Recovering Risk-Neutral Inflation Moments from Option Prices (1Y)

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Abstract

We recover risk-neutral one-year inflation distributions and moments from option prices. We focus on identification and stability (not forecasting) and compare: (i) Breeden–Litzenberger density recovery, (ii) Kitsul–Wright local-polynomial implementation, (iii) model-free BKM moment extraction, and (iv) maximum-entropy density as a robustness benchmark.

1 Objective

- Recover risk-neutral inflation moments (mean, variance, skewness, kurtosis) from 1Y ZC inflation option prices
- Compare methods for identification and numerical stability across time
- Diagnose disagreements using strike coverage and boundary effects

2 Data

- Instruments
  - 1Y zero-coupon inflation caps and floors (cross-section by date and area)
  - 1Y inflation swap inputs: discount factor $B(0,1)$ and swap-implied mean inflation $y_{\pi,1y}$
- Data handling
  - Prices in decimals of notional (unit conversion applied in code)
  - Estimation performed independently for each (date, area)
  - Strike coverage diagnostics: $K_{\min}$, $K_{\max}$, width $K_{\max} - K_{\min}$

3 Preprocessing: call price curve

- Caps interpreted as calls: $C_{\text{cap}}(k)$

- Floors interpreted as puts $P_{\text{floor}}(k)$, converted to calls via put–call parity

$$C(k) = P(k) + B(0,1)\,(y_{\pi,1y} - k)$$

- Merge all call prices; average duplicates by strike
- No-arbitrage stabilization (projection)
  - Non-negativity: $C(k) \geq 0$
  - Monotonicity: $C(k)$ non-increasing in k
  - Convexity: $C''(k) \geq 0$ (required for a valid density)

4 Methods

4.1 Breeden–Litzenberger density (BL)

- Mapping: prices $\rightarrow$ density $\rightarrow$ moments
- Identity

$$f(k) = \frac{1}{B(0,1)} \frac{\partial^2 C(k)}{\partial k^2}$$

- Numerical implementation
  - Regular strike grid + interpolation
  - Finite differences (baseline), sensitive to noise
  - Post-processing: clip $f \geq 0$, normalize $\int f = 1$

4.2 Kitsul–Wright operational density (KW)

- Same mapping and identity as BL
- Key improvement: local polynomial smoothing before differentiation
  - Local quadratic regression with kernel weights
  - Second derivative recovered from local fit
- Post-processing: clip $f \geq 0$, normalize $\int f = 1$

4.3 Bakshi–Kapadia–Madan moments (BKM)

- Mapping: prices $\rightarrow$ moments (no density recovery)
- Uses integral replication identities to compute raw moments from call prices
- Requires wide strike coverage; truncation strongly affects higher moments

4.4 Maximum entropy density (MaxEnt)

- Mapping: prices $\rightarrow$ density $\rightarrow$ moments
- Discretize inflation outcomes $\{\pi_j\}$ and estimate probabilities $\{p_j\}$
- Convex program (regularized)

$$\min_p \; -\sum_j \mathrm{entr}(p_j) + \lambda \|\widehat{C} - C\|^2 \quad \text{s.t.} \quad p_j \geq 0, \; \sum_j p_j = 1$$

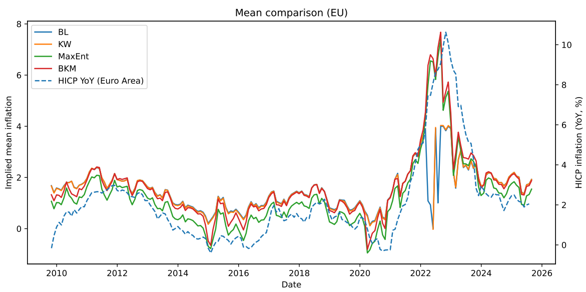
- Role: robustness benchmark under sparse strikes and noisy curvature

