PHSX815_Project1:
I am not too sure those dice are loaded

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1 Introduction

On a sweltering spring day, while enjoying a delicious lunch at Leeway Franks', a mysterious individual approached me and asked if I wanted to play a game. It was a game of dice. Now, this was eerily familiar. So, I inquired about the conditions: "I will pay you a dollar each time you roll a sum that's greater than mine." Okay, that seems reasonable you think: "however, If I roll a sum that's greater than yours, you will pay me two dollar" they continued. This seemed unreasonable; for a fair die, the chances of me winning are the same as theirs.They continued: "I present you with a bag of dice, you can chose the ones to be used for our gamble - two being more special than the others. Will you accept these conditions?" Of course, I agreed: "But, only after I have been given the opportunity to do a bunch of pre-rolls.". So, I took the dice and told the man to meet me there at some future time. Little did they know, I had done this very thing before and was left with more puzzles than answers. This paper is an attempt to answer some of those question. It is organized as followed: Sec. 2 explains the rule of the game as well as the hypotheses. A description of the computer simulation developed to simulate these possibilities is provided in Sec. 3, with an analysis of the outputs included in Sec. 4. Finally, conclusions are presented in Sec. 5.

2 Hypotheses: Are these the golden die?

Our experiment is simple: we will roll some die and the player with the highest sum wins. However, our objective is to seek the biased die as that will increase the chances of us winning more games. How do we go about doing this? We will construct the likelihood function (LH) for the wins for a given set of dice. We seek to find the dice whose distribution is skewed i.e there is a higher probability of winning a greater number of games.

3 Code and Experimental Simulation

We simulate our game as followed: we generate from a binomial distribution two numbers which can be at any of six positions. The locations with the numbers then correspond to the "face" of the dice we rolled. For example, if we notate [0,0,1,1,0,0] as our rolled dice, we would have "rolled" a 3 and 4; thus, our sum would be a 7. Each face has a certain probability: a fair dice would have each side with one-sixth probability each. If any of the faces were being preferred, then this would be reflected in the LH being shifted, stretched or shrink or some combination. However our face probabilities will be sampled from some normal distribution whose location(mean) we provide. Thus, we will generate the "rolls" of my mysterious adversary and I, by first sampling from some normal distribution six numbers (that are then normalized) corresponding to the face probabilities of the dice. However, we have no sense of what is the "real" distribution; for each roll the corresponding output distribution is "all over the place". By this I mean: If we repeat N number of experiments(games) each with n measurements (rolls) then, construct a likelihood of our observable (average number of rolls won per game) then our likelihood will be highly unstable if the initial probabilities vary greatly.

4 Analysis

Let's look at some simulated data. Figure 1 is the simulated average wins for our game of dice. over a number of universes. Meaning, we ran multiple games, over many experiments. The most likely average number of games is given by 51 games, which though not 50 percent, is about half of the games.

