CEM - Computational Electro Magnetics

COMPUTATIONAL
ELECTROMAGNETICS FOR RF AND
MICROWAVE ENGINEERING

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http://www.ATIcourses.com/schedule.htm

http://www.aticourses.com/Computational_Electromagnetics.htm

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Co-funded by the
Erasmus+ Programme
of the European Union

Electromagnetics (EM)

Maxwell's Equations

Faraday's Law:

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}$$

Gauss' Laws:

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \cdot \mathbf{D} = \rho$$

Ampère's Circuital Law:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial}{\partial t}\mathbf{D}$$

Constitutive Equations:

$$\mathbf{B} = \mu \mathbf{H} \qquad \mathbf{D} = \varepsilon \mathbf{E}$$

Actual solution for realistic problems is complex and requires simplifying assumptions and/or numerical approximations

Solutions to Maxwell's equations using numerical approximations is known as the study of Computational Electromagnetics (CEM)

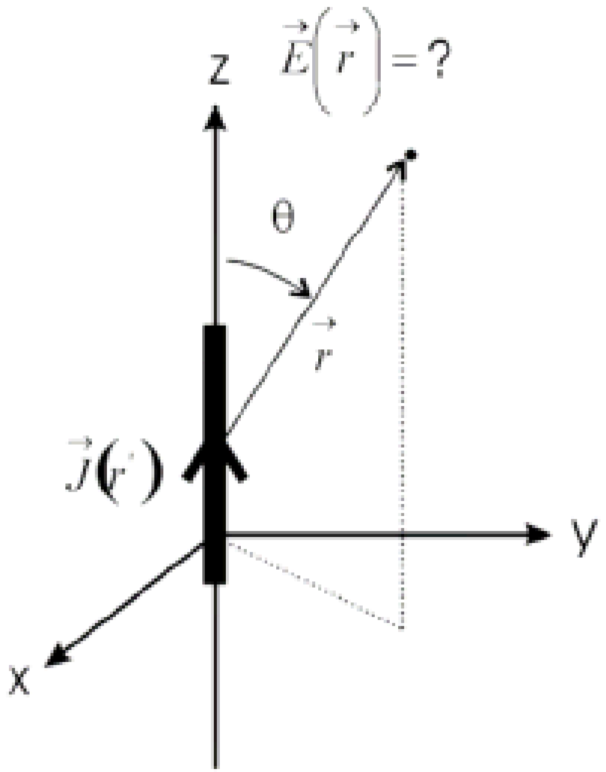
Why Computer-Aided Methods Are Used?

- **Calculation of the radiation of a dipole antenna? Analytical calculations are based on exact or approximate solutions of Maxwell's equations:**

$$\text{Wave equation:}\quad \nabla^2 \vec{A}\left(\vec{r}\right) + k^2 \vec{A}\left(\vec{r}\right) = -\mu \vec{J}(r')$$

$$\text{Vector potential:}\quad \vec{A} = \mu \int_V \frac{\vec{J}(z) e^{-jk\left|\vec{r}-r'\right|}}{4\pi\left|\vec{r}-r'\right|} dV$$

$$\vec{H}\left(\vec{r}\right) = \frac{1}{\mu} \nabla \times \vec{A}\left(\vec{r}\right) \quad ; \quad \vec{E}\left(\vec{r}\right) = \frac{1}{j\omega\varepsilon} \nabla \times \vec{H}\left(\vec{r}\right) \quad \cdots$$

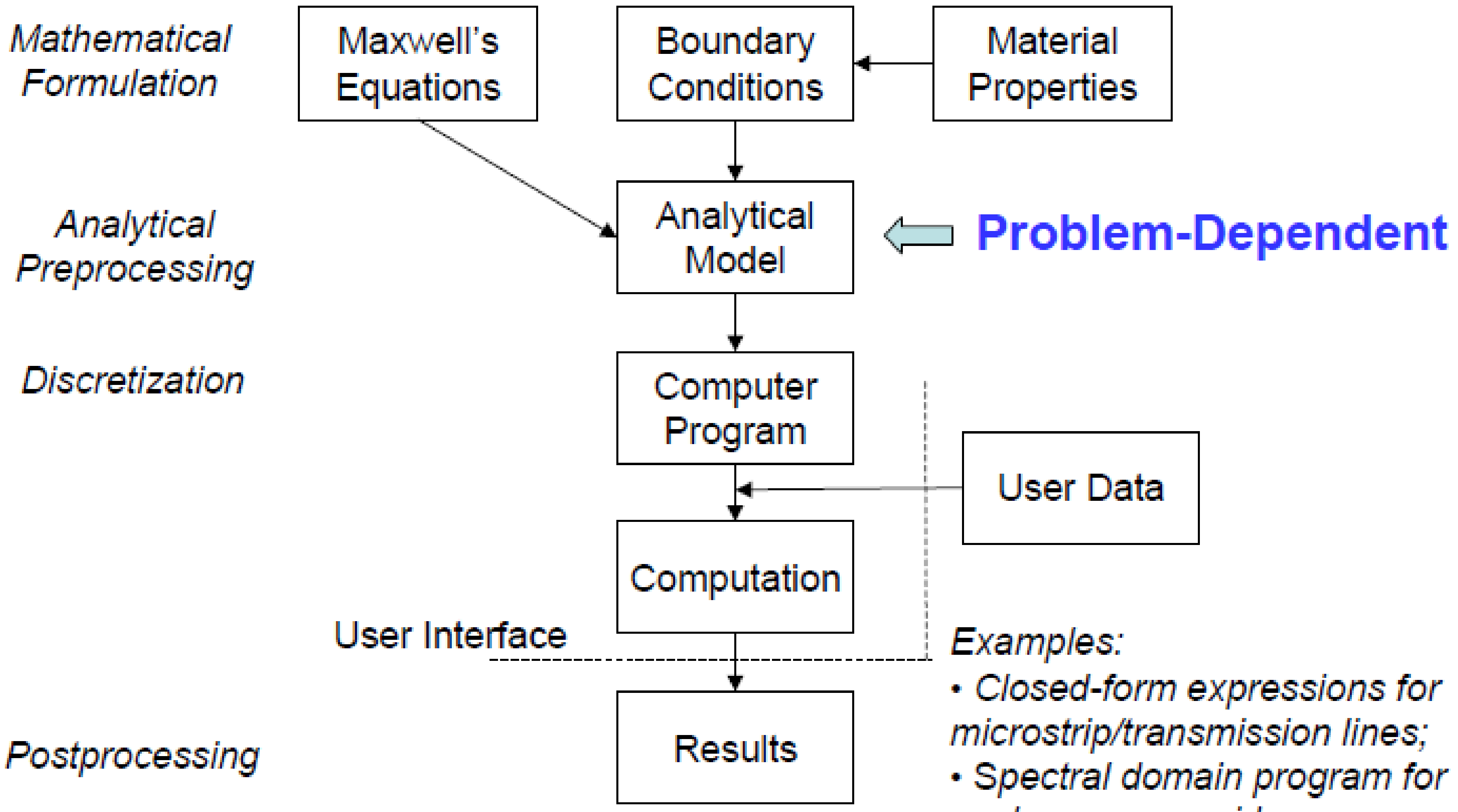


- **Analytical solutions are possible only for simple (often idealized) structures...**

Importance

- Basis for field-theory based and process-oriented CAD (virtual prototyping).
- Only means for dealing with complex (non-canonical) electromagnetic structures.
- Theoretical models must be validated by experiments.
- Theoretical and experimental work are of equal importance.

