

Is delay always harmful to precision?

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Outline

- Time-delayed Coherent feedback
 - Time-delay as a control parameter
 - Non-Markovian environment
- Degenerate Parametric Amplifier (DPA)
 - Without feedback
 - With coherent feedback
- DPA with time-delayed coherent feedback
 - Quantum mechanical model
 - Squeezing spectra
 - Classical analysis

Quantum control

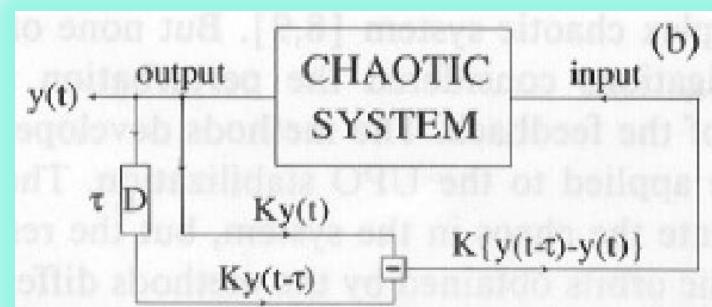


- Open-loop control
 - Reaching the target state relies on the engineering of the desired Hamiltonian
 - No information is obtained about the state of the system
- Closed-loop control
 - Information is obtained about the system's state
 - Adjustment of the parameters accordingly
- Coherent quantum feedback
 - No measurement
 - Coherent evolution of the output preserves the output quantum state
- Measurement-based feedback
 - Coherent classical feedback
 - Measurement backaction

Time-delayed control

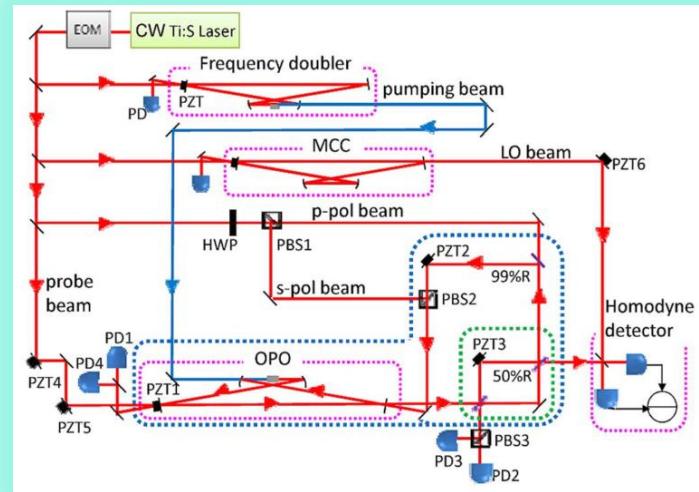


- Classical example
 - Pyragas-type feedback
 - PLA 170:421 (1992), PRE 72:046203 (2005)
 - Extra degrees of freedom
 - Alters the relaxation time-scale
 - Stabilizes unstable periodic orbitals
- Semiclassical
 - Stabilized laser dynamics (IEEE J.Q.El. 16:347 (1980), PRE 78:056213 (2008))
- Quantum
 - Enhanced entanglement (PRA 91:052321 (2015))
 - Recovered Rabi oscillations in the single photon limit (PRL 110:013601 (2013), PRA 92:053801 (2015))
 - Faster convergence to a desired steady state (NJP 16:065004 (2014))



Feasibility of feedback with long time-delay

- Long time-delay: $\kappa\tau \approx \text{const} \cdot 10^0$
 - $\kappa = 10 \text{ MHz} \cdot 2\pi$ (overall cavity decay rate)
- Considered time-delay: $\tau \approx 20\text{ns}$
 - in case of free propagation: $l \approx 5\text{m}$
- Previous experiment
 - feedback loop loss: $L \approx 5 - 10\%$
- Ideas for experimental realization:
 - These values are for the experiment IEEE Trans. Aut. Contr. 57:2045 (2012)
 - Other possibilities: Ringresonator with intrinsic nonlinearities (fabricated from crystals) + optical fibre, Circuit QED, Waveguide constructions



Open quantum system^[1,2]



- Markovian Master Equation:

$$\frac{d}{dt} \rho_S(t) = \mathcal{L} \rho_S(t) = -i[H(t), \rho_S(t)] + \sum_{j=1}^{N^2-1} \gamma_j \left(A_j \rho_S A_j^\dagger - \frac{1}{2} A_j^\dagger A_j \rho_S - \frac{1}{2} \rho_S A_j^\dagger A_j \right)$$

- There is a dynamical map $\rho_S(t) = \Phi(t)\rho_S(0)$, which satisfies the semigroup property $\Phi(t_1)\Phi(t_2) = \Phi(t_1 + t_2)$
- Master equation for a general open quantum system:

$$\frac{d}{dt} \rho_S(t) = -i[H_S(t), \rho_S(t)] + \int_{t_0}^t \mathcal{K}(t, t_1) \rho_S(t_1) dt_1$$

- If there exists an invertible dynamical map $\rho_S(t) = \Phi(t, t_0)\rho_S(t_0)$, a time-local master equation can be constructed in Lindblad-like form.

^[1] S. Whalen, PhD thesis (2015), ^[2] H.-P. Breuer, F. Petruccione The theory of open quantum systems (2007),

Description of the environment^[1,2]



- Interaction Hamiltonian:

$$H_{SEI}(t) = \sum_j (a_j^\dagger B_j(t) + B_j^\dagger(t) a_j)$$

- Environment operators: $B_j(t) = \sum_{\alpha, l_\alpha} \kappa_{j\alpha l_\alpha} \exp(i(\omega_j - \omega_{l_\alpha})t) b_{\alpha l_\alpha}$

- Spectral density:

$$J_{\alpha j k}(\omega) = \sum_{l_\alpha} \kappa_{j\alpha l_\alpha} \kappa_{k\alpha l_\alpha}^* \delta(\omega - \omega_{l_\alpha})$$

- Dissipation kernel:

$$F_{jk}(t_1, t_2) = [B_j(t_1), B_k^\dagger(t_2)] = e^{i(\omega_j t_1 - \omega_k t_2)} \sum_{\alpha} \int_{-\infty}^{\infty} J_{\alpha j k}(\omega) e^{-i\omega(t_1 - t_2)} d\omega$$

- Noise kernel:

$$G_{jk}(t_1, t_2) = \langle B_k^\dagger(t_2) B_j(t_1) \rangle = e^{i(\omega_j t_1 - \omega_k t_2)} \sum_{\alpha} \int_{-\infty}^{\infty} J_{\alpha j k}(\omega) e^{-i\omega(t_1 - t_2)} n_{\alpha}(\omega) d\omega$$

- Where the thermal distribution is: $n_{\alpha}(\omega) = \frac{1}{e^{(\omega - \mu_{\alpha})/T_{\alpha}} - 1}$

^[1] S. Whalen, PhD thesis (2015), ^[2] S. Whalen, H. Carmichael PRA 93:063820 (2016)

Time-delayed CQF model system^[1]



- Components:

- Cavity coupled to an environment
- ID unipartite environment
 - e.g. optical fibre, waveguide, superconducting transmission line, ...

