



THE UNIVERSITY OF
AUCKLAND
Te Whare Wānanga o Tāmaki Makaurau
NEW ZEALAND



DODD-WALLS CENTRE
for Photonic and Quantum Technologies

Manipulating the Squeezing Properties of a Degenerate Parametric Amplifier with Coherent, Time-Delayed Feedback

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Outline

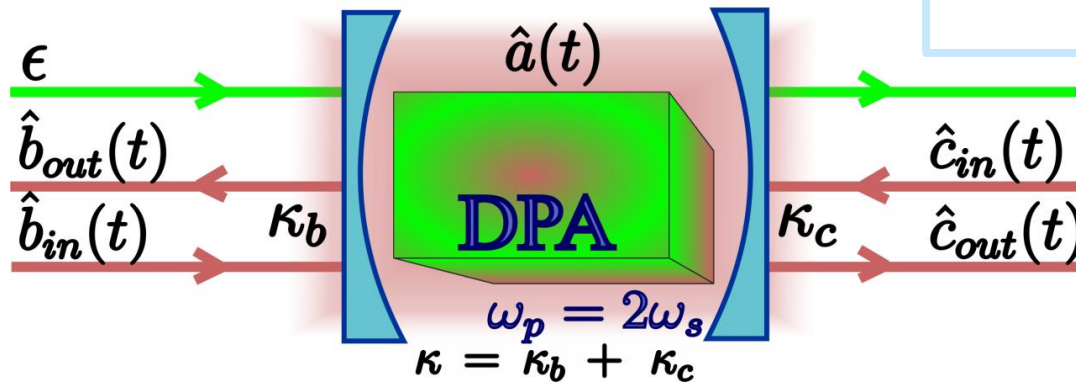
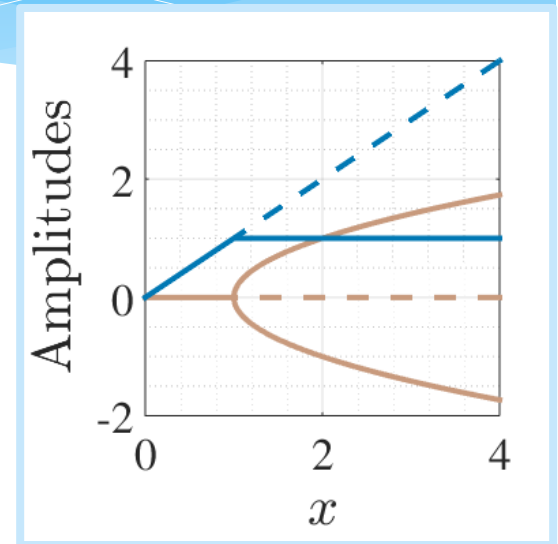
- * Degenerate Parametric Amplifier
 - * Classical dynamics
 - * Quantum treatment below threshold
 - * Squeezing characteristics
- * DPA with time-delayed coherent feedback
 - * Model
 - * Squeezing spectrum
 - * Change in the dynamics
 - * Effects of loss



DPA dynamics & squeezing

- * Classical dynamics:
 - * Saturation of pump: threshold behaviour
 - * Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$
- * Quantum mechanical behaviour^[1]:
 - * Parametric pump approximation ($\epsilon = |\epsilon|e^{i\theta}$)

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$



^[1]PRA, 30:I386 (1984)



DPA dynamics & squeezing

- * Classical dynamics:

- * Saturation of pump: threshold behaviour

- * Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$

- * Quantum mechanical behaviour^[1]:

- * Parametric pump approximation ($\epsilon = |\epsilon|e^{i\theta}$)

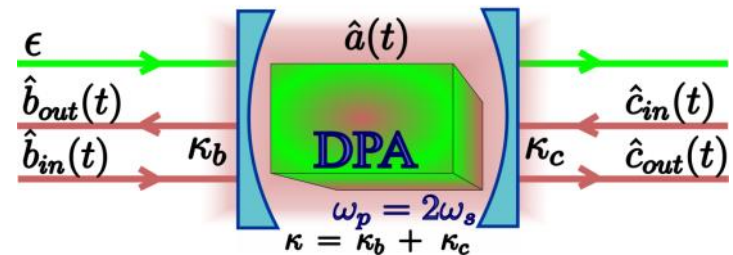
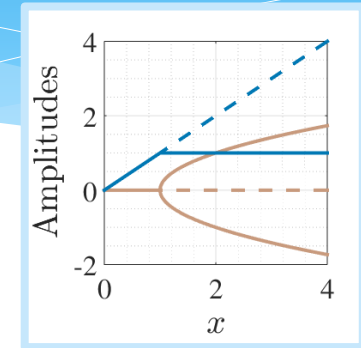
$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

- * One-sided cavity quadrature variance:

$$\chi_{out,\theta+\pi}(0) = \int \left\langle \tilde{X}_{out,\theta'}(0), \tilde{X}_{out,\theta'}(v') \right\rangle dv' = \frac{1}{4} \frac{(\kappa - |\epsilon|)^2}{(\kappa + |\epsilon|)^2}$$

- * Symmetric cavity squeezing on resonance at threshold

$$\chi_{out,\theta+\pi}(0) = \frac{1}{4} \frac{\kappa^2 + |\epsilon|^2}{(\kappa + |\epsilon|)^2}$$



DPA & time-delayed coherent feedback

- * Uncomplicated setup
 - * Time-delay τ , overall phase-shift ϕ , overall loss, decoherence: $L, \hat{\xi}$
- * Equation of motion of the subharmonic mode:

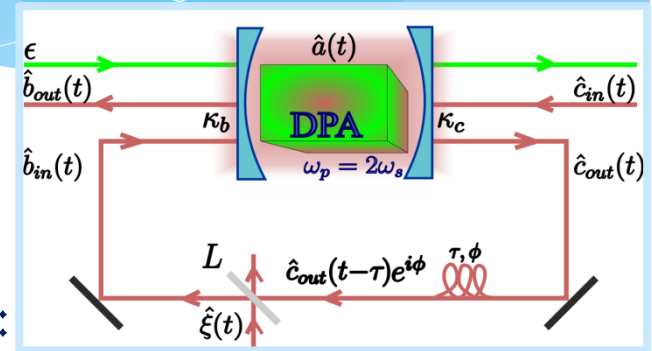
$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - \kappa\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - e^{i\phi}k\hat{a}(t - \tau)$$

