Is delay always harmful to precision?

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Outline

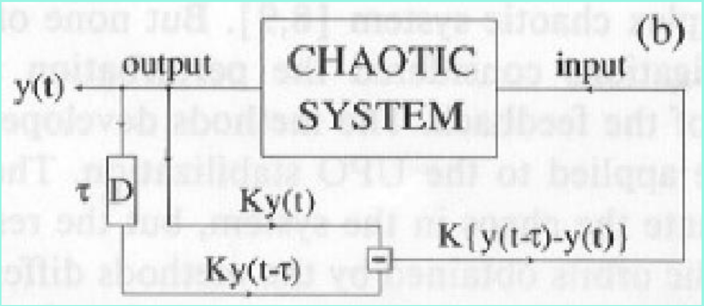
- Time-delayed Coherent feedback
  - Time-delay as a control parameter
  - Non-Markovian environment
- Degenerate Parametric Amplifier (DPA)
  - Without feedback
  - With coherent feedback
- DPA with time-delayed coherent feedback
  - Quantum mechanical model
  - Squeezing spectra
  - Classical analysis

Quantum control

- Open-loop control
  - Reaching the target state relies on the engineering of the desired Hamiltonian
  - No information is obtained about the state of the system
- Closed-loop control
  - Information is obtained about the system's state
  - Adjustment of the parameters accordingly
- Coherent quantum feedback
  - No measurement
  - Coherent evolution of the output preserves the output quantum state
- Measurement-based feedback
  - Coherent classical feedback
  - Measurement backaction

Time-delayed control

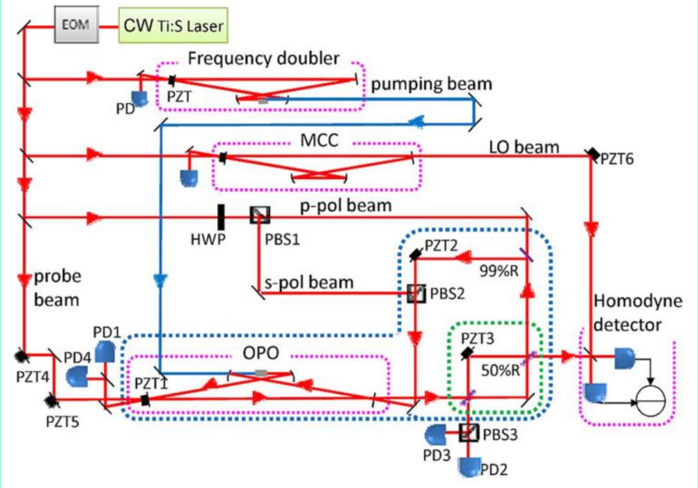
- Classical example
  - Pyragas-type feedback
    - PLA **170**:421 (1992), PRE **72**:046203 (2005)
    - Extra degrees of freedom
    - Alters the relaxation time-scale
    - Stabilizes unstable periodic orbitals



- Semiclassical
  - Stabilized laser dynamics (IEEE J.Q.El. **16**:347 (1980), PRE **78**:056213 (2008))
- Quantum
  - Enhanced entanglement (PRA **91**:052321 (2015))
  - Recovered Rabi oscillations in the single photon limit (PRL **110**:013601 (2013), PRA **92**:053801 (2015))
  - Faster convergence to a desired steady state (NJP **16**:065004 (2014))

Feasibility of feedback with long time-delay

- Long time-delay: $\kappa\tau \approx const \cdot 10^0$
  - $\kappa = 10\ MHz \cdot 2\pi$ (overall cavity decay rate)
- Considered time-delay: $\tau \approx 20ns$
  - in case of free propagation: $l \approx 5m$
- Previous experiment
  - feedback loop loss: $L \approx 5 - 10\%$



- Ideas for experimental realization:
  - These values are for the experiment IEEE Trans. Aut. Contr. **57**:2045 (2012)
  - Other possibilities: Ringresonator with intrinsic nonlinearities (fabricated from crystals) + optical fibre, Circuit QED, Waveguide constructions

Open quantum system[1,2]

- Markovian Master Equation:

$$\frac{d}{dt}\rho_S(t) = \mathcal{L}\rho_S(t) = -i[H(t), \rho_S(t)] + \sum_{j=1}^{N^2-1} \gamma_j \left(A_j \rho_S A_j^\dagger - \frac{1}{2} A_j^\dagger A_j \rho_S - \frac{1}{2} \rho_S A_j^\dagger A_j \right)$$

  - There is a dynamical map $\rho_S(t) = \Phi(t)\rho_S(0)$, which satisfies the semigroup property $\Phi(t_1)\Phi(t_2) = \Phi(t_1 + t_2)$

- Master equation for a general open quantum system:

$$\frac{d}{dt}\rho_S(t) = -i[H_S(t), \rho_S(t)] + \int_{t_0}^{t} \mathcal{K}(t, t_1)\rho_S(t_1)dt_1$$

  - If there exists an invertible dynamical map $\rho_S(t) = \Phi(t, t_0)\rho_S(t_0)$, a time-local master equation can be constructed in Lindblad-like form.

[1] S. Whalen, PhD thesis (2015), [2] H.-P. Breuer, F. Petruccione The theory of open quantum systems (2007),

Description of the environment[1,2]

- Interaction Hamiltonian:

$$H_{SEI}(t) = \sum_j \left(a_j^\dagger B_j(t) + B_j^\dagger(t) a_j\right)$$

  - Environment operators: $B_j(t) = \sum_{\alpha,l_\alpha} \kappa_{j\alpha l_\alpha} \exp\left(i(\omega_j - \omega_{l_\alpha})t\right) b_{\alpha l_\alpha}$

- Spectral density:

$$J_{\alpha jk}(\omega) = \sum_{l_\alpha} \kappa_{j\alpha l_\alpha} \kappa_{k\alpha l_\alpha}^* \, \delta(\omega - \omega_{l_\alpha})$$

- Dissipation kernel:

$$F_{jk}(t_1, t_2) = \left[B_j(t_1), B_k^\dagger(t_2)\right] = e^{i(\omega_j t_1 - \omega_k t_2)} \sum_\alpha \int_{-\infty}^{\infty} J_{\alpha jk}(\omega) e^{-i\omega(t_1 - t_2)} \, d\omega$$

- Noise kernel:

$$G_{jk}(t_1, t_2) = \left\langle B_k^\dagger(t_2) B_j(t_1) \right\rangle = e^{i(\omega_j t_1 - \omega_k t_2)} \sum_\alpha \int_{-\infty}^{\infty} J_{\alpha jk}(\omega) e^{-i\omega(t_1 - t_2)} n_\alpha(\omega) \, d\omega$$

  - Where the thermal distribution is: $n_\alpha(\omega) = \frac{1}{e^{(\omega - \mu_\alpha)/T_\alpha} - 1}$

