

# An introduction to cellular IoT: signal processing aspects of NB-IoT

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# Outline...

## NB-IoT layers

NB-IoT layer functions: PDCP, RLC, MAC  
Congestion control : ACB and EAB  
Power efficiency: PSM, I-DRX, C-DX

## NB-IoT Idle and Connected Mode Procedures

Idle Mode  
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## NB-IoT DL synchronization methods

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## NB-IoT UL synchronization

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## NB-IoT in ISM bands

Characteristics - Alternatives for NB-IoT

## NB-IoT UL Synchronization: Random access design

A new single tone signal with frequency hopping has been designed for NB-IoT physical random access channel (NPRACH).

We describe the new NPRACH design and explain in detail the design rationale.

We further explore possible receiver algorithms for NPRACH detection and time-of-arrival estimation.

Simulation results on NPRACH performance including detection rate, false alarm rate, and time-of-arrival estimation accuracy are also included.

# NB-IoT UL Synchronization: NPRACH Preamble...

Before the NB-IoT random-access process is begun, a UE first uses NPSS to synchronize itself with the symbol timing and carrier frequency of the eNB.

Moreover, by measuring the received NRS power, the UE can determine which of the three coverage classes it belongs to.

Then, from the SIB embedded in the NPDSCH, the UE determines the starting time and length for the transmission of its preamble sequences.

Finally, among the 48 available preamble sequences, the UE selects one sequence and transmits it in NPRACH.

## NB-IoT UL Synchronization: Random access design...

In LTE, a set of random access preambles based on Zadoff-Chu (ZC) sequences is configured within a cell. Though ZC sequences are constant-envelope, the PRACH signal after up-sampling and filtering has a peak-to-average power ratio (PAPR) in the range of 2 to 7 decibels.

The higher the PAPR, the more the required power amplifier backoff. Power amplifier backoff also gives rise to degraded power amplifier efficiency, and thus has a negative impact on device battery life time.

A new single tone PRACH signal with frequency hopping has been designed for NPRACH in NB-IoT. This new NPRACH signal has extremely low PAPR and thus significantly reduces the need for power amplifier backoff.

## NB-IoT UL Synchronization: Random access design...

Consider an OFDMA system with bandwidth  $W$  Hz and  $B$  Hz subcarrier spacing. The FFT size  $L$  is usually chosen larger than  $W/B$  subcarriers. Accordingly, the baseband sampling rate can be conveniently chosen as  $LB$  Hz.

For NPRACH in NB-IoT with  $W = 180$  kHz and  $B = 3.75$  kHz, there are 48 subcarriers available.

The random access preamble design is based on single subcarrier transmission with frequency hopping within a configured NPRACH band within the OFDM resource grid.

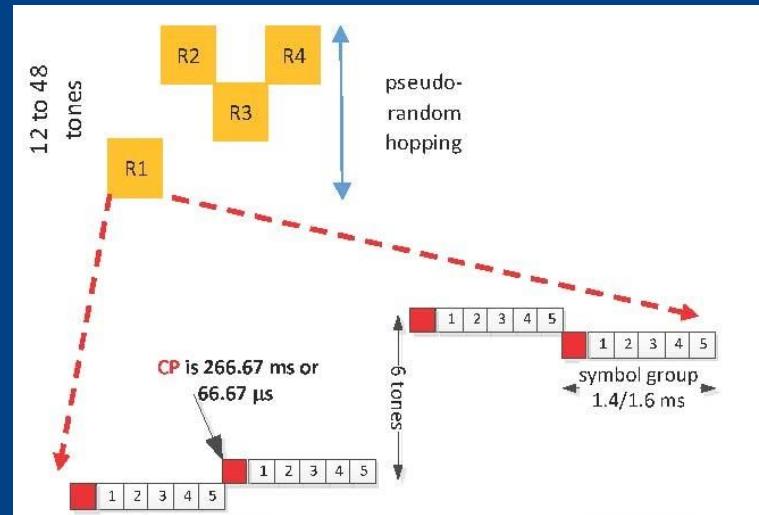


Figure 2-2:NPRACH inter and intra repetition hopping.

# NB-IoT UL Synchronization: Random access design...

## Symbol Group

Classical OFDM symbol structure: a cyclic prefix (CP) portion and a data symbol.

To maintain the orthogonality of the random access transmissions on different subcarriers, CP has to be long enough to accommodate the timing uncertainty in the cell (that can be as large as the maximum round-trip delay plus additional channel delay spread and DL synchronization errors).

NB-IoT systems target large cell deployment. For example, 35 km cell size requires a CP at least as long as 233.3 us (comparable to the duration of a data symbol with  $B = 3.75$  kHz subcarrier spacing).

To reduce CP overhead, we can repeat each  $L$ -sample OFDM symbol  $\xi$  times, and then add a single CP of  $L_{cp}$  samples. **Symbol group (SG)** is  $L_{cp} + \xi L$  samples. Higher  $\xi$ , small CP overhead. However,  $\xi$  should be kept small enough such that channel variation is negligible within SG (to avoid ICI).

# NB-IoT UL Synchronization: Random access design...

## Symbol Group...

A random access preamble consists of  $N$  SG and uses one subcarrier at every SG for transmission.

$N$  is determined by the target operating SNR or the target maximum coupling loss (MCL). Since NB-IoT supports different coverage classes up to 164 dB MCL, networks can configure different values of  $N$  to cater for different coverage requirements.

In NPRACH, two CP lengths are specified: 266.7 us (for large cells with radii in the range 8 to 35 km) and 66.7 us (for cells with radii smaller than 8 km).

The number  $\xi$  of symbols in a SG is specified to be 5.

# NB-IoT UL Synchronization: Random access design...

## Hopping Pattern

The hopping pattern consists of both inner layer fixed size hopping and outer layer pseudo-random hopping.

Outer layer pseudorandom hopping is applied between groups of 4 SG. Inner layer fixed size hopping is applied within every 4 SG.

1<sup>st</sup> level single-subcarrier hopping is used between 1<sup>st</sup> and 2<sup>nd</sup> and between 3<sup>rd</sup> and 4<sup>th</sup> SG. 2<sup>nd</sup> level 6-subcarrier hopping is used between the 2<sup>nd</sup> and 3<sup>rd</sup> SG.

We consider a generic hopping pattern: subcarrier index used by SG  $m$  is  $\Omega(m)$ ,  $m = 0, \dots, N-1$ , where  $\Omega(\cdot)$  is a generic mapping from SG index to subcarrier index.

