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SUMMARY

Vibration & Acoustics
MECA-H411

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Appel à contribution

Synthèse Open Source

Ce document est grandement inspiré de l'excellent cours donné par Patrick Guillaume et Steve Vanlanduit à l'EPB (École Polytechnique de Bruxelles), faculté de l'ULB (Université Libre de Bruxelles). Il est écrit par les auteurs susnommés avec l'aide de tous les autres étudiants et votre aide est la bienvenue ! En effet, il y a toujours moyen de l'améliorer surtout que si le cours change, la synthèse doit être changée en conséquence. On peut retrouver le code source à l'adresse suivante

`https://github.com/nenglebert/Syntheses`

Pour contribuer à cette synthèse, il vous suffira de créer un compte sur *Github.com*. De légères modifications (petites coquilles, orthographe, ...) peuvent directement être faites sur le site ! Vous avez vu une petite faute ? Si oui, la corriger de cette façon ne prendra que quelques secondes, une bonne raison de le faire !

Pour de plus longues modifications, il est intéressant de disposer des fichiers : il vous faudra pour cela installer LaTeX, mais aussi *git*. Si cela pose problème, nous sommes évidemment ouverts à des contributeurs envoyant leur changement par mail ou n'importe quel autre moyen.

Le lien donné ci-dessus contient aussi un `README` contenant de plus amples informations, vous êtes invités à le lire si vous voulez faire avancer ce projet !

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Merci !

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Part I

Vibration

Chapter 1

Discrete systems

1.1 Introduction

Vibrations are found on everything around us, trains, cars and even human body is subject to vibration. Its effects are disturbing because it causes fatigue, loss of performance, no comfort, ... As vibration source we can find the earthquakes, the interaction with road, the wind, the waves, ... The basic terminology for the course is:

- **The source** $F(\omega)$, this characterizes the dynamic forces
- **The path** $H(\omega)$, this characterizes the structural dynamics
- **The response** $X(\omega)$, such that $X(\omega) = H(\omega)F(\omega)$.

Vibrations cause failure, loss of comfort and is harmful for precision operations. We try to suppress it by damping, isolation and structure design.

We have two different approach for analysing a vibration problem. The one called **Signal analysis** or **Fourier analysis** deals with the case where we only have the response of the system to unknown forces. The one called **System analysis** or **Modal analysis** where we stimulate the system with known forces and measure the response, being able to find $H(s)$ (dynamic forces - transfer function of the system).

Basic notions

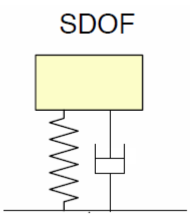

Figure 1.1

Three main forces are acting on bodies:

- the one due to springs, proportional to the displacement: $F = kd$
- the one due to dampers, proportional to the velocity: $F = cv$
- the one due to the mass, proportional to acceleration: $F = ma$.

Notice that we have one resonance frequency for each degree of liberty of each mass.

We can already get some definition, let's consider the free vibration assumed to be always in resonance. We can then define the **period of resonance** T_n, the **resonance frequency** $f_n = \frac{1}{T_n}$ and the **resonance pulsation**:

$$\omega_n = 2\pi f_n = \sqrt{\frac{k}{m}}. \tag{1.1}$$

Notice that if we increase mass, the frequency decreases.

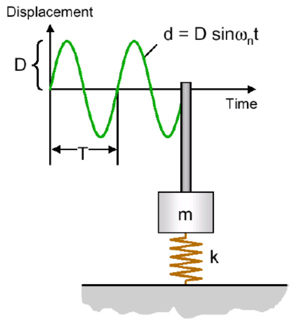

Figure 1.2

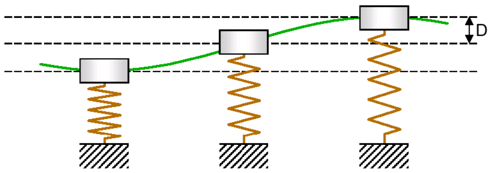

Figure 1.3

Then we have the **energy transfer** between the kinetic energy and the potential energy. Indeed we see that velocity is null on extreme position and max when at the middle. This is written as:

$$\frac{1}{2}mV^2 = \frac{1}{2}kD^2 \tag{1.2}$$

where $V = 2\pi f_n D$. Replacing by this:

$$m(2\pi f_n D)^2 = kD^2 \qquad \Rightarrow f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \tag{1.3}$$