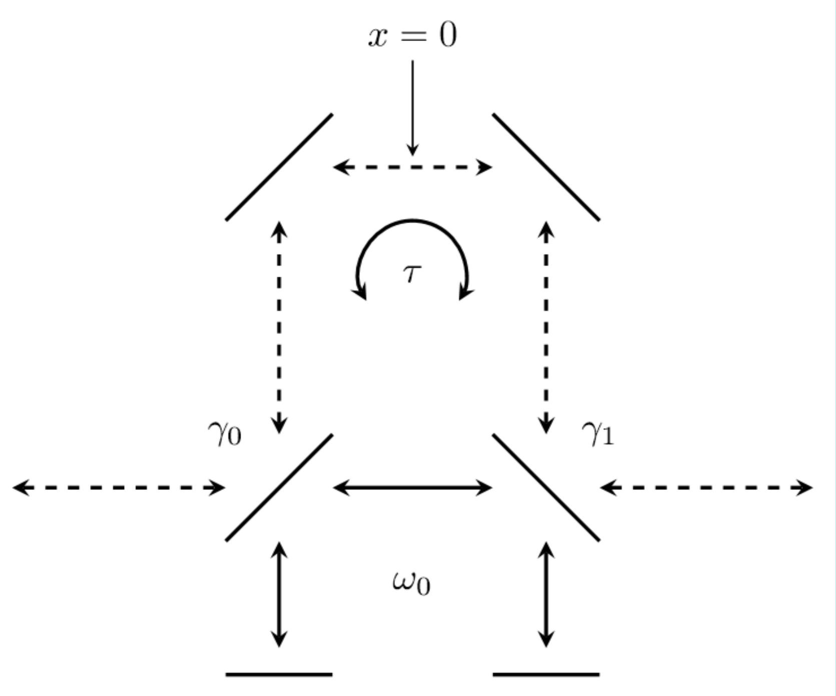
[1] S. Whalen, PhD thesis (2015), [2] S. Whalen, H. Carmichael PRA **93**:063820 (2016)

Time-delayed CQF model system[1]

- Components:
  - Cavity coupled to an environment
  - 1D unipartite environment
    - e.g. optical fibre, waveguide, superconducting transmission line, …
- Lagrangian:

$$\mathcal{L} = \int_{-M/2}^{M/2} \left\{ \frac{1}{2}\left(\frac{\partial}{\partial t}\varphi(x,t)\right)^2 - \frac{1}{2}\left(\frac{\partial}{\partial x}\varphi(x,t)\right)^2 \right\} dx$$

  - Massless scalar field
  - Euler-Lagrange: wave equation



- Interaction Hamiltonian (RWA): $H_{SE} = \sum_{l=-\infty}^{\infty}\left(\kappa_l a^\dagger b_l + \kappa_l^* b_l^\dagger a\right)$
  - With $\kappa_l = \sqrt{\frac{\gamma_0}{2M}} e^{-i\, sgn(l)\left(\frac{|\omega_l|\tau}{2} - \phi_0\right)} + \sqrt{\frac{\gamma_1}{2M}} e^{i\, sgn(l)\left(\frac{|\omega_l|\tau}{2} + \phi_1\right)}$
- Spectral density: $J(\omega) = \theta(\omega + \omega_0)\left(\frac{\gamma_0 + \gamma_1}{2\pi} + \frac{\sqrt{\gamma_0\gamma_1}}{\pi}\cos(\omega\tau + \phi)\right)$

[1] S. Whalen, PhD thesis (2015)

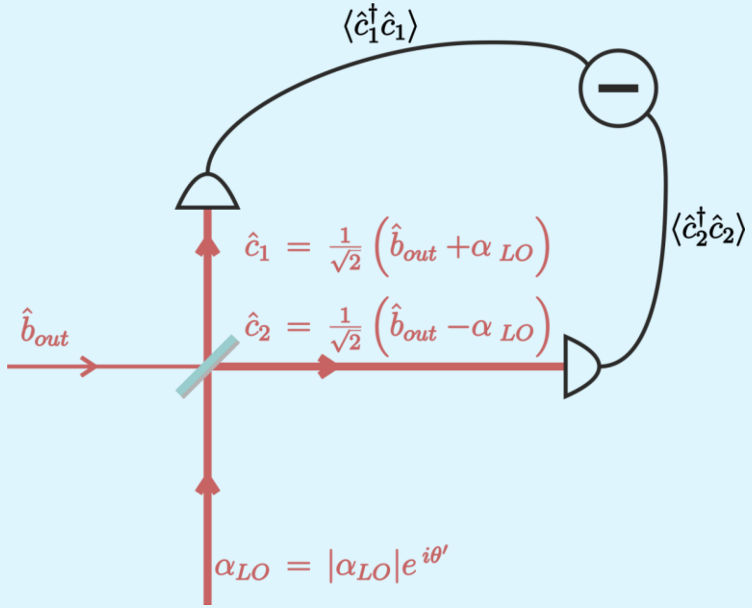
Optical squeezing

- Quadratures: Observables defined by homodyne detection

$$\tilde{X}_{out,\theta'}(\nu) = \frac{1}{2}\left(e^{-i\frac{\theta'}{2}}\tilde{b}_{out}(\nu) + e^{i\frac{\theta'}{2}}\tilde{b}^{\dagger}{}_{out}(-\nu)\right)$$

  - Generalizations of position and momentum

- Local oscillator determines:
  - Quadrature angle ⇔ phase
  - Frequency

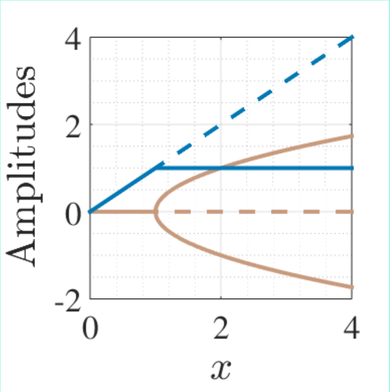


- Optical squeezing:
  - reduction of a quadrature's variance ($\langle\langle a, b\rangle\rangle = \langle ab\rangle - \langle a\rangle\langle b\rangle$) at the expense of the other's

$$\mathcal{X}_{out,\theta'}(\nu) = \int \langle\langle \tilde{X}_{out,\theta'}(\nu), \tilde{X}_{out,\theta'}(\nu')\rangle\rangle d\nu' < \frac{1}{4} \quad \& \quad \mathcal{X}_{out,\theta'+\pi}(\nu) > \frac{1}{4}$$

Degenerate Parametric Amplifier

- Crystal with $\chi^{(2)}$ nonlinearity → parametric down-conversion
- $\omega_p = 2\omega_s$ → phase-sensitive amplification → squeezed light
- Classical dynamics:
  - Saturation of pump: threshold behaviour
  - Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$

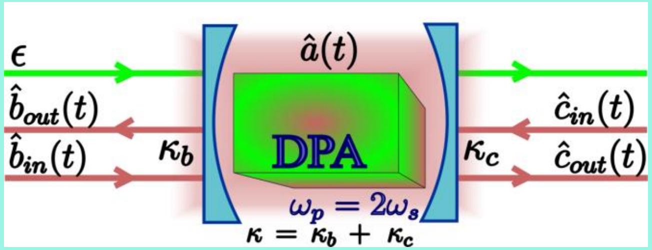


- Quantum mechanical behaviour[1]:
  - Parametric pump approximation $(\epsilon = |\epsilon| e^{i\theta})$

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2} i\hbar\left(\epsilon\left(\hat{a}^\dagger\right)^2 + \epsilon^*(\hat{a})^2\right)$$

  - One-sided cavity quadrature variance:

$$X_{out,\theta+\pi}(0) = \frac{1}{4}\frac{(\kappa - |\epsilon|)^2}{(\kappa + |\epsilon|)^2}$$



  - Symmetric cavity squeezing on resonance:

