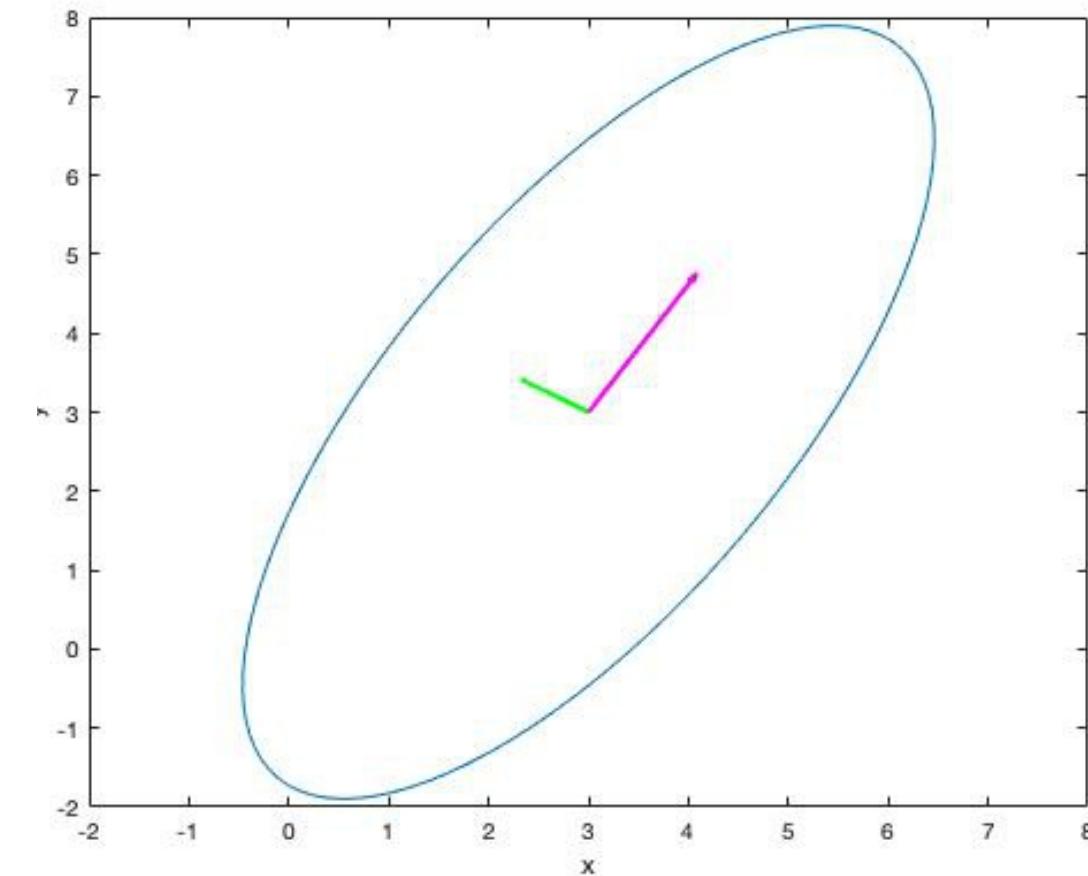
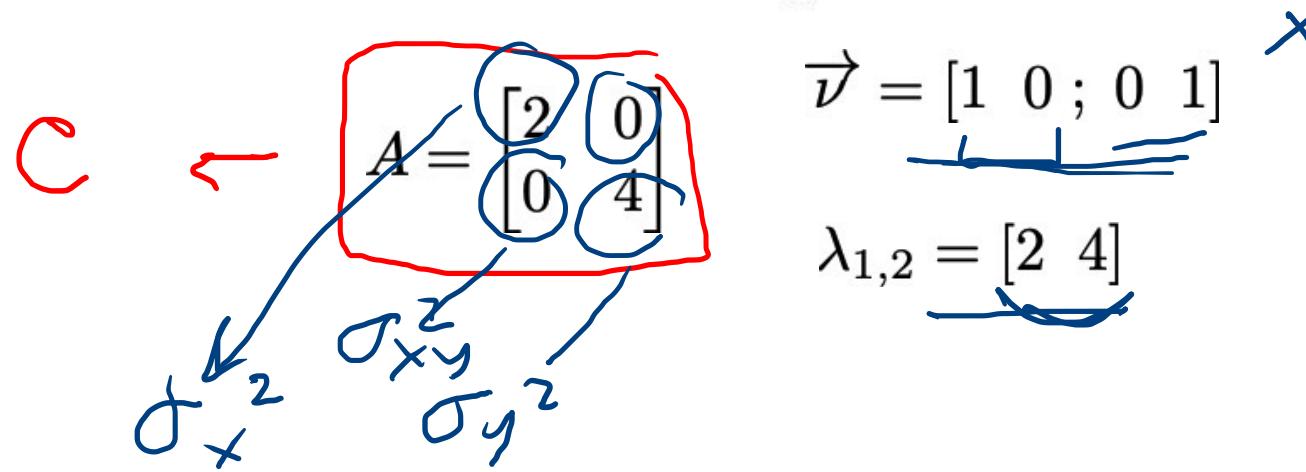
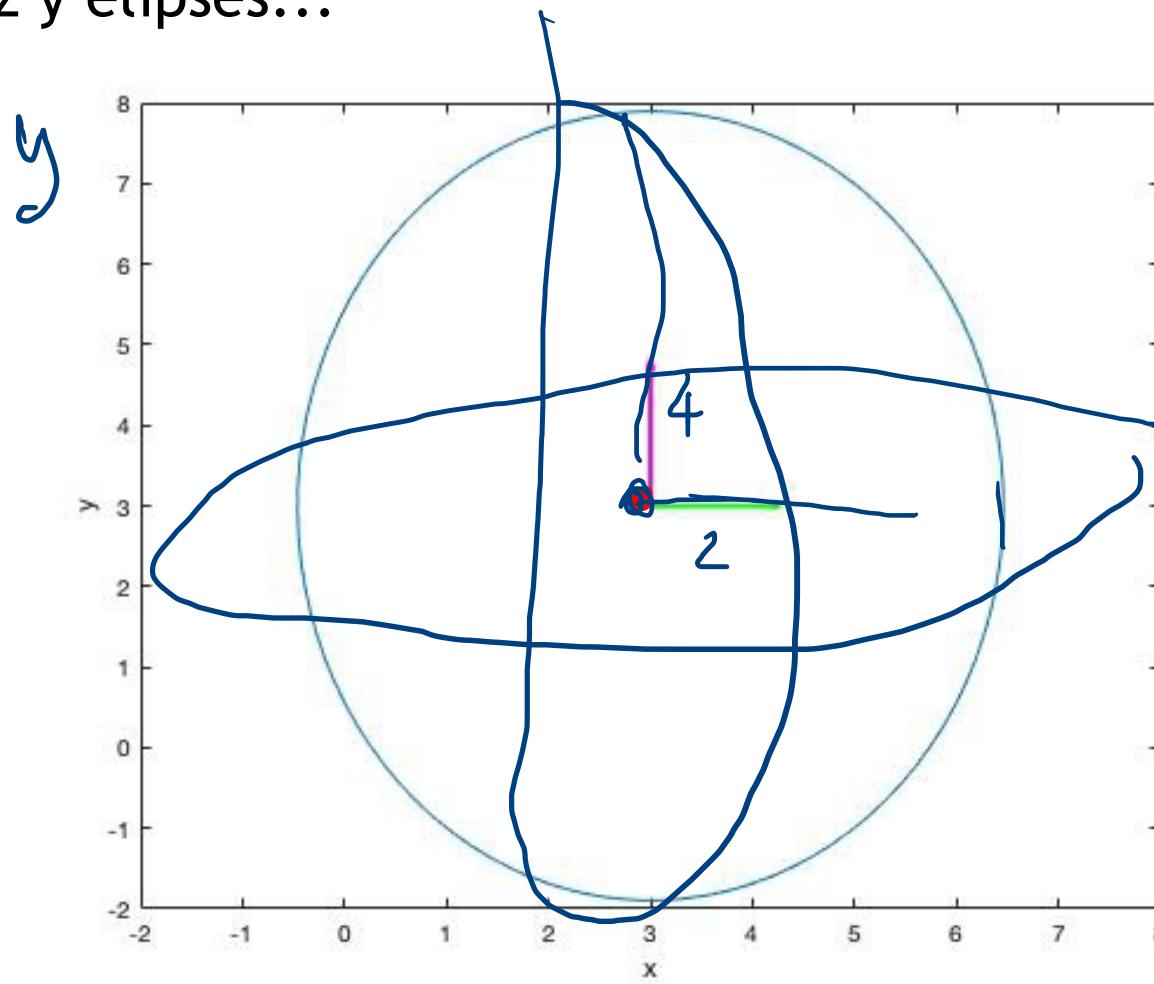