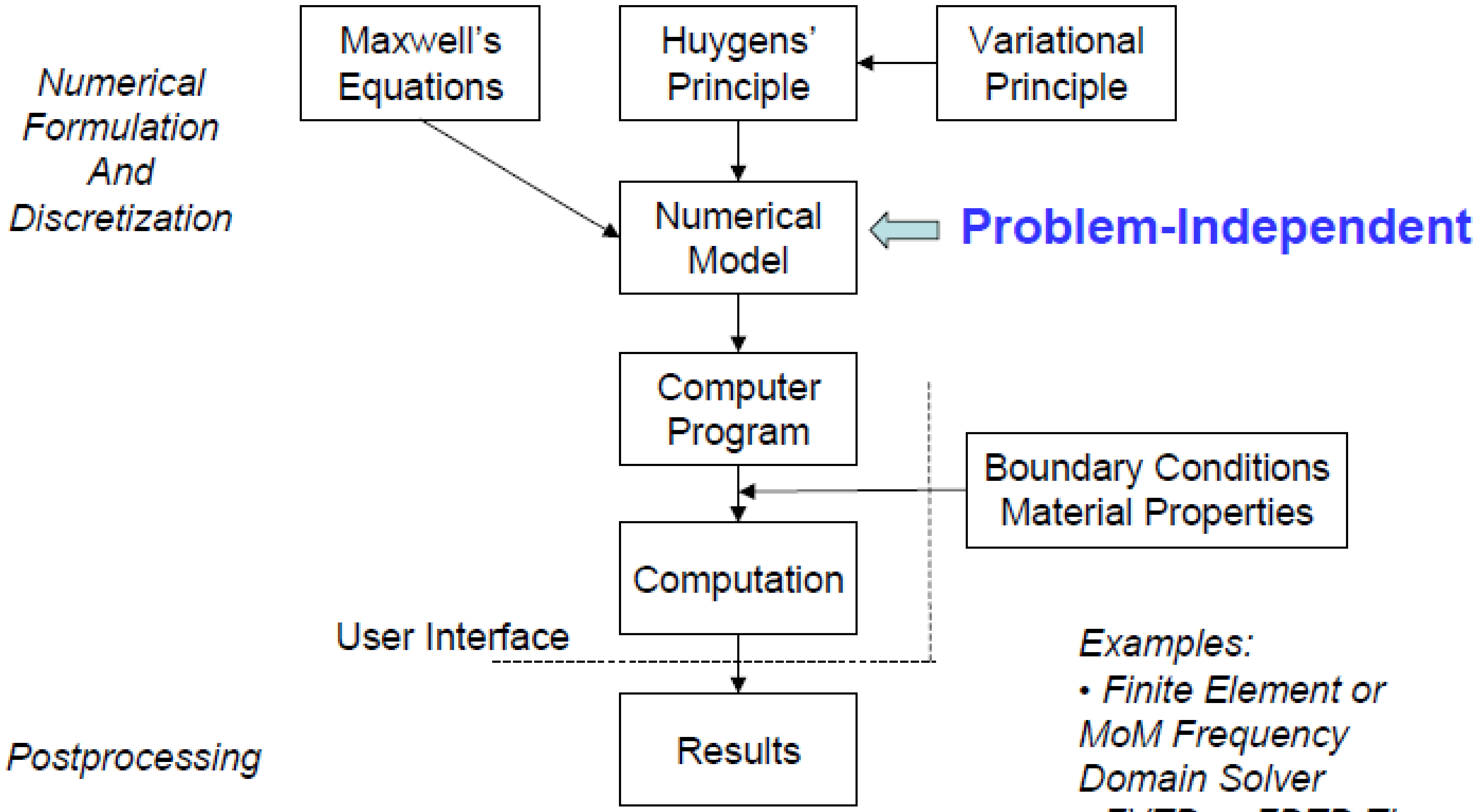
Classic Electromagnetic Solution



Examples:

- *Closed-form expressions for microstrip/transmission lines;*
- *Spectral domain program for coplanar waveguides;*
- Eigenvalue solvers for guided waves and cavities;

Modern Electromagnetic Solution



Examples:

- *Finite Element or MoM Frequency Domain Solver*
- *FVTD or FDTD Time Domain Numerical Simulation*

Electromagnetic Simulators

- An Electromagnetic Simulator is a modeling tool that:
  - solves electromagnetic field problems by numerical analysis;
  - extracts engineering parameters from the field solution and visualize fields and parameters;
  - allows design by means of analysis combined with optimization (PSO, GA, parameterized models, etc.);
  - provides the only viable approach to solving “real world” field problems.

Solving EM Field Problems

- Find electromagnetic field and/or source functions such that they
  - obey Maxwell's equations,
  - satisfy all boundary conditions,
  - satisfy all interface and material conditions,
  - satisfy all excitation conditions.

(In both time and space, or at one frequency in space)

- Field solutions are then unique when tangential fields on conductors and initial conditions are known
- But numerical solution depends on
  - *Physical Modeling Error*
  - *Discretization Error*
  - *Numerical Modeling Error*