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

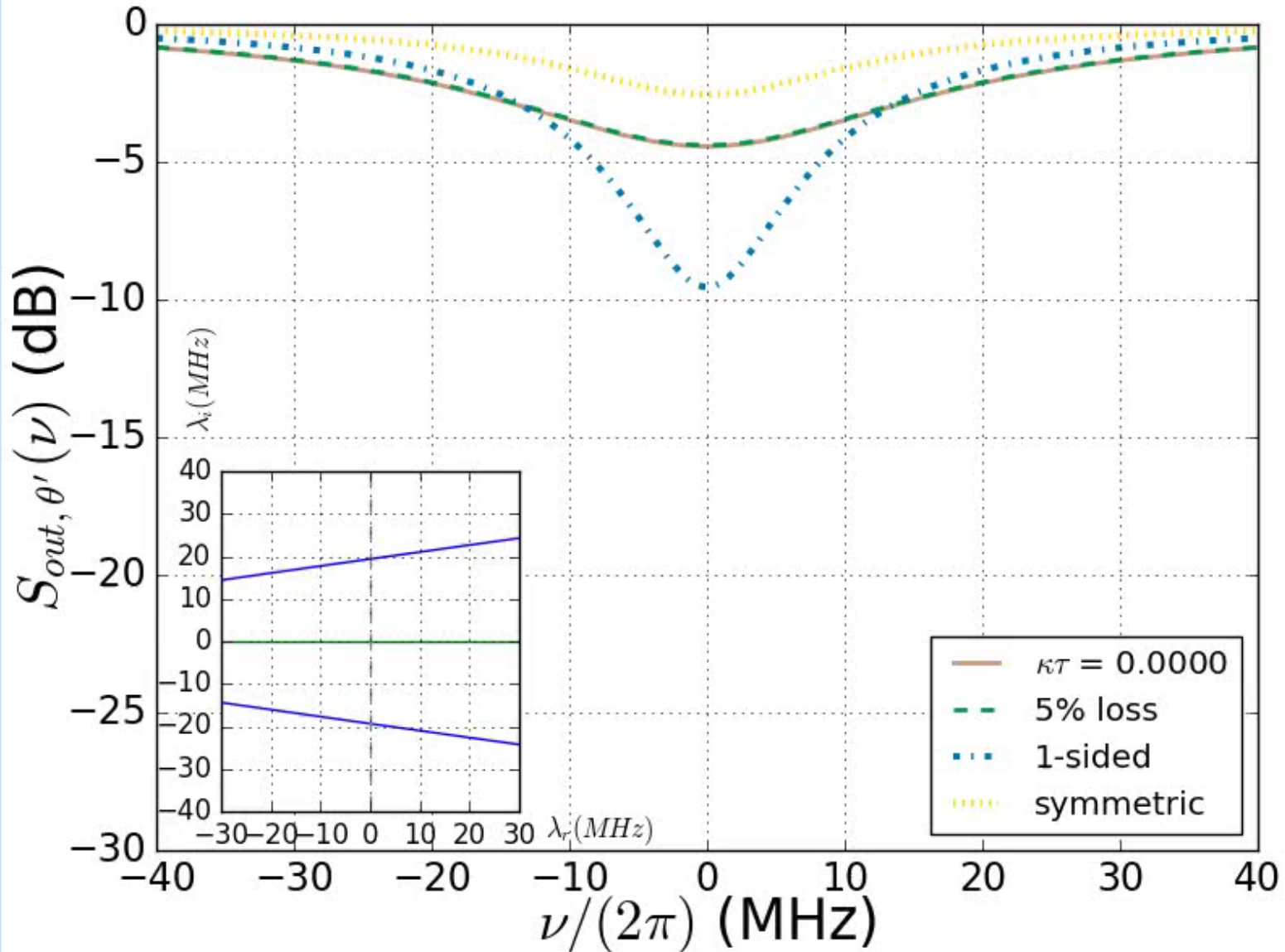
$$\kappa = \kappa_b + \kappa_c, k = 2\sqrt{\kappa_b\kappa_c(1-L)}$$

- * Quadrature variance without delay:

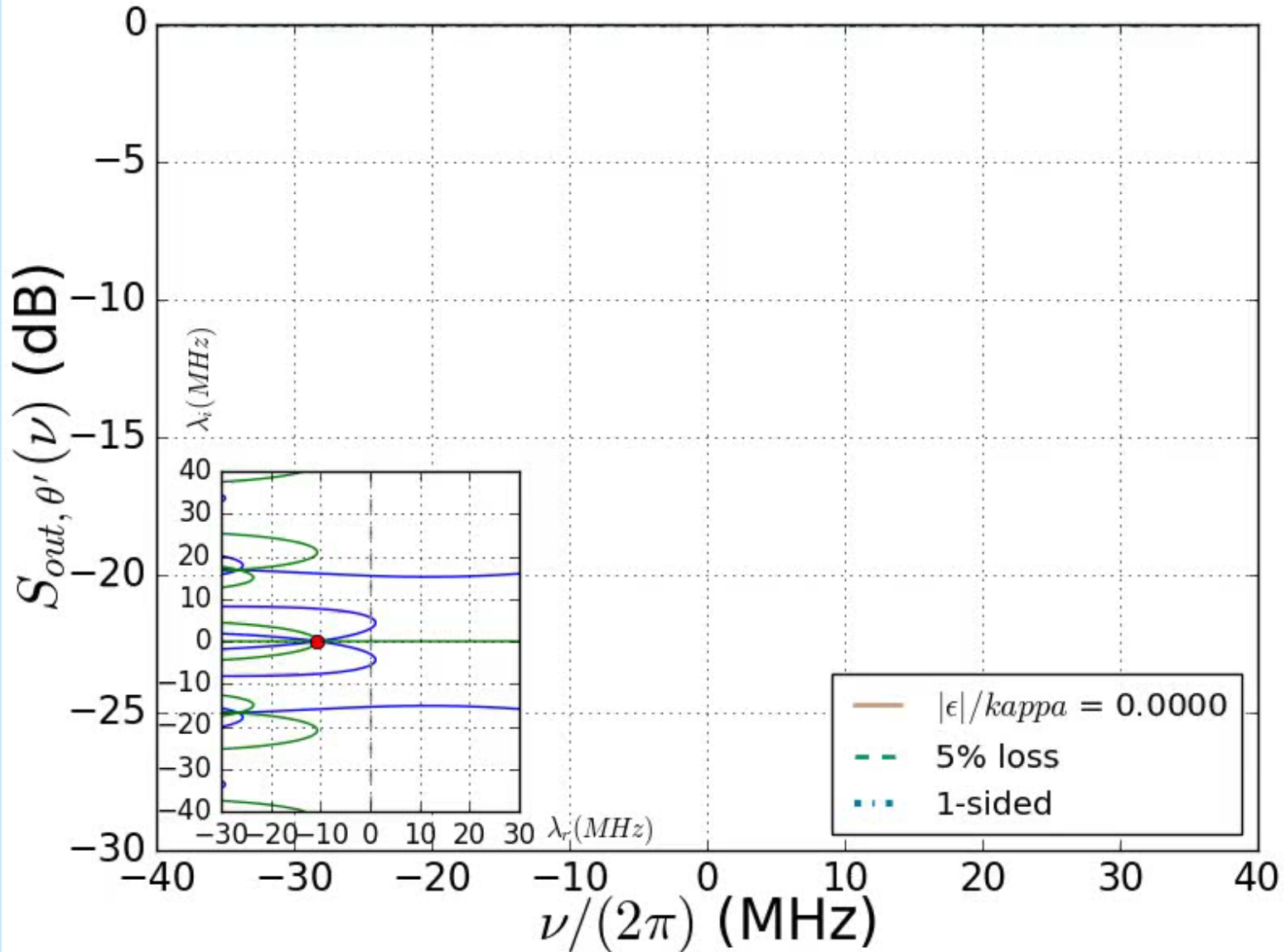
$$\chi_{out, \theta + \pi}(0) = \frac{1}{4} \frac{((\kappa + k \cos \phi) - |\epsilon|)^2}{((\kappa + k \cos \phi) + |\epsilon|)^2}$$



