4) Number of row operations:

1) Division by pivot for each row: 1 → total n

2) $\overset{\text{Multiply}}{\wedge}$ Subtract from rows below: $2(n-i)$, for i^{th} row

Total = $2(n-1 + n-2 + \cdots 0) \to \frac{n(n-1)}{\cancel{2}} \times \cancel{2}$

3) multiply and subtract (back-sub) for each row

$\Rightarrow n(n-1)$

Total $\Rightarrow n + 2n(n-1)$

$\Rightarrow 2n^2 - n$

Each row op is n floating point ops

$\Rightarrow \boxed{2n^3 - n^2}$

6.
a) Minimum floating (double) numbers:

$(-1)^{\text{sign}} (1.b_{51} b_{50} \cdots b_0)_2 \times 2^{e-1023}$

Set $e = 0$

Set $b_0 \cdots b_{51}$ to 0

minimum = 1×2^{-1023}

Maximum:

Set $b_1 \cdots b_{52} = 1$

Set $c = 2^{11} = 2048$; $2048 - 1023 = 1025$

$$\text{maximum} = \left(1 + \sum_{i=1}^{52} 2^{-i}\right) \times 2^{1025}$$

b) $2^{24} =$ [illegible] $110.$

Sign $= 0$
Mantissa $= 1$ $\quad\equiv\quad 1.0 \times 2^{24}$
Exponent $= 24$

$1 \equiv$ Sign $= 0$
Mantissa $= 1$ $\quad = 1 \times 2^{0}$
Exponent $= 0$

$$= \underbrace{0.0 \cdots 01}_{\longleftarrow 24 \longrightarrow} \times 2^{24}$$

Add: $1.0 + 0.00 \cdots 1$

$\Rightarrow 1$ since mantissa has only 23 bits, last bit of $0.0 \cdots 01$ gets truncated.

Final $= \boxed{1 \times 2^{24}}$

C. ~~1.11875~~ 111.875

in binary =, 111 = 1101111

$.875 \times 2 = 1.750$

$.750 \times 2 = 1.5$

$.5 \times 2 = 1.0$

$= (.111)_2$

$\Rightarrow 1101111.111 \times 2^0$

$\Rightarrow 1.101111111 \times 2^6$

$e - 129 = 6$

$e = (135)_{10}$

$= 10000111$

Sign = 0

Bit pattern = $\underbrace{0}_{S}\ \underbrace{101111111 00 \ldots 0}_{M = 21 \text{ bits}}\ \underbrace{0010000111}_{E = 10 \text{ bits}}$

IV. 1 is represented as

$\underbrace{0}_{S}\ \underbrace{111 \ldots 1}_{E}\ \underbrace{00 \ldots 0}_{M}$

This shall be represented as 1 1 0

The smallest exponent is 00 . . . 0

This is equivalent to -127 in single precision
and -1023 in double

when adding $m \times 2^{-127}$ to 1, the
exponent is made equal to 0, and
the mantissa is right shifted by 127 bits.

This would truncate the value completely.

The maximum we can move the
implicit 1 is 23 bits

∴ minimum value is 2^{-23} for single prec.
and 2^{-53} for double prec.

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9. ~~a~~ let $A = (a_{ij})$

$B = (b_{ij})$

$C = (C_{ij})$

a. ~~A(BC) =~~ $A(BC) = (AB)C$

$A(BC)$

~~a~~ Consider BC

$$BC_{ij} = \sum_{k=1}^{n} B_{ik} C_{kj} \quad \text{—— (1)}$$

Similarly

$A(BC)$ ~~A(BC) =~~ $\displaystyle\sum_{k=1}^{n} A_{ik} BC_{kj}$ —— (2)

Substitute l for k

$$A(BC) = \sum_{l=1}^{n} A_{il} BC_{lj} \quad \text{—— (3)}$$

now $BC_{lj} = \displaystyle\sum_{k=1}^{n} B_{lk} C_{kj}$ (sub l for i in (1))

—— (4)

$$\therefore A(BC) = \sum_{l=1}^{n} A_{il} \sum_{k=1}^{n} B_{lk} C_{kj}$$

$$A(BC)_{ij} = \sum_{l=1}^{n} \sum_{k=1}^{n} A_{il} B_{lk} C_{kj}$$

Now Consider

$(AB)C$

Consider AB:

$$AB_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj} \qquad \text{(using (1))}$$

$$(AB)C_{ij} = \sum_{l=1}^{n} AB_{il} C_{lj} \qquad \text{(using (1) and substitutions)}$$

$$= \sum_{l=1}^{n} \left(\sum_{k=1}^{n} A_{ik} B_{kl} \right) C_{lj}$$

$$= \sum_{l=1}^{n} \sum_{k=1}^{n} A_{ik} B_{kl} C_{lj}$$

swap variables l and k

$$(AB)C_{ij} = \sum_{k=1}^{n} \sum_{l=1}^{n} A_{il} B_{lk} C_{kj}$$

$$= A(BC)_{ij}$$

Hence Proved.