We find a waited result. Now, let's show that **increasing damping reduces amplitudes over time**. Take the general newton equation for a linear free system and multiply by $\dot{x}(t)$:

$$m\ddot{x}\dot{x} + b\dot{x}\dot{x} + kx\dot{x} = 0 \qquad \Rightarrow \frac{d}{dt}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) = -b\dot{x}^2 \leq 0 \tag{1.4}$$

1.2 Single degree of freedom oscillator

Given the single degree system here, its free response is given by:

$$m\ddot{x}\dot{x} + b\dot{x}\dot{x} + kx\dot{x} = 0. \tag{1.5}$$

We assume that this differential equation admits a solution of type $x = Ae^{st}$. We can then write the characteristic equation and its eigenvalues as:

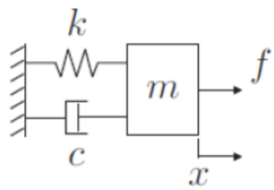

Figure 1.4

$$ms^2 + cx + k = 0, \qquad s = -\frac{c}{2m} \pm j\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}. \tag{1.6}$$

By defining two new quantity, the **natural pulsation** $\omega_n^2 = \frac{k}{m}$ and the **damping ratio** ξ such that $\xi\omega_n = \frac{c}{2m}$, we can rewrite:

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}. \tag{1.7}$$

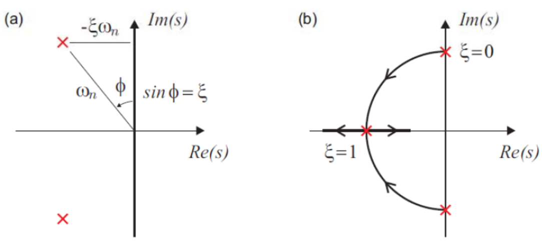

Figure 1.5

We can introduce the **damping pulsation** $\omega_d = \omega_n\sqrt{1-\xi^2}$. This explicitly makes appear the real and imaginary part of s that we plot on a diagram. Notice that the norm and the angle of the complex number are ω_n and $\arcsin\xi$.

The right figure is the **Nyquist diagram**. Last, the final expression for x is:

$$x = e^{-\xi\omega_n t}\left(Ae^{j\omega_d t} + Be^{-j\omega_d t}\right) = e^{-\xi\omega_n t}\left(A_1\cos(\omega_d t) + B_1\sin(\omega_d t)\right) \tag{1.8}$$

where A, B, A_1, B_1 depends on initial conditions.

Impulse response

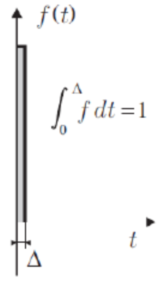

Figure 1.6

Let's now apply an impulse on the system and let's analyse when it is applied during the infinitesimal time Δ, given the initial conditions $x = 0, \dot{x} = 0$. If we integrate the newton equation:

$$\int_0^{\Delta} m\ddot{x}\,dt = \int_0^{\Delta} f\,dt - \cancel{\int_0^{\Delta} c\dot{x}\,dt} - \cancel{\int_0^{\Delta} kx\,dt} = 1 \tag{1.9}$$

where the spring and damping forces cancel as they are finite (infinitesimal integral), the impulse integral =1 by definition. Taking the limit we find new initial conditions:

$$\lim_{\Delta\to 0} m\dot{x}(\Delta) = m\dot{x}(0^+) = 1 \qquad \Rightarrow x(0^+) = 0, \dot{x}(0^+) = \frac{1}{m}. \tag{1.10}$$

The resolution of (1.8) gives the **impulse response**:

$$x(t) = h(t) = \frac{1}{m\omega_d} e^{-\xi\omega_n t}\sin(\omega_d t) \tag{1.11}$$

