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SUMMARY

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**Vibration & Acoustics  
MECA-H411**

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# Appel à contribution

## Synthèse Open Source



Ce document est grandement inspiré de l'excellent cours donné par Patrick GUILLAUME et Steve VANLANDUIT à l'EPB (École Polytechnique de Bruxelles), faculté de l'ULB (Université Libre de Bruxelles). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants et votre aide est la bienvenue ! En effet, il y a toujours moyen de l'améliorer surtout que si le cours change, la synthèse doit être changée en conséquence. On peut retrouver le code source à l'adresse suivante

<https://github.com/nenglebert/Syntheses>

Pour contribuer à cette synthèse, il vous suffira de créer un compte sur *Github.com*. De légères modifications (petites coquilles, orthographe, ...) peuvent directement être faites sur le site ! Vous avez vu une petite faute ? Si oui, la corriger de cette façon ne prendra que quelques secondes, une bonne raison de le faire !

Pour de plus longues modifications, il est intéressant de disposer des fichiers : il vous faudra pour cela installer L<sup>A</sup>T<sub>E</sub>X, mais aussi *git*. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leur changement par mail ou n'importe quel autre moyen.

Le lien donné ci-dessus contient aussi un README contenant de plus amples informations, vous êtes invités à le lire si vous voulez faire avancer ce projet !

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Merci !

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# Part I

# Vibration

# Chapter 1

## Discrete systems

### 1.1 Introduction

Vibrations are found on everything around us, trains, cars and even human body is subject to vibration. Its effects are disturbing because it causes fatigue, loss of performance, no comfort, ... As vibration source we can find the earthquakes, the interaction with road, the wind, the waves, ... The basic terminology for the course is:

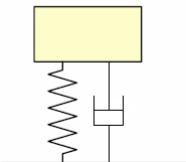
- **The source**  $F(\omega)$ ,  
this characterizes the dynamic forces
- **The path**  $H(\omega)$ ,  
this characterizes the structural dynamics
- **The response**  $X(\omega)$ ,  
such that  $X(\omega) = H(\omega)F(\omega)$ .

Vibrations cause failure, loss of comfort and is harmful for precision operations. We try to suppress it by damping, isolation and structure design.

We have two different approach for analysing a vibration problem. The one called **Signal analysis** or **Fourier analysis** deals with the case where we only have the response of the system to unknown forces. The one called **System analysis** or **Modal analysis** where we stimulate the system with known forces and measure the response, being able to find  $H(s)$  (dynamic forces - transfer function of the system).

### Basic notions

SDOF



Three main forces are acting on bodies:

- the one due to springs, proportional to the displacement:  $F = kd$
- the one due to dampers, proportional to the velocity:  $F = cv$
- the one due to the mass, proportional to acceleration:  $F = ma$ .

Notice that we have one resonance frequency for each degree of liberty of each mass.

Figure 1.1

We can already get some definition, let's consider the free vibration assumed to be always in resonance. We can then define the **period of resonance**  $T_n$ , the **resonance frequency**  $f_n = \frac{1}{T_n}$  and the **resonance pulsation**:

$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}. \quad (1.1)$$

Notice that if we increase mass, the frequency decreases.

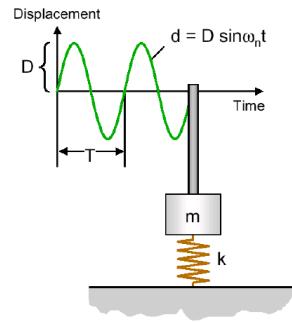


Figure 1.2

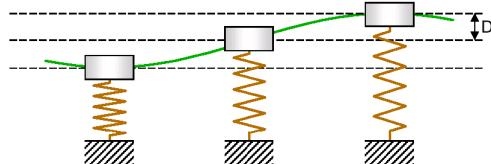


Figure 1.3

where  $V = 2\pi f_n D$ . Replacing by this:

$$m(2\pi f_n D)^2 = kD^2 \Rightarrow f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}. \quad (1.3)$$

We find a waited result. Now, let's show that **increasing damping reduces amplitudes over time**. Take the general newton equation for a linear free system and multiply by  $\dot{x}(t)$ :

$$m\ddot{x}\dot{x} + b\dot{x}\dot{x} + kx\dot{x} = 0 \Rightarrow \frac{d}{dt} \left( \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \right) = -b\dot{x}^2 \leq 0 \quad (1.4)$$

## 1.2 Single degree of freedom oscillator

Given the single degree system here, its free response is given by:

$$m\ddot{x} + b\dot{x} + kx = 0. \quad (1.5)$$

We assume that this differential equation admits a solution of type  $x = Ae^{st}$ . We can then write the characteristic equation and its eigenvalues as:

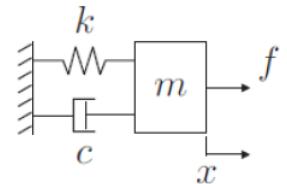
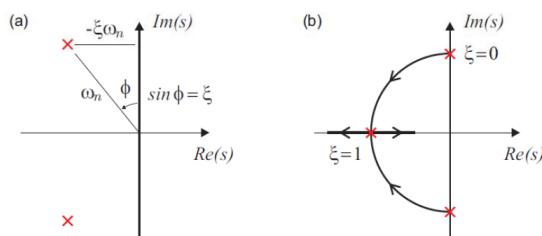


Figure 1.4

$$ms^2 + cx + k = 0, \quad s = -\frac{c}{2m} \pm j\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}. \quad (1.6)$$

By defining two new quantity, the **natural pulsation**  $\omega_n^2 = \frac{k}{m}$  and the **damping ratio**  $\xi$  such that  $\xi\omega_n = \frac{c}{2m}$ , we can rewrite:

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1 - \xi^2}. \quad (1.7)$$



We can introduce the **damping pulsation**  $\omega_d = \omega_n\sqrt{1 - \xi^2}$ . This explicitly makes appear the real and imaginary part of  $s$  that we plot on a diagram. Notice that the norm and the angle of the complex number are  $\omega_n$  and  $\arcsin \xi$ .

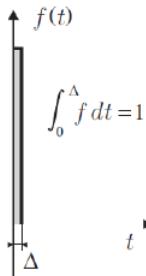
Figure 1.5

The right figure is the **Nyquist diagram**. Last, the final expression for  $x$  is:

$$x = e^{-\xi\omega_n t} (Ae^{j\omega_d t} + Be^{-j\omega_d t}) = e^{-\xi\omega_n t} (A_1 \cos(\omega_d t) + B_1 \sin(\omega_d t)) \quad (1.8)$$

where  $A, B, A_1, B_1$  depends on initial conditions.

### Impulse response



Let's now apply an impulse on the system and let's analyse when it is applied during the infinitesimal time  $\Delta$ , given the initial conditions  $x = 0, \dot{x} = 0$ . If we integrate the newton equation:

$$\int_0^\Delta m\ddot{x} dt = \int_0^\Delta f dt - \int_0^\Delta c\dot{x} dt - \int_0^\Delta kx dt = 1 \quad (1.9)$$

where the spring and damping forces cancel as they are finite (infinitesimal integral), the impulse integral  $=1$  by definition. Taking the limit we find new initial conditions:

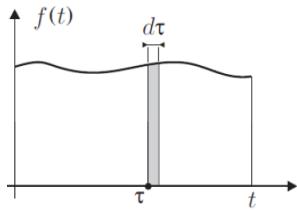
Figure 1.6

$$\lim_{\Delta \rightarrow 0} m\dot{x}(\Delta) = m\dot{x}(0^+) = 1 \quad \Rightarrow x(0^+) = 0, \dot{x}(0^+) = \frac{1}{m}. \quad (1.10)$$

The resolution of (1.8) gives the **impulse response**:

$$x(t) = h(t) = \frac{1}{m\omega_d} e^{-\xi\omega_n t} \sin(\omega_d t) \quad (1.11)$$

## 1.3 Convolution integral



Consider a transfer function  $h(t)$  of a system and the decomposition shown on the figure. The output of the system will be computed with the convolution integral:

$$x = \int_0^t h(t-\tau) f(\tau) d\tau. \quad (1.12)$$

where  $h(t)$  is the **impulse response**. In particular, for a **causal** system:

$$x(t) = \int_{-\infty}^{\infty} h(t-\tau) f(\tau) d\tau = \int_{-\infty}^{\infty} h(\tau) f(t-\tau) d\tau = h(t) * f(t). \quad (1.13)$$

### 1.3.1 Harmonic response

Consider an undamped system to which we apply an harmonic force:

$$m\ddot{x} + kx = Fe^{j\omega t}. \quad (1.14)$$

By considering the Fourier transform  $x(t) = X(\omega)e^{j\omega t}$ , we get:

$$-\omega^2 X(j\omega) + \frac{k}{m} X(j\omega) = \frac{F(j\omega)}{m} \quad \Rightarrow X = \frac{F}{k} \frac{1}{1 - (\omega/\omega)^2} = \frac{F}{k} D(\omega) \quad (1.15)$$

where  $D(\omega)$  is the **dynamic amplification**. For the damped case, we only have to know that  $\xi\omega_n = c/2m$  and we get in the same way as previously:

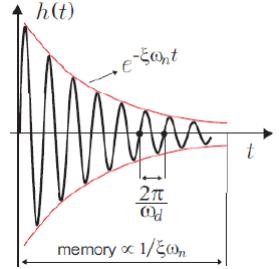


Figure 1.7

$$X = \frac{F}{k} \frac{1}{1 - (\omega/\omega_n)^2 + 2j\omega/\omega_n} = \frac{F}{k} D(\omega). \quad (1.16)$$

The two dynamic amplifications are plotted on the figures below, the second on a Bode diagram where we can clearly see the **Quality factor** defined as  $Q = 1/2\xi$ .