## Matriz y elipses...

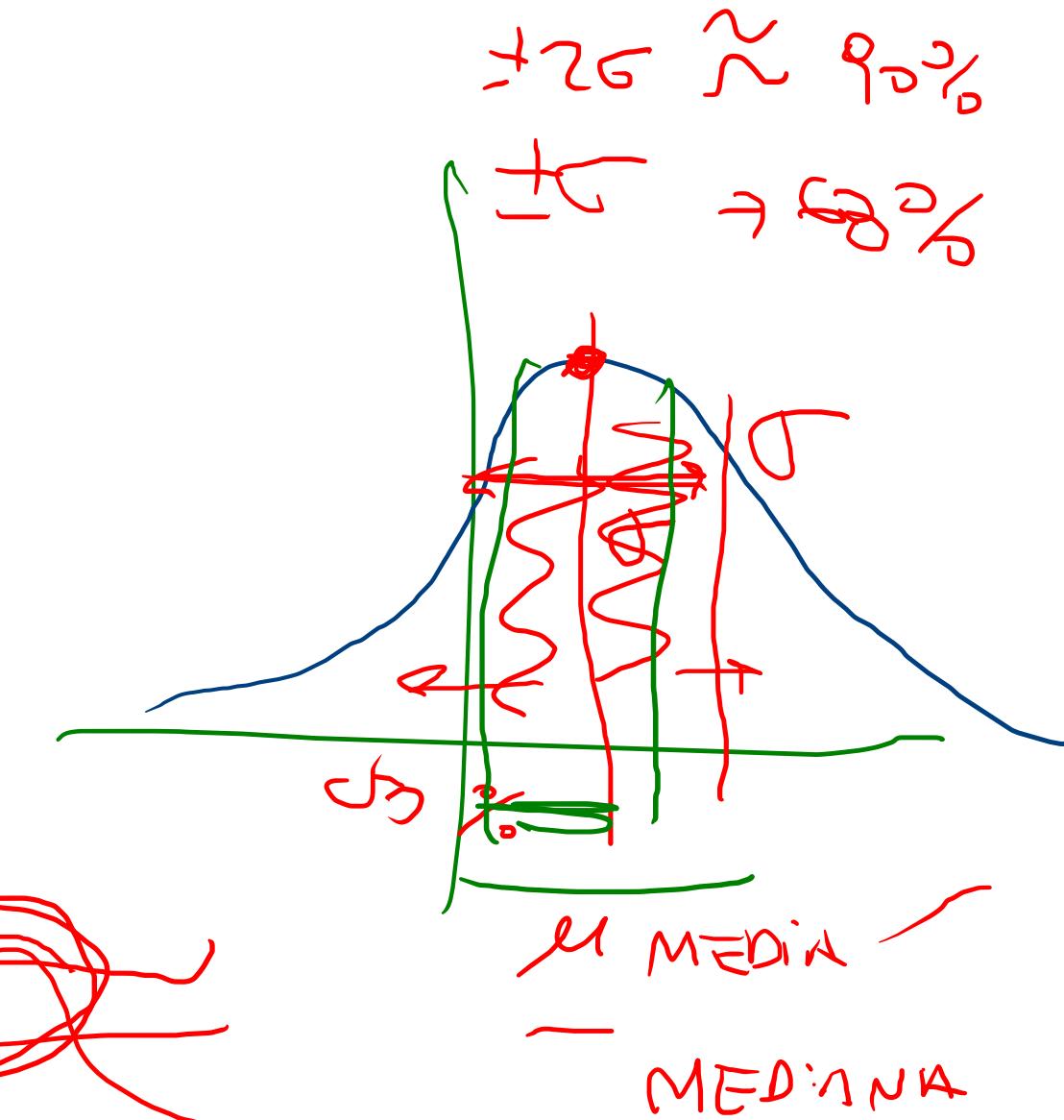
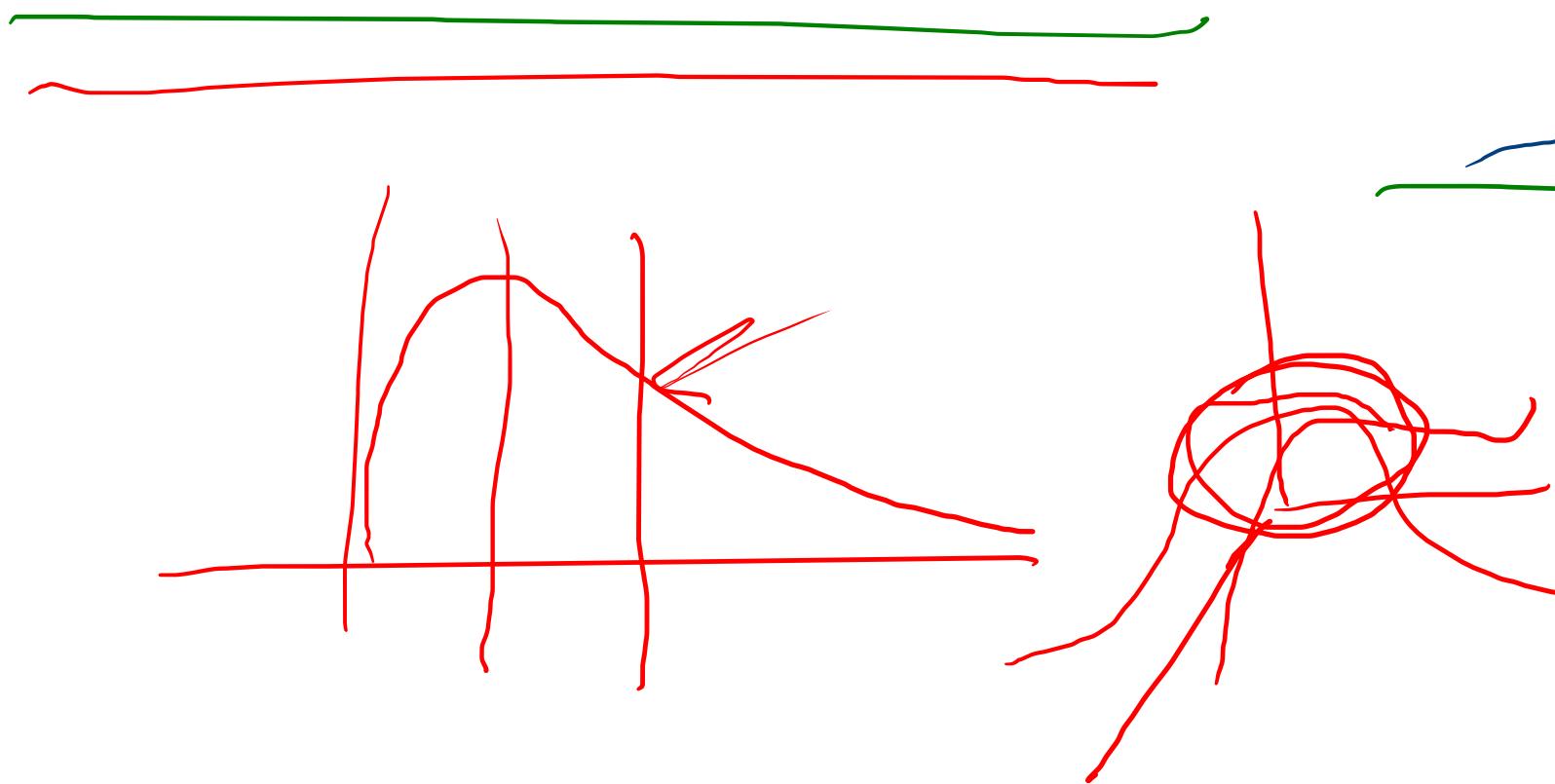


$$A = \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$$

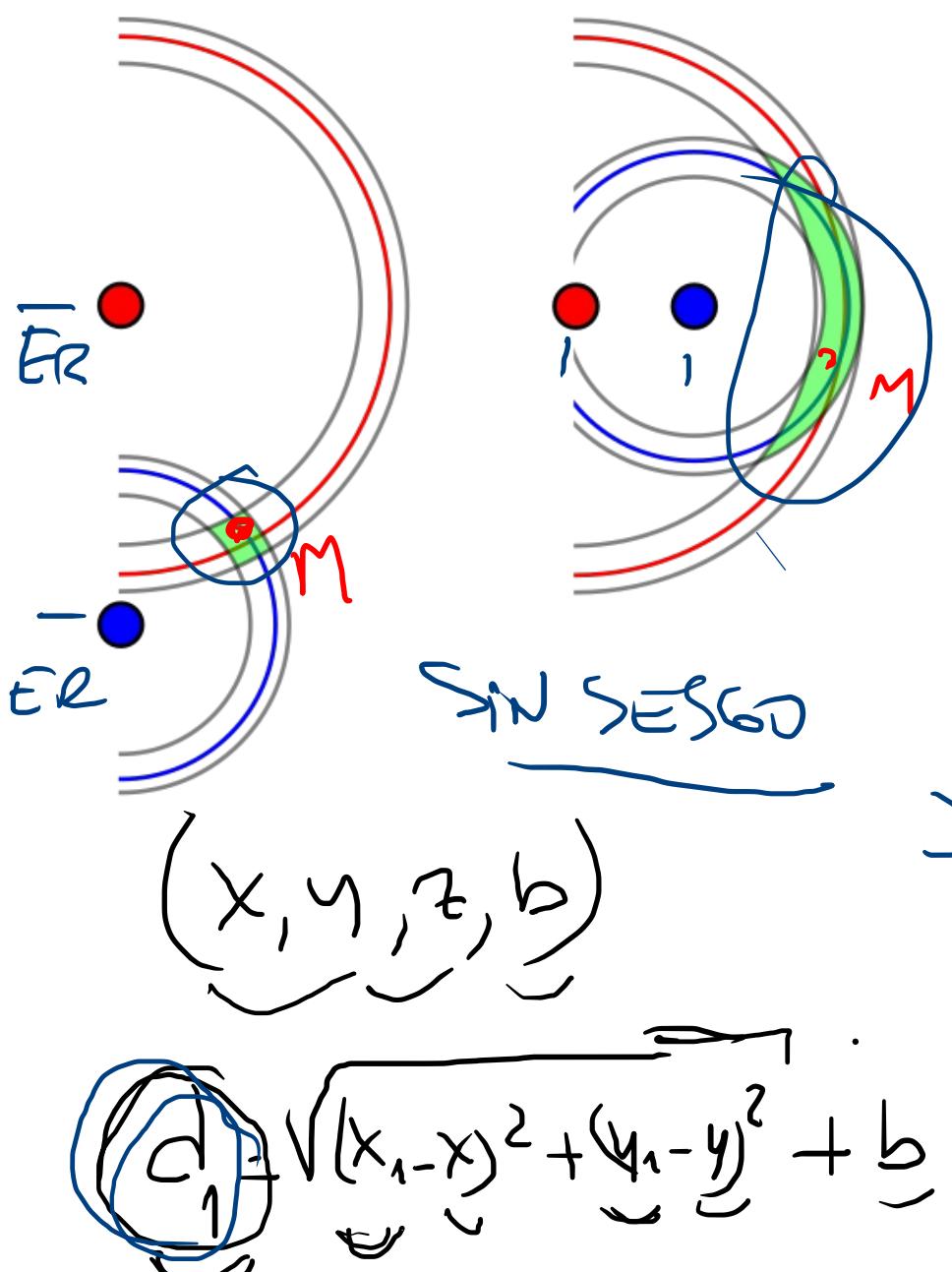
$$\vec{v} = [-0.85 \ 0.53 ; 0.53 \ 0.85]$$

$$\lambda_{1,2} = [0.76 \ 5.24]$$

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$



# Precisión - Dilution of Precision



$$h = \sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 + b}$$

$$\rightarrow \Delta h = \frac{\partial h}{\partial n} \cdot \begin{pmatrix} x \\ y \\ z \\ b \end{pmatrix}$$

$$A = \begin{pmatrix} \Delta h^+ & \Delta h^- \end{pmatrix} \cdot \begin{pmatrix} \Sigma_d \\ \Sigma_{xy} \Sigma_{xz} \Sigma_{yb}^2 \\ \Sigma_{xy}^2 \Sigma_{yz} \Sigma_{yb}^2 \\ \Sigma_{xz}^2 \Sigma_{yz}^2 \Sigma_{zb}^2 \\ \Sigma_{yb}^2 \Sigma_{yz}^2 \Sigma_{zb}^2 \end{pmatrix}$$

