PHSX815_Project3:
I do not know if these dice are loaded.

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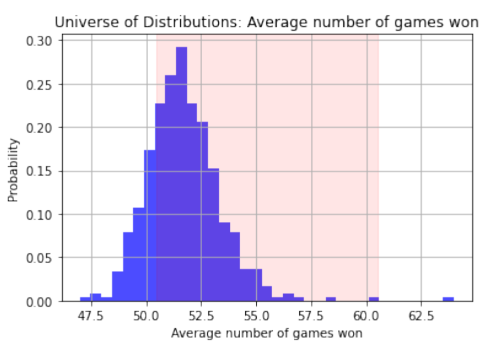
1 Introduction

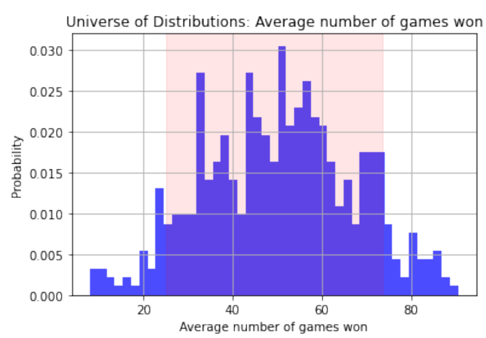
A few days ago, while enjoying a delicious lunch at Leeway Franks, a mysterious individual approached and asked if I wanted to play a game. It was a game of dice. Now, this was eerily familiar. The topic was interesting, but the more I inquire the less sure I became about whether I should take the bet. He loaned me the dice to be used, and I went about simulating. This paper is an attempt to answer some of those question. It is organized as followed: Sec. 2 explains the rule of the game as well as the hypotheses. A description of the computer simulation developed to simulate these possibilities is provided in Sec. 3, with an analysis of the outputs included in Sec. 4. Finally, conclusions are presented in Sec. 5.

2 Hypotheses: Are these the golden die?

Our experiment is simple: we will roll some die and the player with the highest sum wins. However, our objective is to seek the biased die as that will increase the chances of us winning more games. How do we go about doing this? We will construct the likelihood function (LH) for the wins for a given set of dice. We seek to find the dice whose distribution is skewed i.e there is a higher probability of winning a greater number of games. Can we do it within reasonable number of games?



(a) Equal Probabilities

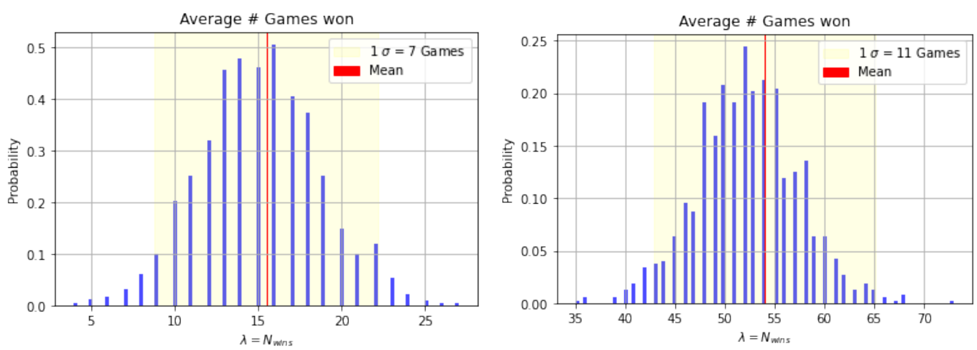


(b) Un-equal Probabilities

Figure 1: Simulations of the most likely average wins per game of dice. To obtain data, we simulated 500 universes where 100 toss were performed per each experiment (1000). It's interesting to note that the distributions are not exactly symmetric about about N = 50. Instead we find naive means of N = μ = 55.5 and 50.5 wins with standard deviations given by σ = 5 and 24.3 with maximum games 64 and 90.5 which is about 2 σ.

3 Code and Experimental Simulation

We simulate our game as followed: we generate from a binomial distribution two numbers which can be at any of six positions. The locations with the numbers then correspond to the "face" of the dice we rolled. For example, if we notate [0,0,1,1,0,0] as our rolled dice, we would have "rolled" a 3 and 4; thus, our sum would be a 7. Each face has a certain probability: a fair dice would have each side with one-sixth probability each. If any of the faces were being preferred, then this would be reflected in the LH being shifted, stretched or shrink or some combination. However, our face probabilities will be sampled from some normal distribution whose location(mean) we provide. Annoyingly, we have no sense of what is the "real" distribution; for each roll the corresponding output distribution is "all over the place". By this I mean: If we repeat N number of experiments(games) each with n measurements (rolls), do this K times then, construct a likelihood of our observable (average number of rolls won per game) then our likelihood will be highly unstable in localizing around the true optimum observable value - if the initial probabilities vary greatly.