Emerging side-peaks



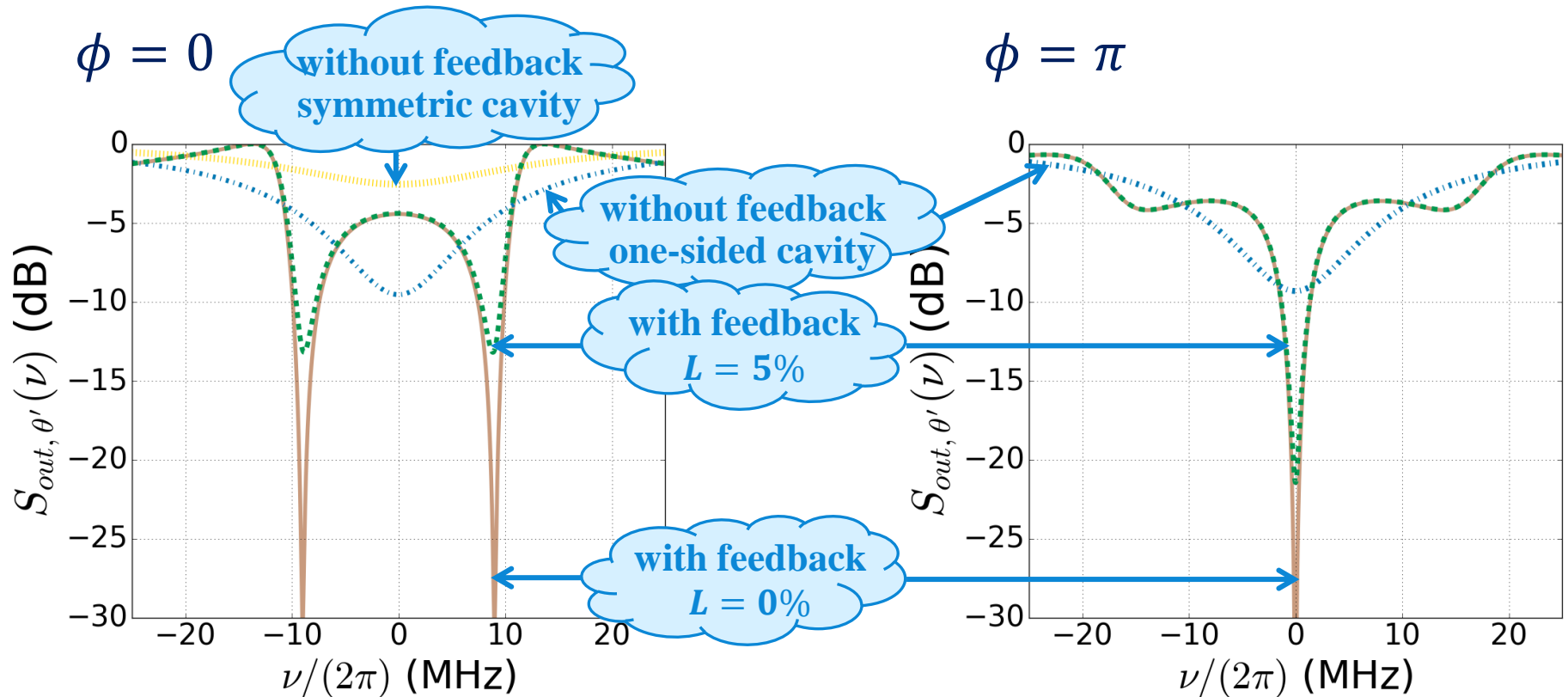
Enhanced squeezing on resonance





New setup with time-delay

- * Transcendental equations
- * Enhanced squeezing with feedback

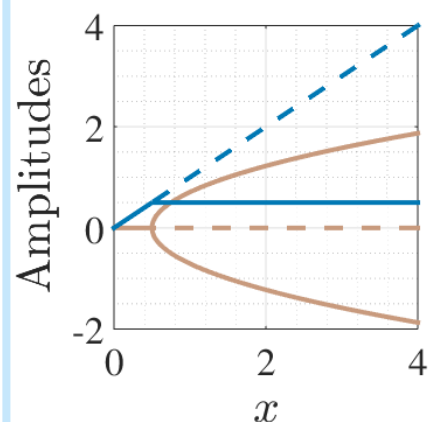


New setup with time-delay

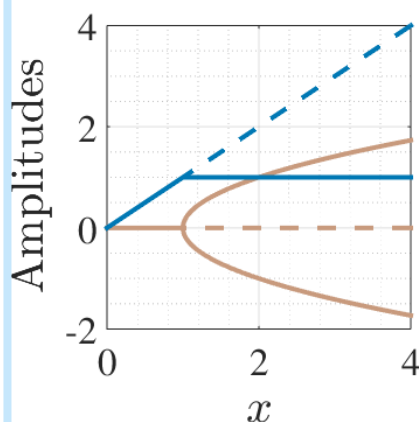
- * Transcendental equations
- * Enhanced squeezing with feedback
- * Pyragas-type feedback:

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - (\kappa - k)\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) + k(\hat{a}(t - \tau) - \hat{a}(t))$$

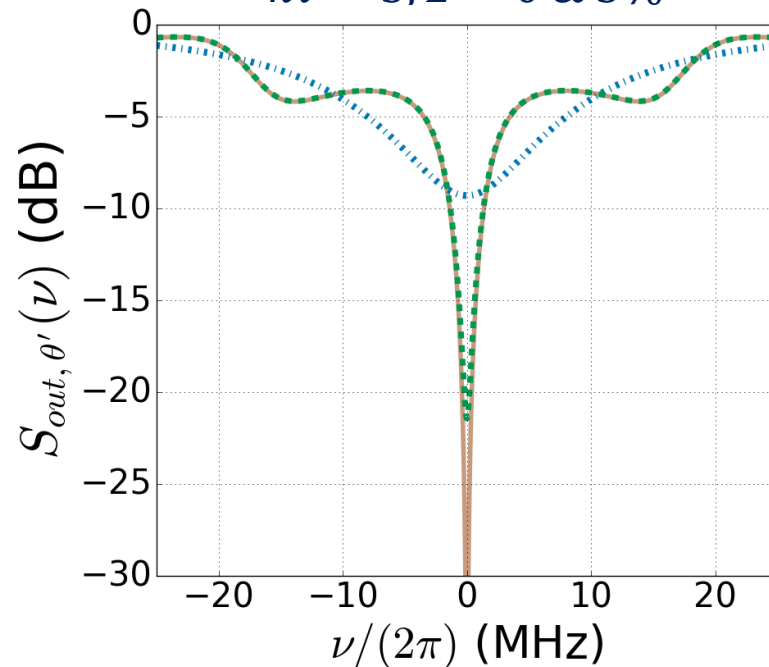
$k = \kappa/2, \phi = \pi$



$k = 0, \phi = 0$



$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa,$
 $\kappa\tau = 3, L = 0 \text{ \& \; } 5\%$

