



THE UNIVERSITY OF  
AUCKLAND  
Te Whare Wānanga o Tāmaki Makaurau  
NEW ZEALAND



DODD-WALLS CENTRE  
for Photonic and Quantum Technologies

# *Manipulating the Squeezing Properties of a Degenerate Parametric Amplifier with Coherent, Time-Delayed Feedback*

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# Outline

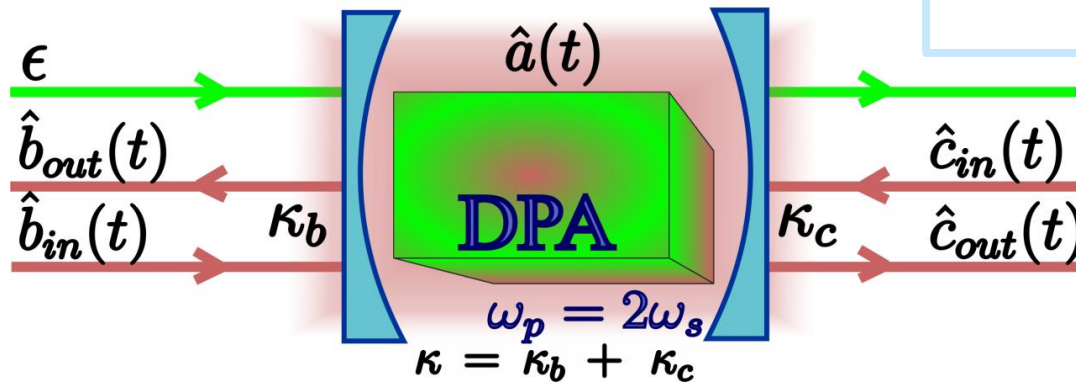
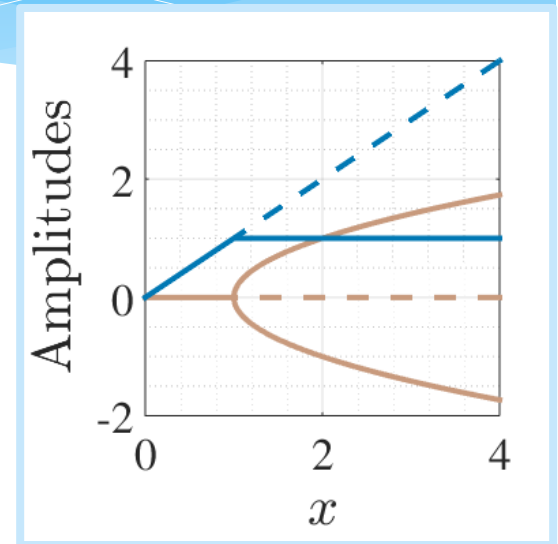
- \* Degenerate Parametric Amplifier
  - \* Classical dynamics
  - \* Quantum treatment below threshold
  - \* Squeezing characteristics
- \* DPA with time-delayed coherent feedback
  - \* Model
  - \* Squeezing spectrum
  - \* Change in the dynamics
  - \* Effects of loss



# DPA dynamics & squeezing

- \* Classical dynamics:
  - \* Saturation of pump: threshold behaviour
  - \* Bifurcation of steady state solutions at  $x = \frac{|\epsilon|}{\kappa} = 1$
- \* Quantum mechanical behaviour<sup>[1]</sup>:
  - \* Parametric pump approximation ( $\epsilon = |\epsilon|e^{i\theta}$ )

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$



<sup>[1]</sup>PRA, 30:I386 (1984)



# DPA dynamics & squeezing

- \* Classical dynamics:

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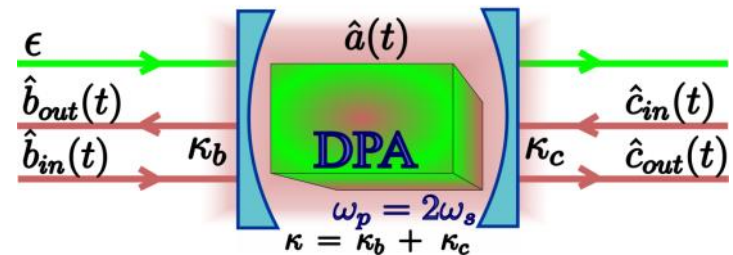
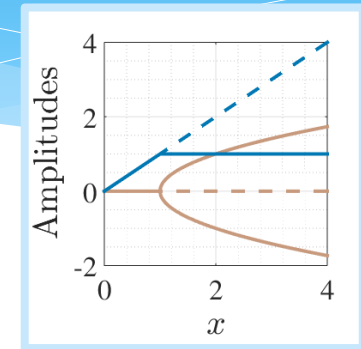
$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

- \* One-sided cavity quadrature variance:

$$\chi_{out,\theta+\pi}(0) = \int \left\langle \tilde{X}_{out,\theta'}(0), \tilde{X}_{out,\theta'}(v') \right\rangle dv' = \frac{1}{4} \frac{(\kappa - |\epsilon|)^2}{(\kappa + |\epsilon|)^2}$$

- \* Symmetric cavity squeezing on resonance at threshold

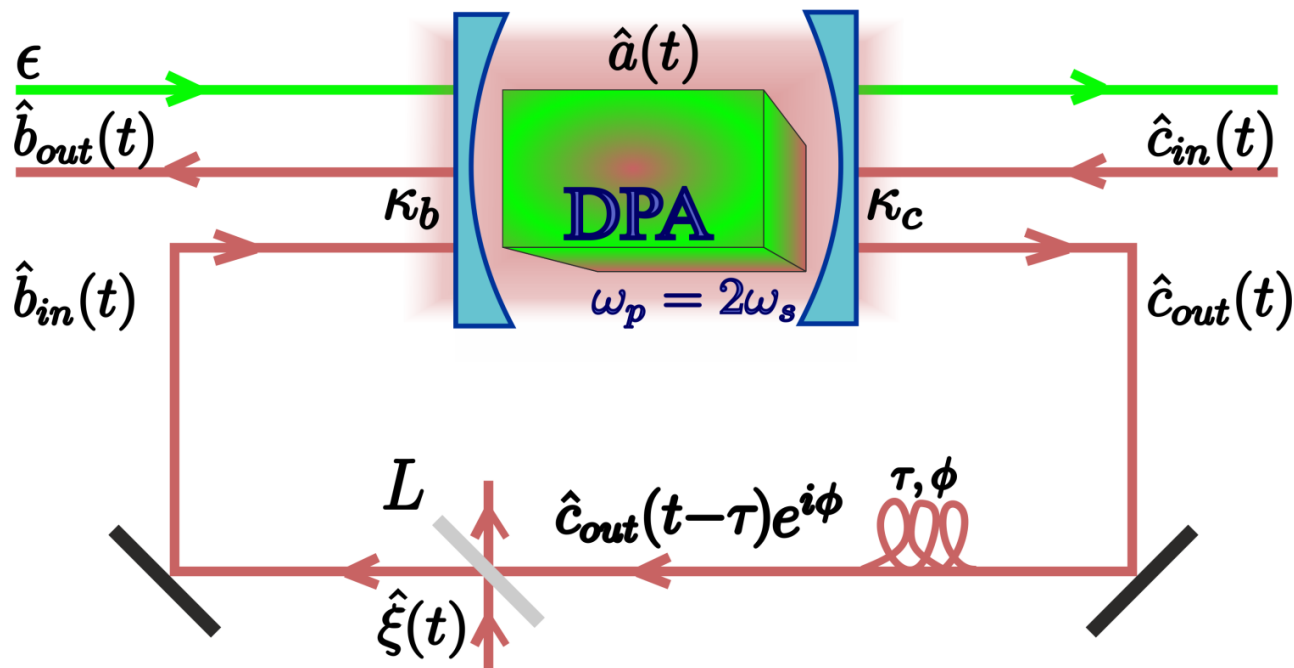
$$\chi_{out,\theta+\pi}(0) = \frac{1}{4} \frac{\kappa^2 + |\epsilon|^2}{(\kappa + |\epsilon|)^2}$$