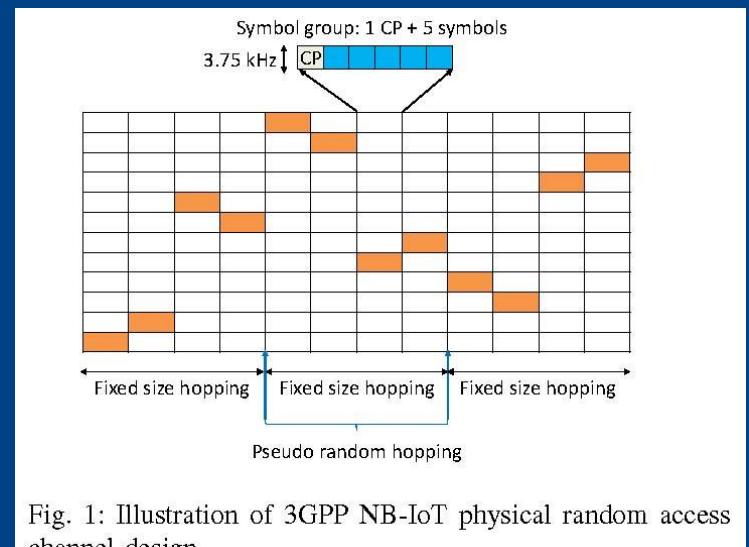


Fig. 1: Illustration of 3GPP NB-IoT physical random access channel design.

# NB-IoT UL Synchronization: Random access design...

## Hopping Pattern...

Assume by now  $\xi = 1$ . Let  $u[m]$ ,  $m=0, \dots N-1$ , the sequence of a random access preamble.

The baseband equivalent digital-domain signal for the random access preamble transmission can be written as

$$s[n; m] = \frac{\sqrt{E}}{L} \sum_k S[k; m] e^{-j2\pi \frac{k}{L} n} \quad n = -L_{cp}, \dots, L-1$$

where  $s[n; m]$  is the  $n$ -th time-domain sample of the  $m$ -th SG,  $E$  is the transmit energy per sample, and  $S[k; m]$  is the symbol on the  $k$ -th subcarrier during the  $m$ -th SG.

If  $h[n; m]$  is the channel coefficient at the  $n$ -th time-domain sample of the  $m$ -th SG,

$$h[n; m] = a[m] \delta[n]$$

where  $a[m]$  is the channel gain at the  $m$ -th SG. The implicit assumption is that the channel is invariant within one SG.

## NB-IoT UL Synchronization: Random access design...

### Hopping Pattern...

Let  $D$  the round-trip delay, i.e. the time-of-arrival (ToA), to be estimated by eNB, for the timing advance command.

We assume the CP is long enough to cover the maximum  $D$ , thus  $D \in [0, \dots, L_{cp}-1]$ .

Then, the  $n$ -th sample of the  $m$ -th SG at the e-NodeB is given by

$$y[n; m] = a[m] \frac{\sqrt{E}}{L} e^{j2\pi\Delta f(n-D+m(L_{cp}+N))} \sum_k S[k; m] e^{j2\pi\frac{k}{L}(n-D)} + v[n; m]$$

where  $\Delta f$  is the residual CFO, normalized by the sampling rate  $NB$ , and  $v[n; m]$  is a complex AWGN  $\mathcal{CN}(0; N_0)$ .

The residual CFO may be due to imperfect carrier frequency estimation in the DL cell search.

# NB-IoT UL Synchronization: Random access design...

## Hopping Pattern...

For each SG with  $\xi = 1$ , the receiver discards the first  $L_{cp}$  samples and performs a FFT on the remaining  $L$  samples of the preamble  $u[m]$ .

At SG  $m$ , if  $\ell = \Omega(m)$ , it can be shown that the received symbol on the  $\ell$ -th subcarrier at SG  $m$  is given by

$$Y[\ell; m] = B(\Delta f, D)a[m]u[m]e^{j2\pi\Delta fm(L_{cp}+L)}e^{-j2\pi\frac{\Omega(m)}{L}D} + V[\ell; m]$$

where

$$B(\Delta f, D) = \sqrt{E}\frac{\sin(L\pi\Delta f)}{L\sin(\pi\Delta f)}e^{j2\pi\Delta f(\frac{L-1}{2}-D)}$$

For  $\xi > 1$ , the difference is that eNB receives  $\xi$  symbols for every SG. With a slight abuse of notation, the  $i$ -th received symbol in SG  $m$  is

$$\tilde{Y}[i; m] = B(\Delta f, D)a[m]u[m]e^{j2\pi\Delta f(m(L_{cp}+\xi L)+iL)}e^{-j2\pi\frac{\Omega(m)}{L}D} + \tilde{V}[i; m]$$

## NB-IoT UL Synchronization: NPRACH design: receiver algorithms

Here we discuss how eNB can detect the random access preamble and estimate the ToA.

We start with ToA estimation and then utilize the corresponding ToA estimation statistic to detect the presence of the preamble.

ToA estimation is to obtain  $D$  from the  $N$  observations  $\tilde{Y}[i; m]$  in the presence of additive noise and  $N+1$  additional unknown parameters  $a[m]$ ,  $m=0, \dots, N-1$  and  $\Delta f$ . Therefore, a joint estimator of all these parameters has high complexity.

To simplify, we make the **approximation of block fading model** :  $a[m]$  do not change in a block of  $Q$  symbol groups but change independently over the blocks. We choose  $Q$  such that the number  $N/Q$  of blocks is an integer. If  $\tilde{a}_g$  is the nominal channel coefficient for block  $g$ , we have

$$a[m] = \tilde{a}_g, \quad \forall m = gQ, \dots, (g+1)Q - 1$$

and  $\{\tilde{a}_g\}$ ,  $g=0, \dots, N/Q-1$  are independent.

## NB-IoT UL Synchronization: NPRACH design: receiver algorithms...

With the block fading assumption, the ToA and residual CFO can be jointly estimated as

$$(D^*, \Delta f^*) = \arg \max_{D, \Delta f} J(D, \Delta f) = \arg \max_{D, \Delta f} \sum_{g=0}^{N/Q-1} |J_g(D, \Delta f)|^2$$

where

$$J_g(D, \Delta f) = \sum_{m=gQ}^{(g+1)Q-1} \sum_{i=0}^{\xi-1} \tilde{Y}[i; m] u^*[m] e^{-j2\pi\Delta f(m(L_{cp} + \xi L) + iL)} e^{j2\pi \frac{\Omega(m)}{L} D}$$

The estimate  $(D^*; \Delta f^*)$  is the one that yields the maximum correlation of the transmitted preamble symbols and the received symbols whose phase shifts due to ToA and residual CFO are corrected by the estimate.