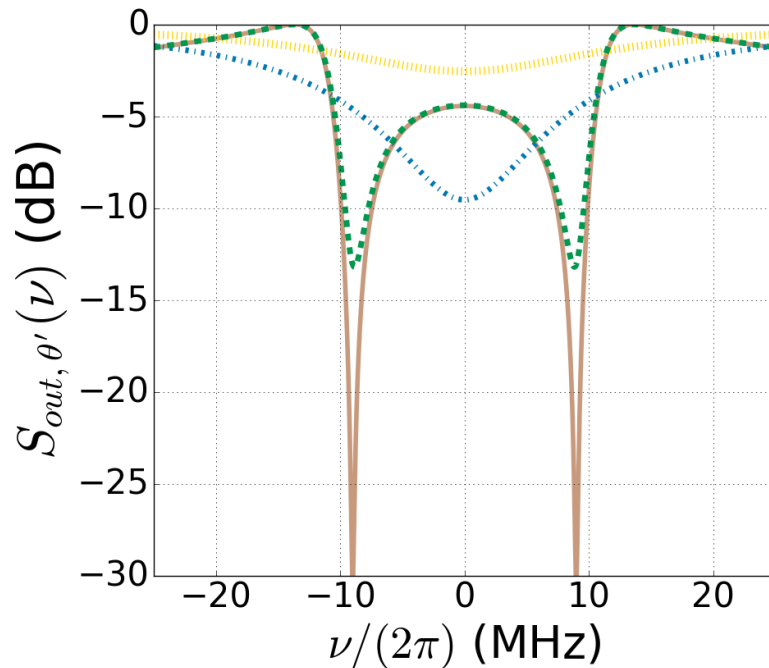




New setup with time-delay

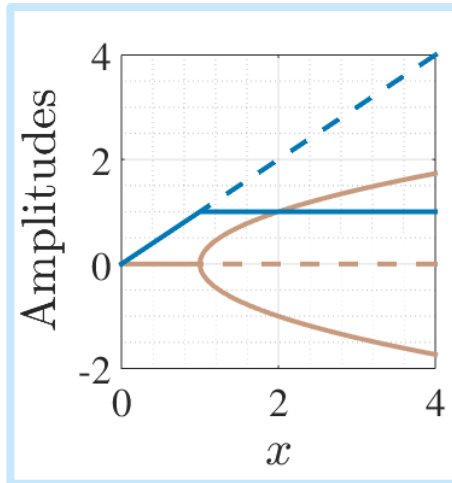
- * Transcendental equations
- * Enhanced squeezing with feedback

$$\phi = 0, \kappa_b = \kappa_c = 0.5\kappa, |\epsilon| = 0.5\kappa, \\ \kappa\tau = 2.3, L = 0 \text{ \& 5\%}$$

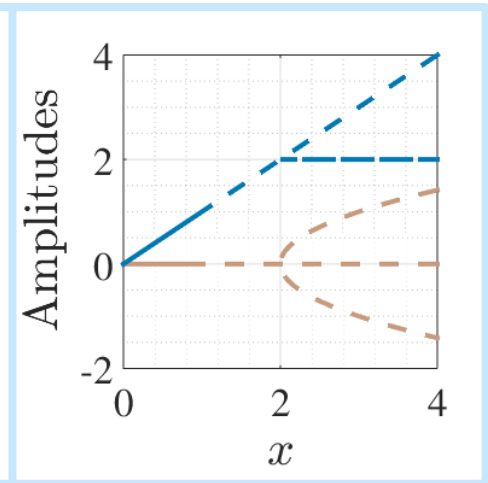


- * Limit cycle is reached on threshold
- * Persistent oscillations with the characteristic frequency

$$k = 0, \phi = 0, \kappa\tau = 0$$



$$k = \kappa, \phi = 0, \kappa\tau = 1.57$$

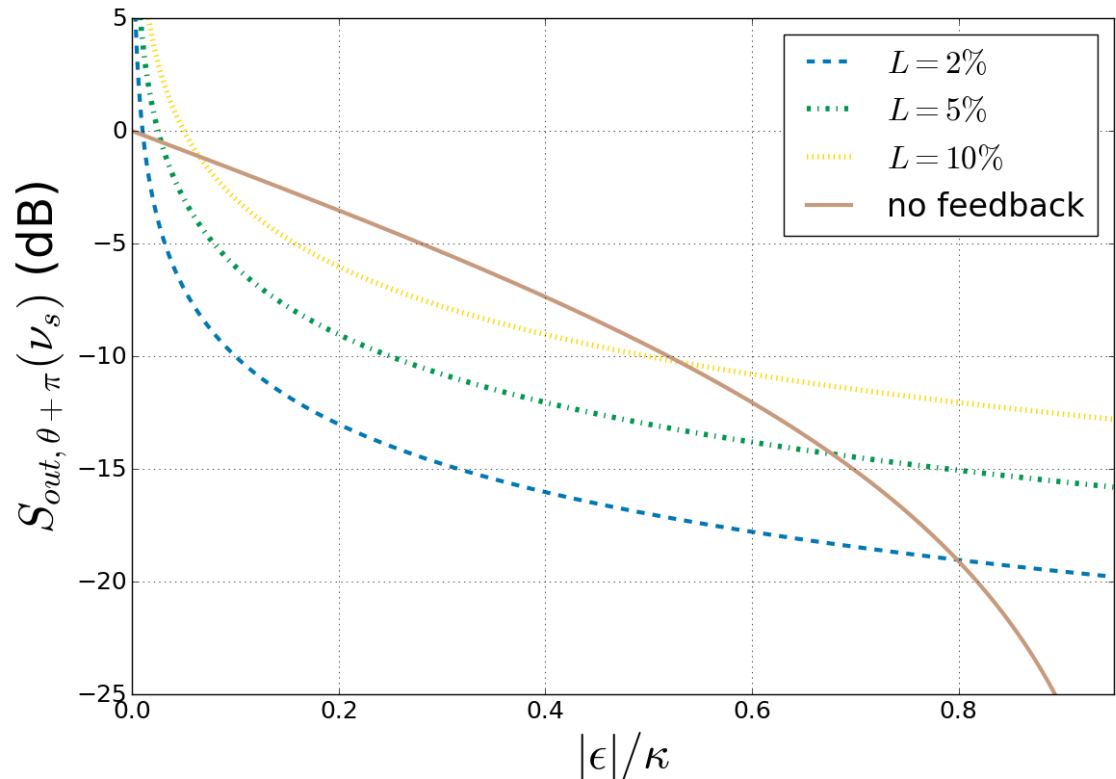


Effects of loss



- * With loss wider range of tunability
- * Quadrature variance at the critical point:

$$\chi_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$$



Summary

- * Coherent feedback:
 - backaction-free way of enhancing useful quantum mechanical behaviours
- * Time-delay:
 - tunable changes in the stability range
- * Time-delayed one-loop setup with a DPA: enhanced squeezing
 - * On and off-resonant squeezing as well
 - * Result of the changing dynamics of the system
- * Loss in the feedback loop:
 - * Decreased squeezing
 - * BUT! wider range of tunability