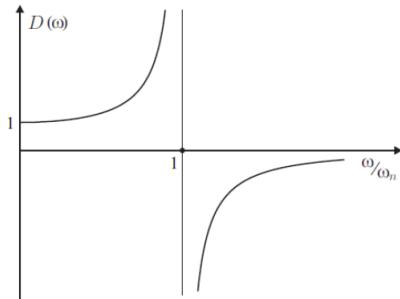


Figure 1.9

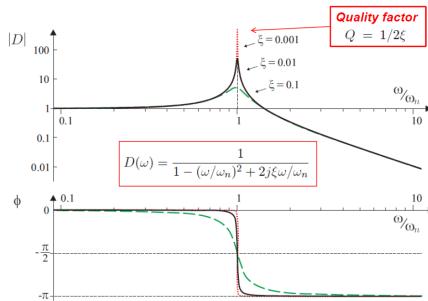


Figure 1.10

### 1.3.2 Frequency response function

Let's remember that the response is given by the convolution integral (1.13), where we will define now  $h(t)$ . Assuming the applied force to be harmonic  $F e^{i\omega t}$ , then proceeding to the Fourier transform of  $x$ , we get:

$$X e^{i\omega t} = \int_{-\infty}^{\infty} h(t) F e^{i\omega(t-\tau)} d\tau \Rightarrow \frac{X(\omega)}{F(\omega)} = \int_{-\infty}^{\infty} h(t) e^{-i\omega\tau} d\tau = H(\omega) \quad (1.17)$$

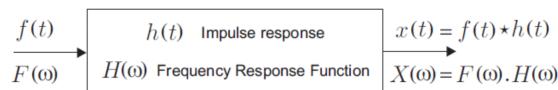


Figure 1.11

where we end up with the fact that **the frequency response function is the Fourier transform of the impulse response**. We can thus avoid the convolution to have a simple multiplication after a Fourier transform.

Here is also a useful theorem where  $|F(\omega)|/2\pi$  is the **energy spectrum** of  $f(t)$ :

#### Parseval theorem

$$\int_{-\infty}^{\infty} f^2(t) dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega. \quad (1.18)$$

### 1.3.3 Discrete Fourier Transform

Consider a signal  $x$  sampled in  $N$  samples, the equivalence of the continuous Fourier transform in discrete domain and the inverse transform are:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i \frac{2k\pi}{N} n} \quad \text{and} \quad X_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{i \frac{2k\pi}{N} n} \quad (1.19)$$

The equivalent Parseval theorem is:

$$\sum_{n=0}^{N-1} |x_n|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X_k|^2 \quad (1.20)$$

And as last definition, we have the root-mean-square value defined as

$$RMS = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} |x_n|^2} = \sqrt{\frac{1}{N^2} \sum_{k=0}^{N-1} |X_k|^2} \quad (1.21)$$

## 1.4 Transient response (Beat)

Consider an undamped oscillator excited by an harmonic force  $F \cos \omega t$  and of impulse response:

$$h(t) = \frac{1}{m\omega_n} \sin \omega_n t \quad (1.22)$$

Applying the convolution integral to this problem and integrating by part we find:

$$x(t) = \int_0^t F \cos(\omega t) h(t - \tau) d\tau = \frac{F}{m} \frac{\cos(\omega t) - \cos(\omega_n t)}{\omega_n^2 - \omega^2} \quad (1.23)$$

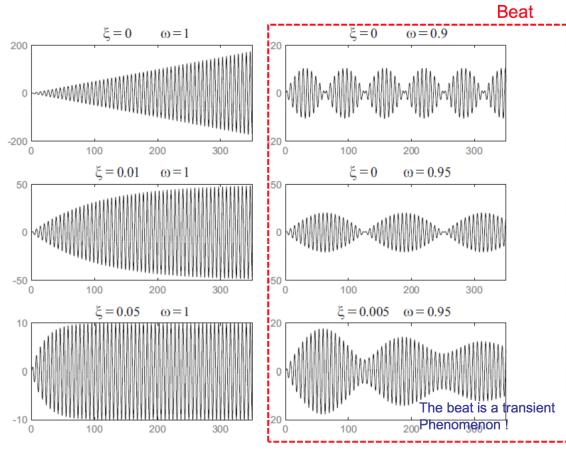


Figure 1.12

as beat is **in fact** a transient phenomenon.

