$$\int \frac{dx}{\sqrt{a^2-x^2}} = \frac{1}{a} \cdot \arcsin \frac{x}{a}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a}} = \ln\left(x + \sqrt{x^2 \pm a}\right) + C$$

$$\left(\ln\left(x + \sqrt{x^2 \pm a}\right)\right)' = \frac{1}{x + \sqrt{x^2 \pm a}} \cdot \left(1 + \frac{1}{2\sqrt{x^2 \pm a}}\right)$$

$$\frac{1}{x^{-1}\sqrt{x^2 \pm \varphi}}\left(\frac{\sqrt{x^2 \pm a} \mp x}{\sqrt{x^2 \pm a}}\right) =$$

$$= \frac{1}{\sqrt{x^2 \pm a}}$$

$$\cancel{2}x) =$$

$$\int \frac{dx}{\sqrt{x^2-1}} = \ln\left(x + \sqrt{x^2-1}\right)$$

$$\int \frac{dx}{\sqrt{x^2+11}} = \ln\left(x + \sqrt{x^2+11}\right)$$

$$\int \frac{dx}{\sqrt{\underbrace{x^2+4x+10}}} = \int \frac{dx}{\sqrt{(x+2)^2+6}} =$$

$$\int \frac{dx}{\sqrt{\underbrace{6x^2+10x+101}}}$$

$$\left(\sqrt{6}\,x + \frac{5}{\sqrt{6}}\right)^2 \pm$$

$$\frac{5}{\sqrt{6}} = \frac{5\sqrt{6}}{6}$$

$$6x^2+10x+101$$

$$\left(\sqrt{6}x+\frac{5}{\sqrt{6}}\right)^2+101-\frac{25}{6}$$

$$6x^2+2\cdot\sqrt{6}x\cdot\frac{5}{\sqrt{6}}+\frac{25}{6}+101-\frac{25}{6}$$

$$\frac{\ln\left(x+2+\sqrt{(x+2)^2+6}\right)}{1}$$

$$\int\frac{dx}{\sqrt{\left(\sqrt{6}x+\frac{5}{\sqrt{6}}\right)^2+101-\frac{25}{6}}}=\frac{\ln\left(\sqrt{6}x+\frac{5\sqrt{6}}{6}+\sqrt{\left(\sqrt{6}x+\frac{5\sqrt{6}}{6}\right)^2+101-\frac{25}{6}}\right)}{\sqrt{6}}$$

$$01-\frac{5\sqrt{6}}{6}$$

$$\int \frac{dx}{\sqrt{7-2x-x^2}} =$$

$$\int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}} = 2$$

$$\int \frac{dx}{\sqrt{8-(x+1)^2}} = \frac{1}{\sqrt{8}} \cdot \arcsin \frac{x+1}{\sqrt{8}}$$

2

$$= \frac{dx}{\sqrt{x-x^2}}$$

$$x-x^2 = -\left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right)$$

$$-\left[\left(x-\frac{1}{2}\right)^2 - \frac{1}{4}\right] = \frac{1}{4} - \left(x-\frac{1}{2}\right)^2$$

$$\arcsin \frac{x-\frac{1}{2}}{1/2}$$

$$\arcsin(2x-1)$$

$$\int \frac{dx}{\sqrt{9x^2+10x+5}} =$$

$$\sqrt{9x^2+10x+5} = \sqrt{\left(3x+\frac{5}{3}\right)^2+5-}$$

$$\int \frac{dx}{\sqrt{\left(3x+\frac{5}{3}\right)^2+5-\frac{25}{9}}} = \frac{\ln\left(3x+\frac{5}{3}+\sqrt{\left(3x+\frac{5}{3}\right)^2+}\right.}{3}$$

$$\int \frac{P_n(x)}{\sqrt{ax^2+bx+c}}\,dx = \underline{P_{n-1}(x)} \cdot \sqrt{ax^2+bx+c} + K \int \frac{dx}{\sqrt{ax^2+bx+c}}$$

$$\frac{25}{9}$$

$$\int \frac{x^2}{\sqrt{x^2+4}}\,dx = (Ax+B) \cdot \sqrt{x^2+4} + K \int \frac{dx}{\sqrt{x^2+4}}$$

$$\frac{20}{9}$$

$$\frac{x^2}{\sqrt{x^2+4}} = A \cdot \sqrt{x^2+4} + (Ax+B) \cdot \frac{1}{\cancel{2}\sqrt{x^2+4}} \cdot \cancel{2}x + K \cdot \frac{1}{\sqrt{x^2+4}}$$