Estimation takes the form of two-dimensional DTFT. As a result, the search for  $(D^*; \Delta f^*)$  can be carried out in the frequency domain by FFT.

## NB-IoT UL Synchronization: NPRACH design: receiver algorithms...

With  $(D^*; \Delta f^*)$  obtained, we next discuss how to determine the presence of the preamble.

In LTE, e-NodeB correlates the received signal with a hypothesis of ZC sequence based random access preamble. If the correlation result exceeds some predetermined threshold, the e-NodeB declares the presence of the preamble; otherwise, it declares that the preamble is not present.

Similar approach can be used for the detection of NPRACH in NB-IoT.

The natural choice of the statistic used to compare against the detection threshold is  $J(D^*; \Delta f^*)$ .

## NB-IoT UL Synchronization: NPRACH design: receiver algorithms...

It is well known that two error events may arise with this threshold-based preamble detection.

- Misdetection: The random access preamble is present, but the statistic  $J(D^*; \Delta f^*)$  does not exceed the detection threshold.
- False alarm: The random access preamble is absent, but the statistic  $J(D^*; \Delta f^*)$  exceeds the detection threshold.

Clearly, there exists a trade-off in setting the detection threshold. Increasing the detection threshold lowers the false alarm rate at the cost of increased likelihood of misdetection.

For random access preamble detection in cellular systems, the detection threshold is usually chosen such that the false alarm rate is below some target.

## NB-IoT UL Synchronization: NPRACH design: some simulations results

The simulation assumptions (3GPP) used are summarized in the Table.

We consider the 3 coverage classes and configure 8, 32, and 128 SG for the 3 coverage classes, respectively.

The target operating SNRs are 14.25 dB, 4.25 dB, and -5.75 dB. According to 3GPP, these three SNR correspond to 144 dB MCL, 154 dB MCL, and 164 dB MCL, respectively.

While 144 dB MCL is the coverage level reachable by GSM/GPRS systems, an additional 20 dB coverage extension (i.e., 164 dB MCL) is targeted in NB-IoT systems.

TABLE I: Simulation Parameters

CP length	266.7 us
Subcarrier spacing	3.75 kHz
Symbol group	1 CP and 5 symbols
NPRACH band	12 subcarriers
Channel model	Typical urban
Doppler spread	1 Hz
Antenna configuration	1 Tx; 2 Rx
Timing uncertainty	Randomly chosen between 0 and CP
Frequency error	Uniformly drawn from $\{-50, 50\}$ Hz
Frequency drift	Uniformly drawn from $\{-22.5, 22.5\}$ Hz/s
No. of iterations	10,000 for detection; 100,000 for false alarm

## NB-IoT UL Synchronization: NPRACH design: some simulations results...

Table II summarizes the misdetection and false alarm probabilities of the NPRACH design under the configured 3 coverage classes.

It can be seen that the detection probabilities exceed 99%, while the false alarm probabilities are well below 0.1%.

Figure 2 shows the distributions of ToA estimation errors for the 3 coverage classes.

It can be seen that the ToA errors are within [-3, 3] us with very high confidence level for all three cases.

TABLE II: Misdetection and false alarm probabilities

	Coverage 1	Coverage 2	Coverage 3
Target SNR	14.25 dB	4.25 dB	-5.75 dB
No. of symbol groups	8	32	128
Misdetection	29/10,000	24/10,000	84/10,000
False alarm	0/100,000	0/100,000	13/100,000

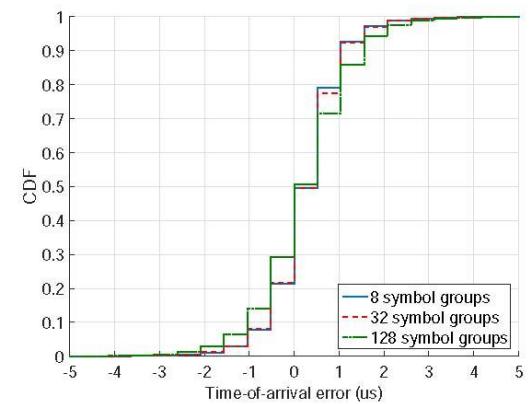


Fig. 2: Distributions of ToA estimation errors

# NB-IoT UL Synchronization: NPRACH: Another approach

## **Detection-based design**

We first estimate the UE power in the frequency domain and apply the Neyman – Pearson criterion to determine an explicit detection threshold for the NPRACH preamble.

Then, we analyze the theoretical detection probability and find the maximum coupling loss (MCL) for different channels (AWGN and Rayleigh).

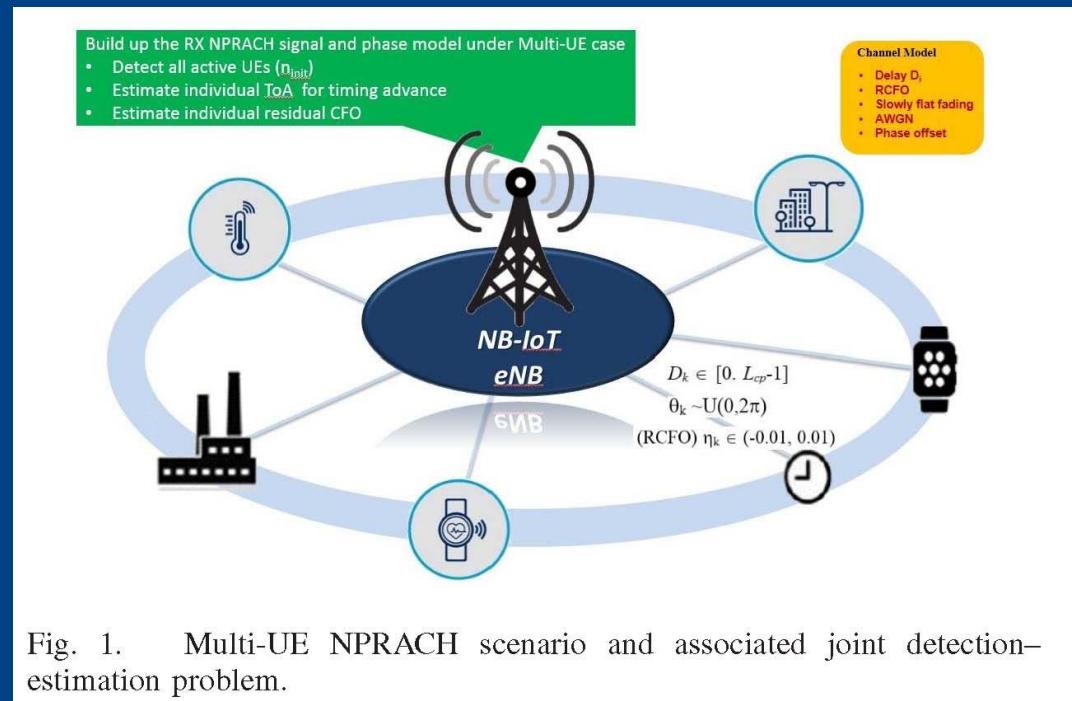
Finally, we exploit the post-FFT phase trace of each detected UE to design an efficient scheme for estimating ToA and residual CFO (RCFO) synchronization parameters.

