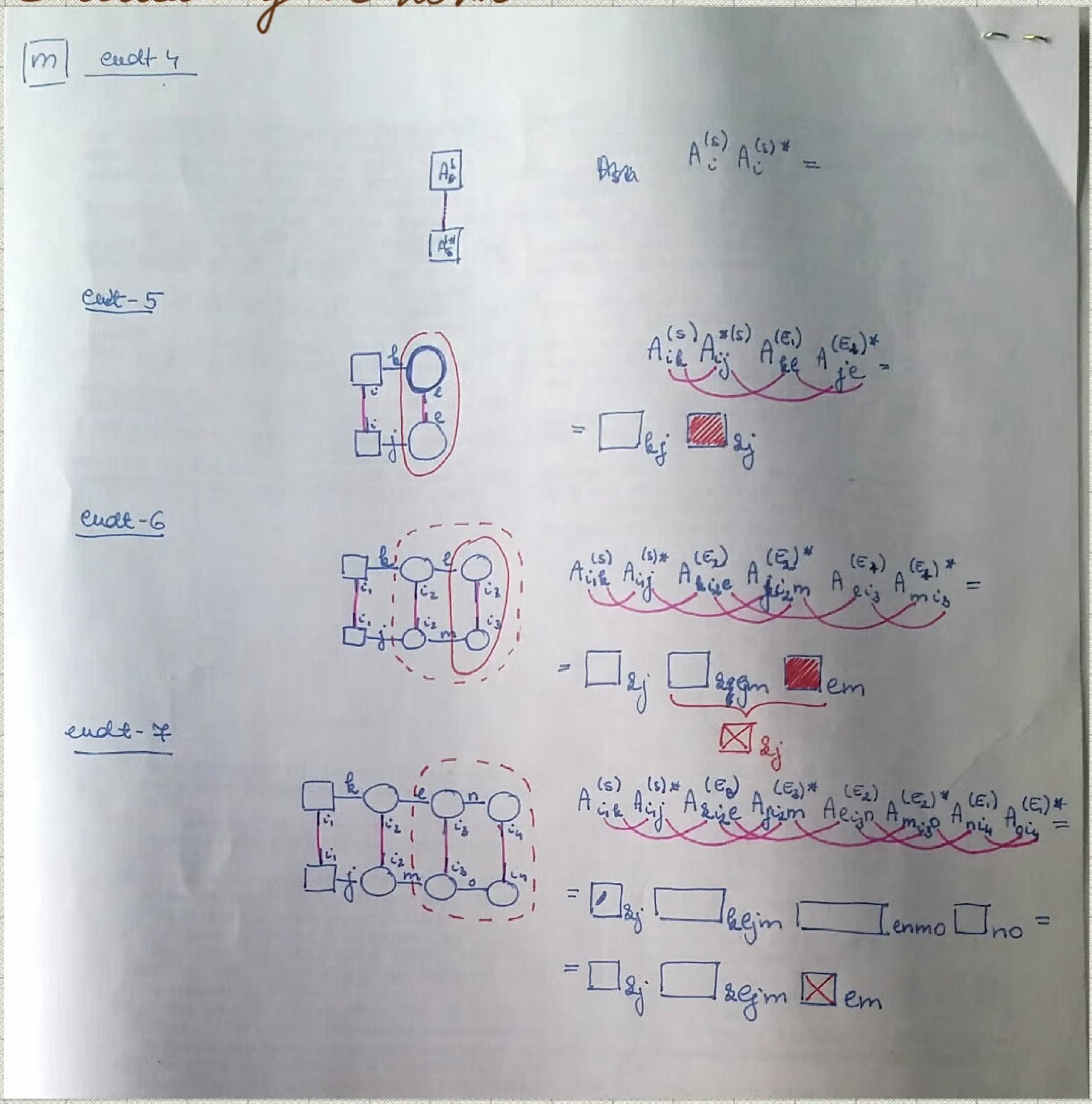
$$\frac{1}{6}M_B^2 M_S = \frac{i\Delta t}{6}\left\{\Omega_e\left[\mathcal{U}_n^{e}\mathbb{1} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_n^{\ell}\mathbb{1}\right]\hat\sigma_- + \left[(\mathcal{U}_n^{e}+1)\mathbb{1} + \mathcal{U}_m^{e}\mathcal{U}_p^{\ell} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + (\mathcal{U}_n^{\ell}+1)\mathbb{1}\right]\left(\Delta_e\hat\sigma_+\hat\sigma_- + \Omega_e\hat\sigma_+\right)\right\}$$