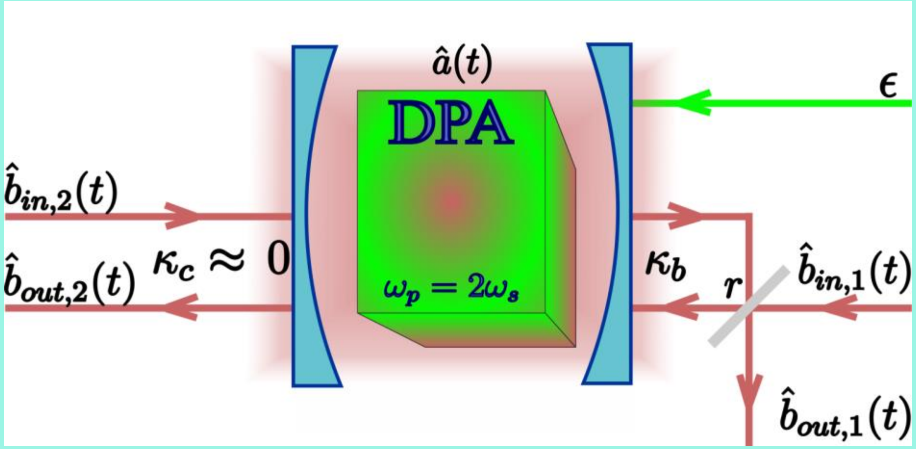
$$X_{out,\theta+\pi}(0) = \frac{1}{4}\frac{\kappa^2 + |\epsilon|^2}{(\kappa + |\epsilon|)^2}$$

[1] M. Collett, C. Gardiner, PRA, **30**:1386 (1984)

Coherent feedback with DPA[1,2]

- Single DPA[1,2]
  - Feedback via beam splitter[1]
  - Undepleted pump approximation
  - Enhanced, tunable squeezing at a given driving strength

    $$\chi_{out,\theta+\pi}(0) = \frac{1}{4}\frac{(\kappa(r)-|\epsilon|)^2}{(\kappa(r)+|\epsilon|)^2}$$
  - Modified threshold
  - Experiment[2] agrees with theory[1]
  - BUT! Performance is limited
    - Efficient $|\epsilon|$ range under 0.6κ in a realistic setup
    - Losses through the perfectly reflecting mirror



- Two DPAs[3]
  - Plant-controller experimental setup
  - Properties depend on the phase shift corresponding to the connecting „wire”
  - Best squeezing shifted from resonance for the „destructive interference” case

[1] PRA, **80**:042107 (2009), [2]IEEE Trans. Auto. Contr., **57**:2045 (2012), [3]Opt. Exp., **21**:18371 (2013)

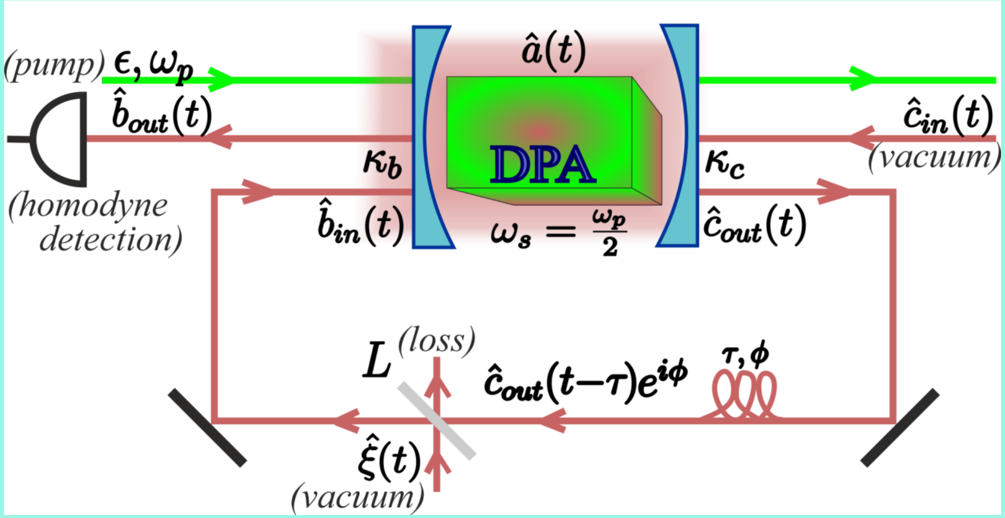
DPA with time-delayed coherent feedback

- Non-Markovian system
  - General equation of motion in case of a single mode:

$$\frac{d}{dt}\hat{a}(t) = -i\Delta\hat{a}(t) + \int_0^t F(t-t_1)\hat{a}(t_1)dt_1 - i\left(\Omega(t)1 + \hat{B}(t)\right)$$

$$F(t-t_1) = [\hat{B}(t), \hat{B}^\dagger(t_1)] =$$
$$= \sqrt{\gamma_0\gamma_1}e^{i\phi}\delta(t-\tau-t_1) + (\gamma_0+\gamma_1)\delta(t-t_1) + \sqrt{\gamma_0\gamma_1}ke^{-i\phi}\delta(t+\tau-t_1)$$

- Uncomplicated setup
  - Characteristics:
    - Time-delay τ
    - Overall phase-shift ϕ
    - Overall loss, decoherence: $L, \hat{\xi}$
    - Homodyne detection with phase θ'
  - $\sqrt{\gamma_0\gamma_1} \to 2\sqrt{\frac{\kappa_b}{\kappa}\frac{\kappa_c}{\kappa}(1-L)}$
  - Overall input field:



$$\hat{B}(t) = \sqrt{2\kappa}\hat{a}_{in}(t) = \sqrt{2\kappa_c}\hat{c}_{in}(t) + \sqrt{2\kappa_b}\overbrace{\left(\sqrt{1-L}e^{i\phi}\hat{c}_{in}(t-\tau) + \sqrt{L}\hat{\xi}(t)\right)}^{\tilde{b}_{in}(t)}$$

[1] N. N., S. Parkins, PRA, **94**:023809 (2016)

DPA with time-delayed coherent feedback

- Equation of motion of the subharmonic mode in the rotating frame

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - \kappa\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - e^{i\phi}k\hat{a}(t-\tau)(\theta(t-\tau))$$

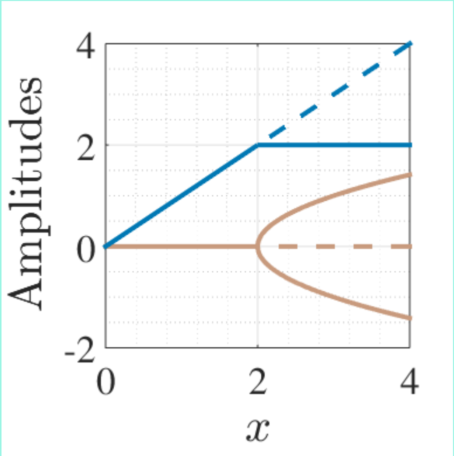
$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

$$\kappa = \kappa_b + \kappa_c, \qquad k = 2\sqrt{\kappa_b\kappa_c(1-L)}$$

- Without delay:
  - Best squeezing on resonance

$$X_{out,\theta+\pi}(0) = \frac{1}{4}\frac{((\kappa + k\cos\phi) - |\epsilon|)^2}{((\kappa + k\cos\phi) + |\epsilon|)^2}$$