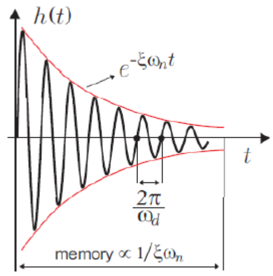

Figure 1.7

1.3 Convolution integral

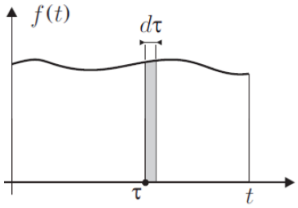

Figure 1.8

Consider a transfer function $h(t)$ of a system and the decomposition shown on the figure. The output of the system will be computed with the convolution integral:

$$x = \int_0^t h(t-\tau) f(\tau)\,d\tau. \tag{1.12}$$

where $h(t)$ is the **impulse response**. In particular, for a **causal** system:

$$x(t) = \int_{-\infty}^{\infty} h(t-\tau) f(\tau)\,d\tau = \int_{-\infty}^{\infty} h(\tau) f(t-\tau)\,d\tau = h(t) * f(t). \tag{1.13}$$

1.3.1 Harmonic response

Consider an undamped system to which we apply an harmonic force:

$$m\ddot{x} + kx = Fe^{j\omega t}. \tag{1.14}$$

By considering the Fourier transform $x(t) = X(\omega)e^{j\omega t}$, we get:

$$-\omega^2 X(j\omega) + \frac{k}{m}X(j\omega) = \frac{F(j\omega)}{m} \qquad \Rightarrow X = \frac{F}{k}\frac{1}{1-(\omega/\omega)^2} = \frac{F}{k}D(\omega) \tag{1.15}$$

where $D(\omega)$ is the **dynamic amplification**. For the damped case, we only have to know that $\xi\omega_n = c/2m$ and we get in the same way as previously:

$$X = \frac{F}{k}\frac{1}{1-(\omega/\omega)^2+2j\omega/\omega_n} = \frac{F}{k}D(\omega). \tag{1.16}$$

The two dynamic amplifications are plotted on the figures below, the second on a Bode diagram where we can clearly see the **Quality factor** defined as $Q = 1/2\xi$.

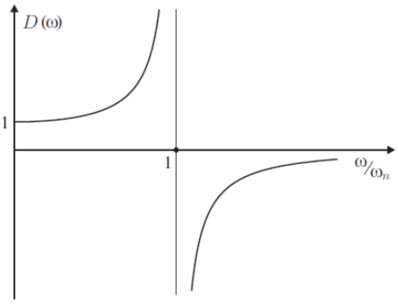

Figure 1.9

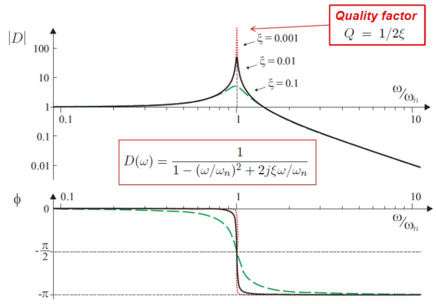

Figure 1.10

1.3.2 Frequency response function

Let's remember that the response is given by the convolution integral (1.13), where we will define now $h(t)$. Assuming the applied force to be harmonic $Fe^{i\omega t}$, then proceeding to the Fourier transform of x, we get:

$$Xe^{i\omega t} = \int_{-\infty}^{\infty} h(t)Fe^{i\omega(t-\tau)}\,d\tau \qquad \Rightarrow \frac{X(\omega)}{F(\omega)} = \int_{-\infty}^{\infty} h(t)e^{-i\omega\tau}\,d\tau = H(\omega) \tag{1.17}$$

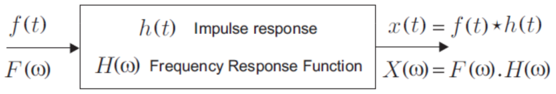

Figure 1.11

where we end up with the fact that **the frequency response function is the Fourier transform of the impulse response**. We can thus avoid the convolution to have a simple multiplication after a Fourier transform. Here is also a useful theorem where $|F(\omega)/2\pi|$ is the **energy spectrum** of $f(t)$:

