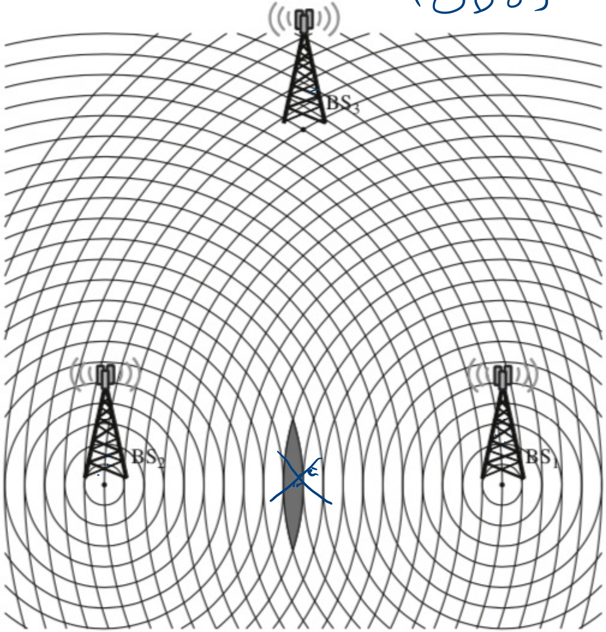
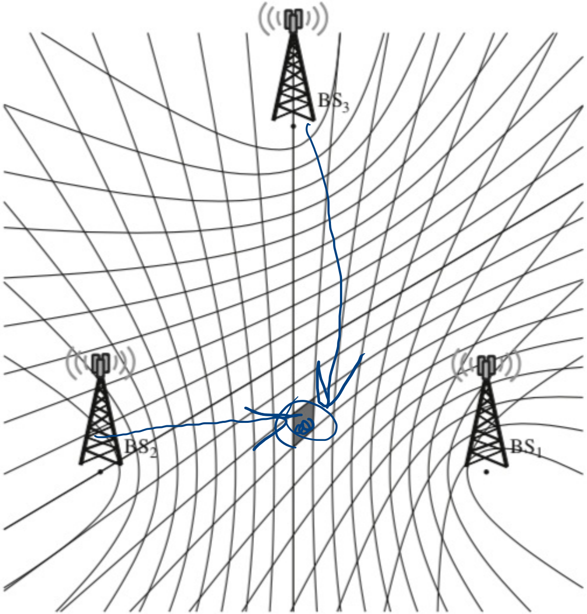
Multilateración Hiperbólica

Sincronización todos



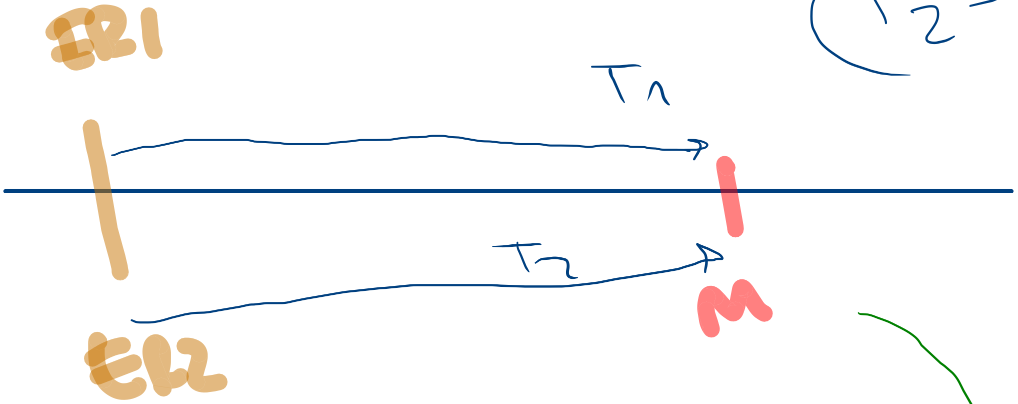
Sincronizar sólo las ER.

