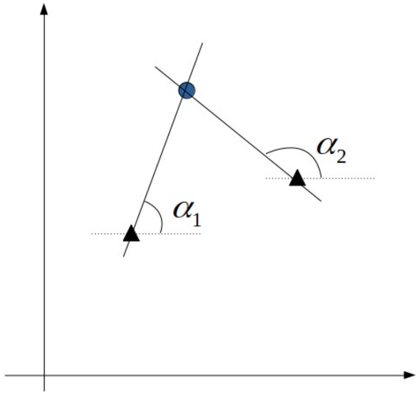
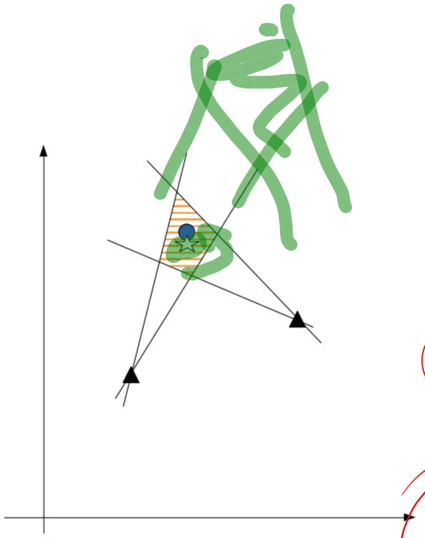
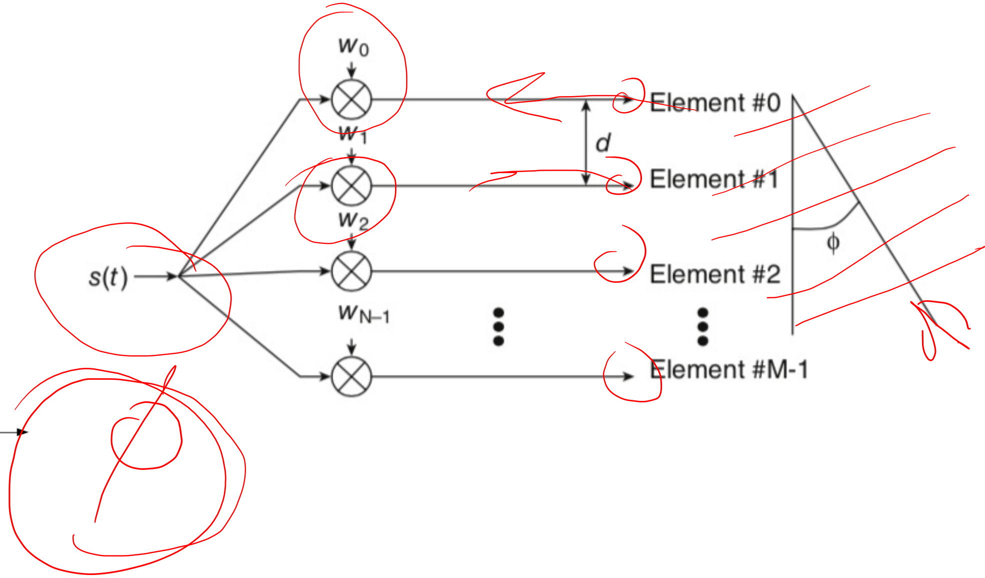
# Multiangulación: DOA/AOA






$$\hat{x} = \frac{[(x_2 \tan\theta_2 - x_1 \tan\theta_1) - (y_2 - y_1)]}{(\tan\theta_2 - \tan\theta_1)}$$

$$\hat{y} = \frac{[(x_2 - x_1)\tan\theta_1 \tan\theta_2 + (y_1 \tan\theta_2 - y_2 \tan\theta_1)]}{(\tan\theta_2 - \tan\theta_1)}$$