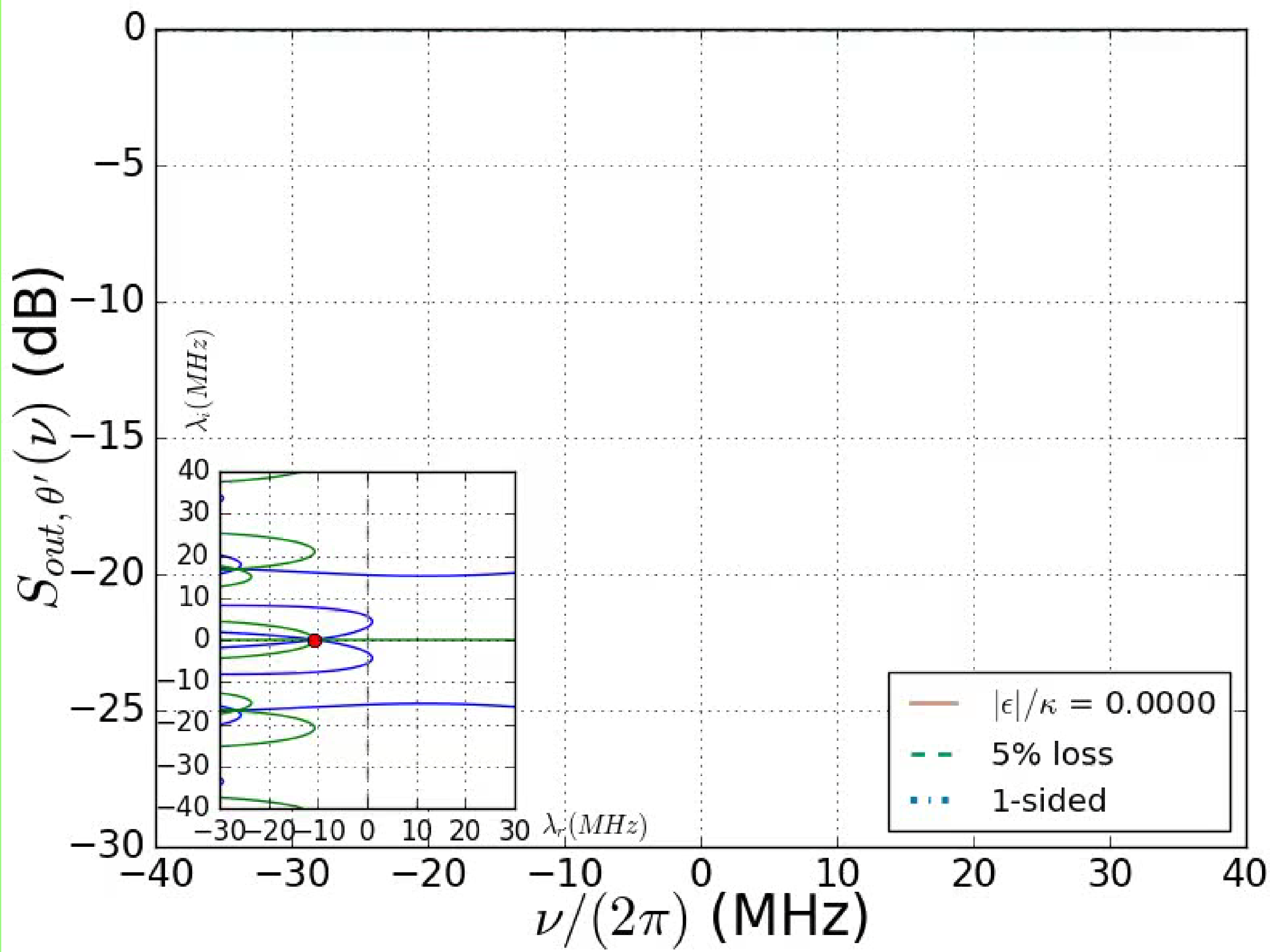
  - Shifted threshold



- With delay:
  - Best squeezing: for $\phi = \pi$ on resonance, for $\phi = 0$ in sidebands
  - Characteristic combination of time-delay, pump strength and frequency

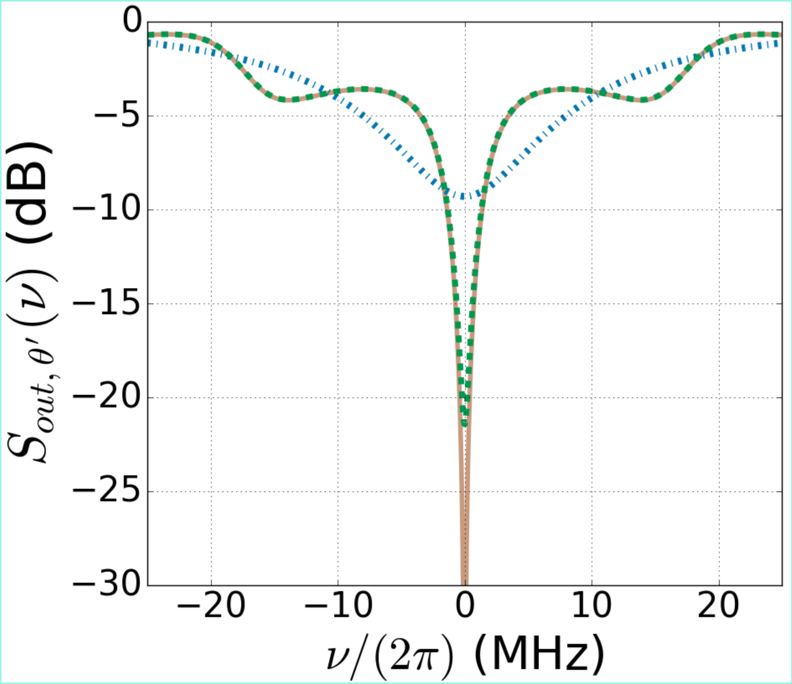
[1] N. N., S. Parkins, PRA, **94**:023809 (2016)

Squeezing spectra for $\phi = \pi$



Squeezing for $\phi = \pi$

- Enhanced squeezing compared to the one-sided case without feedback
- Best squeezing is on resonance
  - Reduced threshold pump power
  - No significant change in case of a finite time-delay
  - Perfect squeezing is related to change in stability
  - $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
- Stable, while $0 \leq |\epsilon| \leq \kappa - k$