- Lagrangian:

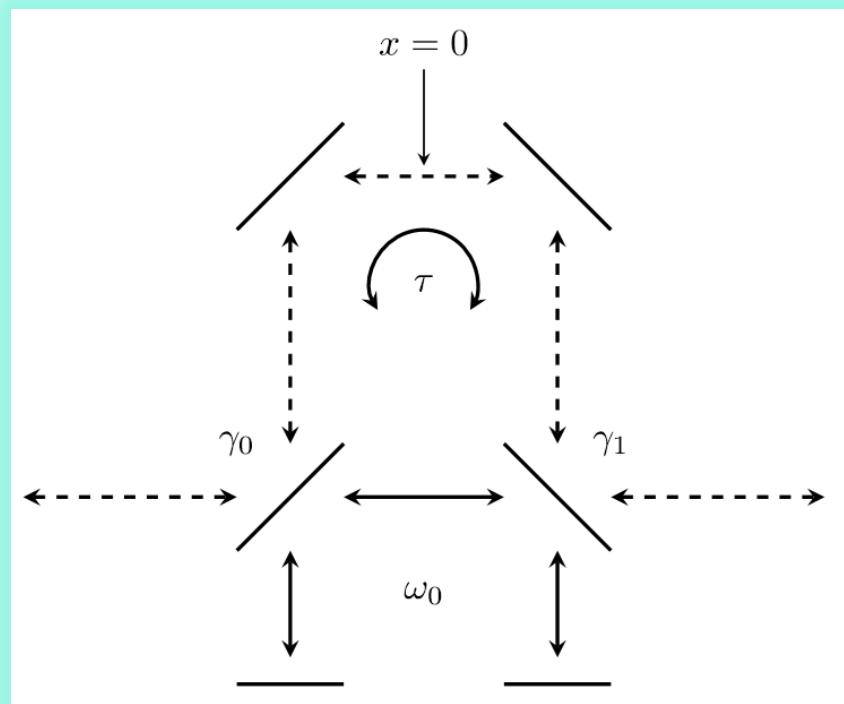
$$\mathcal{L} = \int_{-M/2}^{M/2} \left\{ \frac{1}{2} \left(\frac{\partial}{\partial t} \varphi(x, t) \right)^2 - \frac{1}{2} \left(\frac{\partial}{\partial x} \varphi(x, t) \right)^2 \right\} dx$$

- Massless scalar field
- Euler-Lagrange: wave equation

- Interaction Hamiltonian (RWA): $H_{SE} = \sum_{l=-\infty}^{\infty} (\kappa_l a^\dagger b_l + \kappa_l^* b_l^\dagger a)$

- With $\kappa_l = \sqrt{\frac{\gamma_0}{2M}} e^{-i \operatorname{sgn}(l) \left(\frac{|\omega_l| \tau}{2} - \phi_0 \right)} + \sqrt{\frac{\gamma_1}{2M}} e^{i \operatorname{sgn}(l) \left(\frac{|\omega_l| \tau}{2} + \phi_1 \right)}$

- Spectral density: $J(\omega) = \theta(\omega + \omega_0) \left(\frac{\gamma_0 + \gamma_1}{2\pi} + \frac{\sqrt{\gamma_0 \gamma_1}}{\pi} \cos(\omega \tau + \phi) \right)$



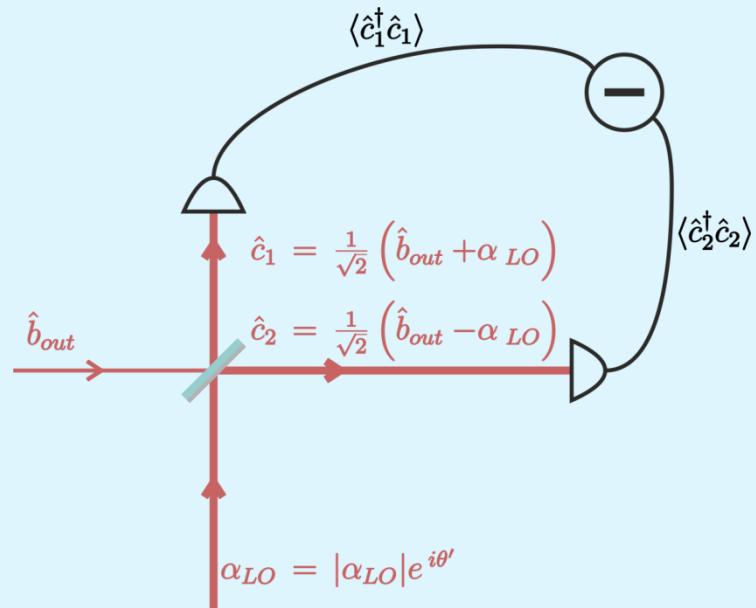
Optical squeezing



- Quadratures: Observables defined by homodyne detection

$$\tilde{X}_{out,\theta'}(\nu) = \frac{1}{2} \left(e^{-i\frac{\theta'}{2}} \tilde{b}_{out}(\nu) + e^{i\frac{\theta'}{2}} \tilde{b}_{out}^\dagger(-\nu) \right)$$

- Generalizations of position and momentum
- Local oscillator determines:
 - Quadrature angle \Leftrightarrow phase
 - Frequency
- Optical squeezing:
 - reduction of a quadrature's variance ($\langle a, b \rangle = \langle ab \rangle - \langle a \rangle \langle b \rangle$) at the expense of the other's



$$\mathcal{X}_{out,\theta'}(\nu) = \int \langle \tilde{X}_{out,\theta'}(\nu), \tilde{X}_{out,\theta'}(\nu') \rangle d\nu' < \frac{1}{4} \quad \& \quad \mathcal{X}_{out,\theta'+\pi}(\nu) > \frac{1}{4}$$

Degenerate Parametric Amplifier



- Crystal with $\chi^{(2)}$ nonlinearity \rightarrow parametric down-conversion
- $\omega_p = 2\omega_s \rightarrow$ phase-sensitive amplification \rightarrow squeezed light
- Classical dynamics:
 - Saturation of pump: threshold behaviour
 - Bifurcation of steady state solutions at $x = \frac{|\epsilon|}{\kappa} = 1$
- Quantum mechanical behaviour^[I]:
 - Parametric pump approximation ($\epsilon = |\epsilon|e^{i\theta}$)

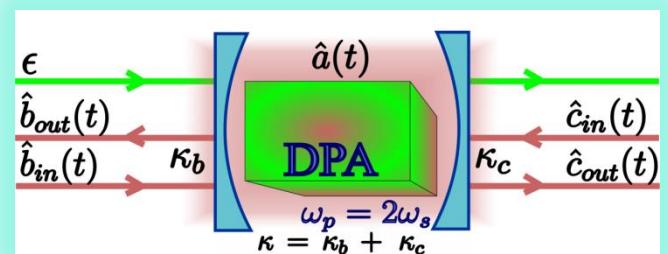
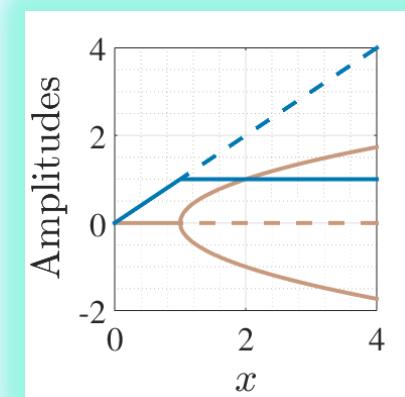
$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

- One-sided cavity quadrature variance:

$$\mathcal{X}_{out,\theta+\pi}(0) = \frac{1}{4}\frac{(\kappa - |\epsilon|)^2}{(\kappa + |\epsilon|)^2}$$

- Symmetric cavity squeezing on resonance:

$$\mathcal{X}_{out,\theta+\pi}(0) = \frac{1}{4}\frac{\kappa^2 + |\epsilon|^2}{(\kappa + |\epsilon|)^2}$$



^[I] M. Collett, C. Gardiner, PRA, 30:1386 (1984)