# NB-IoT UL Synchronization: NPRACH: Detection-based design...

We consider a multi-UE scenario, where  $K$  UEs are assumed to send distinct preamble waveforms simultaneously.

With different round-trip delays (i.e., ToAs) and RCFOs, the  $K$  preamble signals are superimposed and received by the eNB.

Upon receiving the superimposed NPRACH signal, the eNB must detect all  $K$  UE identities and estimate the synchronization parameters (ToA and RCFO) for each UE.



# NB-IoT UL Synchronization: NPRACH: Detection-based design...

NPRACH employs an orthogonal single-tone frequency-hopping, i.e. the one-hop signal segment is a symbol group (SG): 5 repeated symbols and a cyclic prefix (CP).

To simplify the multi-UE scenario, up to  $K \leq 48$  UE can be simultaneously send their NPRACH preambles within the NB-IoT bandwidth of 180 kHz.

$UE_k$  is identified by its  $n_{init}(k)$  parameter in the range 0 – 47, which is used to generate the preamble hopping pattern for consecutive single-tone SG.

Hence, for  $UE_k$  and the  $i$ -th symbol of the  $n$ -th SG, its NPRACH is a single-tone signal with frequency  $f_k(n) \times \Delta f$ , where  $f_k(n) \in [0, 47]$  is the hopping index of the  $n$ -th SG of  $UE_k$ .

The 48 possible frequency-hopping patterns  $\{f_k(n), k=0 - 47, n=0 - (N-1)\}$  have initial subcarrier index  $f_k(0) = n_{init}(k)$ , and NPRACH hopping patterns distinct.

# NB-IoT UL Synchronization: NPRACH: Detection-based design...

The transmitted  $L \times 1$  single-tone baseband signal vector for  $UE_k$  and the i-th symbol of the n-th SG can be expressed as

$$\mathbf{s}_{k,n,i} = [s_{k,n,i}(0), \dots, s_{k,n,i}(L-1)]$$

where

$$s_{k,n,i}(l) = e^{j2\pi f_k(n)l/N}$$

The entire TD signal vector of the n-th SG can thus be expressed as

$$\mathbf{g}_{k,n} = \text{CP}\{\text{REP}_5(\mathbf{s}_{k,n,i})\}$$

where  $\text{REP}_r(\mathbf{a})$  represents the repeated concatenation of the same column vector  $\mathbf{a}$  for  $r$  times and  $\text{CP}\{\cdot\}$  is the CP addition operation, resulting in a length of  $5L + L_{cp}$  for  $\mathbf{g}_{k,n}$ .

Furthermore, 4 SG constitute 1 preamble, and the NPRACH signal may consist of several preambles.

# NB-IoT UL Synchronization: NPRACH received signal model

Next, we construct the received baseband signal model at the eNB in the multi-UE case.

Because the UEs may be located at different distances from the eNB, their NPRACH signals undergo different round-trip propagation delays and thus have different ToAs  $D_k \in [0 \dots L_{cp} - 1]$  in number of samples.

The channel is modeled as a single-tap, flat Rayleigh fading model and varies extremely slowly. Hence, the signal fading of  $UE_k$  can be simply modeled by a complex fading coefficient  $h_k \sim \mathcal{CN}(0, \sigma_k^2)$ , where  $\sigma_k^2$  is the average received signal power of  $UE_k$ .

# NB-IoT UL Synchronization: NPRACH received signal models

Each UE still has a small RCFO  $\eta_k$  normalized to  $\Delta f$ . Therefore, the signal received by the eNB is the superimposed  $K$  NPRACH preambles with all above effects, along with a bandlimited AWGN with power  $\sigma_n^2$ .

The input TD SNR for  $UE_k$  is defined as  $\text{SNR}_i(k) = \sigma_k^2 / \sigma_n^2$

At eNB, the received RF signal is first down-converted. Then, the baseband receiver performs common CP removal and takes the 512-point FFT of each symbol in the SG according to its own eNB timing.

Because the RFCO is relatively small in practice, ICI and MAI can be neglected.

# NB-IoT UL Synchronization: NPRACH received signal models

After some derivation, it can be shown that the post-FFT signal vector of the i-th symbol of the n-th SG has the following  $f_k(n)$ -th FFT bin

$$\begin{aligned} R(n, i, f_k(n)) &= L|h_k|S_L(\eta_k)e^{j\beta_{k,n,i}} + W(n, i) \\ \beta_{k,n,i} &= \frac{2\pi}{L}(\eta_k t(n, i) - D_k f_k(n)) + \theta_k + \phi_k + \varphi_k \end{aligned}$$

where  $S_L(\eta) = \frac{\sin(\pi\eta)}{L \sin(\pi\eta/L)} \cong 1$ ,  $\theta_k \sim U(0, 2\pi)$  is the carrier phase offset,  $\phi_k$  is the phase of the channel coefficient,  $\varphi_k = \pi \eta_k (L-1)/L$ ,  $W(n,i)$  is the post-FFT AWGN sample with power  $L\sigma_n^2$ , and  $t(n, i) = n(5L + L_{cp}) + iL$ .

That means, a faded magnitude term and several phase terms: the first related to RCFO  $\eta_k$ , the 2<sup>nd</sup> term related to ToA  $D_k$  (the remaining are constant over the whole preamble due to the slow fading channel).

# NB-IoT UL Synchronization: NPRACH received signal models

Assuming that  $UE_k$  is absent, we have

$$R(n, i, f_k(n)) = W(n, i)$$

which is distributed as  $C\mathcal{N}(0, \sigma_n^2)$  (independent of the symbols.) However, when  $UE_k$  is present, we have approximately

$$R(n, i, f_k(n)) = L|h_k|e^{j\beta_{k,n,i}} + W(n, i)$$

Note that  $|h_k|e^{j\beta_{k,n,i}} \sim C\mathcal{N}(0, \sigma_k^2)$ , because  $|h_k|$  is Rayleigh with  $E[|h_k|^2] = \sigma_k^2$  and  $\beta_{k,n,i}$  contains the random phase term  $\theta_k \sim U(0, 2\pi)$ .

