Assignment 3

CS6480: Causal Inference and Learning
IIT-Hyderabad
Feb-Apr 2021

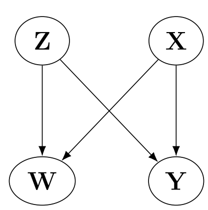
Max Marks: 40
Due: 27th Apr 2021 11:59 pm

Instructions

- Please use Piazza to upload your submission by the deadline mentioned above. Your submission should comprise of a single file, named <Your_Roll_No>_Assign3, with all your solutions.
- For late submissions, 10% is deducted for each day (including weekend) late after an assignment is due. Note that each student begins the course with 5 grace days for late submission of assignments, of which upto 3 grace days can be used for a single assignment. Late submissions will automatically use your grace days balance, if you have any left. You can see your balance on the Marks and Grace Days document, soon to be shared under the course Google drive.
- Please read the department plagiarism policy. Do not engage in any form of cheating - strict penalties will be imposed for both givers and takers. Please talk to instructor or TA if you have concerns.

Questions

1. **(10 points)** Define Collider-bias, M-bias and Selection-bias and illustrate using causal graphs and adjustment sets (6 points). What kind of bias will be observed if we condition on W while estimating the causal effect of X on Y in the following graph (1 point)? How do we estimate the causal effect of X on Y for a given value of W, i.e. how do you find $p(Y = y|do(X = x), W = w)$? Your expression should contain only statistical quantities but not any *do* notations. (3 points)



2. **(5 Points)** The table below lists the results of a hypothetical experiment on 200 people. W, Y are treatment and response variables. X is a covariate. Each row identifies a category of people with the same values of $X, W, Y_{do(W=0)}, Y_{do(W=1)}$, and Y^{obs}. For example, there are 30 people in category 1 and each of these people has $X = 0, W = 0, Y_{do(W=0)} = 4, Y_{do(W=1)} = 6$ and because they are in the control group we have $Y^{obs} = 4$.

Category	Number of People	X	W	$Y_{do(W=0)}$	$Y_{do(W=1)}$	Y^{obs}
1	30	0	0	4	6	4
2	30	0	1	4	6	6
3	10	1	0	4	6	4
4	30	1	1	4	6	6
5	20	0	0	10	12	10
6	20	0	1	10	12	12
7	15	1	0	10	12	10
8	45	1	1	10	12	12

(a) Do you believe that treatment assignment is unconfounded given X for these data? Justify your answer. (2 points)

(b) Assuming the table above reflects the population of interest, what is the propensity score for $X = 0$ and $X = 1$? (2 points)

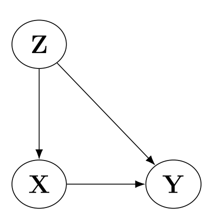
(c) The naive estimate of the Average Treatment Effect compares the observed mean value of Y for those assigned to treatment ($W = 1$) and those assigned to control ($W = 0$). In this case the naive estimate is $9.12 - 6.80 = 2.32$. Carry out a propensity score analysis to estimate the Average Treatment Effect and show that it yields the correct estimate. (2 points)

3. **(6 points)** Define and give examples for the following assumptions of causal inference used in the study of instrumental variables.

- Exclusion Restriction (2 points)
- Relevance (2 points)
- Exogeneity (2 points)

4. **(5 points)** The following table represents the joint distribution over three random variables X, Y, Z which are related as shown in the causal graph below. Using inverse probability weights guided by propensity scores, calculate $\mathbb{E}(Y|\text{do}(X = 1)) - \mathbb{E}(Y|\text{do}(X = 0))$. You can either solve this problem using Python or can be done using pen and paper.

X	Y	Z	p(X,Y,Z)
1	1	1	0.116
1	1	0	0.274
1	0	1	0.009
1	0	0	0.101
0	1	1	0.334
0	1	0	0.079
0	0	1	0.051
0	0	0	0.036



5. **(5 points)** Consider that we are estimating $\psi = \mathbb{E}[Y(1) - Y(0)]$. The direct way of estimating ψ is as follows:

$$\hat{\mu_0} = \mathbb{E}_X[Y|X = x, W = 0]$$
$$\hat{\mu_1} = \mathbb{E}_X[Y|X = x, W = 1]$$
$$\psi = \hat{\mu_1} - \hat{\mu_0}$$

where W is a treatment variable and X is the set of covariates. Starting with $\psi = \mathbb{E}_X[\hat{\mu_1}(X) - \hat{\mu_0}(X)]$, derive an expression that is equivalent to finding ψ through inverse probability weighting (IPW) guided by propensity score $P(W = 1|X = x)$.

6. **(4 points)** Consider the below structural equations of a causal model:

$$X = U_1$$
$$Z = aX + U_2$$
$$Y = bZ$$

where U_1 is a binary variable (0 - not having college education, 1 - having college education), U_2 is also a binary variable (0 - not having professional experience, 1 - having professional experience), Z represents the level of skill needed for a job and Y is the salary.

   (a) If we represent the counterfactual value of Y had X been x as $Y_{X=x}$, write the following in mathematical expression: *Expected salary of individuals with skill level Z = 1, had they received a college education* (1 point)

   (b) If we represent the counterfactual value of Y had X been x as $Y_{X=x}$, Fill the below table of counterfactual quantities.(3 points)

u_1	u_2	$X_{U_1=u_1,U_2=u_2}$	$Z_{U_1=u_1,U_2=u_2}$	$Y_{U_1=u_1,U_2=u_2}$	$Z_{X=0,U_1=u_1,U_2=u_2}$	$Z_{X=1,U_1=u_1,U_2=u_2}$	$Y_{X=0,U_1=u_1,U_2=u_2}$	$Y_{X=1,U_1=u_1,U_2=u_2}$
0	0							
0	1							
1	0							
1	1							

7. **(5 points)** Let X, Y are treatment and response variables. If we represent the counterfactual value of Y had X been x as $Y_{X=x}$, prove that the counterfactual query of the form $E(Y_{X=x}|e)$ is identifiable in linear models given the evidence e.(Assume that e is obtained through the *Abduction*[1] step of counterfactual inference.)

[1] *Causal Inference in Statistics: A Primer* by Judea Pearl, Madelyn Glymour, Nicholas P. Jewell