Vehicle States, Dynamics, and Simulation

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Overview

- Vehicle States
- Vehicle Dynamics Modelling
- Vehicle Dynamics Simulation

Vehicle States

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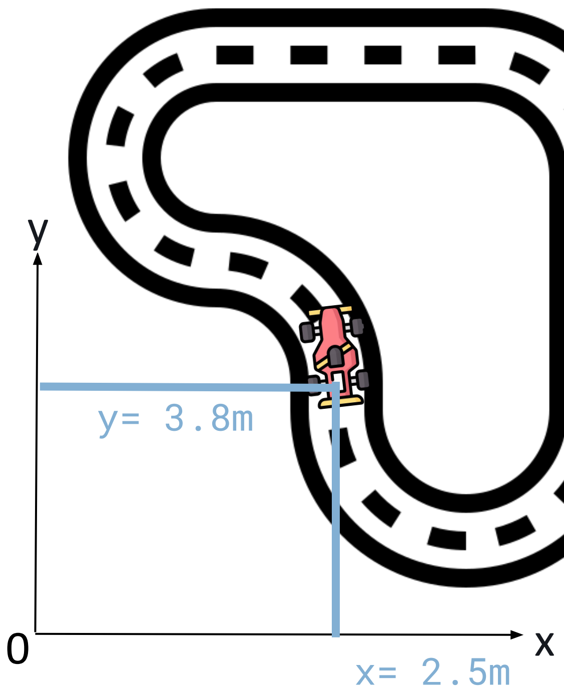
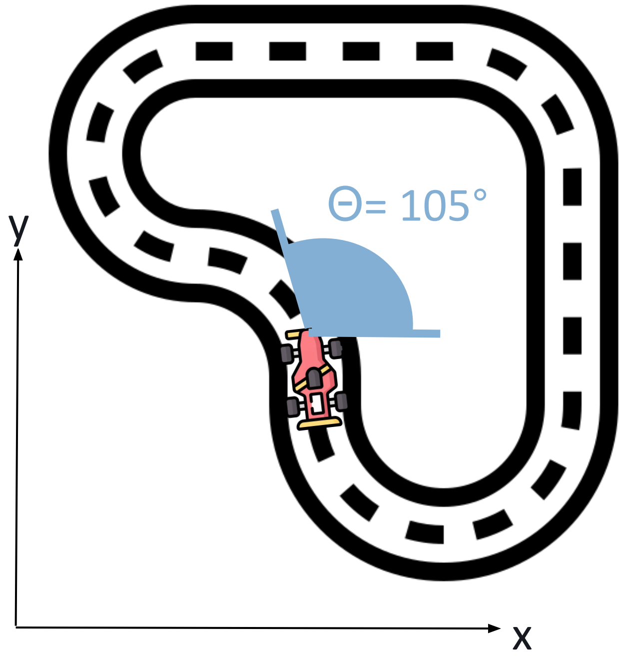
Position



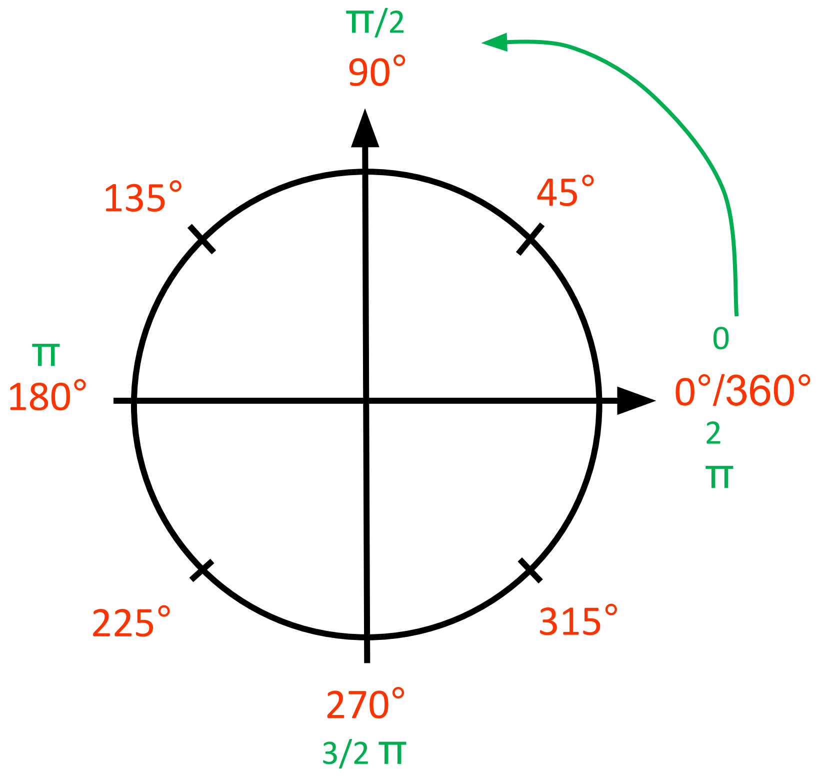
- **Position** defines the translation of the vehicle in some global or local frame
- Respective to the vehicle's center of gravity (CoG), or a pre-defined base frame
- **Normally:** X- and Y- position in **meters**

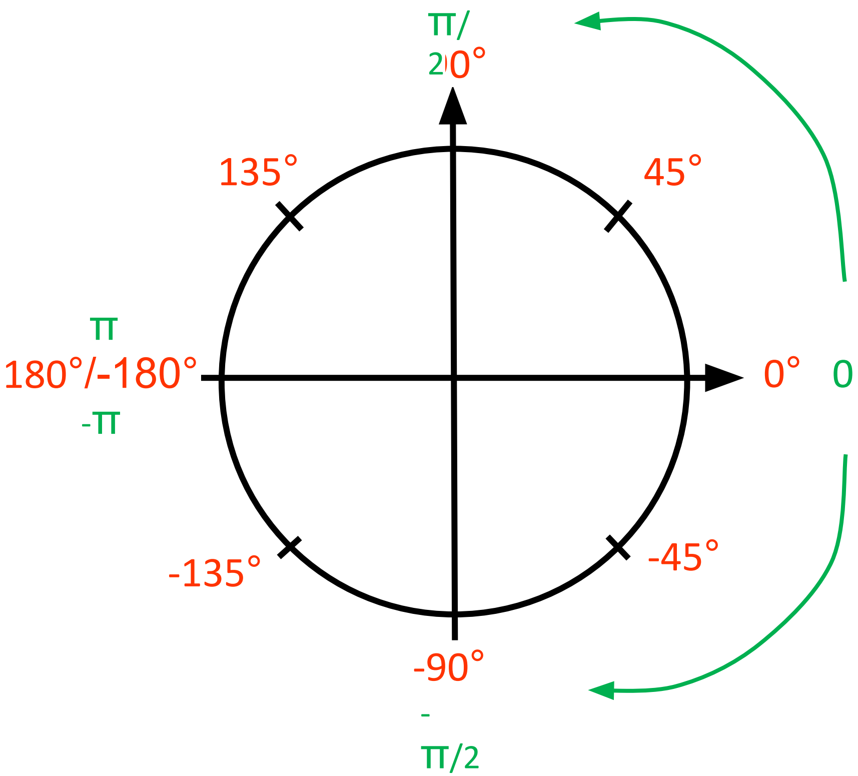
Heading



- **Heading** defines the rotation of the vehicle in some local and global frame
- Usually with respect to the x-axis of the coordinate system of current frame
- When represented as a single angle reading, heading can be displayed in ranges from:
  - $-\pi$ – π = -180° to 180°
  - 0 – 2π = 0° to 360°
- Could be represented as RPY, Rotation matrix, Quaternion, etc.
- Only **rotation**

