

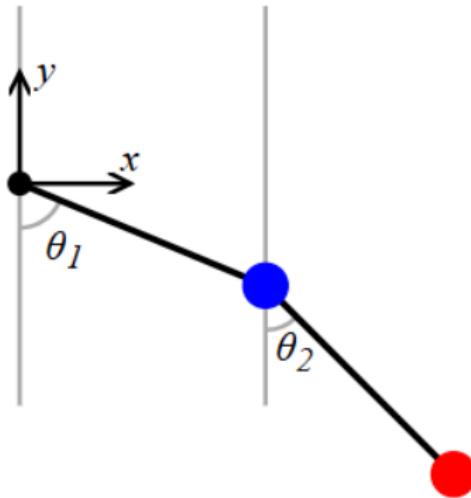
# Animacija dvojnega nihala

Nena Šefman Hodnik, Nina Švigelj

Fakulteta za matematiko in fiziko

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# Predstavitev problema



$$x_1 = l_1 \sin(\theta_1)$$

$$x_2 = l_1 \sin(\theta_1) + l_2 \sin(\theta_2)$$

$$y_1 = -l_1 \cos(\theta_1)$$

$$y_2 = -l_1 \cos(\theta_1) - l_2 \cos(\theta_2)$$

**Slika:** Dvojno nihalo

# Izpeljava diferencialnih enačb za dvojno nihalo

- Potencialna energija

$$V = m_1 g (-l_1 \cos \theta_1) + m_2 g (-l_1 \cos \theta_1 - l_2 \cos \theta_2)$$

- Kinetična energija

$$T = \frac{m_1}{2} (\dot{x}_1^2 + \dot{y}_1^2) + \frac{m_2}{2} (\dot{x}_2^2 + \dot{y}_2^2)$$

- Lagrangeeva enačba

$$\mathcal{L} = T - V$$

$$\begin{aligned} &= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)) \\ &\quad + (m_1 + m_2) l_1 g \cos \theta_1 + m_2 l_2 g \cos \theta_2. \end{aligned}$$

# Izpeljava diferencialnih enačb za dvojno nihalo

- Euler-Lagrangeeve enačbe

$$\frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial \dot{\theta}_i} \right) = \frac{\partial \mathcal{L}}{\partial \theta_i}, \quad i = 1, 2.$$

- Dobimo sistem diferencialnih enačb:

$$(m_1 + m_2)[l_1 \ddot{\theta}_1 + g \sin \theta_1] + m_2 l_2 [\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)] = 0$$
$$m_2 [l_2 \ddot{\theta}_2 + g \sin \theta_2] + m_2 l_1 [\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2)] = 0$$

## Izpeljava diferencialnih enačb za trojno nihalo

- Položaj tretje žogice:

$$x_3 = l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3,$$

$$y_3 = -l_1 \cos \theta_1 - l_2 \cos \theta_2 - l_3 \cos \theta_3,$$

- Po istem postopku kot za dvojno nihalo dobimo sistem

$$\begin{aligned}\theta_1 : \quad & (m_1 + m_2 + m_3)[l_1 \ddot{\theta}_1 + g \sin \theta_1] + (m_2 + m_3)l_2[\ddot{\theta}_2 \cos(\theta_1 - \theta_2) + \dot{\theta}_2^2 \sin(\theta_1 - \theta_2)] \\ & + m_3 l_3[\ddot{\theta}_3 \cos(\theta_1 - \theta_3) + \dot{\theta}_3^2 \sin(\theta_1 - \theta_3)] = 0\end{aligned}$$