# DPA & time-delayed coherent feedback

- \* Uncomplicated setup

- \* Time-delay  $\tau$ , overall phase-shift  $\phi$ , overall loss, decoherence:  
 $L, \hat{\xi}$



# DPA & time-delayed coherent feedback

- \* Uncomplicated setup
  - \* Time-delay  $\tau$ , overall phase-shift  $\phi$ , overall loss, decoherence:  $L, \hat{\xi}$
- \* Equation of motion of the subharmonic mode:

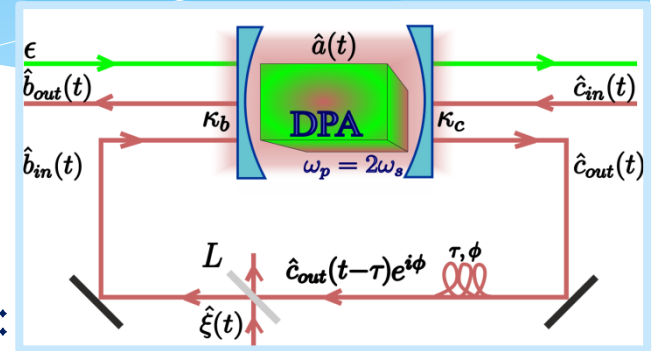
$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - \kappa\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) - e^{i\phi}k\hat{a}(t - \tau)$$

$$\hat{H} = \hbar\Delta\hat{a}^\dagger\hat{a} + \frac{1}{2}i\hbar\left(\epsilon(\hat{a}^\dagger)^2 + \epsilon^*(\hat{a})^2\right)$$

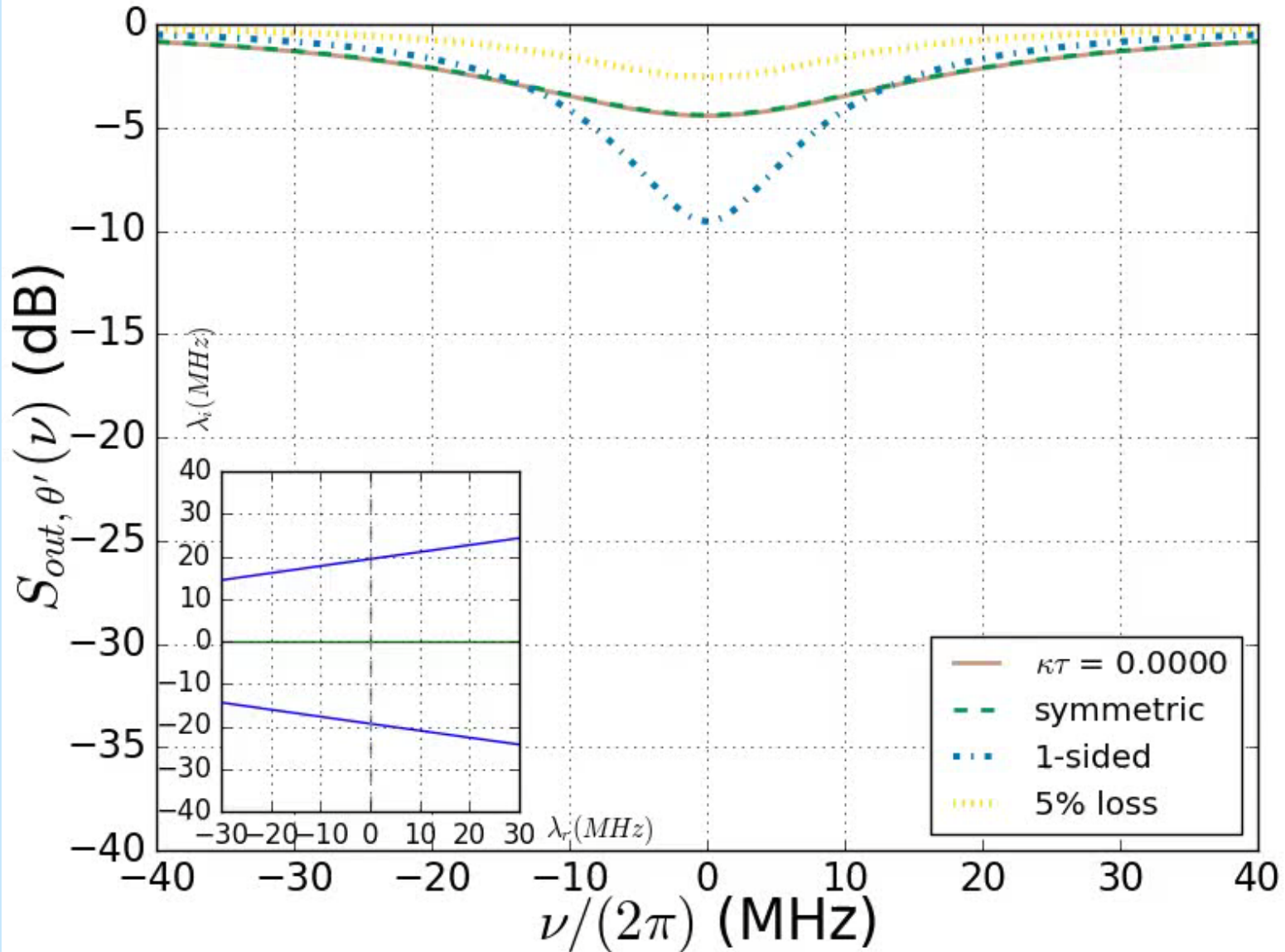
$$\kappa = \kappa_b + \kappa_c, k = 2\sqrt{\kappa_b\kappa_c(1-L)}$$

- \* Quadrature variance without delay:

$$\chi_{out, \theta + \pi}(0) = \frac{1}{4} \frac{((\kappa + k \cos \phi) - |\epsilon|)^2}{((\kappa + k \cos \phi) + |\epsilon|)^2}$$