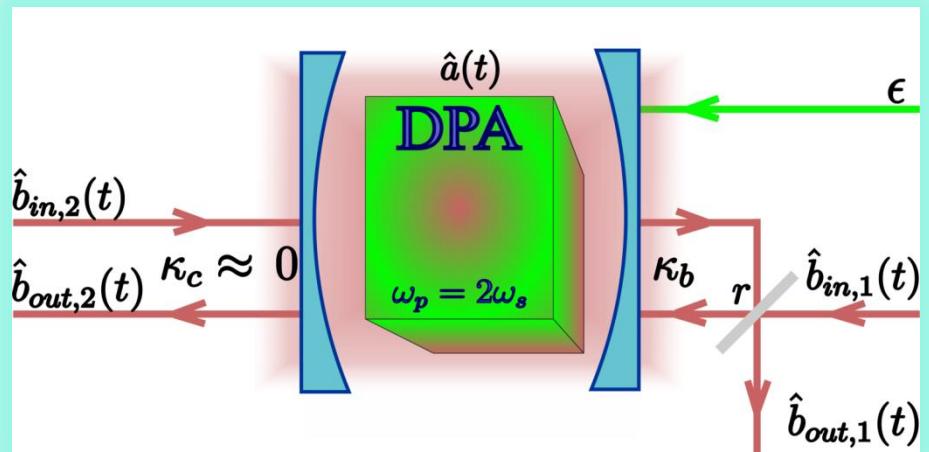
Coherent feedback with DPA^[1,2]



- Single DPA^[1,2]

- Feedback via beam splitter^[1]
- Undepleted pump approximation
- Enhanced, tunable squeezing at a given driving strength

$$x_{out,\theta+\pi}(0) = \frac{1}{4} \frac{(\kappa(r)-|\epsilon|)^2}{(\kappa(r)+|\epsilon|)^2}$$



- Modified threshold
- Experiment^[2] agrees with theory^[1]
- BUT! Performance is limited
 - Efficient $|\epsilon|$ range under 0.6κ in a realistic setup
 - Losses through the perfectly reflecting mirror

- Two DPAs^[3]

- Plant-controller experimental setup
- Properties depend on the phase shift corresponding to the connecting „wire”
- Best squeezing shifted from resonance for the „destructive interference” case

^[1] PRA, 80:042107 (2009), ^[2]IEEE Trans. Auto. Contr., 57:2045 (2012), ^[3]Opt. Exp., 21:18371 (2013)

DPA with time-delayed coherent feedback

- Non-Markovian system
 - General equation of motion in case of a single mode:

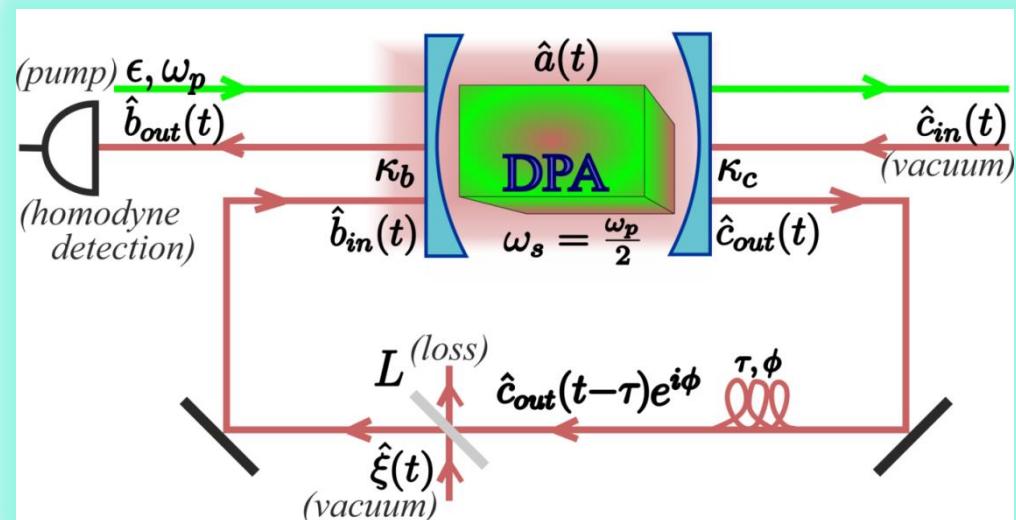
$$\frac{d}{dt} \hat{a}(t) = -i\Delta \hat{a}(t) + \int_0^t F(t-t_1) \hat{a}(t_1) dt_1 - i(\Omega(t)\mathbf{1} + \hat{B}(t))$$

$$F(t-t_1) = [\hat{B}(t), \hat{B}^\dagger(t_1)] =$$

$$= \sqrt{\gamma_0 \gamma_1} e^{i\phi} \delta(t-\tau-t_1) + (\gamma_0 + \gamma_1) \delta(t-t_1) + \sqrt{\gamma_0 \gamma_1} k e^{-i\phi} \delta(t+\tau-t_1)$$

- Uncomplicated setup

- Characteristics:
 - Time-delay τ
 - Overall phase-shift ϕ
 - Overall loss, decoherence: $L, \hat{\xi}$
 - Homodyne detection with phase θ'
- $\sqrt{\gamma_0 \gamma_1} \rightarrow 2 \sqrt{\frac{\kappa_b \kappa_c}{\kappa}} (1-L)$
- Overall input field:



$$\hat{B}(t) = \sqrt{2\kappa} \hat{a}_{in}(t) = \sqrt{2\kappa_c} \hat{c}_{in}(t) + \sqrt{2\kappa_b} \overline{\left(\sqrt{1-L} e^{i\phi} \hat{c}_{in}(t-\tau) + \sqrt{L} \hat{\xi}(t) \right)}$$

[1] N. N., S. Parkins, PRA, 94:023809 (2016)

DPA with time-delayed coherent feedback

- Equation of motion of the subharmonic mode in the rotating frame

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - \kappa\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - e^{i\phi}k\hat{a}(t-\tau)(\theta(t-\tau))$$

$$\begin{aligned}\hat{H} &= \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2) \\ \kappa &= \kappa_b + \kappa_c, \quad k = 2\sqrt{\kappa_b\kappa_c(1-L)}\end{aligned}$$

- Without delay:

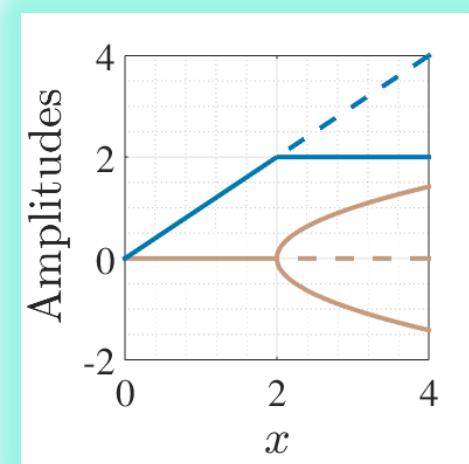
- Best squeezing on resonance

$$\chi_{out,\theta+\pi}(0) = \frac{1}{4} \frac{((\kappa + k \cos \phi) - |\epsilon|)^2}{((\kappa + k \cos \phi) + |\epsilon|)^2}$$