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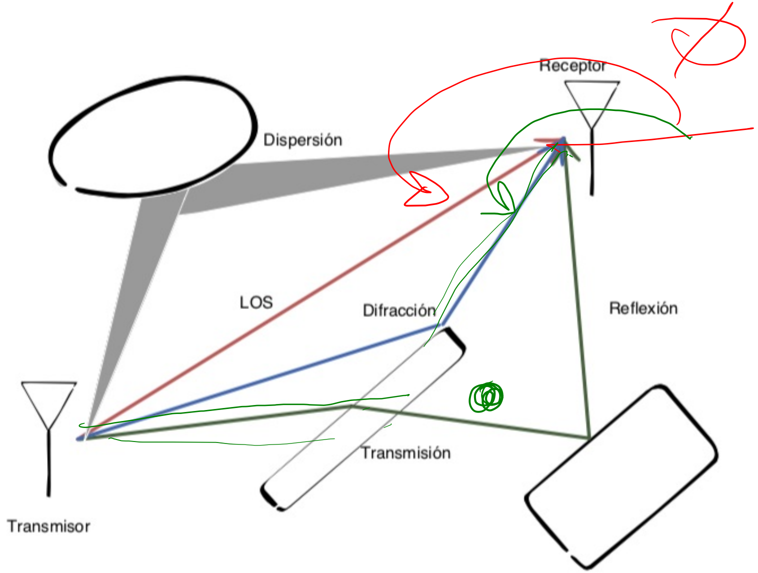
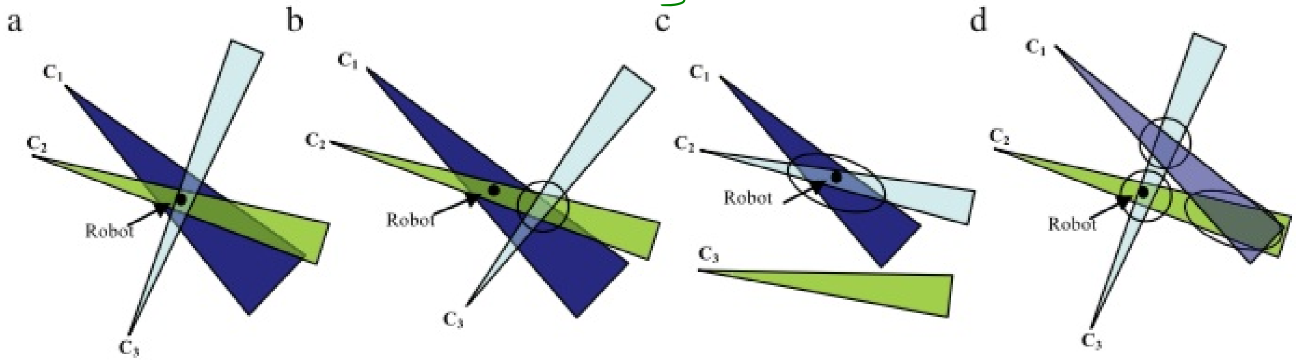



OUTLIERS



# Multiangulación: DOA/AOA

$$\mathbf{s}^0 = [x^0, y^0]^T$$ MÓVIL / OBJETIVO

$$\mathbf{b}_i^0 = [x_i^0, y_i^0]^T$$ $ER_i$ $M$

$$\alpha = [\alpha_1, \alpha_2, \ldots, \alpha_M] = \alpha^0 + n$$ → GAUSIANO MEDIA CERO VARIANZA

$$C_{\alpha\alpha} = \begin{bmatrix} \sigma_{\alpha_1} & 0 & 0 \\ & \sigma_{\alpha_2} & \\ & & \ddots \; \sigma_{\alpha_M} \end{bmatrix}$$

$$\mathbf{b} = [b_1, b_2, \ldots, b_M] = \mathbf{b}^0 + m$$

$$C_{\psi\psi} = \begin{bmatrix} \psi_{11} & \cdots & 0 \\ 0 & & \psi_{MM} \end{bmatrix}$$

$$\mathbf{b}^0 = [b_1^0, b_2^0, \ldots, b_M^0]$$

# Multiangulación: algoritmo ML (Maximum Likelihood)

$$\Theta = [\Theta_1^T, \Theta_2^T]^T = [\mathbf{s}^{0T}, \mathbf{b}_i^{0T}]^T = [x^0, y^0, x_1^0, y_1^0, \cdots, x_M^0, y_M^0]^T$$

$$\boldsymbol{\beta} = [\boldsymbol{\alpha}^{\mathrm{T}}, \mathbf{b}^{\mathrm{T}}]^{\mathrm{T}}$$

$$\hat{\Theta}_{1,ML} = \arg\min_{s_0} [\alpha - g_1(\Theta_1)]^T C_{\alpha\alpha}^{-1} [\alpha - g_1(\Theta_1)]$$