By using Simpson's formula, defining  $\omega + \omega_n = 2\omega_0$  and  $\omega - \omega_n = 2\Delta$ , we get:

$$x(t) = \frac{F}{m} \frac{\sin(\omega_0 t) \sin(\Delta t)}{2\omega_0 \Delta} \quad (1.24)$$

Remark that for  $\Delta \rightarrow 0$ ,  $\frac{\sin \Delta t}{\Delta} = t$ . This term is called the **modulating function** as it grows the response amplitude while the other sinus is confined in  $[-1, 1]$ . We get thus for resonance:

$$x(t) = \frac{F}{m} \frac{\sin(\omega_n t)}{2\omega_n} t. \quad (1.25)$$

The figure shows that the phenomenon known

## 1.5 Multiple degree of freedom

Before going throw the real subject, let's introduce what's the **state space model** (yes CSD). It consists in writing any differential equation in this form of first order equation:

$$\dot{x} = Ax + Bu \quad (1.26)$$

where A, B are matrices and x, u vectors. Let's apply this for an oscillator:

$$\ddot{x} = \frac{f}{m} - 2\xi\omega_n - \omega_n^2 \quad (1.27)$$

We choose as state variable  $x_1 = x$  and  $x_2 = \dot{x}$ , then we only have to rewrite the definition of  $\dot{x}_1$  and  $\dot{x}_2$  as:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 2\xi\omega_n \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{1}{m} \end{Bmatrix} f \quad (1.28)$$

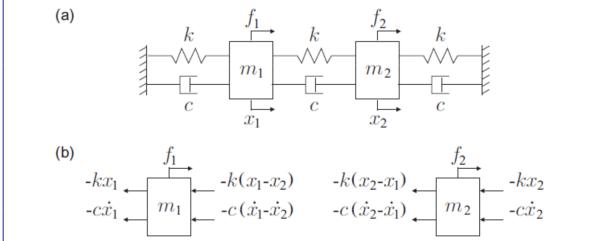


Fig. 2.1. (a) Two degree-of-freedom lumped mass system. (b) Free body diagrams.

$$m_1\ddot{x}_1 = f_1 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) - kx_1 - c\dot{x}_1$$

$$m_2\ddot{x}_2 = f_2 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) - kx_2 - c\dot{x}_2$$

In matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2c & -c \\ -c & 2c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

Figure 1.13

$$\frac{1}{2}\dot{x}^T M \dot{x} \quad \text{and} \quad \frac{1}{2}x^T K x \quad (1.30)$$

## 1.6 Eigenvalue problem

The method to solve this problem is to first consider the free response of the conservative system ( $C = 0$ ):  $M\ddot{x} + Kx = 0$ . A non trivial solution exists if:

$$(K + s^2 M)\phi = 0 \quad (1.31)$$

The eigenvalues  $s$  are solution of  $\det(K + s^2 M) = 0$ . Because  $K$  and  $M$  are symmetric and semi-positive definite, the eigenvalues are purely imaginary:  $s = \pm j\omega$ , this gives:

$$(K - \omega_i M)\phi_i = 0 \quad (1.32)$$

where  $\omega_i$  are the natural pulsations and  $\phi_i$  the mode shapes.

### Orthogonality of the mode shapes

Let's demonstrate that (subtracting with the i,j permuted equation):

$$\begin{aligned} \phi_j^T K \phi_i &= \omega_i \phi_j^T M \phi_i \\ (K - \omega_i M)\phi_i = 0 &\Rightarrow -\phi_i^T K \phi_j = \omega_j \phi_i^T M \phi_j \\ &\Rightarrow \phi_j^T M \phi_i = 0 \quad (\omega_i \neq \omega_j) \end{aligned} \quad (1.33)$$

The conclusion is that the mode shapes corresponding to distinct natural frequencies are orthogonal with respect to  $M$  and  $K$ . We then have the:

### Orthogonality relationships

$$\phi_i^T M \phi_j = \mu_i \delta_{ij} \quad \text{and} \quad \phi_i^T K \phi_j = \mu_i \omega_i^2 \delta_{ij}, \quad \omega_i^2 = \frac{\phi_i^T K \phi_i}{\phi_i^T M \phi_i} \quad (1.34)$$

where  $\mu_i$  is the modal mass and  $\omega_i$  the Rayleigh coefficient

And in matrix form with  $\Phi = (\phi_1, \phi_2, \dots, \phi_n)$ :

$$\Phi^T M \Phi = \text{diag}(\mu_i) \quad \Phi^T K \Phi = \text{diag}(\mu_i \omega_i^2) \quad (1.35)$$

To complete, two remarks:

- If several modes have the same natural frequency, they form a subspace and any vector in this subspace is also solution of the eigenvalue problem.
- Rigid body modes, they have no strain energy so  $u_i^T K u_i = 0$ . They also satisfies  $K u_i = 0$  such that their are also solution of the eigenvalue problem with  $\omega_i = 0$ .

### Free response from initial conditions

As the eigenvalues are imaginary, we have:

$$x = \sum_{i=1}^n (A_i \cos \omega_i t + B_i \sin \omega_i t) \phi_i \quad (1.36)$$

We have so  $2n$  constants to determine using the orthogonality conditions.