*Thank you for Your
attention!*

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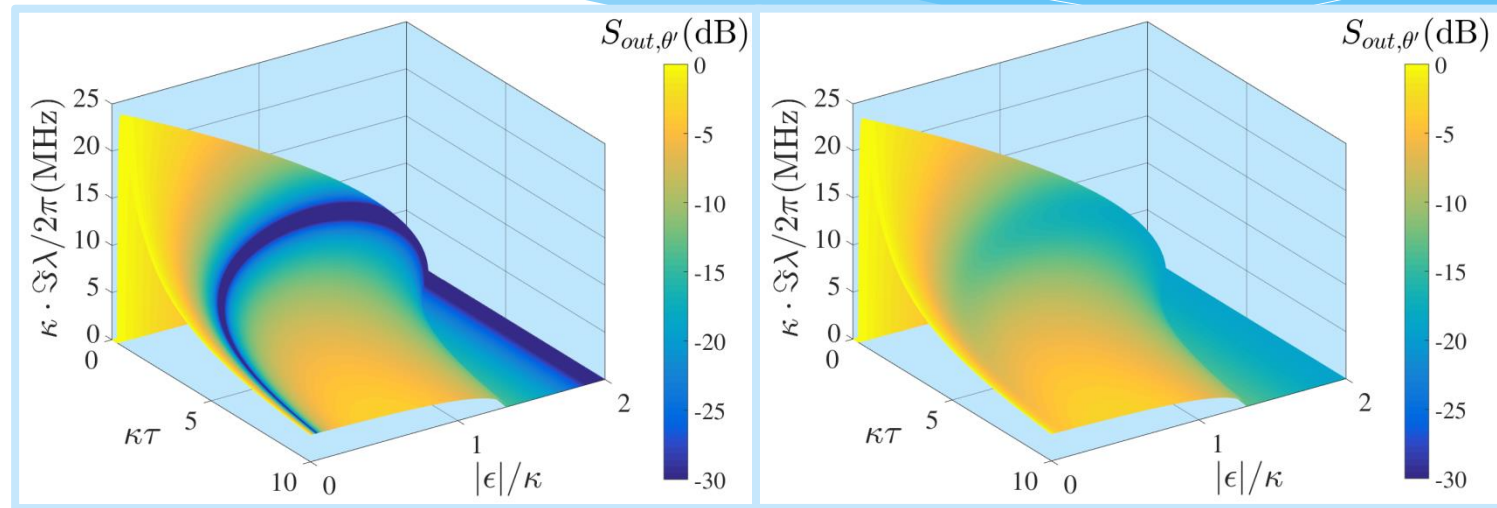
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*Thank you for Your
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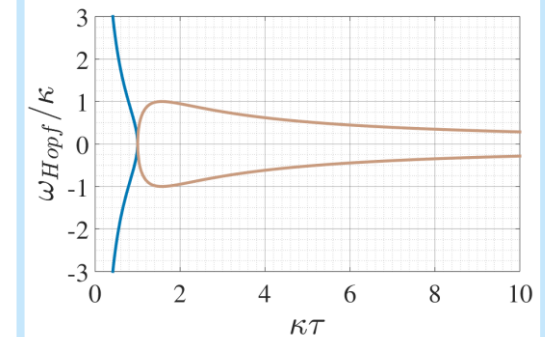
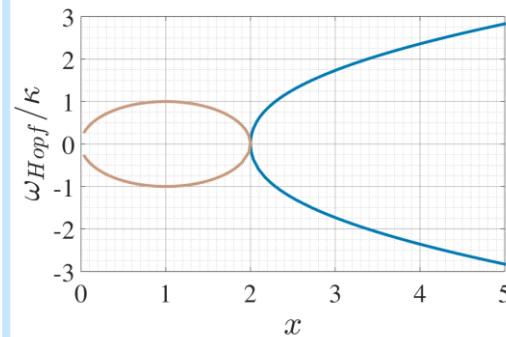
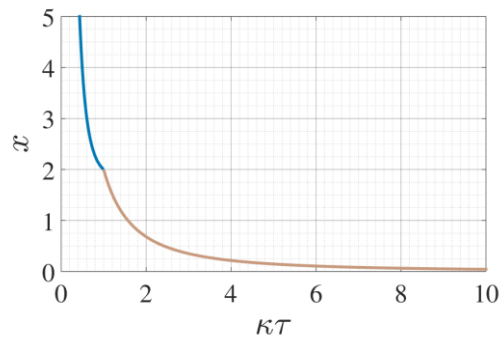




Characteristic frequency range



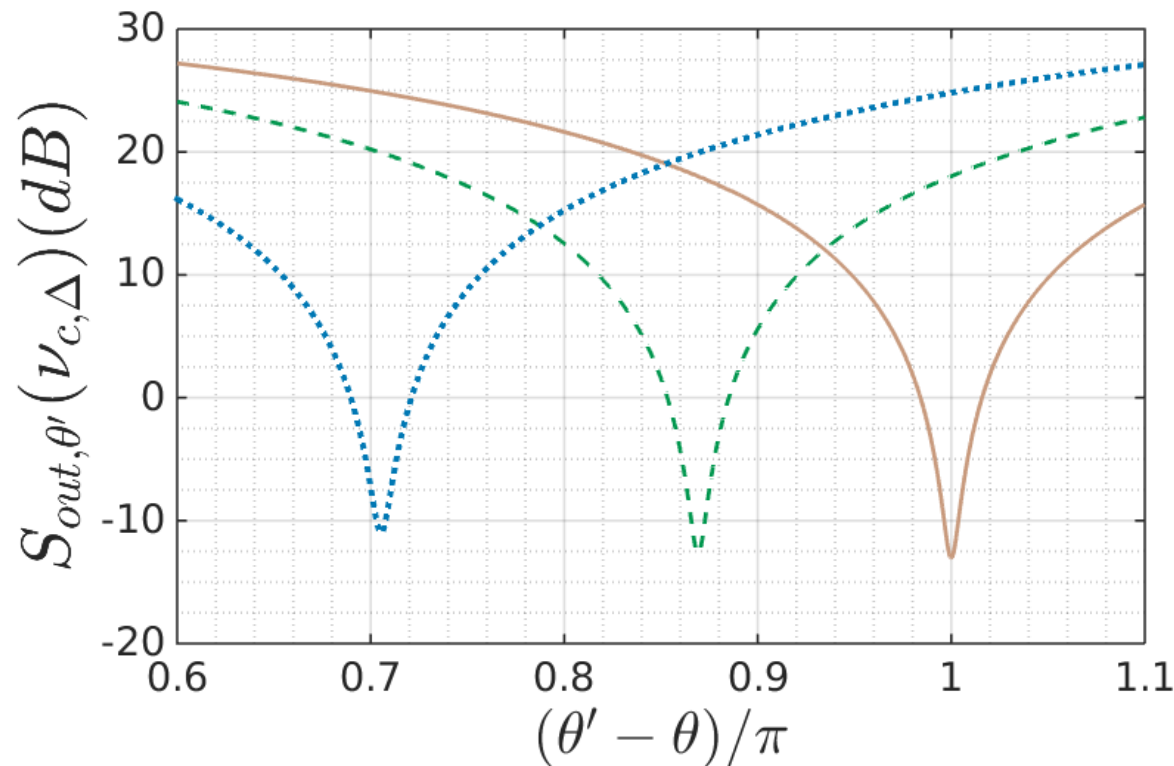
$$0 \leq \nu_s \leq \sqrt{|\epsilon|(2\kappa - |\epsilon|)}, 0 \leq |\epsilon| \leq |\kappa + ke^{i\phi}|$$



