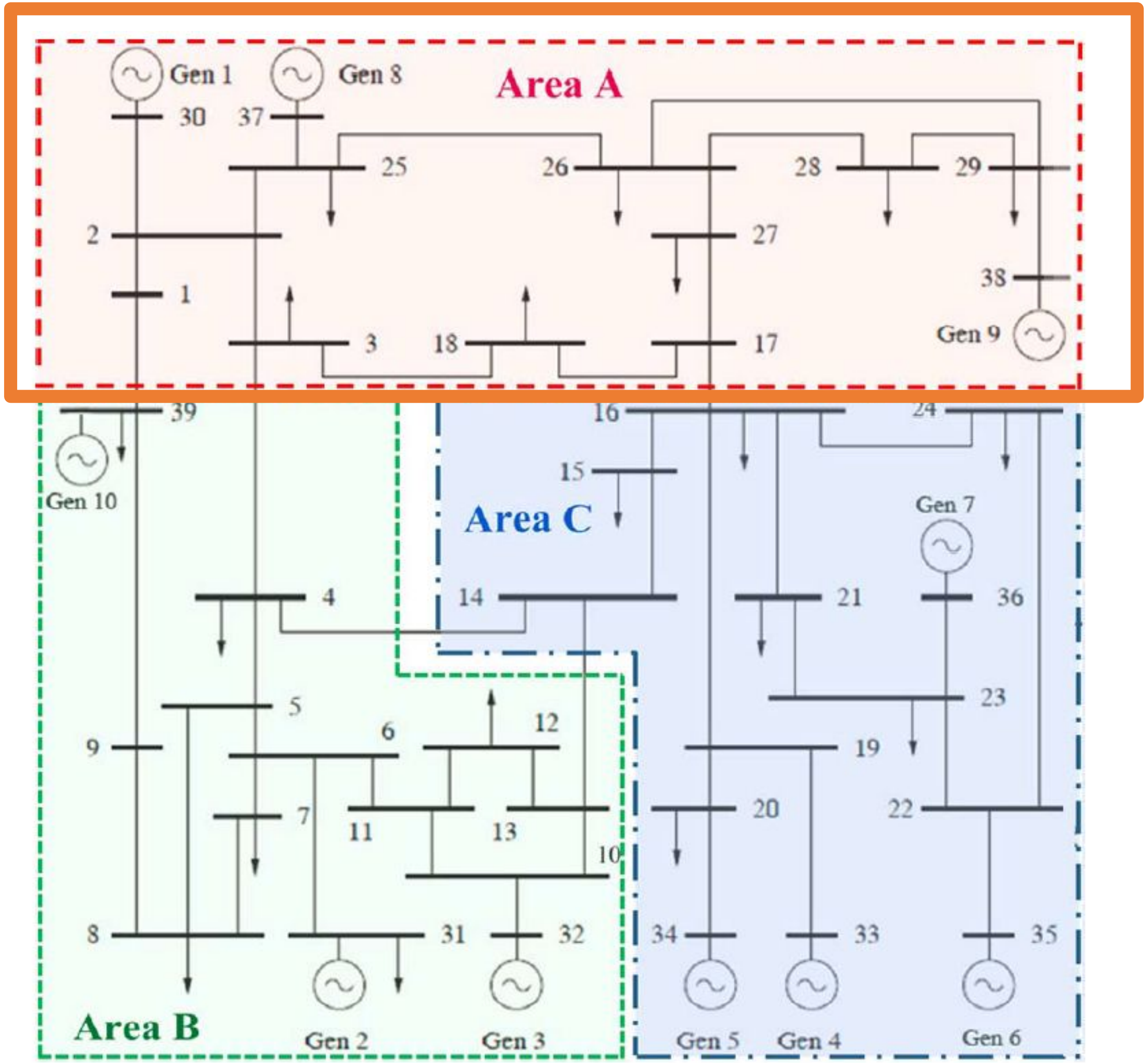
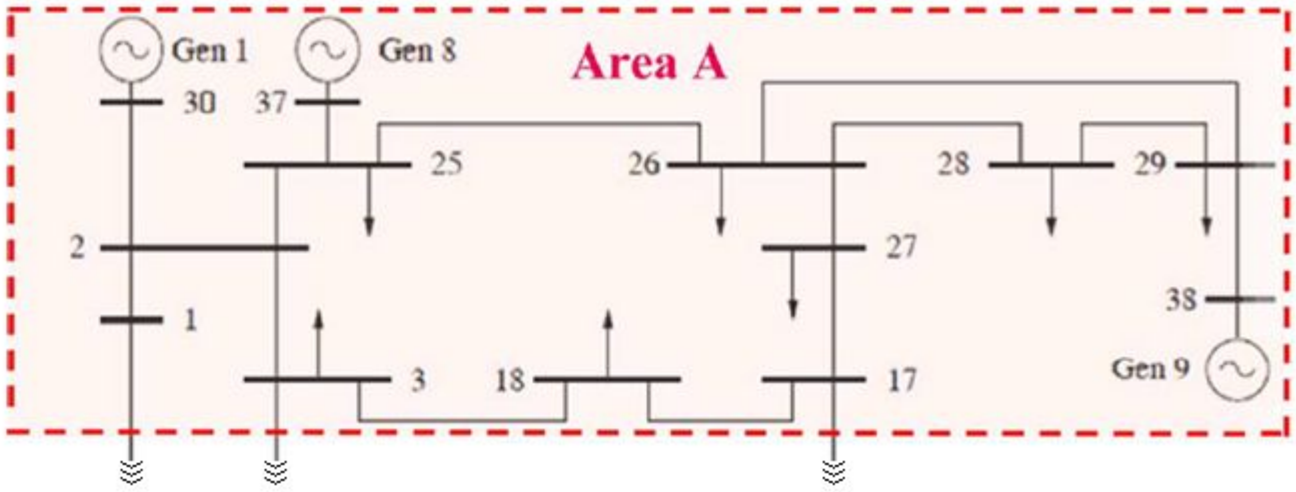
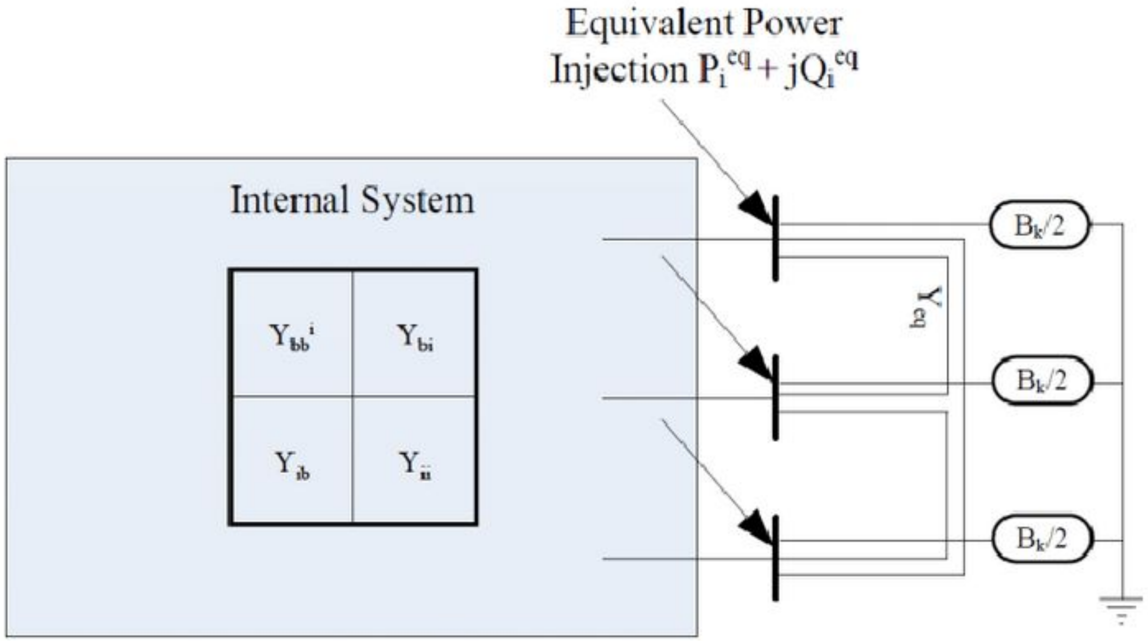