# **Part II**

# **Acoustics**

# Chapter 1

## Fundamental principles of acoustics

### 1.1 Definition and origin of sound

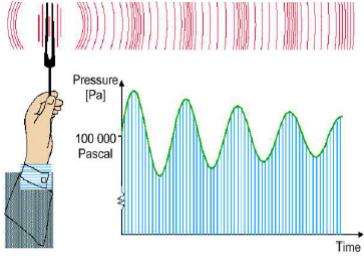


Figure 1.1

20Hz-20kHz, below we speak about infra-sound and above the limit, about ultra-sound.

Vibration when a mechanical excitation is applied on a material, air or water is the main source of sound. We speak about sound when the vibrations in air are perceptible by ear. The example on Figure 1.1 illustrates the vibration of a tuning fork that induces over- and under-pressure in the air around (order of magnitude small compared to ATM). Air particles are moving and describe a wave, a longitudinal wave, meaning that the particles displacement is parallel to the wave direction. Man can hear sound frequencies between 20Hz-20kHz, below we speak about infra-sound and above the limit, about ultra-sound.

### 1.2 Sound levels

#### 1.2.1 The effective sound pressure

The sound perceived with a constant loudness may be both a pure sine tone and a stochastic sound generated by a source:  $p(t)$  is extremely complicated, and yet the human ear have the impression of a constant loudness. The ear seems to be sensitive to the energy of sound waves. This led to the consideration of the **effective**— or **Root-Mean-Square (RMS)** value of the sound pressure, over a (energy of sound) certain time interval, as an measure of intensity:

$$p_{eff} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p^2(t) dt}. \quad (1.1)$$

#### 1.2.2 The dB-scale

It's another way to describe sound intensity. We define the **sound pressure level (SPL)** as:

$$L_p = 20 \log \frac{p}{p_0} \quad (1.2)$$

where  $p_0 = 20\mu Pa$  (threshold of human hearing). Note that every time we multiply the pressure by 10 we are in fact adding 20dB. Figure 1.2 regroups the tricks to use.

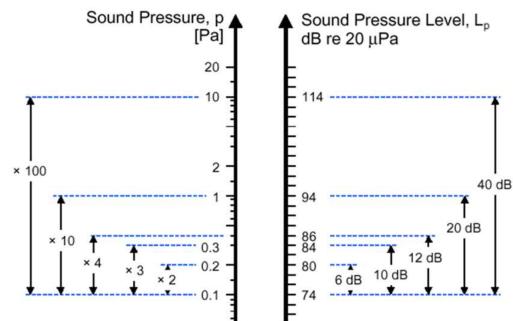


Figure 1.2

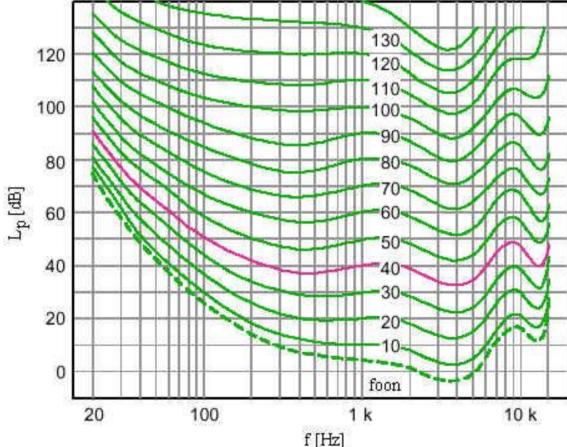


Figure 1.3

We are expecting for example if the loudness increases of 20dB we will perceive a sound 10 times louder. But this is not the case. Figure 1.3 gives the **sensibility** in function of the frequency. We can see that as the most important for human is to communicate by language, the sensibility is higher after 200Hz.

### 1.2.3 Superposition of two sounds

Imagine first that we have two same sources producing a certain amount of sound, how can we compute the total amount of dB? The basic formula is:

$$L_p = 10 \log \left( 10^{L_{p1}/10} + 10^{L_{p2}/10} \right). \quad (1.3)$$

Figure 1.4 regroups some results we can use. But we made the assumption here that we are not in phase or anti-phase. Indeed imagine that we are in phase, in this case the constructive relationship will make the pressure difference double (for example: 2 sources of 80dB in phase  $\rightarrow$  pressure of 2Pa becomes 4Pa and so we add 6dB (Pa  $\times 2 = + 6\text{dB}$ ). In fact the general formula is:

$$p_{rms}^2 = \frac{1}{T} \int_0^T (p_1 + p_2)^2 dt. \quad (1.4)$$

If we consider the two pressure as waves defined by amplitudes, frequencies and phases:  $p_1 = \text{Re}[P_1 e^{i(\omega_1 t + \phi_1)}]$ ,  $p_2 = \text{Re}[P_2 e^{i(\omega_2 t + \phi_2)}]$ , we have a smart way to compute this:

$$p_{rms}^2 = \frac{P_1^2 + P_2^2}{2} + P_1 P_2 \cos(\phi_1 + \phi_2). \quad (1.5)$$

If the last term  $P_1 P_2 \cos(\phi_1 + \phi_2) = 0$ , we speak about **incoherent sources**. In this case we are computing the so known signal addition formula:  $\text{RMS}(\text{total sound})^2 = \text{RMS}(\text{source1})^2 + \text{RMS}(\text{source2})^2$ . If the last term is  $\neq 0$ , the sources are **coherent** and we have to use the definition (1.5).