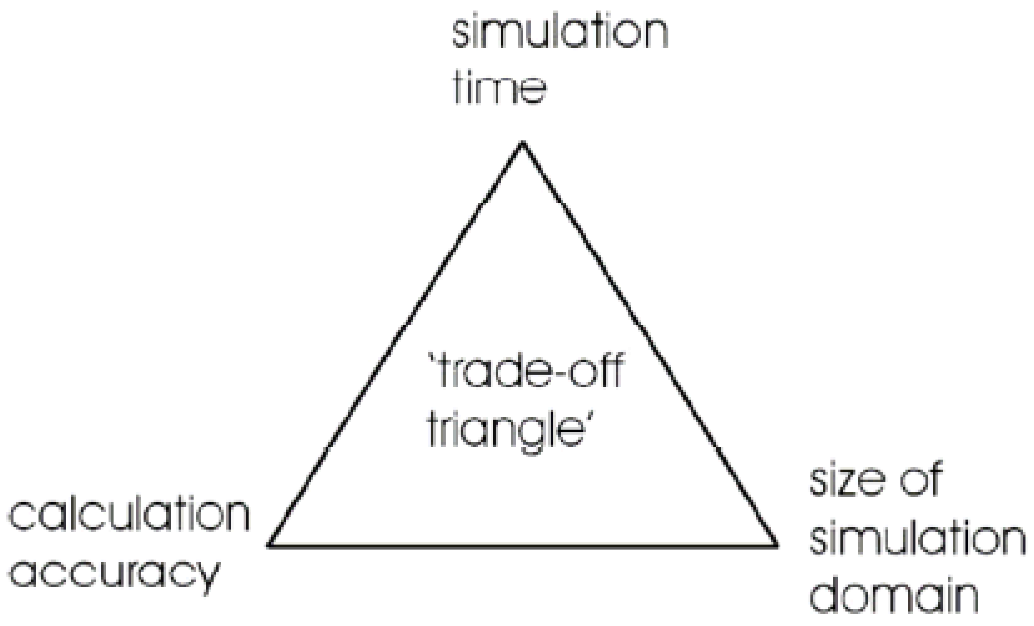
Fundamentals of Simulations

- **Using simulation tools is very useful in radio engineering because efficient and accurate programs make designing easier, less prototypes are needed and time and money are saved**
  - Simulator tools are expensive but they will usually pay the investment back!

- **Use of simulators does NOT make thinking and understanding of radio engineering, electromagnetics and circuit theory "unnecessary" because the user has to**
  - know what he/she puts into the simulator
  - understand the simulation method used because the user chooses the simulation tool and uses "buttons and switches", i.e. chooses the settings of the simulation
  - have suggestive knowledge about expected results because the simulated results can be unreliable if the user has made mistakes
- ***'if you give garbage into the simulator, it throws garbage back to you!'***

Fundamentals of Simulations

- **The user defines specific settings before the simulation is run**
  - To guarantee adequate calculation accuracy, the largest size of a cell, polyhedron or voxel (depending on the method used) has to be **much smaller than the wavelength at the highest frequency <u>in all media</u> and enough small to model <u>sharp variations in slots, near corners, wedges</u>, etc.**
  - Trade-off between simulation time, calculation accuracy and size of the simulation domain needs to be found



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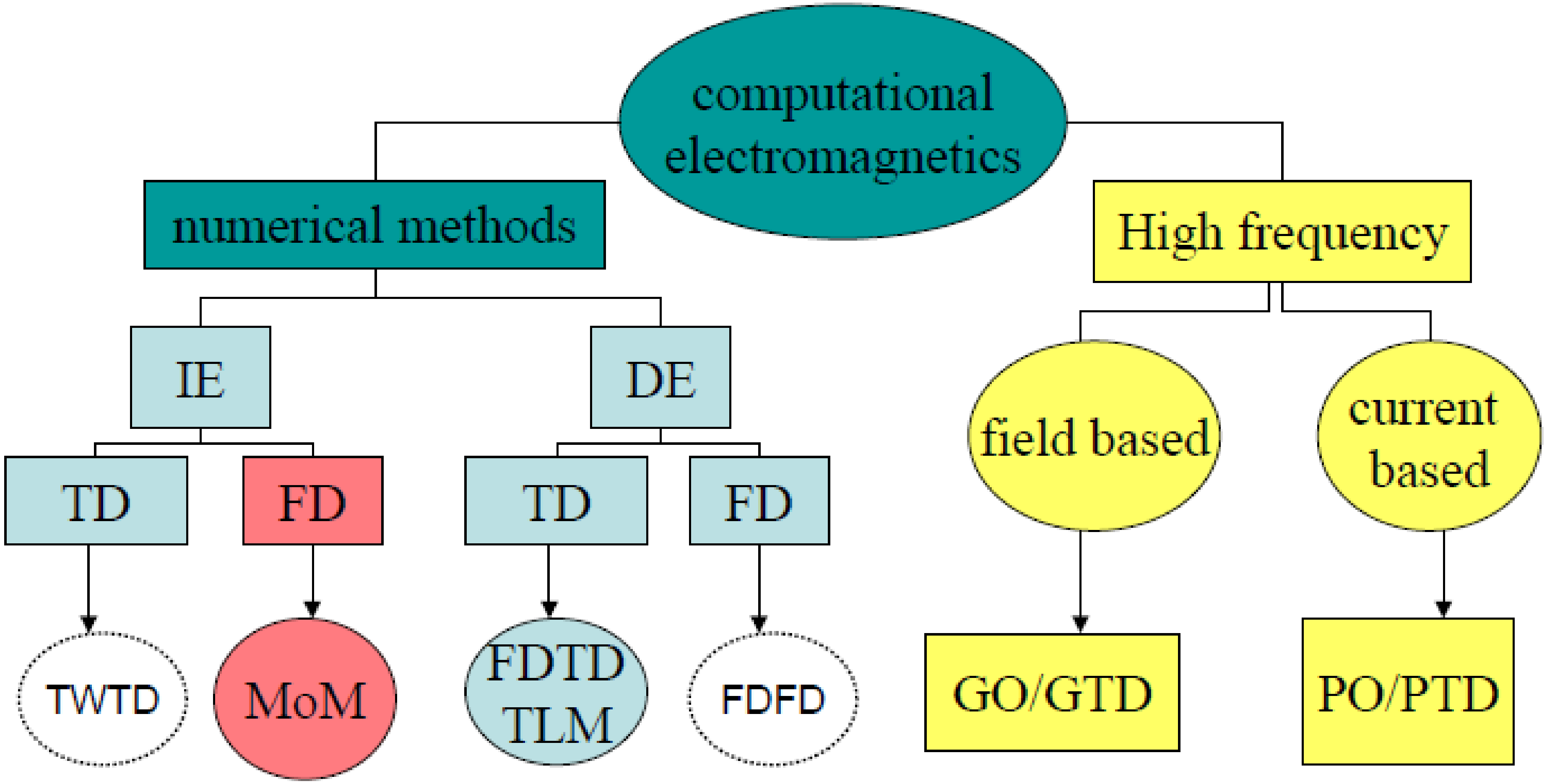
Computational Hierarchy



Classification of Methods

- Maxwell's Equations-based methods

Method	Frequency Domain	Time Domain
Boundary Element	Method of Moments (MoM)	
Finite Element	FEM	
Finite Difference		FDTD

- Optics-based methods

Method
Physical Optics
GTD/UTD

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Classification of Methods

Frequency Domain Methods (Time-Harmonic)

Time Domain Methods (Transient)

Fourier Transform

- *This distinction is based more on human experience than on physical or mathematical considerations.*

- The user requires knowledge of different methods to be able to choose the most suitable design tool and setup the calculation correctly.