Heading





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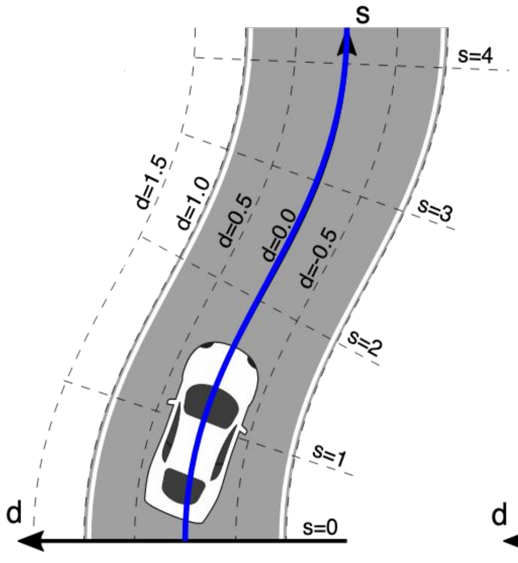
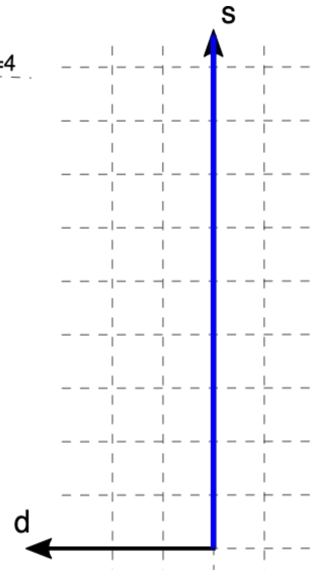
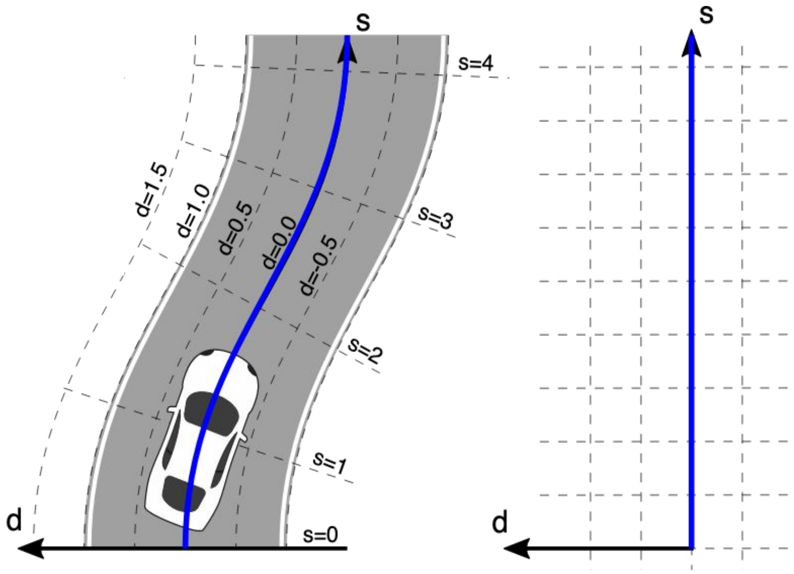
Position in Frenet Frame





- **The Frenet frame** along a curve is a moving coordinate system determined by the tangent line and curvature.
- The frame itself is defined as the coordinate system spanned by a **tangential vector T** and the **normal vector N**, and a **binormal vector B** at any point of the reference line (e.g. centerline of a track).
- (The binormal vector is the cross product of the tangential vector and the normal vector)

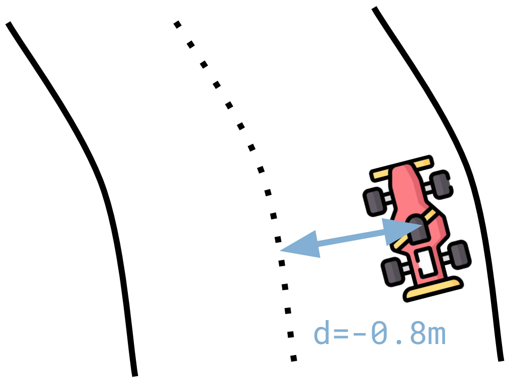
Position in Frenet Frame



- 2 coordinates in 2D.
- The **s** coordinate represents the run length. **Starts with s = 0** at the beginning of the reference path.
- The **d** coordinate represents lateral position relative to the reference path. **Starts with d = 0** for points on the reference path. Measured on the normal vector.
- **d is positive to the left** of the reference path and **negative on the right** of it. (Right hand rule)

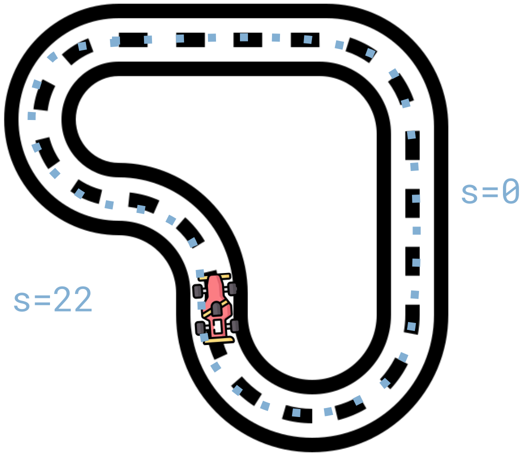
Frenet Frame in Practice

Distance to Centerline



- Displacement (in meters) between the **agent center and the track center**
- The observable maximum displacement occurs when any of the agent's wheels are outside a track border and, depending on the width of the track border

Progress along the track



- An **ordered list** of track-dependent milestones along the track (waypoints).
- Each milestone is described by a coordinate of (x_i ; y_i, …).
- For a looped track, **the first and last waypoints are the same.**

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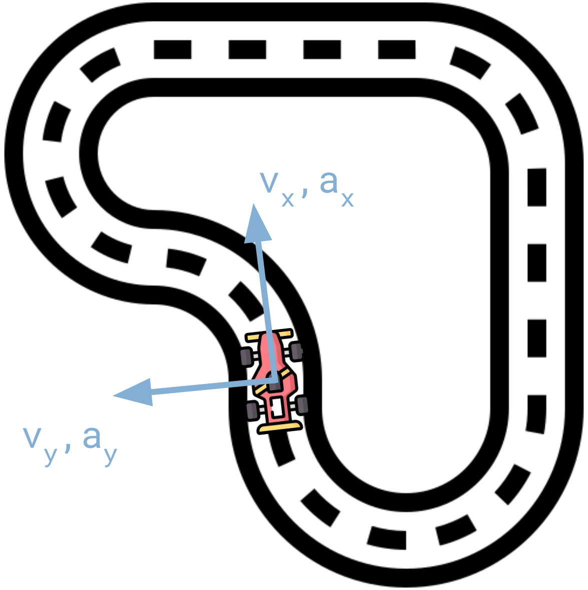
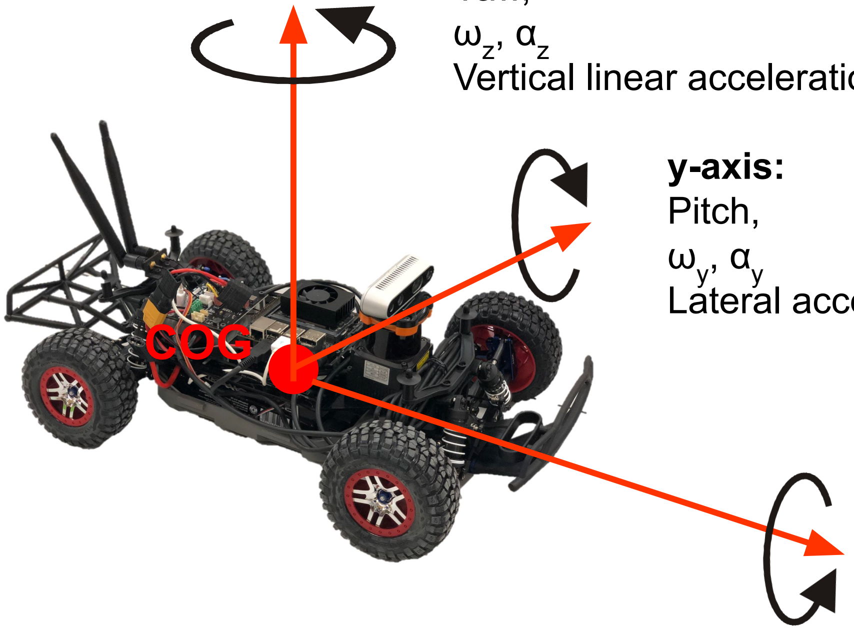
Linear Velocity and Acceleration



- **Linear velocity and acceleration** are measured in the x- and y-, (and z-) axis in the coordinate system of the vehicle.
- For cars: **longitudinal** (x-axis) and **lateral** (y-axis) velocities and accelerations.
- Can be measure with GPS, IMU, wheel speed sensors, pitot sensors, etc.
- Usually velocity in meters per second: m/s, and acceleration in meters per second squared: m/s^2
- (Right hand rule)

Angular Velocity and Acceleration

z-axis:
Yaw,
ω_z, α_z
Vertical linear acceleration (lift), v_z ,a_z



y-axis:
Pitch,
ω_y, α_y
Lateral acceleration v_y,a_y

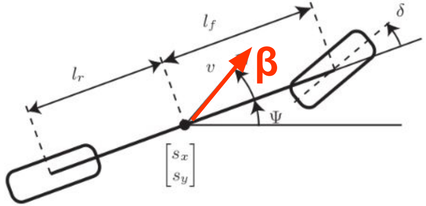
x-axis:
Roll,
ω_x, α_x
Longitudinal acceleration v_x,a_x

Vehicle State - Steering Angle

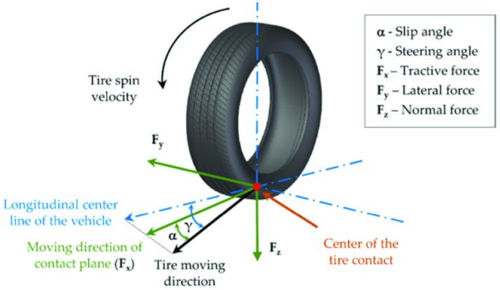
- **Steering angle δ** is the angle formed by the direction the front wheels are pointing at and the vehicle's x-axis.
- Steering angle is the same for both front wheels.
- Usually in radians or degrees.