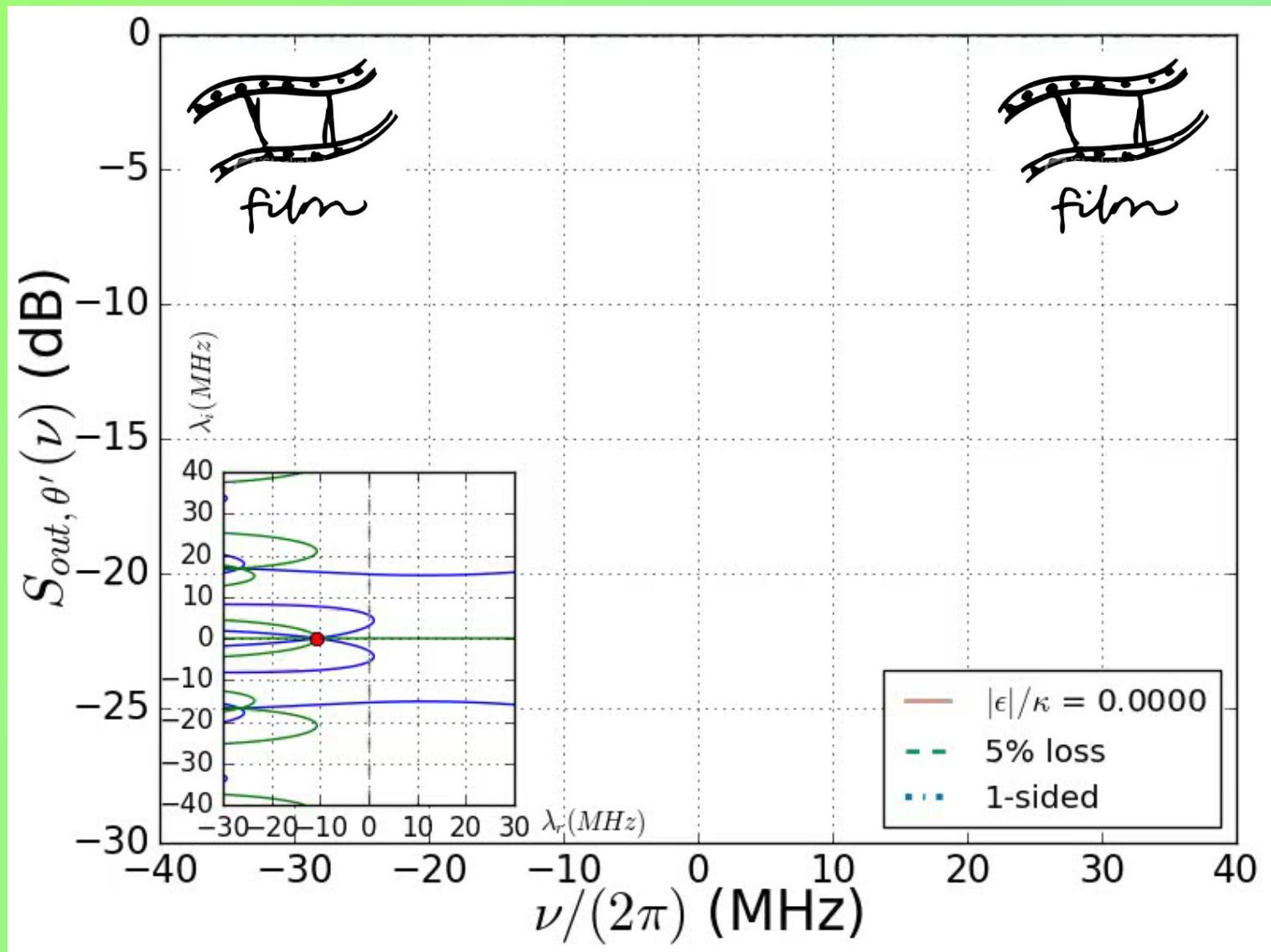
- Shifted threshold

- With delay:

- Best squeezing: for $\phi = \pi$ on resonance, for $\phi = 0$ in sidebands
- Characteristic combination of time-delay, pump strength and frequency



Squeezing spectra for $\phi = \pi$

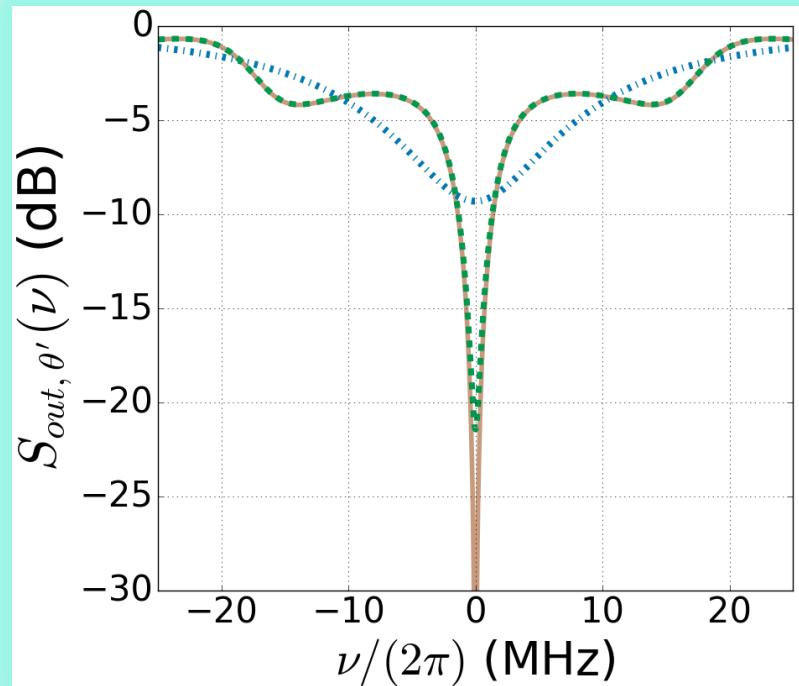


Squeezing for $\phi = \pi$



- Enhanced squeezing compared to the one-sided case without feedback
- Best squeezing is on resonance
 - Reduced threshold pump power
 - No significant change in case of a finite time-delay
 - Perfect squeezing is related to change in stability
 - $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
- Stable, while $0 \leq |\epsilon| \leq \kappa - k$
- Pyragas-type feedback^[I], where the equation of motion:

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - (\kappa - k)\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - k(\hat{a}(t) - \hat{a}(t - \tau))$$