## 1.3 Sound waves



Figure 1.5

conversion of the energy into heat (absorption of air). The wave equation and its solution are:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \quad u(t, x) = \text{Re} \left( U_0 e^{i\omega t} e^{-i\frac{x}{c} i\omega} \right) \quad (1.6)$$

The first exponential contains the pulsation frequency (we look the wave freezed on a point of the environment), and if we freeze and look to the shape we have the space dependence that

is described by the second exponential. The wave-number , the wavelength and the speed of sound respect these relations:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \quad f\lambda = c \quad c = \sqrt{\gamma RT}. \quad (1.7)$$

Frequency	Attenuation in dB, at 30m
2000	0.2
4000	0.7
6000	1.6
8000	2.3
10000	3.2

The loss depends on the frequency, the larger the wavelength the less is the attenuation. The major frequencies are below 1dB of attenuation. When we speak outdoor, the sound is not reduced because of dispersion in air, but because the energy spread in a larger space. Here is the solutions for the plane wave, the spherical wave and the cylindrical wave respectively:

Figure 1.6

$$p(t, x) = Ae^{i\omega t} e^{-ikx} \quad p(t, r) = \frac{A}{r} e^{i\omega t} e^{-ikr} \quad p(t, r) = \frac{A}{\sqrt{r}} e^{i\omega t} e^{-ikr}. \quad (1.8)$$

They only differ from the amplitude. We can see that if the distance double for the two last cases the sound will respectively decrease of 6dB and 3 dB ( $20 \log 2 \approx 3$ ):

$$20 \log \frac{A}{r} = 20 \log A - 20 \log r \quad 20 \log \frac{A}{\sqrt{r}} = 20 \log A - 10 \log r \quad (1.9)$$

This is summarized on Figure 1.7.

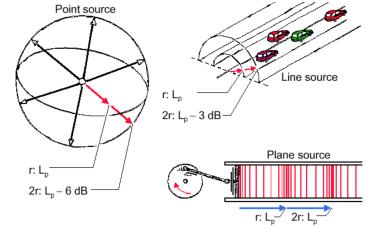


Figure 1.7

## 1.4 Sound power

### 1.4.1 Sound power in Watt

We introduced  $L_p = 20 \log p/p_0$ . We concluded that this was not a good thing to define the sound radiation because of the dependency in distance. We use for this the watt which defines the **energy flux through a closed surface**. We introduce also a  $L_w$  in dB:

$$L_w = 10 \log \frac{W}{W_0} \quad (1.10)$$

where  $W_0 = 10^{-12}$  Watt.

### 1.4.2 Sound intensity

This is the equivalent of the energy flux through a  $1\text{m}^2$  surface, the power per  $\text{m}^2$ . Here also we define a  $L_I$ :

$$L_I = 10 \log \frac{I}{I_0} \quad \text{and} \quad W = \int_S I.dS \quad (1.11)$$

where  $I_0 = 10^{-12}$  Watt/ $\text{m}^2$ . Let's compute the instantaneous intensity:

$$I_{r,inst} = \frac{dE_r}{dtdS} = \frac{F_t dr}{dtdS} = \frac{p_t dS dr}{dtdS} = p_t v_r. \quad (1.12)$$

If we try to specify the mean intensity for plane waves, we know that we have the relationship  $p/v = \rho c$ :

$$I_r = \overline{p v_r} = p_{eff} v_{eff} = \frac{p_{eff}^2}{\rho c} = \rho c v_{eff}^2. \quad (1.13)$$

We have to realize that this is only applicable for plane waves. There is a difference between the sound intensity and sound pressure. The pressure is a scalar value, while  $\mathbf{v}$  is a vector, there is so a direction of propagation.