$$\frac{1}{6}M_S M_B^2 = \frac{i\Delta t}{6}\left\{\Omega_e\left[\mathcal{U}_n^{e}\mathbb{1} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_n^{\ell}\mathbb{1}\right]\hat\sigma_+ + \left[(\mathcal{U}_n^{e}+1)\mathbb{1} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + \mathcal{U}_m^{e}\mathcal{U}_p^{\ell} + (\mathcal{U}_n^{\ell}+1)\mathbb{1}\right]\left(\Delta_e\hat\sigma_+\hat\sigma_- + \Omega_e\hat\sigma_-\right)\right\}$$

$$\frac{1}{6}M_B M_S M_B = \frac{i\Delta t}{6}\left\{\left[\mathcal{U}_n^{e}\mathbb{1} + \mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + \mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + \mathcal{U}_n^{\ell}\mathbb{1}\right]\Delta_e\hat\sigma_-\hat\sigma_+ - \left[\mathcal{U}_m^{e2} + 2\mathcal{U}_m^{e}\mathcal{U}_m^{\ell} + \mathcal{U}_m^{\ell 2}\right]\Omega_e\hat\sigma_+ - \left[\mathcal{U}_p^{e2} + 2\mathcal{U}_p^{e}\mathcal{U}_p^{\ell} + \mathcal{U}_p^{\ell 2}\right]\Omega_e\hat\sigma_-\right\}$$

$$\frac{1}{24}M_B^4 = \frac{1}{24}\Big\{\Big[(\mathcal{U}_n^{e}+1)^2 + (\mathcal{U}_n^{\ell}+1)^2 + (2\mathcal{U}_n^{\ell}+1)\mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + (2(\mathcal{U}_n^{\ell}+1)+1)\mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + (\mathcal{U}_n^{\ell}+1)(2\mathcal{U}_n^{e}+1)\mathbb{1} + (\mathcal{U}_n^{e}+1)(2\mathcal{U}_n^{\ell}+1)\mathbb{1} +$$
$$+ \mathcal{U}_n^{e2}\mathcal{U}_p^{\ell 2} + \mathcal{U}_p^{e2}\mathcal{U}_n^{\ell 2} + (2\mathcal{U}_n^{e}+1)\mathcal{U}_m^{e}\mathcal{U}_p^{\ell} + (2(\mathcal{U}_n^{e}+1)+1)\mathcal{U}_m^{\ell}\mathcal{U}_p^{e}\Big]\hat\sigma_+\hat\sigma_- +$$
$$+\Big[\mathcal{U}_n^{e2} + \mathcal{U}_n^{\ell 2} + (2\mathcal{U}_n^{\ell}+1)\mathcal{U}_p^{\ell}\mathcal{U}_m^{e} + (2(\mathcal{U}_n^{\ell}-1)+1)\mathcal{U}_m^{\ell}\mathcal{U}_p^{e} + (2\mathcal{U}_n^{e}+1)\mathcal{U}_n^{\ell} + (2\mathcal{U}_n^{\ell}+1)\mathcal{U}_n^{e} + \mathcal{U}_m^{\ell 2}\mathcal{U}_p^{e2} + \mathcal{U}_p^{\ell 2}\mathcal{U}_m^{e2} +$$
$$+ (2\mathcal{U}_n^{e}+1)\mathcal{U}_p^{e}\mathcal{U}_m^{\ell} + (2(\mathcal{U}_n^{e}-1)+1)\mathcal{U}_m^{e}\mathcal{U}_p^{\ell}\Big]\hat\sigma_-\hat\sigma_+\Big\}$$

Calculating the norm



m cudt 4

$A_i^{(s)} A_i^{(s)*} =$

cudt-5

$A^{(s)}_{ik} A^{*(s)}_{ij} A^{(E_1)}_{ke} A^{(E_1)*}_{je} =$

$= \square_{kj} \blacksquare_{kj}$

cudt-6

$A^{(s)}_{ik} A^{(s)*}_{ij} A^{(E_2)}_{k i_2 e} A^{(E_2)*}_{j i_2 m} A^{(E_1)}_{e i_3} A^{(E_1)*}_{m i_3} =$