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Classification of Methods

1D Methods: Fields and voltage/current vary in **one** space dimension (Transmission Line Problems)
(…Touchstone, Supercompact, SPICE…)

2D Methods: Fields and currents vary in **two** space dimensions (Cross-section problems, TE_{n0} waveguide problems)
(…FEM-2D, MEFiSTo-2D…)

2 1/2 D Methods: Fields vary in **three** space dimensions, currents vary in **two** space dimensions (Planar multilayer circuits)
(…Sonnet, Momentum, Ensemble...) frequency domain

3D Methods: Fields and currents vary in **three** space dimensions (General propagation, scattering and radiation problems)
(…HFSS, FEKO, CST, XFDTD, GEMS, GEMACS…)

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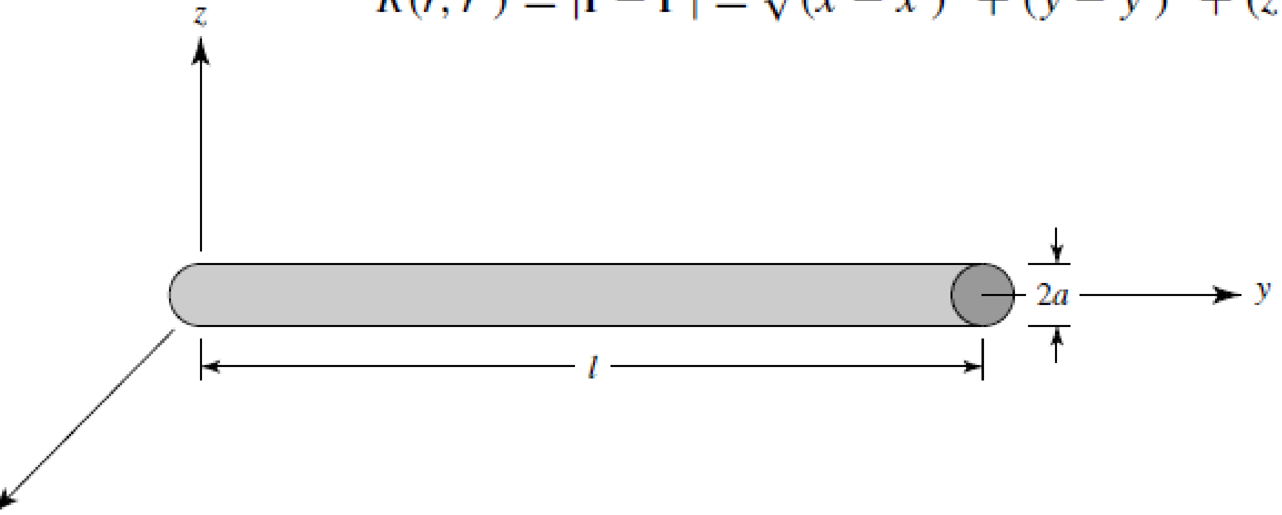
Method of Moments (MoM)

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_{\substack{\text{source} \\ \text{(charge)}}} \frac{\rho(r')}{R} dl'$$

$$R(r, r') = |\mathbf{r} - \mathbf{r}'| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$



Problema directo

$$1 = \frac{1}{4\pi\epsilon_0} \int_0^1 \frac{\rho(y')}{R(y, y')} dy', \quad 0 \le y \le l$$

$$R(y, y') = R(r, r')|_{x=z=0} = \sqrt{(y - y')^2 + a^2}$$

y' denotes the source coordinates.

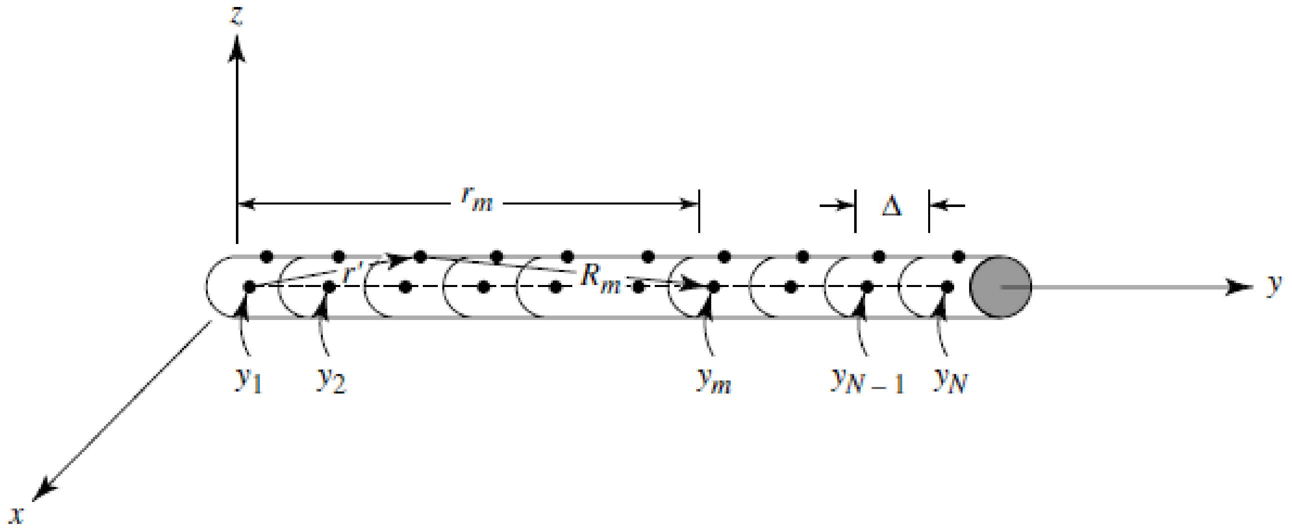
y denotes the observation coordinates.

