

The programmer's primary responsibility is correctness of code. Efficiency of code must always be a secondary concern. Good programmers have a repertoire of algorithms that can be applied to many problems and have an understanding of efficiency.

## KEY TERMS

asymptotic dominance (p. 544)	linear probing (p. 567)
average case analysis (p. 561)	linear search (p. 556)
benchmark (p. 577)	linear time (p. 548)
best case analysis (p. 560)	load factor (p. 571)
big-O notation (p. 545)	logarithmic time (p. 548)
binary search (p. 557)	midsquare technique (p. 566)
bubble sort (p. 575)	$N \log N$ sort (p. 587)
chained hash table (p. 570)	$N^2$ sorts (p. 577)
clustering (p. 568)	order of a function (p. 545)
code tuning (p. 578)	perfect hash function (p. 565)
collision (p. 565)	performance analysis (p. 543)
collision resolution strategy (p. 567)	pivot (p. 581)
complexity theory (p. 542)	polynomial time (p. 548)
constant time (p. 548)	profiler (p. 579)
cost function (p. 543)	quadratic time (p. 548)
cubic time (p. 548)	quicksort (p. 579)
division technique (p. 566)	radix sort (p. 591)
dominance (p. 544)	search key (p. 556)
exchange sorts (p. 575)	space complexity (p. 543)
exponential time (p. 548)	space efficiency (p. 542)
hash function (p. 564)	straight selection sort (p. 573)
hash table (p. 565)	synonyms (p. 565)
hashing (p. 564)	time complexity (p. 543)
in-place sorts (p. 593)	time efficiency (p. 542)
intellectual efficiency (p. 590)	worst case analysis (p. 560)

## EXERCISES

**12.1** For each pair of functions  $T1$  and  $T2$  below, tell whether  $T1$  dominates  $T2$ ,  $T2$  dominates  $T1$ , or neither dominates the other. (The letter  $T$  is often used to suggest execution Time of an algorithm.)

- $T1(X) = 2 * X^2$   
 $T2(X) = 2 * X$
- $T1(X) = 5 * X^2$   
 $T2(X) = 2 * X^3$
- $T1(Y) = 55 * Y^{15} + 3 * Y^4 + 7500$   
 $T2(Y) = Y^{16} + 2$
- $T1(Z) = (Z + 3) * (Z + 5)$   
 $T2(Z) = 250 * Z$
- $T1(Z) = (Z + 7) * (Z + 9)$   
 $T2(Z) = 2 * Z^3$
- $T1(N) = 37 * N^2$   
 $T2(N) = N \log N$

- g.  $T1(N) = 37 \cdot N^2$   
 $T2(N) = N^2 \log N$
- h.  $T1(N) = \log_2 N$   
 $T2(N) = \log_3 N$
- i.  $T1(N) = 84$   
 $T2(N) = \log_9 N$
- j.  $T1(X) = \frac{3 \cdot X + 2}{X}$   
 $T2(X) = 766 \cdot X$
- k.  $T1(X) = \frac{X^4 + X^2 - 17}{X^3}$

$$T2(X) = X^2$$

- l.  $T1(W) = W^9$   
 $T2(W) = 9^W$
- m.  $T1(W) = W!$   
 $T2(W) = 67^W$
- n.  $T1(V) = V^V$   
 $T2(V) = 25^V$
- o.  $T1(X, Y) = 2 \cdot X^3 \cdot Y$   
 $T2(X, Y) = 3 \cdot X \cdot Y$
- p.  $T1(X, Y) = X^2 + 75$   
 $T2(X, Y) = 7 \cdot Y$
- q.  $T1(V, W) = W^3 + W + 74$   
 $T2(V, W) = V + 18$

**12.2** Using big-O notation, give the order of each function in Exercise 12.1.

**12.3** Rank the following big-O measures from greatest to least. (Recall that  $O(f) > O(g)$  if and only if  $f$  dominates  $g$ .)