$$x^2 = A(x^2+4) + (Ax+B)x + K$$

$$\begin{array}{l|l} x^2 & 1 = 2A \\ x & 0 = B \\ & 0 = 4A + K \end{array}$$

$x + C$

$$\frac{dx}{\sqrt{x^2+4}} = \frac{1}{2} x \cdot \sqrt{x^2+4} - 2 \ln\left(x + \sqrt{x^2+4}\right)$$

$$\frac{1}{\sqrt{x^2+4}} \Big/ \sqrt{}'$$

$A = 1/2$

$B = 0$

$K = -2$

$$\int \frac{x^2}{\sqrt{5-x^2}}\,dx = ($$

$$\frac{x^2}{\sqrt{5-x^2}} = A\sqrt{5-$$

$$x^2 = A(5-x^2$$

$1 = -2A \qquad 0 = -$

$A = -\frac{1}{2} \qquad B = 0$

$$(Ax+B)\sqrt{5-x^2}+k\int\frac{dx}{\sqrt{5-x^2}}=-\frac{1}{2}x\sqrt{5-x^2}+\frac{5}{2}\cdot\frac{1}{\sqrt5}\cdot\arcsin\frac{x}{\sqrt5}$$

$$x^2 - \frac{(Ax+B)\cdot x}{\sqrt{5-x^2}}+k\left(\frac{1}{\sqrt{5-x^2}}\right) \Big/ \cdot\sqrt{5-x^2}$$

$$^2) - (Ax+B)x+k$$

$$B \qquad 0=5A+k$$

$$0 \qquad 0=-\frac{5}{2}+k$$

$$k=\frac{5}{2}$$

$$\int \frac{\sqrt{x^2+x+1}}{1}\,dx = \int \frac{x^2+x+1}{\sqrt{x^2+x+1}}\,dx = (Ax+B)\sqrt{x^2+x+1} + k\int \frac{dx}{\sqrt{x^2+x+1}} =$$

$$\frac{x^2+x+1}{\sqrt{x^2+x+1}} = A\sqrt{x^2+x+1} + (Ax+B)\frac{1}{2\sqrt{x^2+x+1}}(2x+1) + \frac{K}{\sqrt{x^2+x+1}}$$

$$x^2+x+1 = A(x^2+x+1) + (Ax+B)\frac{2x+1}{2} + K$$

$$2x^2+2x+2 = 2A(x^2+x+1) + (Ax+B)(2x+1) + 2K$$

$$\begin{array}{l|l} x^2 & 2=4A \\ x & 2=2A+2B+A \\ & 2=2A+B+2K \end{array}$$

$$2 = 1+\frac{1}{4}+2K$$

$$\begin{cases} A=\frac{1}{2} \\ B=\frac{1}{4} \\ K=\frac{3}{8} \end{cases}$$

$$(x+\sqrt{6})^2+c=$$

$$2\cdot 1\cdot\sqrt{6}=1$$

$$6=\frac{1}{4}$$

$$c+6=1$$

$$c=\frac{3}{4}$$

$$\left(\frac{1}{2}x+\frac{1}{4}\right)\sqrt{x^2+x+1}+\frac{3}{8}\ln\left(x+\frac{1}{2}+\sqrt{x^2+x+1}\right)+C$$

$$\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$$

$$\int x^2\cdot\sqrt{9-x^2}\,dx=\int\frac{x^2(9-x^2)}{\sqrt{9-x^2}}\,dx$$

$$\int\frac{9x^2-x^4}{\sqrt{9-x^2}}\,dx=\left(Ax^3+Bx^2+Cx+D\right)\cdot\sqrt{9-x^2}+K\int\frac{dx}{\sqrt{9-x^2}}$$

$$\frac{9x^2-x^4}{\sqrt{9-x^2}}=(3Ax^2+2Bx+C)\sqrt{9-x^2}+(Ax^3+Bx^2+Cx+D)\cdot\frac{(-\cancel{2}x)}{\cancel{2}\sqrt{9-x^2}}+K\cdot\frac{-}{\sqrt{9}}$$