Different Slip Angles

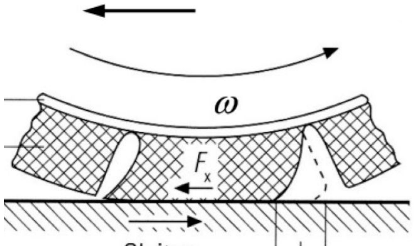
- Sideslip angle **β**



- Slip Angle **α**

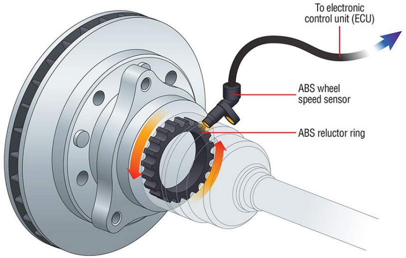


- Wheelslip ratio **s**



Measuring Vehicle States

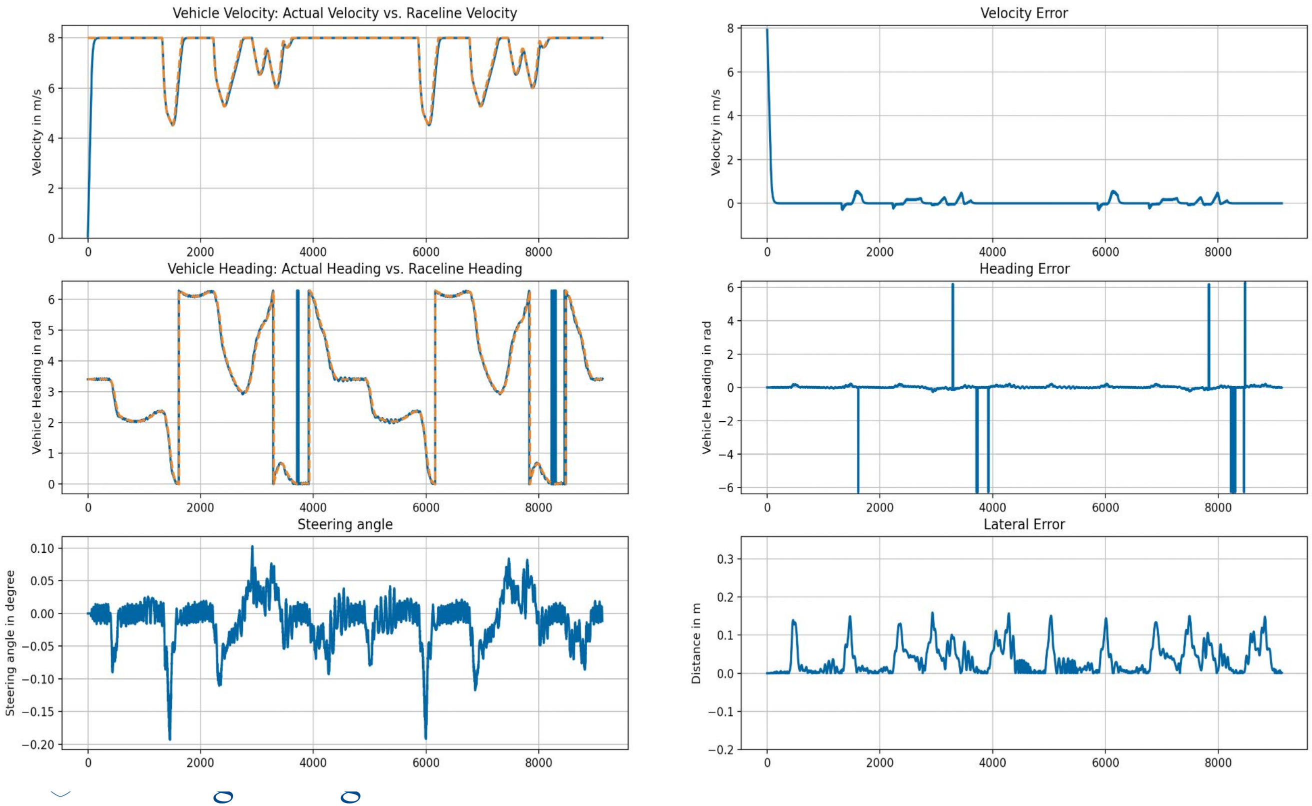
- Vehicle states can be derived from different sensor
    - GPS (Global Positioning System)



    - Odometry: Gyrometer, Wheel Speed Sensor
    - Camera

    - Lidar

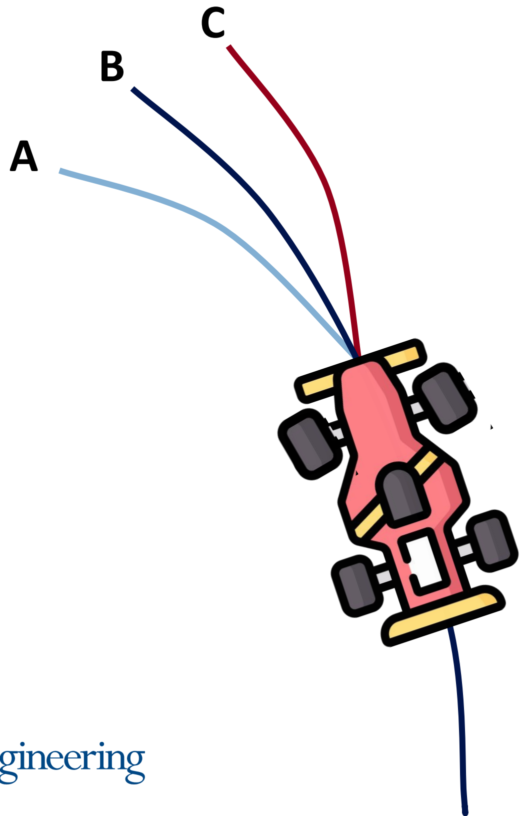
Important - Visualize your States



- Visualize
- Show trends
- Calculate errors
- Learn to Interpret the signals
- Plotjuggler, matplotlib, rqt_plot, etc.

Vehicle Dynamics Modelling

Why Modelling



Which trajectory is the car following?