$|\epsilon| = 0.49\kappa, \kappa_b = 0.933\kappa,$
$\kappa\tau = 3, L = 0$ & 5%

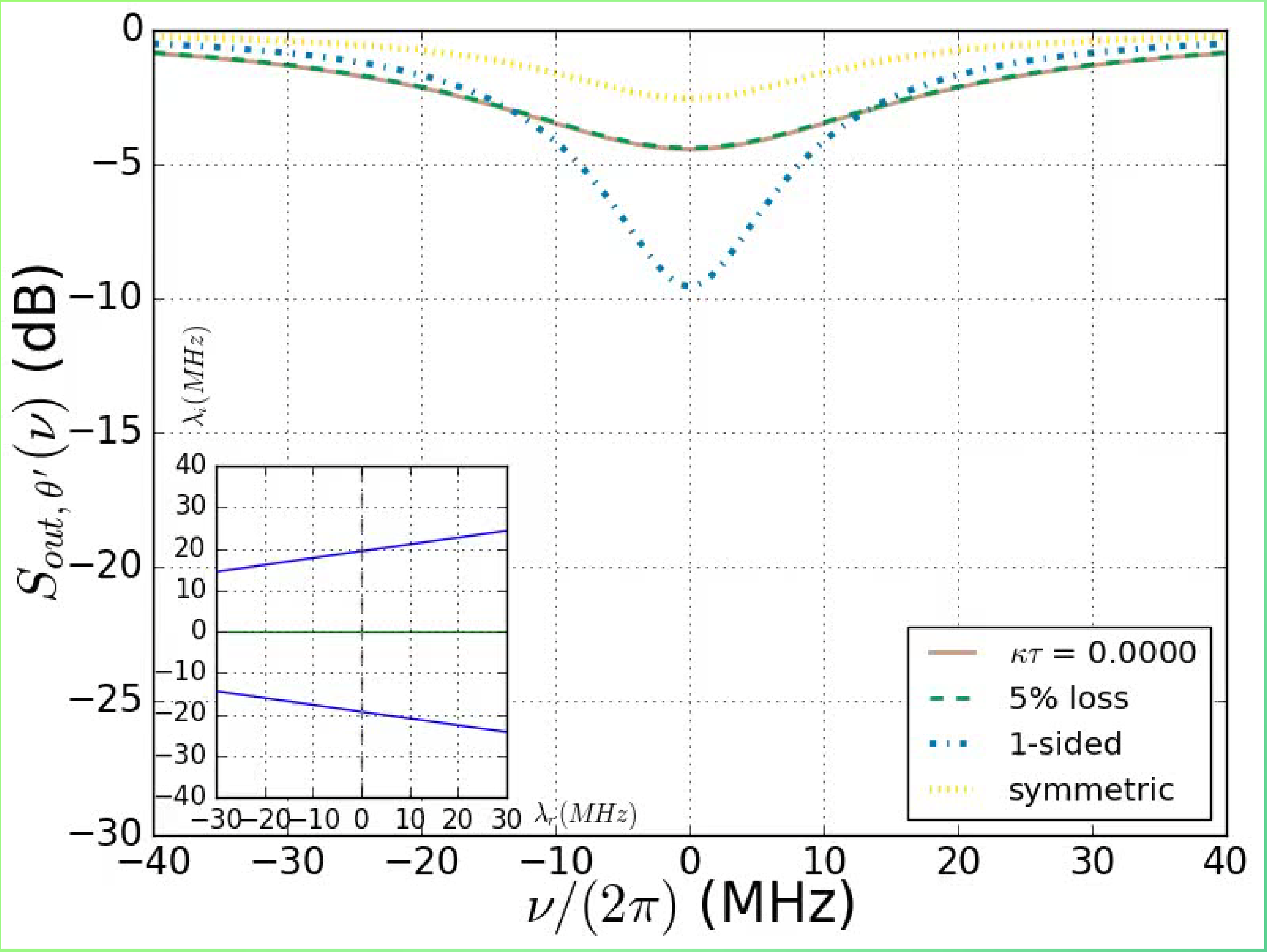
- Pyragas-type feedback[I], where the equation of motion:

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - (\kappa - k)\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - k\big(\hat{a}(t) - \hat{a}(t-\tau)\big)$$

$$\boxed{\begin{aligned} |\epsilon_\Delta| &= \sqrt{|\epsilon|^2 - \Delta^2} \\ k &= 2\sqrt{\kappa_b\kappa_c(1-L)} \end{aligned}}$$

[I] K. Pyragas, PLA, **170**:421 (1992)

Squeezing spectra for $\phi = 0$



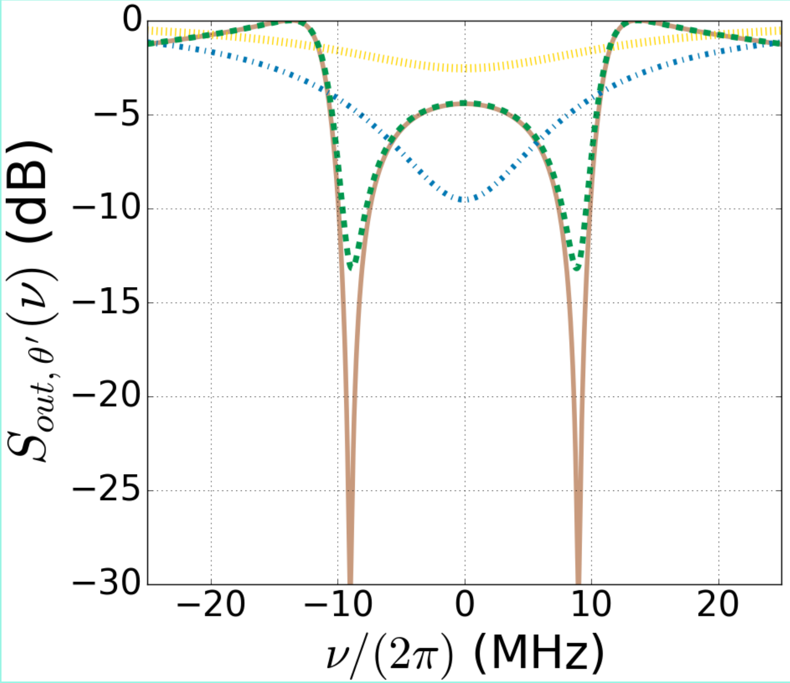
Squeezing for $\phi = 0$

- Enhanced squeezing for a symmetric cavity compared to the one-sided case without feedback
- Best squeezing is off-resonant
  - Reduced threshold pump power
  - Time-delay dependent peaks
  - Perfect squeezing is related to change in stability
- Characteristic quantities:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$$

$$\tau_{c,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right)}{\nu_{s,\Delta}}$$

$$\nu_{c,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$$



$$|\epsilon| = 0.5\kappa, \kappa_b = \kappa_c = 0.5\kappa,$$
$$\kappa\tau = 2.3, L = 0 \;\&\; 5\%$$

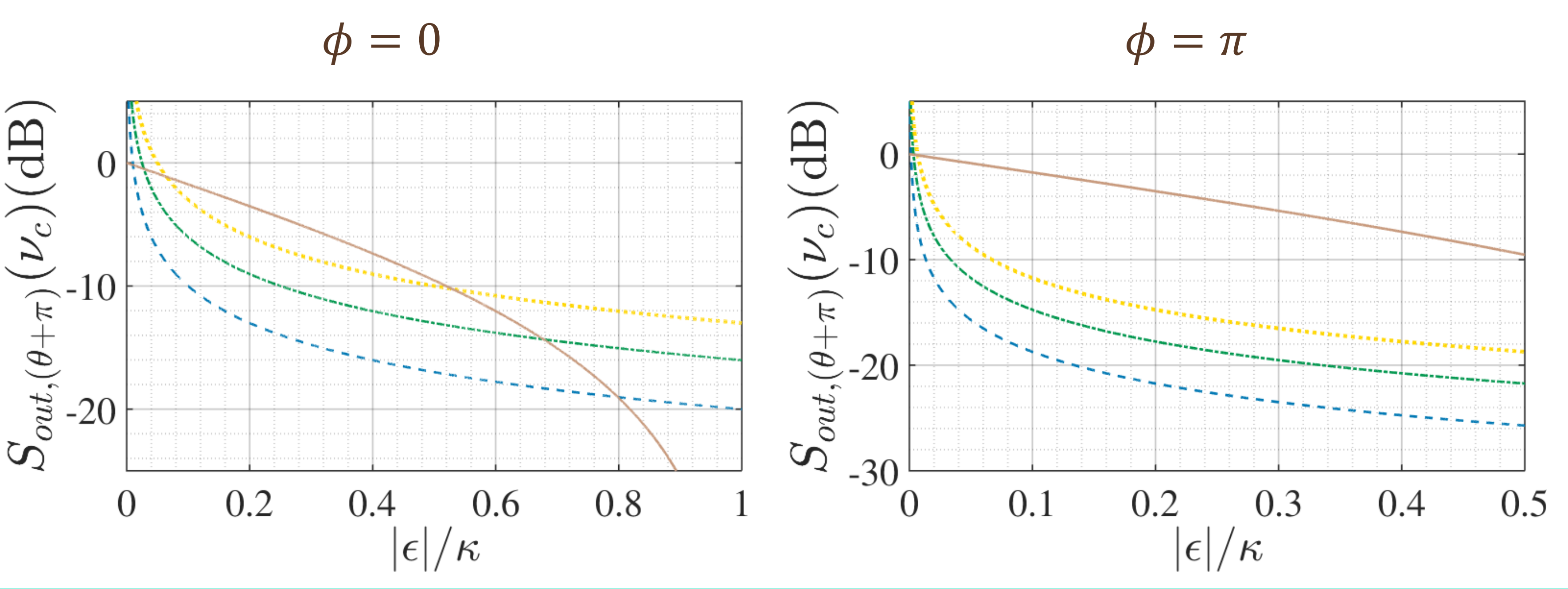
$$|\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2}$$
$$k = 2\sqrt{\kappa_b \kappa_c (1 - L)}$$

