Lab 3 – Performance analysis

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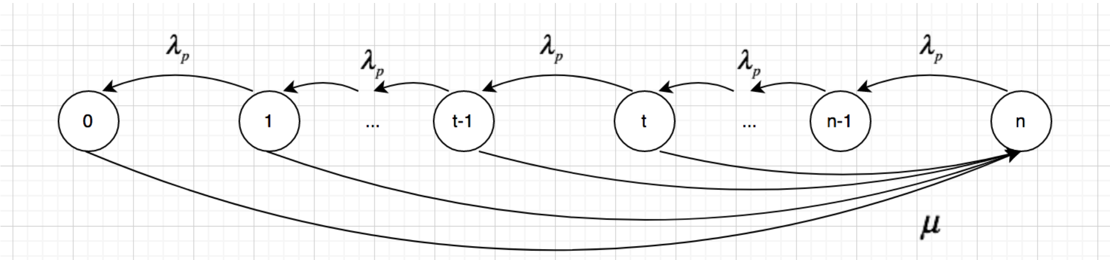
III.A.I

The system state in each section id the current stock s_i, and the state space is the capacity N_i +1. In other words, the state space contains N_i +1 nodes, one node for all the system states including one node when the stock is zero. The system state changes either when it gets refilled, or a customer takes something from the stock. The transition intensities are $\lambda_p = 1 min^{-1}$ for taking items and μ for refill intensity. In the figure below, we se that only if we are in state t (threshold) or below, the shelf can be refilled with intensity μ.

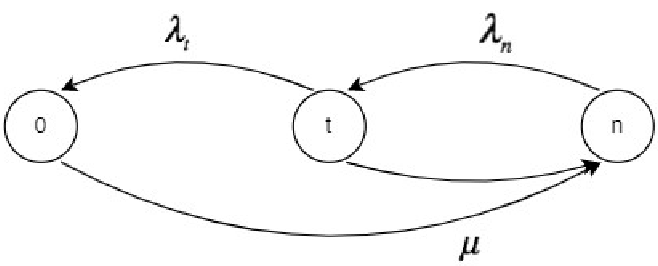


III.A.2

We can aggregate the graph above to only 3 nodes. We have three main states:

1. We have more in stock than the refill threshold and at least 3 items. (n)
2. We have less in stock than the refill threshold, but more than 3 items. (t)
3. We have less than three items. (0)

The Markov model then looks like this:



III.A.3

When we combine multiple states will the new intensities be the arrival intensity times the probability of being in the state where that intensity originated. The new intensities are shown in the picture to the right, where N_i are the shelf capacity and u_i are the average refilling time.

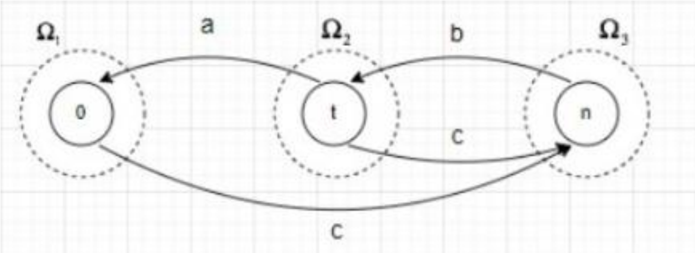
$$\lambda_n = \frac{\lambda_p}{N_i - t} = \frac{\lambda_p}{N_i - (p_i \cdot N_i)}$$

$$\lambda_t = \frac{\lambda_p}{t-2} = \frac{\lambda_p}{(p_i \cdot N_i) - 2}$$

$$\mu = \frac{1}{u_i}$$

III.A.4, III.A.5

We substitute λ_t, λ_n, and μ with a, b, and c and get the equation below.



$$\Omega_1 \rightarrow c \cdot p_0 = a \cdot p_t$$
$$\Omega_2 \rightarrow a \cdot p_t + c \cdot p_t = b \cdot p_n$$
$$\Omega_3 \rightarrow c \cdot p_t + c \cdot p_0 = b \cdot p_n$$

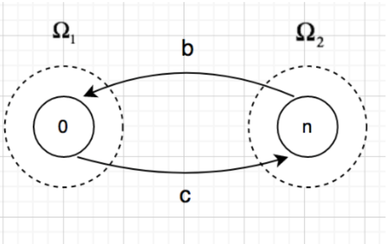
When solving for the probabilities we get the equations to the right.