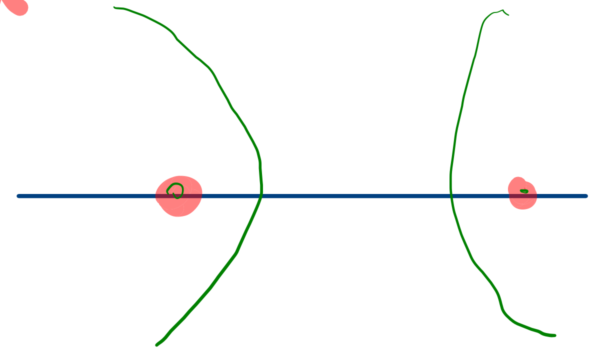


ΔT_{12}

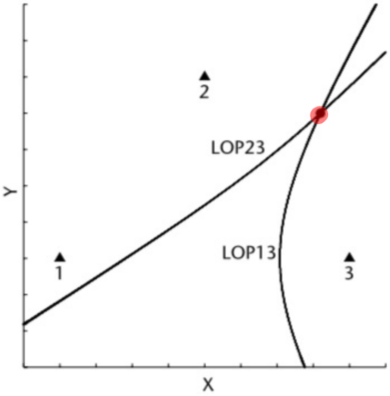
$$(T_2 - T_1)(\Delta t \times c)$$

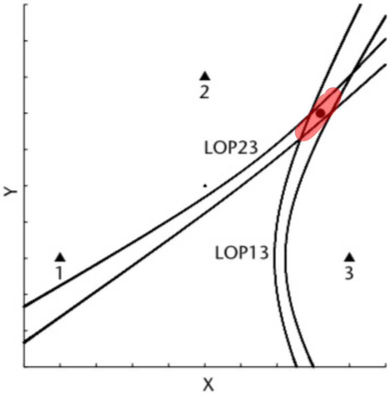
$$\Delta d$$





Multilateración Hiperbólica



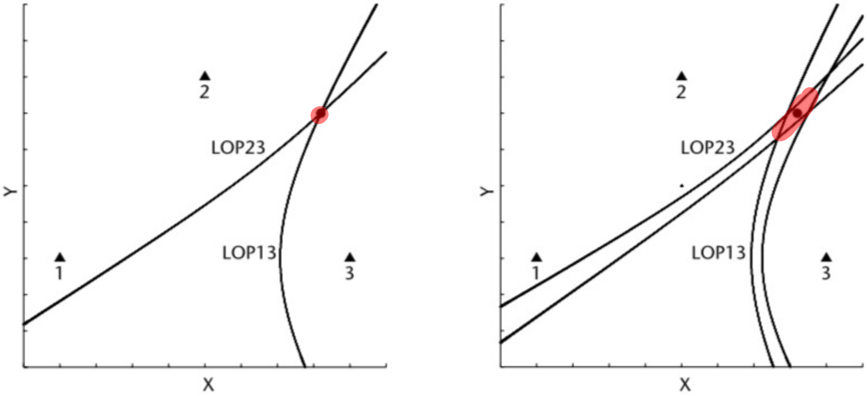


$$\Delta d = c\,\Delta t$$

$$\Delta d = c(\Delta t + n)$$

$$\hat{\Delta t}$$

Multilateración Hiperbólica



$$\Delta d = c\,\Delta t$$

$$\Delta d = c(\Delta t + n)$$

$$\widehat{\Delta t}$$

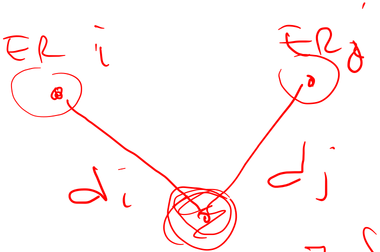
No requiere sincronización de reloj entre el móvil y las ER

TDOA

Requiere sincronización de reloj entre las ER, puede requerir marcas de tiempo, requiere mayor ancho de banda

Multilateración Hiperbólica

$$\Delta\hat{r}_k = \overbrace{(d_i - d_j)}^{F(S)} = \overbrace{c(\Delta t_{ij} + n_{ij})}^{\hat{\Delta t}}$$



$$\begin{bmatrix} d_1-d_2 \\ d_1-d_3 \\ \vdots \\ d_1-d_N \end{bmatrix} = \hat{\Delta R} = \begin{bmatrix} c(\Delta t_{12}+n_{12}) \\ c(\Delta t_{13}+n_{13}) \\ c(\Delta t_{14}+n_{14}) \\ \vdots \\ c(\Delta t_{1N}+n_{1N}) \end{bmatrix} = c\left(\begin{bmatrix} \Delta t_{12} \\ \Delta t_{13} \\ \vdots \\ \Delta t_{1N} \end{bmatrix} + \begin{bmatrix} n_{12} \\ n_{13} \\ \vdots \\ n_{1N} \end{bmatrix}\right)$$