Problema inverso

Method of Moments (MoM)

$$\rho(y') = \sum_{n=1}^{N} a_n g_n(y')$$

$$4\pi\epsilon_0 = \int_0^l \frac{1}{R(y, y')} \left[\sum_{n=1}^{N} a_n g_n(y') \right] dy'$$



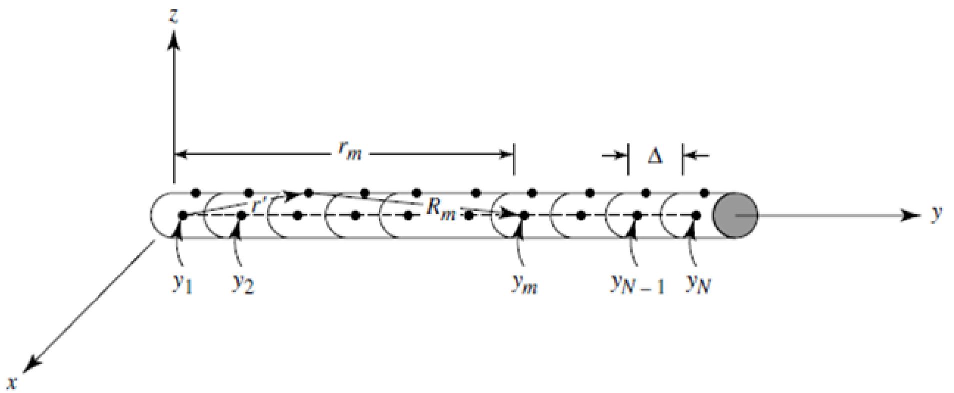
$$4\pi\epsilon_0 = \sum_{n=1}^{N} a_n \int_0^l \frac{g_n(y')}{\sqrt{(y-y')^2 + a^2}} dy'$$

Se aproxima la distribución de cargas (función continua) por una suma de funciones conocidas ***gn(y')*** y coeficientes ***an*** desconocidos.

Method of Moments (MoM)

$$g_n(y') = \begin{cases} 0 & y' < (n-1)\Delta \\ 1 & (n-1)\Delta \le y' \le n\Delta \\ 0 & n\Delta < y' \end{cases}$$

$$4\pi\epsilon_0 = a_1 \int_0^{\Delta} \frac{g_1(y')}{R(y_m, y')} dy' + a_2 \int_{\Delta}^{2\Delta} \frac{g_2(y')}{R(y_m, y')} dy' + \dots$$
$$+ a_n \int_{(n-1)\Delta}^{n\Delta} \frac{g_n(y')}{R(y_m, y')} dy' + \cdots + a_N \int_{(N-1)\Delta}^{l} \frac{g_N(y')}{R(y_m, y')} dy'$$



La forma más sencilla es suponer que en cada tramo Δ la distribución es constante (y amplitud ***an*** desconocida).

Las integrales están en función de ***y'*** se pueden evaluar pero cada término queda en función del punto de observación ***ym.***

Method of Moments (MoM)

$$4\pi\epsilon_0 = a_1 \int_0^{\Delta} \frac{g_1(y')}{R(y_m, y')} dy' + a_2 \int_{\Delta}^{2\Delta} \frac{g_2(y')}{R(y_m, y')} dy' + \dots$$
$$+ a_n \int_{(n-1)\Delta}^{n\Delta} \frac{g_n(y')}{R(y_m, y')} dy' + \dots + a_N \int_{(N-1)\Delta}^{l} \frac{g_N(y')}{R(y_m, y')} dy'$$

$$4\pi\epsilon_0 = a_1 \int_0^{\Delta} \frac{g_1(y')}{R(y_1, y')} dy' + \dots + a_N \int_{(N-1)\Delta}^{l} \frac{g_N(y')}{R(y_1, y')} dy'$$
$$\vdots$$
$$4\pi\epsilon_0 = a_1 \int_0^{\Delta} \frac{g_1(y')}{R(y_N, y')} dy' + \dots + a_N \int_{(N-1)\Delta}^{l} \frac{g_N(y')}{R(y_N, y')} dy'$$

Tomando **N** puntos de observación ***ym*** diferentes se plantea un sistema de **NxN** ecuaciones en las cuales las integrales se puede evaluar (se conoce ***y'*** e ***ym***)

Method of Moments (MoM)

$$Z_{mn} = \int_0^l \frac{g_n(y')}{\sqrt{(y_m - y')^2 + a^2}}\, dy'$$

$$= \int_{(n-1)\Delta}^{n\Delta} \frac{1}{\sqrt{(y_m - y')^2 + a^2}}\, dy'$$

$$Z_{mn} = \begin{cases} 2\ln\left(\dfrac{\frac{\Delta}{2} + \sqrt{a^2 + \left(\frac{\Delta}{2}\right)^2}}{a}\right) & m = n \\ \ln\left\{\dfrac{d_{mn}^+ + [(d_{mn}^+)^2 + a^2]^{1/2}}{d_{mn}^- + [(d_{mn}^-)^2 + a^2]^{1/2}}\right\} & m \neq n \text{ but } |m-n| \leq 2 \\ \ln\left(\dfrac{d_{mn}^+}{d_{mn}^-}\right) & |m-n| > 2 \end{cases}$$