$= \square_{kj} \underbrace{\square_{kejm} \blacksquare_{em}}_{\boxtimes_{kj}}$

cudt-7

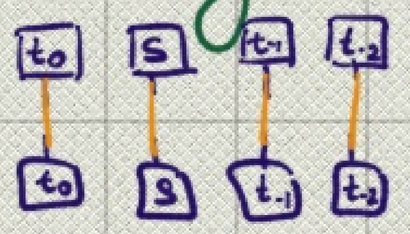
$A^{(s)}_{ik} A^{(s)*}_{ij} A^{(E_3)}_{k i_2 e} A^{(E_3)*}_{j i_2 m} A^{(E_2)}_{e i_3 n} A^{(E_2)*}_{m i_3 o} A^{(E_1)}_{n i_4} A^{(E_1)*}_{o i_4} =$

$= \square_{kj} \square_{kejm} \square_{enmo} \square_{no} =$

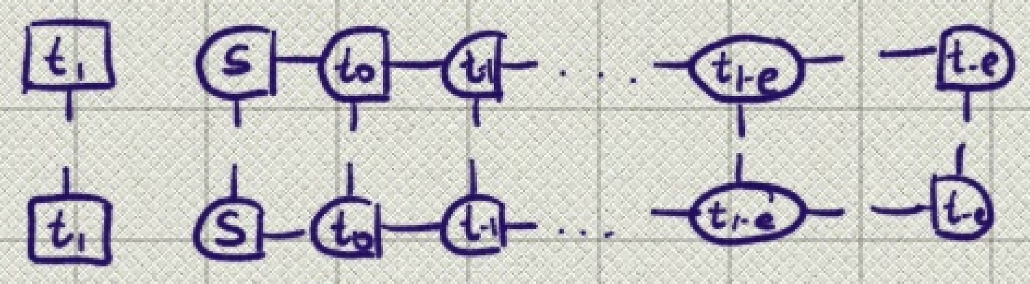
$= \square_{kj} \square_{kejm} \boxtimes_{em}$