$$\boxed{\hat{\Delta R} = c(\Delta T + N)}$$

$$F(S) = \begin{bmatrix} d_1 - d_2 \\ d_1 - d_3 \\ \vdots \\ d_1 - d_N \end{bmatrix} = \begin{bmatrix} \sqrt{(x-x_1)^2+(y-y_1)^2} - \sqrt{(x-x_2)^2+(y-y_2)^2} \\ \sqrt{(x-x_1)^2+(y-y_1)^2} - \sqrt{(x-x_3)^2+(y-y_3)^2} \\ \vdots \\ \sqrt{(x-x_1)^2+(y-y_1)^2} - \sqrt{(x-x_N)^2+(y-y_N)^2} \end{bmatrix}$$

$$S = \begin{bmatrix} x \\ y \end{bmatrix} \qquad Q_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

$$\Delta\Omega = F(S) + N$$

$$\hat{\Delta R} - F(s) = N \qquad \text{¿} \; S = \{x_i\}?$$

$$\underset{S}{\operatorname{argmin}} \left(\overline{\hat{\Delta R} - F(S)}\right)^T \left(\Delta R - F(S)\right)$$

$$\hat{S} = \underset{S}{\operatorname{argmin}} \left(N^T N \right)$$

$$\hat{S} = \underset{S}{\operatorname{argmin}} \left[\left(\hat{\Delta R} - F(S)\right)^T \left(\hat{\Delta R} - F(S)\right) \right]$$

$$\begin{bmatrix} c(\Delta T + n) \\ \vdots \end{bmatrix} \qquad \begin{bmatrix} \sqrt{x_1 - x} & \cdots & \sqrt{x_2 - x} & \cdots \\ x_1 - x & \cdots & \sqrt{x_N - x} & \cdots \end{bmatrix}$$

$$\hat{S} = \underset{S}{\arg\min} \left((\hat{\Delta R} - F(S))^T (\hat{\Delta R} - F(S)) \right)$$

expansion en S_0 (cond. inicial)

$$\hat{F}(S) = F(S_0) + \frac{\partial f}{\partial x}(x - x_0) + \frac{\partial f}{\partial y}(y - y_0) + \cdots$$

$$\hat{F}(S) = F(S_0) + \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial x} \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

$$\hat{F}(S) = \begin{bmatrix} f_1(s_0) \\ f_2(s_0) \\ \vdots \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial x} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \\ \vdots & \\ \frac{\partial f_N}{\partial x} & \frac{\partial f_N}{\partial y} \end{bmatrix}_{S_0} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

$$\hat{F}(S) = \begin{bmatrix} f_1(s_0) \\ f_2(s_0) \\ \vdots \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \\ \vdots & \\ \frac{\partial f_N}{\partial x} & \frac{\partial f_N}{\partial y} \end{bmatrix} \begin{bmatrix} x_1 - x_0 & x_2 - x_0 & \cdots \\ y_1 - y_0 & y_2 - y_0 & \cdots \end{bmatrix}$$

$$\hat{F}(s) = F(s_0) + H\Big|_{s=s_0} (s - s_0) = \underbrace{F(s_0)} + H \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$$

$$\hat{S} = \underset{S}{\arg\min} \left\{ \left(\hat{\Delta R} - F(s_0) - H(s - s_0)\right)^T \left(\hat{\Delta R} - F(s_0) - H(s - s_0)\right) \right\}$$

$$\hat{S} = \underset{S}{\operatorname{argmin}} \left\{ \left(\hat{\Delta R} - F(S_0) - H(S - S_0)\right)^T \left(\hat{\Delta R} - F(S_0) - H(S - S_0)\right) \right\}$$

$$\hat{S} = \underset{S}{\operatorname{argmin}} \left\{ \left[\underbrace{\left(\hat{\Delta R} - F(S_0) + HS_0\right)}_{\tilde{\Delta R}} - HS \right]^T \left[\cdots \right] \right\}$$

$$\hat{S} = \underset{S}{\operatorname{argmin}} \left\{ \left[\tilde{\Delta R} - HS \right]^T \left[\tilde{\Delta R} - HS \right] \right\} \quad ¿S?$$