5 Diagnostics and KW shutdown rule

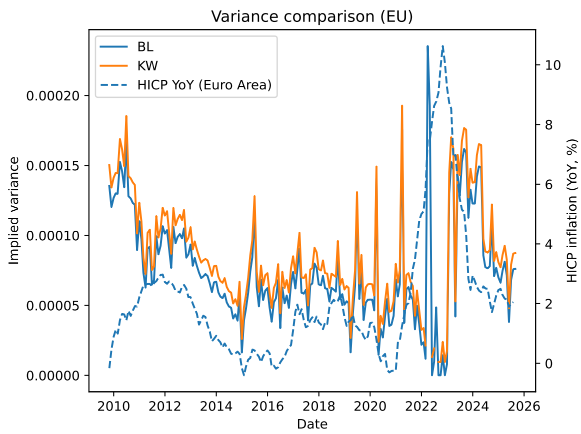
- Identification window for KW: $\pi \in [-1\%, 5\%]$
- Boundary-driven estimates detected using edge probability mass
  - Left edge mass: $\int_{-1\%}^{-1\%+\varepsilon} f_{\mathrm{KW}}(\pi)\, d\pi$
  - Right edge mass: $\int_{5\%-\varepsilon}^{5\%} f_{\mathrm{KW}}(\pi)\, d\pi$
- If either edge mass exceeds a threshold, KW outputs are suppressed (set to NaN)
  - Moments (mean/variance/skewness/kurtosis)
  - Tail probabilities $P(\pi < 0)$ and $P(\pi > 4\%)$
- Motivation: reduce spurious signals caused by boundary effects and weak tail identification

6 Results: moments over time

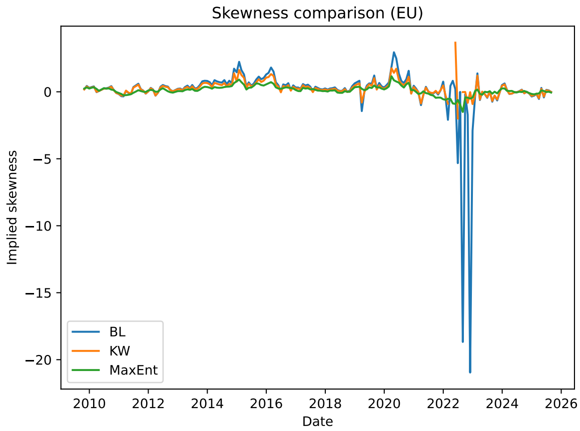
- Main objective: stability and agreement across methods
- Interpretation priority: mean and variance (best identified); higher moments (weak identification)



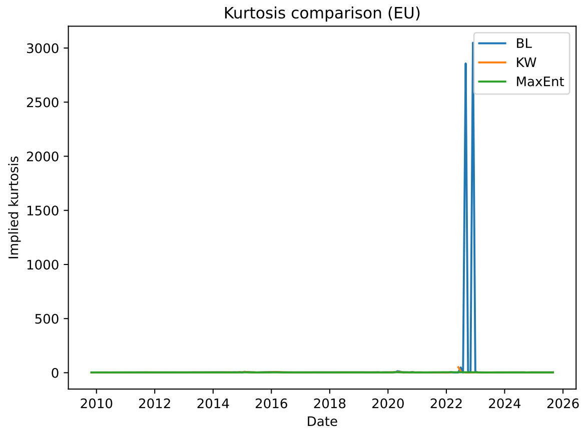
(a) Implied mean (with realized HICP overlay)