with feedback

t=0

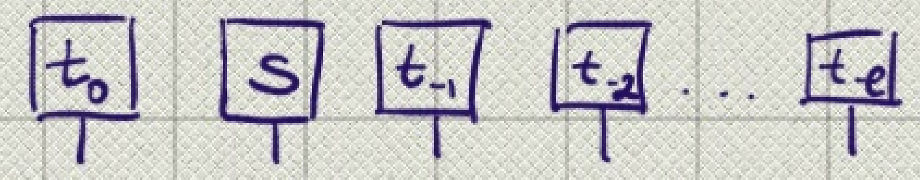


t=1



Adding feedback

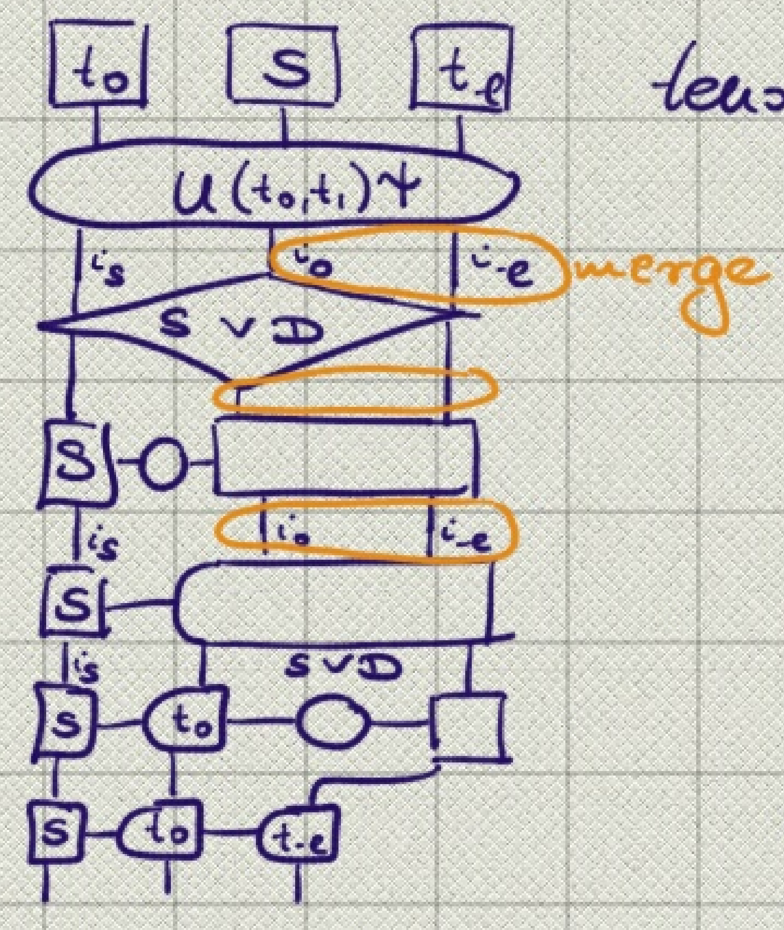
$t = 0$



$e \hat{=} 3$

independent bins

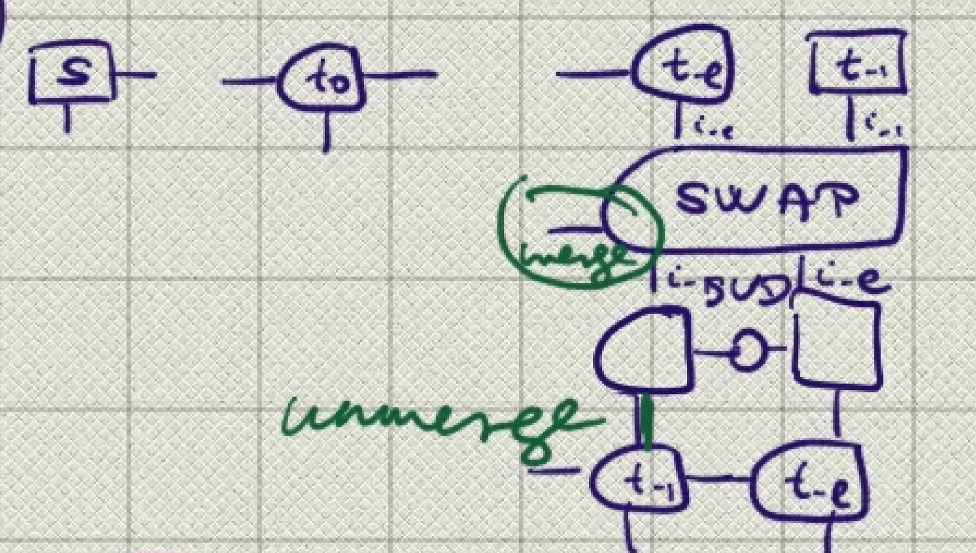
3.)



tensor prod

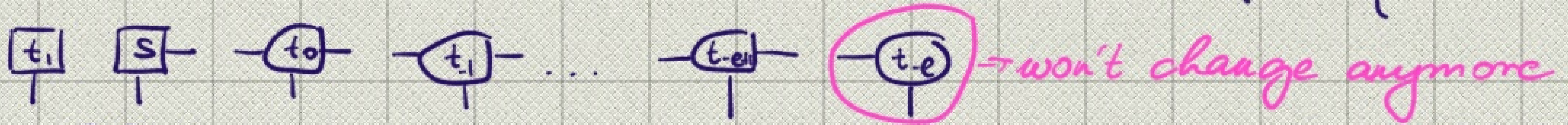
OC & SWAP-step not needed

4.)