Parseval theorem

$$\int_{-\infty}^{\infty} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\,d\omega. \tag{1.18}$$

1.3.3 Discrete Fourier Transform

Consider a signal x sampled in N samples, the equivalence of the continuous Fourier transform in discrete domain and the inverse transform are:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i\frac{2k\pi}{N}n} \quad \text{and} \quad X_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{i\frac{2k\pi}{N}n} \tag{1.19}$$

The equivalent Parseval theorem is:

$$\sum_{n=0}^{N-1} |x_n|^2 = \frac{1}{N}\sum_{k=0}^{N_1} |X_k|^2 \tag{1.20}$$

And as last definition, we have the root-mean-square value defined as

$$RMS = \sqrt{\frac{1}{N}\sum_{n=0}^{N-1}|x_n|^2} = \sqrt{\frac{1}{N^2}\sum_{k=0}^{N-1}|X_n|^2} \tag{1.21}$$

1.4 Transient response (Beat)

Consider an undamped oscillator excited by an harmonic force $F\cos\omega t$ and of impulse response:

$$h(t) = \frac{1}{m\omega_n}\sin\omega_n t \tag{1.22}$$

Applying the convolution integral to this problem and integrating by part we find:

$$x(t) = \int_0^t F\cos(\omega t)h(t - /tau)\,d\tau = \frac{F}{m}\frac{\cos(\omega t) - \cos(\omega_n t)}{\omega_n^2 - \omega^2} \tag{1.23}$$

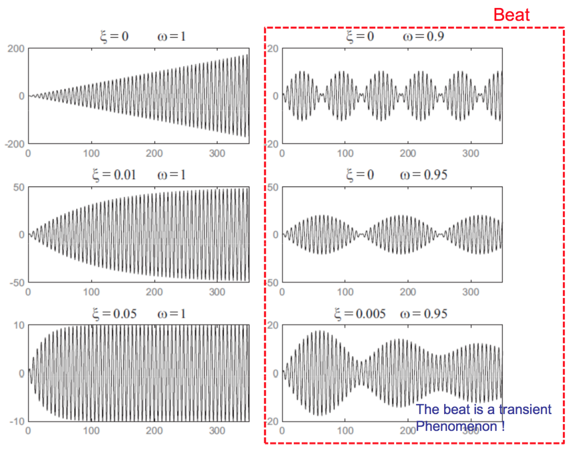

Figure 1.12

By using Simpson's formula, defining $\omega + \omega_n = 2\omega_0$ and $\omega - \omega_n = 2\Delta$, we get:

$$x(t) = \frac{F}{m}\frac{\sin(\omega_0 t)\sin(\Delta t)}{2\omega_0\Delta} \tag{1.24}$$

Remark that for $\Delta \to 0$, $\frac{\sin\Delta t}{\Delta} = t$. This term is called the **modulating function** as it grows the response amplitude while the other sinus is confined in $[-1, 1]$. We get thus for resonance:

$$x(t) = \frac{F}{m}\frac{\sin(\omega_n t)}{2\omega_n}t. \tag{1.25}$$

The figure shows that the phenomenon known as beat is **in fact** a transient phenomenon.

1.5 Multiple degree of freedom

Before going throw the real subject, let's introduce what's the **state space model** (yes CSD). It consists in writing any differential equation in this form of first order equation:

$$\dot{x} = Ax + Bu \tag{1.26}$$

where A, B are matrices and x, u vectors. Let's apply this for an oscillator:

$$\ddot{x} = \frac{f}{m} - 2\xi\omega_n - \omega_n^2 \tag{1.27}$$

We choose as state variable $x_1 = x$ and $x_2 = \dot{x}$, then we only have to rewrite the definition of $\dot{x}_1$ and $\dot{x}_2$ as:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 2\xi\omega \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \frac{1}{m} \end{Bmatrix} f \tag{1.28}$$