Effect of loss in the feedback loop

- Quadrature variance in the lossy case:

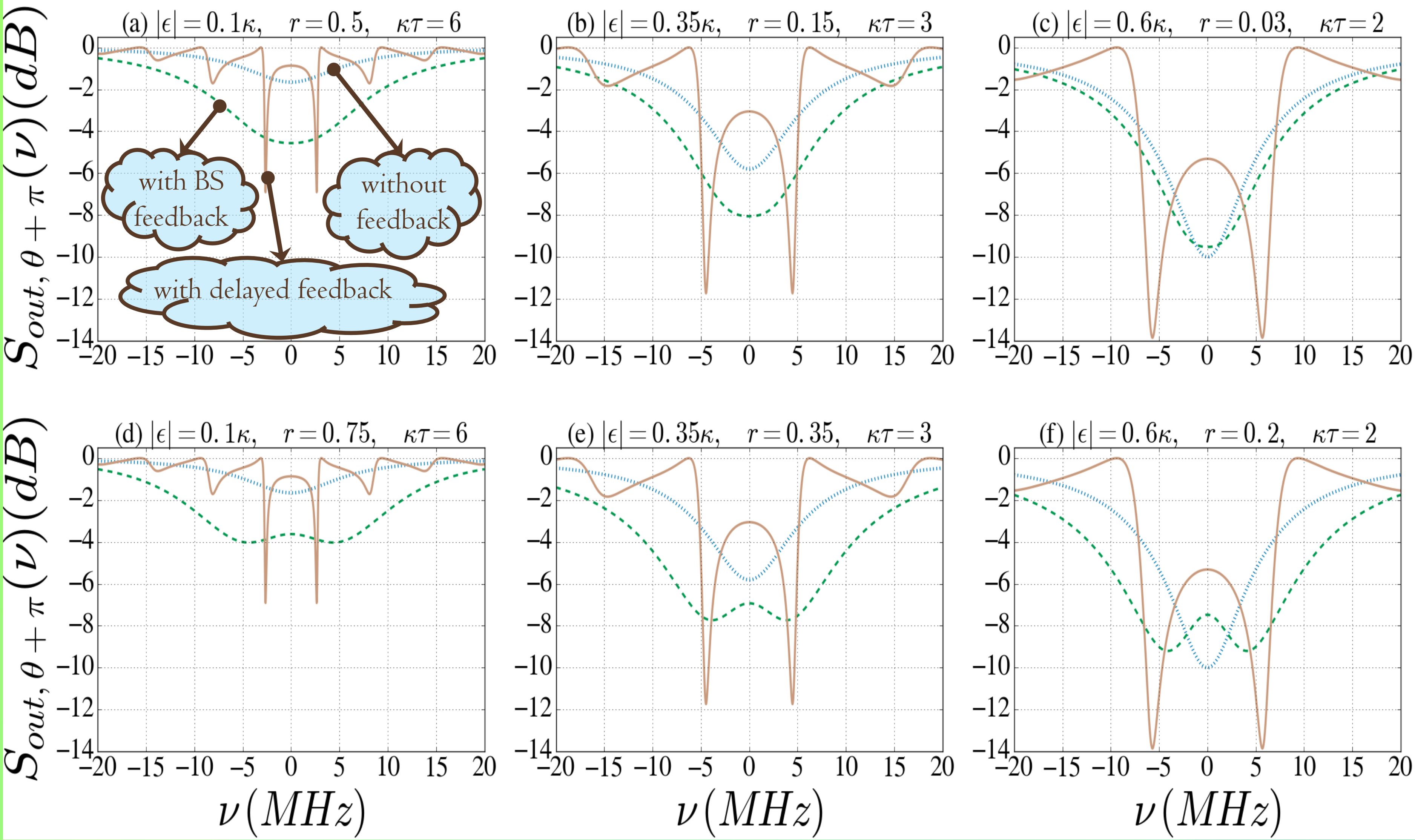
$$X_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4}\frac{L\kappa_c}{|\epsilon_\Delta|}$$

- Wider range of fluctuations are allowed in time-delay, detuning, phase shift, etc.



[1] N. N., S. Parkins, PRA, **94**:023809 (2016)

Comparison with previous results[1,2]

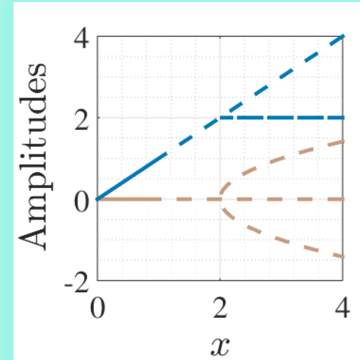


[1] J. E. Gough, S. Wildfeuer, PRA, **80**:042107 (2009), [2] A. Furusawa's group, IEEE Trans. Auto. Contr., **57**:2045 (2012)

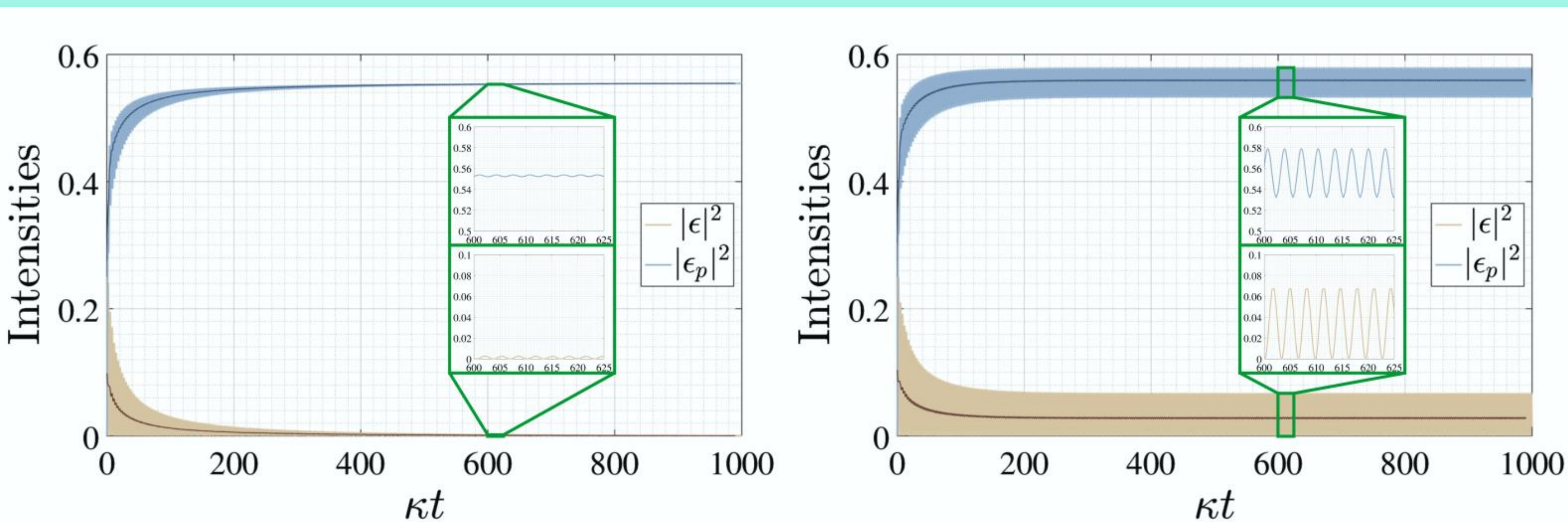
Classical analysis[1]

$$\dot{\varepsilon} = -\kappa\varepsilon(t) + \kappa\varepsilon^*(t)\varepsilon_p(\mathrm{t}) - \mathrm{e}^{i\phi}k\varepsilon(\mathrm{t}-\tau)$$

$$\dot{\varepsilon_p} = -\kappa_p\big(\varepsilon_p(t) + \varepsilon^2(t) - x\big)^{[2]}$$