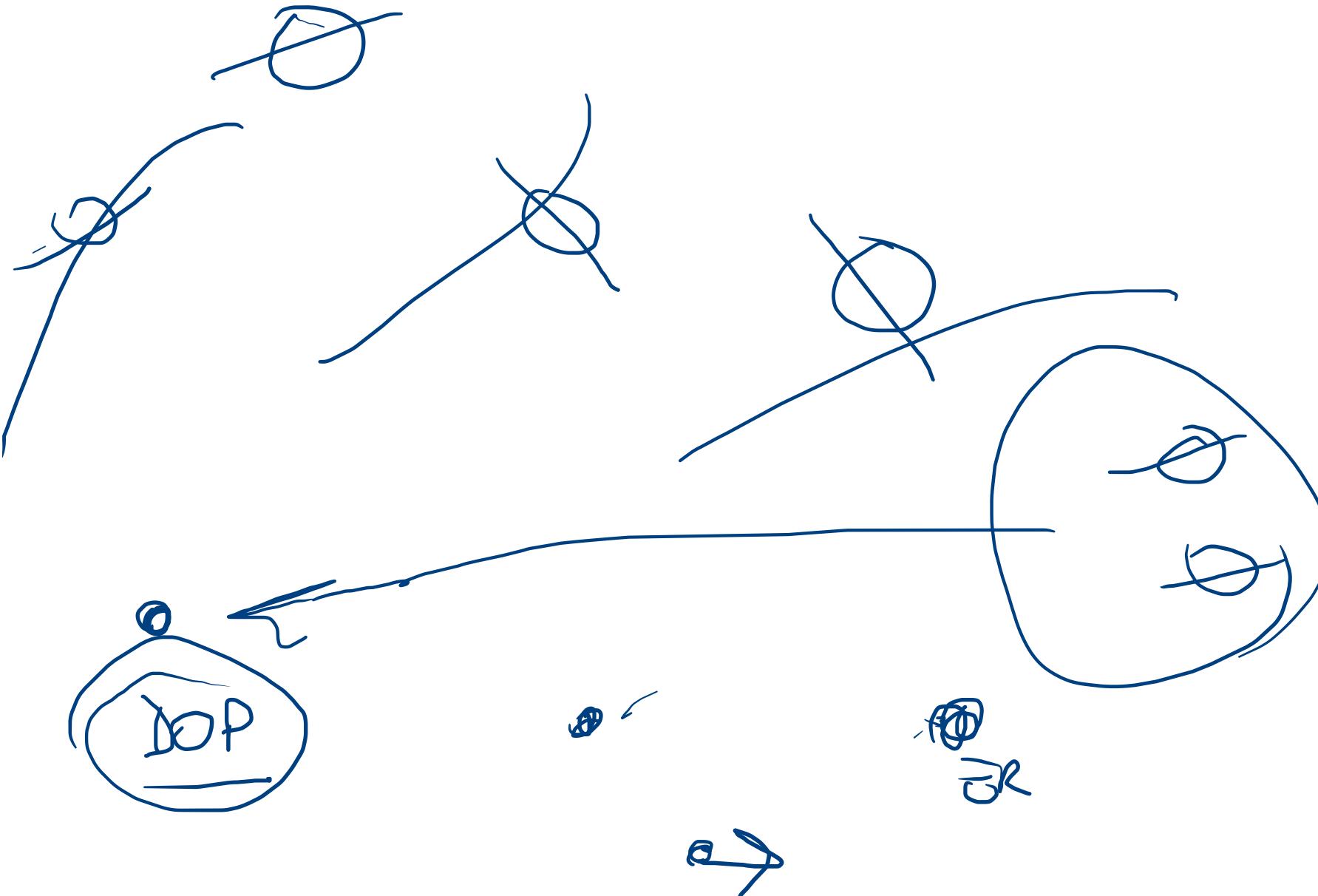
$$\text{DOPG} = \sqrt{\Sigma_x^2 + \Sigma_y^2 + \Sigma_z^2 + \Sigma_b^2}$$

$$\text{DOP}_X = \sqrt{\Sigma_x^2}$$

$$\text{DOP}_H = \sqrt{\Sigma_x^2 + \Sigma_y^2}$$

$$\text{DOP}_Z = \sqrt{\Sigma_z^2}$$

$$\text{DOP}_B = \sqrt{\Sigma_b^2}$$



# Circular Error Probability - Cramer Rao Lower Bound

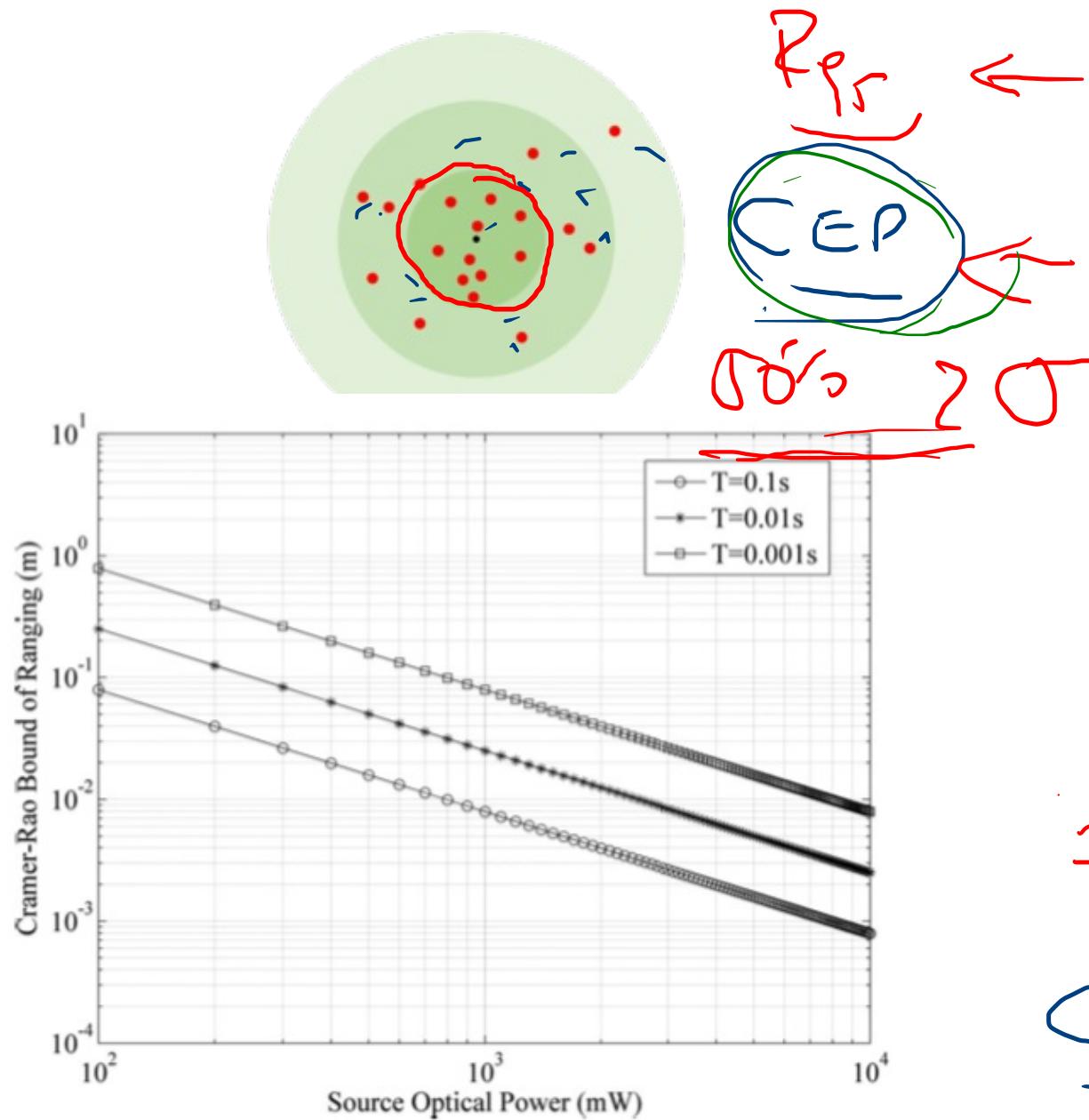
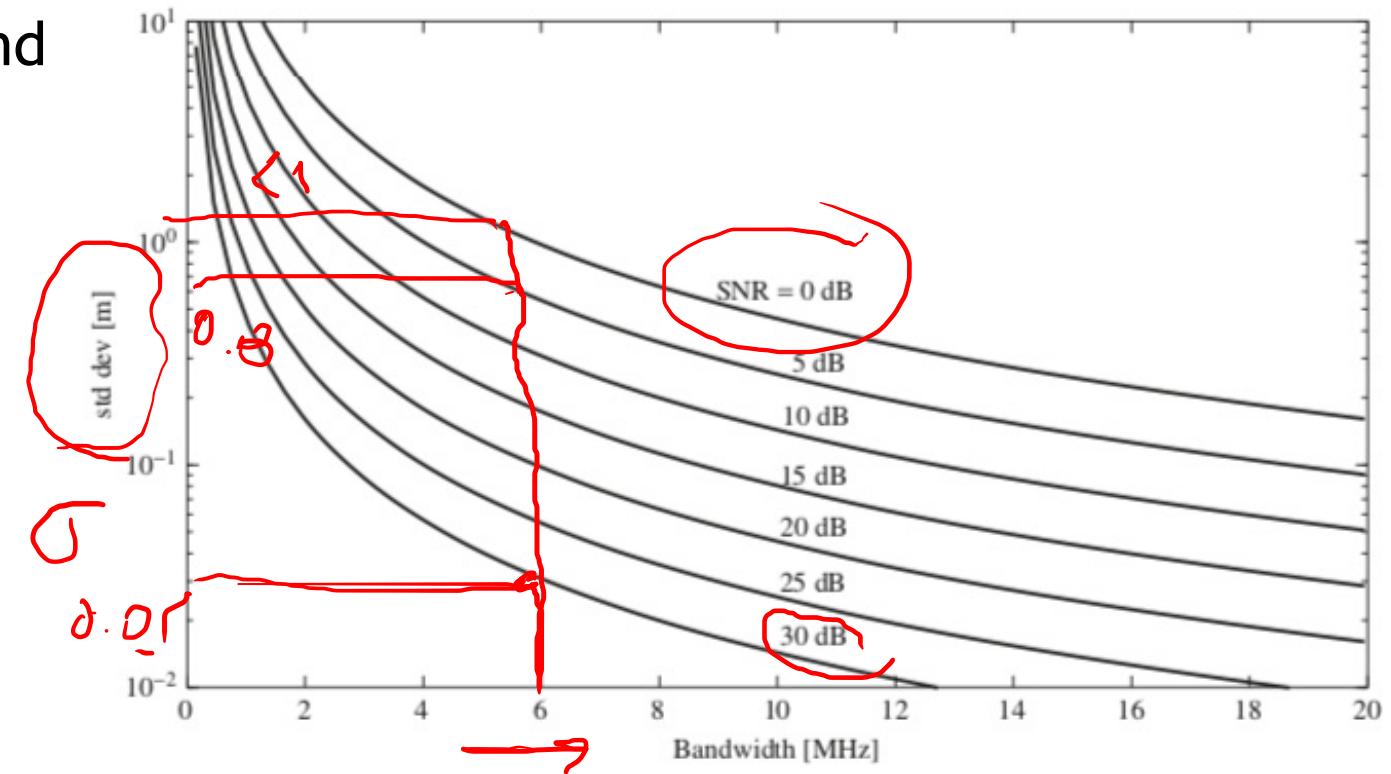


