PHSX815_Project1:
Are those die loaded?

Neno Fuller

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1 Introduction

On a wonderful summer day, while enjoying a delicious lunch at Leeway Franks, a mysterious individual approaches and asks you to play a game of dice. You inquire about the conditions: "I will pay you a dollar each time you roll a sum that's greater than mine." Okay, that seems reasonable you think: "however, If I roll a sum that's greater than yours, you will pay me two dollar" they continued. This seems unreasonable to you; for a fair die, the chances of you winning are the same as theirs. To pique your interest further they continued: "I present you with a bag of dice, you can chose the ones to be used for our gamble - two being more special than the others. Will you accept these conditions?" You think for a minute and agree: "But, only after I've been given the opportunity to do a bunch of pre-rolls.", you reply. You take the dice, and while leaving noticed the me snacking on a hoogie: "I'll give you two leeway burgers if you can find the golden dice" I agreed and went about testing. This paper is an attempt to do so and is organized as follows: Sec. 2 explains the rule of the game as well as the hypotheses. A description of the computer simulation developed to simulate these possibilities is provided in Sec. 3, with an analysis of the outputs included in Sec. 4. Finally, conclusions are presented in Sec. 5.

2 Hypotheses: Are these the golden die?

Our experiment is simple: we will roll some die and the player with the highest sum wins. However, our objective is to seek the biased die as that will increase the chances of us winning more games. How do we go about doing this? We will construct the likelihood function (LH) for the wins for a given set of dice. We seek to find the dice whose distribution is skewed i.e there is a higher probability of winning a greater number of games. Is the overall game's win distribution centered at rolls/2?

3 Code and Experimental Simulation

Our code is composed of two simulations: that of a fair roll and a biased rolled. We simulate our game as followed: we generate from a binomial distribution two numbers which can be at any of six positions. The locations with the number then correspond to the "face" of the dice we rolled. For example, if we notate [0,0,1,1,0,0] as our rolled dice, we would have "rolled" a 3 and 4; thus, our sum would be a 7. Each face has a certain probability: a fair dice would have each side with one-sixth probability each. If any of the faces were being preferred, then this would be reflected in the LH being shifted, stretched or shrink or some combination. Let's look at some simulated data. Figure 1 is the simulated average wins for our game of dice. Note it's as expected for a fair game of dice where each player is simply rolling a pair of dice and obtaining the sums. Each player has the same probability of obtaining a large sum.

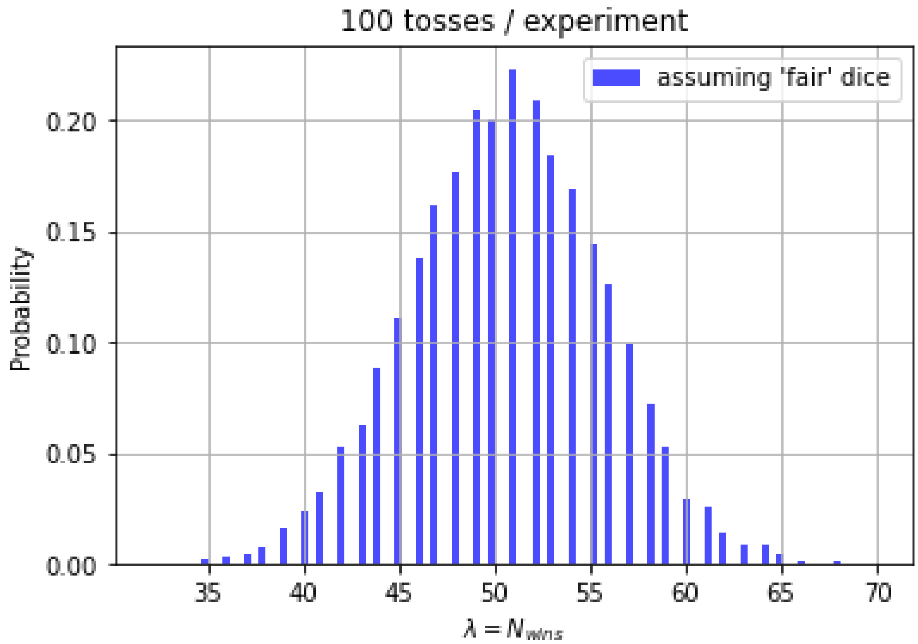


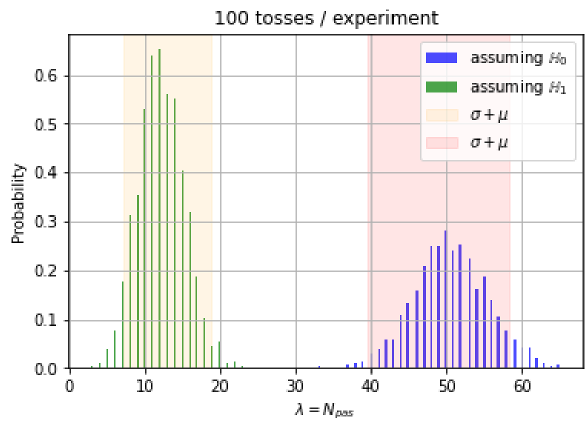
Figure 1: Simulations of the average wins per game of dice. Here each face has an equal probability P = 1/6th. To obtain data, we simulated 100 toss per each experiment for which there were 100,000. It's interesting to note that the distribution is not exactly symmetric about about N = 50. Instead we find a mean N = μ = 52.5 wins with standard deviations given be σ = 10.2 games with the maximum wins being 70. which is about 2 σ.

