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② $Pr(A) = \binom{7}{6}\left(\frac{1}{2}\right)^7 + \binom{7}{7}\left(\frac{1}{2}\right)^7$

$Pr(B) = \binom{7}{5}\left(\frac{1}{2}\right)^7 + \binom{7}{6}\left(\frac{1}{2}\right)^7 + \binom{7}{7}\left(\frac{1}{2}\right)^7$

$Pr(C) = 1 - \left(\binom{7}{0}\left(\frac{1}{2}\right)^7 + \binom{7}{1}\left(\frac{1}{2}\right)^7\right)$

$Pr(D) = 1 - \left(\binom{7}{0}\left(\frac{1}{2}\right)^7 + \binom{7}{1}\left(\frac{1}{2}\right)^7 + \binom{7}{2}\left(\frac{1}{2}\right)^7 + \binom{7}{3}\left(\frac{1}{2}\right)^7\right)$

$$Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} = \frac{Pr(A)}{Pr(B)} = \frac{\binom{7}{6}\left(\frac{1}{2}\right)^7 + \binom{7}{7}\left(\frac{1}{2}\right)^7}{\binom{7}{5}\left(\frac{1}{2}\right)^7 + \binom{7}{6}\left(\frac{1}{2}\right)^7 + \binom{7}{7}\left(\frac{1}{2}\right)^7} = \frac{0.0625}{0.2265625}$$

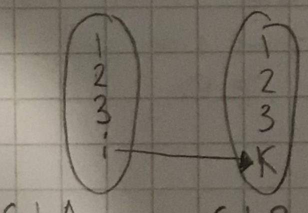
≈ 0.2759

$\approx 27.59\%$

$$Pr(C|D) = \frac{Pr(C \cap D)}{Pr(D)} = \longrightarrow \frac{\left(\binom{7}{4}\left(\frac{1}{2}\right)^7 + \binom{7}{5}\left(\frac{1}{2}\right)^7\right)}{1 - \left(\binom{7}{0}\left(\frac{1}{2}\right)^7 + \binom{7}{1}\left(\frac{1}{2}\right)^7 + \binom{7}{2}\left(\frac{1}{2}\right)^7 + \binom{7}{3}\left(\frac{1}{2}\right)^7\right)}$$

$$= \frac{0.4375}{0.5} = 0.875 = 87.50\%$$

③ $n \geq 2,\ m \geq 1$ $\quad f: A \to B \quad A_{ik} = \text{"}f(i) = k\text{"}$



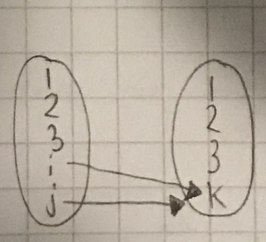
Set A = n (or n-1) Set B = m

Total # of functions defined as $|S|$.

$|S| = m^n$ possible functions

$|A_ik| = m^{n-1}$, because i is a fixed integer mapping to another fixed integer (k). This removes an element from Set A (n)

$$\therefore Pr(A_ik) = \frac{|A_ik|}{|S|} = \frac{m^{n-1}}{m^n} = \frac{1}{m}$$



Set A = n (or n-2) Set B = m

We know A_{jk} is a fixed integer such as A_ik, so it is also defined as $\frac{1}{m}$ mapping to k.

$$\therefore Pr(A_ik \cap A_jk) = \frac{|A_ik \cap A_jk|}{|S|} = \frac{m^{n-2}}{m^n} = \frac{1}{m^2}$$

Both events $Pr(A_ik)$ and $Pr(A_jk)$ have the same Pr, but the events occur independently.

4. $Pr(A) = 1 - \left(\frac{5}{6}\right)^6$
≈ 0.6651
$\approx 66.51\%$

Using the complement rule we eliminate all the non-relevant events (rolling 1-5) to recieve the Pr(A).

$Pr(B) = 1 - \left(\frac{5}{6}\right)^{12} - \binom{12}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11}$
≈ 0.6635
$\approx 66.35\%$

Once again using the complement rule to eliminate the events that do not concern us: rolling zero 6's in 12 rolls and rolling 1 6 in 12 rolls.