(a) Un-Equal Probabilities

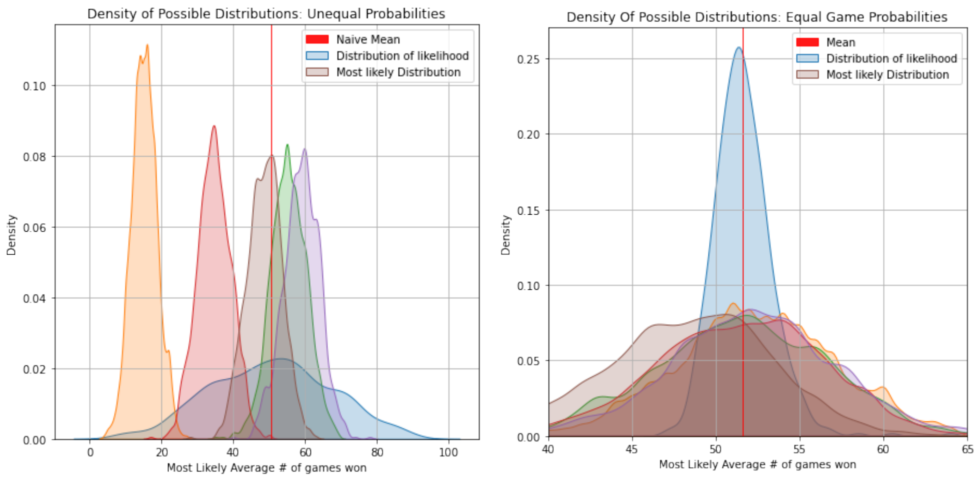
(b) Equal Probabilities

Figure 2: Sampled likelihood functions from our universe of games.

4 Analysis

Let's look at some simulated data. Figure 1 is the simulated average wins for our game of dice over a number of trials (universes): meaning, we ran multiple games, over many experiments. We did two cases: we allowed the house and player to have equal probabilities (figure 1.a) and the unequal (figure 1.b). For the equal case, the most likely number of games won is strongly localized around 51.5 games (the highest probability) with extreme cases to the right; however, it is not the mean of our distribution! In the unequal case, we find something interesting: The most likely average number of games is given by 50.5 games, which though not 50 percent, is about half of the games. Moreover, we have a large spread in the possible most likely wins for our game. Though, as with the equal case, there is a higher tendency for games with means close to 50, to appear.

To gain further clarity on what is happening in our simulation, we plot a few representative distributions from our two different simulations. In figure 2, We can see the manifestation of the face probabilities in the locations of the means: the naive means being 17 and 54 games respectively. We also perform two-sample Kolmogorov–Smirnov test on the different possible representative distributions as can be discern in Figure 4. We assume a significance of 5 and in all cases are able to differentiate between the different possible distributions, the strongest being for those distributions whose μ correspond to the highest probability in our universe of distribution! (Equal and unequal) Remember, we



(a) Un-Equal Probabilities

(b) Equal Probabilities

Figure 3: Density plots of our game. Note for the the equal case, the distributions are strongly localized around a preferential means whereas for the un-equal there is a greater spread in possible scenarios.

are probabilistic-ally sampling face probabilities. Thus our game distribution will shift and morph. We see this in figure 3, where we have density plots of the universal distribution vs a few representative distributions from the sets. From figure 4, it thus stands to reason we would need a 19 percent chance of being "wrong" in rejecting the correct null ("the average number of games won is about rolls/2", "this dice is highly skewed towards losing" etc) to gauge whether a particular value is the "correct" one in positing alternative distributions. As such, if one were to simply rolls dice a priori without any sense of the face probabilities, then chances are, unless we have a large enough sample space, our distribution seems to be indicating that our chances of winning would be about 50/50.

5 Conclusion

Now, none of this changes the fact that if we employ the dice roller-3000 and were able to construct a probability distribution of the dice, if the dice demonstrate extreme behaviors, then we should choose those (if it falls on the right side of the spectrum) for our-self and give the other (on the opposite spectrum) to our adversary. However, what these results indicate,is that we cannot be too certain about the true face probabilities unless we have a rather large sample space. Though we saw that the average number of wins varied, there was a most likely average wins which corresponded to our null. Thus, we simply needed enough data to confidently say "Yes, this is correct." With that in mind, I am still not satisfied. More work most be done to convince myself as to whether I should still take the bet! Now, on to the next question: what is the probability of you being a dice gambler, given you eat at Leeway Franks.

