Manipulating the Squeezing Properties of a Degenerate Parametric Amplifier with Coherent, Time-Delayed Feedback

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Outline

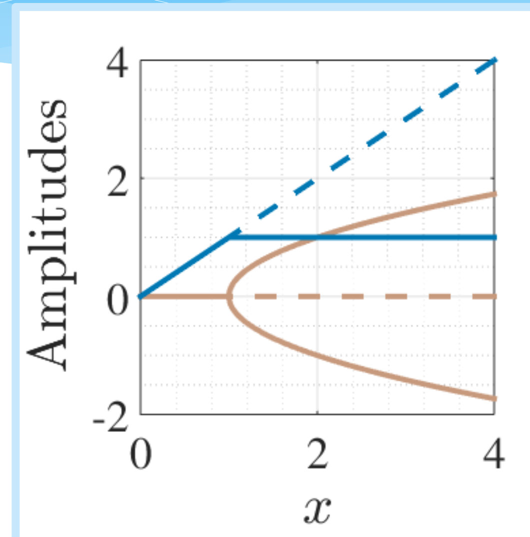
* Degenerate Parametric Amplifier
  * Classical dynamics
  * Quantum treatment below threshold
  * Squeezing characteristics

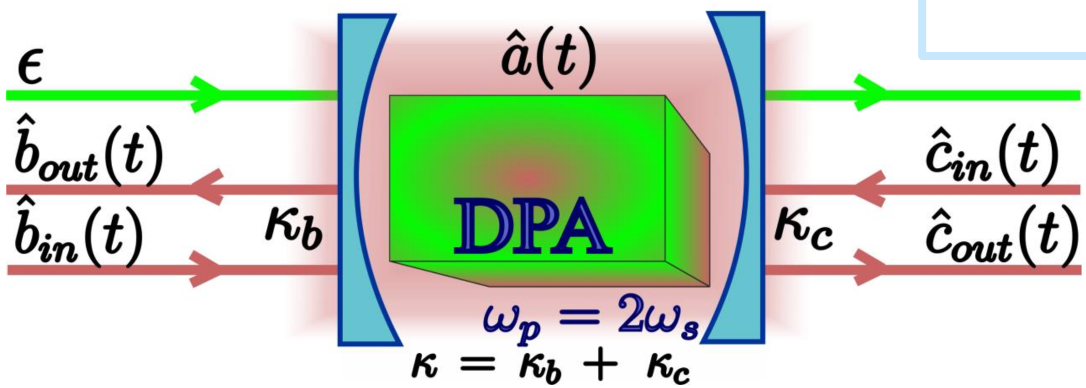
* DPA with time-delayed coherent feedback
  * Model
  * Squeezing spectrum
  * Change in the dynamics
  * Effects of loss

DPA dynamics & squeezing

* Classical dynamics:
  * Saturation of pump: threshold behaviour
  * Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$
* Quantum mechanical behaviour[1]:
  * Parametric pump approximation $\left(\epsilon = |\epsilon| e^{i\theta}\right)$

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon\left(\hat{a}^\dagger\right)^2 + \epsilon^*(\hat{a})^2\right)$$

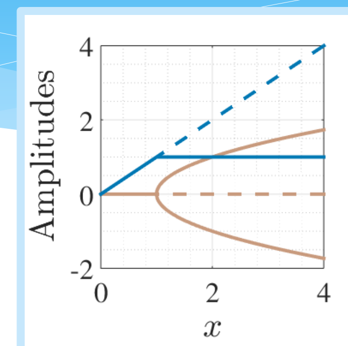




[1]PRA, **30**:1386 (1984)

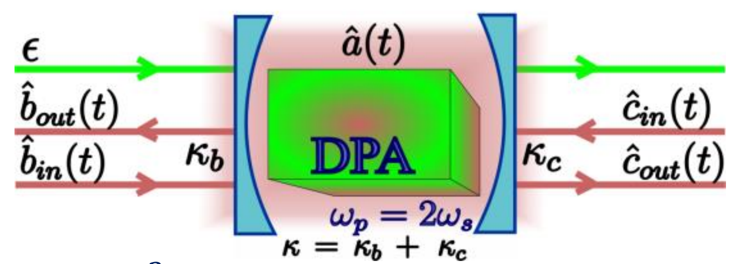
DPA dynamics & squeezing

- Classical dynamics:
  - Saturation of pump: threshold behaviour
  - Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$



- Quantum mechanical behaviour[1]:
  - Parametric pump approximation $\left(\epsilon = |\epsilon| e^{i\theta}\right)$

$$\hat{H} = \hbar\Delta\hat{a}^{\dagger}\hat{a} + \frac{1}{2} i\hbar\left(\epsilon\left(\hat{a}^{\dagger}\right)^{2} + \epsilon^{*}(\hat{a})^{2}\right)$$



  - One-sided cavity quadrature variance:

$$\chi_{out,\theta+\pi}(0) = \int \left\langle \widetilde{X}_{out,\theta'}(0), \widetilde{X}_{out,\theta'}(\nu') \right\rangle d\nu' = \frac{1}{4}\frac{(\kappa - |\epsilon|)^2}{(\kappa + |\epsilon|)^2}$$

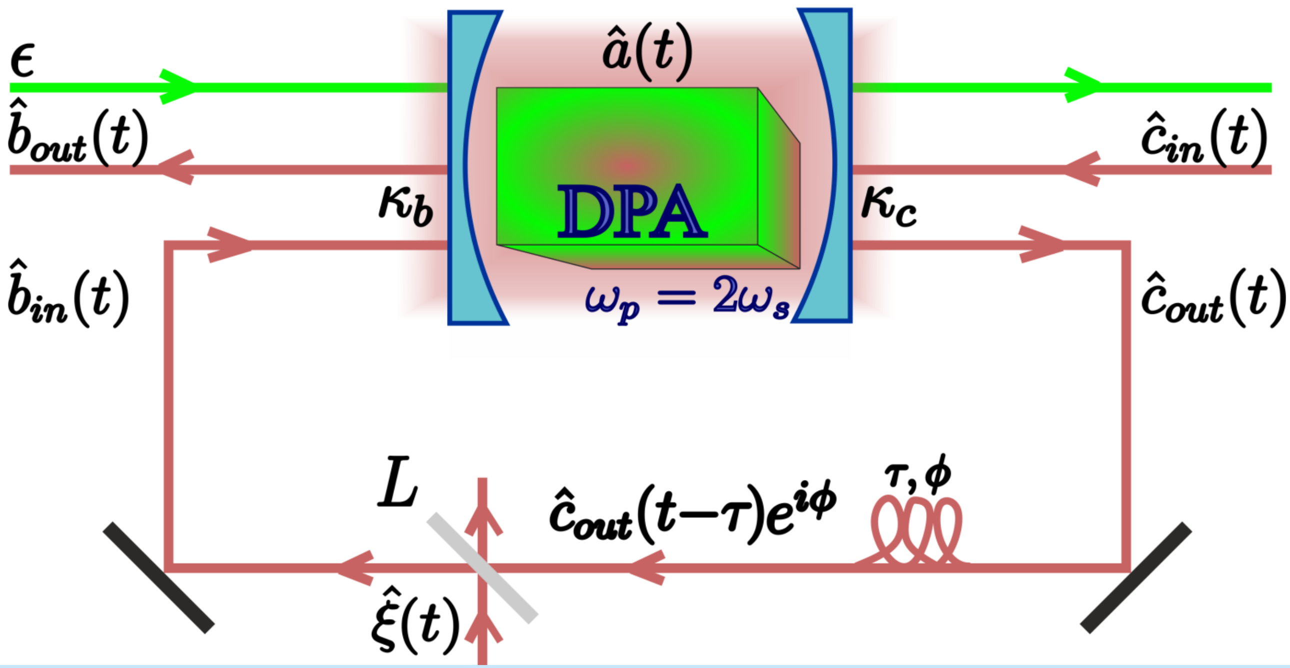
  - Symmetric cavity squeezing on resonance at threshold

$$\chi_{out,\theta+\pi}(0) = \frac{1}{4}\frac{\kappa^2 + |\epsilon|^2}{(\kappa + |\epsilon|)^2}$$