Hence,  $R(n, i, f_k(n))$  will be distributed as  $C\mathcal{N}(0, L^2\sigma_k^2 + L\sigma_n^2)$ .

# NB-IoT UL Synchronization: NPRACH detection – estimation problem

Following NPRACH signal model and formulation, the twofold detection-estimation problem can now be stated:

- 1) *Detection*: Obtain the presence and identity parameters  $\{n_{init}(k)\}$  of the active UE with performance requirements of  $P_F \leq 0.1\%$  and  $P_D \leq 99\%$ .
- 2) *Estimation*: For each detected UE, find its ToA and RCFO parameters accurately and efficiently.

For 1), we calculate the sufficient test statistics (average power) for each possible UE and then determine the optimal threshold as a function of the preamble length in number of SGs. This is done using the Neyman-Pearson criterion.

For 2), we explore the phase terms in

$$R(n, i, f_k(n)) = L|h_k|e^{j\beta_{k,n,i}} + W(n, i)$$

# NB-IoT UL Synchronization: NPRACH detection & threshold setting

Because there are 48 distinct possible hopping patterns, we first retrieve all post-FFT bins for each UE as the UE's test dataset:

$$Q_k = \{R(n, i, f_k(n)) , n=0, \dots, N-1; i=0, \dots, 4\}$$

for  $UE_k$ , where  $N=4P$  is the number of SGs and  $P$  is the repetition number of the preambles.  $N$  can also be regarded as the decision delay in the UE detection process.

For different coverages, the eNB can specify  $P$  as: 1, 2, 4, 8, 16, 32, 64 or 128; then, the corresponding decision delays become:  $N= 4, 8, 16, 32, 64, 126, 256$  or 512.

Hence, if  $UE_k$  has a low receiving SNR, more data should be collected (i.e., large  $N$ ), leading to longer decision delay for distant or weakly powered UEs.

## NB-IoT UL Synchronization: NPRACH detection & threshold setting...

The hopping patterns are mutually orthogonal; thus, the superimposed NPRACH detection problem can be decoupled into a parallel of single-UE detection problems.

To test for the presence of  $UE_k$ , we collect the energy of the post-FFT bins along the hopping pattern of  $UE_k$  for  $N$  SGs, and note that the sufficient test statistics is

$$P_k(N) = \frac{1}{5N} \sum_{n=0}^{N-1} \sum_{i=0}^4 |R(n, i, f_k(n))|^2$$

$UE_k$  present

The decision rule for testing for the presence of  $UE_k$  is

$P_k(N)$	$>$	$\lambda$
	$<$	
$UE_k$ absent		

The ROC or test performance is described in terms of

$$P_F = \Pr(P_k(N) > \lambda / UE_k \text{ is absent}) \quad P_D = \Pr(P_k(N) > \lambda / UE_k \text{ is present})$$

Hence, the task of NPRACH detection problem is to determine the threshold  $\lambda$ .

# NB-IoT UL Synchronization: NPRACH detection & threshold setting...

We formulate the detection problem in terms of the Neyman-Pearson criterion as

$$\max\{P_D\}, \text{ such that } P_F \leq \alpha$$

For the NB-IoT case, we choose  $\alpha = 0.1\%$ .

Considering the Neyman-Pearson criterion, we derive the threshold  $\lambda$  as a function of the decision delay  $N$ , along with two other parameters:

- 1) The given false alarm significance level  $\alpha$  and
- 2) The post-FFT noise power  $P_n = L \sigma_n^2$ .

In practice, it is reasonable assumed that  $P_n$  can be measured at the receiver.

# NB-IoT UL Synchronization: NPRACH detection & threshold setting...

Then, the threshold  $\lambda$  should satisfy the relation

$$\int_{\lambda}^{\infty} g_n(x)dx = \alpha$$

where  $g_n(x)$  is the pdf of the test statistics  $P_k(N)$  in the noise-only case.

Because  $P_k(N)$  is the average power of  $10N$  independent real Gaussian RVs with identical distribution  $\mathcal{N}(0, P_n/2)$ , the pdf  $g_n(x)$  is given by a **scaled central Chi-squared distribution with  $10N$  degrees of freedom**

$$g_n(x) = \kappa f_c(\kappa x; 10N)$$

where

$$f_c(x; m) = \frac{1}{2^{m/2}\Gamma(m/2)} x^{\frac{m}{2}-1} e^{-\frac{x}{2}} u(x)$$

is the standard  $\chi_m^2$  pdf with  $m$  degrees of freedom and  $\kappa = 10N/P_n$  is a scale factor.

# NB-IoT UL Synchronization: NPRACH detection & threshold setting...

The cumulative distribution function (CDF) of the standard  $\chi_m^2$  pdf is given by

$$F_c(x; m) = \int_0^x f_c(\lambda; m) d\lambda = \frac{1}{\Gamma(\frac{m}{2})} \gamma\left(\frac{m}{2}, \frac{x}{2}\right) u(x)$$

where  $\gamma(s, t) = \int_0^x t^{s-1} e^{-t} dt$  is the lower incomplete Gamma function.

# NB-IoT UL Synchronization: NPRACH detection & threshold setting...

Given the false alarm level  $\alpha$  and the decision delay  $N$ , we can explicitly determine the optimal Neyman – Pearson threshold  $\lambda_0$  as

$$\lambda_0 = \frac{1}{\kappa} F_c^{-1}(1 - \alpha; 10N) = P_n \times \left( \frac{1}{10N} F_c^{-1}(1 - \alpha; 10N) \right) = P_n \times A(N; \alpha)$$

To obtain  $\lambda_0$  for a given  $\alpha$  and  $N$ , we can first measure the post-FFT average noise power  $P_n$  by using the noise-only resource grids.

Then multiplying the  $P_n$  estimate by the factor  $A(N; \alpha)$  gives the optimal threshold  $\lambda_0$ .

As illustrated, the factor is always larger than one but decreases as  $N$  increases.