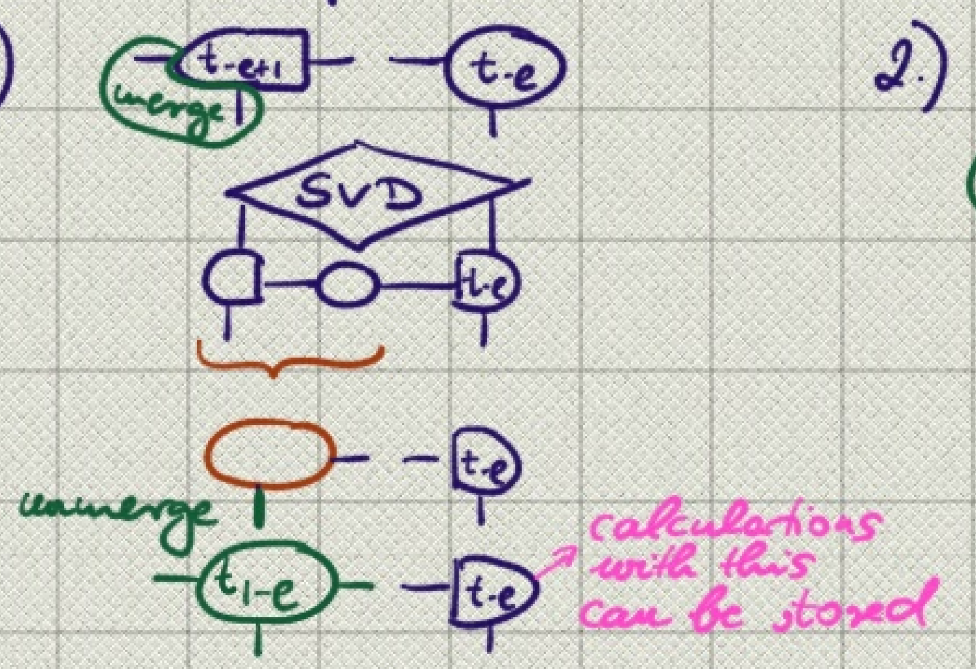
tensor prod

$t = 1$



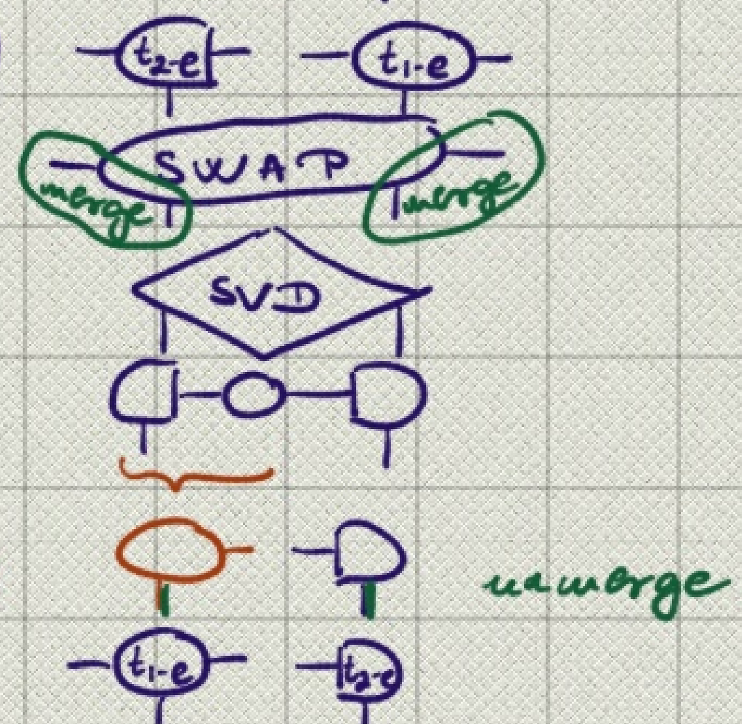
OC-step

1.)