- Threshold: changing stability
  - Pitchfork bifurcation: different steady states
  - Hopf bifurcation: reaching a limit cycle
- Imaginary eigenvalue becomes the frequency of persistent oscillations



[1]Matlab package DDE-BIFTOOL, and solver dde23,[2]H. Carmichael: Statistical Methods in Quantum Optics 2 (2009)

Summary

- Coherent quantum feedback:
  - Backaction-free way of controlling a quantum systems
- Influence of time-delay:
  - Memory effect in the environment
  - Changes in the stability landscape
- Time-delayed one-loop setup with a DPA: dramatically enhanced squeezing
  - Enhancement even compared to previous feedback setups
  - Result of the changing dynamics of the system
- Loss in the feedback loop decreases the degree of squeezing, but also provides higher tunability
- Future plans:
  - General linear model for two different modes
  - Nonlinear systems

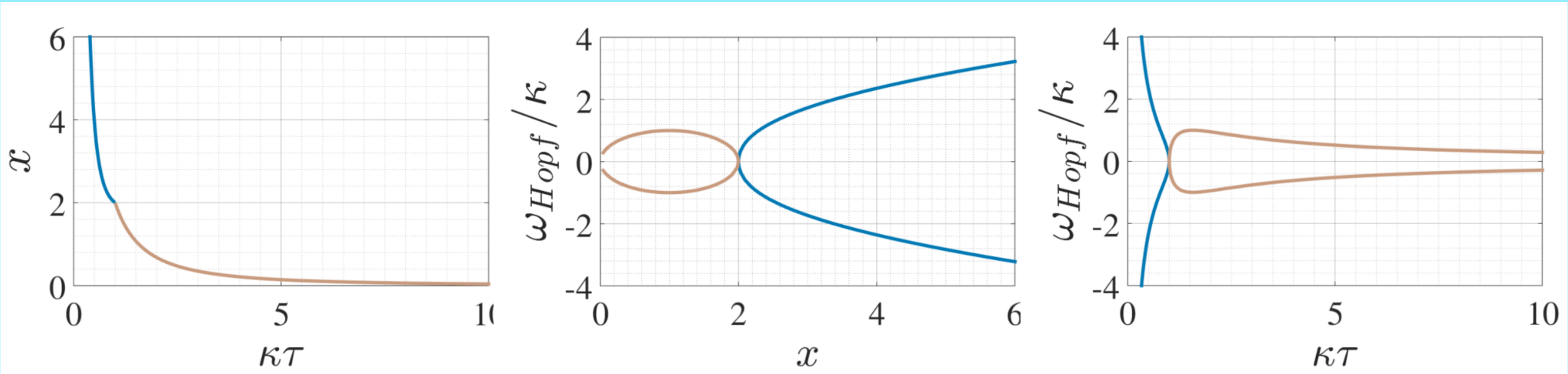
1 2 3 4 5 6 7 8 9 10 11 12 13

14 15 16 17 18

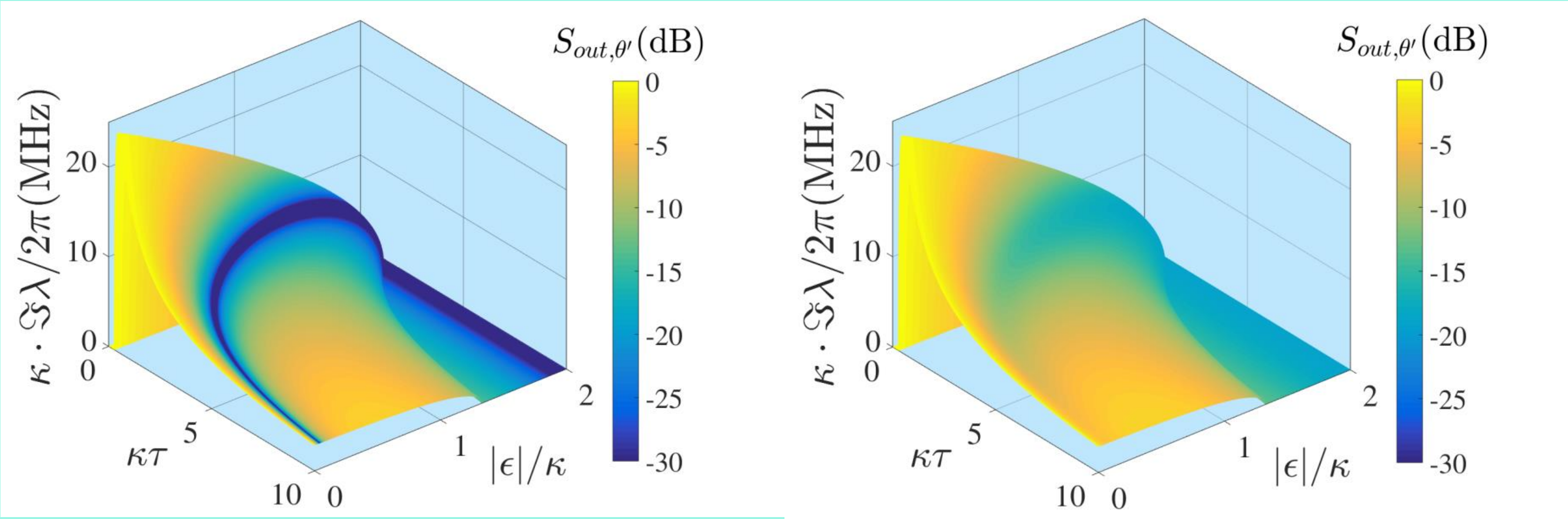
Thank you for your attention!