$$g_1(\Theta_1) = [g_{1,1}(\Theta_1), \cdots, g_{1,M}(\Theta)]$$

$$g_{1,i}(\Theta) = \operatorname{tg}^{-1} \frac{y_0 - y_i}{x_0 - x_i}$$

$$\hat{\theta}_{1,ML} = \arg\min \left[\alpha - g_1(\theta_1)\right]^T C_{\alpha\alpha}^{-1} \left[\alpha - g_1(\theta_1)\right]$$

$$\hat{\theta}_{1,k+1} = \hat{\theta}_{1,k} + \left(H_{1k}^T C_{\alpha\alpha} H_{1k}\right)^{-1} H_{1k}^T C_{\alpha\alpha} \left(\alpha - g_1(\theta_{1,k})\right)$$

$$g_1(\theta_{1,k}) = \begin{bmatrix} g^{-1}\dfrac{y_k - y_1}{x_k - x_1} \\ \vdots \\ g^{-1}\dfrac{y_k - y_M}{x_k - x_M} \end{bmatrix}$$

G error

$$H_{1k} = \frac{\partial \theta_1}{\partial \theta_1} \Big/ \theta_{1,k}$$

$$\begin{bmatrix} \partial & \partial \\ \partial & \partial \end{bmatrix}$$

Multiangulación: algoritmo LS (Least Square)

$$\alpha_i^0 = \tan^{-1} \frac{y^0 - y_i^0}{x^0 - x_i^0}$$

$$\operatorname{tg} \alpha_i^0 = \frac{y^0 - y_i^0}{x^0 - x_i^0} = \frac{\operatorname{sen} \alpha_i^0}{\cos \alpha_i^0}$$

$$(y^0 - y_i^0) \cos \alpha_i^0 = (x^0 - x_i^0) \operatorname{sen} \alpha_i^0$$

$$x_i^0 \operatorname{sen} \alpha_i^0 - y_i^0 \cos \alpha_i = x^0 \operatorname{sen} \alpha_i^0 - y^0 \cos \alpha_i^0$$

$$H = G \cdot S$$

$$\begin{bmatrix} x_i^0 \operatorname{sen} \alpha_i^0 - y_i^0 \cos \alpha_i^0 \\ \vdots \\ x_m^0 \operatorname{sen} \alpha_m^0 - y_m^0 \cos \alpha_m^0 \end{bmatrix} = \begin{bmatrix} \operatorname{sen} \alpha_i^0 & -\cos \alpha_i^0 \\ \vdots & \\ \operatorname{sen} \alpha_m^0 & -\cos \alpha_m^0 \end{bmatrix} \begin{bmatrix} x^0 \\ y^0 \end{bmatrix}$$

$$H = G \cdot S$$

$$\begin{bmatrix} x_1^0 \sin\alpha_1^0 - y_1^0 \cos\alpha_1^0 \\ \vdots \\ x_m^0 \sin\alpha_m^0 - y_m^0 \cos\alpha_m^0 \end{bmatrix} = \begin{bmatrix} \sin\alpha_1^0 & -\cos\alpha_1^0 \\ \vdots & \\ \sin\alpha_m^0 & -\cos\alpha_m^0 \end{bmatrix} \begin{bmatrix} x^0 \\ y^0 \end{bmatrix}$$

$$\begin{array}{cc} G & H \\ M \times 2 & M \times 1 \end{array}$$

$$\boxed{\hat{S}_{LS} = \left(G^T G\right)^{-1} G^T H}$$

$$2 \times M \times M \times 1$$

$$2 \times 1$$

# Multiangulación: algoritmo TLS (Total Least Square)

$$\hat{\Theta}_{1,TLS} = (\mathbf{G}^T\mathbf{G} - \sigma_s^2\mathbf{I})^{-1}\mathbf{G}^T\mathbf{h}$$