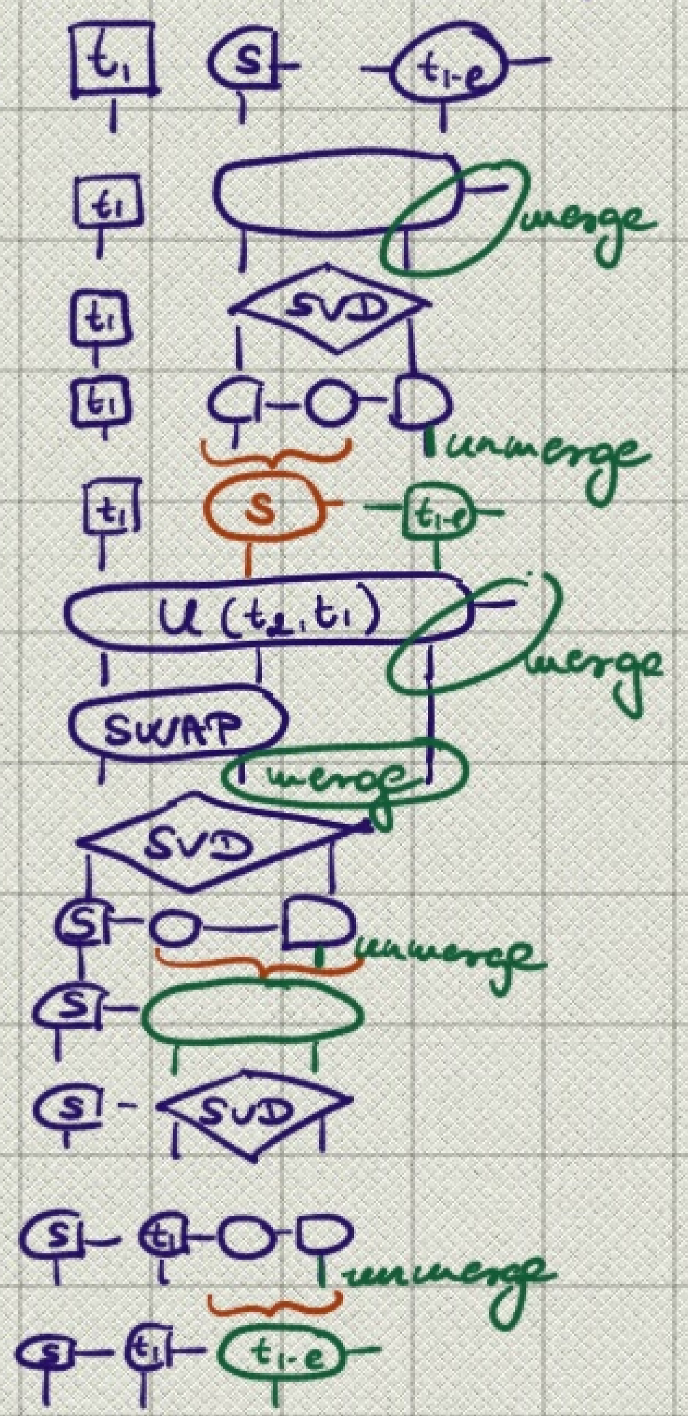
SWAP-step

2.)



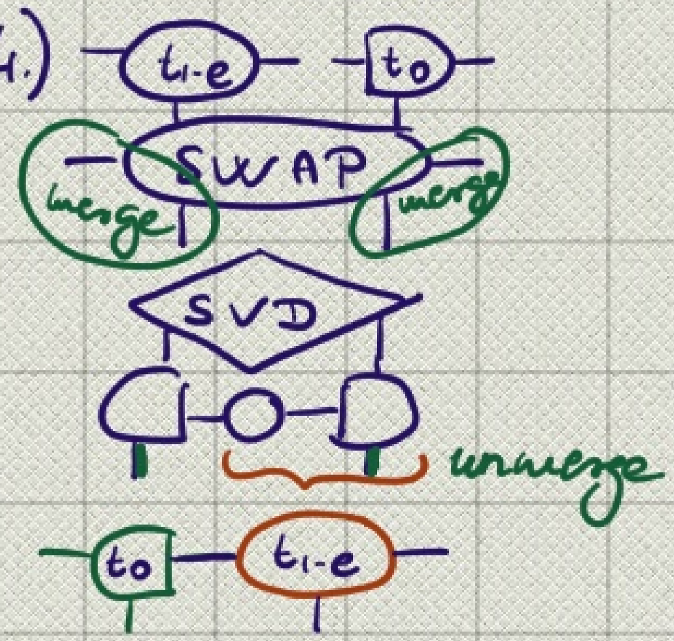
EVOLUTION step

3.)