$Pr(C) = 1 - \left(\frac{5}{6}\right)^{18} - \binom{18}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{17} - \binom{18}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{16}$
≈ 0.5973
$\approx 59.73\%$

Using the component rule we remove the event of zero, one and two 6's in 18 dice rolls. This is the smallest probability!

5. A_s = "the sum of the results equals s" $x > 0$ $y > 0$

$$\frac{x}{y} + \frac{y}{x} \geq 2 \rightarrow \frac{x^2}{xy} + \frac{y^2}{xy} \geq 2 \rightarrow \frac{x^2 + y^2}{xy} \geq 2 \rightarrow x^2 + y^2 \geq 2xy$$

$$x^2 - 2xy + y^2 \geq 0 \rightarrow (x-y)^2 \geq 0$$

$\therefore$ because $x > 0$ and $y > 0$ and we have $(x-y)^2$ we know left-hand side will always be ≥ 0.

5-Part 2

Let p_i denote a red dice roll.
Let q_i denote a blue dice roll.

$Pr(A_2)$: $p_1 \cdot q_1 = a$ OR $p_1 = \frac{a}{q_1}$ OR $q_1 = \frac{a}{p_1}$

You roll one on each colored die.

$Pr(A_{12})$: $p_6 \cdot q_6 = a$ OR $p_6 = \frac{a}{q_6}$ OR $q_6 = \frac{a}{q_6}$

You roll six on each colore die.

$$Pr(A_7): (p_1 \cdot q_6) + (p_2 q_5) + (p_3 q_4) + (p_4 q_3) + (p_5 q_2) + (p_6 q_1) \underbrace{\geq 2a}_{\text{by definition}}$$

To help prove $Pr(A_7) \geq 2a$ and to get us to more familar we can remove unneccessary dice rolls to add up to 7.

$Pr(A_7): (p_1 q_6) + (p_6 q_1) \geq 2a$

We have seen these terms when defining $Pr(A_2)$ and $Pr(A_{12})$.

Remember: $q_6 = \frac{a}{p_6}$ AND $q_1 = \frac{a}{p_1}$

$\therefore p_1\left(\frac{a}{p_6}\right) + p_6\left(\frac{a}{p_1}\right) \geq 2a$

$\frac{p_1 a}{p_6} + \frac{p_6 a}{p_1} \geq 2a$

$a\left(\frac{p_1}{p_6} + \frac{p_6}{p_1}\right) \geq 2a$

$\frac{p_1}{p_6} + \frac{p_6}{p_1} \geq 2$

$\therefore$ because we have seen this similar formula before $\left(\frac{x}{y} + \frac{y}{x} \geq 2\right)$ and we have proved the inequality beforehand $(x-y)^2 \geq 0$, we can say the same here.

5 Part C

To prove the choice of $p_1, p_2, \dots, p_6, q_1, q_2, \dots, q_6$ such that for any $s \in \{2, 3, \dots, 12\}$, $Pr(A_s) = 1/11$, we can use a proof by contradiction.

Assume false for $Pr(A_s) = 1/11$.

We know that $a = 1/11$, so if $Pr(A_7) \geq 2\left(\frac{1}{11}\right)$ or $\frac{2}{11}$, we see that the inequality is true, because $Pr(A_7) \geq \frac{2}{11}$ is also true.

6 $Pr(J)$ = The probability that John is hired.

$$\underbrace{\left(\binom{3}{3}\left(\frac{1}{n}\right)^3 \times \left(1-\frac{1}{n}\right)^0\right)}_{\text{Refers to the probability of all 3 committee members ranking John first,}} + \underbrace{\left(\binom{3}{2}\left(\frac{1}{n}\right)^2 \times \left(1-\frac{1}{n}\right)^1\right)}_{\text{Refers to the probability of 2 committe members ranking John first,}}$$

7 The $Pr(A) = \frac{3^2-1}{3^4} = \frac{8}{81}$

3^4 refers to the sample space of the boys and choices.

The $Pr(B) = \frac{3^2-1}{3^4} = \frac{8}{81}$

3^2 refers to, the fact two boys need to look at each other.

The $Pr(C) =$