$$d_{mn}^+ = l_m + \frac{\Delta}{2}$$

$$d_{mn}^- = l_m - \frac{\Delta}{2}$$

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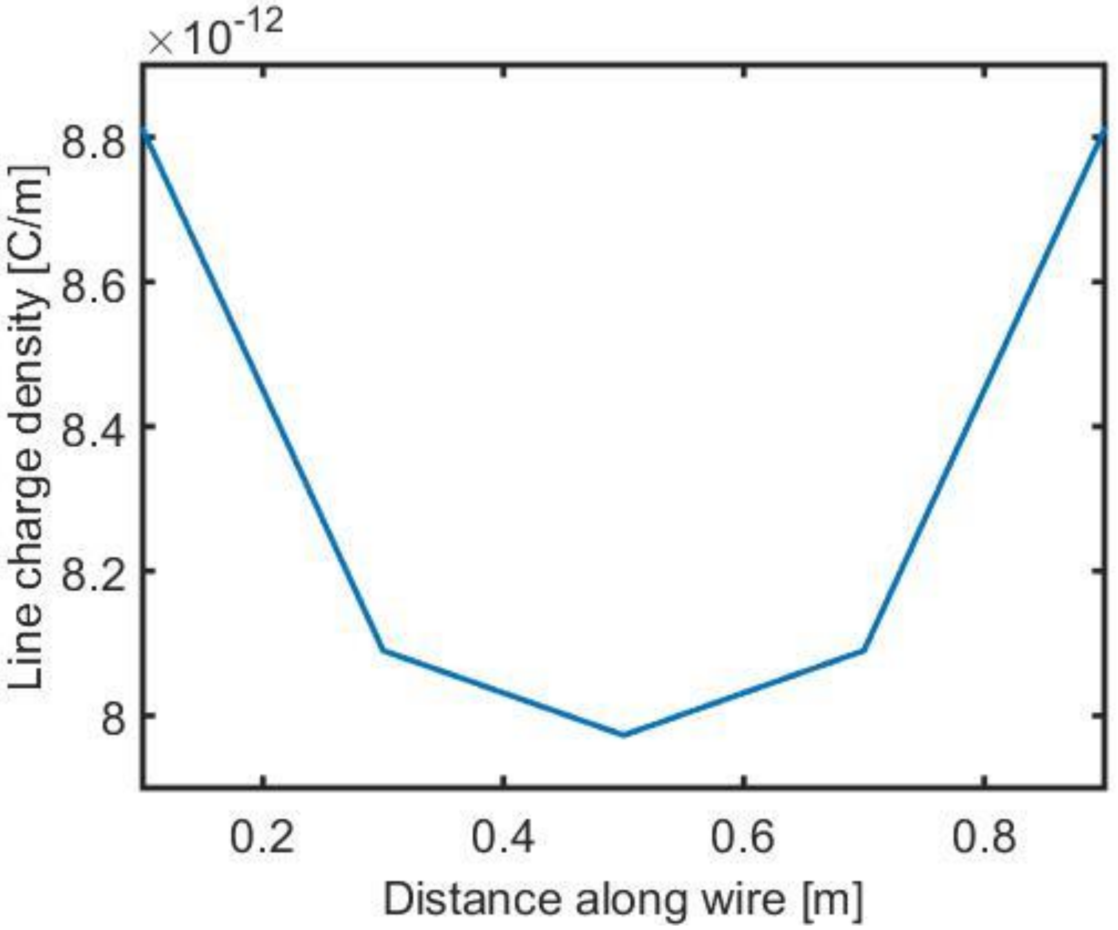
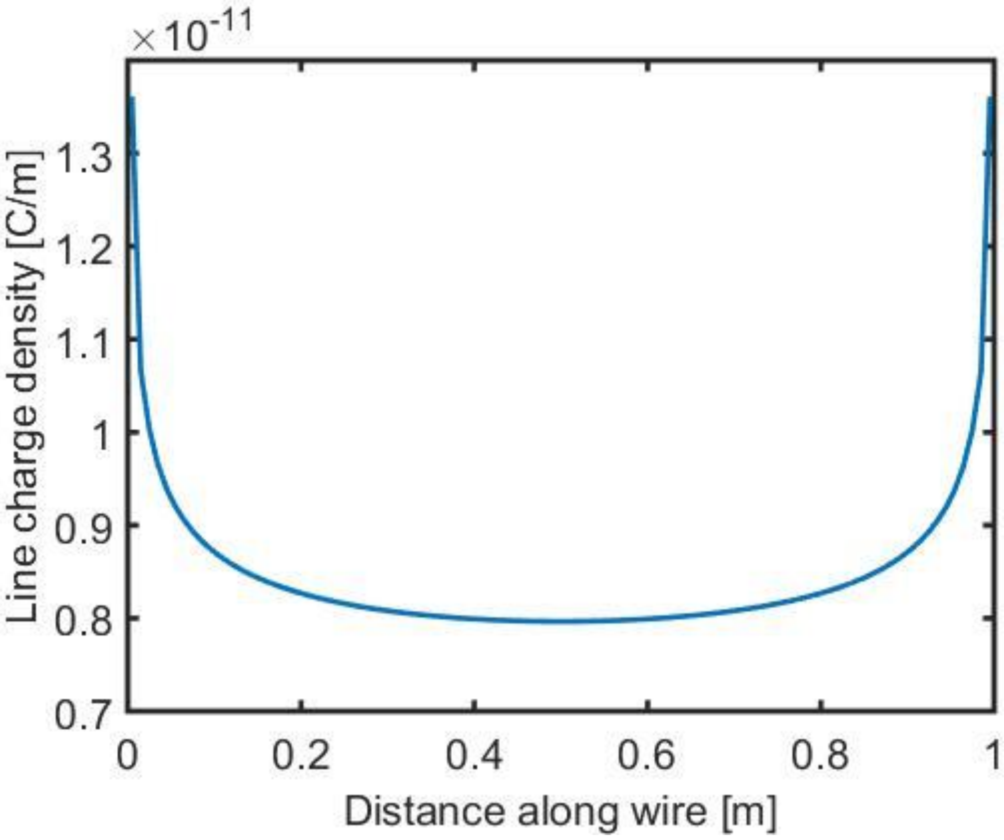
Method of Moments (MoM)

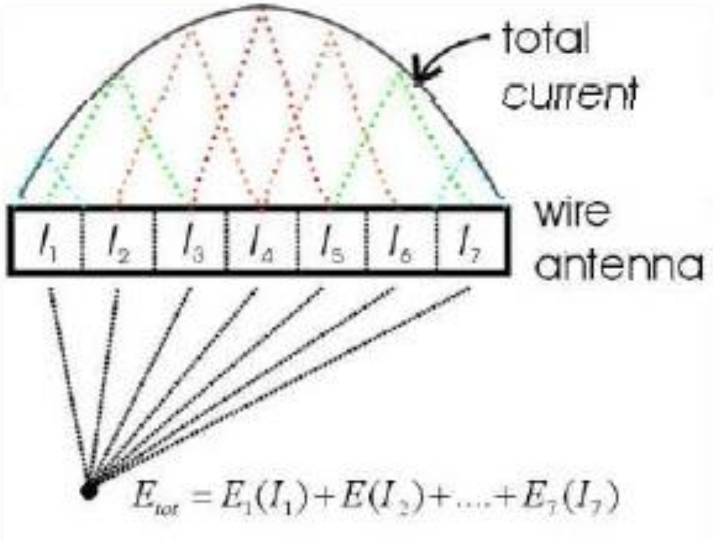




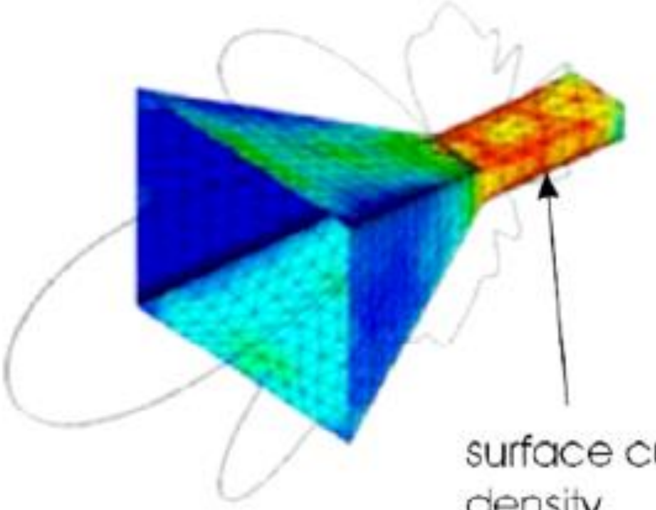
La rutina de Matlab static_mom.m del libro de Davidson resuelve el problema para distintos N.