[1]PRA, **30**:1386 (1984)

DPA & time-delayed coherent feedback[1]

* Uncomplicated setup
  * Time-delay τ, overall phase-shift ϕ, overall loss, decoherence: $L, \hat{\xi}$



[1] arXiv: 1606.00178

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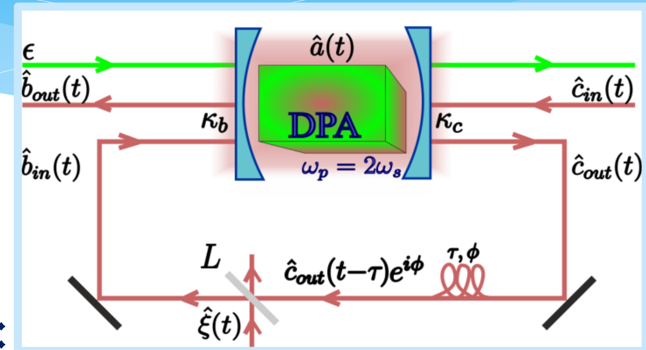


* Equation of motion of the subharmonic mode:

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - \kappa\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - e^{i\phi}k\hat{a}(t-\tau)$$

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon\left(\hat{a}^\dagger\right)^2 + \epsilon^*(\hat{a})^2\right)$$

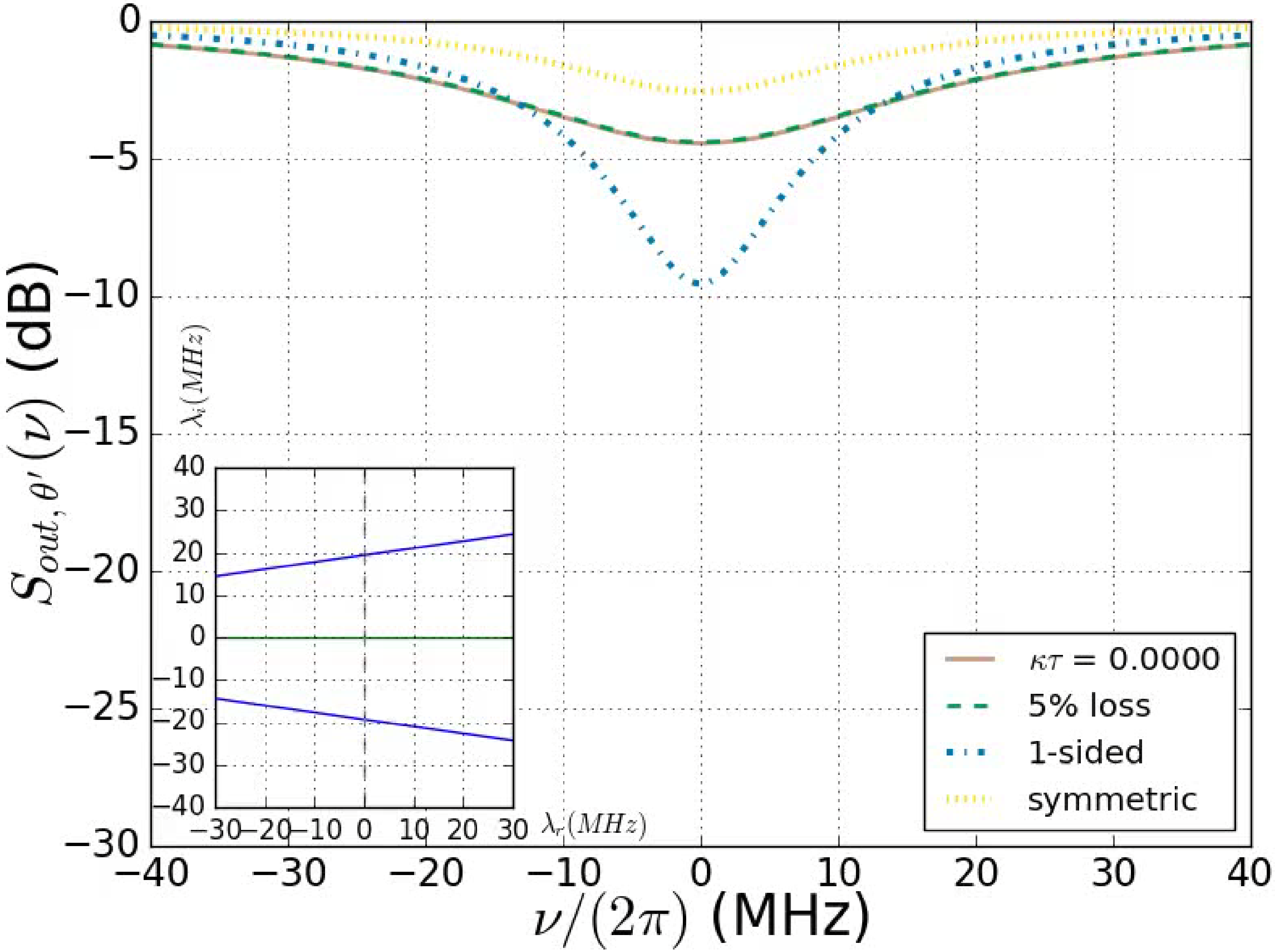
$$\kappa = \kappa_b + \kappa_c, k = 2\sqrt{\kappa_b\kappa_c(1-L)}$$

* Quadrature variance without delay:

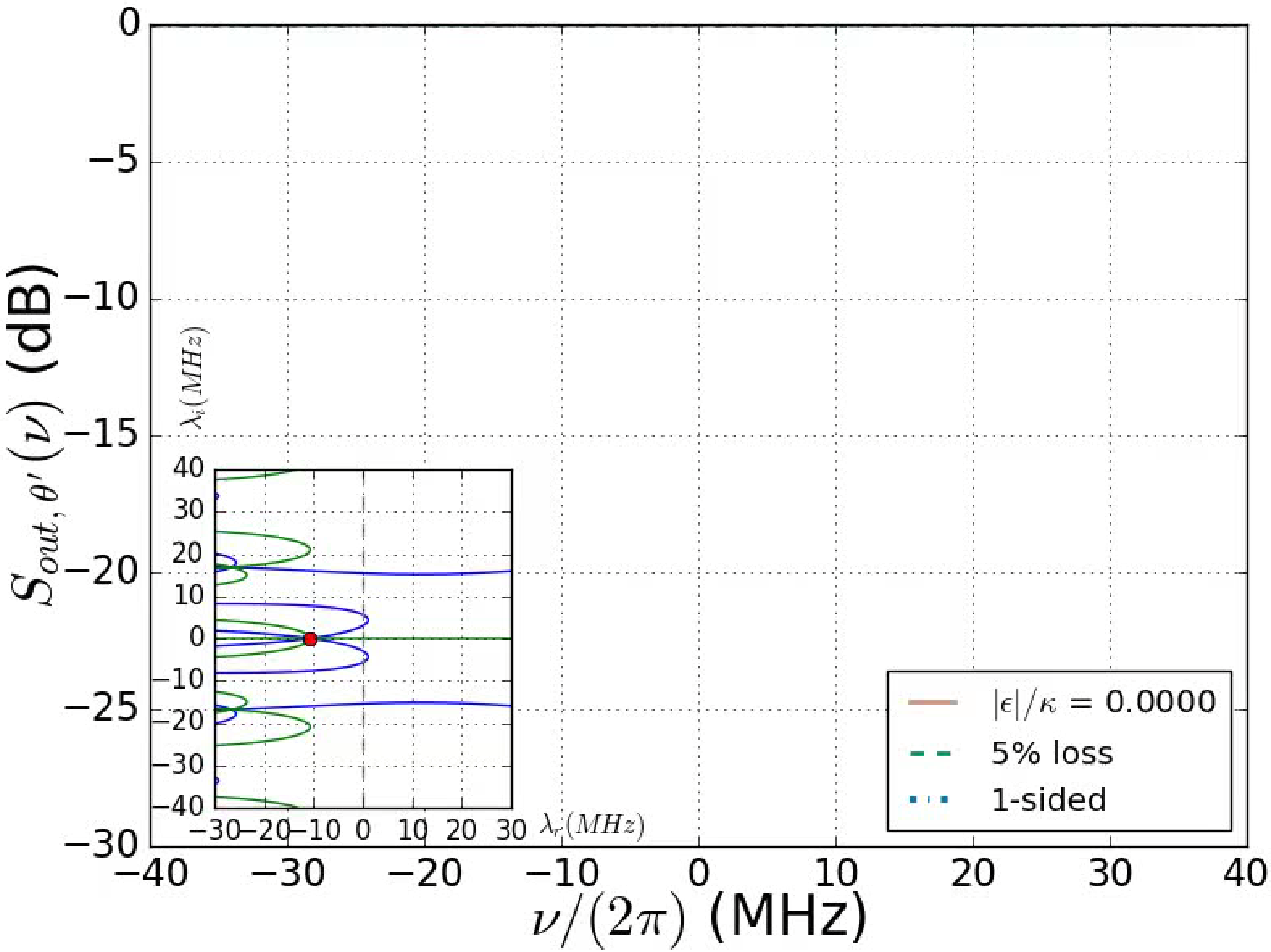
$$X_{out,\theta+\pi}(0) = \frac{1}{4}\frac{((\kappa + k\cos\phi) - |\epsilon|)^2}{((\kappa + k\cos\phi) + |\epsilon|)^2}$$