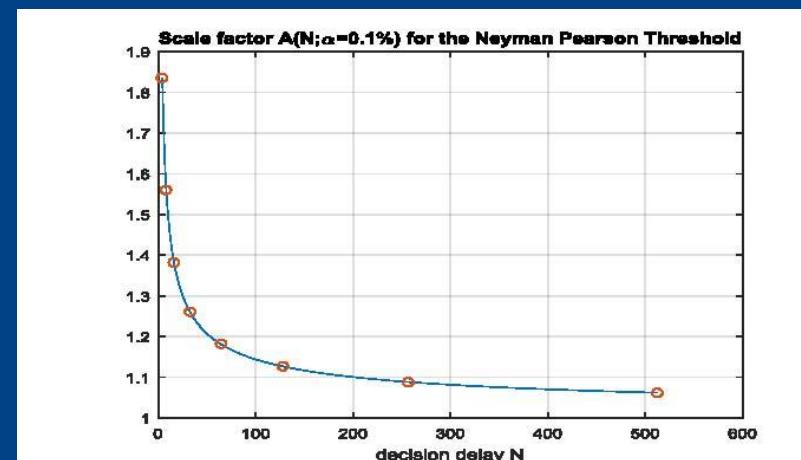


Fig. 3. Plot of  $A(N; \alpha = 0.1\%)$  versus decision delay  $N$ .

# NB-IoT UL Synchronization: Detection in AWGN and Rayleigh channels

In the AWGN channel case, we set  $h_k = 1$ , and  $R(n, i, f_k(n))$  has the nonzero mean  $Le^{j\beta_{k,n,i}}$ . Hence, the post-FFT signal power is  $P_0 = L^2$ , not random. The statistics  $P_k(N)$  under the signal-plus-noise case are characterized by the scaled non central Chi-square pdf as

$$g_s(x) = \kappa f_{nc}(\kappa x; 10N, \gamma_0)$$

where

$$f_{nc}(x; m, \gamma_0) = \frac{1}{2} e^{-\frac{x+\gamma_0}{2}} \left( \frac{x}{\gamma_0} \right)^{m/4-1/2} I_{m/2-1}(\sqrt{\gamma_0 x}) u(x)$$

is the standard non central Chi-square pdf with  $m$  degrees of freedom, in which the non centrality parameter  $\gamma_0$  is given by

$$\gamma_0 = 10N \times P_0/P_n = 10NL/\sigma_n^2$$

and  $I_v(x)$  is the modified Bessel function of the first kind with degree  $v$ .

# NB-IoT UL Synchronization: Detection in AWGN and Rayleigh channels...

The CDF of  $f_{nc}(x; m, \gamma_0)$  is given by

$$F_{nc}(x; m, \gamma_0) = \int_0^x f_{nc}(\lambda; m, \gamma_0) d\lambda = 1 - Q_{\frac{m}{2}}(\sqrt{\gamma_0}, \sqrt{x})$$

where

$$Q_M(a, b) = \int_b^\infty x \left(\frac{x}{a}\right)^{M-1} e^{-\left(\frac{x^2+a^2}{2}\right)} I_{M-1}(ax) dx$$

is the generalized Marcum Q-function.

# NB-IoT UL Synchronization: Detection in AWGN and Rayleigh channels...

The theoretical detection probability  $P_D$  under the AWGN channel can be derived as

$$P_{D,AWGN}(\gamma_0) = Q_{5N}(\sqrt{\gamma_0}, \sqrt{10N\lambda_0/P_n})$$

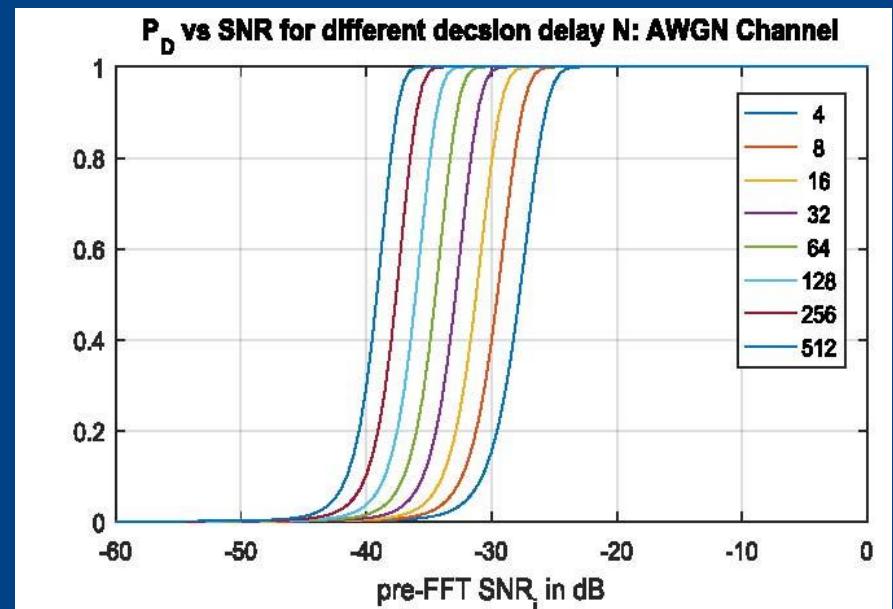


Fig. 4.  $P_{D,AWGN}$  versus  $\text{SNR}_i$  for different decision delays  $N$ .

# NB-IoT UL Synchronization: Detection in AWGN and Rayleigh channels...

Based on  $P_{D,AWGN}(\gamma_0)$ , we can compute  $P_D$  under the Rayleigh fading channel.

Assuming  $\sigma_k^2 = 1$  without loss of generality,  $\rho = |h_k|^2$  is an exponential RV with unity mean.

The mean post-FFT signal power  $P_s = E[\rho P_0] = P_0$  remains unchanged, and the non centrality parameter becomes  $\gamma = \rho \gamma_0$ .

Hence, the  $P_D$  can be found by averaging  $P_{D,AWGN}(\gamma_0)$  over the pdf of  $\rho$ , which results in

$$P_{D,Fading}(\gamma_0) = \int_0^\infty e^{-\rho} Q_{5N}(\sqrt{\rho\gamma_0}, \sqrt{10N\lambda_0/P_n}) d\rho$$

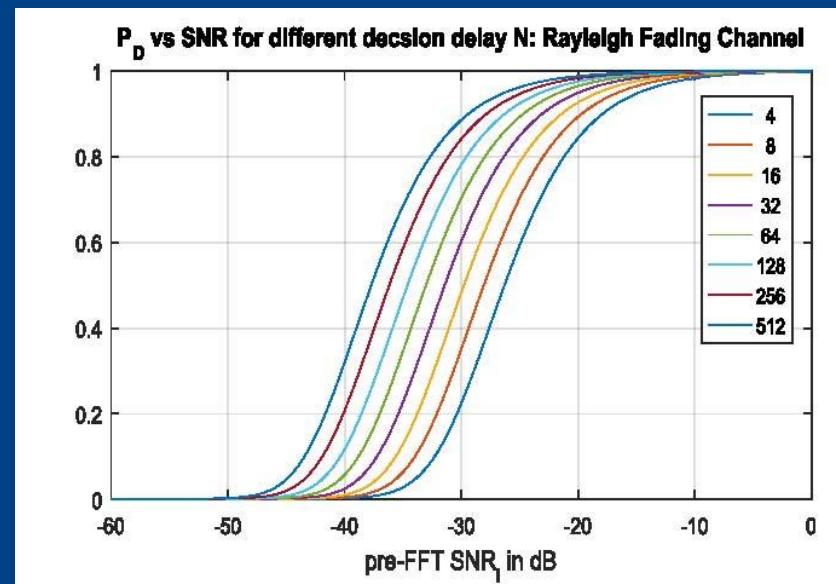


Fig. 5.  $P_{D,Fading}$  versus  $\text{SNR}_i$  for different decision delays.