1. New England 39-bus system with 3 areas.

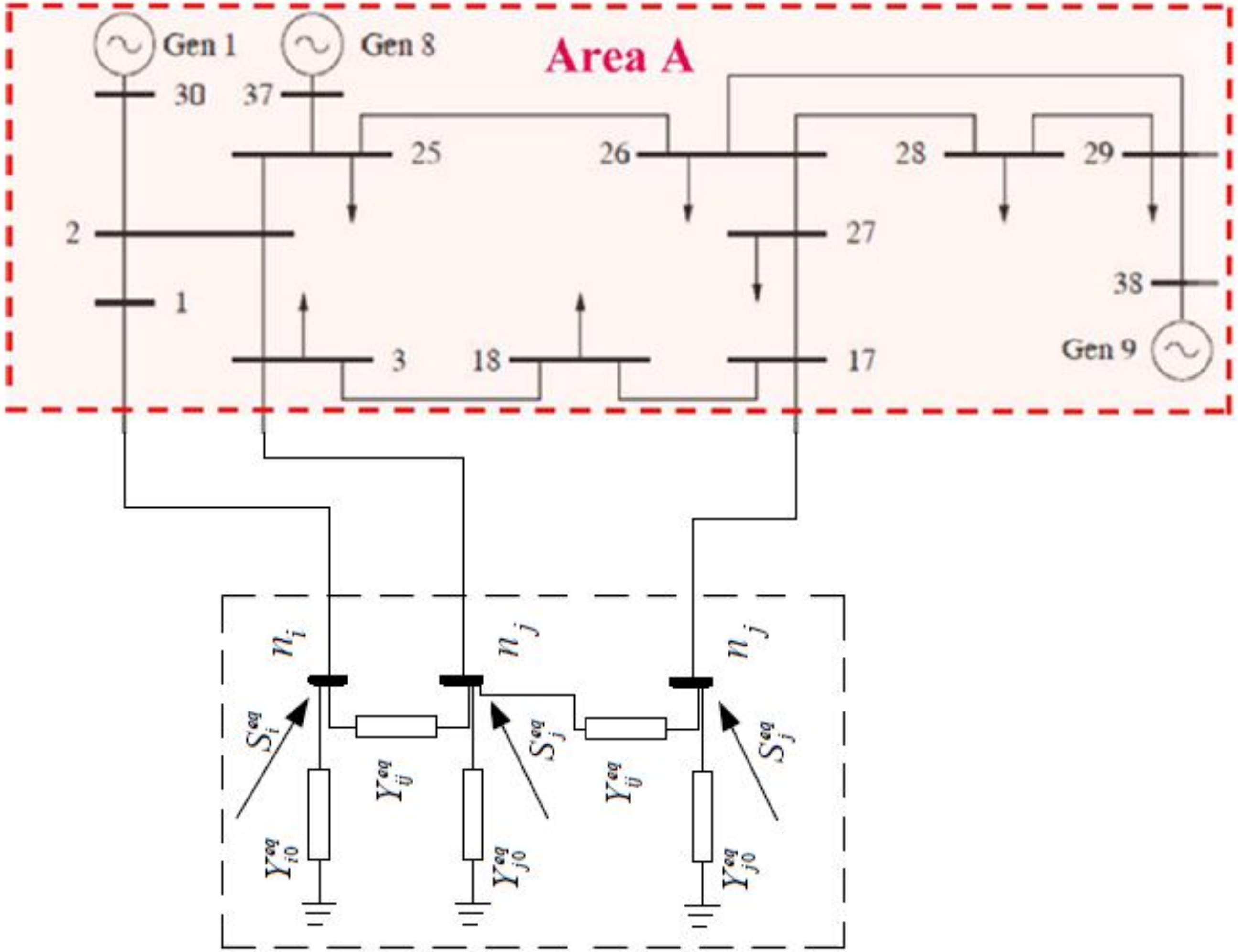
Análisis para el área A



Análisis para el área A

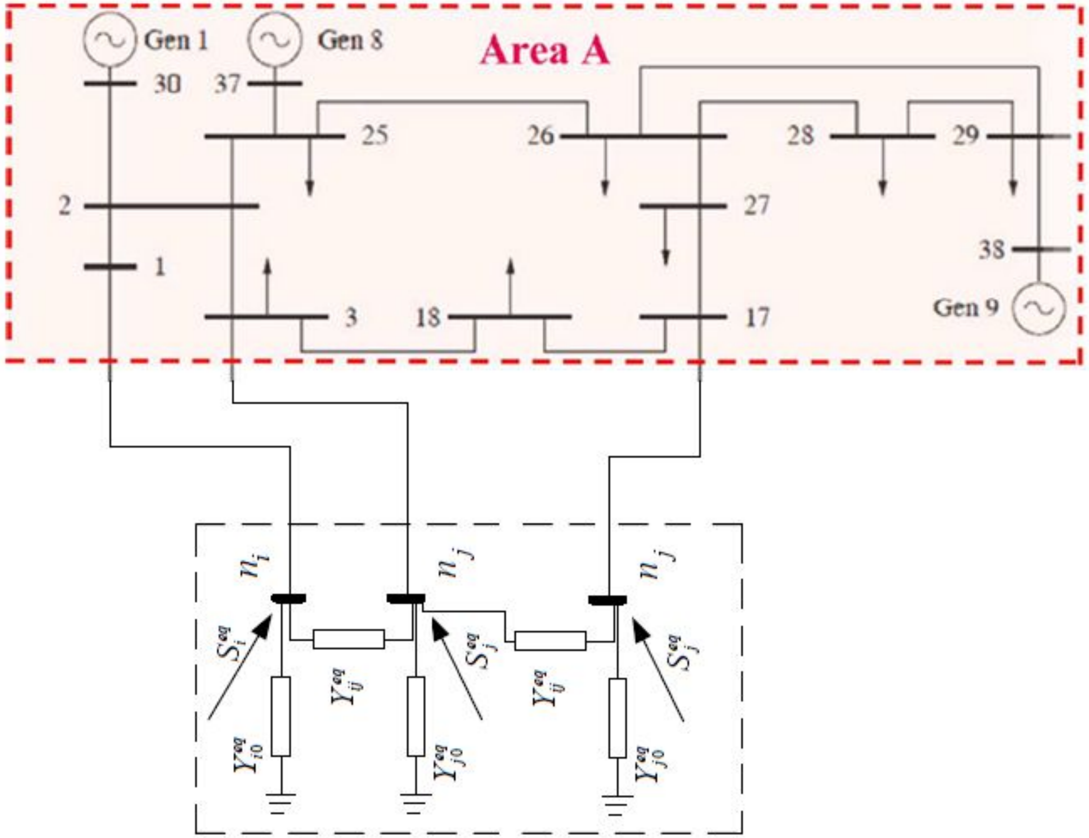


Ward Equivalent



Análisis para el área A

Análisis para el área A



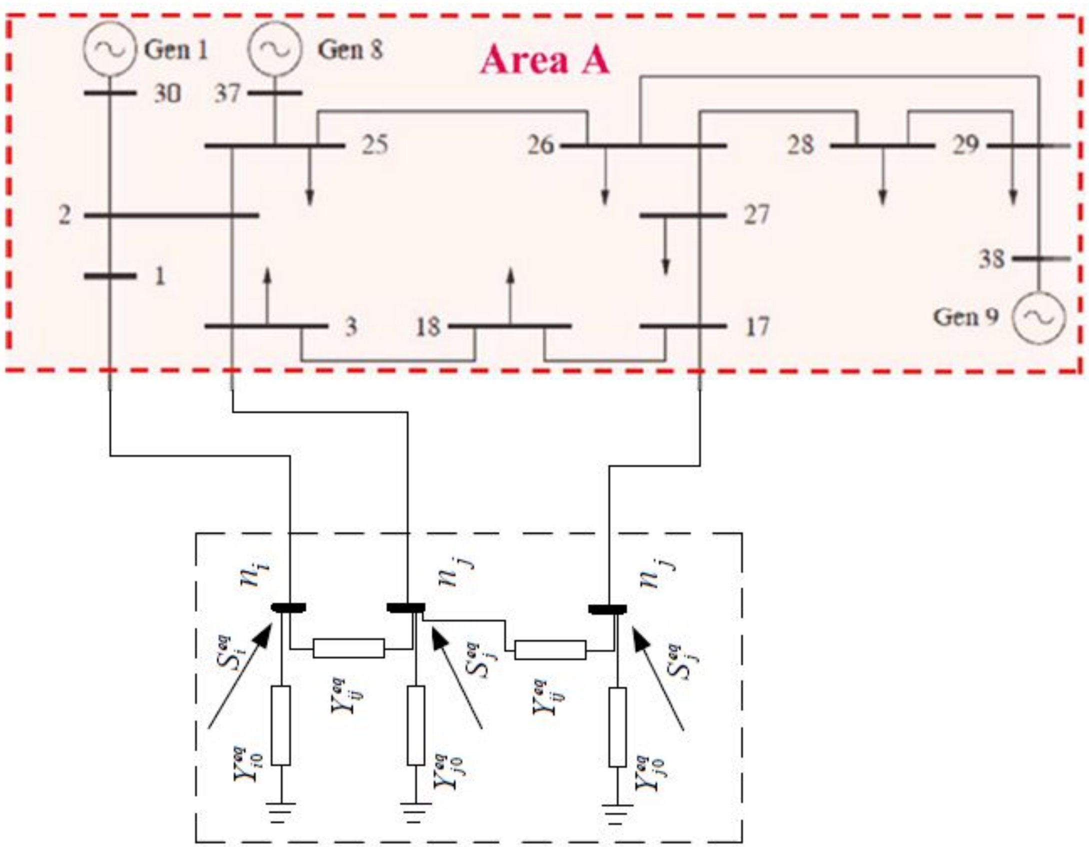
$$\text{Min}\ (\beta_1 \text{TVD} + \beta_2 \text{TQD} + \beta_3 \text{TQG})$$

$$\text{TVD} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_p} \left(v_{it} - V_{ref,it}\right)^2$$

$$\text{TQD} = \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{S}} \left(n_{kt}^{\mathcal{S}} - n_{k(t-1)}^{\mathcal{S}}\right)^2$$

$$\text{TQG} = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (q_{gt}^{\mathcal{G}})^2$$