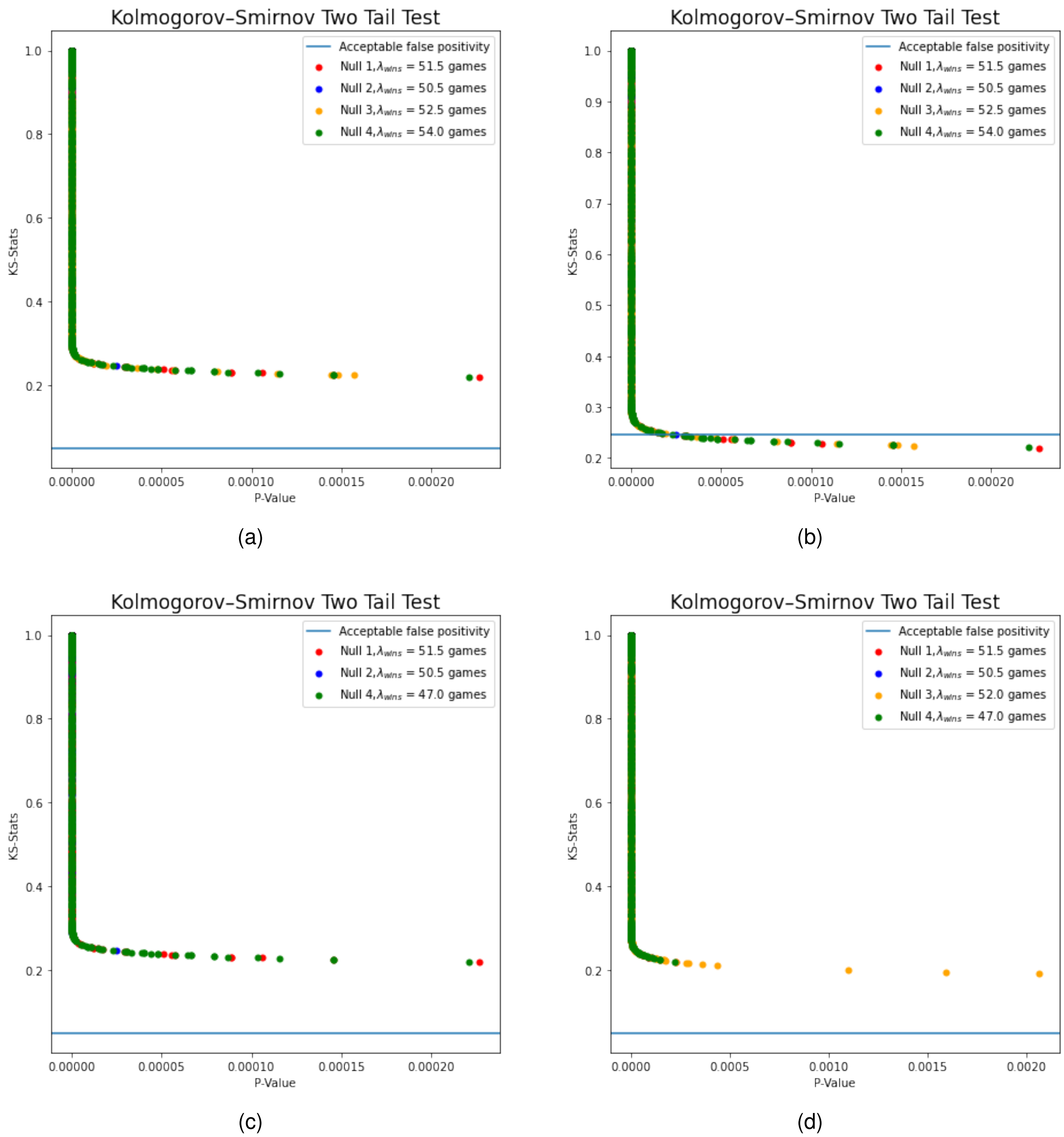


Figure 4: KS Test on our two different simulation types assuming different representative nulls. a) K.S test on the equal probability simulation. The horizontal line represents a 5 percent alpha value. b) Same as a) except the horizontal line represents the minimum KS values of our distribution of games. c) and d) are the same for as a) and b) with un-equal probabilities.

References

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