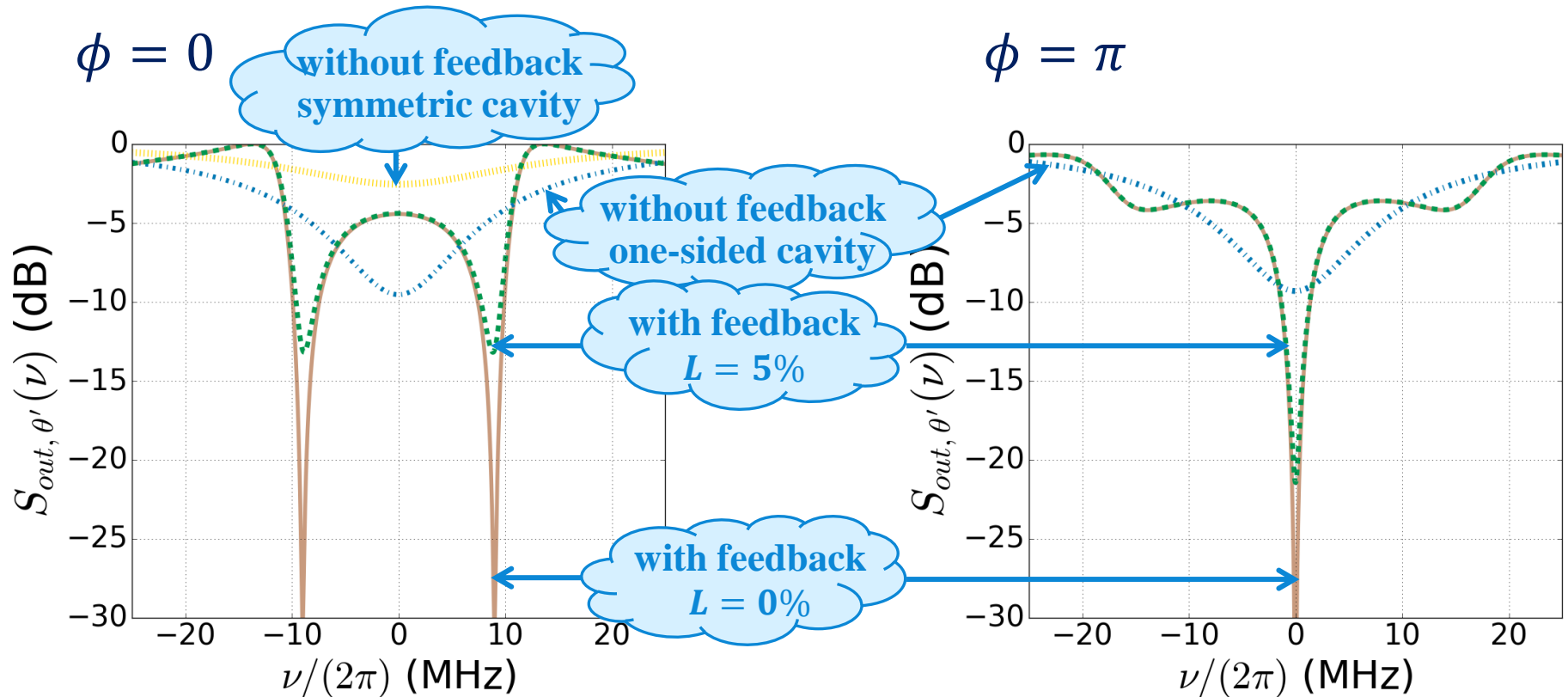
# Emerging side-peaks





# New setup with time-delay

- \* Transcendental equations
- \* Enhanced squeezing with feedback



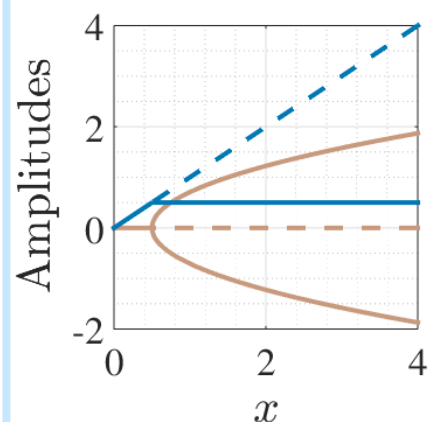


# New setup with time-delay

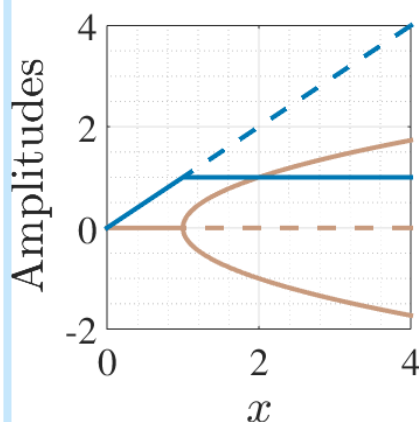
- \* Transcendental equations
- \* Enhanced squeezing with feedback
- \* Pyragas-type feedback:

$$\frac{d\hat{a}}{dt} = i[\hat{H}, \hat{a}(t)] - (\kappa - k)\hat{a}(t) - \sqrt{2\kappa}\hat{a}_{in}(t) + k(\hat{a}(t - \tau) - \hat{a}(t))$$

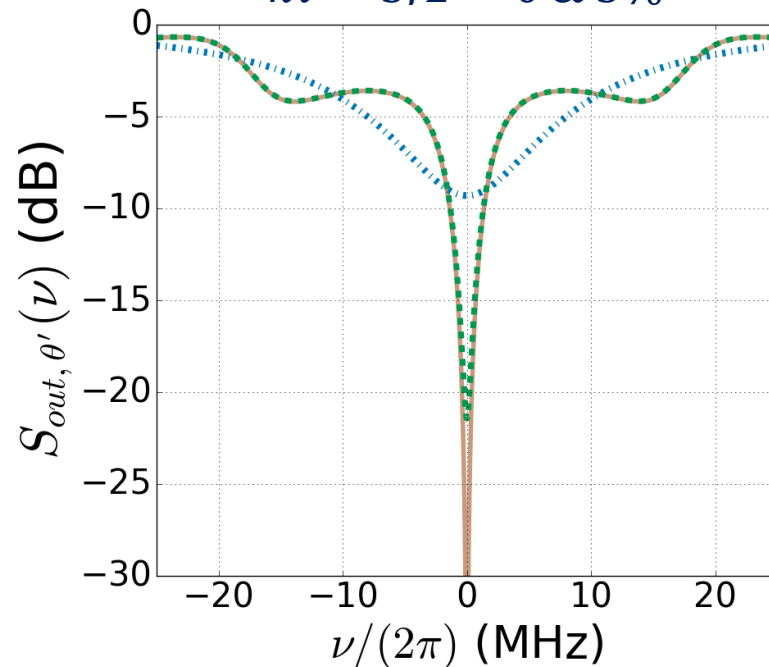
$k = \kappa/2, \phi = \pi$



$k = 0, \phi = 0$



$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa,$   
 $\kappa\tau = 3, L = 0 \text{ \& } 5\%$