Method of Moments (MoM)

- **Only the conducting surfaces of the structure are discretized**
  - Current in each cell is calculated
  - Method is suited best for metallic objects such as antennas
  - Nearby objects (such as the user) and dielectric parts are difficult to solve



- **Frequency-domain method**
  - Every single frequency point needs to be solved separately



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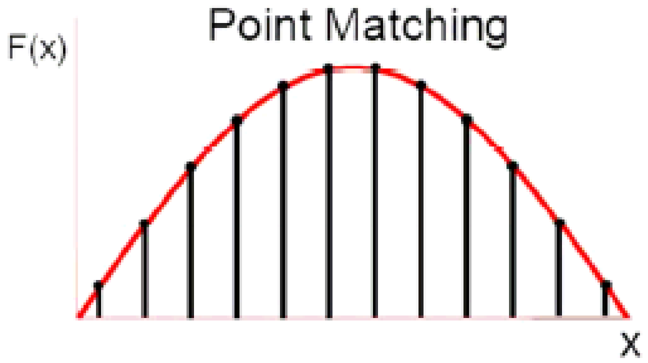
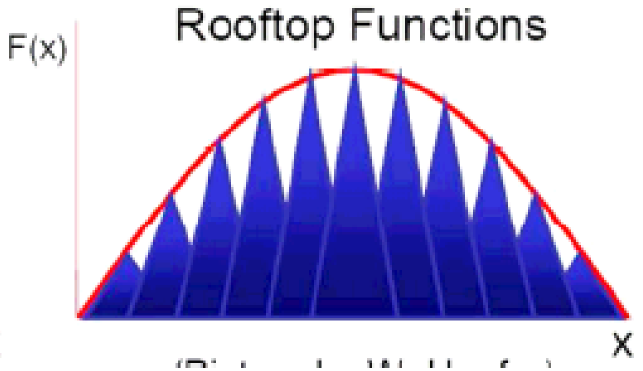
Method of Moments (MoM)

- **Only conducting surfaces are discretized**
- **Basis-function expansions, possibility to choose various functions**
- **Various possibilities to choose test functions (collocation method, Galerkin's method)**
- **Set of linear equations**
- **Dense matrix**
- **Well suited for wire antennas (1D approximation of current distribition), patch antennas, metal bodies (like cars)**





Finite Element Method (FEM)

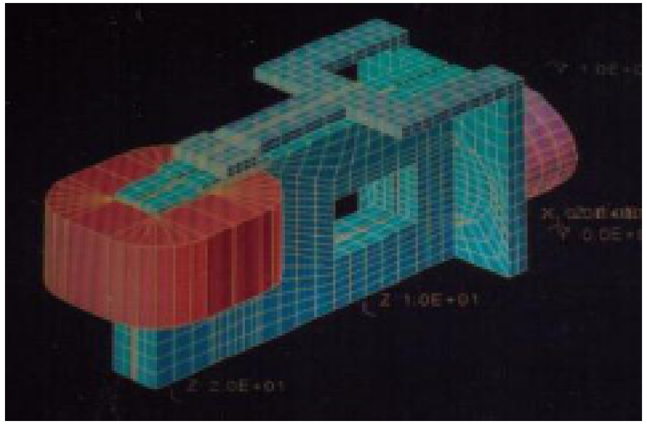
- Variational method: Minimizing an energy functional

<u>**3D time harmonic EM problem→**</u>

$$F = \int_V \left\{ \frac{\mu |H|^2}{2} + \frac{\varepsilon |E|^2}{2} - \frac{J \cdot E}{2j\omega} \right\} dV$$

energy stored in magnetic & electric fields

energy dissipated/supplied by conduction currents



Procedures of FEM Analysis:

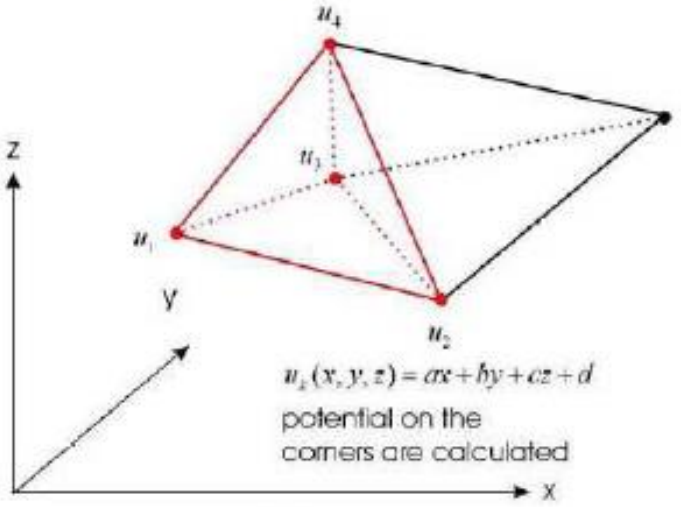
- Discretizing solution regions into finite number of *subregions* or *elements*
- Deriving governing equations (elemental equation) for a typical element
- Assembling of all elements in the solution region to form matrix equation
- Solving the system of equations obtained.

Solving the Generalized Poisson Equation Using the Finite-Difference Method (FDM)

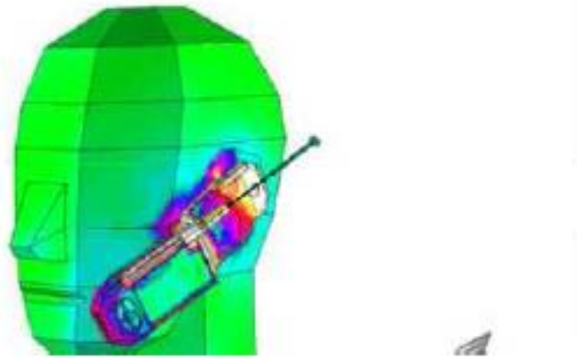
Introduction to the Finite Element Method
James R. Nagel
Department of Electrical and Computer Engineering
University of Utah, Salt Lake City, Utah
April 4, 2012

Finite Element Method (FEM)

- **Whole simulation domain is discretized by using polyhedrons**
  - Suitable method for arbitrary shaped objects
  - Dielectric materials can be treated easily
  - Method suited e.g. for simulating a mobile phone antenna with covers, battery, screen, etc.
  - Simulations with large simulation domains take long time