$$\boxed{\hat{S} = (H^T H)^{-1} H^T \tilde{\Delta R}}$$

$$\boxed{\hat{S} = (H^T H)^{-1} H^T \widetilde{\Delta\Omega}}$$

$$\hat{S} = (H^T H)^{-1} H^T \left(\hat{\Delta R} - F(s_0) + H s_0 \right)$$

$$\hat{S}_1 = (H^T H)^{-1} H^T \left(\hat{\Delta R} - F(s_0) \right) + s_0$$

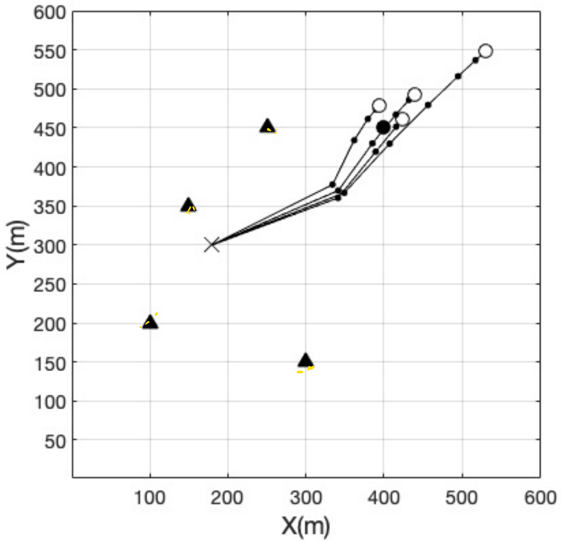
$$\boxed{\hat{S}_{k+1} = \hat{S}_k + (H^T H)^{-1} H^T_{s_k} \left(\hat{\Delta R} - F(s_k) \right)}$$

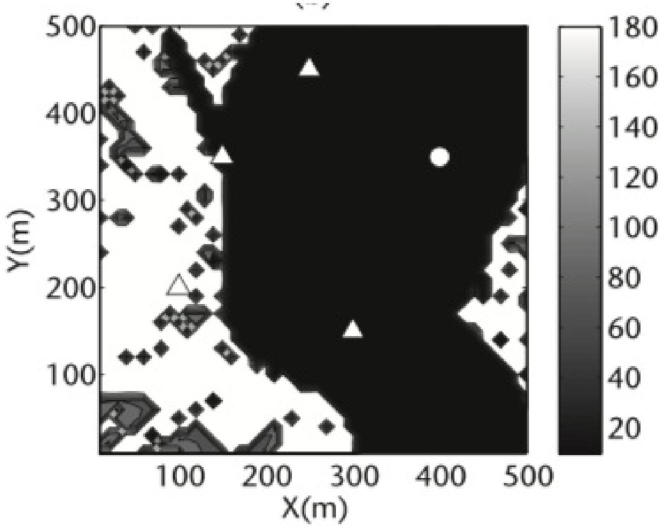
Multilateración Hiperbólica

$$\hat{S} = \arg\min_{s} \{(\Delta\hat{R} - F(s))^T (\Delta\hat{R} - F(s))\}$$

Multilateración Hiperbólica: Ejercicio

Correr el programa ch02fig11.m para la misma CI y luego para distintas condiciones inciales fuera del poligono que forman las ER y el móvil dentro y fuera también.





Multilateración Hiperbólica: Filtro de Kalman

$x, y, \dot{x}, \dot{y}$

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k\,\hat{\mathbf{x}}_{k-1|k-1}$$

$$\mathbf{P}_{k|k-1} = \mathbf{F}_k\,\mathbf{P}_{k-1|k-1}\,\mathbf{F}_k^T + \mathbf{G}_k\,\mathbf{Q}_k\,\mathbf{G}_k^T$$

$$\nabla h = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & 0 & 0 \end{bmatrix}$$

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_k\,[\hat{\mathbf{z}}_k - \hat{\mathbf{h}}(\hat{\mathbf{x}}_{k|k-1})]$$

$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_k\mathbf{S}_k\mathbf{W}_k^T$$

$$h = \begin{bmatrix} \sqrt{(x-x_1)^2+(y-y_1)^2} - \sqrt{(x-x_2)^2+(y-y_2)^2} \\ \\ \end{bmatrix}$$

$$\mathbf{W}_k = \mathbf{P}_{k|k-1}(\nabla^T\mathbf{h}_k^x)\,\mathbf{S}_k^{-1}$$

$$\mathbf{S}_k = (\nabla\mathbf{h}_k^x)\,\mathbf{P}_{k|k-1}\,(\nabla^T\mathbf{h}_k^x) + \mathbf{R}_k$$

