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3 (a) $f(x) = -x^3 + 2x^2 + x + 2$

$f'(x) = -3x^2 + 4x + 1$

$f''(x) = -6x + 4 \geq 0$ when $x \leq \frac{2}{3}$

→ convex when $x \leq \frac{2}{3}$

(b) Gradient $\nabla f(x) = \begin{bmatrix} 3x_1^2 - 1 \\ 4x_2 \end{bmatrix}$ Hessian $\nabla^2 f(x) = \begin{bmatrix} 6x_1 & 0 \\ 0 & 4 \end{bmatrix}$

To find eigenvalues: → bad

$\begin{vmatrix} 6x_1 - \lambda & 0 \\ 0 & 4 - \lambda \end{vmatrix} = 0$, $24x_1 - (6x_1 + 4)\lambda + \lambda^2 = 0$

We then use leading principal minors

$|6x_1| > 0$ if $x_1 > 0$, $\begin{vmatrix} 6x_1 & 0 \\ 0 & 4 \end{vmatrix} = 24x_1 > 0$ if $x_1 > 0$

⇒ convex when $x_1 > 0$

(c) Gradient $\nabla f(x) = \begin{bmatrix} 2x_1x_3 \\ 2x_2 + 2x_3 \\ x_1^2 + 2x_2 \end{bmatrix}$ Hessian $\nabla^2 f(x) = \begin{bmatrix} 2x_3 & 0 & 2x_1 \\ 0 & 2 & 2 \\ 2x_1 & 2 & 0 \end{bmatrix}$

(3×3) ~~$-4x_1^2 - 8x_3 > 0$~~ (1×1) 且 $2x_3 \geq 0$, $2 \geq 0$, $0 \geq 0$ ，if $x_3 \geq 0$

(2×2) $4x_3 \geq 0$ if $x_3 \geq 0$, $-4x_1^2 \geq 0$ when $x_1 = 0$, $-4 \geq 0$ 不存在

⇒ use first order: $x_3 > 0$ 且 $x_1^2 + x_3 < 0$

⇒ 無解, no convex area

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