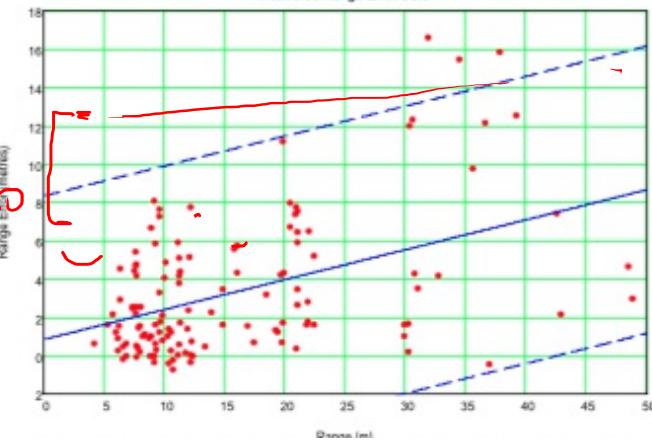
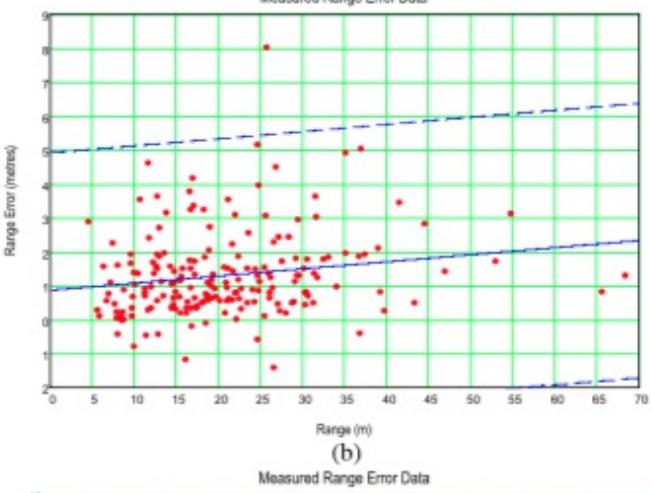
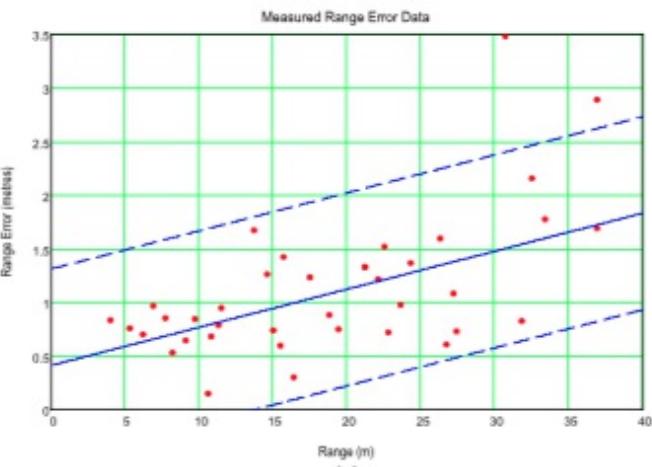
Fig. 4. CRB versus the emitted optical power with various time durations.



Koorapaty, Havish. "Cramer-Rao bounds for time of arrival estimation in cellular systems." 2004 IEEE 59th Vehicular Technology Conference. VTC 2004-Spring (IEEE Cat. No. 04CH37514). Vol. 5. IEEE, 2004.

handwritten notes:

- Range error
- $P(|X_1 - \bar{X}| < 0.5)$
- CRLB: Costa inferior de error



## Evaluación a partir de medidas

Sharp, Ian, and Kegen Yu. "Indoor TOA error measurement, modeling, and analysis." IEEE Transactions on Instrumentation and Measurement 63.9 (2014): 2129-2144.

