Algorithmsanalysis	Section	02
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Homework1	Name	정보윤

1. $2^{n+1} = O(2^n)$? $0 \le 2^{n+1} \le c_1 2^n$, $(n_0 > 0,\ n \ge n_0,\ c_1 > 0) \rightarrow 2^n > 0$ $\therefore 2^{n+1-n} \le c_1 2^{n-n}$

$\therefore 2 \le c_1$, so $n \rightarrow \infty$ c_1 can be defined $\therefore$ $\boxed{\text{true}}$

2. $2^{2n} = O(2^n)$? $0 \le 2^{2n} \le c_1 2^n$, $(n_0 > 0,\ n \ge n_0,\ c_1 > 0) \rightarrow 2^n > 0$ $\therefore 2^{2n-n} \le c_1 2^{n-n}$

$\therefore 2^n \le c_1$, so $n \rightarrow \infty$ c_1 can't be defined $\boxed{\therefore \text{False}}$

3. Prove the statement "For two functions f(n) and g(n), we have $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$."

sol) $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$, $(n_0 > 0,\ n \ge n_0,\ c_1 > 0,\ c_2 > 0)$ is $f(n) = \Theta(g(n))$

so, $f(n) = O(g(n)) \rightarrow 0 \le f(n) \le c_2 g(n)$, $(n_0 > 0,\ n \ge n_0,\ c_2 > 0) \Rightarrow$ upper

and $f(n) = \Omega(g(n)) \rightarrow 0 \le c_1 g(n) \le f(n)$, $(n_0 > 0,\ n \ge n_0,\ c_1 > 0) \Rightarrow$ lower

$\therefore O(g(n)) \cap \Omega(g(n))$ means $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$, $\Rightarrow \boxed{f(n) = \Theta(g(n))}$

4. Prove that the running time of an algorithm is $\Theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$.

sol) $0 \le c_1 g(n) \le f(n) \le c_2 g(n)$: $\Theta(g(n))$, When running time is $\boxed{O(g(n)),\ \text{it is bounded by } c_2 g(n) \rightarrow \text{worst case}}$

when running time is $\boxed{\Omega(g(n)),\ \text{it is bounded below by } c_1 g(n) \rightarrow \text{best case}}$

5. Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

$f_1(n) = o(g(n)) \rightarrow 0 \le f_1(n) < c g(n)$, $(n_0 > 0,\ n \ge n_0,\ c > 0)$
$f_2(n) = \omega(g(n)) \rightarrow 0 \le c g(n) < f_2(n)$, $(n_0 > 0,\ n \ge n_0,\ c > 0)$
$\Rightarrow 0 \le f_1(n) < c g(n) < f_2(n)$

$\boxed{\therefore f_1(n) \ne f_2(n) \text{ means that there's no intersection}}$

6. For a given function $g(n,m)$, we denote by $O(g(n,m))$ the set of functions

$O(g(n,m)) = \{f(n,m)$: there exist positive constructs c, n_0, and m_0 such that

$0 \le f(n,m) \le cg(n,m)$ for all $n \ge n_0$ and $m \ge m_0\}$

Give corresponding definitions for $\Omega(g(n,m))$ and $\Theta(g(n,m))$

sol)

$\Omega(g(n,m)) = \{f(n,m)$: there exist positive constructs C, n_0, and m_0 such that
$0 \le cg(n,m) \le f(n,m)$ for all $n \ge n_0$ and $m \ge m_0\}$

$\Theta(g(n,m)) = \{f(n,m)$: there exist positive constructs C_1, C_2, n_0 and m_0 such that
$0 \le C_1 g(n,m) \le f(n,m) \le C_2 g(n,m)$ for all $n \ge n_0$ and $m \ge m_0\}$