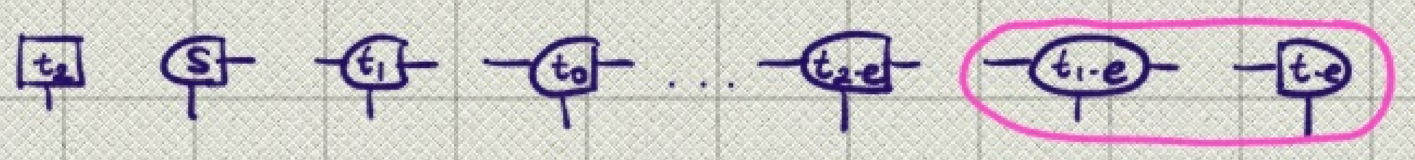


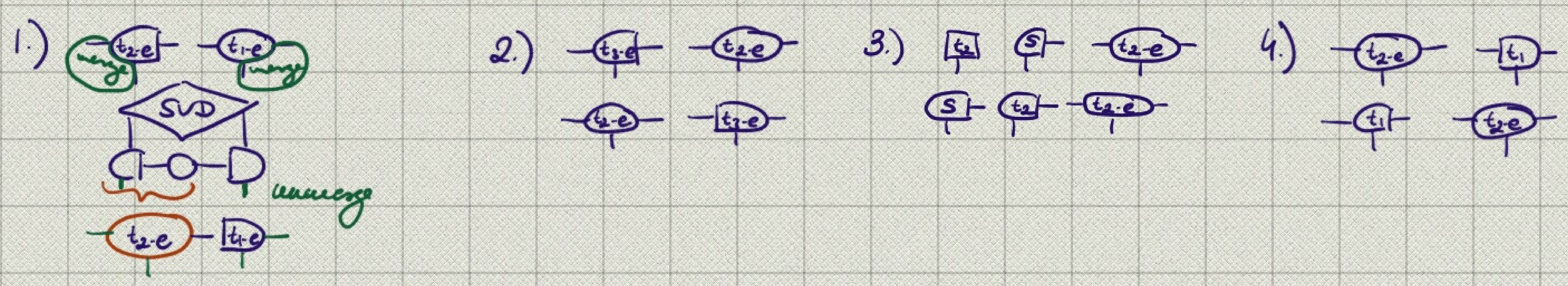
UNSWAP-step

4.)



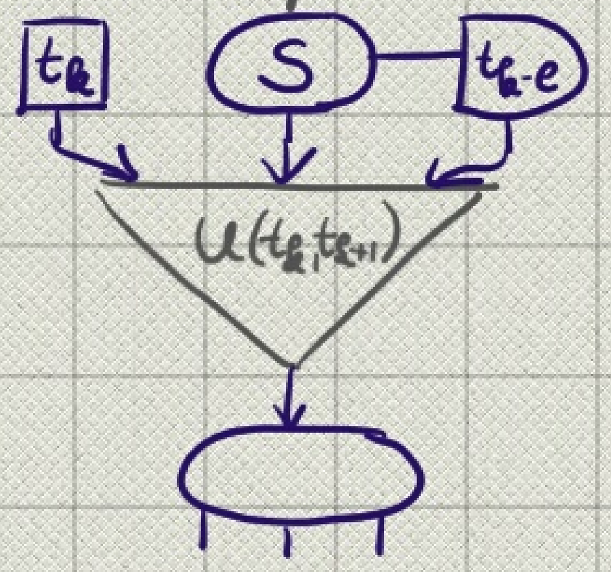
$t = 2$





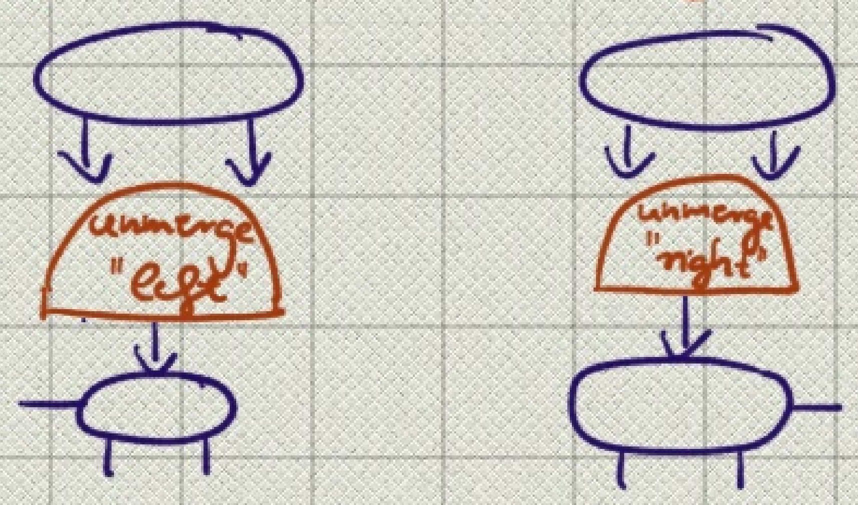
Blocks that I've defined :