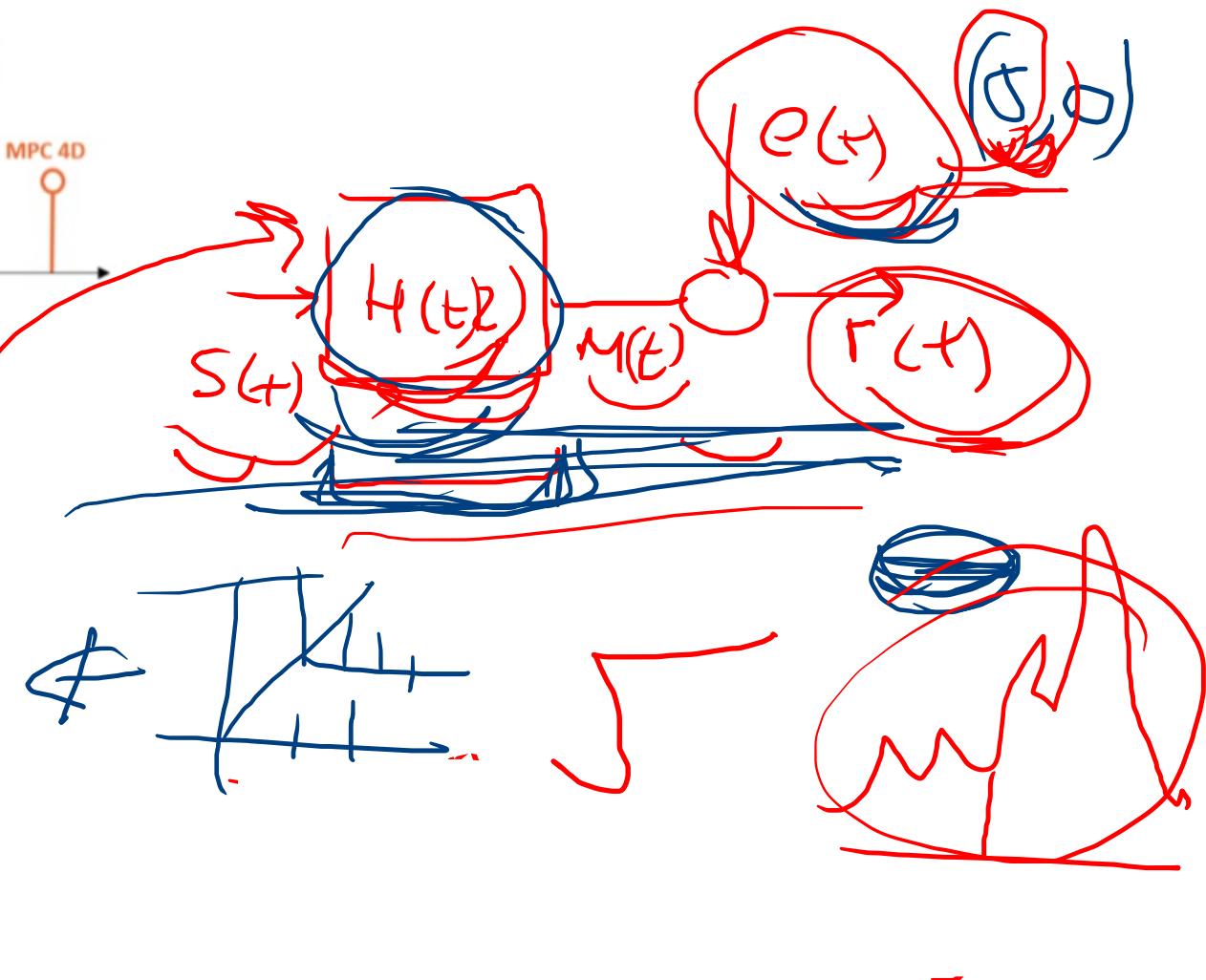
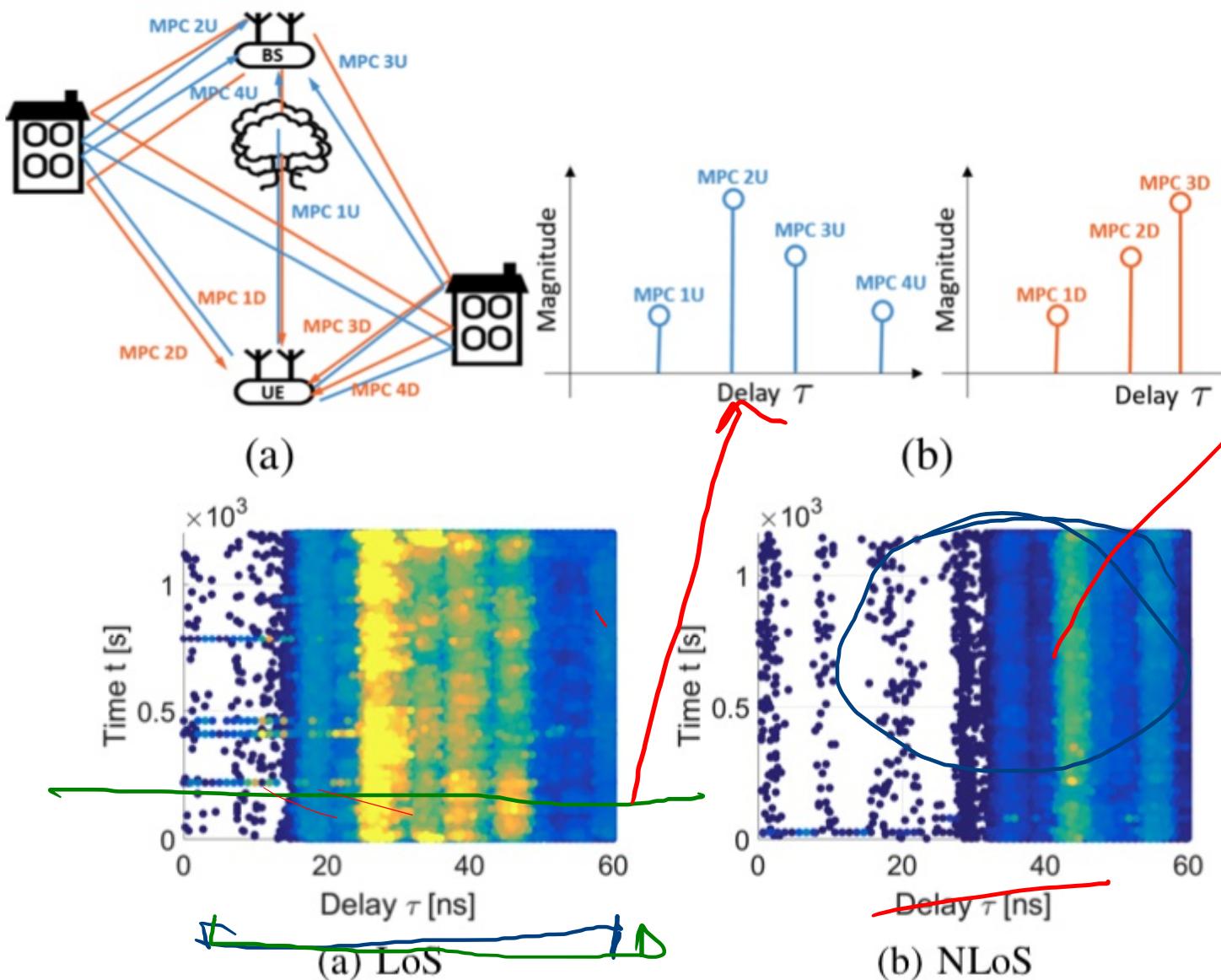
Parameter	UWB	WASP	PLS
RF Frequency	5 GHz	5.8 GHz	2.4 GHz
RF Channel	500 MHz	125 MHz	60 MHz
Bandwidth (3 dB)	250 MHz	65 MHz	40 MHz
Pulse rise-time	3 ns (0.9 m)	13.5 ns (4 m)	25 ns (7.5 m)
Coverage Area	800 m <sup>2</sup>	10000 m <sup>2</sup>	1500 m <sup>2</sup>

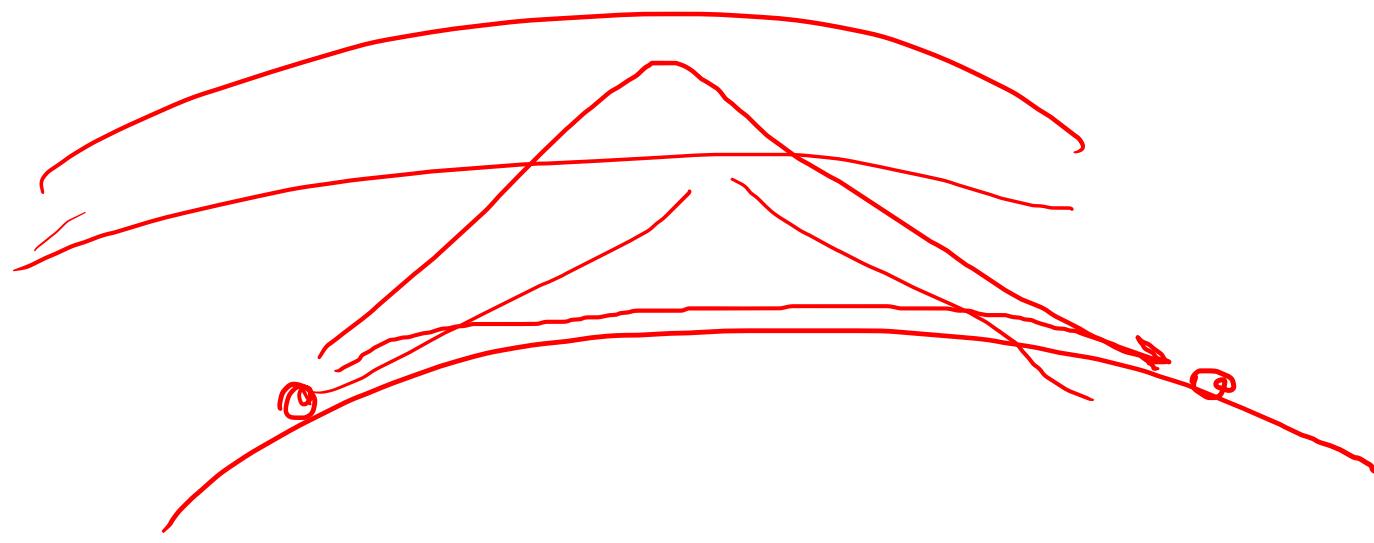
$$E_{TOT} = E(t) + E(R, N_p, O_s)$$

$$E_{TOT} = \Delta o + m \times$$

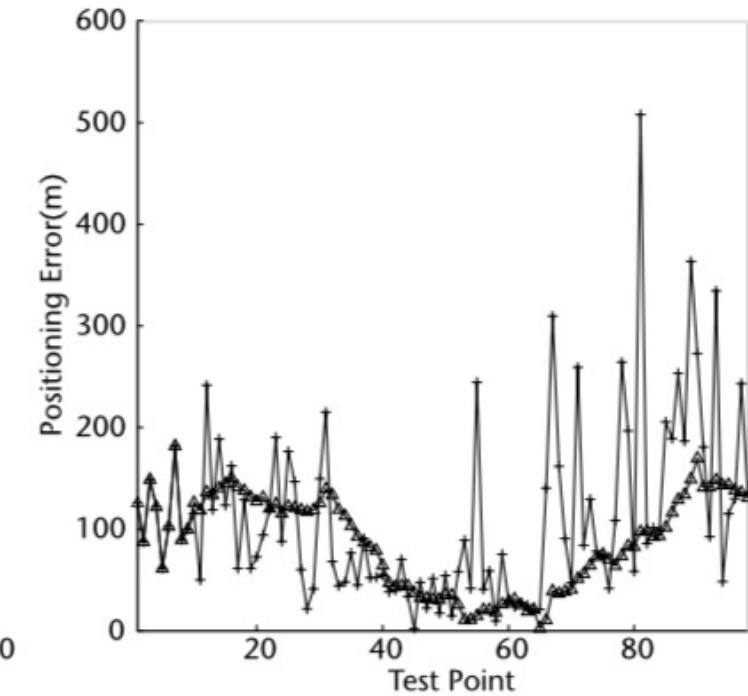
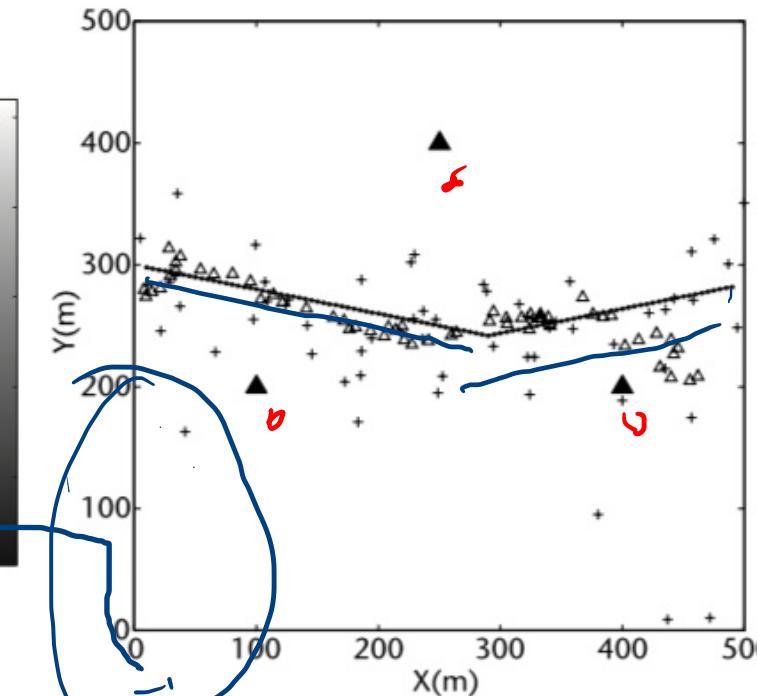
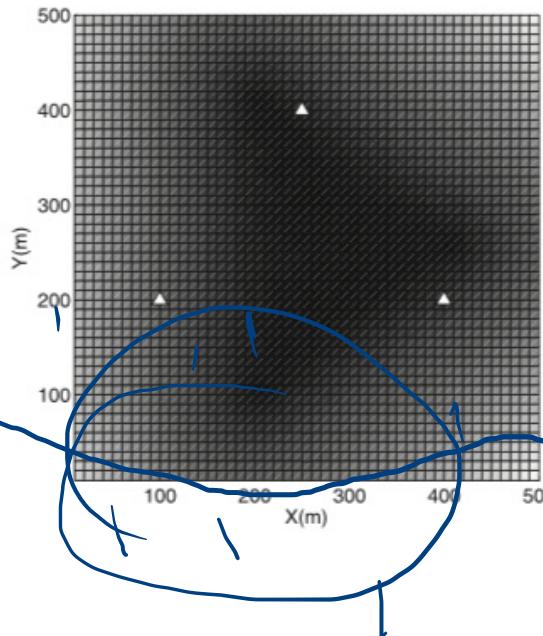
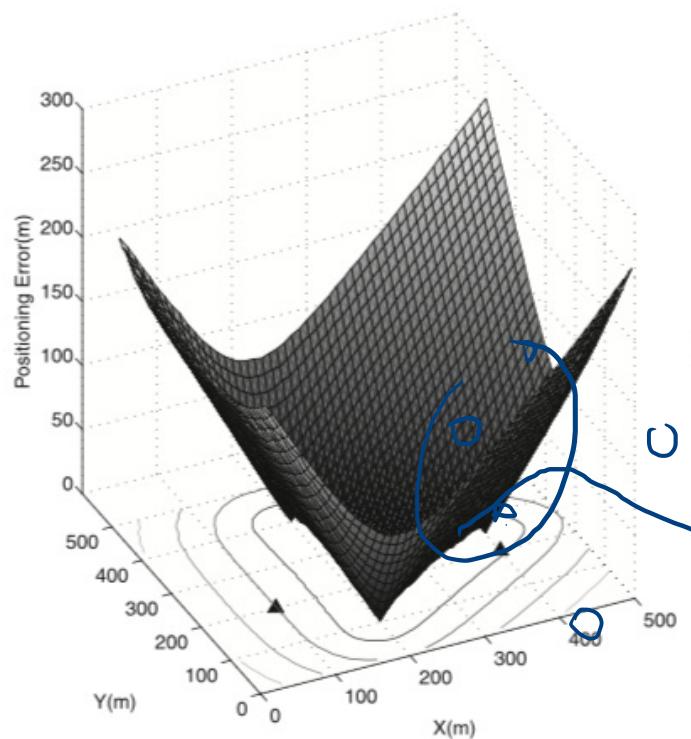
# Evaluación por simulación