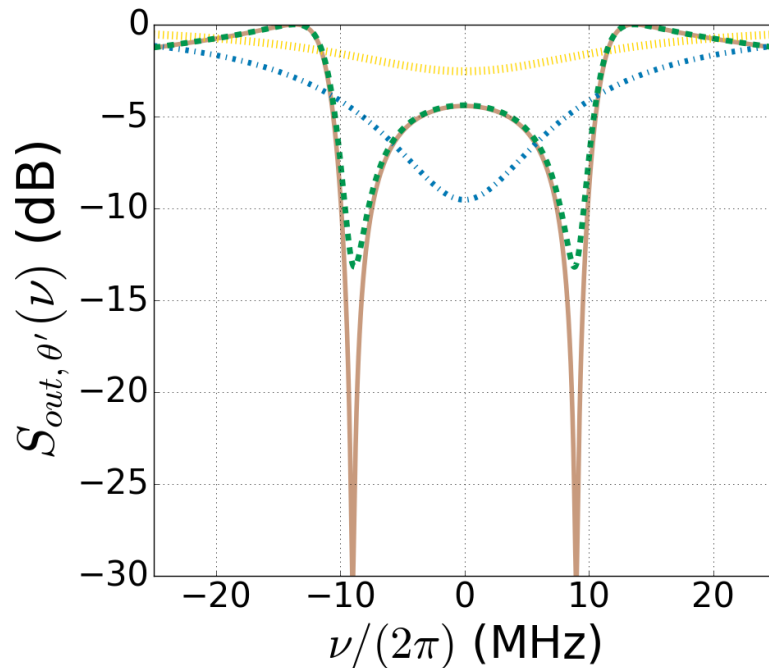




# New setup with time-delay

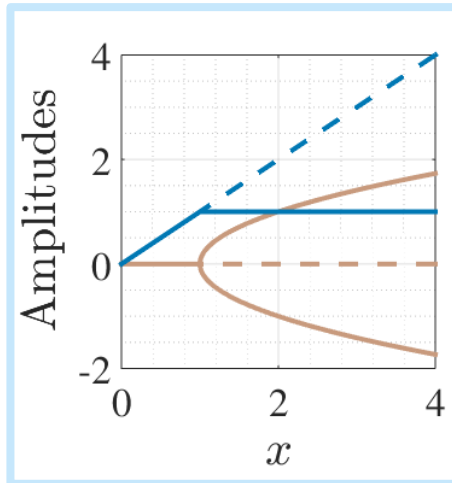
- \* Transcendental equations
- \* Enhanced squeezing with feedback

$$\phi = 0, \kappa_b = \kappa_c = 0.5\kappa, |\epsilon| = 0.5\kappa, \\ \kappa\tau = 2.3, L = 0 \text{ \& 5\%}$$

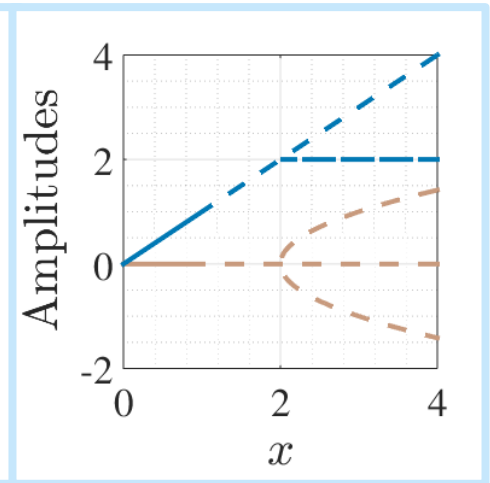


- \* Limit cycle is reached on threshold
- \* Persistent oscillations with the characteristic frequency

$$k = 0, \phi = 0, \kappa\tau = 0$$



$$k = \kappa, \phi = 0, \kappa\tau = 1.57$$

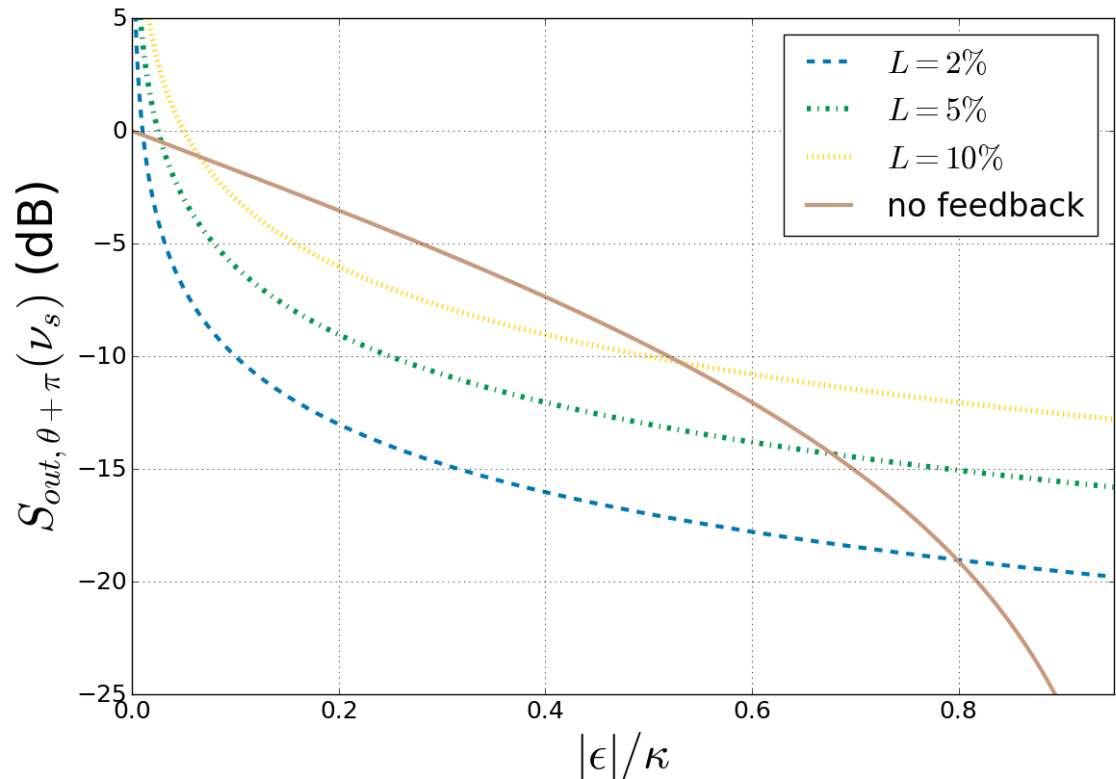


# Effects of loss



- \* With loss wider range of tunability
- \* Quadrature variance at the critical point:

$$\chi_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$$



# Summary

- \* Coherent feedback:
  - backaction-free way of enhancing useful quantum mechanical behaviours
- \* Time-delay:
  - tunable changes in the stability range
- \* Time-delayed one-loop setup with a DPA: enhanced squeezing
  - \* On and off-resonant squeezing as well
  - \* Result of the changing dynamics of the system
- \* Loss in the feedback loop:
  - \* Decreased squeezing
  - \* BUT! wider range of tunability