$$9x^2-x^4=(3Ax^2+2Bx+C)(9-x^2)+(Ax^3+Bx^2+Cx+D)\cdot(-x)+K$$

$$-1=-3A-A \qquad 0=-2B-B \qquad 9=-C+27A-C$$

$$A=\frac{1}{4} \qquad B=0 \qquad 9-\frac{27}{4}=-2C$$

$$\frac{9}{4}=-2C$$

$$C=-\frac{9}{8}$$

$$\frac{x}{x^2}$$

$$\frac{1}{\sqrt{9-x^2}}$$

$$\int \frac{9x^2-x^4}{\sqrt{9-x^2}}\,dx = \left(\frac{1}{4}x^3-\frac{9}{8}x\right)\sqrt{9-x^2}+\frac{81}{8}\cdot\frac{1}{3}\arcsin$$

$$0 = 18B - D$$

$$D = 0$$

$$0 = 9C + K$$

$$\frac{81}{8} = K$$

$$\int \frac{P_n(x)}{(x-d)^m\cdot\sqrt{ax^2+bx+c}}\,dx$$

$$x-d=\frac{1}{t}$$

$$\int \frac{dx}{x\sqrt{x^2+3}}$$

$$\int \frac{dx}{x^4 \cdot \sqrt{x^2-2}} = -$$

$$x = \frac{1}{t}$$

$$dx = -\frac{1}{t^2}\,dt$$

$$\frac{x}{3}$$

$$x = \frac{1}{t}$$

$$dx = -\frac{1}{t^2}\,dt$$

$$= -\int \frac{\frac{1}{t^2}\,dt}{\frac{1}{t}\cdot\sqrt{\frac{1}{t^2}+3}}$$

$$= -\int \frac{dt}{\cancel{t}\cdot\sqrt{\frac{1+3t^2}{\cancel{t^2}}}}$$

$$= -\int \frac{dt}{\sqrt{3t^2+1}} \qquad t = 1/x$$

$$= -\frac{1}{\sqrt{3}}\ln\left(\sqrt{3}t + \sqrt{3t^2+1}\right)$$

$$-\int \frac{\frac{1}{t^2}dt}{\frac{1}{t^4}\sqrt{\frac{1}{t^2}-2}} = -\int \frac{t^2 dt}{\sqrt{\frac{1-2t^2}{t^2}}} = -\int \frac{t^3 dt}{\sqrt{1-2t^2}} = -\left(-\frac{1}{6}t^2-\frac{1}{6}\right)\sqrt{1-2t^2} = \frac{1}{6}(t^2+1)\sqrt{1-2t^2}$$

$$= \frac{1}{6}\cdot\frac{1+x}{x}$$

$$\int \frac{t^3 dt}{\sqrt{1-2t^2}} = \left(At^2+Bt+C\right)\sqrt{1-2t^2} + k\int \frac{dt}{\sqrt{1-2t^2}}$$

$$\frac{t^3}{\sqrt{1-2t^2}} = (2At+B)\sqrt{1-2t^2} + (At^2+Bt+C)\cdot\frac{1}{2\sqrt{1-2t^2}}\cdot(-4t) + \frac{K}{\sqrt{1-2t^2}}$$

$$t^3 = (2At+B)(1-2t^2) - 2(At^2+Bt+C)t + K$$

$$\begin{array}{l|l}
t^3 & 1 = -4A - 2A \\
t^2 & 0 = -2B - 2B \\
t & 0 = 2A - 2C \\
 & 0 = B + K
\end{array}
\qquad
\begin{array}{l}
A = -\frac{1}{6} \\
B = 0 \\
C = -\frac{1}{6} \\
K = 0
\end{array}$$

$$\ldots^2 = \frac{1}{6}\left(\frac{1}{x^2}+1\right)\sqrt{1-\frac{2}{x^2}} =$$

$$\ldots^2 \sqrt{\frac{x^2-2}{x^2}} = \frac{1}{6}\cdot\frac{1+x^2}{x^3}\sqrt{x^2-2}$$

$$-\frac{1}{2}\int\frac{-2x}{\sqrt{\underset{\sim\sim}{1-x^2}}}\,dx = -\frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

$$1-x^2=t$$

$$-2x\,dx=dt$$

$$=-\frac{1}{2}2t^{1/2}=-\sqrt{t}=-\sqrt{1-x^2}$$

$$\boxed{\int\frac{f'(x)}{\sqrt{f(x)}}\,dx = \ldots}$$

$$\left(2\sqrt{f(x)}\right)' = 2\cdot\frac{1}{2\sqrt{f\ldots}}$$

$$\boxed{2\sqrt{f(x)}}$$

$$f'(x)$$

$$(x)$$

$$\int \frac{x+3}{\sqrt{4x^2+4x-3}}\,dx$$

$$\int \sqrt{4x^2-4x+3}\,dx$$

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$

$$\int \sqrt{x^2+3}\,dx$$