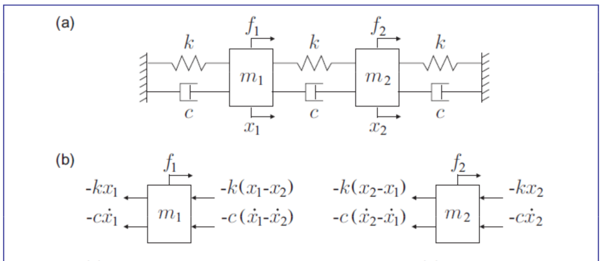

Fig. 2.1. (a) Two degree-of-freedom lumped mass system. (b) Free body diagrams.

$$m_1\ddot{x}_1 = f_1 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) - kx_1 - c\dot{x}_1$$

$$m_2\ddot{x}_2 = f_2 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) - kx_2 - c\dot{x}_2$$

'n matrix form:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2c & -c \\ -c & 2c \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}$$

Figure 1.13

Let's now look at the multiple degree case on Figure 1.13. The first thing to do is to put everything in matrix form. We see that we have something in the form:

$$M\ddot{x} + C\dot{x} + Kx = f. \tag{1.29}$$

The matrices M, C, K are symmetric and semi-positive definite: $K = K^T$ and $x^T Kx \geq 0,\ \forall x$. In particular, we can compute the kinetic energy and the strain energy as:

$$\frac{1}{2}\dot{x}^T M\dot{x} \qquad \text{and} \qquad \frac{1}{2}x^T Kx \tag{1.30}$$

1.6 Eigenvalue problem

The method to solve this problem is to first consider the free response of the conservative system ($C = 0$): $M\ddot{x} + Kx = 0$. A non trivial solution exists if:

$$(K + s^2 M)\phi = 0 \tag{1.31}$$

The eigenvalues s are solution of $\det(K + s^2 M) = 0$ Because K and M are symmetric and semi-positive definite, the eigenvalues are purely imaginary: $s = \pm j\omega$, this gives:

$$(K - \omega_i M)\phi_i = 0 \tag{1.32}$$

where ω_i are the natural pulsations and ϕ_i the mode shapes.

Orthogonality of the mode shapes

Let's demonstrate that (substracting with the i,j permuted equation):

$$(K - \omega_i M)\phi_i = 0 \qquad \Rightarrow \begin{array}{c} \phi_j^T K\phi_i = \omega_i \phi_j^T M\phi_i \\ \underline{-\phi_i^T K\phi_j = \omega_j \phi_i^T M\phi_j} \\ 0 = (\omega_i^2 - \omega_j^2)\phi_j^T M\phi_i \end{array} \qquad \Rightarrow \phi_j^T M\phi_i = 0 \qquad (\omega_i \neq \omega_j) \tag{1.33}$$

The conclusion is that the mode shapes corresponding to distinct natural frequencies are orthogonal with respect to M and K. We then have the:

Orthogonality relationships

$$\phi_i^T M\phi_j = \mu_i \delta_{ij} \qquad \textit{and} \qquad \phi_i^T K\phi_j = \mu_i \omega_i^2 \delta_{ij}, \qquad \omega_i^2 = \frac{\phi_i^T K\phi_i}{\phi_i^T M\phi_i} \tag{1.34}$$

where μ_i is the modal mass and ω_i the Rayleigh coefficient

.

And in matrix form with $\Phi = (\phi_1, \phi_2, ..., \phi_n)$:

$$\boxed{\Phi^T M \Phi = diag(\mu_i) \qquad \Phi^T K \Phi = diag(\mu_i \omega_i^2)} \tag{1.35}$$

To complete, two remarks:

- If several modes have the same natural frequency, they form a subspace and any vector in this subspace is also solution of the eigenvalue problem.

- Rigid body modes, they have no strain energy so $u_i^T K u_i = 0$. They also satisfies $K u_i = 0$ such that their are also solution of the eigenvalue problem with $\omega_i = 0$.

Free response from initial conditions

As the eigenvalues are imaginary, we have:

$$x = \sum_{i=1}^{n} (A_i \cos \omega_i t + B_i \sin \omega_i t) \phi_i \tag{1.36}$$

We have so 2n constants to determine using the orthogonality conditions.

