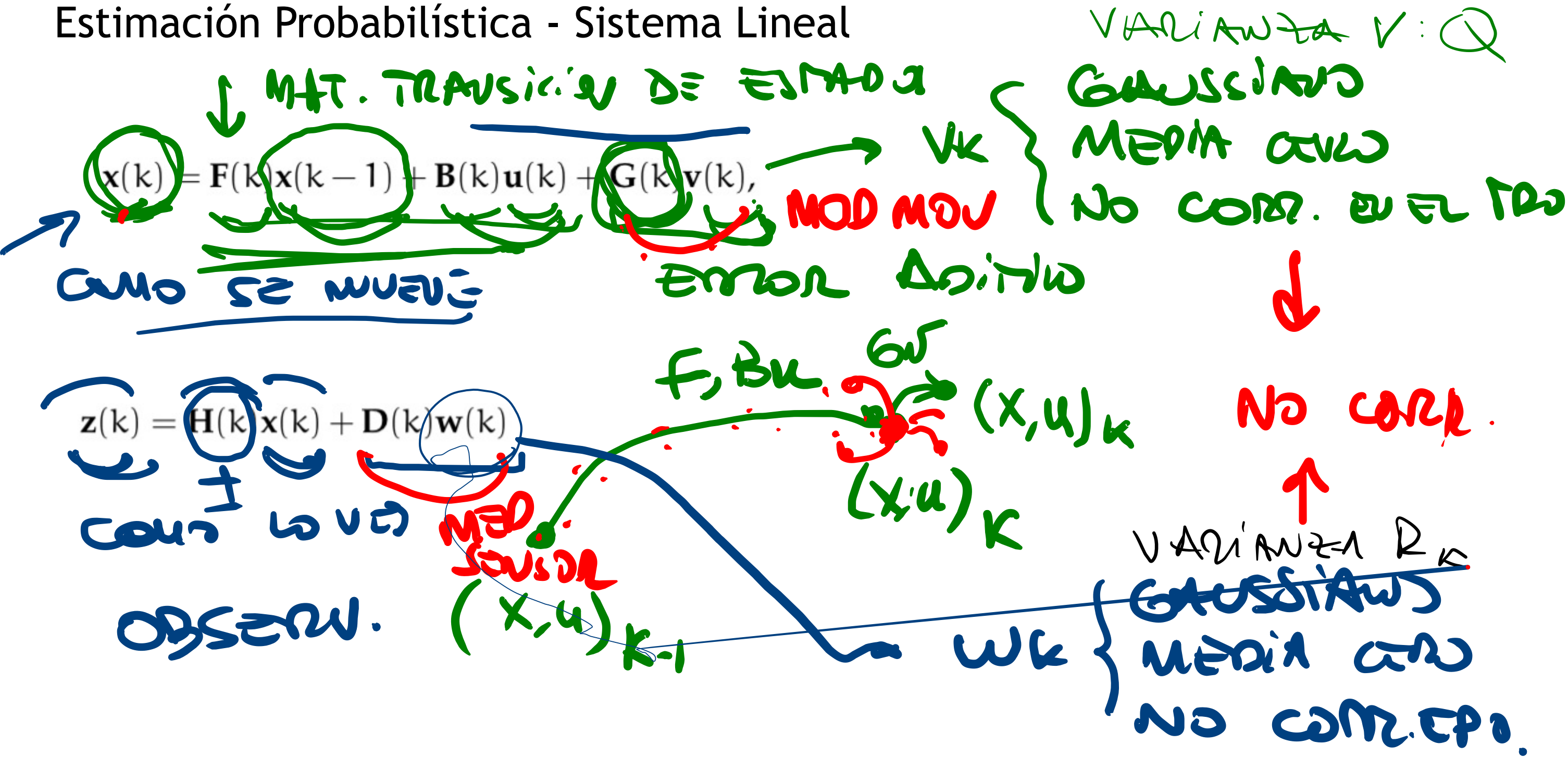