*Thank you for Your  
attention!*

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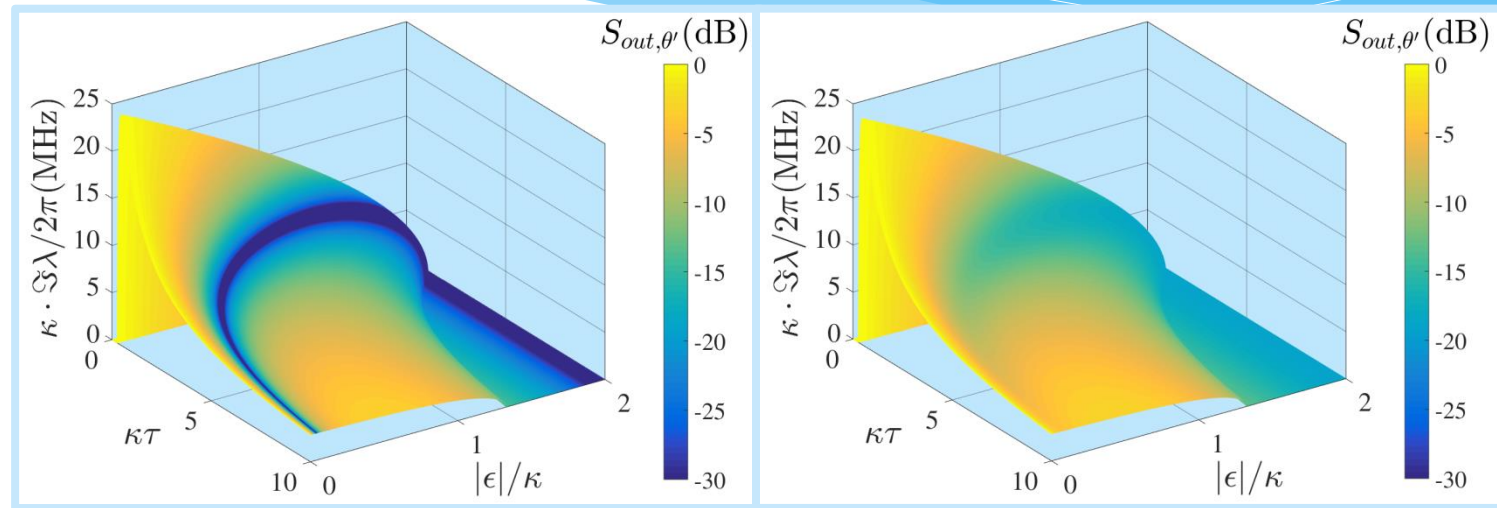
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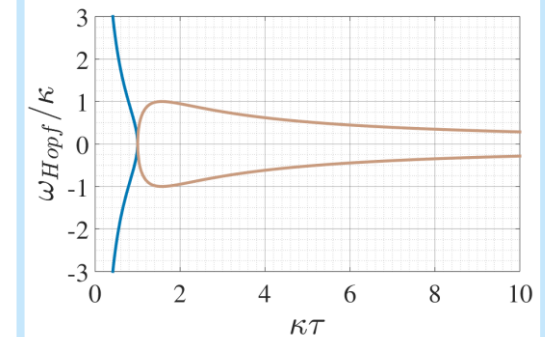
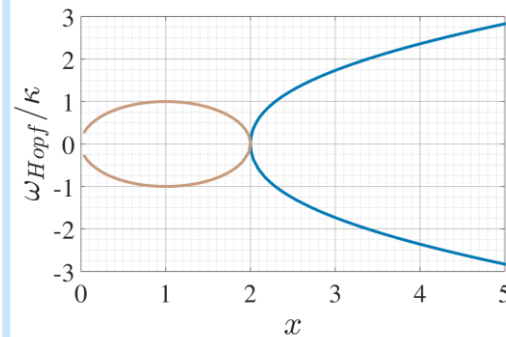
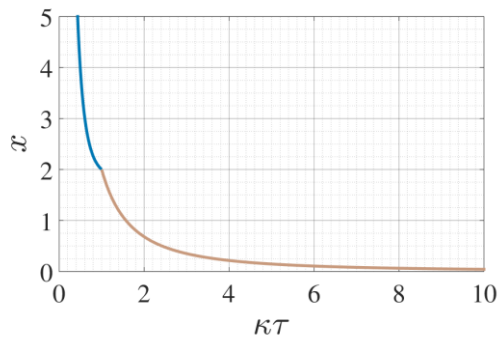




# Characteristic frequency range



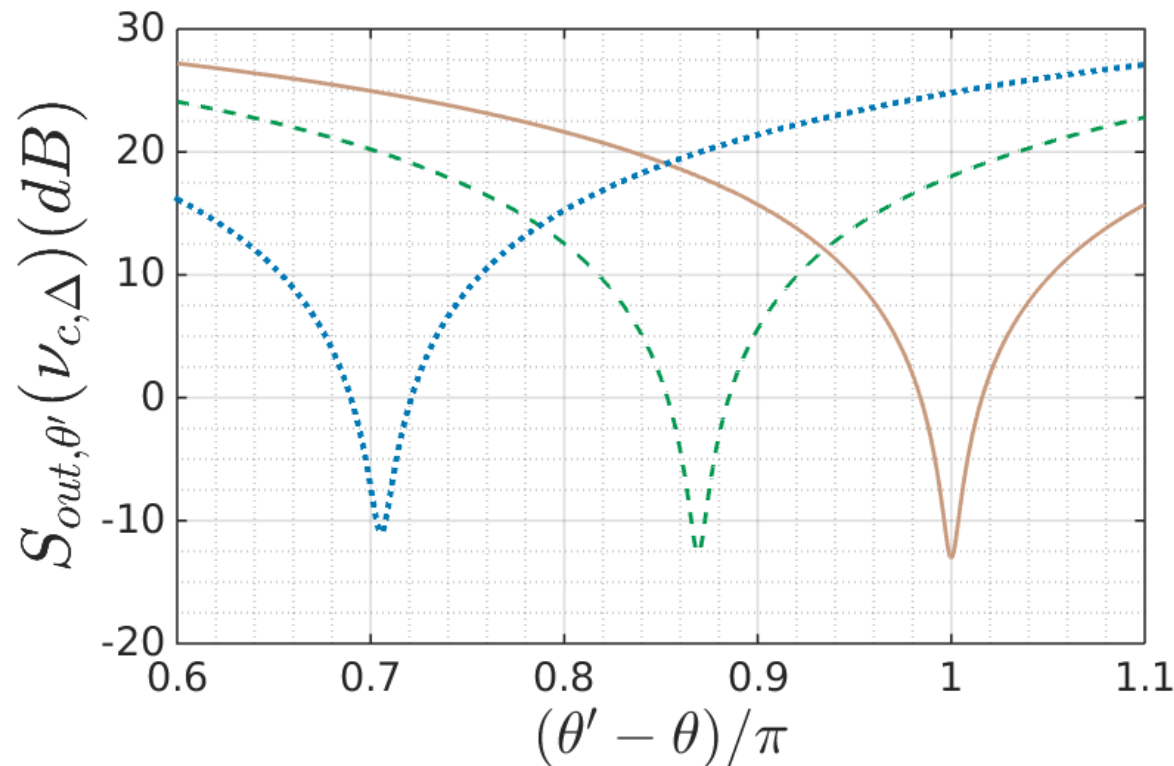
$$0 \leq \nu_s \leq \sqrt{|\epsilon|(2\kappa - |\epsilon|)}, 0 \leq |\epsilon| \leq |\kappa + ke^{i\phi}|$$