- **Frequency-domain method**



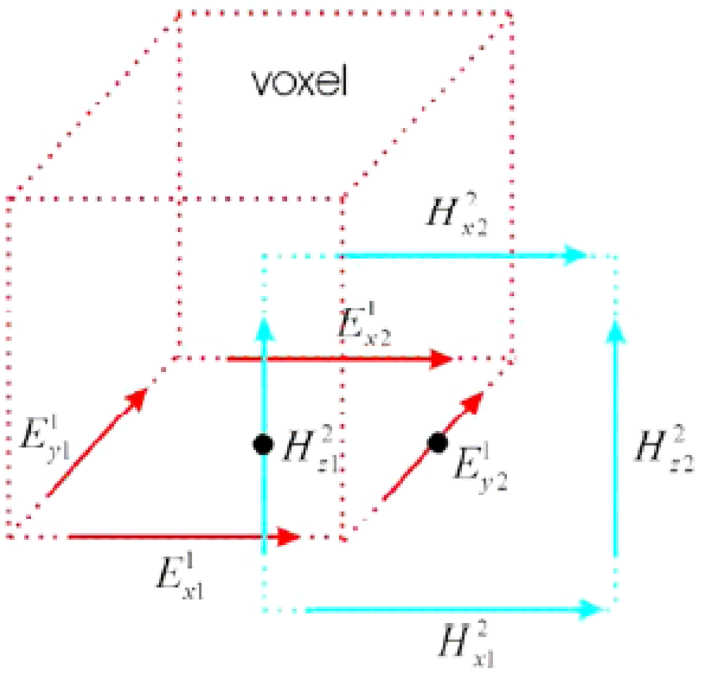
- ***Popular simulators based on FEM is HFSS by Ansoft and Comsol Multiphysics by Comsol***

Finite Difference Time Domain (FDTD)

- **The whole simulation domain is discretized by using cubes (called *voxels*)**
  - Dielectric materials easy to model
  - Problems with arbitrary (e.g. round-shaped) objects

- **Time-domain method**
  - Good for broadband simulations
  - Low-loss resonant structures store energy, and simulation time is long

- **Popular FDTD-based simulators are *SEMCAD-X* by Speag and *Microwave Studio* by CST**



magnetic field component H^2_{z1} is calculated from the surrounding electric field components E^1_{x1}, E^1_{y1}, E^1_{x2} and E^1_{y1} by using discretized Maxwell's equations

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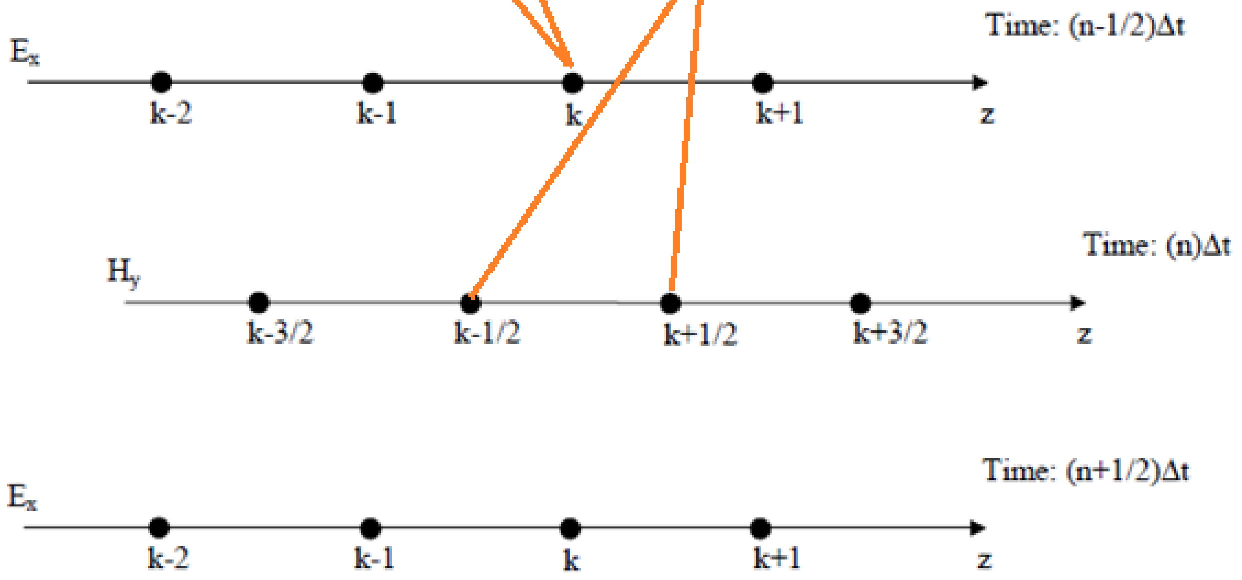
Finite Difference Time Domain (FDTD)

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}$$

$$\frac{E_x^{n+1/2}(k) - E_x^{n-1/2}(k)}{\Delta t} = -\frac{1}{\varepsilon_0}\frac{H_y^{n}(k+1/2) - H_y^{n}(k-1/2)}{\Delta z}$$

$$\frac{H_y^{n+1}(k+1/2) - H_y^{n}(k+1/2)}{\Delta t} = -\frac{1}{\mu_0}\frac{E_x^{n+1/2}(k+1) - E_x^{n+1/2}(k)}{\Delta z}$$



Finite Difference Time Domain (FDTD)

$$E_x^{n+1/2}(k) = E_x^{n-1/2}(k) + \frac{1}{\sqrt{\mu_0\varepsilon_0}}\frac{\Delta t}{\Delta z}\left(H_y^n(k-1/2) - H_y^n(k+1/2)\right)$$

$$\Delta t \leq \frac{\Delta z}{c_0}$$

$$H_y^{n+1}(k+1/2) = H_y^n(k+1/2) + \frac{1}{\sqrt{\mu_0\varepsilon_0}}\frac{\Delta t}{\Delta z}\left(E_x^{n+1/2}(k) - E_x^{n+1/2}(k+1)\right).$$

```
% Main loop (Loop C)
for t = 1:numSteps
  % (Loop A)
  % Calculate Ex field. Note that first
  % cell is skipped in the loop, since we
  % need to access k-1
  for k = 2:numCells
    Ex(k) = Ex(k) - 0.5*(Hy(k)-Hy(k-1));
  end
  %Ex(numCells)=0;% Condición de borde en el extremo para CC
  % Add E field source excitation with a Gaussian pulse.
  Ex(1) = exp(-0.5*((t0-t)/spread)^2); % Hard source
  % (Loop B)
  % Calculate Hy field. Note that last
  % cell is skipped in the loop, since we
  % need to access k+1
  for k = 1:numCells-1
    Hy(k) = Hy(k) - 0.5*(Ex(k+1)-Ex(k));
  end
  % Vectores que almacenan el campo E en ambos extremos y en el medio
  El(t)=Ex(numCells);
  Em(t)=Ex(numCells/2);
  Es(t)=Ex(1);
```

Comparison

	MoM	FEM	FDTD
Discretization	Only wires or surfaces	Entire domain (tetrahedron)	Entire domain (cube)
Solution method	FD, linear equations, full matrix	FD, linear equations, sparse matrix	TD, iterations
Boundary conditions	No need for special BC	Absorbing boundary conditions	Absorbing boundary conditions
Numerical effort	$\sim N^3$	$\sim N^2$	$\sim N$

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