[1]arXiv: 1606.00178

Emerging side-peaks

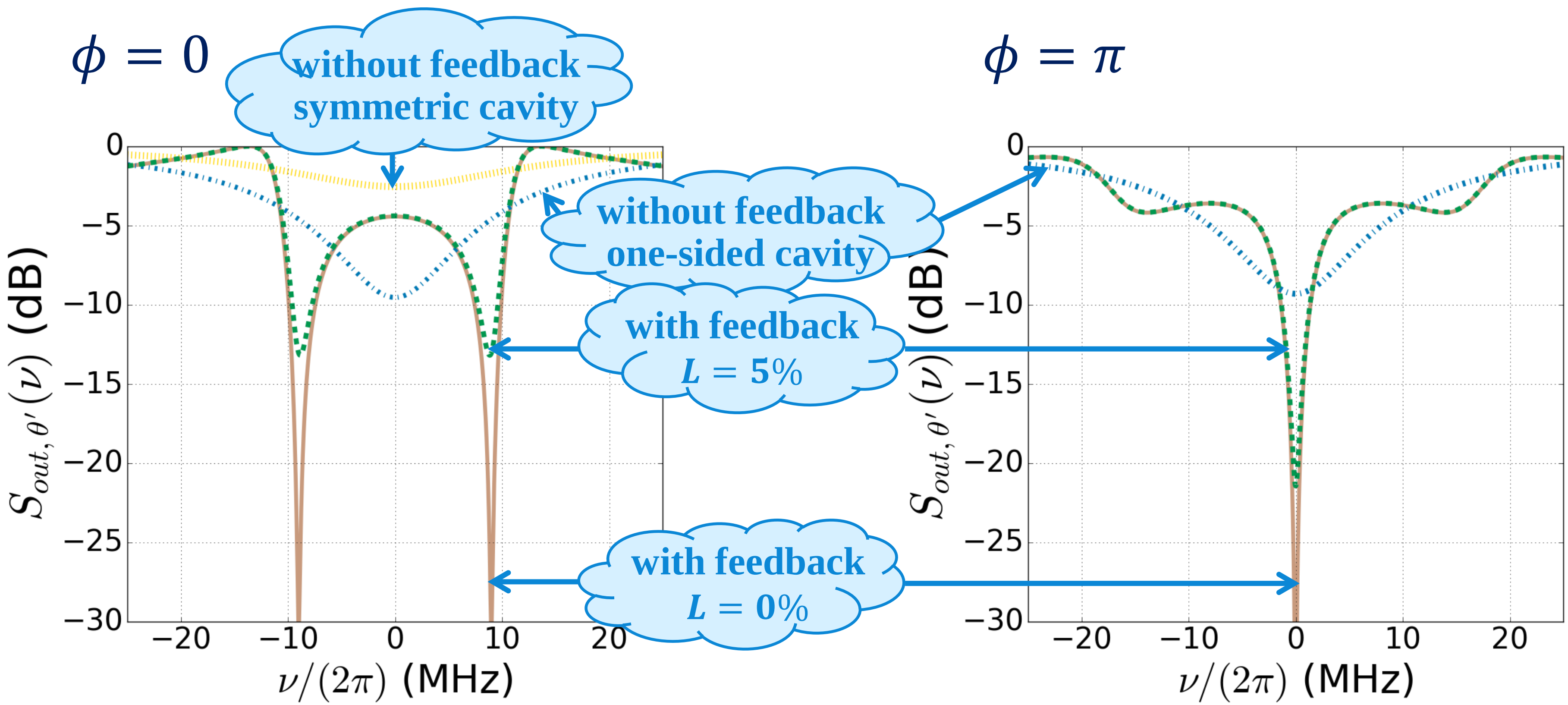


Enhanced squeezing on resonance



New setup with time-delay

- Transcendental equations
- Enhanced squeezing with feedback

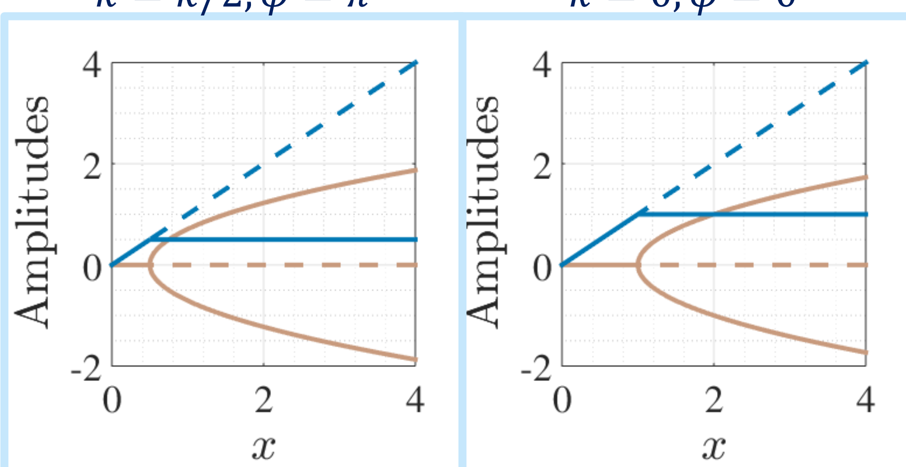


New setup with time-delay

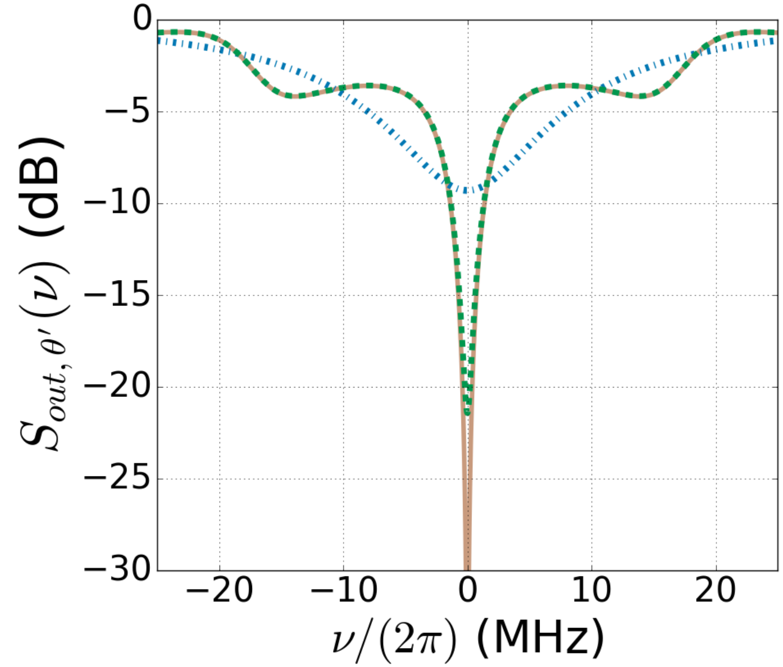
- Transcendental equations
- Enhanced squeezing with feedback
- Pyragas-type feedback:

$$\frac{d\hat{a}}{dt} = i\left[\hat{H}, \hat{a}(t)\right] - (\kappa - k)\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) +$$
$$+k\left(\hat{a}(t-\tau) - \hat{a}(t)\right)$$