Análisis para el área A

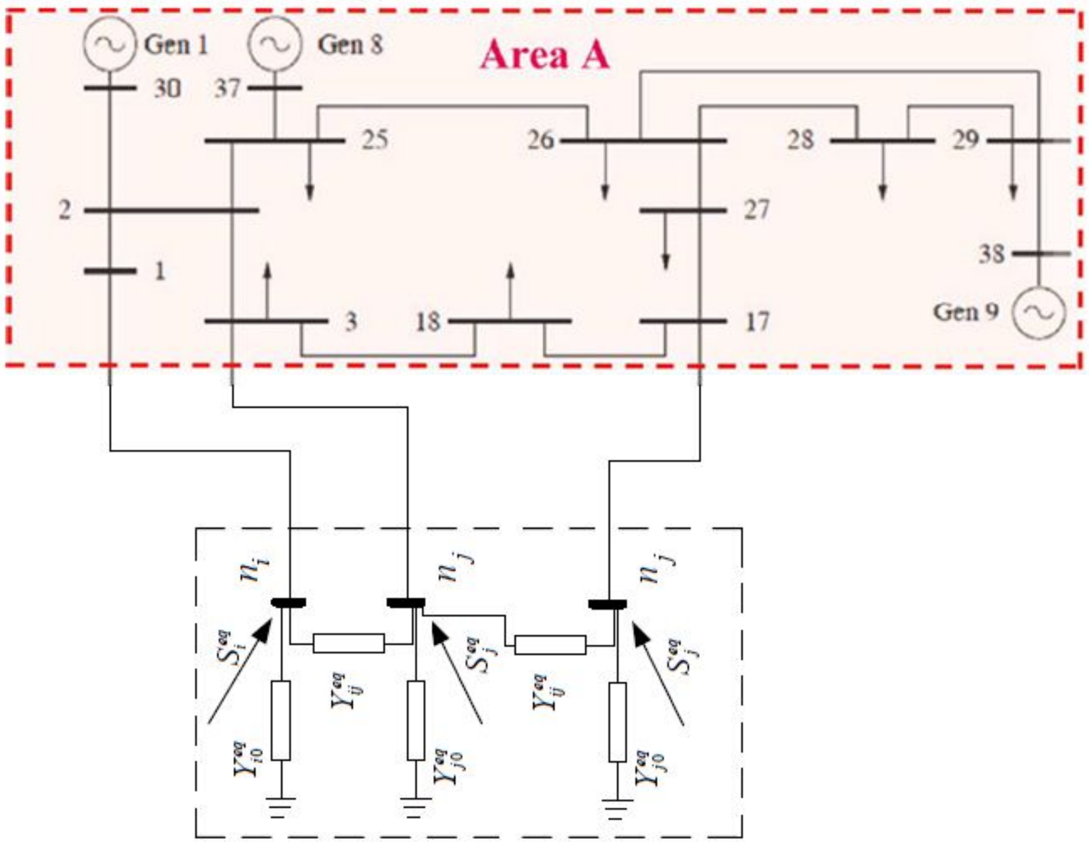


$$\text{Min}\ (\beta_1 \text{TVD} + \beta_2 \text{TQD} + \beta_3 \text{TQG})$$

$$\text{TVD} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}_p} \left(v_{it} - V_{ref,it}\right)^2$$

$$TVD = \sum_{t \epsilon \tau} \sum_{a \epsilon A} \sum_{i \epsilon \mathcal{N}_{pA}} \left(\mathcal{V}_{iat - V_{ref,iat}}\right)^2$$

Análisis para el área A

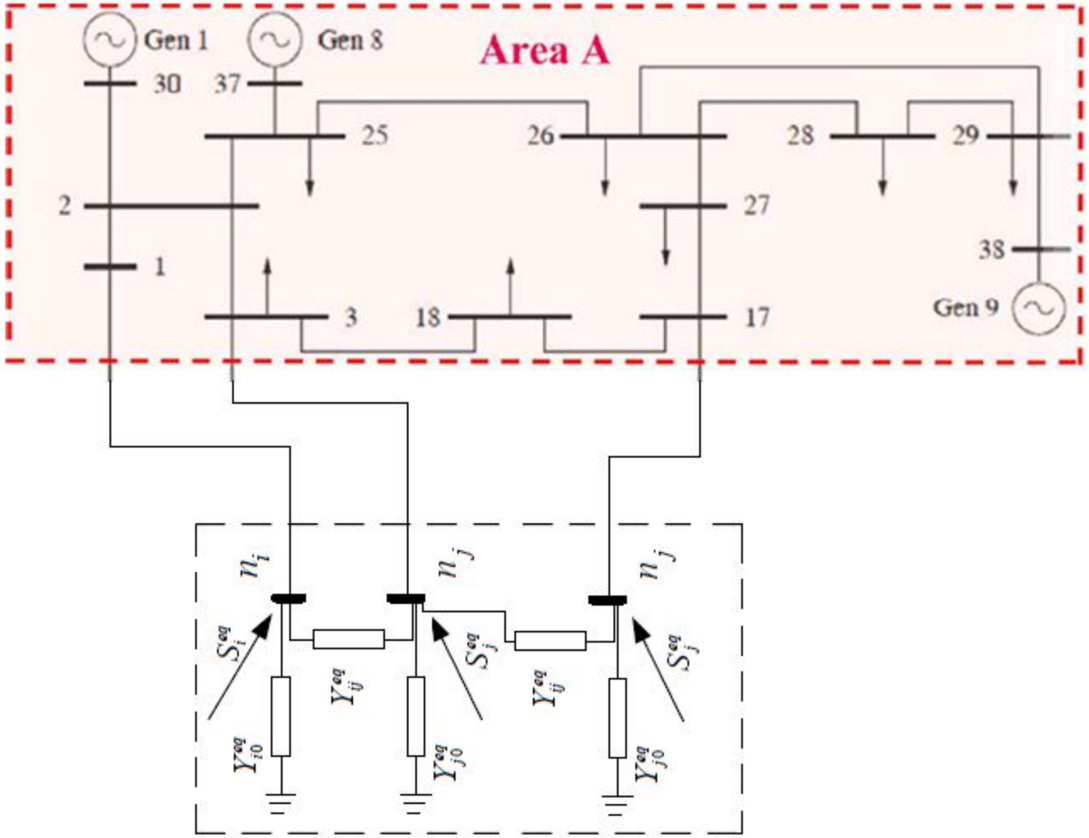


$$\text{Min}\ (\beta_1 \text{TVD} + \beta_2 \text{TQD} + \beta_3 \text{TQG})$$

$$\text{TQD} = \sum_{t \in \mathcal{T}} \sum_{k \in \mathcal{S}} \left(n_{kt}^{\mathcal{S}} - n_{k(t-1)}^{\mathcal{S}} \right)^2$$

$$TVD = \sum_{t \epsilon \tau} \sum_{a \epsilon A} \sum_{k \epsilon \mathcal{S} A} \left(n_{kat}^{\mathcal{S}} - n_{ka(t-1)}^{\mathcal{S}} \right)^2$$