$\sigma_s \rightarrow$ MENOR VALOR SINGULAR DE $[G^T H]$

NN

$A_{MN} = U \Sigma V^T$

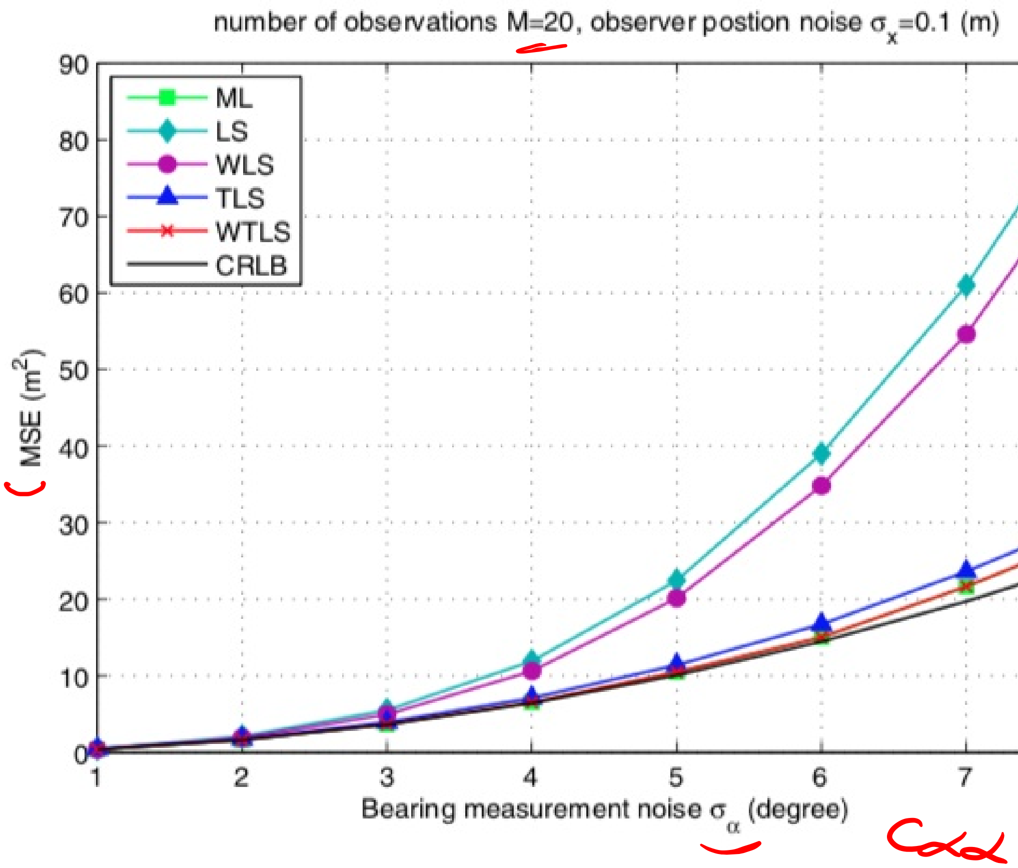
VALORES SING.

$A_{NN} = \rightarrow$ AUTOVAL.

AUTOVECT

$$\begin{bmatrix} \ddots & & 0 \\ & \ddots & \\ 0 & & \ddots \end{bmatrix} \rightarrow \text{VALORES SING.}$$

# Multiangulación: comparación



Multiangulación: Estimador Bayesiano seudo-lineal

$$p(\mathbf{s}|\boldsymbol{\alpha}) = \frac{p(\boldsymbol{\alpha}|\mathbf{s}) \cdot p(\mathbf{s_0})}{p(\boldsymbol{\alpha})}$$

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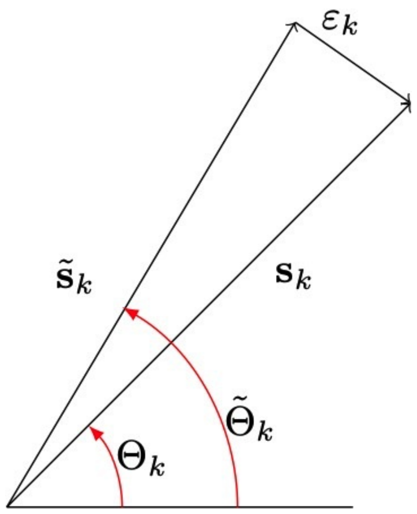
# Multiangulación: Estimador Bayesiano seudo-lineal



# Multiangulación: Estimador Bayesiano seudo-lineal

$$\hat{\mathbf{s}}_{BPLE} = \mathbf{s}_{mean} + (\mathbf{C}_{ss}^{-1} + \mathbf{G}^T \mathbf{C}_{\eta\eta}^{-1} \mathbf{G})^{-1} \mathbf{G}^T \mathbf{C}_{\eta\eta}^{-1} (\mathbf{h} - \mathbf{G} \cdot \mathbf{s}_{mean})$$