$k = \kappa/2, \phi = \pi$ $\qquad$ $k = 0, \phi = 0$



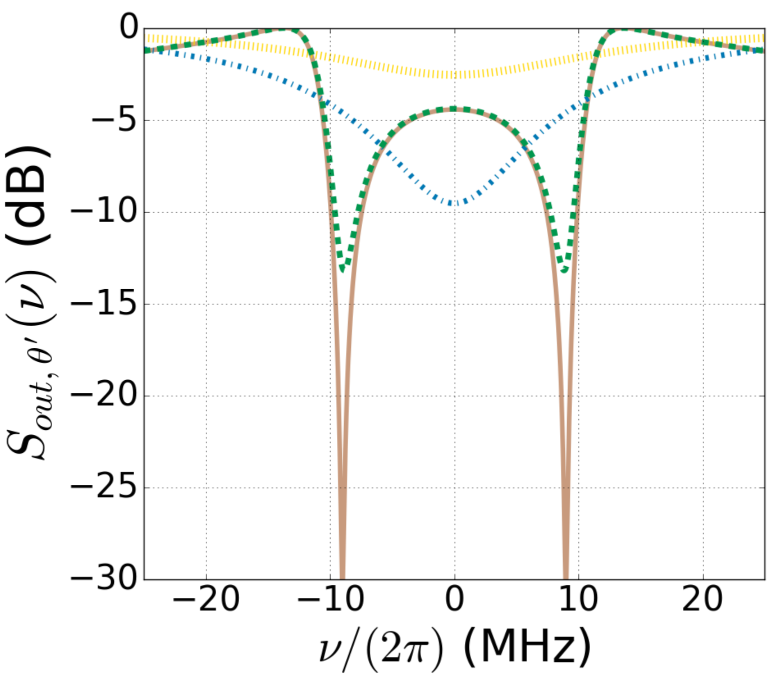
$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa,$
$\kappa\tau = 3, L = 0 \,\&\, 5\%$



New setup with time-delay

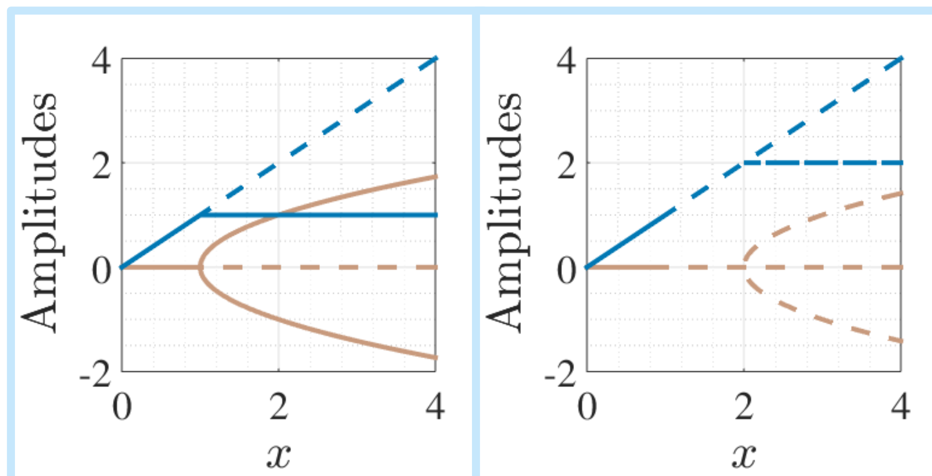
* Transcendental equations
* Enhanced squeezing with feedback

$$\phi = 0, \kappa_b = \kappa_c = 0.5\kappa, |\epsilon| = 0.5\kappa,$$
$$\kappa\tau = 2.3, L = 0 \ \&\ 5\%$$



* Limit cycle is reached on threshold
* Persistent oscillations with the characteristic frequency

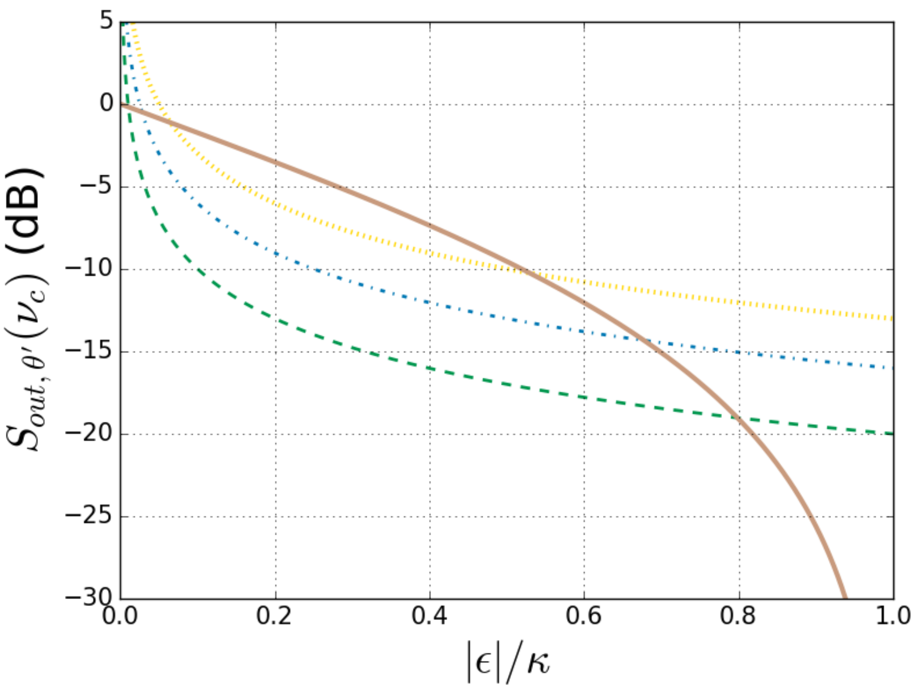
$k = 0, \phi = 0, \kappa\tau = 0$ $\quad$ $k = \kappa, \phi = 0, \kappa\tau = 1.57$



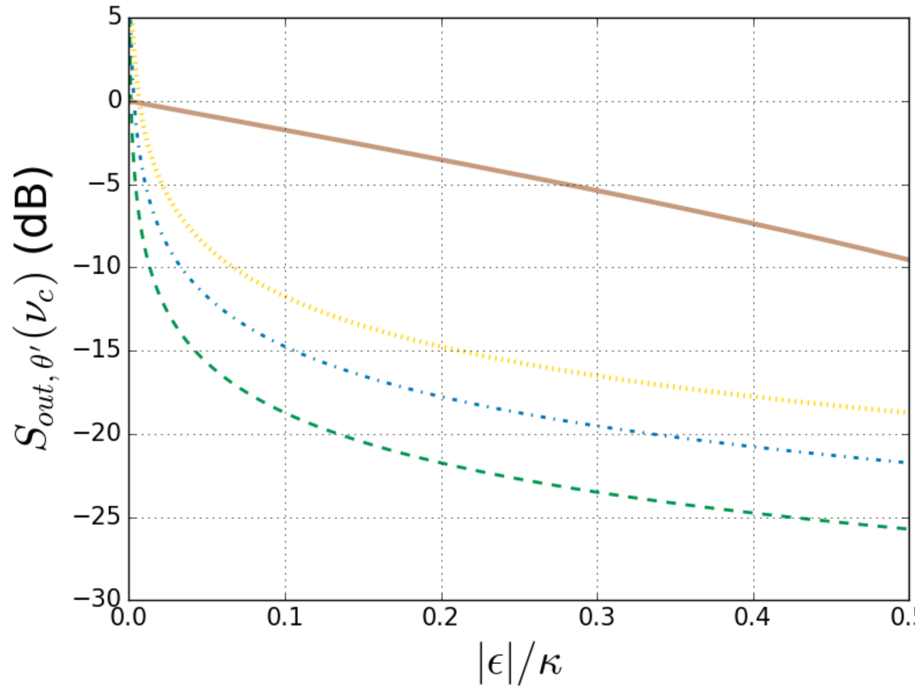
Effects of loss

* With loss wider range of tunability
* Quadrature variance at the critical point: $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4}\frac{L\kappa_c}{|\epsilon_\Delta|}$