### 1.4.3 Relationship between sound power and sound pressure

If I have a uniform sound wave we have  $W = IS$ , and far away from a source, we can assume the sound-wave to be planar:

$$W = \frac{p_{eff}^2 4\pi r^2}{\rho c} \quad p_{eff} = \sqrt{\frac{\rho c W}{4\pi r^2}}. \quad (1.14)$$

This allows us to assume that in air and in ground respectively:

$$L_p = L_W - 10 \log(4\pi r^2) \quad \text{and} \quad L_p = L_W - 10 \log(2\pi r^2). \quad (1.15)$$

# Chapter 2

## The human hearing system

### 2.1 Anatomy of the ear

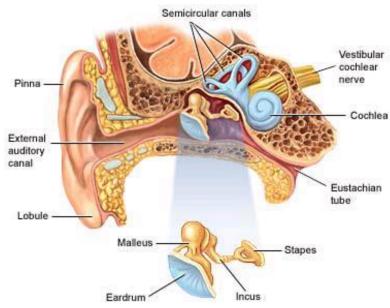


Figure 2.1

which amplifies the sound. Then the eardrum, thin member that starts to vibrate, then we have the ear-bones that moves and transmit the vibrations to the cochlea. When a strong sound stimulant enters, a muscle attached to the stapes contracts to limit the movement of the middle ear (acoustic reflex).

Inside the cochlea we have a liquid (the endolymph), it is divided into two tubes by the cochlear tube and are connected at the end of the spiral. When a sound comes to the eardrum, the ossicles compress the upper channel, where pressure waves propagate. We have water in the cochlea so if we don't have the bones that do the mechanism, we don't hear anything as the sound impedance is different in water and air. The transversal component of the wave exerts its force directly on the cochlear duct where the **organ of Corti is located**.

This one is the organ that allows to perceive sound. This organ is in fact situated on the **basilar membrane**. The organ of Corti consists of hair cells internally arranged in a row and externally arranged in 3 to 5 rows.

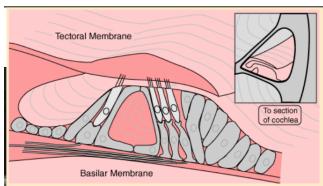


Figure 2.3

The hearing organ is sketched in Figure 2.1 and consists of three parts: the external hearing organ, the middle ear and the inner ear. The external hearing organ consists of the pinna (also called auricle), the external auditory canal and the ear drum. The middle ear consists of the hammer (malleus), anvil (incus), stirrup (stapes) and also the eardrum. The inner ear consists of the cochlea.

The sound first comes to the auricle, then the external auditory canal which has a 3cm length and has a resonance frequency

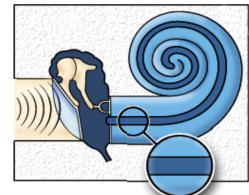


Figure 2.2

The hairs of the cells are in direct contact with a heavy membrane, called the **tektorial membrane**. When the basilar membrane is compressed, the contact of the hairs with the tektorial membrane will be lost beginning with the external rows. Every time the contact is broken or recovered, the electrical potential of the cells is changed, then transmitted to the brain via the fibres of the cochlear nerve. In the brain they are decoded and converted into a perception of sound.

Due to the interaction between the waves in the two canals, the maximal displacement of the basilar membrane becomes larger as the incoming tone gets lower. The displacement of the basilar membrane is proportional to the intensity of the incoming sound.

# Chapter 3

## Measuring sound

### 3.1 The measurement microphone

The measuring device is called a **sonometer** and is generally composed of a microphone, an amplifier, a conditioner and a display. We denote "type x" to specify the accuracy of the device, the more accurate being type 0 and then 1, 2, ...

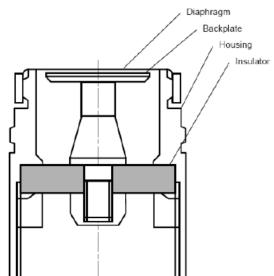


Figure 3.1

Figure 3.1 represents the principle of a condenser microphone. It is composed of two charged plates, one being the thin membrane in contact with the incoming sound waves, the second is the backplate which is the base plate. An electrical charge is created by means of a voltage supply. The sound makes the distance between the plates change and thus implies a capacity change and a voltage change according to:

$$Q = CV, C = \epsilon \frac{A}{d} \quad \Rightarrow \Delta V = \frac{Q}{\epsilon A} \Delta d. \quad (3.1)$$

The size of the microphone is important. The smaller the microphone, the higher is the range of frequency. Indeed, if we take the formula  $f\lambda = c$ , we've seen that the wave length is smaller when the frequency is higher, for example if we have 1000 Hz  $\rightarrow \lambda = 3$  cm. We need to have a wave homogeneity on the membrane. So for the example we gave, we need to have a membrane smaller than 1.5 cm. On the other hand, larger microphones have bigger sensibility and allow to hear less loud sounds.