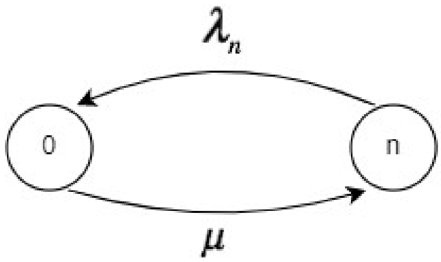
$$p_0 = \frac{a \cdot b}{(a+c)(b+c)}$$
$$p_t = \frac{b \cdot c}{(a+c)(b+c)}$$
$$p_n = \frac{c}{b+c}$$

There is an edge case for section 5 when there is one employee. The threshold for refilling is $p_i = 0.05*E$ (E=number of employees). For 1 employee and $N_i = 40$ we have that the threshold is 2<3 → We can skip node t. Therefore, we have a unique model for section 5, without the t node.

With the same substitutions and solving for the probabilities for section 5 we get:

$$\Omega_1 = \Omega_2 \rightarrow b \cdot p_n = p_0 \cdot c$$
$$p_0 + p_n = 1$$
$$p_n = \frac{c}{b+c}, p_0 = \frac{b}{b+c}$$





Edge case for section 5

Here are the results when calulating for 1 employee. The calulations are done in the appended python script lab3.py. v is equal to 1- the probability of beeing in state 0, meaning that the custumer won't get any items.

Section	A	B	C	v	MOS
0	0.333333	0.010526	0.016667	0.631336	1
1	0.181818	0.007018	0.027778	0.825048	3
2	2.000000	0.021053	0.023810	0.530726	1
3	0.181818	0.007018	0.023810	0.798716	2
4	0.500000	0.013158	0.033333	0.734670	2
5	inf	0.016667	0.612245	0.387755	1
6	0.095238	0.011111	0.246098	0.753902	2

III.B.1

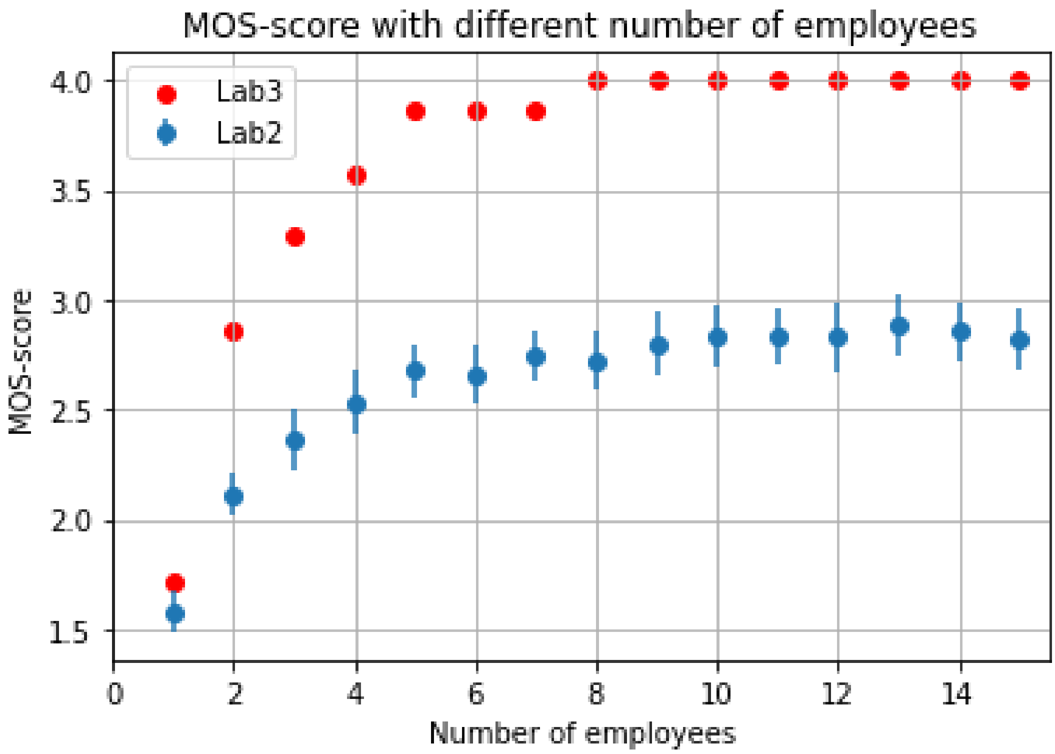
Since the three states, 0, t, and n, remains the same, the state description also remains the same. However, the refill transition will change based on the number of employees.