# Detuning and quadrature angle

- \* Local oscillator phase:

$$\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_{\Delta}|}\right)$$



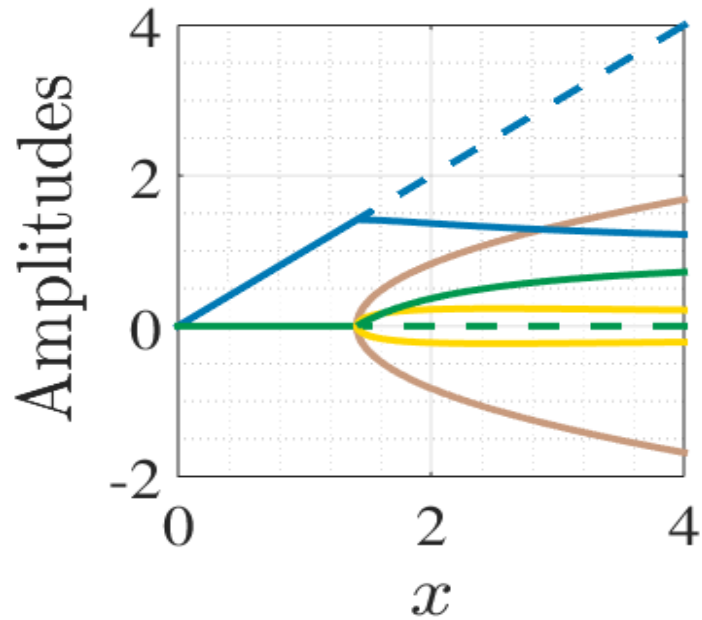
$$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2},$$
$$\kappa\tau = 2.418,$$
$$\nu_{c,\Delta} = 0.866\kappa,$$
$$L = 5\%,$$

$\Delta = 0$  (solid),  
 $0.1\kappa$  (dashed)  
 $0.2\kappa$  (dotted)





# Effects of the phase shift

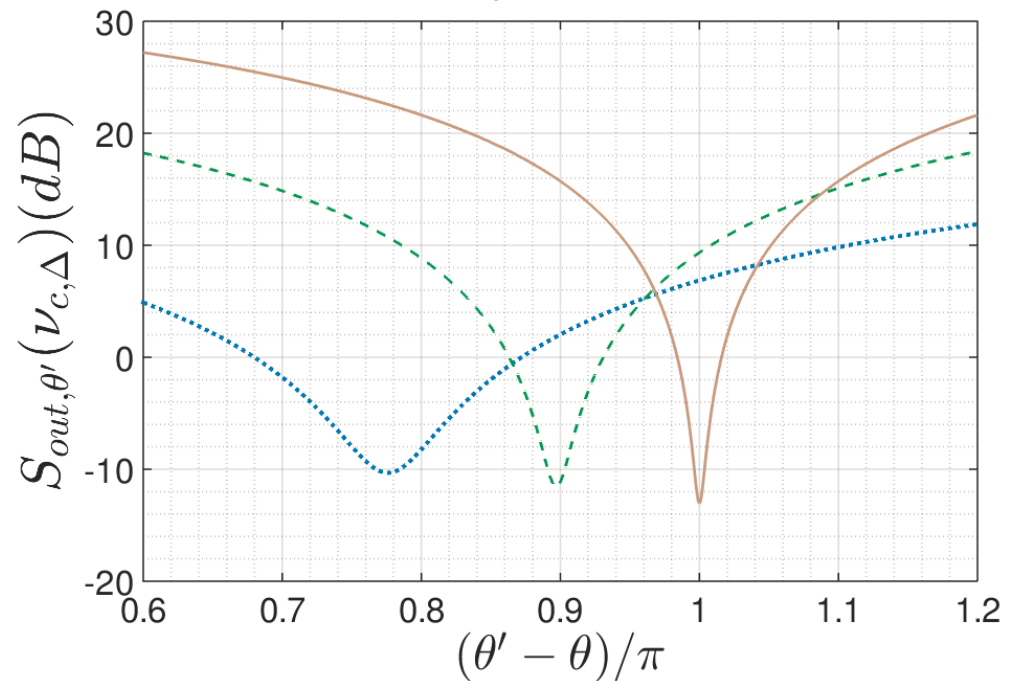


$$k = \kappa, \phi = \pi/2$$

$\varepsilon_p$ : real (blue), imag (green)

$\varepsilon$ : real (brown), imag (yellow)

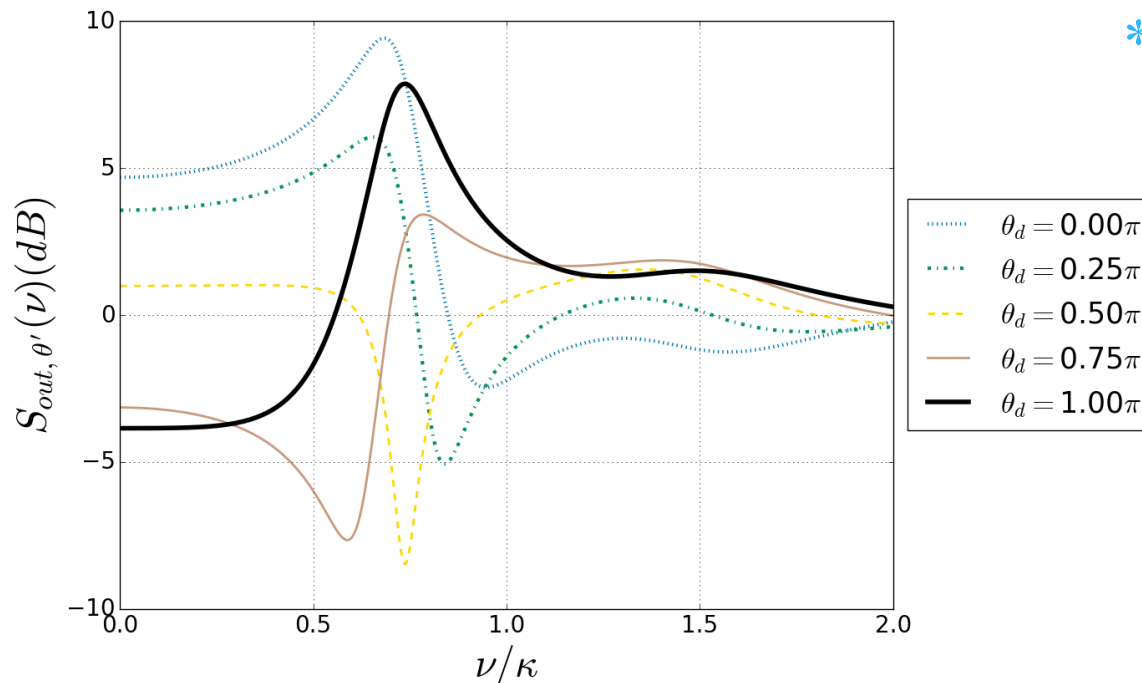
$$|\epsilon| = \frac{\kappa}{2}, \kappa_b = \kappa_c = \frac{\kappa}{2}, \kappa\tau = 2.418, \nu_{c,\Delta} = 0.866\kappa, \\ L = 5\%, \quad \phi = -0.1\pi, 0.05\pi \text{ \& } 0$$



# Gravitational waves



- \* Adding  $\pi$  to the local oscillator phase in our case gives the required characteristics



- \* Frequency of the best squeezing: changing with the local oscillator phase
- \* Lower frequencies: around  $\theta' - \theta = 0$
- \* Higher frequencies: around  $\theta' - \theta = \pi$