$\phi = 0$



$\phi = \pi$



Summary

* Coherent feedback:

  backaction-free way of enhancing useful quantum mechanical behaviours

* Time-delay:

  tunable changes in the stability range

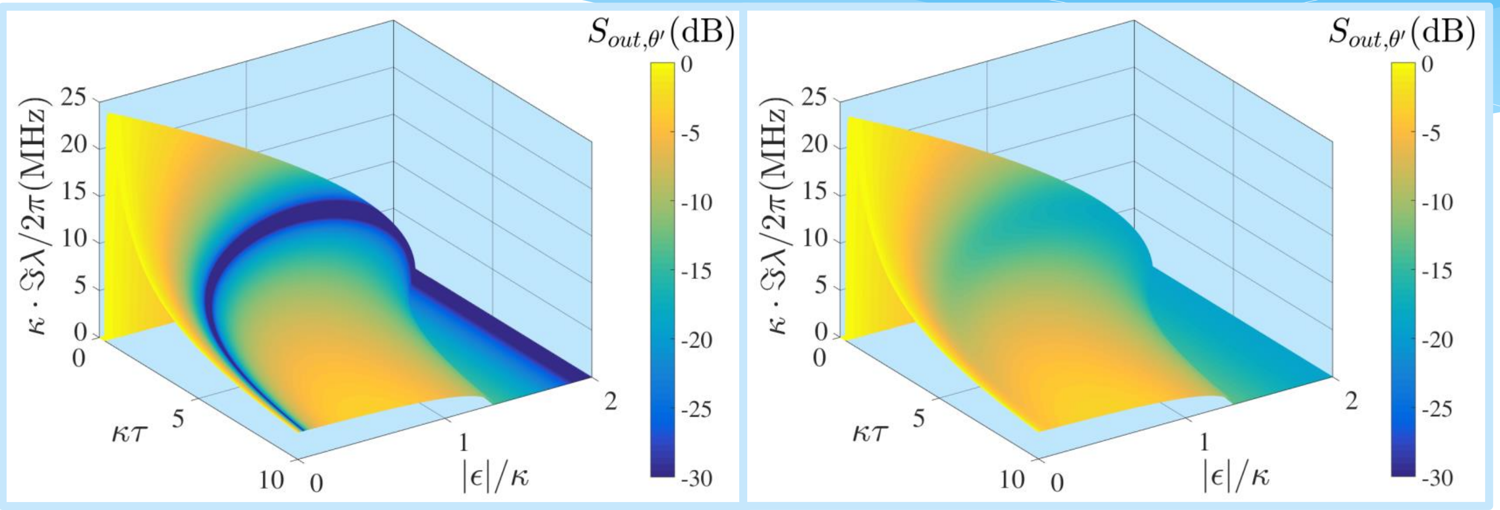
* Time-delayed one-loop setup with a DPA: enhanced squeezing
  * On and off-resonant squeezing as well
  * Result of the changing dynamics of the system
* Loss in the feedback loop:
  * Decreased squeezing
  * BUT! wider range of tunability

Thank you for Your attention!

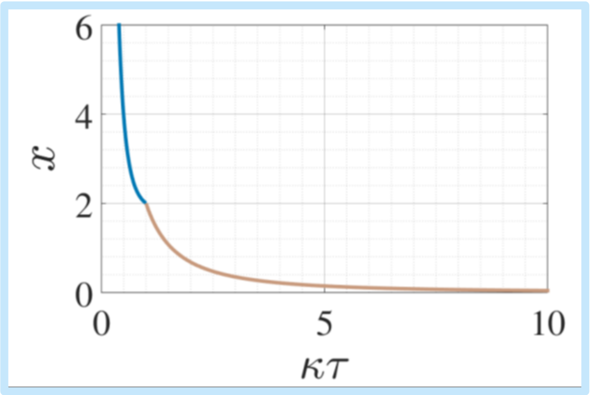
1 2 3 4 5 6 7 8 9

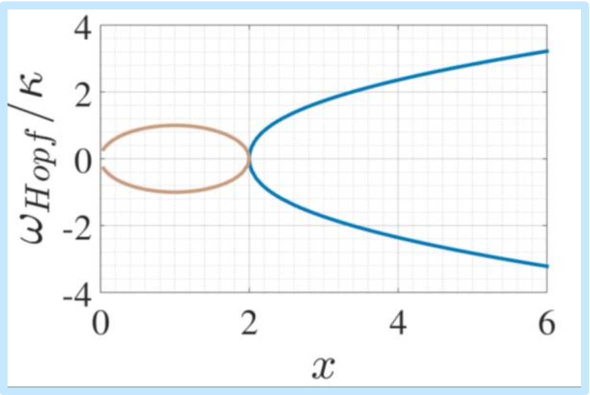
Thank you for Your attention!

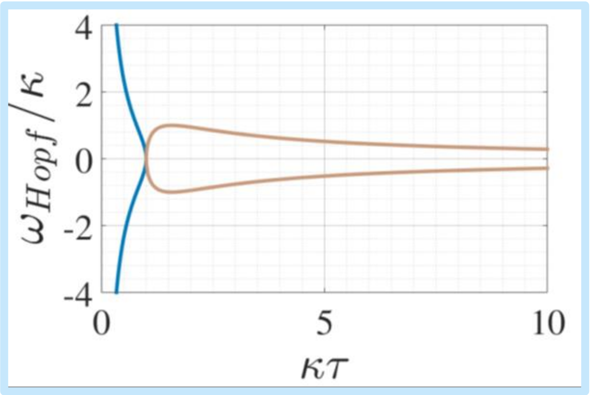
Characteristic frequency range



$$0 \le \nu_s \le \sqrt{|\epsilon|(2\kappa - |\epsilon|)}, 0 \le |\epsilon| \le \left|\kappa + k e^{i\phi}\right|$$