Bergström, Andreas, et al. "TOA estimation improvements in multipath environments by measurement error models." 2017 IEEE 28th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC). IEEE, 2017.





# Evaluación por simulación

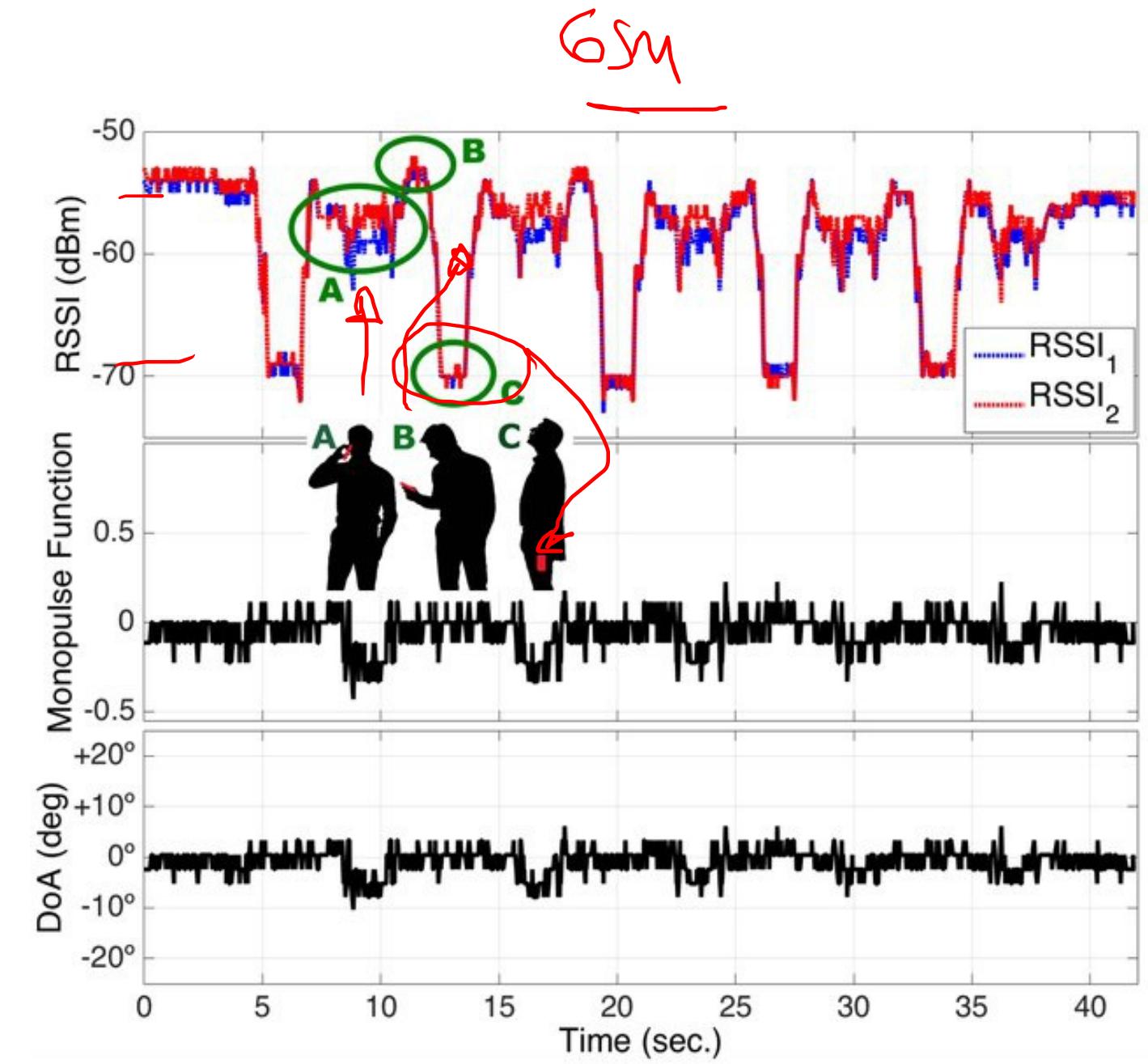
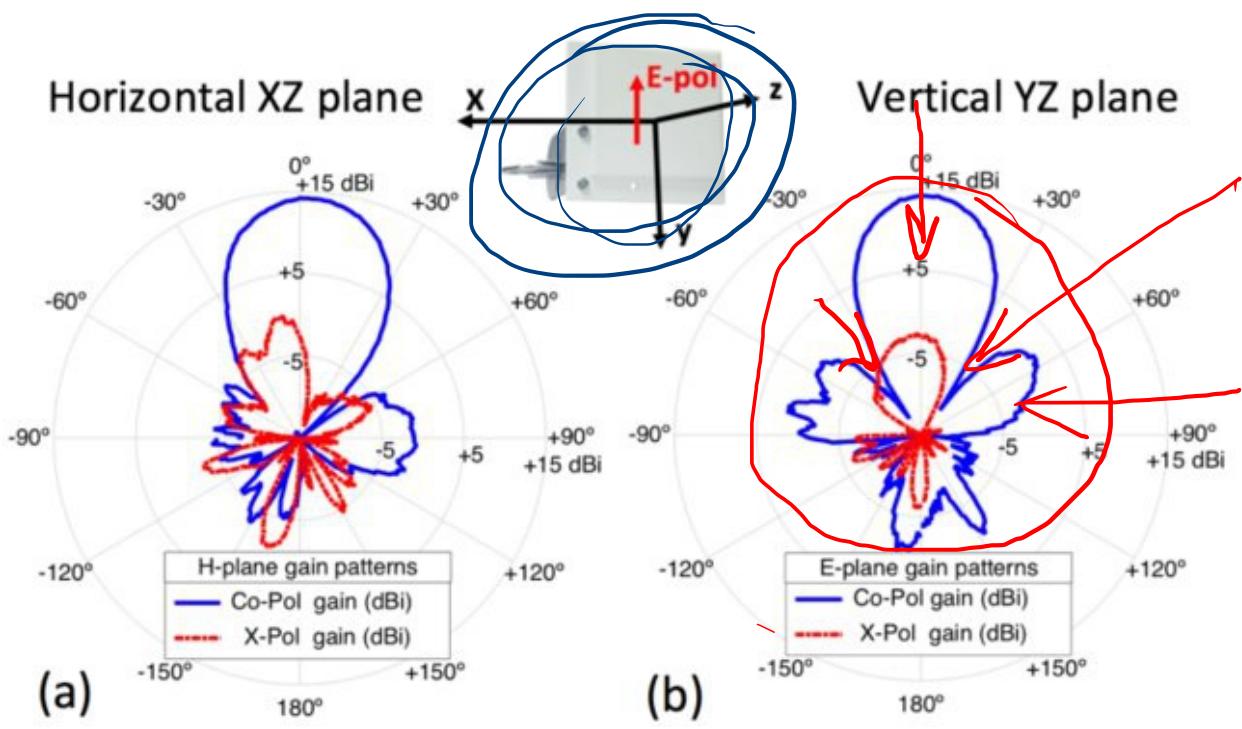


Ejercicio: Programa ch02fig05\_06.m  
(\toaErrorMaps\toaNlosErrorMap.m)