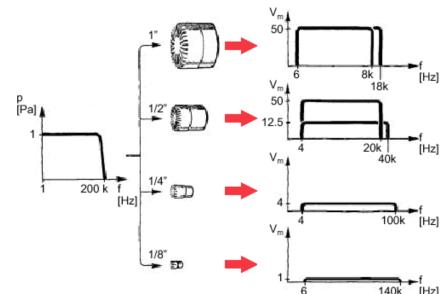


Figure 3.2

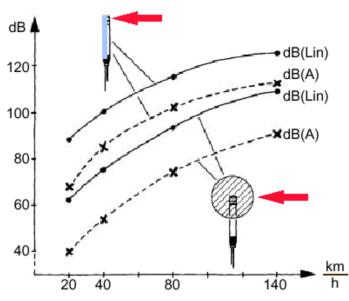


Figure 3.3

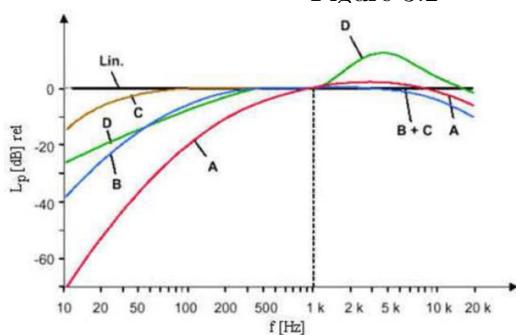


Figure 3.4

The wind is a polluting noise that has great effects. We can see that above 40 km/h the influence is enormous. To avoid the parasites, we use a ball windscreen and an A-weighting to soften the signal. This is shown on Figure 3.3. On Figure 3.4 are shown the different weightings used in practice, the dbA is accomplished by adjusting the sound level in every band to the frequency sensitivity of the human ear for soft sound (40 dB). According to the practical case, one of them will be used. For example if we organize a party, the presence of a lot of low frequencies forces the use of dB(A).

We can have devices that gives the sound level by frequency band. In this case the device has a calibration chart and need to be calibrated sometimes.

Another qualification unit is the **sound pressure level**, which is the exposure level. This is comparable to the RMS value but we integrate this in a certain range of time without dividing:

$$SEL = 10 \log \left( \int_0^T 10^{L/10} dt \right) \quad (3.2)$$

we use in this case a **dose meter**.

## 3.2 Sound intensity measurement

We have seen that  $I_x = pv_x$ , where p can be measured by means of a microphone but the air particles velocity is more difficult. We can do this:

$$F_x = ma \quad \Rightarrow \rho \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} \quad \Rightarrow v_x = -\frac{1}{\rho} \int \frac{\partial p}{\partial x} \quad (3.3)$$

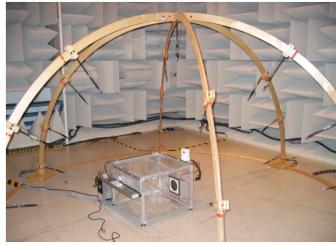
In fact to compute the intensity we use the discretized function:

$$I_x = \overline{pv_x} = -\frac{1}{2\rho\Delta r} \overline{(p_A + p_B)} \int (p_A - p_B) dt \quad (3.4)$$

where  $p_A, p_B$  are the pressure at two neighboring positions. The intensity meter consists so in two microphones separated by a certain distance. This device can be used to determine the direction of the source as the intensity is a vectorial quantity. Let's specify that the frequency range is rather limited, its upper limit is given by  $f_U = \frac{c}{4D}$ .

## 3.3 Sound power measurement

### 3.3.1 Using microphones



A first method to determine the power is the use of microphone in a delimited volume, but this method can only be applied in no wall conditions, no reflection (semi-anechoic room)! The surrounding surface can be a parallelepipedum, an hemisphere or a sphere, so we can use the formula  $W : IS$ . If the microphones are far from the emitting body (plane wave assumption), we can calculate the power using:

Figure 3.5

$$L_w = \bar{L}_p + 10 \log \frac{S}{S_0} - K_1 - K_2 \quad \text{and } \bar{L}_p = 20 \lg \frac{1}{N} \sum_{i=1}^N 10^{0.1 L_{pi}} \quad (3.5)$$

If I come near the machine, the waves are not plane waves anymore.

### 3.3.2 Comparison method

In this method, we are not limited about the volume anymore, the microphones have to be placed far enough too. We are using a known source which diffuses a known power  $L'_W$ . We then perform a test in the acoustic hard room and compute the spacial averages  $L_{pm}$  from the  $L_{pi}$  of the unknown source. Then we do the same for the known source and we can compute the unknown power as:

$$L_W = L'_W + (L_{pm} - L'_{pm}). \quad (3.6)$$

### 3.3.3 Using sound intensity

In this method we directly measure the intensity using an intensity meter and not anymore the pressure. We have to define a surrounding surface that we discretize. Then we get the intensity on these points and compute the discrete powers  $W_i = I_{ni}S_i$  that we use to get the average power level:

$$L_W = 10 \log \sum_{i=1}^N \frac{W_i}{W_0}. \quad (3.7)$$

The advantages of this method are: elimination of background noise, in situation measurements possible, nearfield measurements and the arbitrary surface. The disadvantages are: limited frequency range, directivity and the cost of the device.

## 3.4 Question (written exam)

- Explain the working principle of the human hearing system.
- Discuss the working principle of a sonometer and an intensity meter.
- Explain how one can measure sound power in practice.