Detuning and quadrature angle

- * Local oscillator phase:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_{\Delta}|}\right)$$

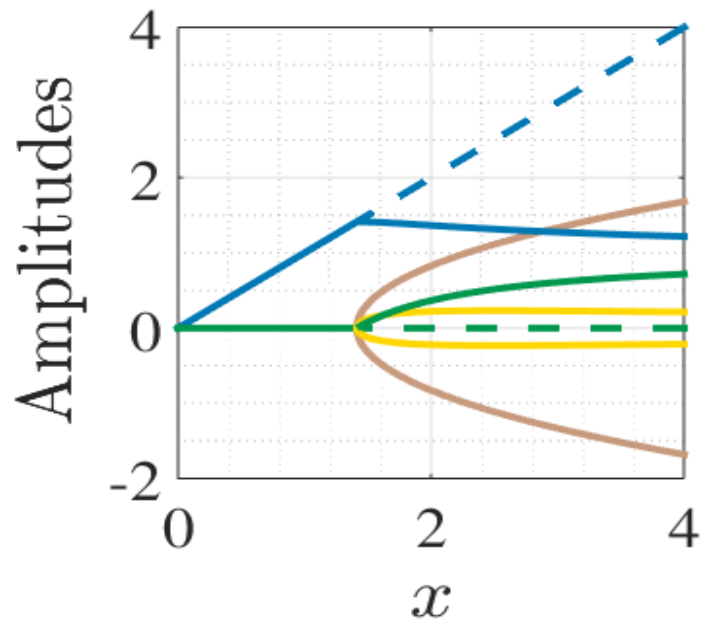


$$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2},$$
$$\kappa\tau = 2.418,$$
$$\nu_{c,\Delta} = 0.866\kappa,$$
$$L = 5\%,$$

$\Delta = 0$ (solid),
 0.1κ (dashed)
 0.2κ (dotted)



Effects of the phase shift

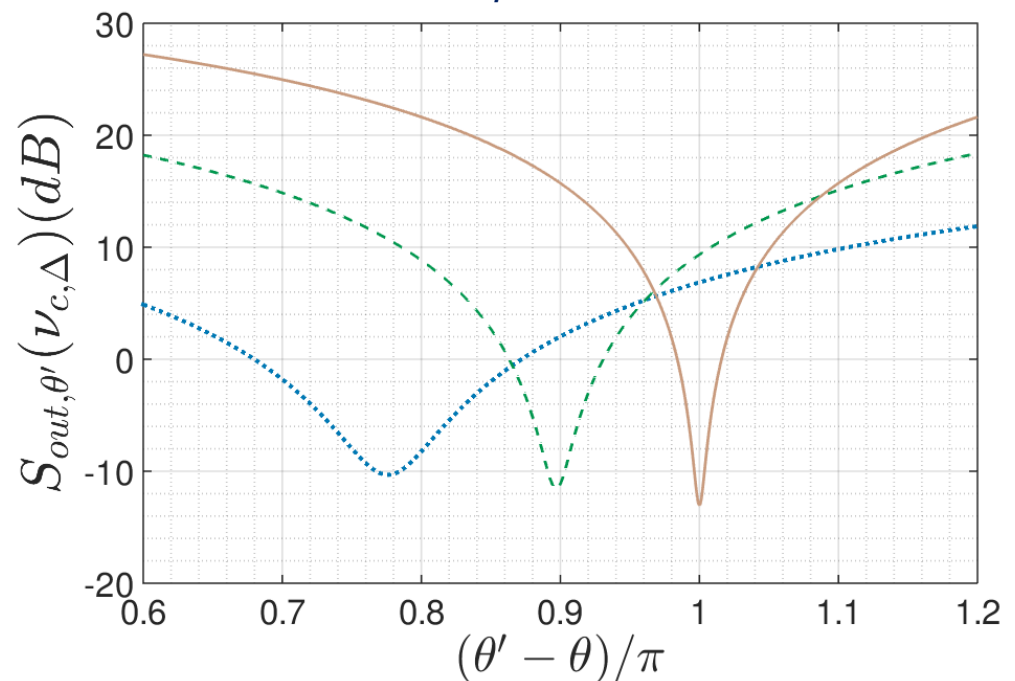


$$k = \kappa, \phi = \pi/2$$

ε_p : real (blue), imag (green)

ε : real (brown), imag (yellow)

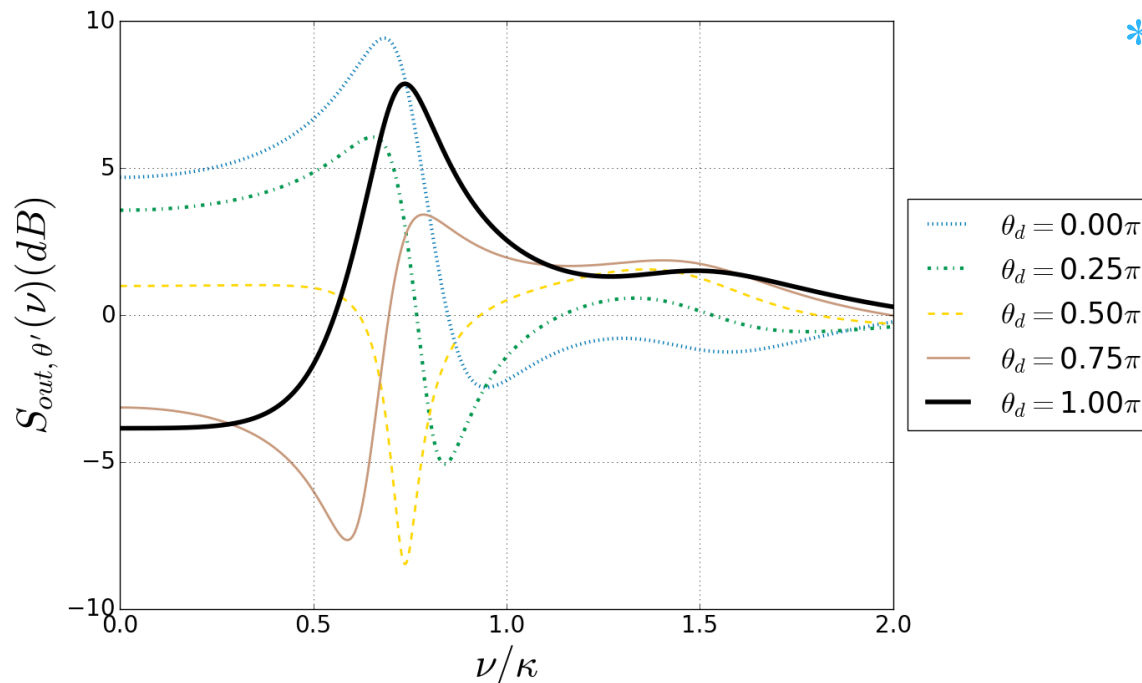
$$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2}, \kappa\tau = 2.418, \nu_{c,\Delta} = 0.866\kappa, \\ L = 5\%, \quad \phi = -0.1\pi, 0.05\pi \text{ \& } 0$$