We can not tell

We need information about:

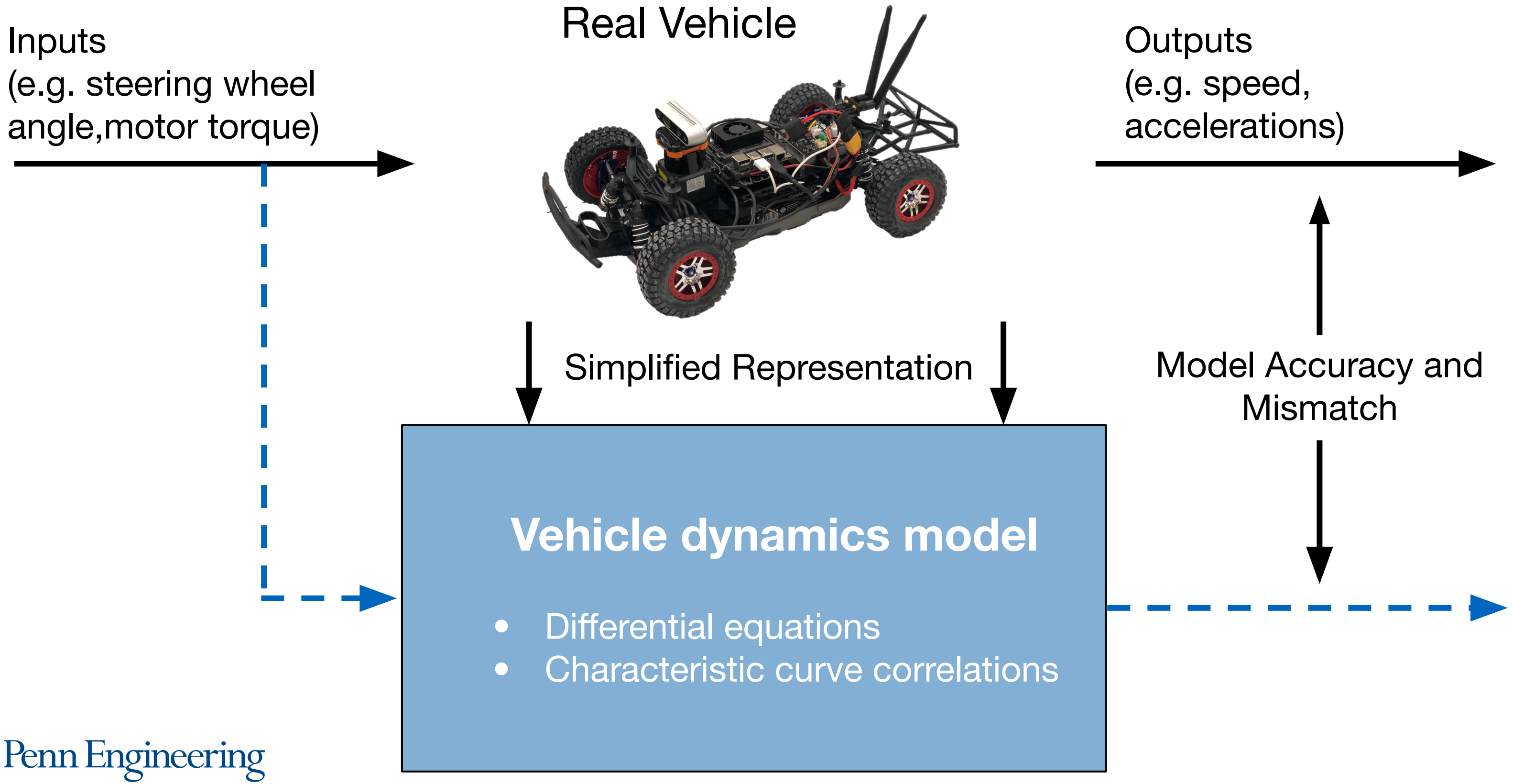
Velocity

Acceleration

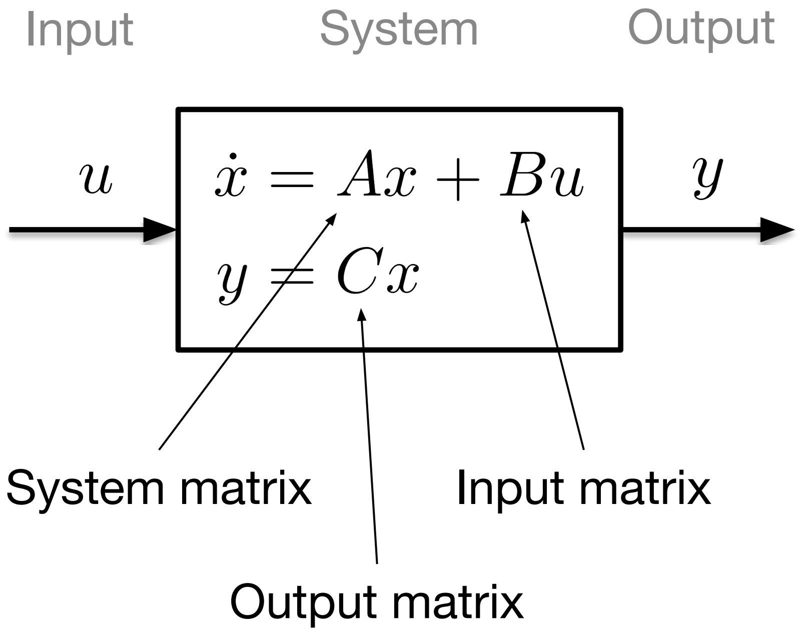
Steering angle

Vehicle: Mass, Length, Width, ….

Why Modelling?

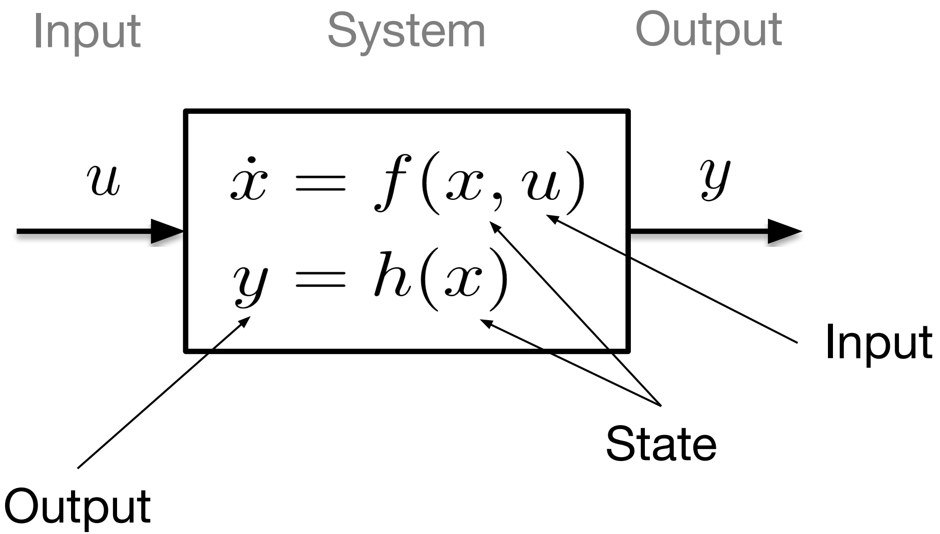


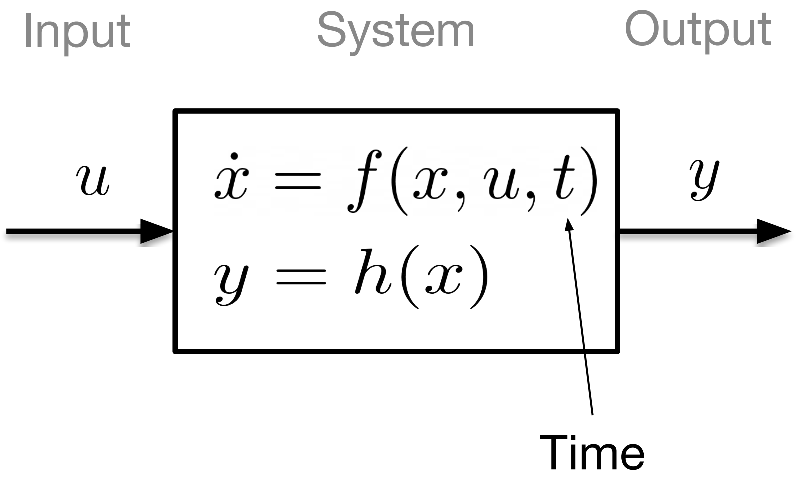
System Dynamics



- A dynamic system is a mathematical description of the relation between an *input u* and an *output y* signal. This description is usually given in form of an ODE (Ordinary Differential Equation).
- The system has ***states x*.** These are variables which allow us to **formulate the system behavior** in form of a set of first-order ODE for each of this variables.
- The standard system description consists of the ***system dynamics*** and the ***output equation***. The former describes the timely behavior of the states as a reaction to the inputs and the initial state. The latter describes the relation between the state and the output.