Análisis para el área A

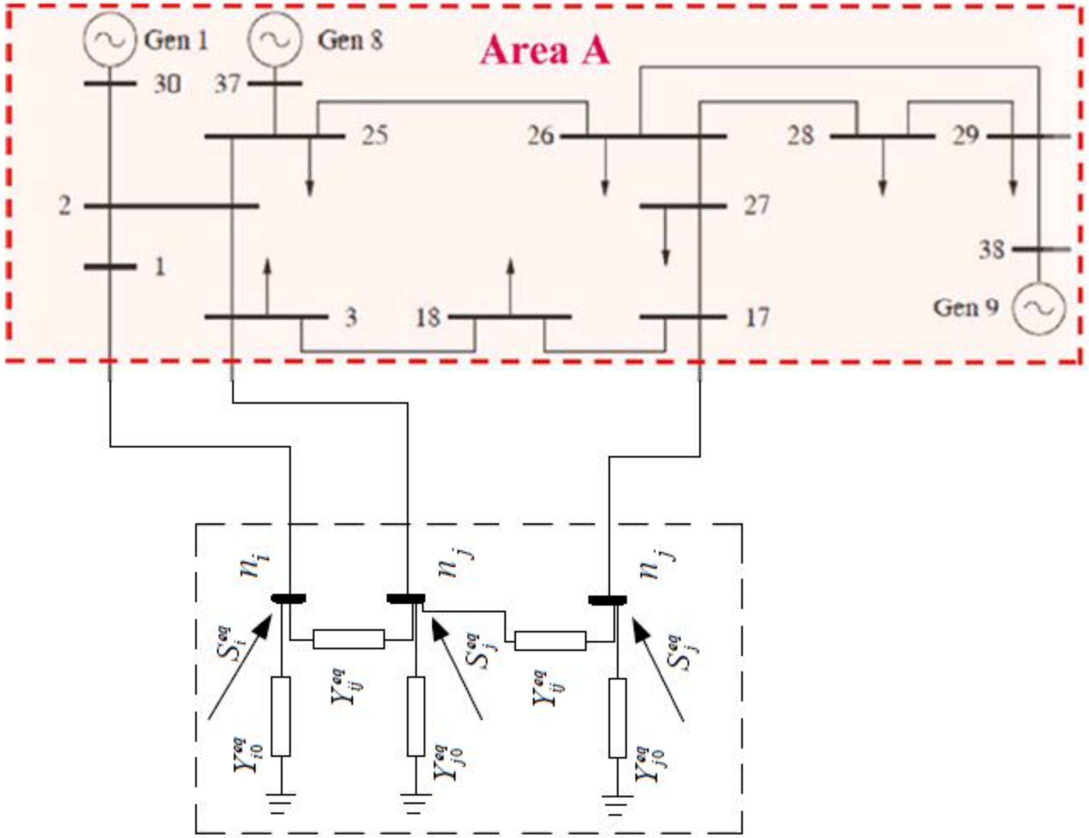


$$\mathrm{Min}\,(\beta_1 \mathrm{TVD} + \beta_2 \mathrm{TQD} + \beta_3 \mathrm{TQG})$$

$$\mathrm{TQG} = \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} (q_{gt}^{\mathcal{G}})^2$$

$$TVD = \sum_{t \epsilon \tau} \sum_{a \epsilon A} \sum_{g \epsilon \mathcal{G}A} \left(q_{gat}^{\mathcal{G}}\right)^2$$

Análisis para el área A

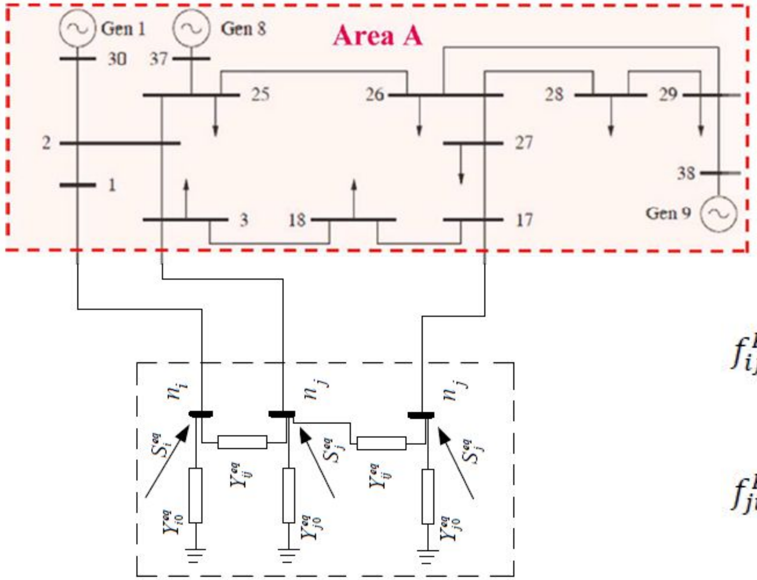


$$\text{Min}\ (\beta_1 \text{TVD} + \beta_2 \text{TQD} + \beta_3 \text{TQG})$$

$$TVD = \sum_{t\epsilon\tau} \sum_{a\epsilon A} \sum_{i\epsilon \mathcal{N}_{pA}} \left(\mathcal{V}_{iat - V_{ref,iat}}\right)^2$$

$$TVD = \sum_{t\epsilon\tau} \sum_{a\epsilon A} \sum_{k\epsilon \mathcal{S}A} \left(n^{\mathcal{S}}_{kat} - n^{\mathcal{S}}_{ka(t-1)}\right)^2$$

$$TVD = \sum_{t\epsilon\tau} \sum_{a\epsilon A} \sum_{g\epsilon \mathcal{G}A} \left(q^{\mathcal{G}}_{gat}\right)^2$$



Análisis para el área A - Restricciones

$$f^{P}_{ijct} = \frac{1}{\alpha^2_{ijct}} G^{\mathcal{L}}_{ijc} v^2_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{it} - \theta_{jt} - \phi_{ijc}) + B^{\mathcal{L}}_{ijc} \sin(\theta_{it} - \theta_{jt} - \phi_{ijc}) \right) \quad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$f^{P}_{jict} = G^{\mathcal{L}}_{ijc} v^2_{jt} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{jt} - \theta_{it} + \phi_{ijc}) + B^{\mathcal{L}}_{ijc} \sin(\theta_{jt} - \theta_{it} + \phi_{ijc}) \right) \quad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

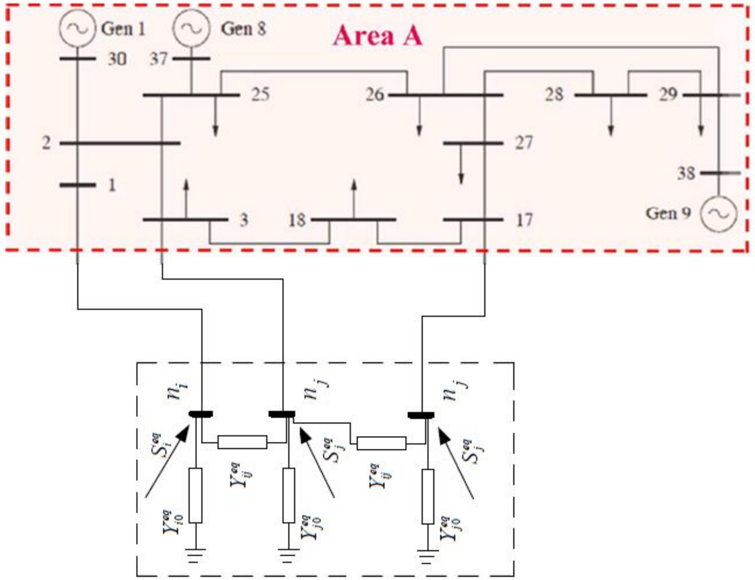
$$f^{Q}_{ijct} = -\frac{1}{\alpha^2_{ijct}} \left(B^{\mathcal{L}}_{ijc} + \frac{B^{C}_{ijc}}{2} \right) v^2_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{it} - \theta_{jt} - \phi_{ijc}) - B^{\mathcal{L}}_{ijc} \sin(\theta_{it} - \theta_{jt} - \phi_{ijc}) \right) \quad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$f^{Q}_{jict} = -\left(B^{\mathcal{L}}_{ijc} + \frac{B^{C}_{ijc}}{2} \right) v^2_{jt} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{jt} - \theta_{it} + \phi_{ijc}) - B^{\mathcal{L}}_{ijc} \sin(\theta_{jt} - \theta_{it} + \phi_{ijc}) \right) \quad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$\sum_{g \in \mathcal{G}_{i(i \neq slack)}} P^{\mathcal{G}}_{gt} + \sum_{g \in \mathcal{G}_{slack}} p^{\mathcal{G}}_{gt} - \sum_{(jc):ijc \in \mathcal{L}} f^{P}_{ijct} - \sum_{(jc):jic \in \mathcal{L}} f^{P}_{jict} - D^{P}_{it} - {v_{it}}^2 G^{\mathcal{E}}_{i} = 0 \quad , \forall i \in \mathcal{N}; t \in \mathcal{T}$$