4 Analysis

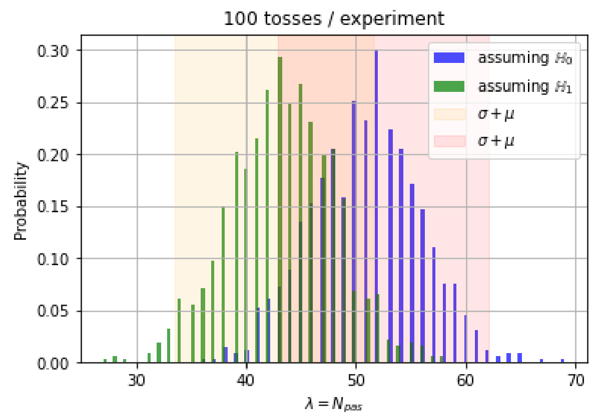
To emphasize what we seek out for our golden die, we simulated four scenarios: dice with comparable face probabilities to the left and right; dice whose lower face's are biased favourably and vice versa: favorable higher faces. We see these four situations in Figure 2.

The figures represent the physical situation of biasing the faces in such a way as to limit the range of maximum obtainable sum. For example, the LH in 2.a represents an over-representation of the lower faces: the highest roll is 3 and corresponding possible sum 6. This is more than half the general fair probability where one can obtain a six and corresponding sum 12. The others are of similar nature: the distribution moves in accordance to the range of permitted sum values. It it worth noting that most of the data is within 2 sigmas of the mean.

To shine further clarity, we employ the log-likelihood. Figure 3 contains the LLR for the four simulations. Each LLR was obtained using a Bernoulli distribution: the result of a roll was in essence a yes or no. First note the skewness: the distributions were never centered at zero except for when they were biased similarly. The more polarized distributions were biased about either surely winning or losing a game or two on average. In all cases we could differentiate between the two dice rather confidently: the powers of each distribution in comparison to the null were about 0.85, 0.812, 0.852, 0.684 respectively. With this in mind, distributions 2.a) and 2.d) - those produced with extreme face probabilities demonstrate what we would look for in a pair of golden dice.



(a)



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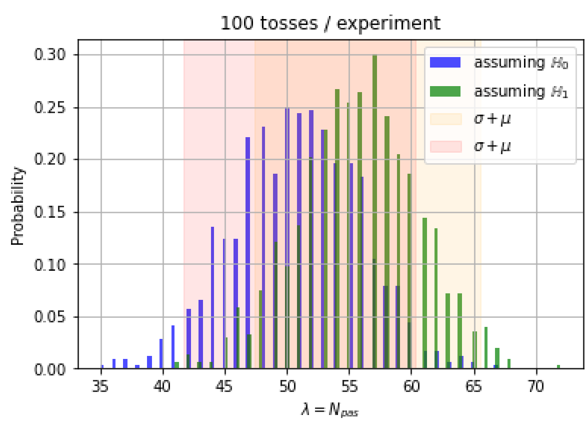

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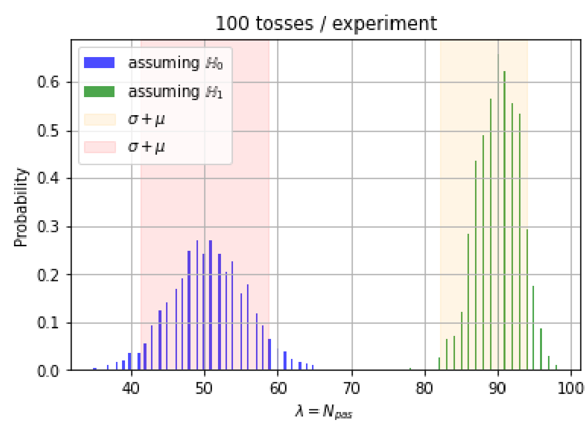

(d)

Figure 2: The LH for four different biased dice. From left to right: simulated data from dice whose faces had probabilities a) [3/9, 3/9, 3/9, 0, 0, 0], b)[1/9, 2/9, 2/9, 2/9, 2/9, 0], c) [1/9, 1/9, 2/9, 2/9, 2/9, 1/9] and d) [0, 0, 0, 3/9, 3/9, 3/9]. Note the corresponding shifts in the distribution. For the extreme cases, where one either lost or won the majority of the games, we find means μ = 13, 42, 56.5 and 88 games respectively.