System Dynamics





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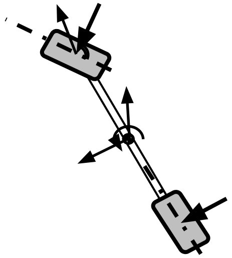
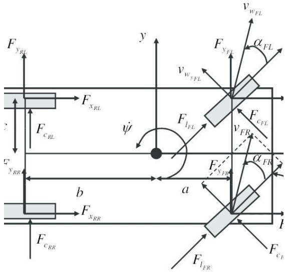
Vehicle Dynamics Model Types

Accuracy

Single Track Model

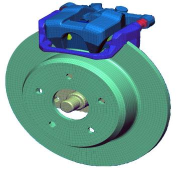


Double Track Model



Multi-Body Simulation

Finite Elements Simulation



Analytical models

Numbers of Parameters

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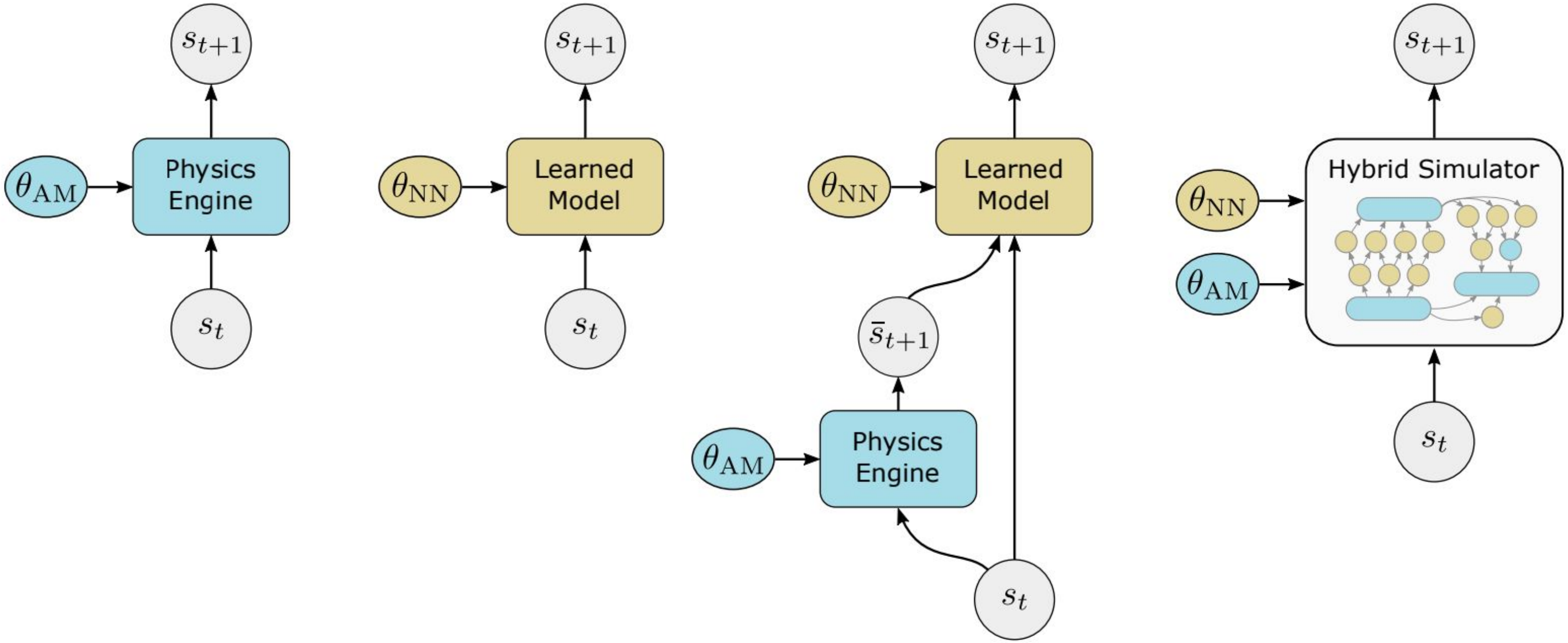
Vehicle Dynamics Model Types

Analytical Model | Data-driven Model | Residual Physics | Hybrid



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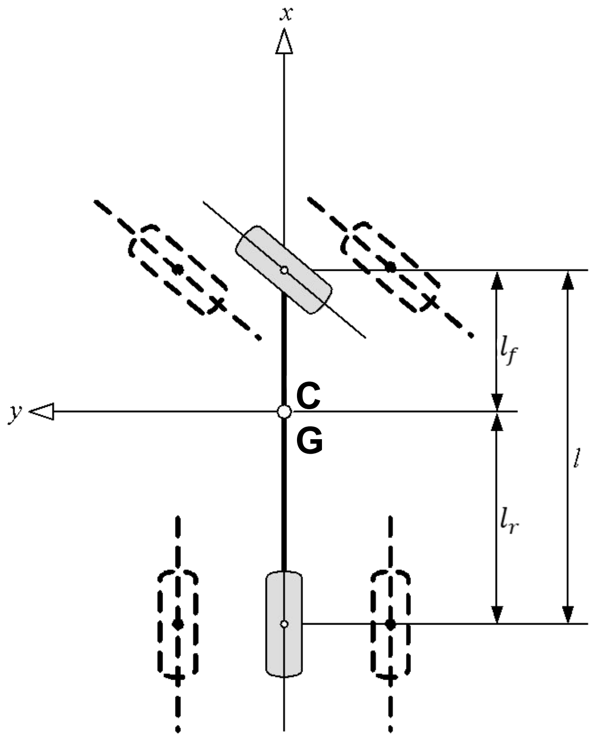
E. Heiden, D. Millard, E. Coumans, Y. Sheng and G. S. Sukhatme, "NeuralSim: Augmenting Differentiable Simulators with Neural Networks," *2021 IEEE International Conference on Robotics and Automation (ICRA)*

Single Track Models

Single Track Model

Simplifications:

- Wheels of one axle are combined
- Center of gravity is at road level
- No rolling
- No pitching
- No wheel load differences left/right
- No vertical dynamics



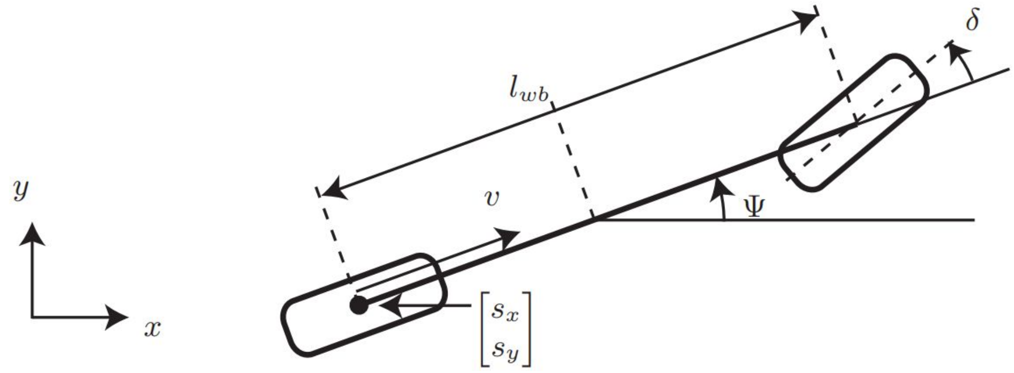
Kinematic Single Track Model

Simplifications:

- No tire dynamics, hence **no lateral forces/accelerations and slip angles**

At slow velocities, especially when cornering slowly, kinematic model is usually accurate enough for simulation.

Ackermann steering modeled around an instantaneous pole.



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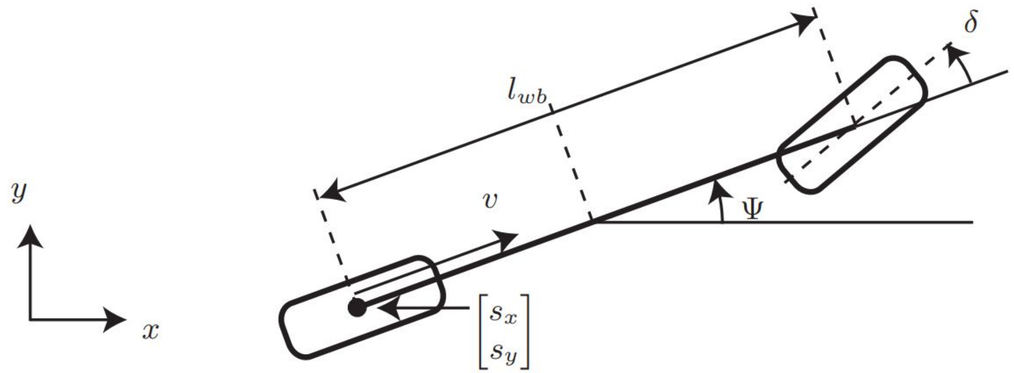
Kinematic Single Track Model

$$x_1 = s_x, \quad x_2 = s_y, \quad x_3 = \delta, \quad x_4 = v, \quad x_5 = \Psi$$

$$u_1 = v_\delta, \quad u_2 = a_{\text{long}}$$

$$\begin{aligned}
\dot{x}_1 &= x_4 \cos(x_5) \\
\dot{x}_2 &= x_4 \sin(x_5) \\
\dot{x}_3 &= f_{\text{steer}}(x_3, u_1) \\
\dot{x}_4 &= f_{acc}(x_4, u_2) \\
\dot{x}_5 &= \frac{x_4}{l_{wb}} \tan(x_3)
\end{aligned}$$