$Z$   
---  
Prop.  $Z_m \rightarrow Z_{rus}$

$\sim$ . Lognormal  $X \Rightarrow \log X$

Uso de RSS (RAN 60) Ee - m (TOA)

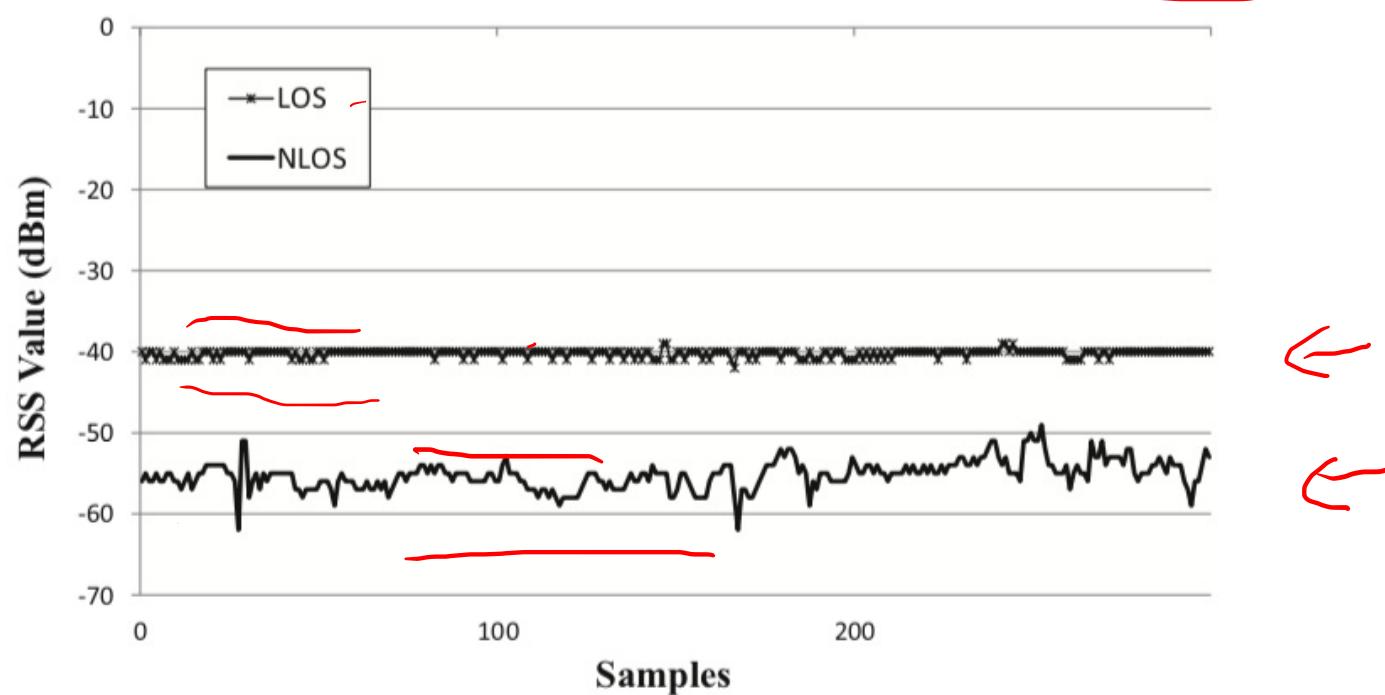


RADIO SIGNAL STRENGTH

Fácil de implementar, barato, se puede utilizar con varias tecnologías.

# RSS

Propenso a desvanecimiento multicamino y al ruido ambiental, menor precisión de localización, la info es mas usada en otras técnicas (fingerprint)



# Triangulación usando RSS

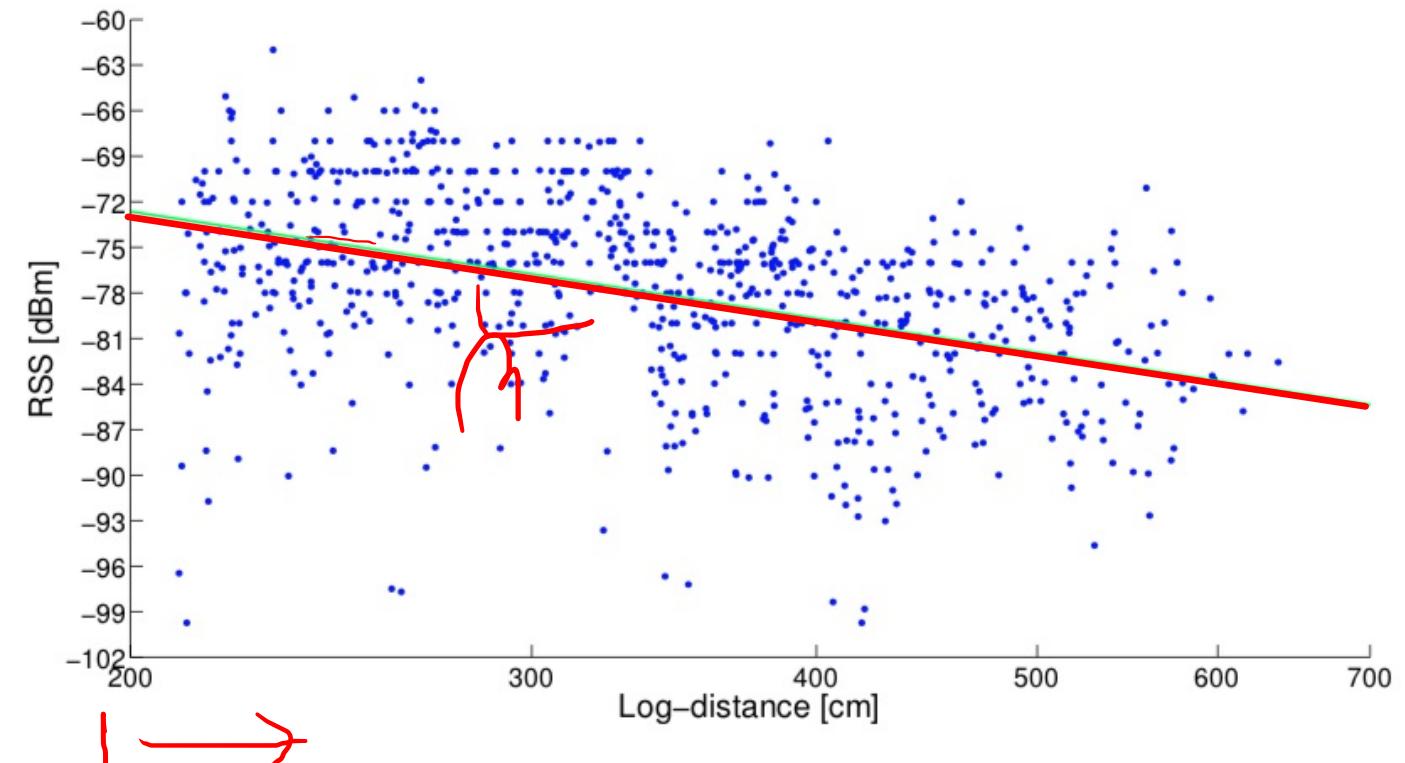
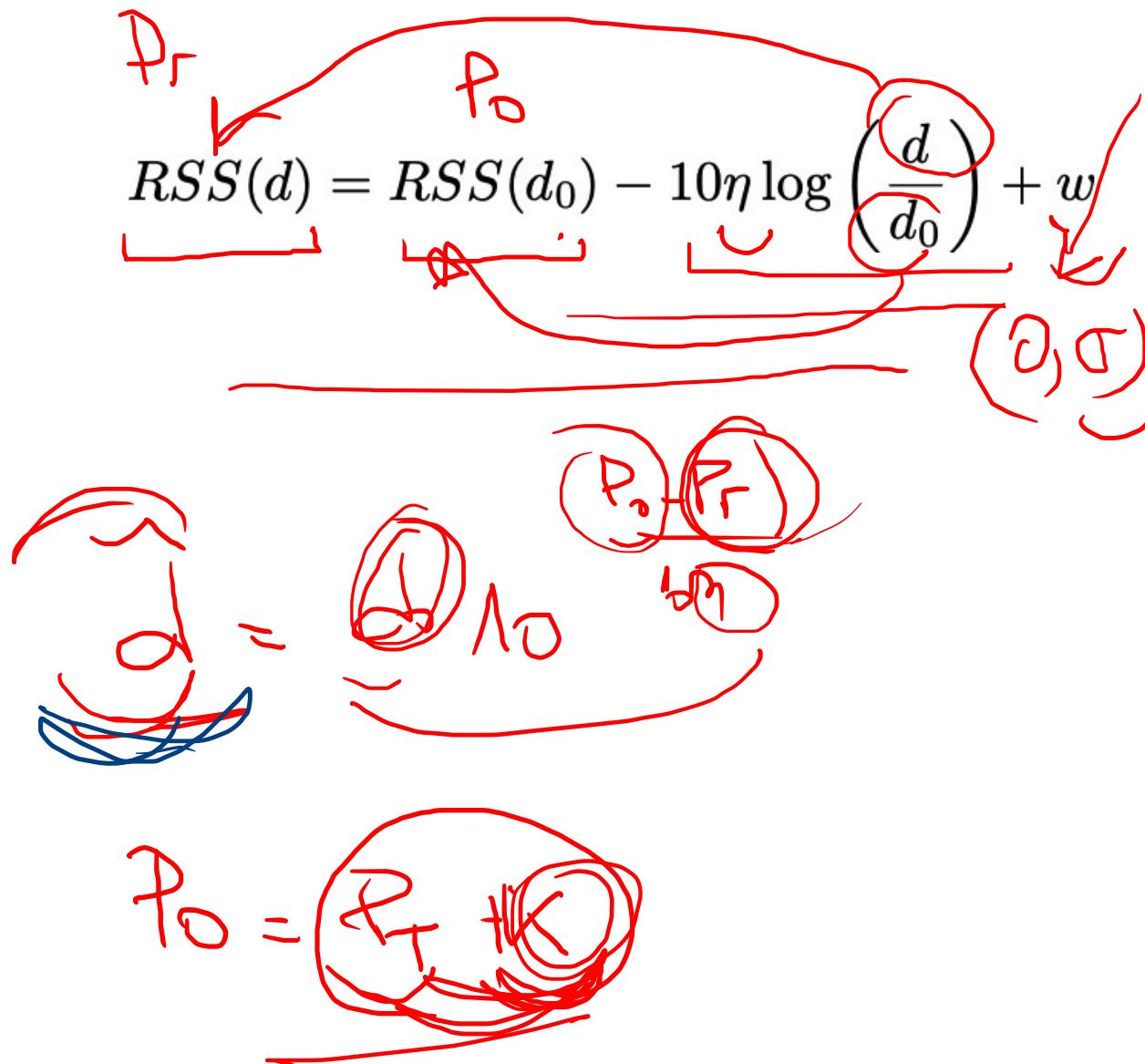