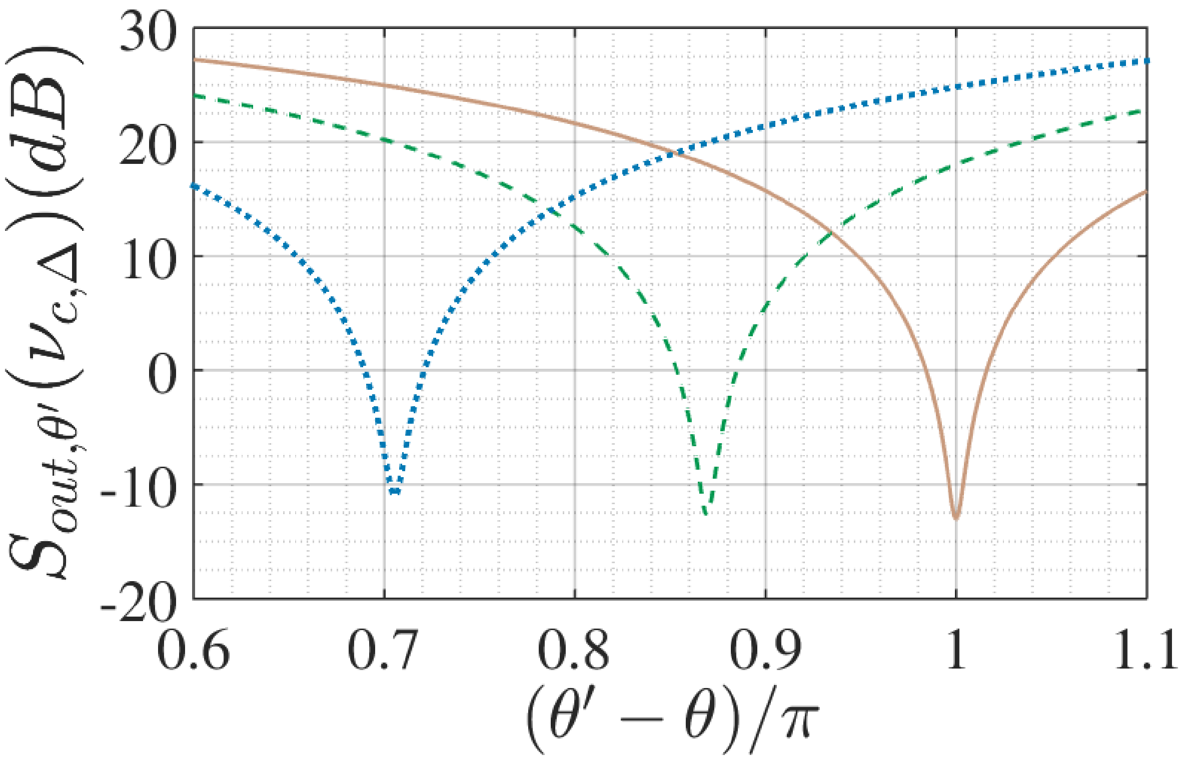




Detuning and quadrature angle

* Local oscillator phase:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$$



$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2},$

$\kappa\tau = 2.418,$

$\nu_{c,\Delta} = 0.866\kappa,$

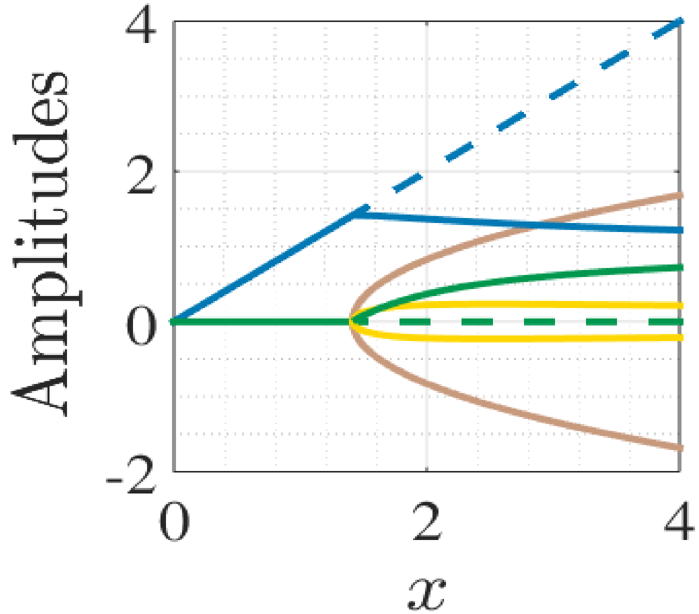
$L = 5\%,$

$\Delta = 0\ (solid),$

$0.1\kappa\ (dashed)$

$0.2\kappa\ (dotted)$

Effects of the phase shift



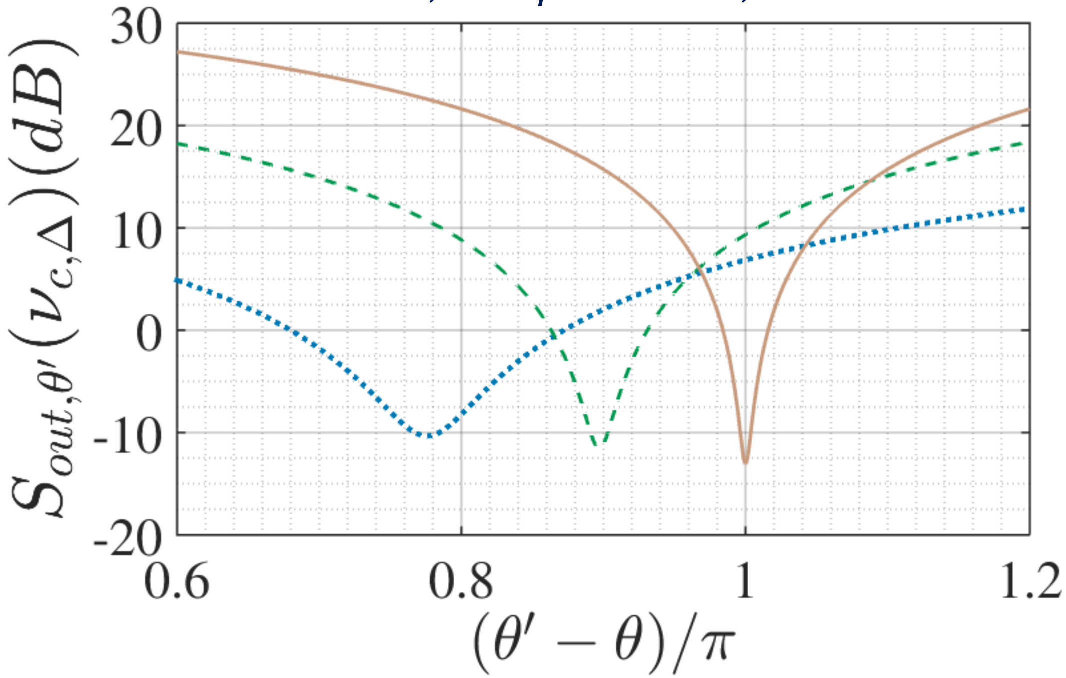
$k = \kappa, \phi = \pi/2$

ε_p: real (blue), imag (green)

ε: real (brown), imag (yellow)

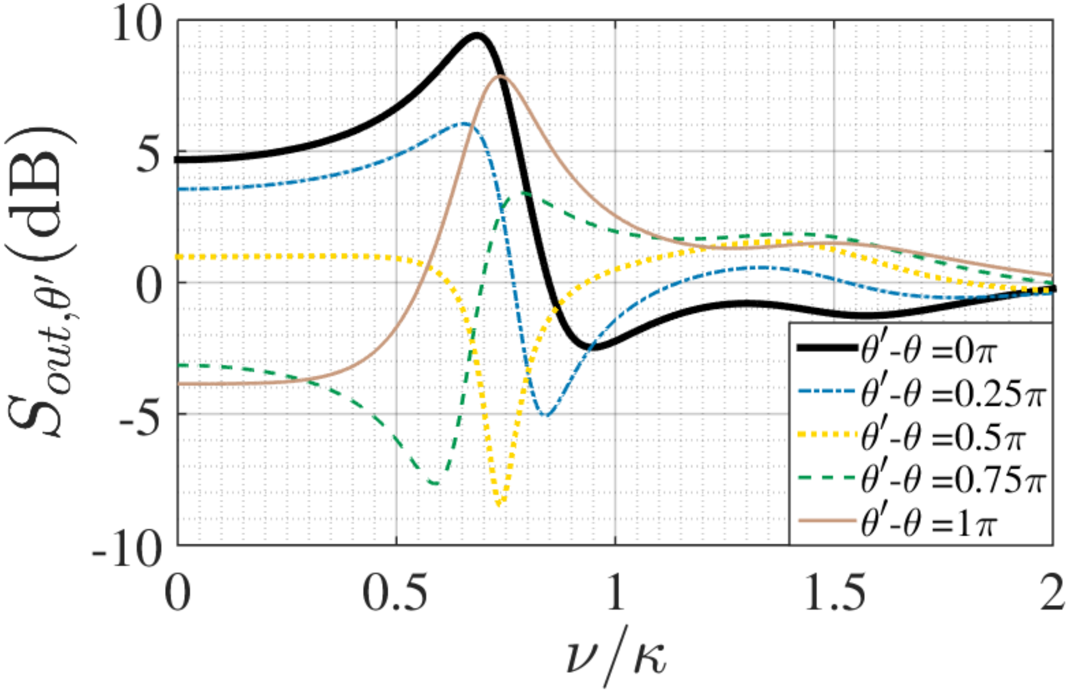
$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2}, \kappa\tau = 2.418, \nu_{c,\Delta} = 0.866\kappa,$

$L = 5\%, \quad \phi = -0.1\pi, -0.05\pi$ & 0



Gravitational waves

* Adding π to the local oscillator phase in our case gives the required characteristics



* Frequency of the best squeezing: changing with the local oscillator phase
  * Lower frequencies: around $\theta' - \theta = 0$
  * Higher frequencies: around $\theta' - \theta = \pi$