$$\sum_{g \in \mathcal{G}_i} q^{\mathcal{G}}_{gt} + {v_{it}}^2 \sum_{k \in \mathcal{S}_i} B^{\mathcal{S}}_{k} n^{\mathcal{S}}_{kt} - \sum_{(jc):ijc \in \mathcal{L}} f^{Q}_{ijct} - \sum_{(jc):jic \in \mathcal{L}} f^{Q}_{jict} - D^{Q}_{it} + {v_{it}}^2 B^{\mathcal{E}}_{i} = 0 \quad , \forall i \in \mathcal{N}; t \in \mathcal{T}$$

Análisis para el área A - Restricciones

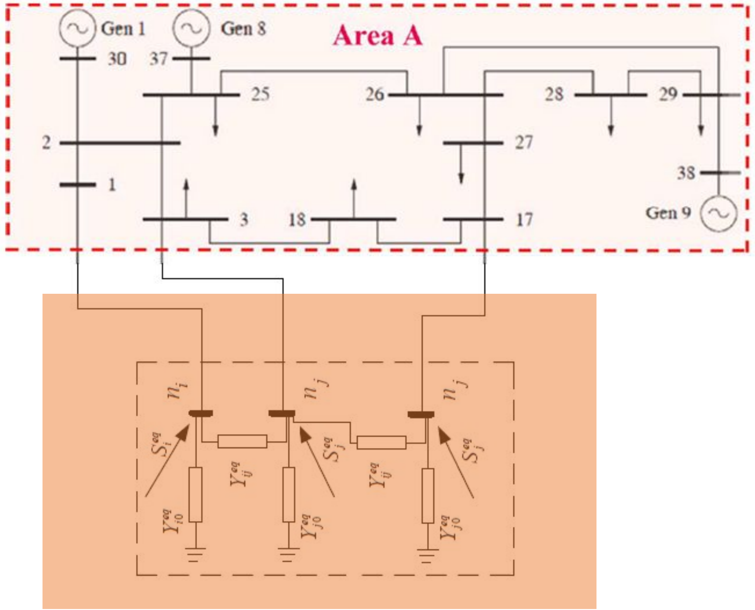


$$f^{P}_{ijct} = \frac{1}{\alpha^{2}_{ijct}} G^{\mathcal{L}}_{ijc} v^{2}_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt}\left(G^{\mathcal{L}}_{ijc}\cos(\theta_{it}-\theta_{jt}-\phi_{ijc}) + B^{\mathcal{L}}_{ijc}\sin(\theta_{it}-\theta_{jt}-\phi_{ijc})\right) \quad ,\forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$f^{Q}_{ijct} = -\frac{1}{\alpha^{2}_{ijct}}\left(B^{\mathcal{L}}_{ijc} + \frac{B^{C}_{ijc}}{2}\right) v^{2}_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt}\left(G^{\mathcal{L}}_{ijc}\cos(\theta_{it}-\theta_{jt}-\phi_{ijc}) - B^{\mathcal{L}}_{ijc}\sin(\theta_{it}-\theta_{jt}-\phi_{ijc})\right) \quad ,\forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$\sum_{g\in\mathcal{G}_{i(i\neq slack)}} P^{\mathcal{G}}_{gt} + \sum_{g\in\mathcal{G}_{slack}} p^{\mathcal{G}}_{gt} - \sum_{(jc):ijc\in\mathcal{L}} f^{P}_{ijct} - \sum_{(jc):jic\in\mathcal{L}} f^{P}_{jict} - D^{P}_{it} - {v_{it}}^{2} G^{\mathcal{E}}_{i} = 0 \quad ,\forall i \in \mathcal{N}; t \in \mathcal{T}$$

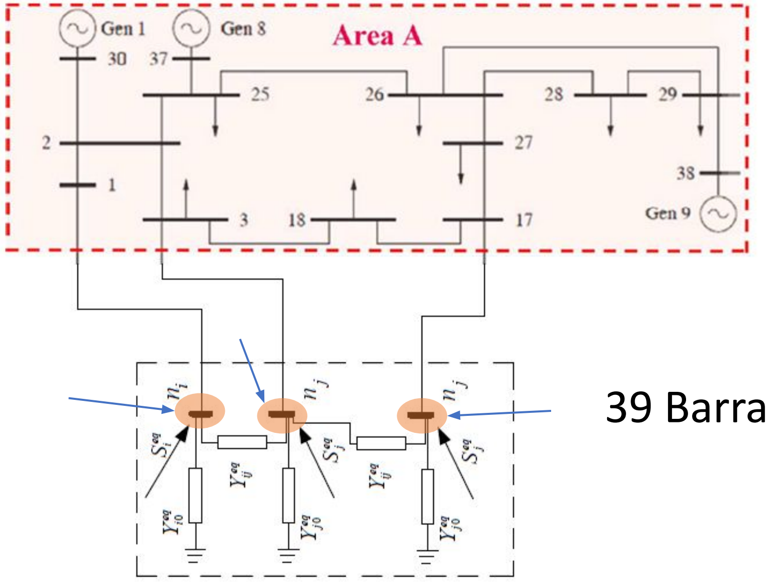
$$\sum_{g\in\mathcal{G}_{i}} q^{\mathcal{G}}_{gt} + {v_{it}}^{2} \sum_{k\in\mathcal{S}_{i}} B^{\mathcal{S}}_{k} n^{\mathcal{S}}_{kt} - \sum_{(jc):ijc\in\mathcal{L}} f^{Q}_{ijct} - \sum_{(jc):jic\in\mathcal{L}} f^{Q}_{jict} - D^{Q}_{it} + {v_{it}}^{2} B^{\mathcal{E}}_{i} = 0 \quad ,\forall i \in \mathcal{N}; t \in \mathcal{T}$$