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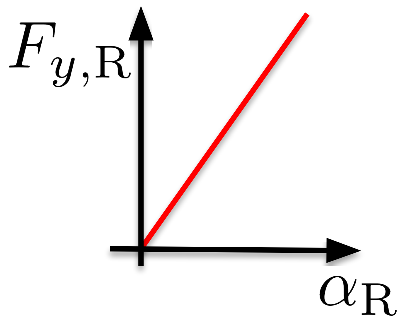
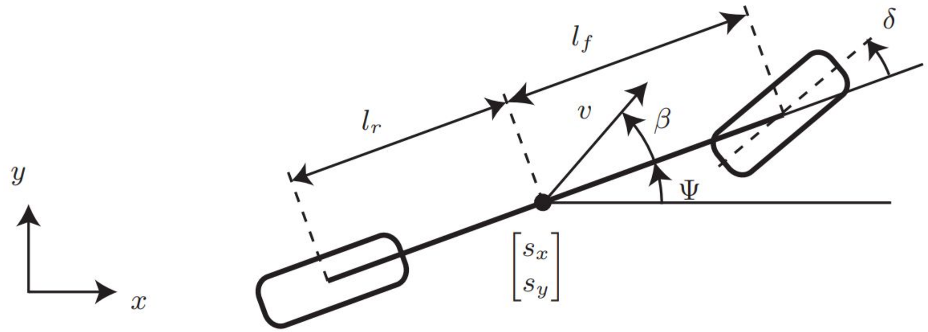
Dynamic Single Track Model - Linear Tire Model

Single Track Model with Linear Tire Model:

- Consider important effects such as understeer or oversteer.
- Introduction of tire forces
  - A tire can apply lateral and longitudinal tyre forces
  - A tire can apply more forces if there is a higher friction coefficient μ
  - **Linear** relation between tire force and side slip angle
  - Model the tire dynamics with the **cornering stiffness C**, or the **cornering stiffness coefficient Cs**





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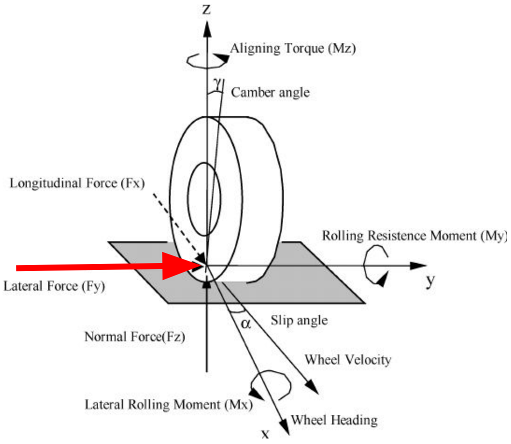
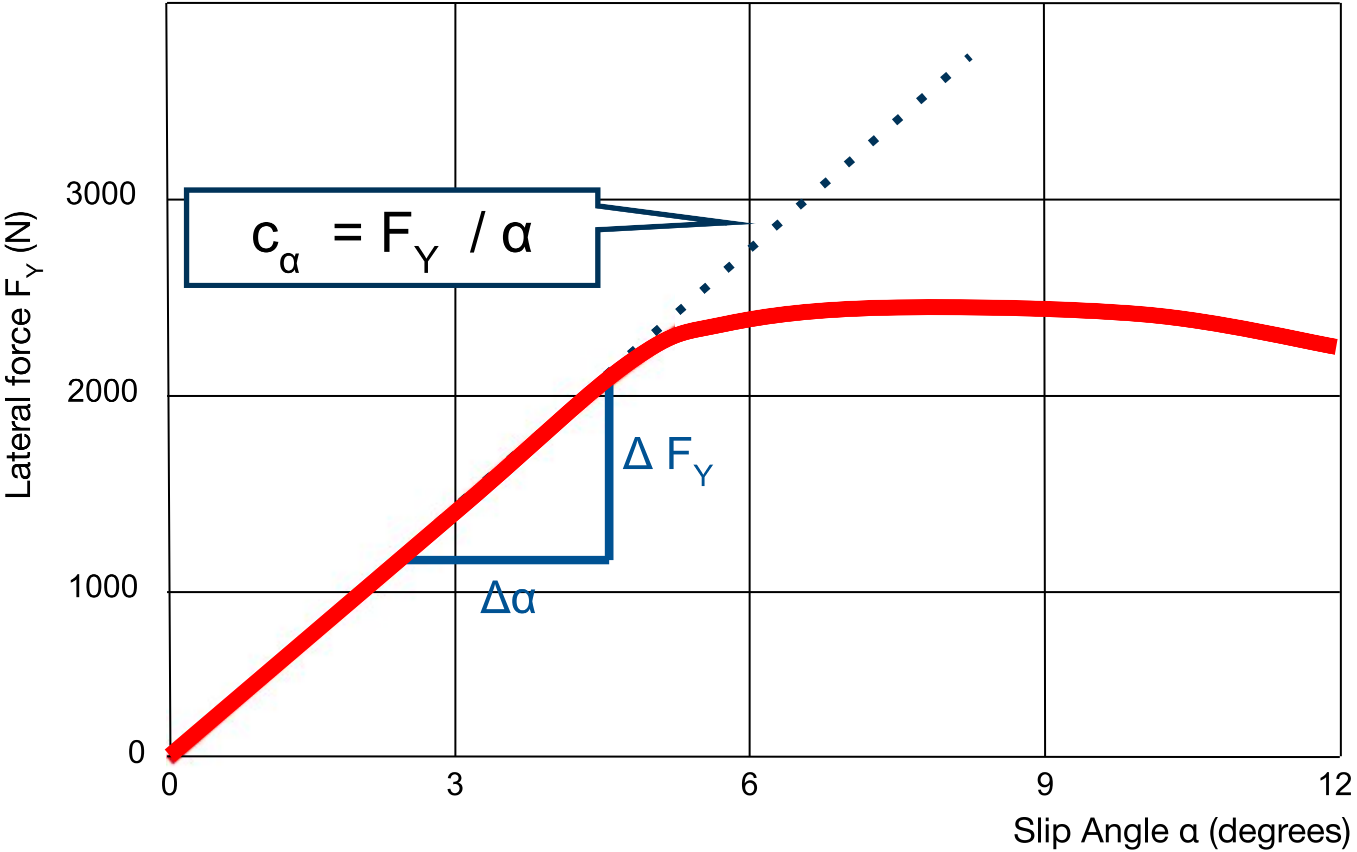
Linear Tire Model

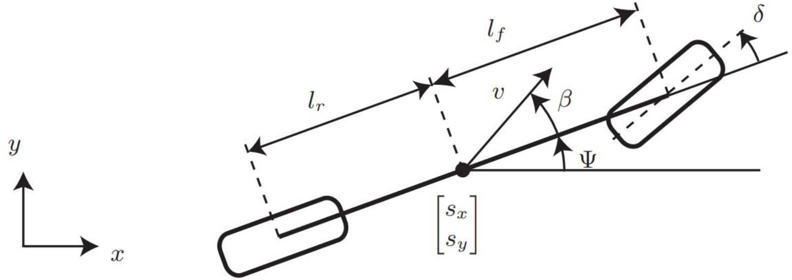




Dynamic Single Track Model - Linear Tire Model

$$x_1 = s_x, \quad x_2 = s_y, \quad x_3 = \delta, \quad x_4 = v, \quad x_5 = \Psi, \quad x_6 = \dot{\Psi}, \quad x_7 = \beta$$