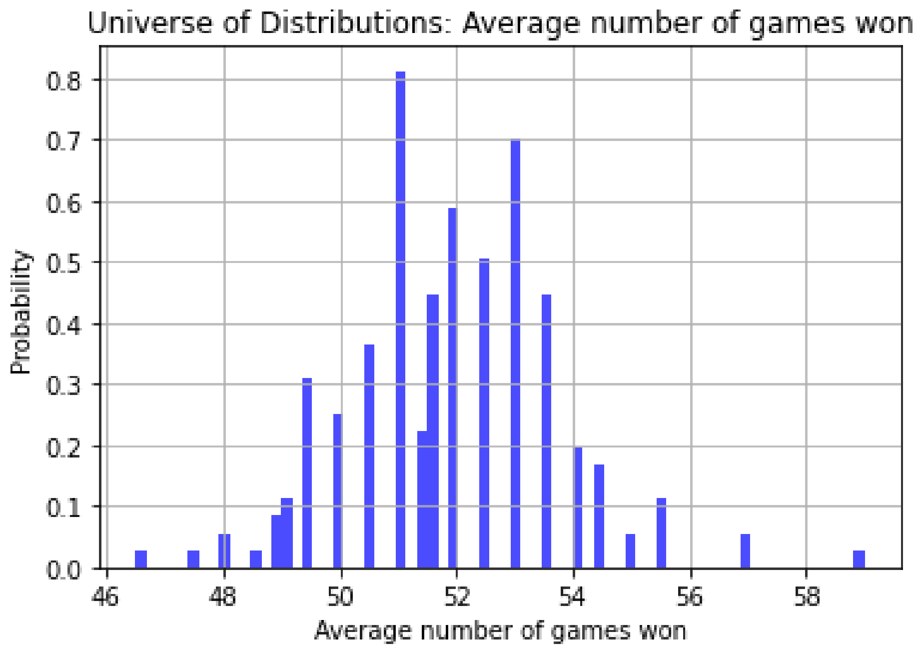
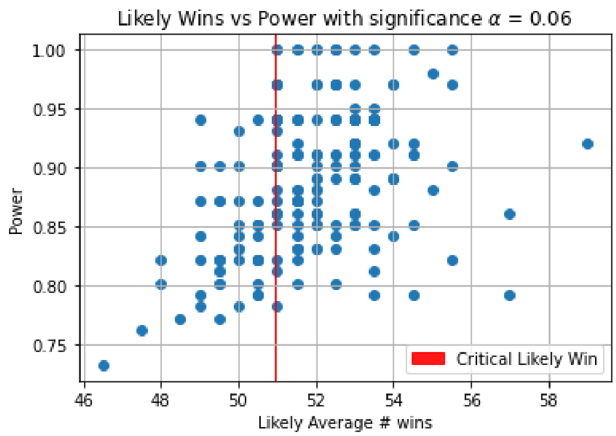


Figure 1: Simulations of the most likely average wins per game of dice. To obtain data, we simulated 200 universes where 100 toss were performed per each experiment (1000). It's interesting to note that the distribution is not exactly symmetric about about N = 50. Instead we find a mean N = μ = 51 wins with standard deviations given be σ = 4 games with the maximum wins being 59. which is about 2 σ.

We can further extract data from the multiverse of games. Note the above chance of getting a distribution with the given mean is about 81 percent, given our distribution over different number of universes of experiments. Remember, we are probabilistic-ally sampling face probabilities. It thus stands to reason we would need a 19 percent chance of being "wrong" in rejecting the correct null ("the average number of games won is about rolls/2") to gauge whether a particular value is the "correct" one in positing alternative distributions. We thus perform a significance: one at 6 percent and the other 18.8 percent between our standard (most likely distribution) and our multiverse of distributions.

Figure 2 is a plot of the Most likely average wins vs the power of our test. We assume a null



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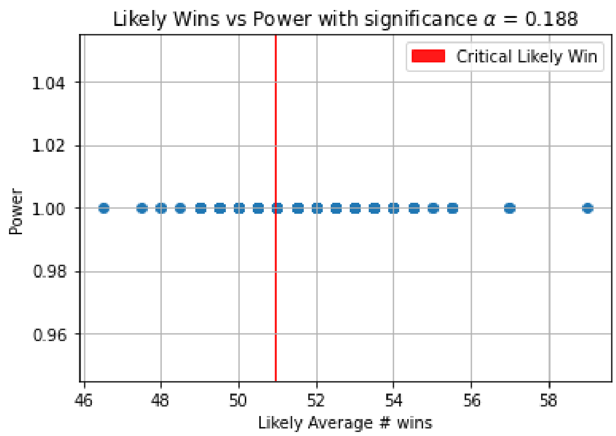

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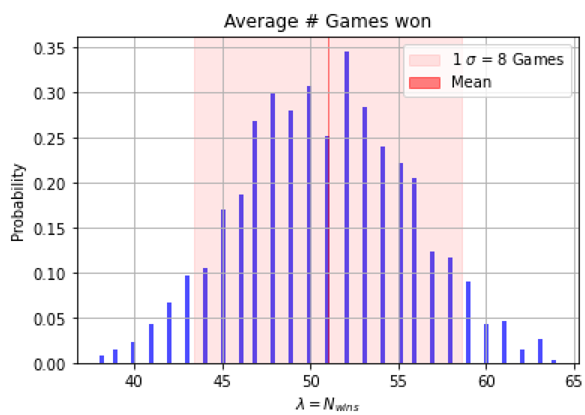
Figure 2: Significance testing over two different levels. The null is given by the most optimal distribution in our universe of distributions.