# Coherent feedback with DPA

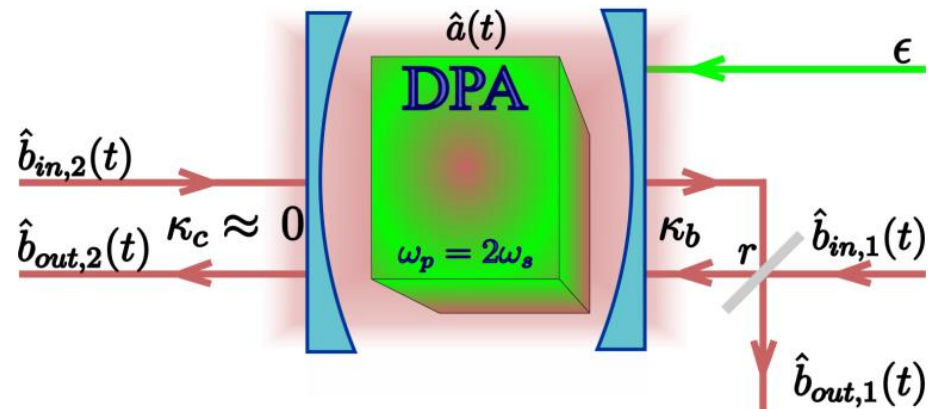


## \* Single DPA

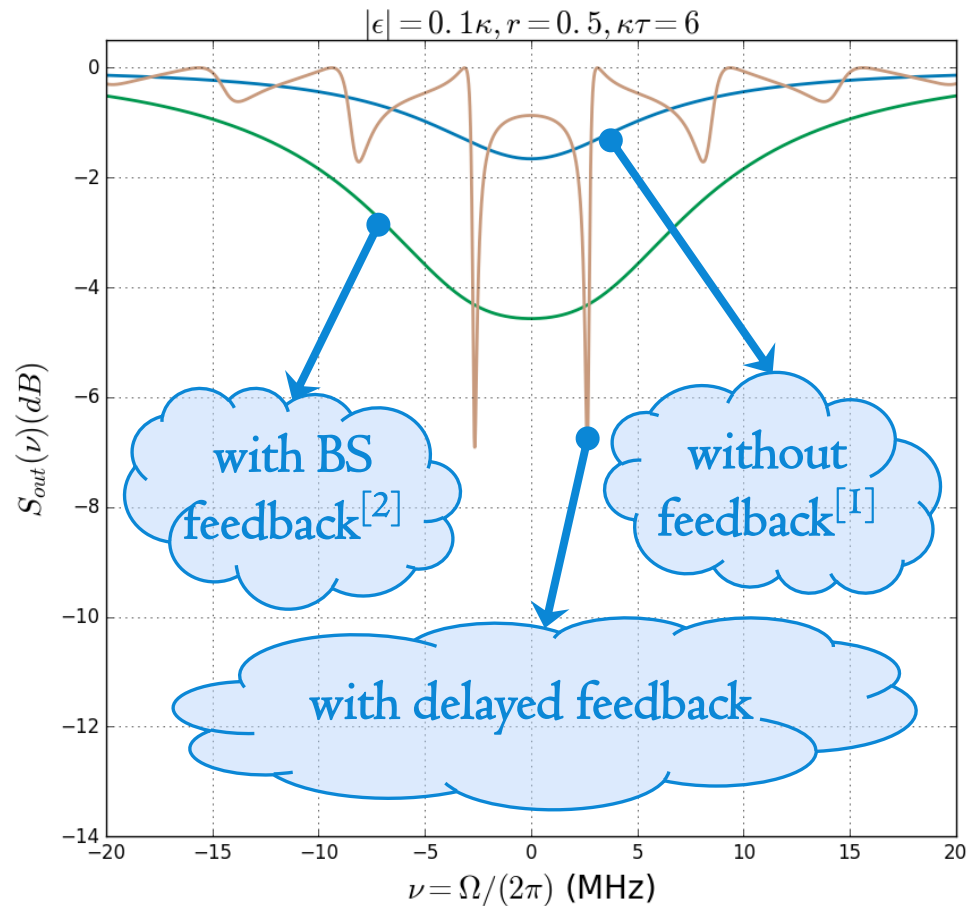
- \* Feedback via beam splitter<sup>[1]</sup>
- \* Enhanced, tunable squeezing at a given driving strength

$$\mathcal{X}_{out,\theta+\pi}(0) = \frac{1}{4} \frac{(\kappa(r) - |\epsilon|)^2}{(\kappa(r) + |\epsilon|)^2}$$

- \* Modified threshold
- \* Experiment<sup>[2]</sup> agrees with theory<sup>[1]</sup>
- \* BUT! Performance is limited
  - \* efficient  $|\epsilon|$  range under  $0.6\kappa$
  - \* losses through the „perfectly reflecting mirror”