Análisis para el área A - Restricciones

$$f_{ijct}^{P} = \frac{1}{\alpha_{ijct}^{2}} G_{ijc}^{\mathcal{L}} v_{it}^{2} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G_{ijc}^{\mathcal{L}} \cos(\theta_{it} - \theta_{jt} - \phi_{ijc}) + B_{ijc}^{\mathcal{L}} \sin(\theta_{it} - \theta_{jt} - \phi_{ijc}) \right) ,$$

$$f_{ijcat}^{P} = \frac{1}{\alpha_{ijcat}^{2}} G_{ijca}^{\mathcal{L}} \mathcal{V}_{iat}^{2} - \frac{1}{\alpha_{ijcat}} \mathcal{V}_{iat} \mathcal{V}_{jat} (G_{ijca}^{\mathcal{L}} \cos(\theta_{iat} - \theta_{jat} - \phi_{ijca})) + B_{ijca}^{\mathcal{L}} si\,n\left(\theta_{iat} - \theta_{jat} - \phi_{ijca}\right) , \forall a \epsilon A$$



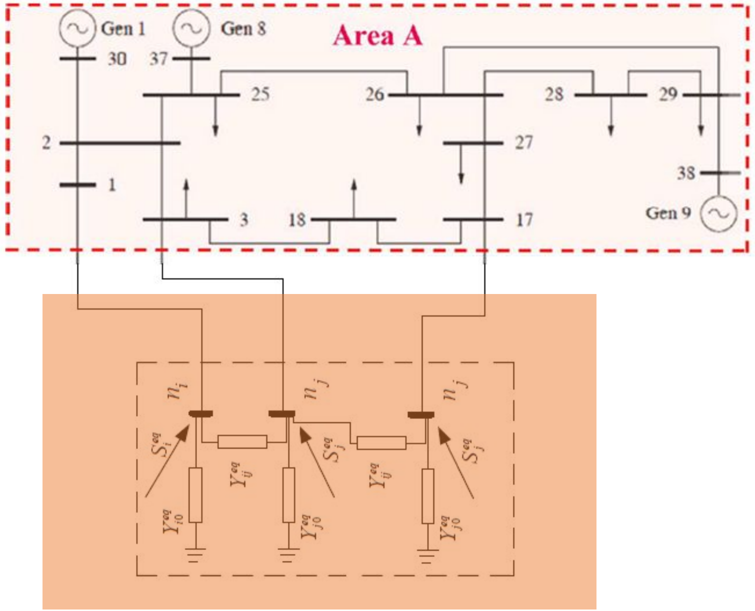
Análisis para el área A - Restricciones

39 Barras -> 16 Barras (Área A)

$$f^{P}_{ijct} = \frac{1}{\alpha^2_{ijct}} G^{\mathcal{L}}_{ijc} v^2_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{it} - \theta_{jt} - \phi_{ijc}) + B^{\mathcal{L}}_{ijc} \sin(\theta_{it} - \theta_{jt} - \phi_{ijc}) \right) \qquad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

$$f^{P}_{ijcat} = \frac{1}{\alpha^2_{ijcat}} G^{\mathcal{L}}_{ijca} \mathcal{V}^2_{iat} - \frac{1}{\alpha_{ijcat}} \mathcal{V}_{iat} \mathcal{V}_{jat} (G^{\mathcal{L}}_{ijca} \cos(\theta_{iat} - \theta_{jat} - \phi_{ijca})) + B^{\mathcal{L}}_{ijca}\, sin\left(\theta_{iat} - \theta_{jat} - \phi_{ijca}\right), \forall a \epsilon A$$

$$f^{Q}_{ijct} = -\frac{1}{\alpha^2_{ijct}} \left(B^{\mathcal{L}}_{ijc} + \frac{B^{C}_{ijc}}{2} \right) v^2_{it} - \frac{1}{\alpha_{ijct}} v_{it} v_{jt} \left(G^{\mathcal{L}}_{ijc} \cos(\theta_{it} - \theta_{jt} - \phi_{ijc}) - B^{\mathcal{L}}_{ijc} \sin(\theta_{it} - \theta_{jt} - \phi_{ijc}) \right) \qquad , \forall ijc \in \mathcal{L}; t \in \mathcal{T}$$

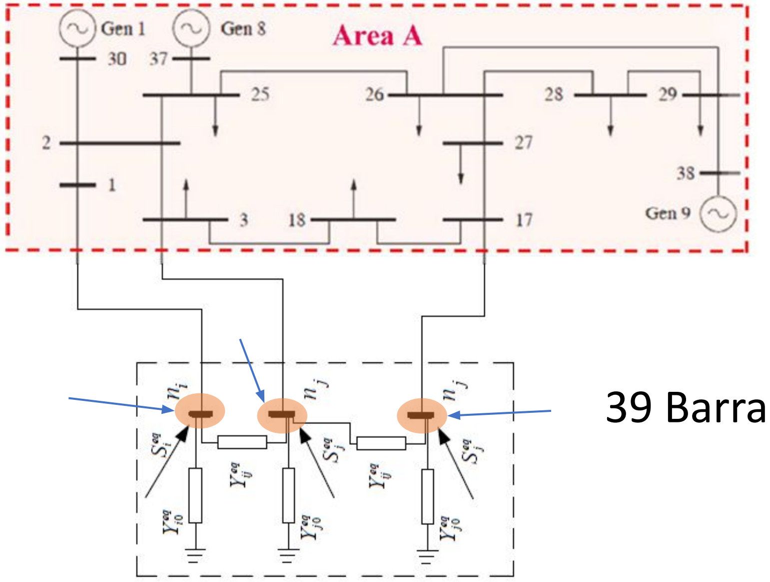


Análisis para el área A - Restricciones

$$\sum_{g\in\mathcal{G}_{i(i\neq slack)}} P_{gt}^{\mathcal{G}} + \sum_{g\in\mathcal{G}_{slack}} p_{gt}^{\mathcal{G}} - \sum_{(jc):ijc\in\mathcal{L}} f_{ijct}^{P} - \sum_{(jc):jic\in\mathcal{L}} f_{jict}^{P} - D_{it}^{P} - v_{it}{}^{2} G_{i}^{\mathcal{E}} = 0 \qquad ,\forall i\in\mathcal{N};t\in\mathcal{T}$$

$$\sum_{g\in\mathcal{G}(i\neq slack):a\epsilon A} P_{gat}^{\mathcal{G}} + \sum_{g\in\mathcal{G}slack:a\epsilon A} P_{gat}^{\mathcal{G}} - \sum_{(jc):ijc\in\mathcal{L}:a\epsilon A} f_{ijcat}^{p} - D_{iat}^{P} - \mathcal{V}_{iat}^{2} G_{ia}^{\mathcal{E}} = 0$$