distribution given by the most likely and tested the others based on this. As we can see, we are able to differentiate between the distributions. We also see that there are other similar distributions, within a sigma, that would be viable candidate distributions for our game. To shine further clarity, we look at a few representative distributions. Figure 3 contains plots of distributions of varying means. Note, the change in probability for the the most likely average game. As such, if one were to simply rolls dice a priori without any sense of the face probabilities, then chances are, given a large enough sample space, our distribution seems to be indicating that our chances of winning would be about 50/50.

5 Conclusion

Now, none of this changes the fact that if we employ the dice roller-3000 and were able to construct a probability distribution of the dice, if the dice demonstrate extreme behaviors, then we should choose those (if it falls on the right side of the spectrum) for our-self. However, what this indicates however,is that we cannot be too certain about the true face probabilities unless we have a rather large sample space. Though we saw that the average number of wins varied, there was a most likely average wins which corresponded to our null. Thus, we simply needed enough data to confidently say "Yes, this is correct." With that in mind, I was still not satisfied. More work most be done to convince myself as to whether I should take the bet!



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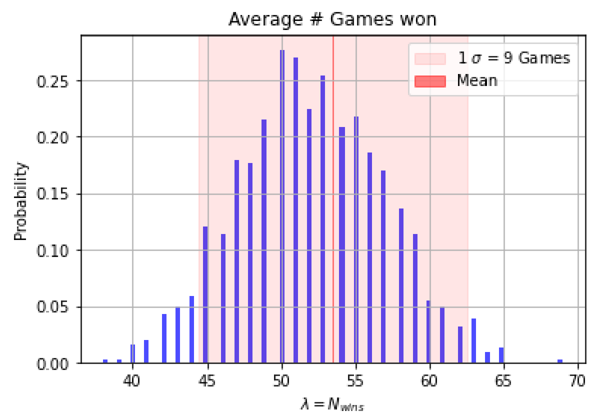

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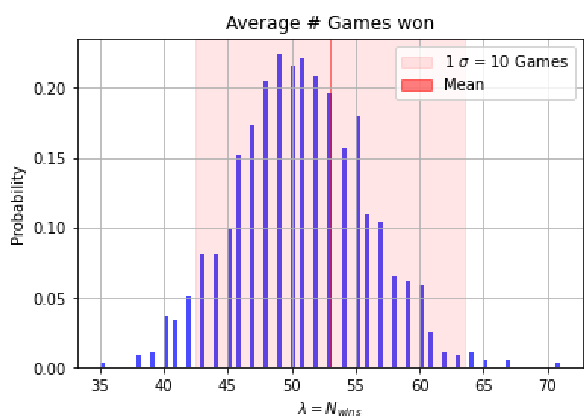

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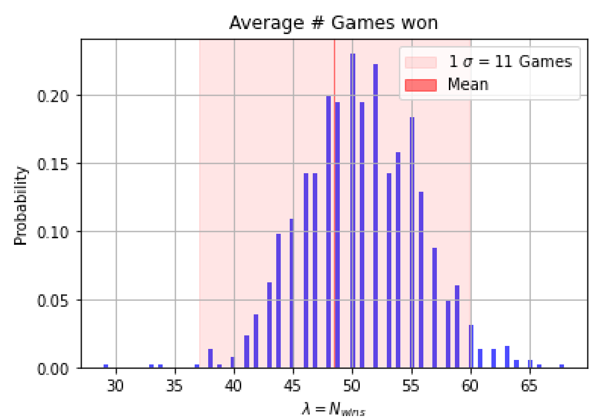

(d)

Figure 3: Sample of four possible game distributions. Each with corresponding means given by μ = 51,53,53.49 games respectively

References