1.) time-evolution step $\mathcal{U}$ to 2nd order:

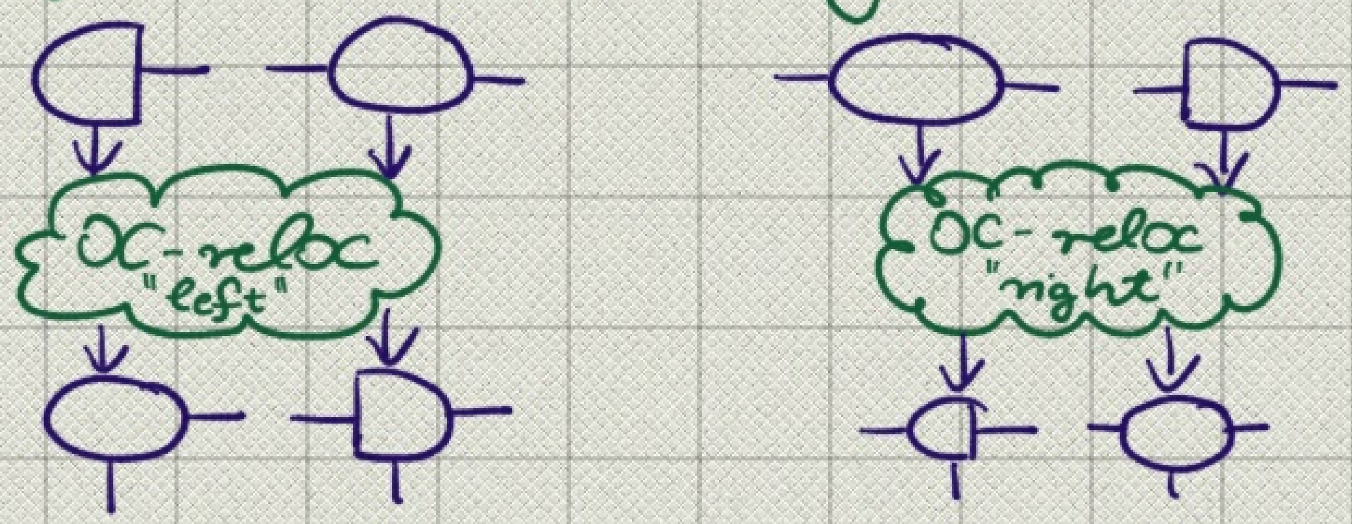


2.) merging of indices

3.) undoing the merge of indices



4.) shifting the Orthogonality Centre (OC)



t=0
1.)
t0
S
t-e
U(t0, t1)
merge "right"
cut "right"
SVD
S
unmerge "right"
merge "left"
cut "left"
SVD
"left"
S
t0
t-e
2.)
(l-1) times
t-e
t-1
'left'
cut "left"
SVD
'left'
t-1
t-e
3.)
t1-e
t-e
OC - shift "left"
t1-e
t-e
3.)
t1
S
t1-e
U(t1, t2)
"right"
"right"
cut "right"
SVD
S
"right"
"left"
cut "left"
SVD
"left"
"right"
S
t1
t1-e
t=1
1.)
(l-1) times
t2-e
t1-e
"both"
cut "both" "OC left"
SVD
"right"
t2-e
"left"
t1-e
t2-e
2.)
S
t1-e
OC-shift "left"
S
t1-e
4.)
t1-e
t0
(l-1) times
"both"
cut "both" "OC right"
"left"
t0
"right"
t1-e
5.)
t2-e
t1-e
OC-shift "left"
t2-e
t1-e
t=b general
1.)
tb+1-e
tb-e