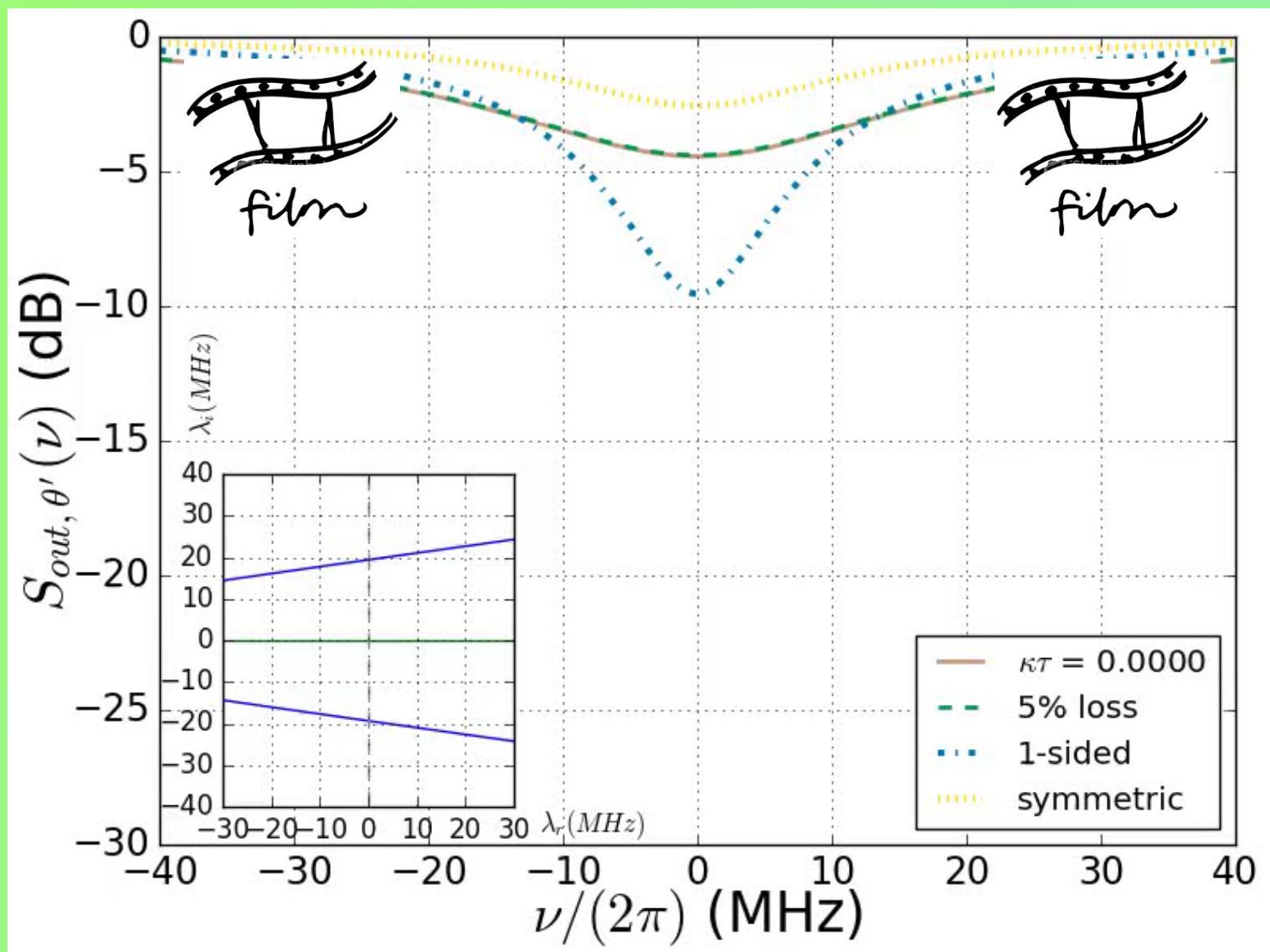


$$|\epsilon| = 0.49\kappa, \kappa_b = 0.933\kappa, \\ \kappa\tau = 3, L = 0 \text{ \& } 5\%$$

$$|\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2} \\ k = 2\sqrt{\kappa_b \kappa_c (1 - L)}$$

^[I] K. Pyragas, PLA, I70:421 (1992)

Squeezing spectra for $\phi = 0$



Squeezing for $\phi = 0$

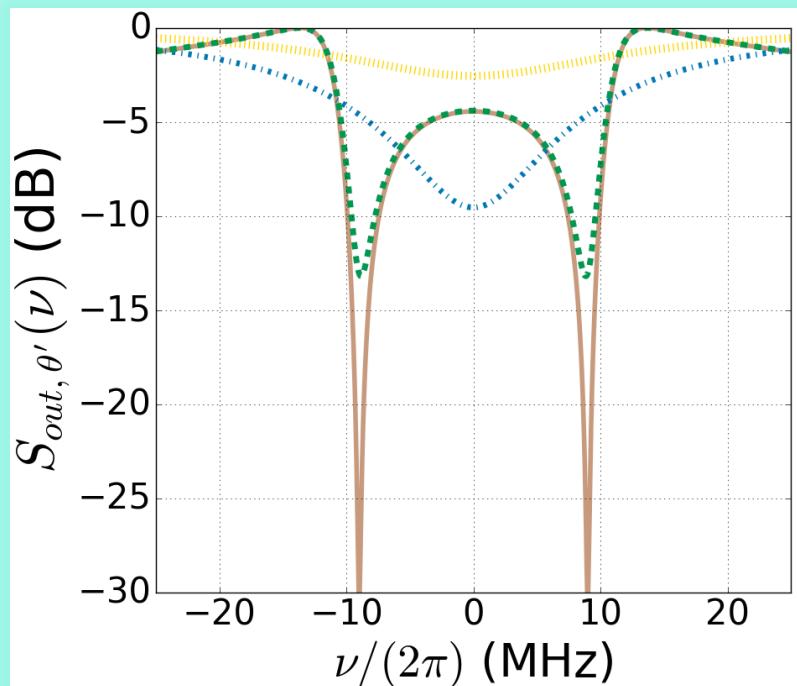


- Enhanced squeezing for a symmetric cavity compared to the one-sided case without feedback
- Best squeezing is off-resonant
 - Reduced threshold pump power
 - Time-delay dependent peaks
 - Perfect squeezing is related to change in stability
- Characteristic quantities:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$$

$$\tau_{c,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right)}{\nu_{s,\Delta}}$$

$$\nu_{c,\Delta} = \pm \sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$$



$$|\epsilon| = 0.5\kappa, \kappa_b = \kappa_c = 0.5\kappa, \\ \kappa\tau = 2.3, L = 0 \text{ & } 5\%$$

$$|\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2} \\ k = 2\sqrt{\kappa_b \kappa_c (1 - L)}$$