Análisis para el área A - Restricciones



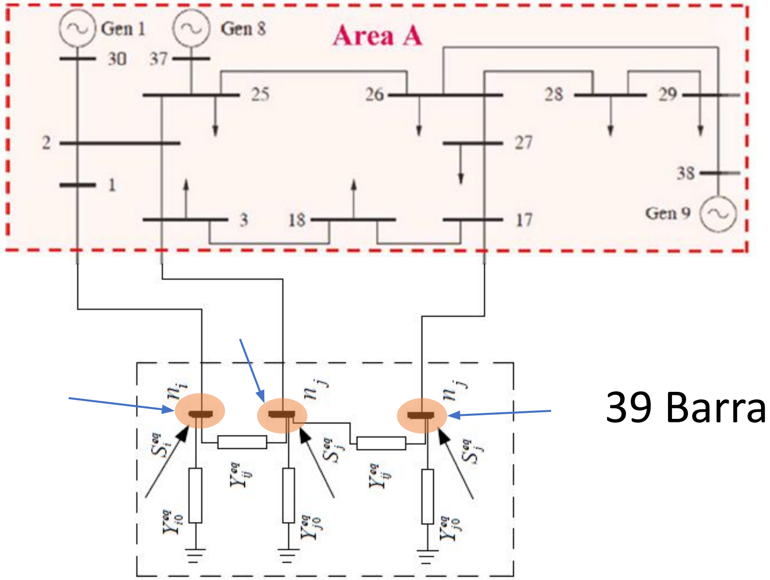
39 Barras -> 16 Barras (Área A)

$$\sum_{g\in\mathcal{G}_{i(i\neq slack)}} P^{\mathcal{G}}_{gt} + \sum_{g\in\mathcal{G}_{slack}} p^{\mathcal{G}}_{gt} - \sum_{(jc):ijc\in\mathcal{L}} f^{P}_{ijct} - \sum_{(jc):jic\in\mathcal{L}} f^{P}_{jict} - D^{P}_{it} - v_{it}{}^{2} G^{\mathcal{E}}_{i} = 0 \qquad ,\forall i\in\mathcal{N}; t\in\mathcal{T}$$

$$\sum_{g\in\mathcal{G}(i\neq slack):a\epsilon A} P^{\mathcal{G}}_{gat} + \sum_{g\in\mathcal{G}slack:a\epsilon A} P^{\mathcal{G}}_{gat} - \sum_{(jc):ijc\in\mathcal{L}:a\epsilon A} f^{p}_{ijcat} - D^{P}_{iat} - \mathcal{V}^{2}_{iat} G^{\mathcal{E}}_{ia} = 0$$

$$\sum_{g\in\mathcal{G}_i} q^{\mathcal{G}}_{gt} + v_{it}{}^{2} \sum_{k\in\mathcal{S}_i} B^{\mathcal{S}}_{k} n^{\mathcal{S}}_{kt} - \sum_{(jc):ijc\in\mathcal{L}} f^{Q}_{ijct} - \sum_{(jc):jic\in\mathcal{L}} f^{Q}_{jict} - D^{Q}_{it} + v_{it}{}^{2} B^{\mathcal{E}}_{i} = 0 \qquad ,\forall i\in\mathcal{N}; t\in\mathcal{T}$$

Análisis para el área A - Ward



39 Barras -> 16 Barras (Área A)

Actualización del equivalente Ward.

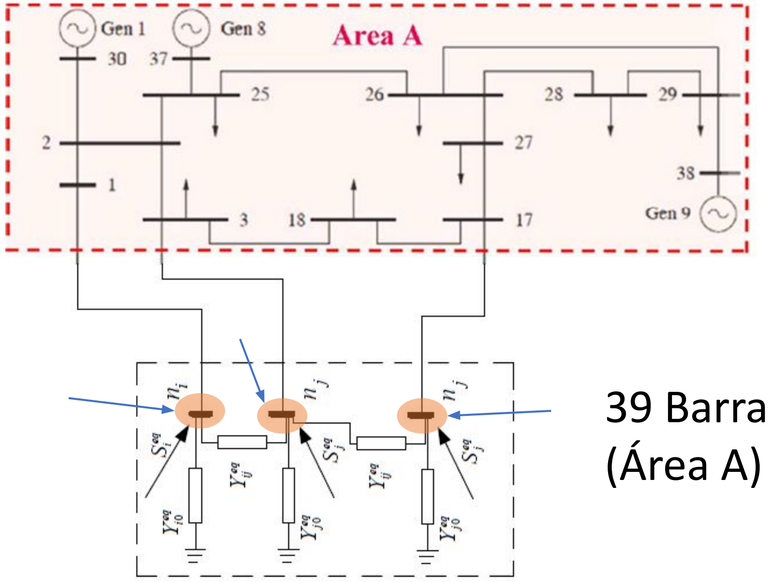
$$\tilde{\boldsymbol{Y}}_{BB}^{(0)} = \boldsymbol{Y}_{BB}^{(0)} - \boldsymbol{Y}_{BE}^{(0)} (\boldsymbol{Y}_{EE}^{(0)})^{-1} \boldsymbol{Y}_{EB}^{(0)}$$

Where $\boldsymbol{Y}_{BB}^{(0)}$ is the initial admittance matrix of boundary buses in any area, $\boldsymbol{Y}_{BE}^{(0)}$ is the initial admittance matrix between boundary buses and external buses, and $\boldsymbol{Y}_{EB}^{(0)} = (\boldsymbol{Y}_{BE}^{(0)})^{T}$, $\boldsymbol{Y}_{EE}^{(0)}$ is the initial admittance matrix of external network, $\tilde{\boldsymbol{Y}}_{BB}^{(0)}$ is the initial equivalent admittance matrix of boundary buses.

At the k-th main circle, assuming $\boldsymbol{Y}_{EE}^{(k)} = (\boldsymbol{Y}_{EE}^{(0)} + \Delta \boldsymbol{Y}_{EE}^{(k)})$, $\boldsymbol{Y}_{BE}^{(k)} = \boldsymbol{Y}_{BE}^{(0)} + \Delta \boldsymbol{Y}_{BE}^{(k)}$, $\boldsymbol{Y}_{EB}^{(k)} = \boldsymbol{Y}_{EB}^{(0)} + \Delta \boldsymbol{Y}_{EB}^{(k)}$, the new equivalent admittance matrix $\tilde{\boldsymbol{Y}}_{BB}^{(k)}$ is calculated as follow:

$$\tilde{\boldsymbol{Y}}_{BB}^{(k)} = \boldsymbol{Y}_{BB}^{(0)} - (\boldsymbol{Y}_{BE}^{(0)} + \Delta \boldsymbol{Y}_{BE}^{(k)})(\boldsymbol{Y}_{EE}^{(0)} + \Delta \boldsymbol{Y}_{EE}^{(k)})^{-1}(\boldsymbol{Y}_{EB}^{(0)} + \Delta \boldsymbol{Y}_{EB}^{(k)}) \quad (4)$$

Análisis para el área A - Flujo de información entre áreas



39 Barras -> 16 Barras (Área A)