Gravitational waves



- * Adding π to the local oscillator phase in our case gives the required characteristics



- * Frequency of the best squeezing: changing with the local oscillator phase
- * Lower frequencies: around $\theta' - \theta = 0$
- * Higher frequencies: around $\theta' - \theta = \pi$

Coherent feedback with DPA

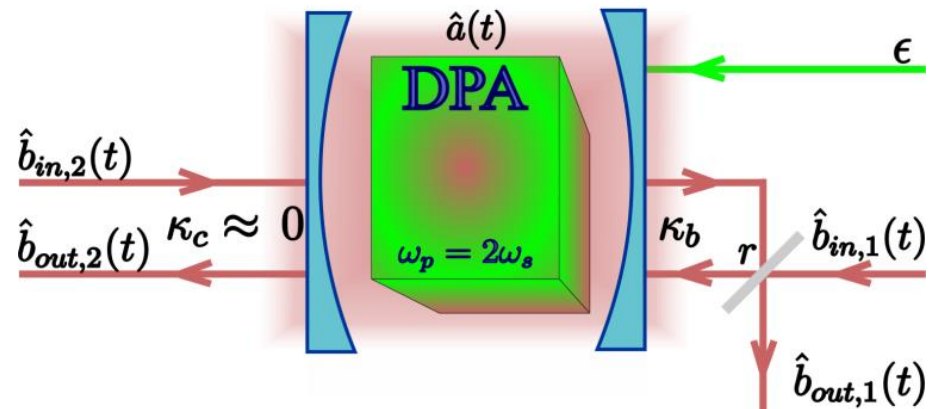


* Single DPA

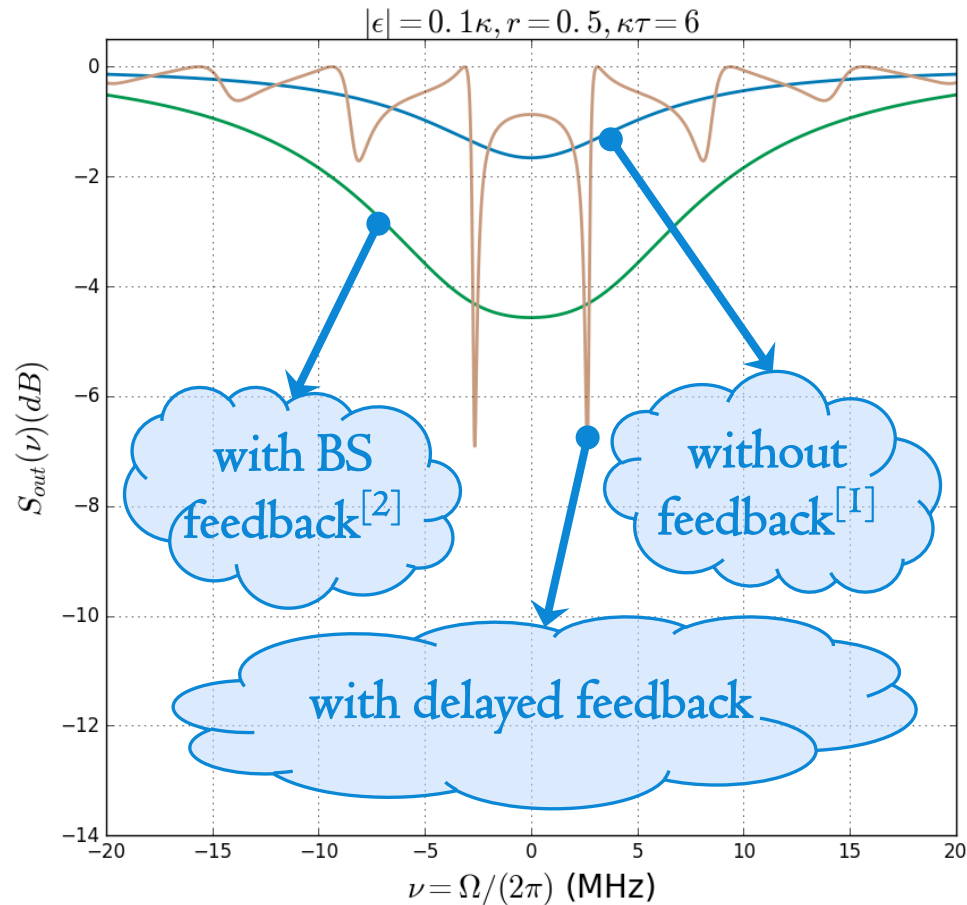
- * Feedback via beam splitter^[1]
- * Enhanced, tunable squeezing at a given driving strength

$$\mathcal{X}_{out,\theta+\pi}(0) = \frac{1}{4} \frac{(\kappa(r) - |\epsilon|)^2}{(\kappa(r) + |\epsilon|)^2}$$

- * Modified threshold
- * Experiment^[2] agrees with theory^[1]
- * BUT! Performance is limited
 - * efficient $|\epsilon|$ range under 0.6κ
 - * losses through the „perfectly reflecting mirror”