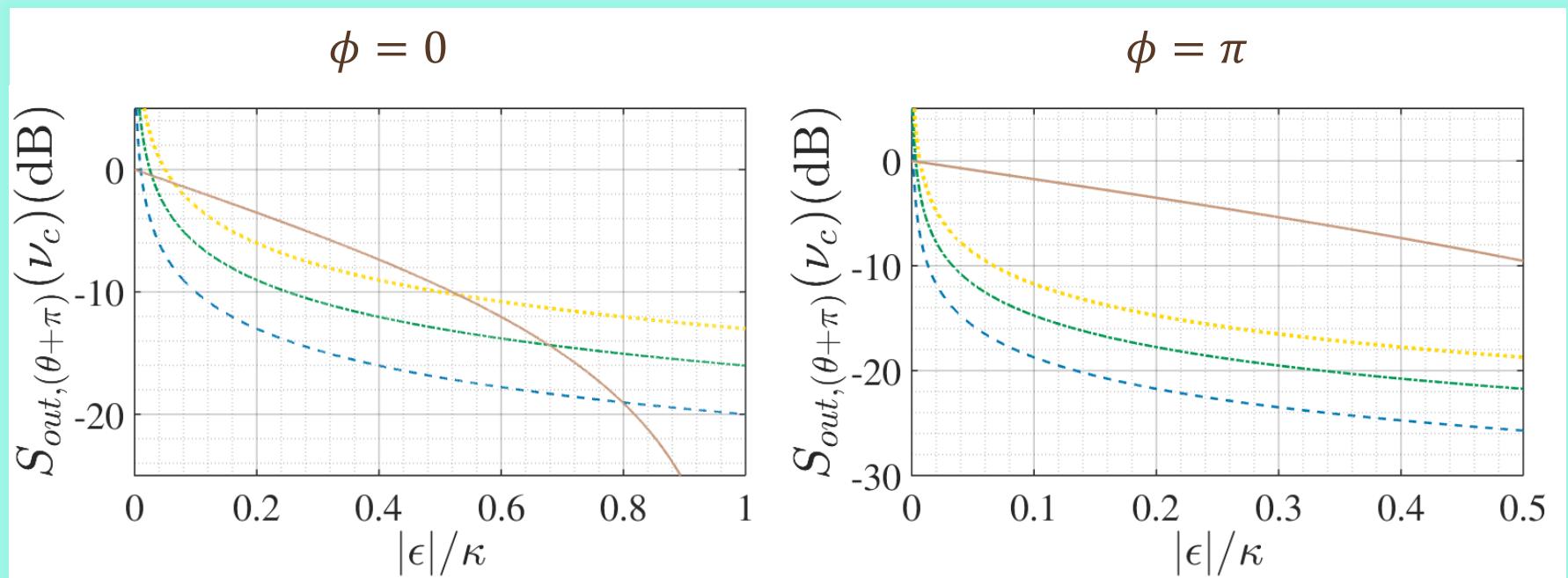
Effect of loss in the feedback loop



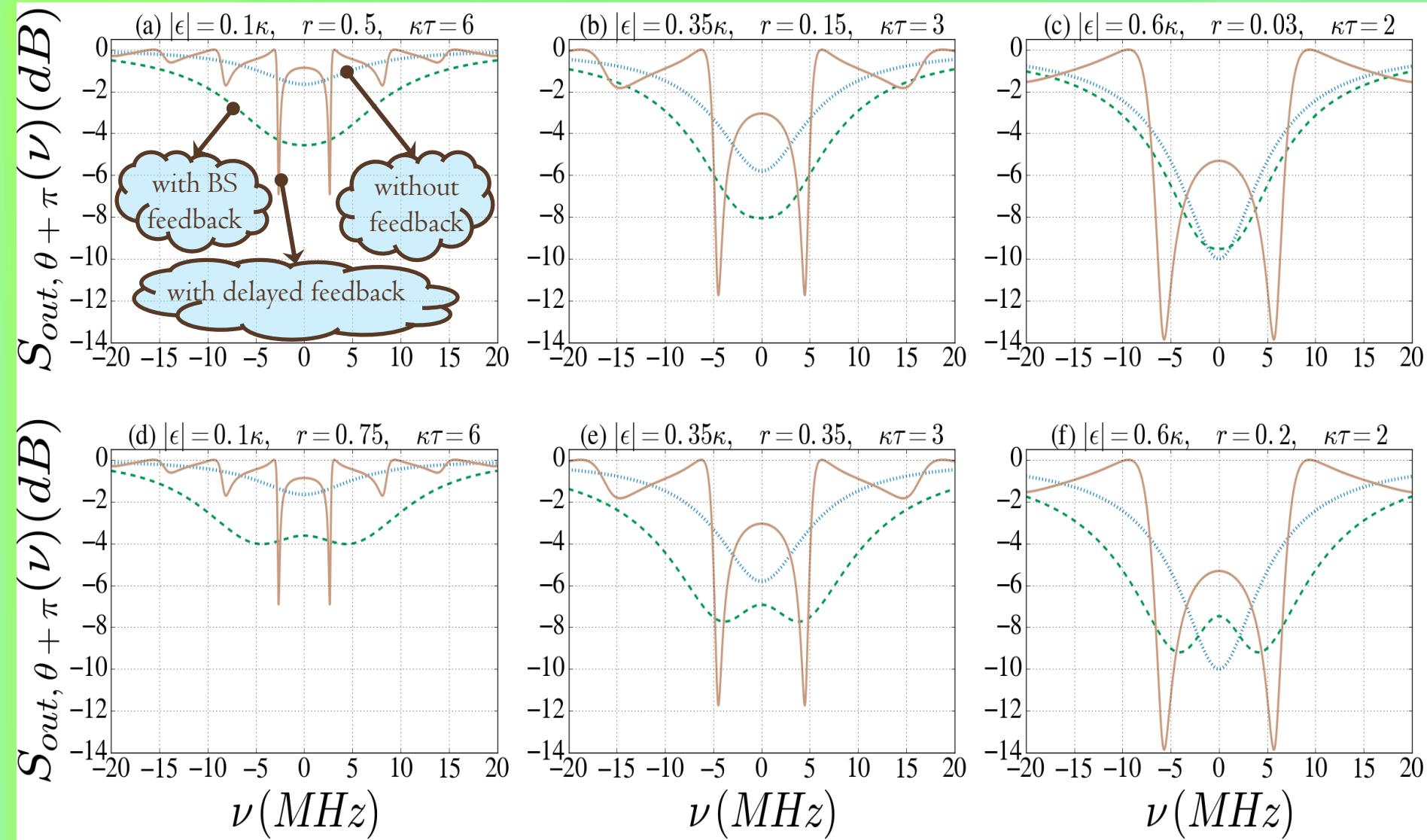
- Quadrature variance in the lossy case:

$$\chi_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$$

- Wider range of fluctuations are allowed in time-delay, detuning, phase shift, etc.



Comparison with previous results [1,2]



[¹] J. E. Gough, S. Wildfeuer, PRA, 80:042107 (2009), [²] A. Furusawa's group, IEEE Trans. Auto. Contr., 57:2045 (2012)

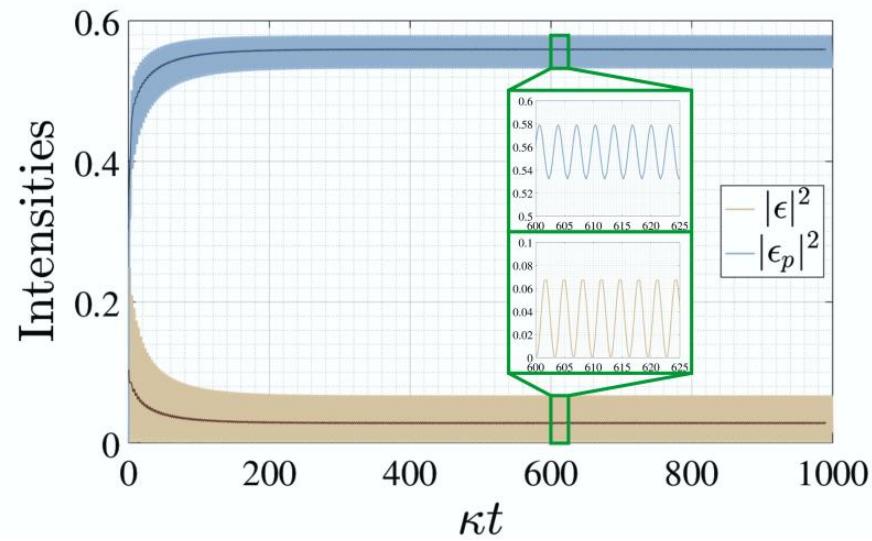
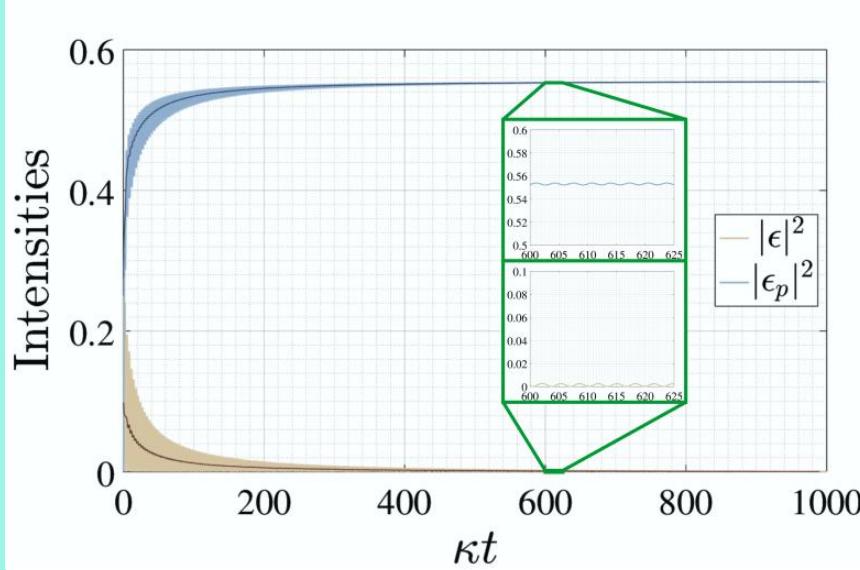
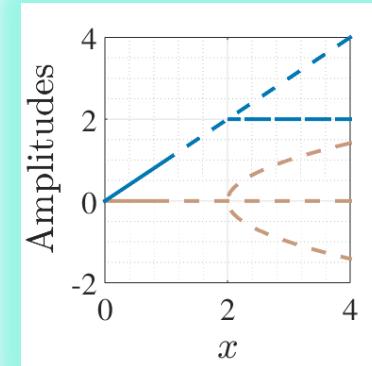
Classical analysis^[1]



$$\dot{\varepsilon} = -\kappa\varepsilon(t) + \kappa\varepsilon^*(t)\varepsilon_p(t) - e^{i\phi}k\varepsilon(t-\tau)$$

$$\dot{\varepsilon}_p = -\kappa_p(\varepsilon_p(t) + \varepsilon^2(t) - x)^{[2]}$$

