16.930 Advanced Topics in Numerical Methods for PDEs
Massachusetts Institute of Technology – Spring 2015

Project 1 – 1D Stabilized FEM

Handed Out: Feb. 15, 2015 *Due: March 2, 2015 submitted on-line by 12pm (noon)*

The objective of this project is to gain experience with different stabilization methods for finite element methods, including both continuous Galerkin (CG), discontinuous Galerkin (DG), and hybridized discontinuous Galerkin (HDG). Distributed with this project are a set of notes that provide details of the weak forms associated with these different methods for the convection-diffusion-reaction equation: We will begin with solving the (linear) convection-diffusion-reaction equation with Dirichlet boundary conditions.

We will consider the following three test cases:

Poisson: $\nu = 1$, $b = c = 0$, $f = x^2$ and $u(0) = 0$ and $u(1) = 1$.

Reaction-diffusion: $\nu = 10^{-2}$, $b = 1$, $c = 0$, $f = 0$, $u(0) = 0$ and $u(1) = 1$.

Convection-diffusion: $\nu = 10^{-4}$, $b = 0$, $c = 1$, $f = 0$, $u(0) = 0$ and $u(1) = 1$.

Your Tasks

- Your are to develop a 1D FEM code (Matlab is highly preferred) that implements all of the following discretization methods:

  1) CG (without stabilization),
  2) CG with either GLS or VMS stabilization
  3) DG using the BR2 diffusion discretization
  4) HDG

  Implement elements of order $p = 1$ (linear) and $p = 3$ (cubic). Use Gaussian quadrature (of sufficient order) for all integrals. A direct solver should be used (no iterative techniques).

- For all test cases and each discretization, you are to perform a study of the $L^2(\Omega)$ and $H^1(\Omega)$ error convergence using uniform meshes for $p = 1$ and $p = 3$. Use a series of at least 4 meshes for each combination of test case, discretization, and p. Choose the mesh series to allow good comparison amongst different discretizations and p for a given test case.

Project deliverables

- You should submit your source directory (in an archive).
- In a short report (submitted as a single PDF), provide the following:

  1) For all test cases, discretization methods, and polynomial order p: plots of the log of the error norm versus the log of the mesh size h. Construct the plots so as to allow easy comparison of different discretizations and p.
  2) Tables of the observed rate of convergence for all of these combinations.