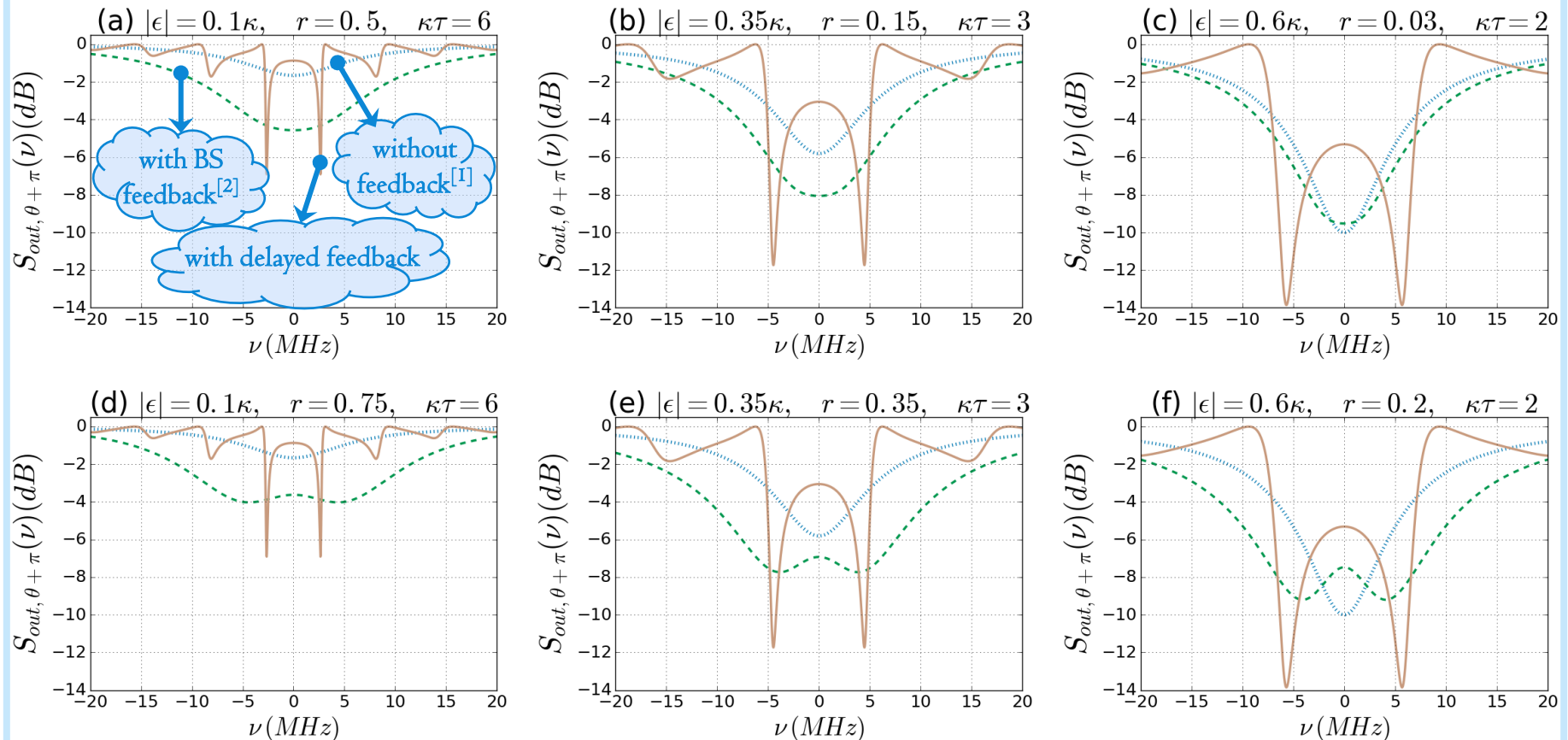


# Comparison with previous results<sup>[1,2]</sup>



<sup>[1]</sup>PRA, 30:1386 (1984), <sup>[2]</sup>IEEE Trans. Auto. Contr., 57:2045 (2012)

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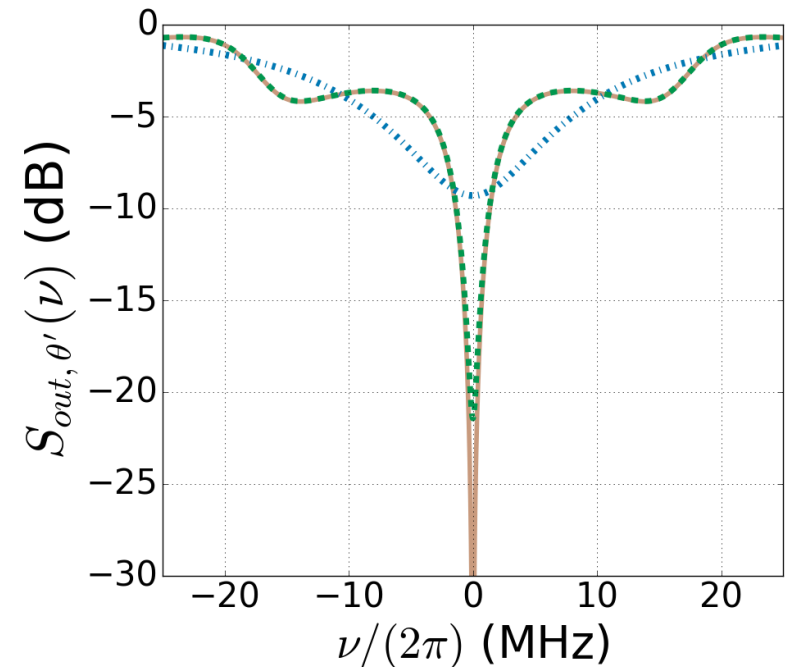
<sup>[1]</sup>PRA, 30:1386 (1984), <sup>[2]</sup>IEEE Trans. Auto. Contr., 57:2045 (2012)



# New setup with time-delay

- \* Transcendental equations
- \* Enhanced squeezing with feedback
- \* Properties:
  - \* Best squeezing is on resonance
  - \* Effects of time-delay:
    - \* Narrower spectrum
    - \* Emerging small side-peaks
  - \* Perfect squeezing at stability change
    - \* Condition:  $\kappa - k = |\epsilon_\Delta| = \sqrt{|\epsilon|^2 - \Delta^2}$
    - \* Local phase:  $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
    - \* Variance:  $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$

$$\phi = \pi, \kappa_b = 0.93\kappa, |\epsilon| = 0.49\kappa, \\ \kappa\tau = 3, L = 0 \text{ \& \; } 5\%$$

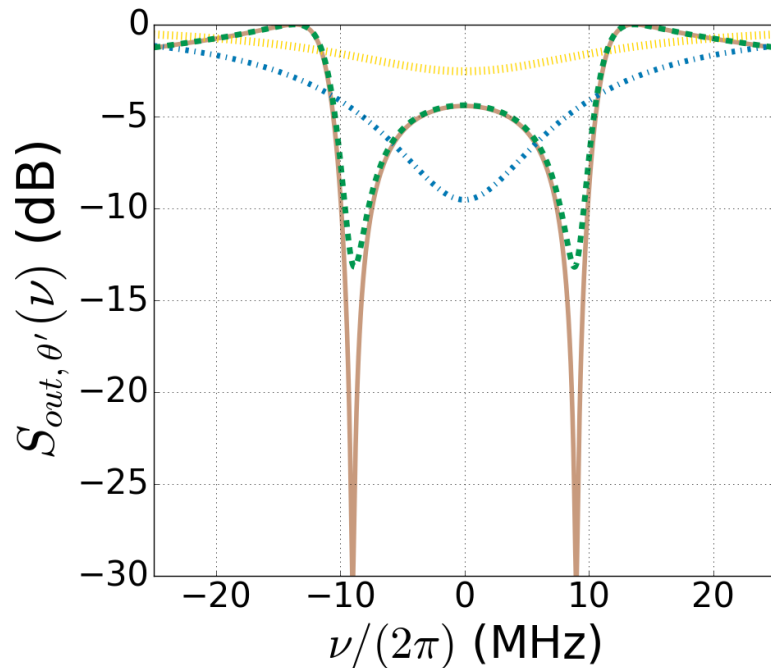




# New setup with time-delay

- \* Transcendental equations
- \* Enhanced squeezing with feedback

$$\phi = 0, \kappa_b = \kappa_c = 0.5\kappa, |\epsilon| = 0.5\kappa, \\ \kappa\tau = 2.3, L = 0 \text{ \& 5\%}$$



## \* Properties:

- \* Best squeezing is off-resonant
- \* Perfect squeezing at stability change
  - \* Local phase:  $\theta' = \theta - \pi + \arcsin\left(\frac{\Delta}{|\epsilon_\Delta|}\right)$
  - \* Time-delay:  $\tau_{s,n,\Delta} = \frac{\arccos\left(\frac{|\epsilon_\Delta| - \kappa}{k}\right) + 2n\pi}{\nu_{s,\Delta}}$
  - \* Frequency:  $\nu_{s,\Delta} = \pm\sqrt{k^2 - (|\epsilon_\Delta| - \kappa)^2}$
  - \* Variance:  $\mathcal{X}_{out,\theta'}(\nu_{s,\Delta}) = \frac{1}{4} \frac{L\kappa_c}{|\epsilon_\Delta|}$

# Classical analysis

- \* Originally: pitchfork bifurcation
- \* Time-delay: Changing stability of steady state solutions

↓  
Hopf bifurcation

- \* Imaginary eigenvalue becomes the frequency of persistent oscillations