- $O(N)$
- $O(N^3)$
- $O(4^N)$
- $O(\log_4 N)$
- $O(\log_5 N)$
- $O(N^2)$
- $O(1)$
- $O(N \log_3 N)$
- $O(N^2 \log_3 N)$

**12.4** Classify each big-O measure of Exercise 12.3 as one (or more) of the following: constant, linear, quadratic, cubic, logarithmic, polynomial, or exponential.

**12.5** Determine the order of each function below, and classify each result as constant, linear, quadratic, cubic, logarithmic, or exponential.

- a.  $T(N) = 3 \cdot N^2 + N$
- b.  $T(N) = 55 \cdot N^3 + 77 \cdot N^2 + 99$
- c.  $T(N) = 2^N \cdot N^2$
- d.  $T(N) = 7501$
- e.  $T(N) = \log N + 46 \cdot N$
- f.  $T(N) = 3^{(N+1)}$
- g.  $T(N) = \frac{N \log N}{2 + N}$
- h.  $T(N) = \frac{3 \cdot N^4 + 4 \cdot N^3}{5 \cdot N^2 + N}$
- i.  $T(N) = \frac{17^N}{N^2}$

**12.6** Using big-O notation, estimate the running time of each of the following algorithms. You may assume that all variables are of type `int`.

- a. 

```
for (i = 1; i <= X; i++)
    for (j = 1; j <= X; j++)
        for (k = 1; k <= X; k++) {
            // Five assignment instructions
        }
```
- b. 

```
for (i = 10; i <= X; i++) {
    // Two assignment instructions
    for (j = 15; j <= X; j++) {
        for (k = 1; k <= X; k++) {
            // Five assignment instructions
        }
        // Seven assignment instructions
    }
}
```
- c. 

```
for (i = 1; i <= X; i++) {
    for (j = 1; j <= X; j++) {
        // Twenty assignment instructions
    }
    for (j = 1; j <= X; j++)
        if (j % 2 == 1)
            for (k = 1; k <= X; k++) {
                // Five assignment instructions
            }
}
```

```

d. for (i = 1; i <= X; i++) {
    for (j = i; j <= X; j++) {
        // Six assignment instructions
    }
    if (i % 2 == 1) {
        // Four assignment instructions
    }
}

e. for (i = 1; i <= X; i++)
    for (j = 1; j <= Y; j++)
        for (k = 1; k <= X; k++) {
            // Two assignment instructions
        }

f. i = 1;
   while (i <= X) {
       // Three assignment instructions
       j = 17;
       while (j <= 100) {
           // Two assignment instructions
           j++;
       }
       i++;
   }

g. i = X;
   do {
       // Three assignment instructions
       j = 1;
       while (j <= X) {
           // Two assignment instructions
           j++;
       }
       i--;
   } while (i >= 1);

h. i = 1;
   while (i <= X) {
       // Three assignment instructions
       j = 1;
       while (j <= X) {
           // Two assignment instructions
           j = j * 2;
       }
       i++;
   }

```

```

i. i = X;
   while (i >= 1) {
       // Three assignment instructions
       j = 1;
       while (j <= X) {
           // Two assignment instructions
           j = j + 2;
       }
       i = i / 3;
   }

```

**12.7** Using big-O notation, estimate the running time of each of the following recursive functions.

```

a. void RecA( int X )
{
    // Some task requiring constant time
    if (X > 0)
        RecA(X-1);
}

b. void RecB( int X )
{
    int i;

    for (i = 1; i <= X; i++) {
        // Some task requiring constant time
    }
    if (X > 1)
        RecB(X-1);
}

c. void RecC( int X )
{
    int i;

    for (i = 1; i <= X; i++) {
        // Some task requiring constant time
    }
    if (X > 1)
        RecC(X/2);
}

d. void RecD( int X )
{
    int i;

    // Some task requiring constant time
    for (i = 1; i <= X; i++)
        if (X > 1)
            RecD(X-1);
}

```