1.723 – Computational Methods for Flow in Porous Media

Homework #6

Due on April 30, 2015

Consider one-dimensional two-phase immiscible flow in porous media, as described by the Buckley-Leverett equation with capillarity:

$$\phi \partial_t S + \partial_x [u_T f(S) - f(S)\lambda_o(S)(-\partial_x P_c(S))] = 0, \tag{1}$$

where S is the water saturation, u_T is the total velocity (assumed constant), $P_c(S)$ is the capillary pressure curve (a monotonically decreasing function of saturation), $f(S) = \lambda_w(S)/\lambda_T(S)$ is the fractional flow function, $\lambda_w(S) = k\, k_{rw}(S)/\mu_w$ and $\lambda_o(S) = k\, k_{ro}(S)/\mu_o$ are the water and oil mobilities, k_{rw} and k_{ro} are the water and oil relative permeability curves. Consider the following functional forms for the constitutive relations:

$$k_{rw} = S^2, \tag{2}$$

$$k_{ro} = (1-S)^2, \tag{3}$$

$$P_c = P_c^e S^{-2}, \tag{4}$$

$$M = \mu_o/\mu_w = 2. \tag{5}$$

Consider the following 'natural' boundary conditions in terms of the water advective flux $F = u_T f(S)$:

$$F|_{x=0} = F(S_0), \quad \text{with } S_0 = 1, \tag{6}$$

$$F|_{x=L} = F(S_L), \quad \text{with } S_L \text{ from inside the domain.} \tag{7}$$

1. (2 points) Write down the mathematical problem in dimensionless form, and identify the key dimensionless parameter, the capillary number Ca.

2. (4 points) Solve the equation using the finite volume method with appropriate discretizations of the advective and diffusive fluxes, and *Forward Euler time-stepping*. Write down the equation for a generic inner gridblock i. Plot the saturation profiles at well chosen times, for three different values of the capillary number: $\mathrm{Ca} = 10^8$, 10^4 and 10^2. Use a fine-enough grid ($N = 100$ or 1000 gridblocks), and comment on the solution and on the time step restrictions for each case.

3. (4 points) Solve the equation using the finite volume method with appropriate discretizations of the advective and diffusive fluxes, and *Backward Euler time-stepping.* Write down the equation for a generic inner gridblock i, and the structure of the system of nonlinear equations. Use Newton iteration to solve the nonlinear system of equations at each time step. Compare the solution with that of the previous case, and comment on the time-step restrictions and convergence of the nonlinear iteration.