Fig. 3. Experimental RSS measurements for all anchors and related LSM model. Model parameters are  $\eta = 2.3$ ,  $K = -17.2$ ,  $\rho = 0.42$ ,  $\sigma_w = 6$  dB

ZR

$$X \quad X = B$$

$$\hat{X} = (A^T A)^{-1} A^T B$$

$$A = \begin{bmatrix} 2 & 6 \\ 2 & 5.99999 \end{bmatrix}$$

$$B = \begin{bmatrix} 8,000 \\ 8,000 \end{bmatrix} \rightarrow B =$$

$$A = \begin{bmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ \vdots & \vdots \\ 2(x_N - x_1) & 2(y_N - y_1) \end{bmatrix} \quad N \times 2$$

$$B = \begin{bmatrix} d_2^2 - d_1^2 + x_2^2 + x_1^2 + y_2^2 + y_1^2 \\ \vdots \\ d_N^2 - d_1^2 + x_N^2 + x_1^2 + y_N^2 + y_1^2 \end{bmatrix}$$

①  $\hat{x} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$  ②  $x = \begin{bmatrix} 304 \\ -100 \end{bmatrix}$

1 - N medidas RSS

2 - DRDEWS  $\theta = \overline{\text{MAYOR AMEJOR}}$

3 - Encjo 3 (4) y armo A

4 - Calculo tu  $\text{COND}(A) < U$  cond  $(A^T A)^{-1} A^T$

5 - Encjo 3 (4) mejor armo. —  $\tilde{x} = x_k + \Delta x$

3 -  $E R_1 \dots E R_4$

$$\omega_i = \frac{p_i}{\sum_{j=1}^N p_j}$$

$$\begin{cases} A_1 = E R_1 E R_2 E R_3 \\ A_2 = E R_2 E R_1 E R_4 \\ A_3 = E R_3 E R_4 E R_1 \\ A_4 = E R_4 E R_1 E R_2 \end{cases} \quad \begin{cases} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{cases}$$

cond > U

$$\tilde{x} = \frac{1}{M} \sum_{i=1}^M \tilde{x}_i w_i$$

$$w_i = \frac{1}{B} \sum \omega_i = 1$$

## RSS - Minimos cuadrados iterativo

$$\hat{d}_i^2 = (x_i - \bar{x})^2 + (y_i - \bar{y})^2$$

$\frac{\hat{d}_i}{\hat{d}_i} = 10 \frac{P_0 - P_{r,i}}{10n}$

1 -  $(x_0, y_0)$

2 -  $(x_k, y_k) = (x_{k-1}, y_{k-1}) + (\Delta x, \Delta y)_k$

3 -  $\sqrt{\Delta x^2 + \Delta y^2}$  *step*

$$S_{di} = \frac{(x_i - \bar{x}) \delta x + (y_i - \bar{y}) \delta y}{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}} = \frac{x_i - \bar{x}}{\sqrt{\sum}} \delta x + \frac{y_i - \bar{y}}{\sqrt{\sum}} \delta y$$

$$\begin{bmatrix} S_{d1} \\ S_{d2} \\ \vdots \\ S_{dn} \end{bmatrix} = \begin{bmatrix} \frac{x_1 - \bar{x}}{\sqrt{\sum}} & \frac{y_1 - \bar{y}}{\sqrt{\sum}} \\ \vdots & \vdots \\ \frac{x_n - \bar{x}}{\sqrt{\sum}} & \frac{y_n - \bar{y}}{\sqrt{\sum}} \end{bmatrix} \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

$$S_{di} = \bar{d}_i - \bar{d}_{k-1}$$

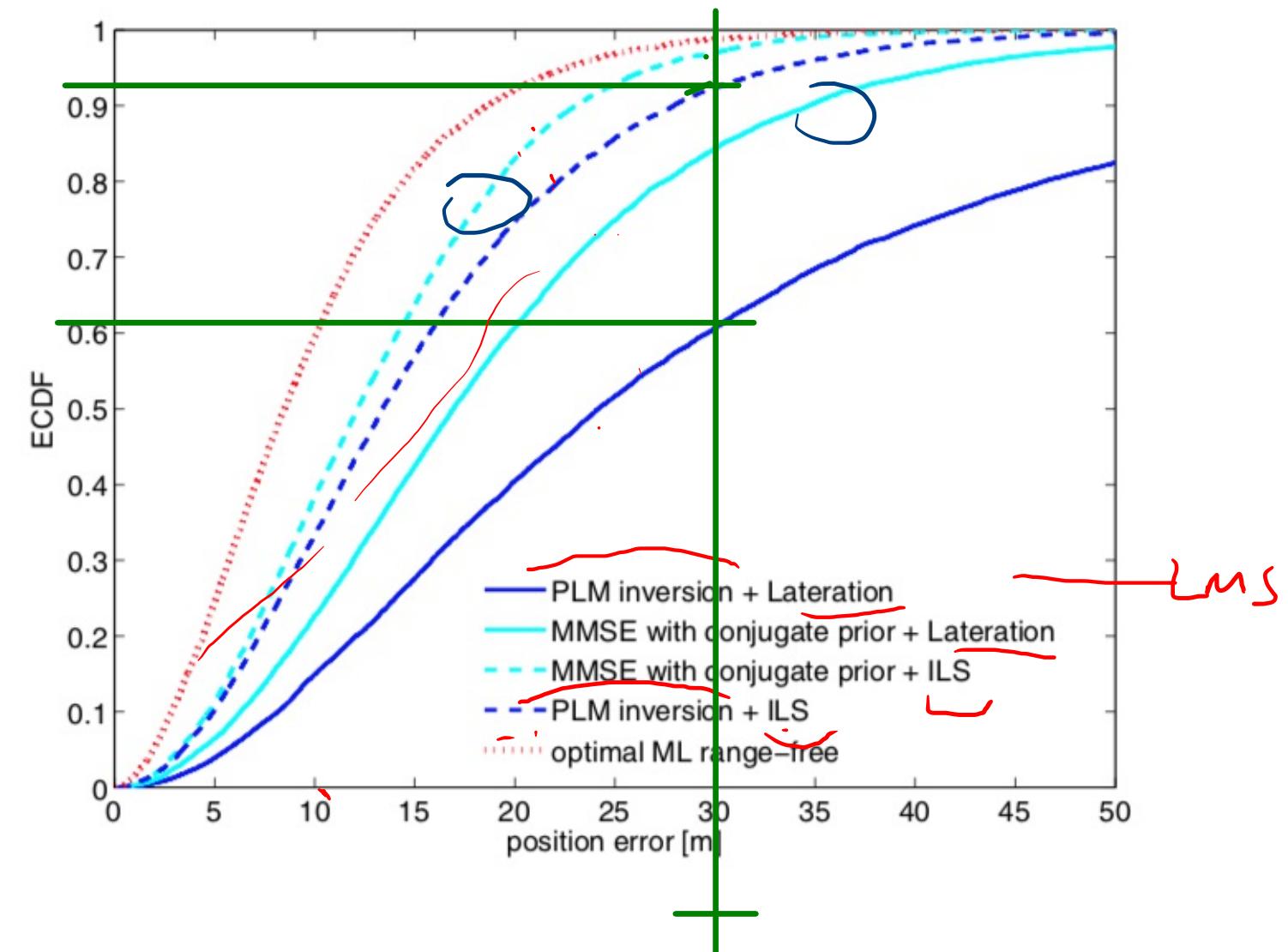
$$\begin{bmatrix} \delta x \\ \delta y \end{bmatrix} = \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \\ \vdots \\ \bar{d}_n \end{bmatrix} - \begin{bmatrix} \bar{d}_{k-1} \\ \bar{d}_{k-1} \\ \vdots \\ \bar{d}_{k-1} \end{bmatrix}$$

$(A^T A)^{-1} A^T$

$(x_{k-1}, y_{k-1})$

# RSS - Minimos cuadrados iterativo

Coluccia, Angelo, and Fabio Ricciato. "RSS-based localization via Bayesian ranging and iterative least squares positioning." IEEE Communications Letters 18.5 (2014): 873-876.



## RSS - Algoritmo Bayesiano

$$P_{r,i} = \theta_i + n_i \sim N(\bar{\theta}, \sigma^2)$$

$$\theta_i = P_0 - 10\eta \log \frac{d_i}{d_0} \sim \mathcal{U}(\bar{\theta}_i, \sigma_\theta)$$

$$p(\theta_i | r_i) = \frac{p(r_i | \theta_i) \cdot p(\theta_i)}{p(r_i)}$$

likelihood  
versimilitud

$$d_i = 10^{\frac{P_0 - \hat{\theta}_i}{10\eta}}$$

$$\hat{\theta}_i = \hat{\beta} \bar{r}_i + (1 - \hat{\beta}) \bar{r}_j$$

$$\hat{\beta} = \frac{\sigma^2}{\sigma^2 + \eta^2}$$

$$\bar{r} = \frac{1}{n} \sum_i r_i$$

$$\gamma = \max(0, \zeta^2 - \sigma^2)$$

$$\zeta^2 = \frac{1}{n} \sum_i (r_i - \bar{r})^2$$

LMS, A, B  
X  
 (model selection)  
 ITERA,  $\Delta x, \Delta y$