# NB-IoT UL Synchronization: MCL & coverage analysis

According to the NB-IoT standard, a UE has 3 coverage classes, which use different numbers of preamble repetitions.

Additionally, the NB-IoT requires the detection probability to be no less than 99 %. Hence, we first find the corresponding pre-FFT  $SNR_i$ ,  $SNR_{99}$ , that satisfies  $P_D = 99\%$ .

We obtain the required  $SNR_{99}$  for different values of  $N$  from previous figures.

TABLE I  
REQUIRED  $SNR_{99}$  CL AND CELL COVERAGE

<b><i>N</i></b>	<b>4</b>	<b>8</b>	<b>16</b>	<b>32</b>	<b>64</b>	<b>128</b>	<b>256</b>	<b>512</b>
<b><math>SNR_{99}</math> dB (AWGN)</b>	-24	-26	-28	-30	-31	-33	-35	-37
<b><math>SNR_{99}</math> dB (Fading)</b>	-8	-9	-11	-13	-14	-16	-17	-19
<b><math>CL</math> dB (Fading)</b>	153	154	156	158	159	161	162	164 <b>*MCL</b>

## NB-IoT UL Synchronization: MCL & coverage analysis

For example, a very low  $SNR_{99} = -37$  dB is identified for the longest decision delay of  $N=512$  and  $SNR_{99} = -24$  dB for  $N=4$  under the AWGN channel.

However, under the Rayleigh fading channel, the required  $SNR_{99}$  is significantly degraded to -19 dB for  $N=512$  and -8 dB for  $N=4$ .

The 5<sup>th</sup> row in the Table lists the coupling loss (CL) in dB, which in turn determines the maximum UE-to-eNB distance or cell coverage  $d$  in km presented in the last row.

To find the MCL, we take the extreme cell edge case. It is assumed that a UE belonging to coverage class 3 transmits its NPRACH with the longest  $N=512$  configuration at the NB-IoT maximum UE power of  $P_T = 23$  dBm.

# NB-IoT UL Synchronization: MCL & coverage analysis...

At room temperature (290 K), the RX noise power spectral density floor is -174 dBm/Hz, so the noise power within the system bandwidth (180 kHz) is  $\sigma_n^2 = -121.4$  dBm.

The MCL for the Rayleigh fading channel can then be found using the following link analysis:

$$\text{MCL}(dB) = P_t - (\sigma_n^2 + \text{SNR}_{99}) = 23 - (-121.4 - 19) \cong 164 \text{ dB}$$

Moreover, assuming a simple log-distance path loss model

$$\text{PL}(dB) = PL_0 + 10n \times \log(d)$$

With the path loss exponent of  $n=4$  and the 1-km reference path loss  $PL_0 = 101.7$  dB, the MCL can finally be translated into the maximum cell coverage in km

$$d = 10^{\frac{MCL - PL_0}{10n}} \cong 34 \text{ km}$$

# NB-IoT UL Synchronization: TOA and RCFO estimation

The synchronization algorithm follows the detection scheme, and for all detected UEs, their ToA and RCFO parameters can be estimated in parallel.

Assuming  $UE_k$  detected, we begin with its unwrapped phase trace of  $R(n, i, f_k(n))$ ,

$$q_{k,n,i} = \text{unwrap}\{\arg\{R(n, i, f_k(n))\}\}$$

In the noise-free case, the phase trace is given by

$$q_{k,n,i} = \beta_{k,n,i} = -2D_k f_k(n) - 2\pi\eta_k t(n, i) + C$$

where  $C$  is a constant phase.

Because the first two phase terms are directly related to the ToA and RCFO, the phase trace  $q_{k,n,i}$  can be exploited through suitable phase differences and averaging operation.

## NB-IoT UL Synchronization: TOA and RCFO estimation...

First, for each SG of 5 symbols, there are 4 adjacent phase differences that can be calculated, yielding

$$\epsilon_{k,n,i} = q_{k,n,i+1} - q_{k,n,i} \quad i = 1, 2, 3, 4$$

Then, by averaging  $\epsilon_{k,n,i}$  over all n and l in the preamble, we can obtain the RCFO estimate for  $UE_k$  as

$$\hat{\eta}_k = \frac{1}{2\pi} \times \frac{1}{4N} \sum_{n=0}^{N-1} \sum_{i=1}^4 \epsilon_{k,n,i}$$

## NB-IoT UL Synchronization: TOA and RCFO estimation...

Next, to estimate the ToA parameter, we first subtract the RCFO-induced term from  $q_{k,n,i}$  yielding

$$\bar{q}_{k,n,i} = q_{k,n,i} - 2\pi\hat{\eta}_k\Delta f t_{n,i}$$

where  $t_{n,i} = [(5n + i)L + (n + 1)L_{cp}]T_s$  is the starting time of the  $i$ -th symbol of the  $n$ -th SG. Let the mean for each SG be expressed as

$$z_{k,n} = \frac{1}{5} \sum_{i=0}^4 \bar{q}_{k,n,i}$$

Then, we can find the ToA estimate by dividing the total absolute differences in  $z_{k,n}$  by the total hopping-frequency absolute differences as

$$\hat{D}_k = \frac{1}{2\pi T_s} \times \frac{\sum_{n=1}^{N-1} |z_{k,n+1} - z_{k,n}|}{\Delta f \times \sum_{n=1}^{N-1} |f_k(n+1) - f_k(n)|}$$

# NB-IoT UL Synchronization: Estimation performance comparison

We compare the PD-based algorithm with the 2-D search method to estimate  $(D_k, \eta_k)$ .