Part II

Acoustics

Chapter 1

Fundamental principles of acoustics

1.1 Definition and origin of sound

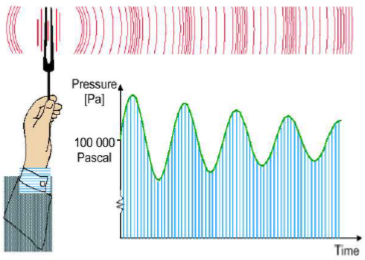

Figure 1.1

Vibration when a mechanical excitation is applied on a material, air or water is the main source of sound. We speak about sound when the vibrations in air are perceptible by ear. The example on Figure 1.1 illustrates the vibration of a tuning fork that induces over- and under-pressure in the air around (order of magnitude small compared to ATM). Air particles are moving and describe a wave, a longitudinal wave, meaning that the particles displacement is parallel to the wave direction. Man can hear sound frequencies between 20Hz-20kHz, below we speak about infra-sound and above the limit, about ultra-sound.

1.2 Sound levels

1.2.1 The effective sound pressure

The sound perceived with a constant loudness may be both a pure sine tone and a stochastic sound generated by a source: p(t) is extremely complicated, and yet the human ear have the impression of a constant loudness. The ear seems to be sensitive to the energy of sound waves. This led to the consideration of the **effective**– or **Root-Mean-Square (RMS)** value of the sound pressure, over a (energy of sound) certain time interval, as an measure of intensity:

$$p_{eff} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p^2(t)\, dt}. \tag{1.1}$$

1.2.2 The dB-scale

It's another way to describe sound intensity. We define the **sound pressure level (SPL)** as:

$$L_p = 20 \log \frac{p}{p_0} \tag{1.2}$$

where $p_0 = 20\mu Pa$ (threshold of human hearing). Note that every time we multiply the pressure by 10 we are in fact adding 20dB. Figure 1.2 regroups the tricks to use.

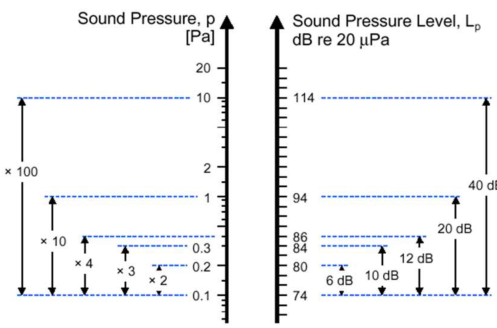

Figure 1.2

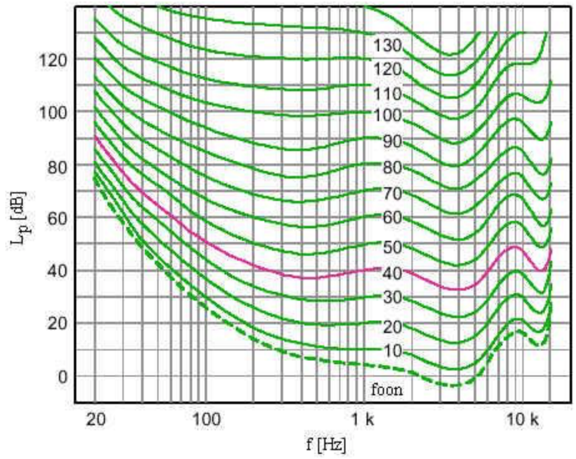

Figure 1.3

We are expecting for example if the loudness increases of 20dB we will perceive a sound 10 times louder. But this is not the case. Figure 1.3 gives the **sensibility** in function of the frequency. We can see that as the most important for human is to communicate by language, the sensibility is higher after 200Hz.