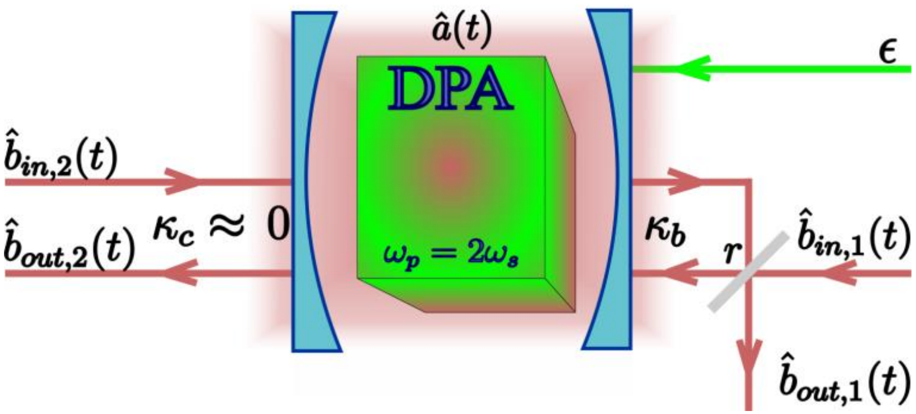
[1]PLA, **170**:421 (1992), [2]NJP **16**:065004 (2014),

Coherent feedback with DPA

- Single DPA
  - Feedback via beam splitter[1]
  - Enhanced, tunable squeezing at a given driving strength

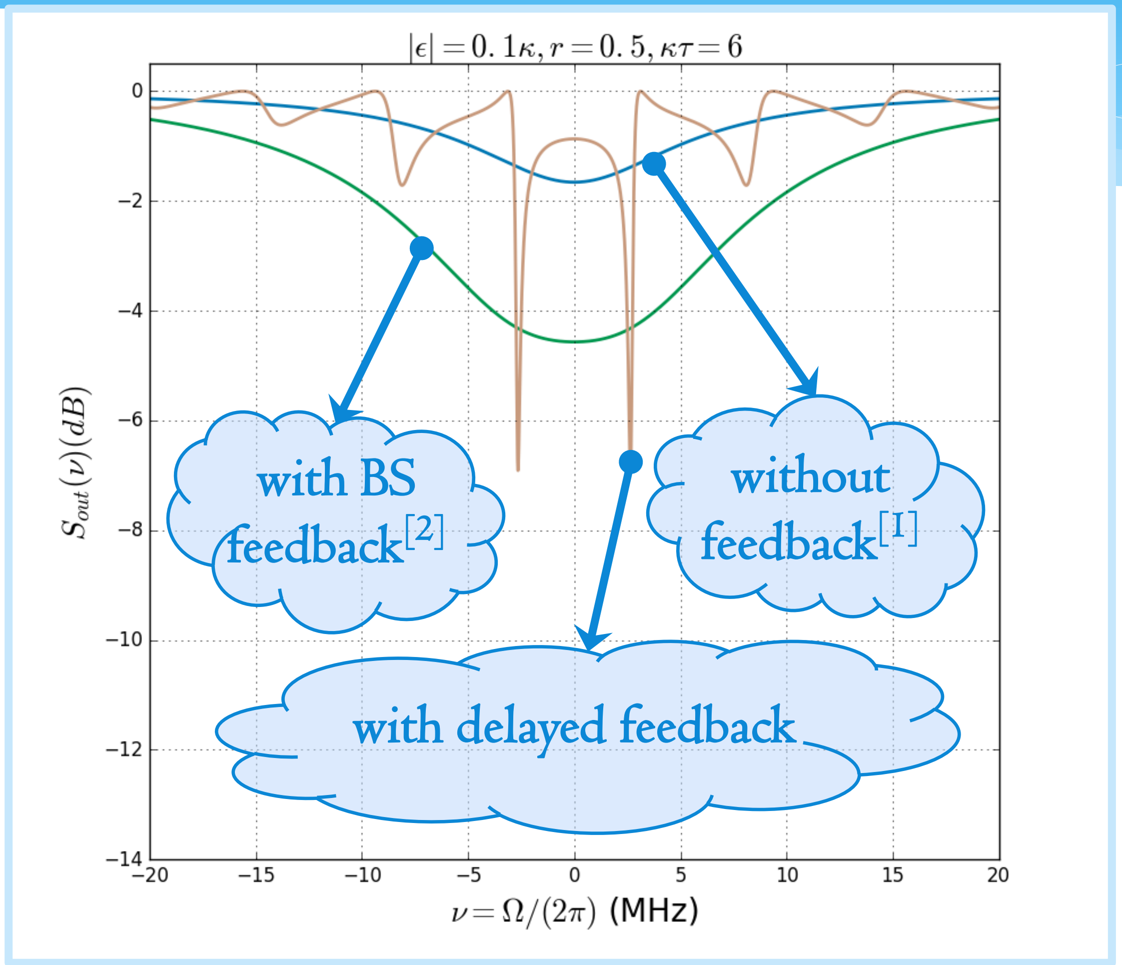
$$\mathcal{X}_{out,\theta+\pi}(0) = \frac{1}{4}\frac{(\kappa(r)-|\epsilon|)^2}{(\kappa(r)+|\epsilon|)^2}$$

  - Modified threshold
  - Experiment[2] agrees with theory[1]
  - BUT! Performance is limited
    - efficient $|\epsilon|$ range under 0.6κ
    - losses through the „perfectly reflecting mirror"



[1]PRA, **80**:042107 (2009), [2]IEEE Trans. Auto. Contr., **57**:2045 (2012)

Comparison with previous results[1,2]



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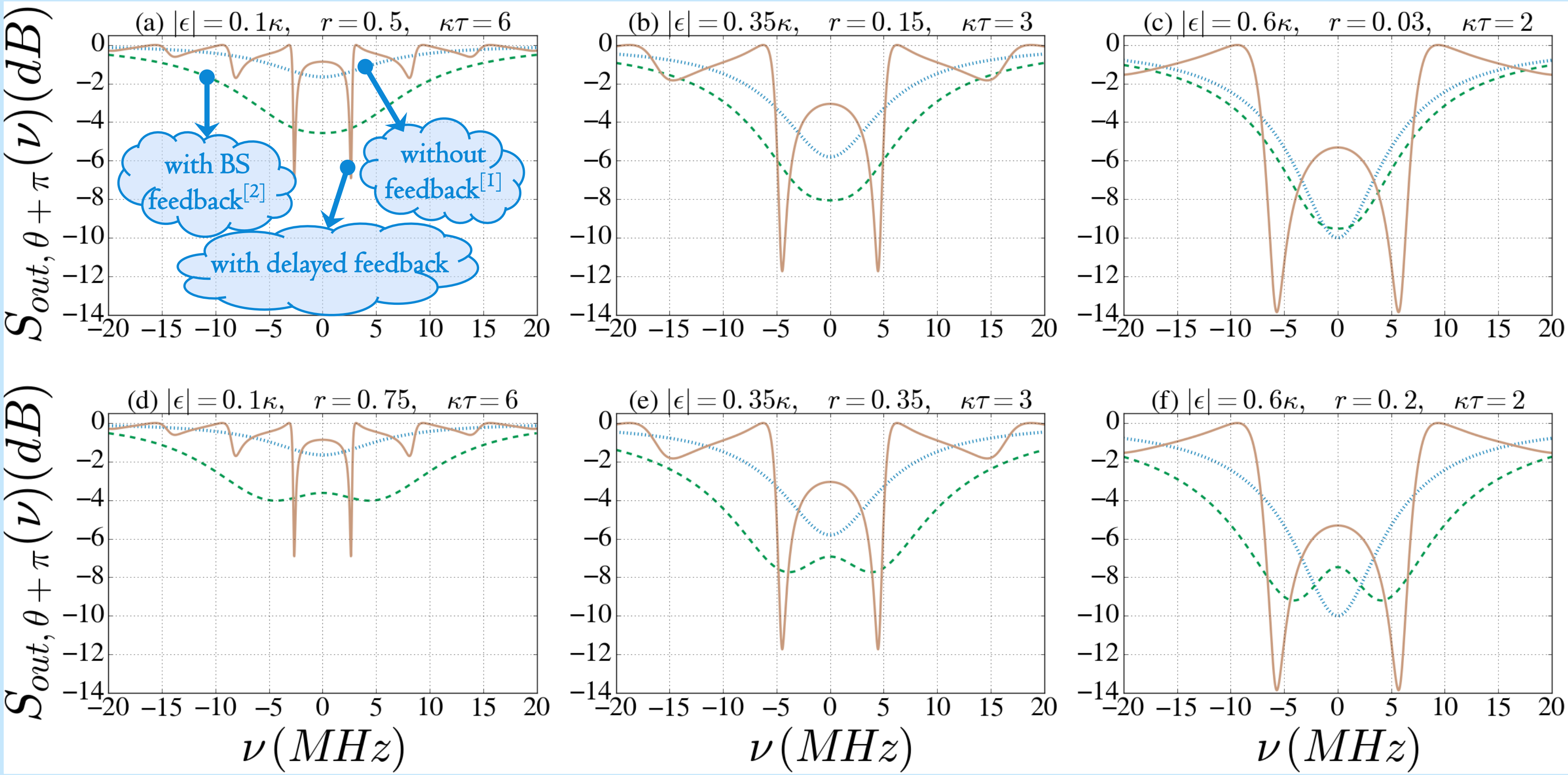