- Threshold: changing stability
 - Pitchfork bifurcation: different steady states
 - Hopf bifurcation: reaching a limit cycle
- Imaginary eigenvalue becomes the frequency of persistent oscillations



^[1]Matlab package DDE-BIFTOOL, and solver dde23, ^[2]H. Carmichael: Statistical Methods in Quantum Optics 2 (2009)

Summary

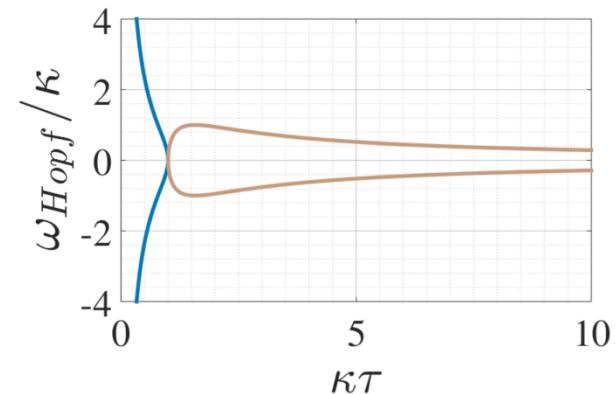
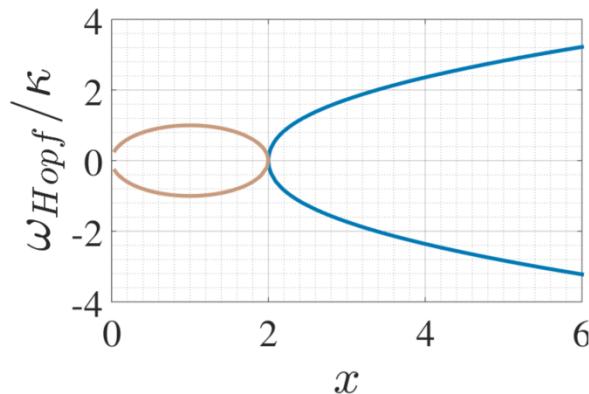
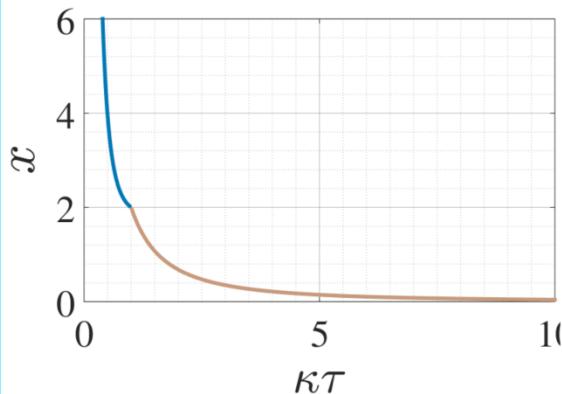
- Coherent quantum feedback:
 - Backaction-free way of controlling a quantum systems
- Influence of time-delay:
 - Memory effect in the environment
 - Changes in the stability landscape
- Time-delayed one-loop setup with a DPA: dramatically enhanced squeezing
 - Enhancement even compared to previous feedback setups
 - Result of the changing dynamics of the system
- Loss in the feedback loop decreases the degree of squeezing, but also provides higher tunability
- Future plans:
 - General linear model for two different modes
 - Nonlinear systems

1 2 3 4 5 6 7 8 9 10 11 12 13
14 15 16 17 18

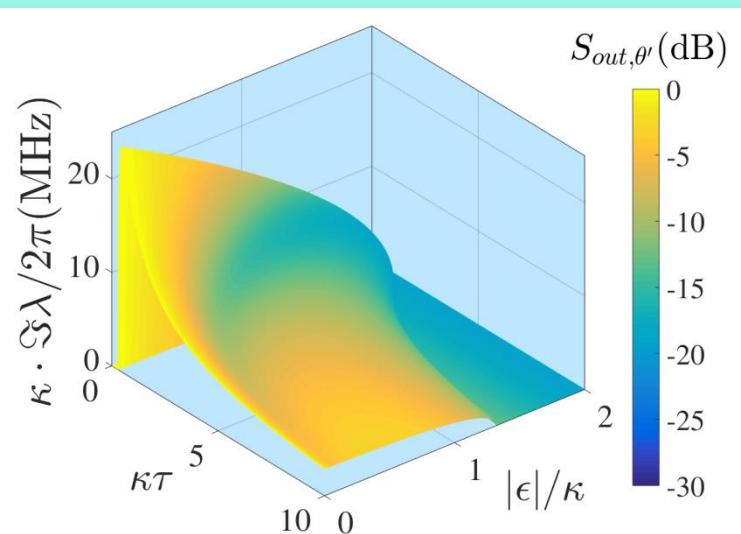
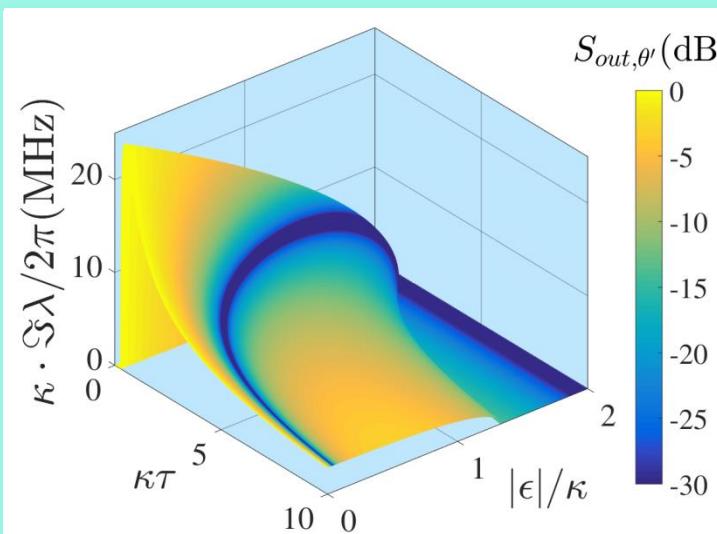
*Thank you for your
attention!*



Tunability



$$0 \leq v_s \leq \sqrt{|\epsilon|(2\kappa - |\epsilon|)}, 0 \leq |\epsilon| \leq |\kappa + ke^{i\phi}|$$