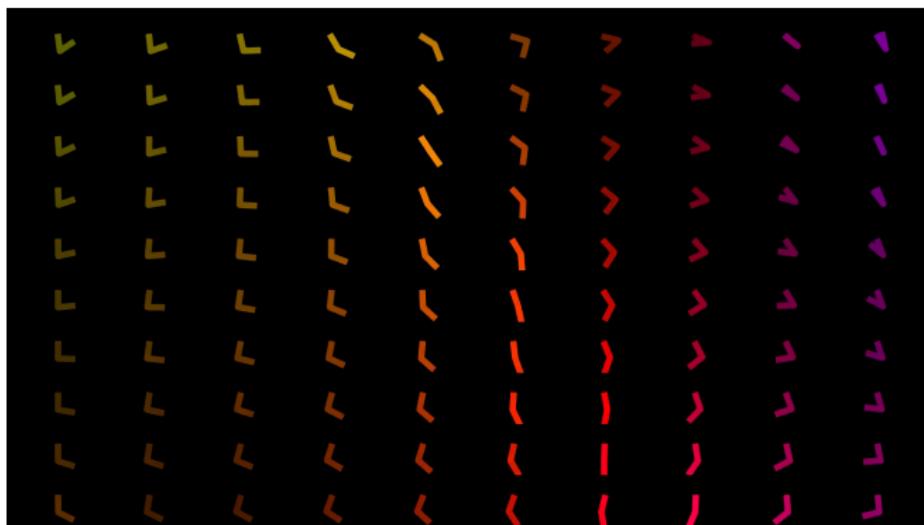
$$\begin{aligned}\theta_2 : \quad & (m_2 + m_3)[l_2 \ddot{\theta}_2 + g \sin \theta_2] + (m_2 + m_3)l_1[\ddot{\theta}_1 \cos(\theta_1 - \theta_2) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_2)] \\ & + m_3 l_3[\ddot{\theta}_3 \cos(\theta_2 - \theta_3) + \dot{\theta}_3^2 \sin(\theta_2 - \theta_3)] = 0\end{aligned}$$

$$\begin{aligned}\theta_3 : \quad & m_3[l_3 \ddot{\theta}_3 - g \sin \theta_3] + m_3 l_1[\ddot{\theta}_1 \cos(\theta_1 - \theta_3) - \dot{\theta}_1^2 \sin(\theta_1 - \theta_3)] \\ & + m_3 l_2[\ddot{\theta}_2 \cos(\theta_2 - \theta_3) - \dot{\theta}_2^2 \sin(\theta_2 - \theta_3)] = 0\end{aligned}$$

# Osnovno risanje dvojnega in trojnega nihala

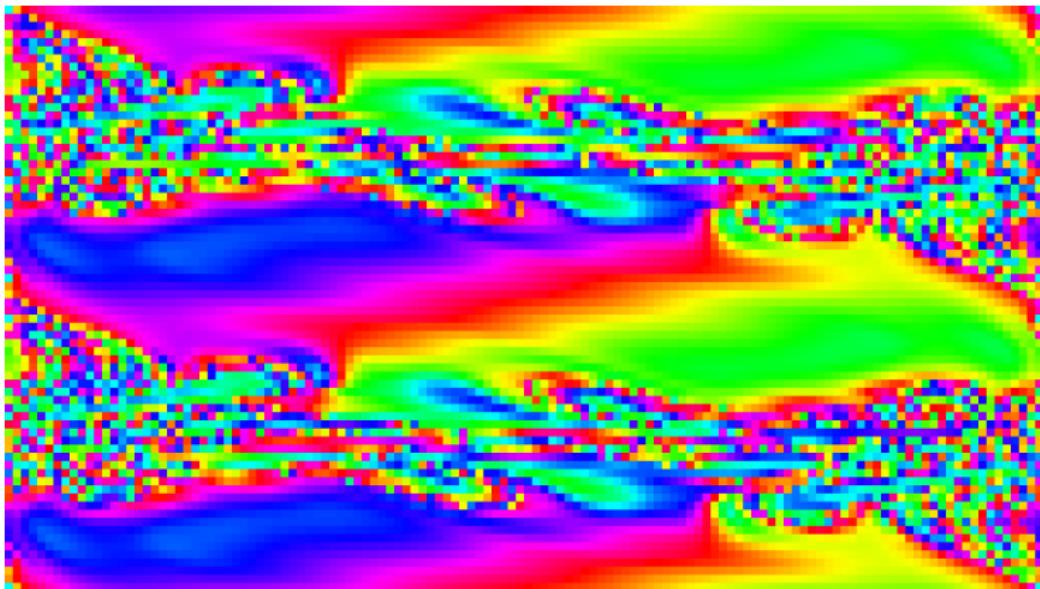


## Dvojna nihala v prostoru



Slika:  $10 \times 10$  Dvojno nihalo

## Dvojna nihala kot kvadratki



## Dvojna nihala kot kvadratki



## Dvojna nihala v koordinatnem sistemu $\theta_1, \theta_2$