Multilateración Hiperbólica: Método de Lavenberg-Marquardt

$$\underbrace{\mathbf{x}_{k+1}}_{S_{k+1}} = \underbrace{\mathbf{x}_k}_{S_k} + (\underbrace{\mathbf{A}}^k + \lambda^k \mathbf{I})^{-1} \cdot \mathbf{g}_k$$

$$S_k = S_k + (H^T H)^{-1} H^T (\hat{\Delta R} - F(S_k))$$

$$A^k = \left[\phi_k^T \Sigma^{-1} \phi_k \right]$$

$$\phi_k = \begin{bmatrix} \frac{x_k - x_1}{\sqrt{(x_k - y_1)^2 + (y_k - y_1)^2}} & \frac{y_k - y_1}{\sqrt{\quad}} \\ \vdots & \\ \frac{x_k - x_N}{} & \end{bmatrix}$$

$$\underbrace{\mathbf{x}_{k+1}}_{S_{k+1}} = \underbrace{\mathbf{x}_k}_{S_k} + (\mathbf{A}^k + \lambda^k \mathbf{I})^{-1} \cdot \mathbf{g}_k$$

$$S_k = S_k + (H^T H)^{-1} H^T (\Delta \hat{R} - F(s_k))$$

$$\Lambda_k = [\phi^T \Sigma^{-1} \phi]$$

$$g_k = \phi^T \Sigma^{-1} (d - \hat{d}_k)$$

ΔR

dist. est.

dist medidas

$(\hat{d}_1 - \hat{d}_2)$

$\sqrt{(x_1 - x_k)^2 + (y_1 - y_k)^2}$

H^T $\quad$ $F(s_k)$

$$\lambda^k = \sum \max \{ A_k / i,i \}$$

km. $\quad 2 \sim 5$

$$\begin{bmatrix} 0 & & & & \\ & 0 & & & \\ & & 0 & & \\ & & & 0 & \\ & & & & 0 \end{bmatrix}$$

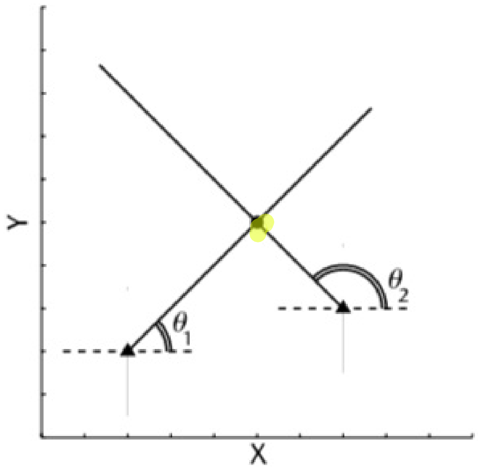
$$\rho = \frac{\varepsilon(x_k) - \varepsilon(x_{k+1})}{H_k^T (d^k H_k + g_k)} \rightarrow \text{escalar}$$

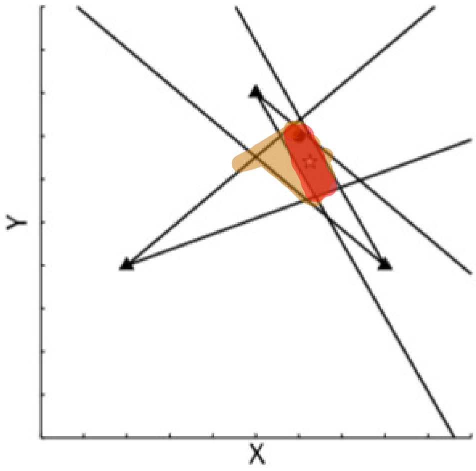
$$\varepsilon(x_k) = [d - \hat{d}_k]^T \Sigma^{-1} [d - \hat{d}_k]$$