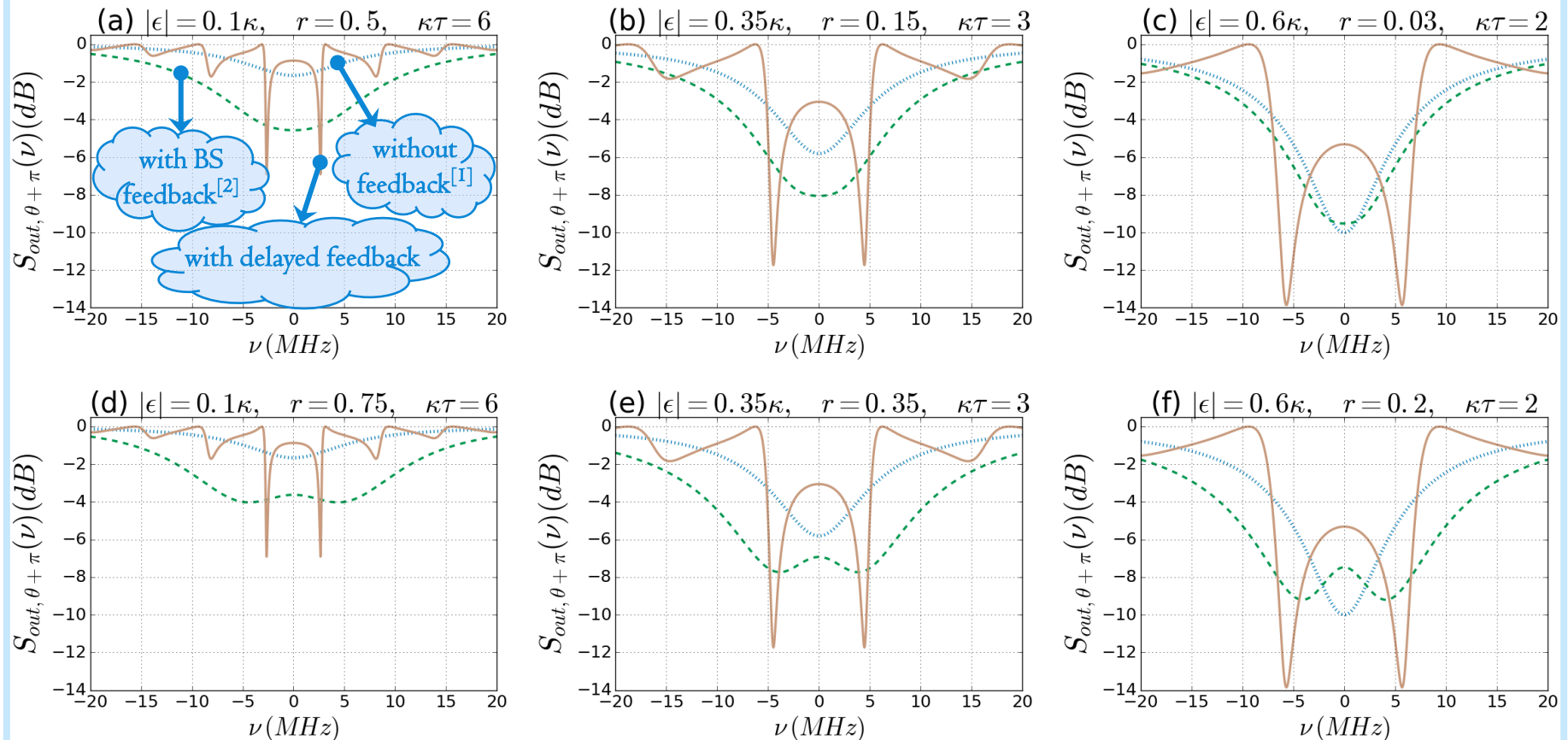


Comparison with previous results^[1,2]



^[1]PRA, 30:1386 (1984), ^[2]IEEE Trans. Auto. Contr., 57:2045 (2012)

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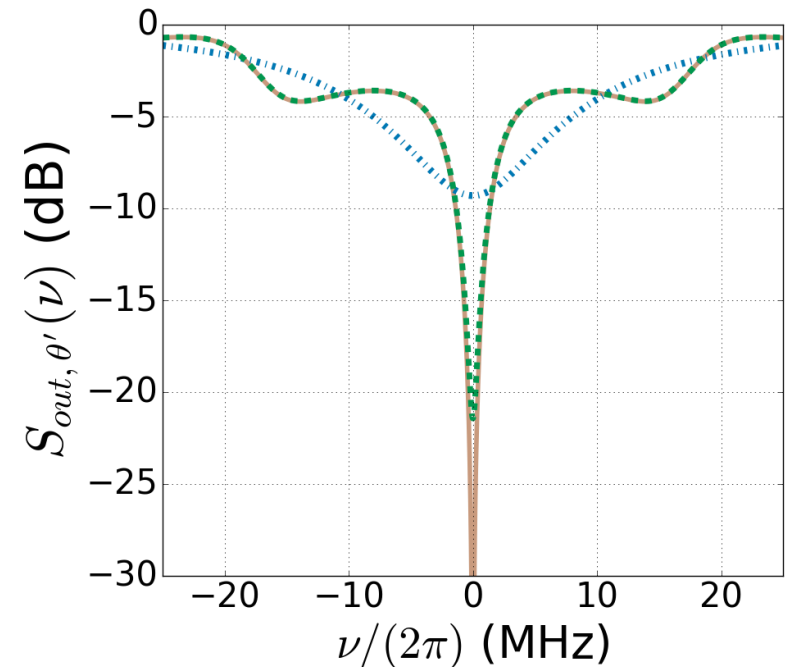
^[1]PRA, 30:1386 (1984), ^[2]IEEE Trans. Auto. Contr., 57:2045 (2012)



New setup with time-delay

- * Transcendental equations
- * Enhanced squeezing with feedback
- * Properties:
 - * Best squeezing is on resonance
 - * Effects of time-delay:
 - * Narrower spectrum
 - * Emerging small side-peaks
 - * Perfect squeezing at stability change
 - * Condition: $\kappa - k = |\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2}$
 - * Local phase: $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
 - * Variance: $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$

$$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa, \\ \kappa\tau = 3, L = 0 \text{ \& \; } 5\%$$

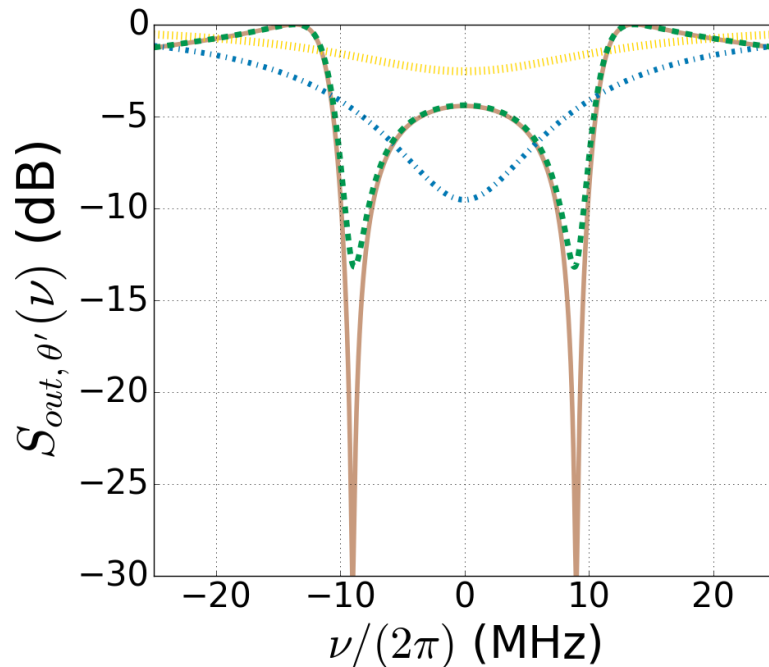




New setup with time-delay

- * Transcendental equations
- * Enhanced squeezing with feedback

$$\phi = 0, \kappa_b = \kappa_c = 0.5\kappa, |\epsilon| = 0.5\kappa, \\ \kappa\tau = 2.3, L = 0 \text{ \& 5\%}$$



* Properties:

- * Best squeezing is off-resonant
- * Perfect squeezing at stability change
 - * Local phase: $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
 - * Time-delay: $\tau_{s,n,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right) + 2n\pi}{\nu_{s,\Delta}}$
 - * Frequency: $\nu_{s,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$
 - * Variance: $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$

Classical analysis

- * Originally: pitchfork bifurcation
- * Time-delay: Changing stability of steady state solutions
↓
Hopf bifurcation
- * Imaginary eigenvalue becomes the frequency of persistent oscillations