b. $A(B+C) = AB + \cancel{BC}\ AC$

$(B+C)_{ij} = B_{ij} + C_{ij}$

$A(B+C)_{ij} = \sum_{k=1}^{n} A_{ik} (B+C)_{kj}$

$= \sum_{k=1}^{n} A_{ik} (B_{kj} + C_{kj})$

$= \sum_{k=1}^{n} (A_{ik} B_{kj} + A_{ik} C_{kj})$

$\Rightarrow \sum_{k=1}^{n} A_{ik} B_{kj} + \sum_{k=1}^{n} A_{ik} C_{kj}$

$\Rightarrow AB_{ij} + AC_{ij}$

Hence proved.

c) Lower triangular matrix has all elements above the ~~of~~ diagonal to be 0.

if element a_{ij} is on the diagonal. then $i = j$

if its above the diagonal then $i < j$

$A^T_{ij} = A_{ji}$

if $j < i$, then $a_{ji} = 0$

$\therefore$ when $i > j$ $a^T_{ij} = 0$

$\therefore$ its a ~~lo~~ upper triangular matrix.

d. $(A+B)' = A' + B'$

$(A+B)_{ij} = (a_{ij} + b_{ij})$

$(a+b)'_{ij} = (a+b)_{ji} = a_{ji} + b_{ji} = (a+b)_{ji}$

$\therefore (a+b)' = a' + b'$.

e) $AB_{ij} = \sum_{k=1}^{n} \cancel{a_{ki} b_{jk}}\, a_{ik} b_{kj}$

$(AB)'_{ij} = AB_{ji} = \sum_{k=1}^{n} a_{jk} b_{ki}$

$= \sum_{k=1}^{n} b'_{ik} a'_{kj}$

$= \cancel{\sum_{k=1}^{n} b'_{ki} a'_{jk}}$

$= A(B'A')_{ij}$

hence proved.

10) let $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$

Step I: Divide 1^{st} row by a_{11}

$$\begin{pmatrix} 1 & \frac{a_{12}}{a_{11}} & \frac{a_{13}}{a_{11}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

2 : make elements below 1 0 :

$$\begin{pmatrix} 1 & \frac{a_{12}}{a_{11}} & \frac{a_{13}}{a_{11}} \\ 0 & a_{22} - \frac{a_{12}}{a_{11}} \times a_{21} & a_{23} - \frac{a_{13}}{a_{11}} \times a_{21} \\ 0 & a_{32} - \frac{a_{12}}{a_{11}} \times a_{31} & a_{33} - \frac{a_{13}}{a_{11}} \times a_{31} \end{pmatrix}$$

3: make second pivot $=1$

$$\begin{pmatrix} 1 & \frac{a_{12}}{a_{11}} & \frac{a_{13}}{a_{11}} \\ 0 & 1 & \dfrac{a_{23} - \frac{a_{13}}{a_{11}} \times a_{21}}{a_{22} - \frac{a_{12}}{a_{11}} \times a_{21}} \\ 0 & a_{32} - \frac{a_{12} a_{31}}{a_{11}} & \frac{a_{33} - a_{13} a_{31}}{a_{11}} \end{pmatrix}$$

Can be written as:

$$\begin{bmatrix} 1 & \dfrac{a_{12}}{a_{11}} & \dfrac{a_{13}}{a_{11}} \\[2ex] 0 & 1 & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \\[4ex] 0 & \dfrac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{a_{11}} & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}}{a_{11}} \end{bmatrix}$$

4: make values below 1 = 0

$$\begin{bmatrix} 1 & \dfrac{a_{12}}{a_{11}} & \dfrac{a_{13}}{a_{11}} \\[2ex] 0 & 1 & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \\[4ex] 0 & 0 & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}}{a_{11}} - \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \cdot \dfrac{\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}}{a_{11}} \end{bmatrix}$$

3. Divide last row:

$$\underline{R} = \begin{pmatrix} 1 & \dfrac{a_{12}}{a_{11}} & \dfrac{a_{13}}{a_{11}} \\ 0 & 1 & \dfrac{\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \\ 0 & 0 & 1 \end{pmatrix}$$

Now $AX = R$

$X = A^{-1}R$

$$\underline{A^{-1}} = \frac{1}{|A|}\begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{12} \\ a_{33} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ \begin{vmatrix} a_{23} & a_{21} \\ a_{33} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{13} & a_{11} \\ a_{23} & a_{21} \end{vmatrix} \\ \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$\boxed{X = A^{-1}R}$$