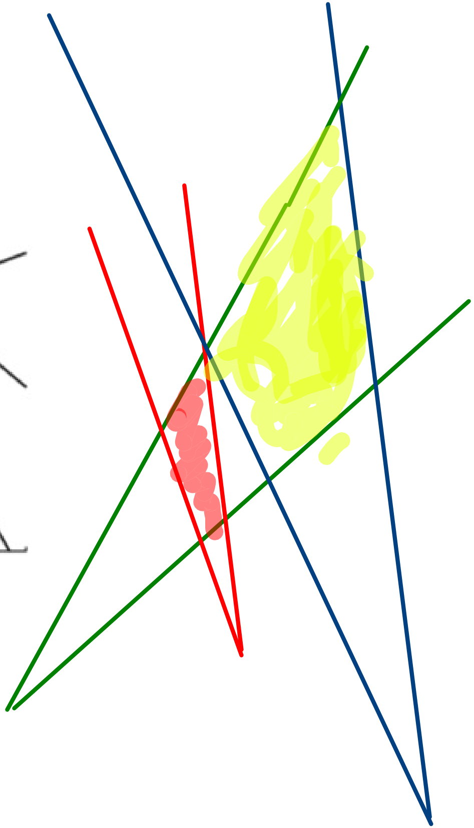
si $\rho > 0$ → usa el alg. normal

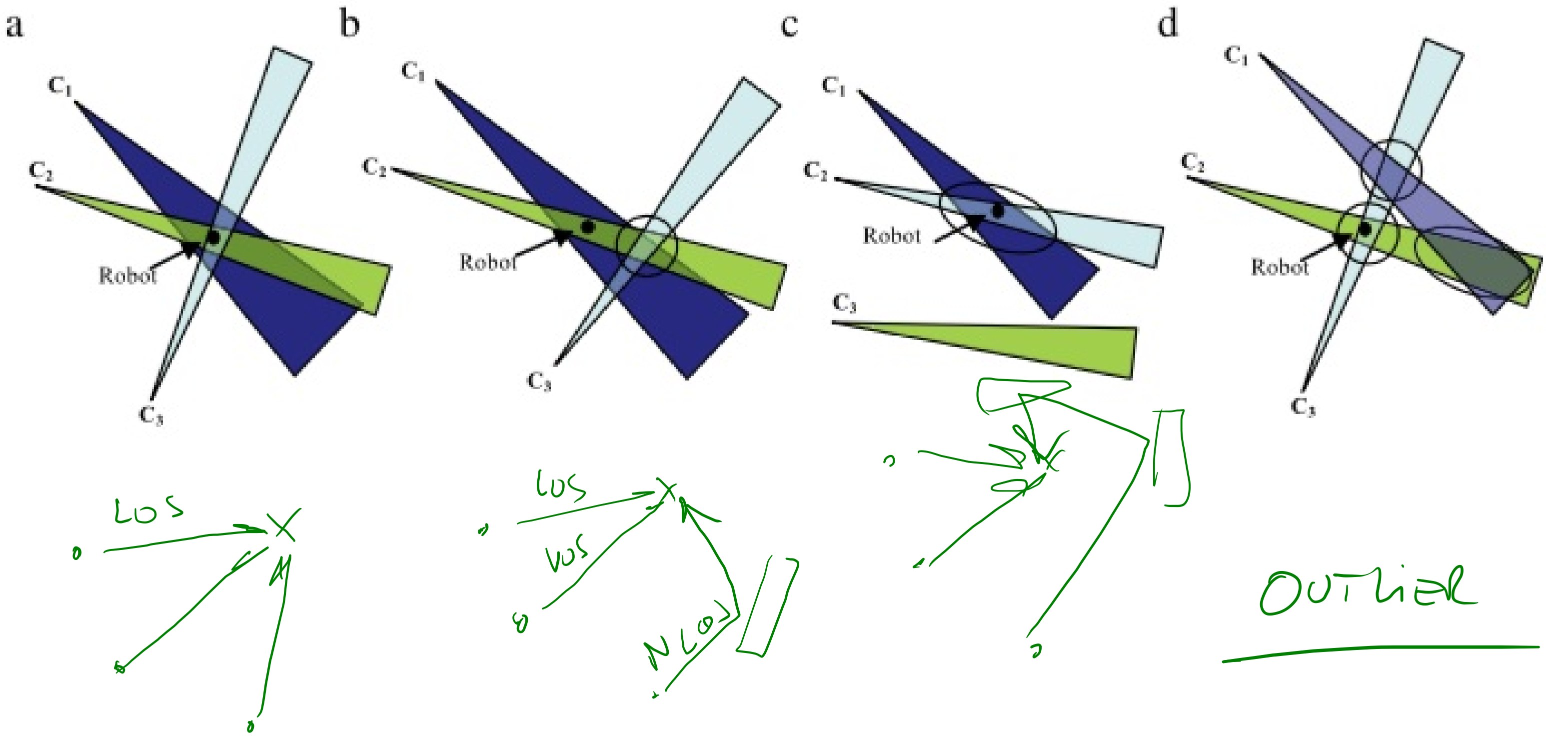
$\rho \leq 0$ $\quad \Delta_{k+1} = \Delta_k \quad g_{k+1} = g_k$

Multiangulación: DOA/AOA








a
b
c
d
C1
C2
C3
Robot
LOS
LOS
LOS
NLOS
OUTLIER