1.2.3 Superposition of two sounds

Imagine first that we have to same sources producing a certain amount of sound, how can we compute the total amount of dB? The basic formula is:

$$L_p = 10 \log \left(10^{L_{p1}/10} + 10^{L_{p2}/10} \right). \tag{1.3}$$

Figure 1.4 regroups some results we can use. But we made the assumption here that we are not in phase or anti-phase. Indeed imagine that we are in phase, in this case the constructive relationship will make the pressure difference double (for example: 2 sources of 80dB in phase → pressure of 2Pa becomes 4Pa and so we add 6dB (Pa x2 = + 6dB). In fact the general formula is:

Difference	0	1	2	3	4	5	6	7	8	9	10	12
Add	3	2,5	2,1	1,8	1,4	1,2	1,0	0,8	0,6	0,5	0,4	0,2

Figure 1.4

$$p_{rms}^2 = \frac{1}{T} \int_0^T (p_1 + p_2)^2 \, dt. \tag{1.4}$$

If we consider the two pressure as waves defined by amplitudes, frequencies and phases: $p_1 = Re[P_1 e^{i(\omega_1 t + \phi_1)}], p_2 = Re[P_2 e^{i(\omega_2 t + \phi_2)}]$, we have a smart way to compute this:

$$p_{rms}^2 = \frac{P_1^2 + P_2^2}{2} + P_1 P_2 \cos(\phi_1 + \phi_2). \tag{1.5}$$

If the last term $P_1 P_2 \cos(\phi_1 + \phi_2) = 0$, we speak about **incoherent sources**. In this case we are computing the so known signal addition formula: RMS(total sound)^2 = RMS(source1)^2 + RMS(source2)^2. If the last term is $\neq 0$, the sources are **coherent** and we have to use the definition (1.5).

1.3 Sound waves

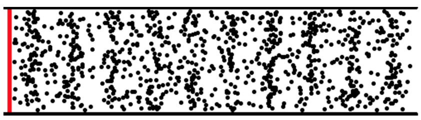

Figure 1.5

How does the sound propagates? Let's assume that we have a vibration, then the air particles in contact with the membrane are moved, that movement is then transmitted to neighboring particles. The pressure is transmitted but it is decreasing. It is decreasing because we have conversion of the energy into heat (absorption of air). The wave equation and its solution are:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} \qquad u(t, x) = Re \left(U_0 e^{i\omega t} e^{-i \frac{x}{c} i\omega} \right) \tag{1.6}$$

The first exponential contains the pulsation frequency (we look the wave freezed on a point of the environment), and if we freeze and look to the shape we have the space dependence that is described by the second exponential. The wave-number , the wavelength and the speed of sound respect these relations:

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} \qquad f\lambda = c \qquad c = \sqrt{\gamma RT}. \tag{1.7}$$

Frequency	Attenuation in dB, at 30m
2000	0.2
4000	0.7
6000	1.6
8000	2.3
10000	3.2

Figure 1.6

The loss depends on the frequency, the larger the wavelength the less is the attenuation. The major frequencies are below 1dB of attenuation. When we speak outdoor, the sound is not reduced because of dispersion in air, but because the energy spread in a larger space. Here is the solutions for the plane wave, the spherical wave and the cylindrical wave respectively:

$$p(t,x) = Ae^{i\omega t}e^{-ikx} \qquad p(t,r) = \frac{A}{r}e^{i\omega t}e^{-ikr} \qquad p(t,r) = \frac{A}{\sqrt{r}}e^{i\omega t}e^{-ikr}. \tag{1.8}$$

They only differ from the amplitude. We can see that if the distance double for the two last cases the sound will respectively decrease of 6dB and 3 dB ($20 \log 2 \approx 3$):

$$20 \log \frac{A}{r} = 20 \log A - 20 \log r \qquad 20 \log \frac{A}{\sqrt{r}} = 20 \log A - 10 \log r \tag{1.9}$$

This is summarized on Figure 1.7.

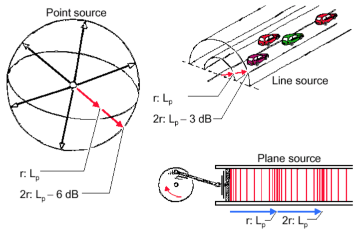

Figure 1.7

1.4 Sound power

1.4.1 Sound power in Watt

We introduced $L_p = 20 \log p/p_0$. We concluded that this was not a good thing to define the sound radiation because of the dependency in distance. We use for this the watt which defines the **energy flux through a closed surface**. We introduce also a L_w in dB:

$$L_w = 10 \log \frac{W}{W_0} \tag{1.10}$$

where $W_0 = 10^{-12}$ Watt.