III.B.2

λ_t and λ_n will be impacted, because p_i will be employee count dependent, but the equations won't change. Below are the new refill intensity.

$$\mu = \frac{E}{u_i}$$

III.C.1



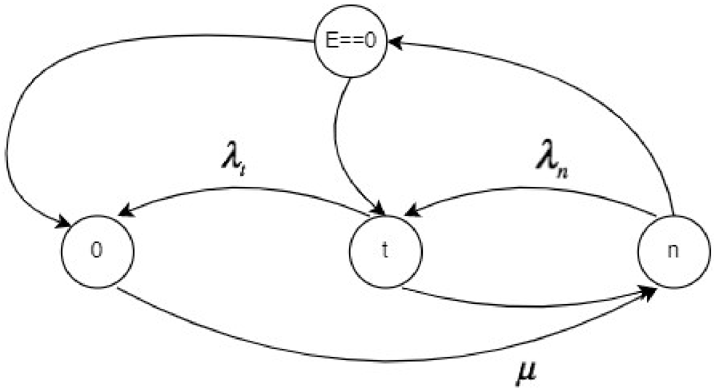
The calculations are done in lab3.ipynb and lab3, See the attached files. The clear difference is the large gap between the scores. Lab3 peaks at 4, while lab2 peaks at around 2.8. This is because of how the simulation is done versus the modelling in this lab. In lab2 the employees and customers move around with multiple timeouts to simulate movement, grabbing items and refilling items. Therefore, there will be more occurrences where the shelves are empty when a customer arrives. In addition, we removed the queuing element from the MOS-score in lab3. However, this did not impact the MOS-score with the specific simulation parameters.

In lab3 we have calculated the MOS-score from only non-random variables. This means that the results will be the same each time we calculate the MOS-score, and no std are presented. Something else worth noting about the lab3 results are that a score of 5 simply is impossible. The score is based on the probability of being in state 0, but this probability won't be 0 and thereby leading to a MOS-score<5. This means that even though the performance increases with the number of employees, the MOS-score does not show it and should be modified when using it in lab3. The MOS-score will therefore always be 4 when E>7.

Lab 2 follows the same trend as lab3 as it increases rapidly when the employee count rises and flattens when the employee count becomes larger. As only one employee can be at a station at the time, increasing the employee count won't highly affect the MOS-score after reaching 9-10 employees. To conclude, the two labs share the same trend in MOS-score, but for different reasons.

III.D.1

When refilling, we should include some sort of probability that an employee is busy. The first thing we need to assume is whether two employees can work at one station at the same time. Assuming only one employee works at a station at the time, and there is no impact of moving from and to sections, we would have a similar model for E>6. However, we need to decide for E<7. We could for instance add a fourth node in our model, like in the example below. If there are no employees available when the shelf reaches its threshold, we will move into a waiting state, where we will be until an employee are available for refilling. Depending on the state of the stock, we will either move into node 0 or t.



This makes our system significantly more complex as we must come up with a probability of an employee is not available when a section reaches its threshold. In other words, a transition from n to E=0. This depends on N_i, E and the other sections. Parameter such as u_i for all sections will impact this transition as it affect the time an employee is busy. We also have to solve for the probability that the stock is larger or lower than 3 when moving out of the state E=0.