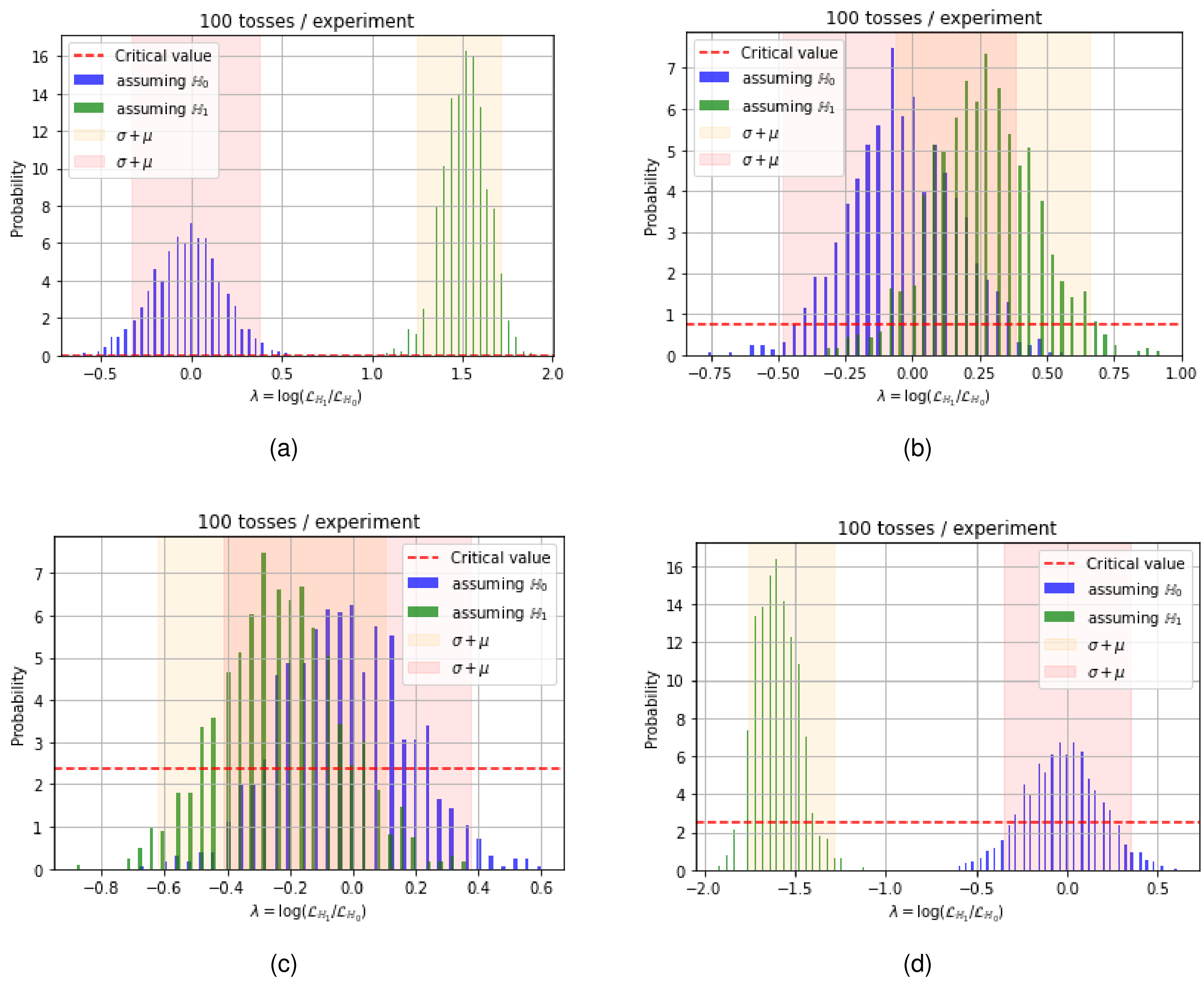

Figure 3: The LLR for our four biased distributions. Each was tested against the null assuming α = 0.06,0.06,0.06 and 0.009 with corresponding β = 0.148,0.188,0.148,0.316

5 Conclusion

With this in mind, the author proceeded to the engineering department where he happened to meet a lovely chap with a dice-roller 3000. To his amazement, they were able to accomplish this task after a few arduous sessions. The data demonstrated a very loaded die. It was a great day. On that stormy day at Leeways, you preceded to win 23 dollars over the course of a few games. Secretly though, I worried how conclusiveness my results were. Would they change if I were to sample from an continuous distribution as oppose to discrete with individual hypothesis being testing? Indeed, there were a number of questionable assumption for the the model. In any-case, experimentally the results bore fruit.

References