$$u_1 = v_\delta, \quad u_2 = a_{\mathrm{long}}$$

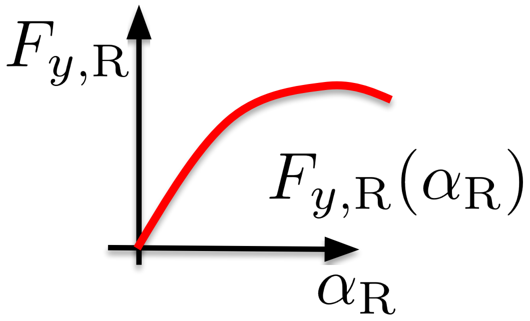


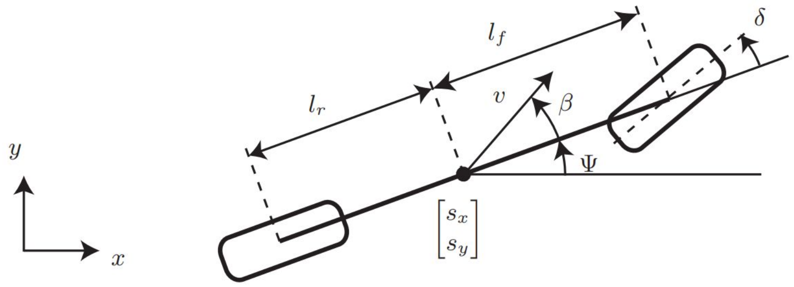
$$\begin{aligned}
\dot{x}_1 &= x_4 \cos\left(x_5 + x_7\right)\\
\dot{x}_2 &= x_4 \sin\left(x_5 + x_7\right)\\
\dot{x}_3 &= f_{\mathrm{steer}}\ \left(x_3, u_1\right)\\
\dot{x}_4 &= f_{acc}\left(x_4, u_2\right)\\
\dot{x}_5 &= x_6,\\
\dot{x}_6 &= \frac{\mu m}{I_z\left(l_r + l_f\right)}\left(l_f C_{S,f}\left(g l_r - u_2 h_{cg}\right) x_3 + \left(l_r C_{S,r}\left(g l_f + u_2 h_{cg}\right) - l_f C_{S,f}\left(g l_r - u_2 h_{cg}\right)\right) x_7\right.\\
&\quad \left. - \left(l_f^2 C_{S,f}\left(g l_r - u_2 h_{cg}\right) + l_r^2 C_{S,r}\left(g l_f + u_2 h_{cg}\right)\right)\frac{x_6}{x_4}\right)\\
\dot{x}_7 &= \frac{\mu}{x_4\left(l_r + l_f\right)}\left(C_{S,f}\left(g l_r - u_2 h_{cg}\right) x_3 - \left(C_{S,r}\left(g l_f + u_2 h_{cg}\right) + C_{S,f}\left(g l_r - u_2 h_{cg}\right)\right) x_7\right.\\
&\quad \left. + \left(C_{S,r}\left(g l_f + u_2 h_{cg}\right) l_r - C_{S,f}\left(g l_r - u_2 h_{cg}\right) l_f\right)\frac{x_6}{x_4}\right) - x_6
\end{aligned}$$

Dynamic Single Track Model - Nonlinear Tire Model

Single Track Model with Nonlinear Tire Model:

- Based on the ST with linear tire model
- Extension of tire forces:
    - No small angle approximations
    - Considers longitudinal tire forces and longitudinal slip on the front and rear wheels
    - Nonlinear relation between tire force and side slip angle
    - Model the tire dynamics with the special models:
        - Pacejka Magic Formula
        - Fiala
        - …





Dynamic Single Track Model - Nonlinear Tire Model

$$x_1 = s_x, \quad x_2 = s_y, \quad x_3 = \delta, \quad x_4 = v, \quad x_5 = \Psi, \quad x_6 = \Psi, \quad x_7 = \beta, \quad x_8 = \omega_f, \quad x_9 = \omega_r$$

$$u_1 = v_\delta, \quad u_2 = a_{\text{long}}$$

$$\dot{x}_1 = x_4 \cos(x_7 + x_5)$$

$$\dot{x}_2 = x_4 \sin(x_7 + x_5)$$

$$\dot{x}_3 = f_{\text{steer}}\ (x_3, u_1)$$

$$\dot{x}_4 = \frac{1}{m}\left(-F_{s,f}\sin(x_3 - x_7) + F_{s,r}\sin x_7 + F_{l,r}\cos(x_7) + F_{l,f}\cos(x_3 - x_7)\right)$$

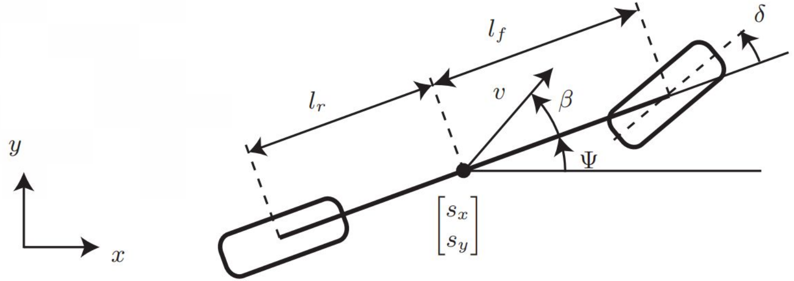
$$\dot{x}_5 = x_6$$

$$\ddot{x}_6 = \frac{1}{I_z}\left(F_{s,f}\cos(x_3)\, l_f - F_{s,r} l_r + F_{l,f}\sin(x_3)\, l_f\right)$$

$$\dot{x}_7 = -\dot{x}_5 + \frac{1}{m x_4}\left(F_{s,f}\cos(x_3 - x_7) + F_{s,r}\cos(x_7) - F_{l,r}\sin(x_7) + F_{l,f}\sin(x_3 - x_7)\right)$$

$$\dot{x}_8 = \frac{1}{I_{y,w}}\left(-R_w F_{l,f} + T_{s,b} T_B + T_{s,e} T_E\right)$$

$$\dot{x}_9 = \frac{1}{I_{y,w}}\left(-R_w F_{l,r} + (1 - T_{s,b})\, T_B + (1 - T_{s,e})\, T_E\right)$$



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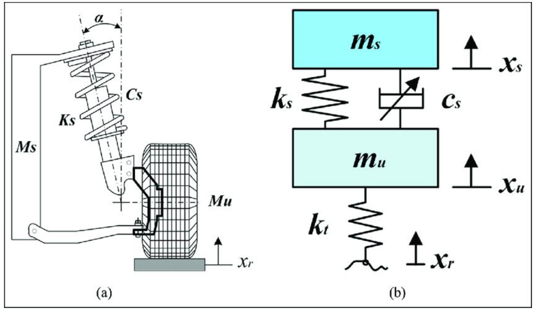
Allen, R. Wade, et al. "Vehicle dynamic stability and rollover." *NASA STI/Recon Technical Report N* 93 (1992): 30828.

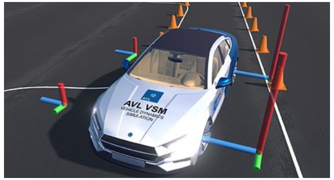
Multi-body Model

- Each wheel is modelled individually with sprung and unsprung mass
- Each wheel has individual wheel load
- Each wheel has individual lat./long forces
- Center of gravity is at certain height
  - Rolling
  - Pitching
  - Wheel load differences left/right
  - Axle load differences front/rear
- Vertical dynamics
- Tyre modelling with different nonlinear models, similar to nonlinear ST

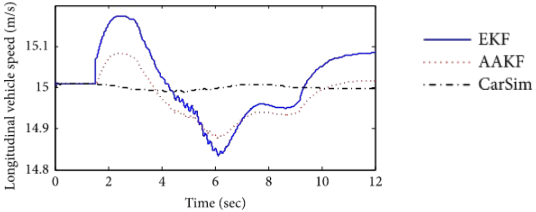


Example:
https://gitlab.lrz.de/tum-cps/commonroad-vehicle-models/-/blob/master/vehicleModels_commonRoad.pdf
Section **9**

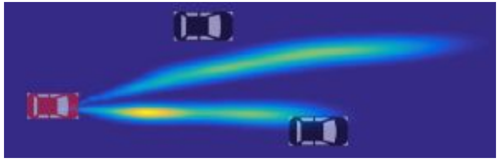
Vehicle Dynamics - Use Cases



- **Vehicle Dynamics Simulation** - Provide the correct behavior for your vehicle in Simulation



- **State Estimation:** Calculate how your vehicle has moved



- **Behavior Prediction:** Calculate how other vehicles will move/behave

- **Model-Based Algorithms:**
    - Model Predictive Control (MPC)
    - Model based Reinforcement Learning