Tunability



$$0 \leq \nu_s \leq \sqrt{|\epsilon|(2\kappa - |\epsilon|)}, 0 \leq |\epsilon| \leq \left|\kappa + ke^{i\phi}\right|$$



Effects of detuning

- Characteristic quantities:

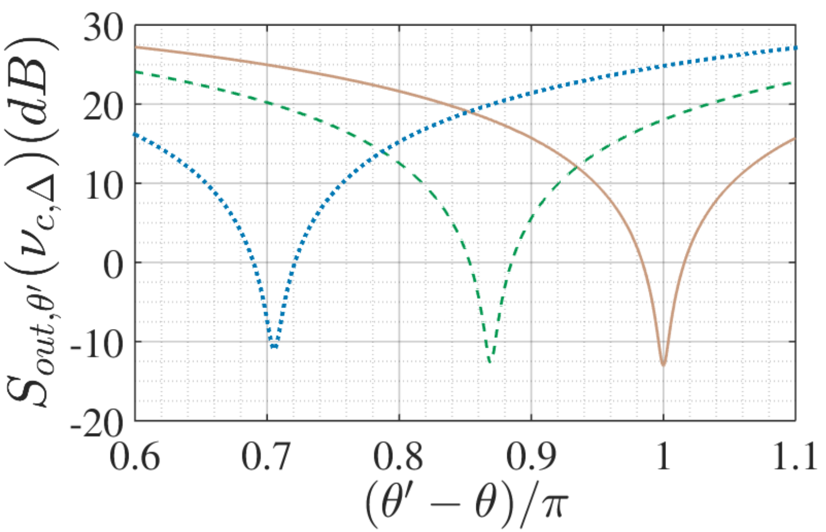
$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$$

$$\tau_{s,n,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right) + 2n\pi}{\nu_{s,\Delta}}$$

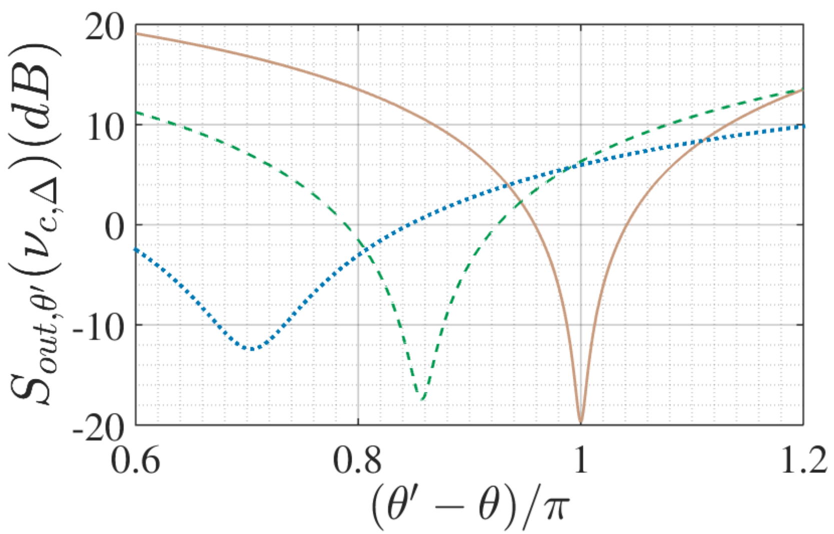
$$\nu_{s,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$$

- In the figures below:
  - $|\epsilon| = const.$
    - $\Delta = 0$ (solid)
    - $\Delta = 0.1\kappa$ (dashed)
    - $\Delta = 0.2\kappa$ (dotted)
  - Optimal frequency and time-delay

$\phi = 0$



$\phi = \pi$



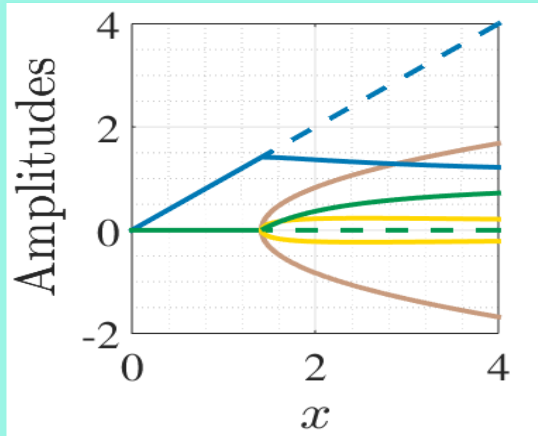
Effects of a phase shift

- In this figure:

  $$k = \kappa, \phi = \frac{\pi}{2}$$

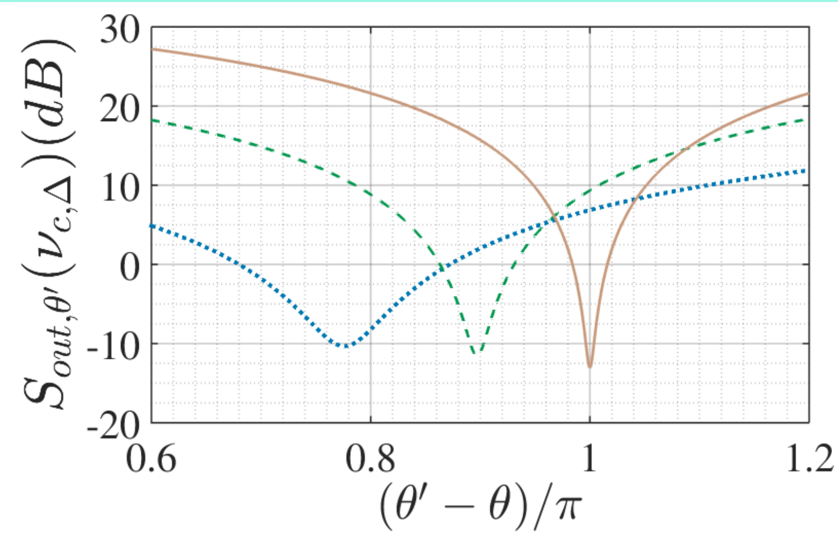
  ε_p: real (blue), imag (green)

  ε: real (brown), imag (yellow)

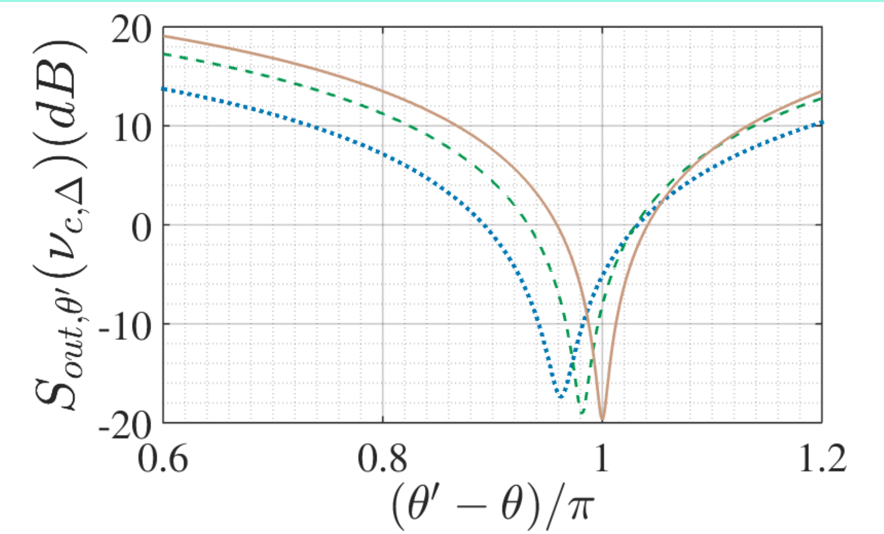


- In the figures below:
  - $|\epsilon| = const.$
    - $\phi = \phi_0 - 0.1\pi$ (dotted)
    - $\phi = \phi_0 - 0.05\pi$ (dashed)
    - $\phi = \phi_0$ (solid)
  - Optimal frequency and time-delay

$\phi_0 = 0$



$\phi_0 = \pi$



Frequency-dependent squeezing

- Adding π to the local oscillator phase in our case gives the following frequency-dependence of the optimal quadrature angle for the maximum squeezing:
  - Low frequencies: around $\theta' - \theta = 0$
  - High frequencies: around $\theta' - \theta = \pi$
- Characteristic that is required for the new generation of gravitational wave detectors