Effects of detuning



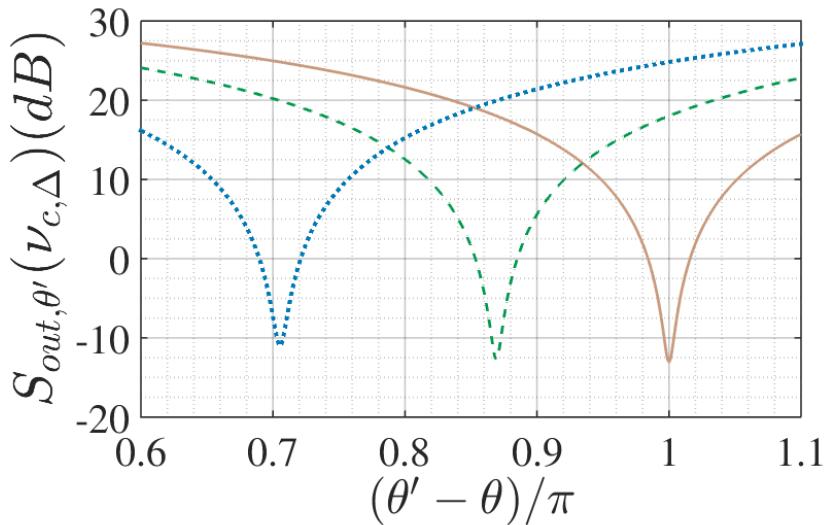
- Characteristic quantities:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$$

$$\tau_{s,n,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right) + 2n\pi}{v_{s,\Delta}}$$

$$v_{s,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$$

$$\phi = 0$$



- In the figures below:

- $|\epsilon| = \text{const.}$

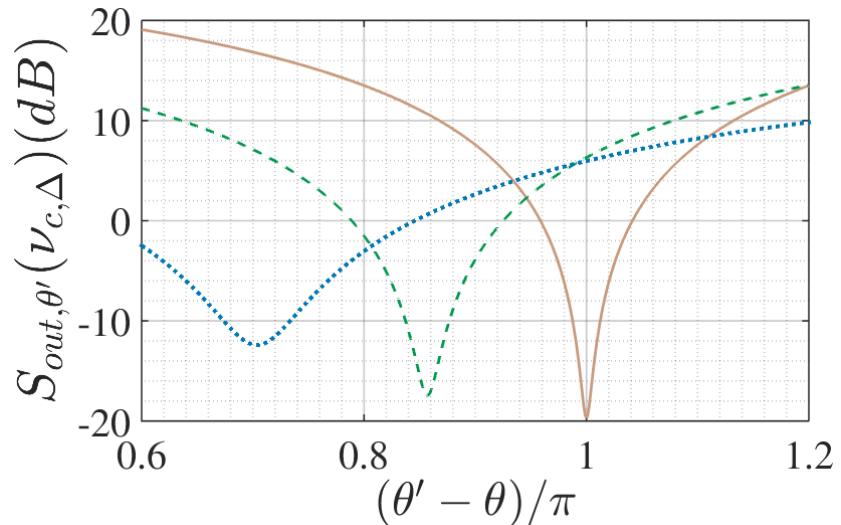
- $\Delta = 0$ (solid)

- $\Delta = 0.1\kappa$ (dashed)

- $\Delta = 0.2\kappa$ (dotted)

- Optimal frequency and time-delay

$$\phi = \pi$$



Effects of a phase shift

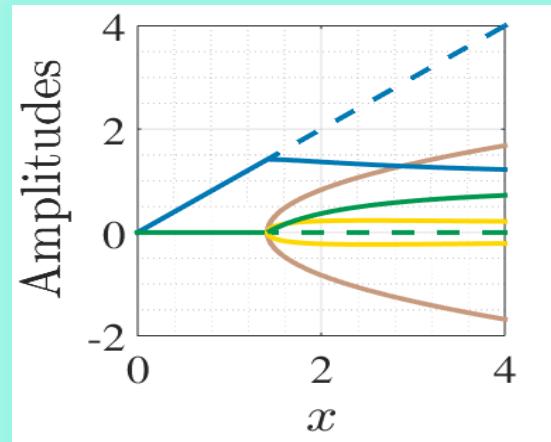


- In this figure:

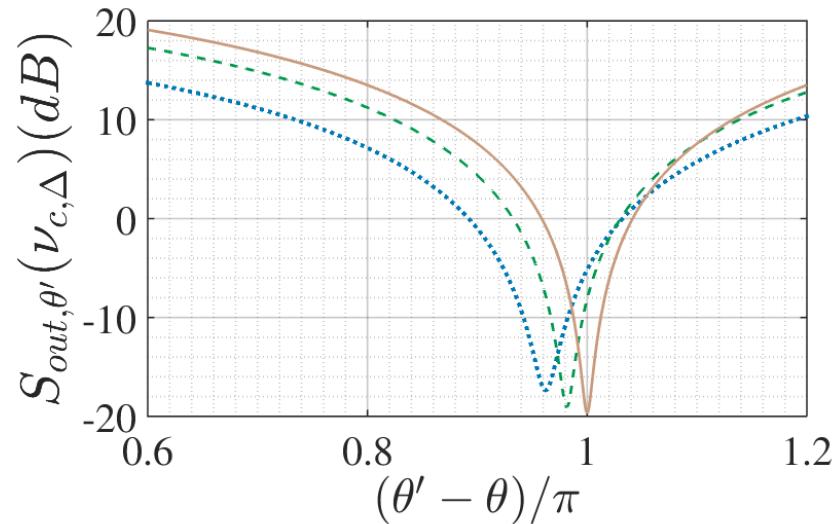
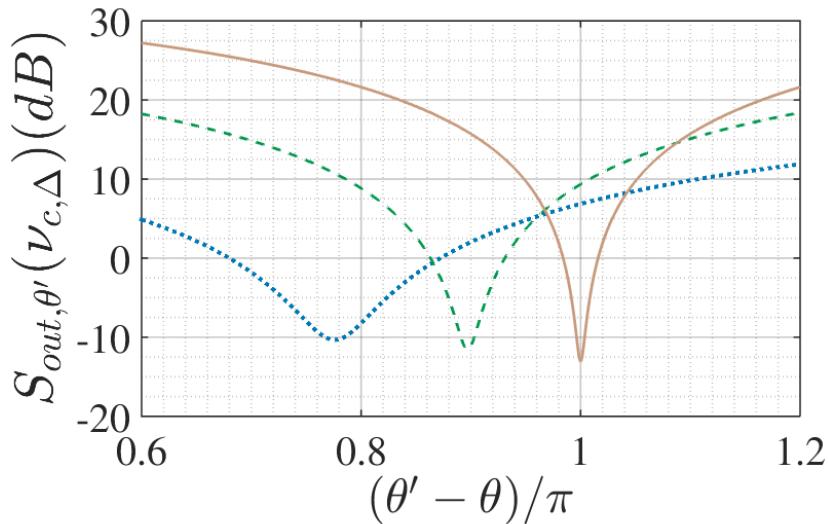
$$k = \kappa, \phi = \frac{\pi}{2}$$

ε_p : real (blue), imag (green)

ε : real (brown), imag (yellow)



$$\phi_0 = 0$$



- In the figures below:

- $|\epsilon| = \text{const.}$
 - $\phi = \phi_0 - 0.1\pi$ (dotted)
 - $\phi = \phi_0 - 0.05\pi$ (dashed)
 - $\phi = \phi_0$ (solid)
- Optimal frequency and time-delay

$$\phi_0 = \pi$$

Frequency-dependent squeezing



- Adding π to the local oscillator phase in our case gives the following frequency-dependence of the optimal quadrature angle for the maximum squeezing:
 - Low frequencies: around $\theta' - \theta = 0$
 - High frequencies: around $\theta' - \theta = \pi$
- Characteristic that is required for the new generation of gravitational wave detectors