To find the global maximum for the 2-D search method:

- We guess a ToA parameter  $D_k$  and multiply the inverse  $D_k$ -induced phase term to the data set  $\{R(n, i, f_k(n))\}$ .
- Taking the 1D-FFT of the phase-corrected data set, the FFT peak gives the RCFO candidate  $\eta_k$ .
- Repeating the 1D-FFT search for another  $D_k$  and comparing all the 1D-FFT peaks, we get the global maximum.

Thus, the 2-D search method can be regarded as the optimal scheme for performance comparison (despite that it has very high complexity).

# NB-IoT UL Synchronization: Estimation performance comparison...

Channels considered: flat Rayleigh fading channel and typical urban (TU) multipath fading channel.

For the 2-D grid search, we set the ToA grid size  $D_k = 1$  sample, and the 1D-FFT length is 1024 for RCFO searching.

3 coverage classes are simulated:  $N = 4, 32$ , and  $128$  corresponding to:  $SNR_i = 14.25, 4.25$ , and  $-5.75$  dB.

We set a tolerance region of  $\text{tol1} = 20$  Hz for the RCFO estimation, i.e., only consider trials with RCFO error of no greater than 20 Hz.

We also set a tolerance region of  $\text{tol2} = 5$  samples for ToA estimation. Because the sampling period is  $T_s = 1/1.92E6 = 0.52 \mu\text{s}$ ,  $\text{tol2}$  is equivalent to  $2.6 \mu\text{s}$ .

# NB-IoT UL Synchronization: Estimation performance comparison...

For the cases of CC#1 and CC#2, it is seen that our suboptimal low-complexity PD method can achieve low RCFO/ToA RMSEs which are close to those of the optimal high-complexity 2-D method.

However, the PD method suffers from a higher failure than the 2-D method, especially for CC#3 with low  $SNR_i$ .

SETTINGS AND SIMULATION RESULT FOR THE THREE COVERAGE CLASSES ( $TOL_1 = 20$  Hz) ( $TOL_2 = 5 T_s$ )

Coverage Class (CC)		CC#1	CC#2	CC#3
No. of SGs $N$		4	32	128
$SNR_i$ (Fading Ch.)		14.25 dB	4.25 dB	-5.75 dB
Flat ch - RCFO RMSE (Hz) / Failure no.	PD	1.51 Hz/10	1.57 Hz/18	2.49 Hz/69
	2D	1.22 Hz/2	1.05 Hz/0	1.05 Hz/1
Flat ch - ToA RMSE ( $\mu$ s) / Failures no.	PD	0.15 $\mu$ s/15	0.14 $\mu$ s/21	0.34 $\mu$ s/216
	2D	0.22 $\mu$ s/3	0.18 $\mu$ s/0	0.27 $\mu$ s/1
TU ch - RCFO RMSE (Hz) / Failure no.	PD	1.48 Hz/14	1.66 Hz/15	2.35 Hz/70
	2D	1.34 Hz/6	1.04 Hz/0	1.05 Hz/0
TU ch - ToA RMSE ( $\mu$ s) / Failures no.	PD	0.39 $\mu$ s/52	0.38 $\mu$ s/64	0.42 $\mu$ s/257
	2D	0.43 $\mu$ s/30	0.41 $\mu$ s/35	0.44 $\mu$ s/51

# NB-IoT UL Synchronization: Estimation performance comparison...

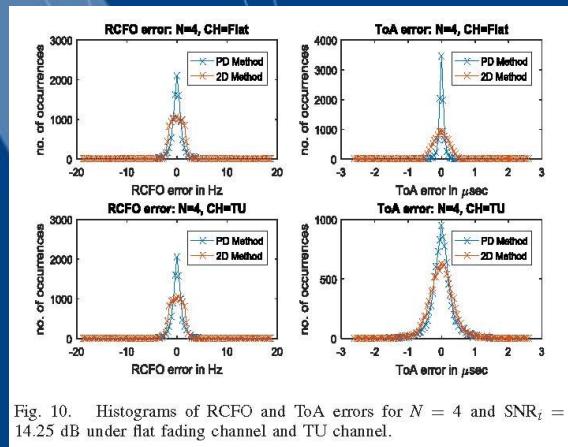


Fig. 10. Histograms of RCFO and ToA errors for  $N = 4$  and  $\text{SNR}_i = 14.25 \text{ dB}$  under flat fading channel and TU channel.

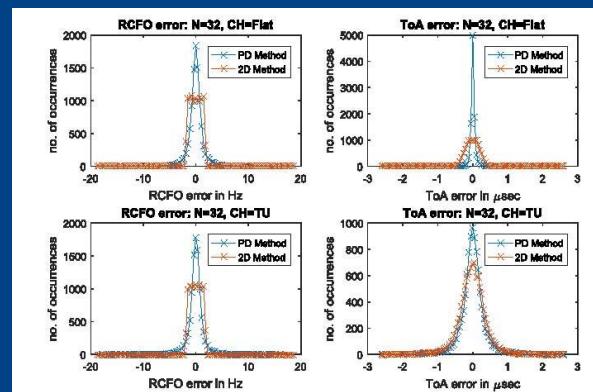


Fig. 11. Histograms of RCFO and ToA errors for  $N = 32$  and  $\text{SNR}_i = 4.25 \text{ dB}$  under flat fading channel and TU channel.

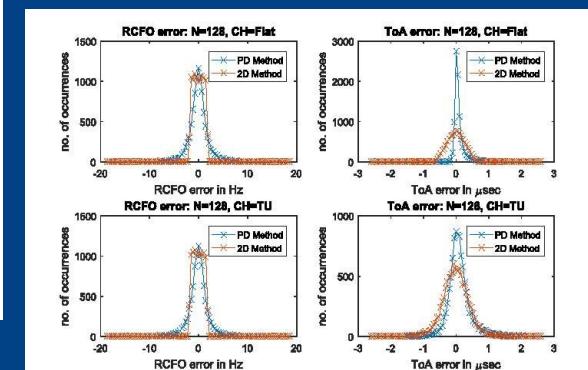


Fig. 12. Histograms of RCFO and ToA errors for  $N = 128$  and  $\text{SNR}_i = -5.75 \text{ dB}$  under flat fading channel and TU channel.

# Conclusions

- Uplink random access signal, in particular the preamble design and the properties required, is discussed in first place.
- Algorithms to detect the NPRACH and estimate ToA and residual CFO are discussed.
- A first solution is based on a 2-dimensional FFT (high complexity).
- A 2<sup>nd</sup> algorithm uses a detection based design: estimate the UE power in frequency domain and use an statistic criterion to detects NPRACH preamble.
- Later, based on the post-FFT phase trace obtain estimates of ToA and residual CFO.