1.4.2 Sound intensity

This is the equivalent of the energy flux through a 1m^2 surface, the power per m^2. Here also we define a L_I:

$$L_I = 10 \log \frac{I}{I_0} \qquad and \qquad W = \int_S I.dS \tag{1.11}$$

where $I_0 = 10^{-12}$ Watt/m^2. Let's compute the instantaneous intensity:

$$I_{r,inst} = \frac{dE_r}{dtdS} = \frac{F_t dr}{dtdS} = \frac{p_t dS dr}{dtdS} = p_t v_r. \tag{1.12}$$

If we try to specify the mean intensity for plane waves, we know that we have the relationship $p/v = \rho c$:

$$I_r = \overline{pv_r} = p_{eff} v_{eff} = \frac{p_{eff}^2}{\rho c} = \rho c v_{eff}^2. \tag{1.13}$$

We have to realize that this is only applicable for plane waves. There is a difference between the sound intensity and sound pressure. The pressure is a scalar value, while v is a vector, there is so a direction of propagation.

1.4.3 Relationship between sound power and sound pressure

If I have a uniform sound wave we have $W = IS$, and far away from a source, we can assume the sound-wave to be planar:

$$W = \frac{p_{eff}^2 4\pi r^2}{\rho c} \qquad p_{eff} = \sqrt{\frac{\rho c W}{4\pi r^2}}. \tag{1.14}$$

This allows us to assume that in air and in ground respectively:

$$L_p = L_W - 10\log(4\pi r^2) \qquad and \qquad L_p = L_W - 10\log(2\pi r^2). \tag{1.15}$$

Chapter 2

The human hearing system

2.1 Anatomy of the ear

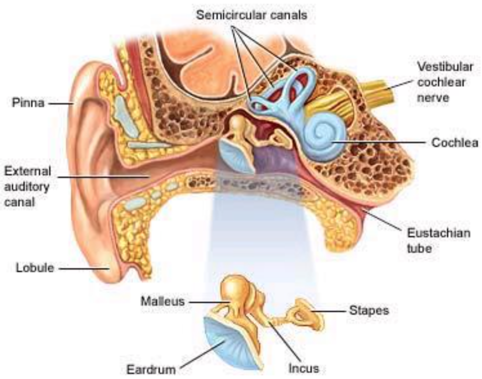

Figure 2.1

The hearing organ is sketched in Figure 2.1 and consists of three parts: the external hearing organ, the middle ear and the inner ear. The external hearing organ consists of the pinna (also called auricle), the external auditory canal and the ear drum. The middle ear consists of the hammer (malleus), anvil (incus), stirrup (stapes) and also the eardrum. The inner ear consists of the cochlea.

The sound first comes to the auricle, then the external auditory canal which has a 3cm length and has a resonance frequency which amplifies the sound. Then the eardrum, thin member that starts to vibrate, then we have the ear-bones that moves and transmit the vibrations to the cochlea. When a strong sound stimulant enters, a muscle attached to the stapes contracts to limit the movement of the middle ear (acoustic reflex).

Inside the cochlea we have a liquid (the endolymph), it is divided into two tubes by the cochlear tube and are connected at the end of the spiral. When a sound comes to the eardrum, the ossicles compress the upper channel, where pressure waves propagate. We have water in the cochlea so if we don't have the bones that do the mechanism, we don't hear anything as the sound impedance is different in water and air. The transversal component of the wave exerts its force directly on the cochlear duct where the **organ of Corti is located**.

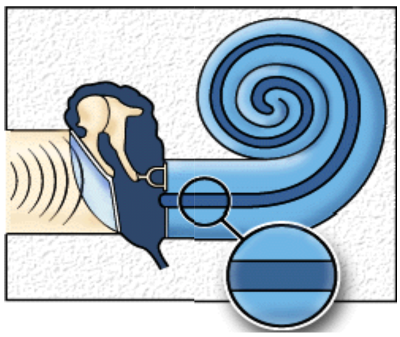

Figure 2.2