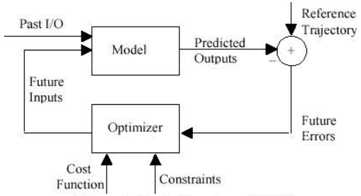


Figure 1: Basic Structure of MPC

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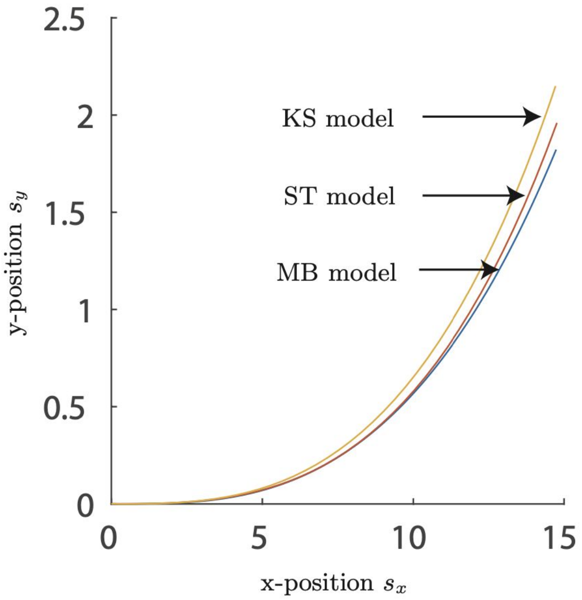
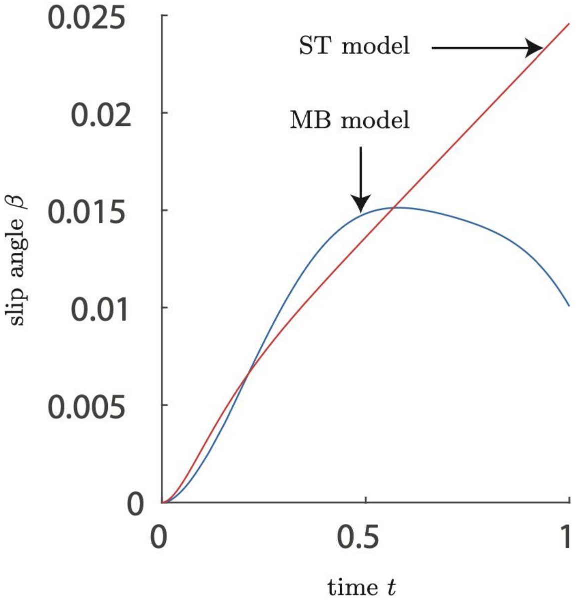
Example: Cornering



(a) Path of center of gravity.



(b) Slip angle.

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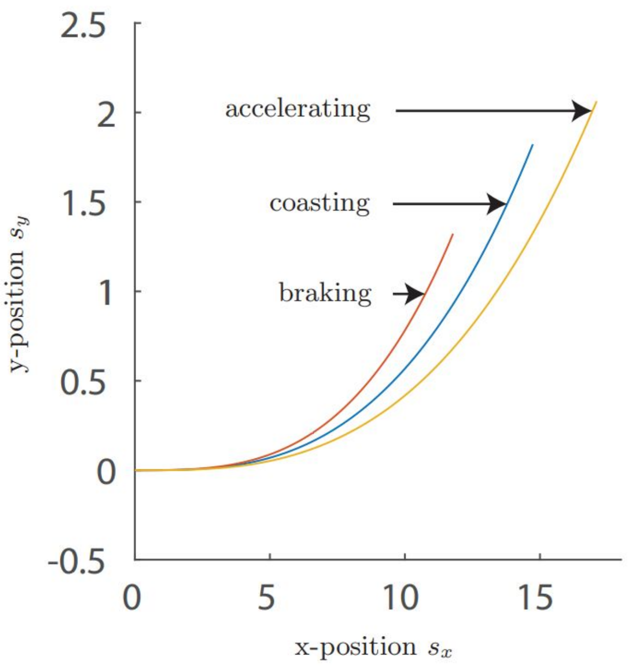
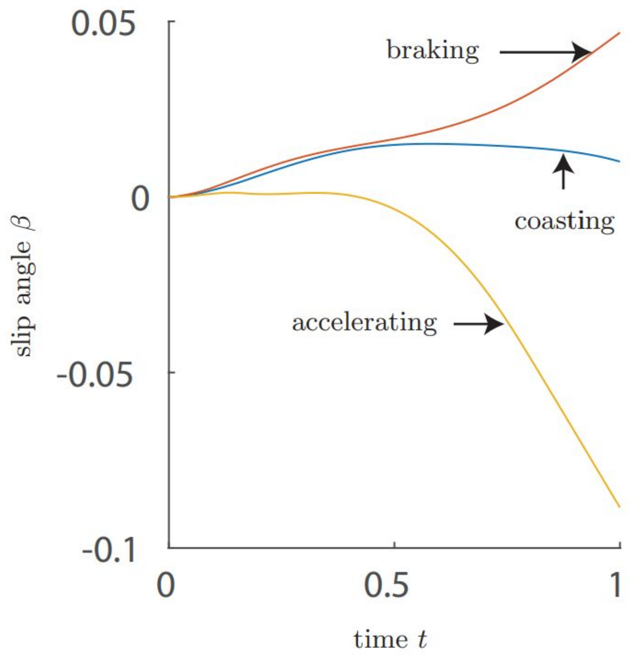
Example: Over and Under Steering



(a) Path of center of gravity.



(b) Slip angle.

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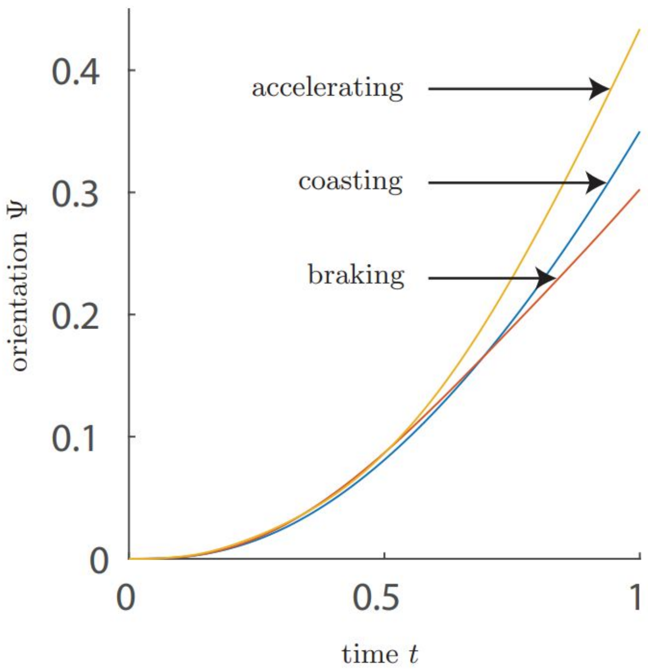
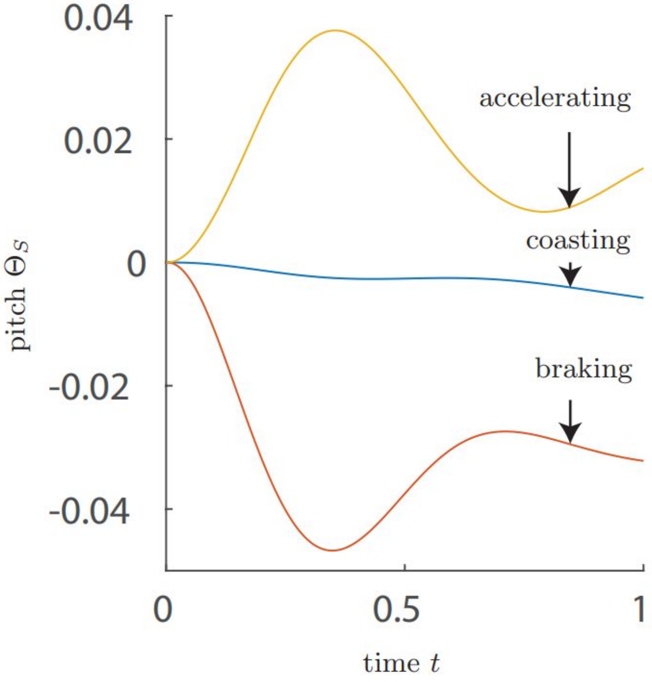
Example: Over and Under Steering



(c) Orientation.



(d) Pitch.

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Questions?