Problem2

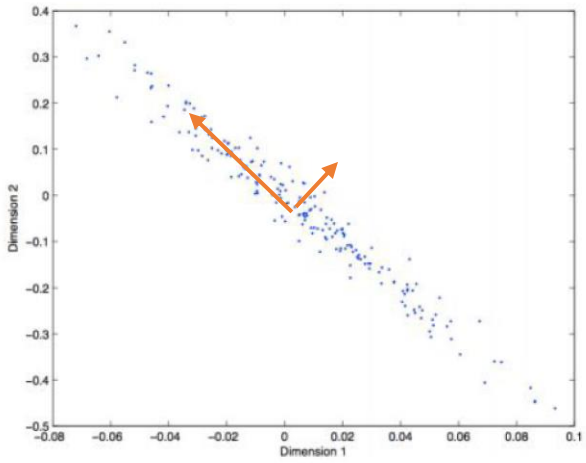
1. You have to communicate a signal in a language that has 3 symbols A, B and C. The probability of observing A is 50% while that of observing B and C is 25% each. Design an appropriate encoding for this language. What is the entropy of this signal in bits?

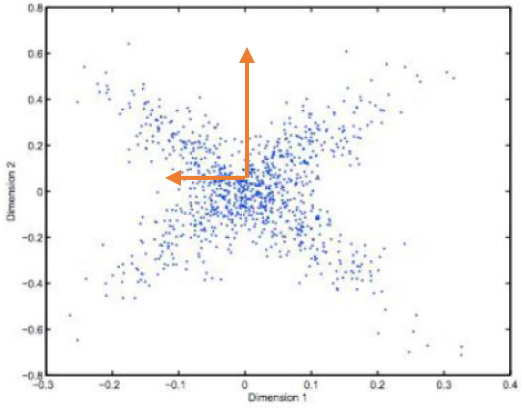
A: 0
B: 10
C: 01
Entropy: -1/2 * log(1/2) - ¼ * log(1/4)*2 = 1.5

2. Show that the K-means procedure can be viewed as a special case of the EM algorithm applied to an appropriate mixture of Gaussian densities model.

K means is using the mean value of the points as the center. According to the central limit theorem, the center will form a Gaussian distribution, which covers the points in the cluster.

3. Plot the direction of the first and second PCA components in the figures given .




The longer arrow is first.
Firstly, eigen vectors will be perpendicular to each other. and Y axis has a larger scale, so having a larger eigen value.

4. for a there's no difference between first 4 algorithm as they all use distance for clustering, EM will also work but running time would be slower.
For c, EM will work as first 4 will cluster by distance and as a result the x shape plot will be cluster by half instead of by two lines.
For b, EM will work better as the borderline in the middle is very close to each other, using distance will have a hard time to determine which cluster the points belong to.

5. States: silent(0), laughing(1)
Actions: (incense, organ play) 0, (no incence, organ play) 1, (no incence, no organ play) 2, (incense, no organ play) 3
Reward: R(0,0) = 1, R(0,1) = -1, R(1,0) = 1, R(1,1) = -1
Transition matrix: it's all deterministic transition with 1 probability.
For policy iteration: p(1) = 3 ->v(0) = -1-> p(1) change to 0 and v(0) change to 0.1 -> p(0) is 3 and v(1) change to -1 -> p(0) = 1 and converge.

Advice to wits end is to play organ and light incense every other minite.

6. Q(s,a1) = 1 + gamma * V(s1)

Q(s,a2) = 1 + gamma * V(s2)
R(s1,a1) = R(s1,a2) = R(s2,a2) = 1.5
R(s2,a3) =3
Assuming gamma = 0.9
For total exploration:
V(s1) = 0.5*(1.5+1.5) = 1.5
V(s2) = 0.5*(1.5+3) = 2.25
So Q(s,a1) = 2.35; Q(s,a2) = 3.205
For greedy V(s1) = 1.5; V(s2) = 3
So Q(s,a1) = 2.35: Q(s,a2) = 3.7

7. Grids

| | | |
|---|---|---|
| S<br>S | 2<br>S | 3<br>S |
| 4<br>S | 5<br>S<br>9 | 6<br>S<br>8 |
| 7<br>E | 8<br>E<br>10 | G |

8. For first grid, both upper left and lower right could be nash equilibrium
   For second, there is no nash equilibrium
   For the third one, both upper left and lower right could be nash equilibrium.