(b) Implied variance comparison



(a) Implied skewness comparison



(b) Implied kurtosis comparison

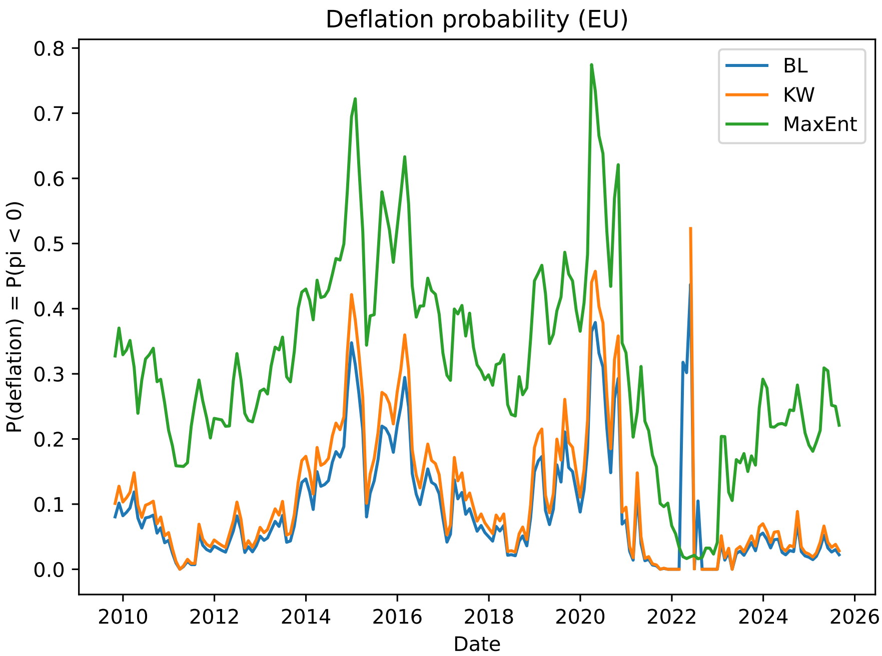


Figure 3: Deflation probability $P(\pi < 0)$ (KW suppressed when boundary-driven)

7 Density-level comparison (EU)

- Compare recovered densities (BL vs KW vs MaxEnt) on representative dates
- Visual diagnostic for tail behavior and boundary sensitivity

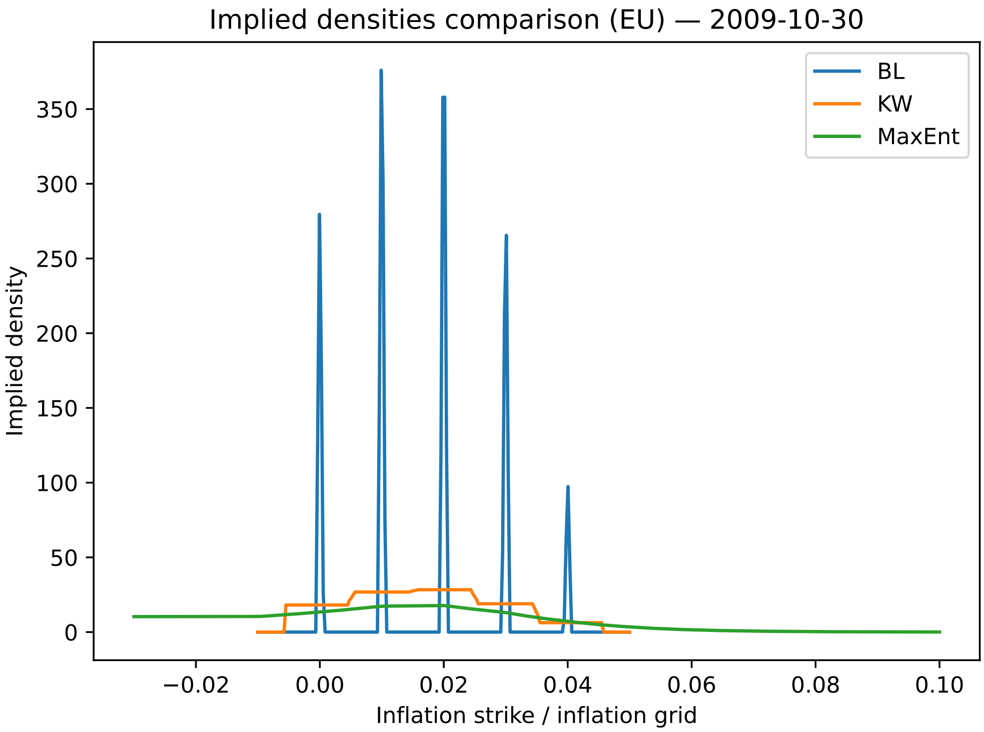


Figure 4: Implied density comparison (EU): BL vs KW vs MaxEnt

8 Main conclusions

- Mean: strongly identified and stable across methods
- Variance: moderately identified; sensitive to coverage but reasonably stable under KW/MaxEnt
- Skewness and kurtosis: weakly identified; highly sensitive to tails and numerical regularization
- KW vs BL: KW improves numerical stability by smoothing before curvature extraction
- MaxEnt: conservative robustness benchmark under sparse strikes
- BKM: diagnostic only; truncation dominates higher moments in limited strike ranges

9 LLM disclosure

- Tools: ChatGPT; Gemini
- Usage: coding support (debugging), LaTeX formatting assistance, methodological clarification

10 References

References

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