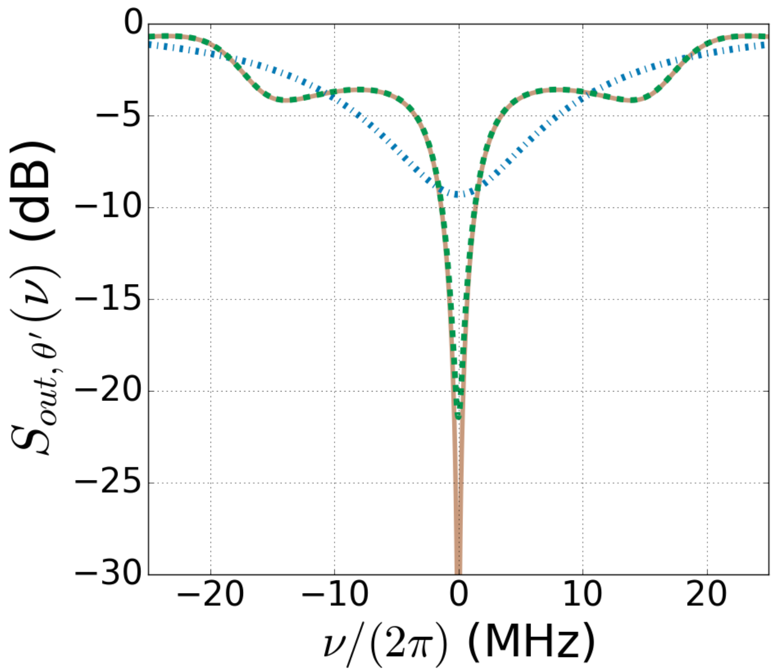
[1]PRA, **30**:1386 (1984), [2]IEEE Trans. Auto. Contr., **57**:2045 (2012)

New setup with time-delay

* Transcendental equations
* Enhanced squeezing with feedback

* Properties:
  * Best squeezing is on resonance
  * Effects of time-delay:
    * Narrower spectrum
    * Emerging small side-peaks
  * Perfect squeezing at stability change
    * Condition: $\kappa - k = |\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2}$
    * Local phase: $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
    * Variance: $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4}\frac{L\kappa_c}{|\epsilon_\Delta|}$

$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa,$
$\kappa\tau = 3, L = 0\ \&\ 5\%$



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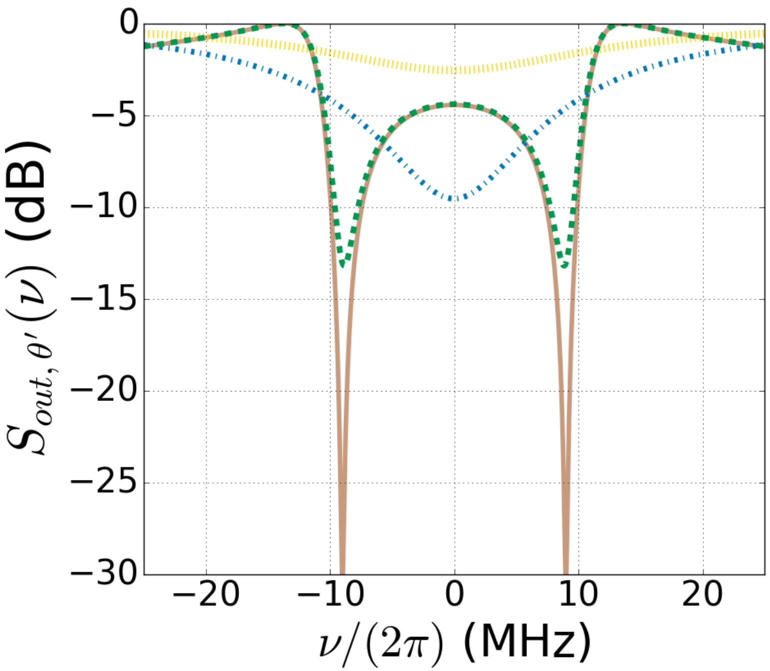
$\kappa\tau = 2.3, L = 0 \,\&\, 5\%$



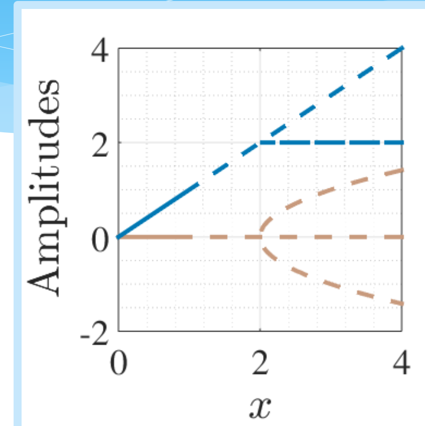
* Properties:
  * Best squeezing is off-resonant
  * Perfect squeezing at stability change
    * Local phase: $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
    * Time-delay: $\tau_{s,n,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta|-\kappa}{k}\right)+2n\pi}{\nu_{s,\Delta}}$
    * Frequency: $\nu_{s,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$
    * Variance: $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4}\frac{L\kappa_c}{|\epsilon_\Delta|}$

Classical analysis

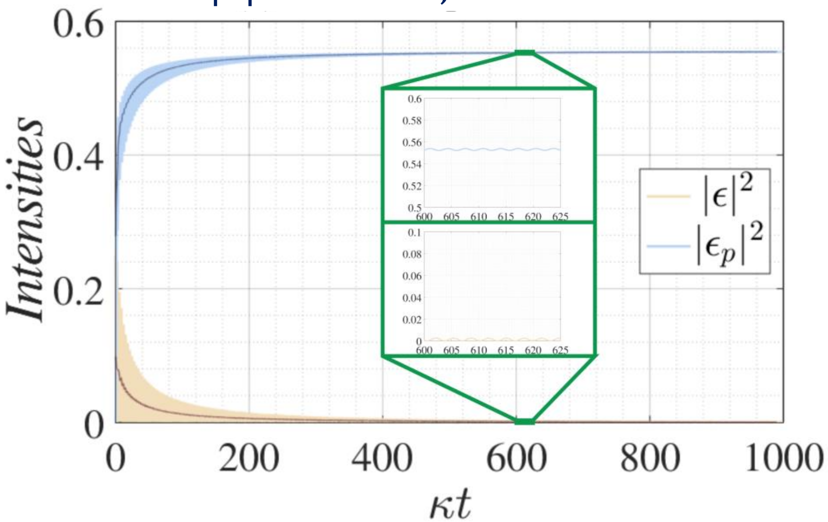
* Originally: pitchfork bifurcation
* Time-delay: Changing stability of steady state solutions

  ↓

  Hopf bifurcation
* Imaginary eigenvalue becomes the frequency of persistent oscillations



$|\epsilon| = 0.745\kappa, \kappa\tau = 1.8833$



$|\epsilon| = 0.78\kappa, \kappa\tau = 1.8833$