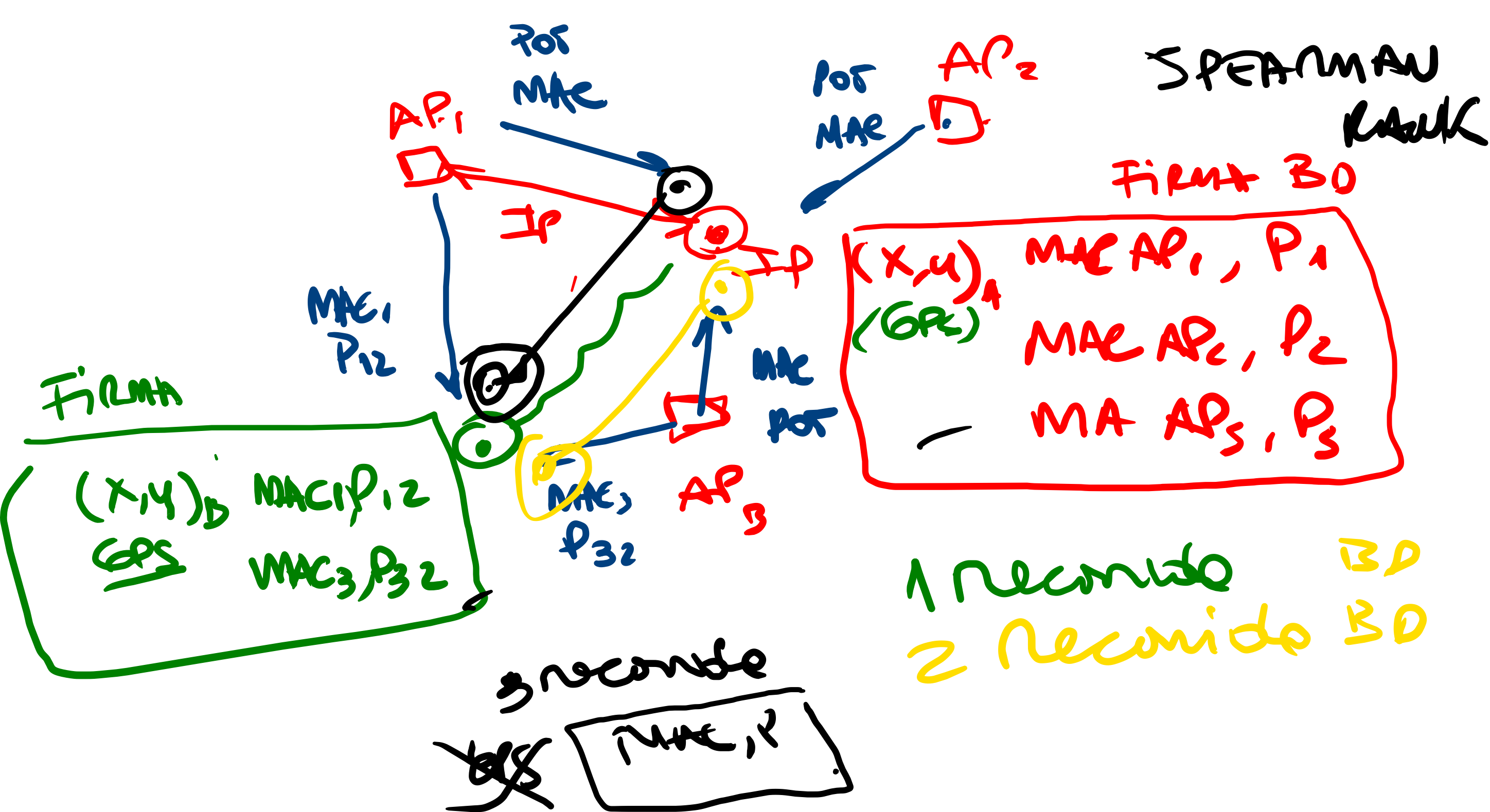
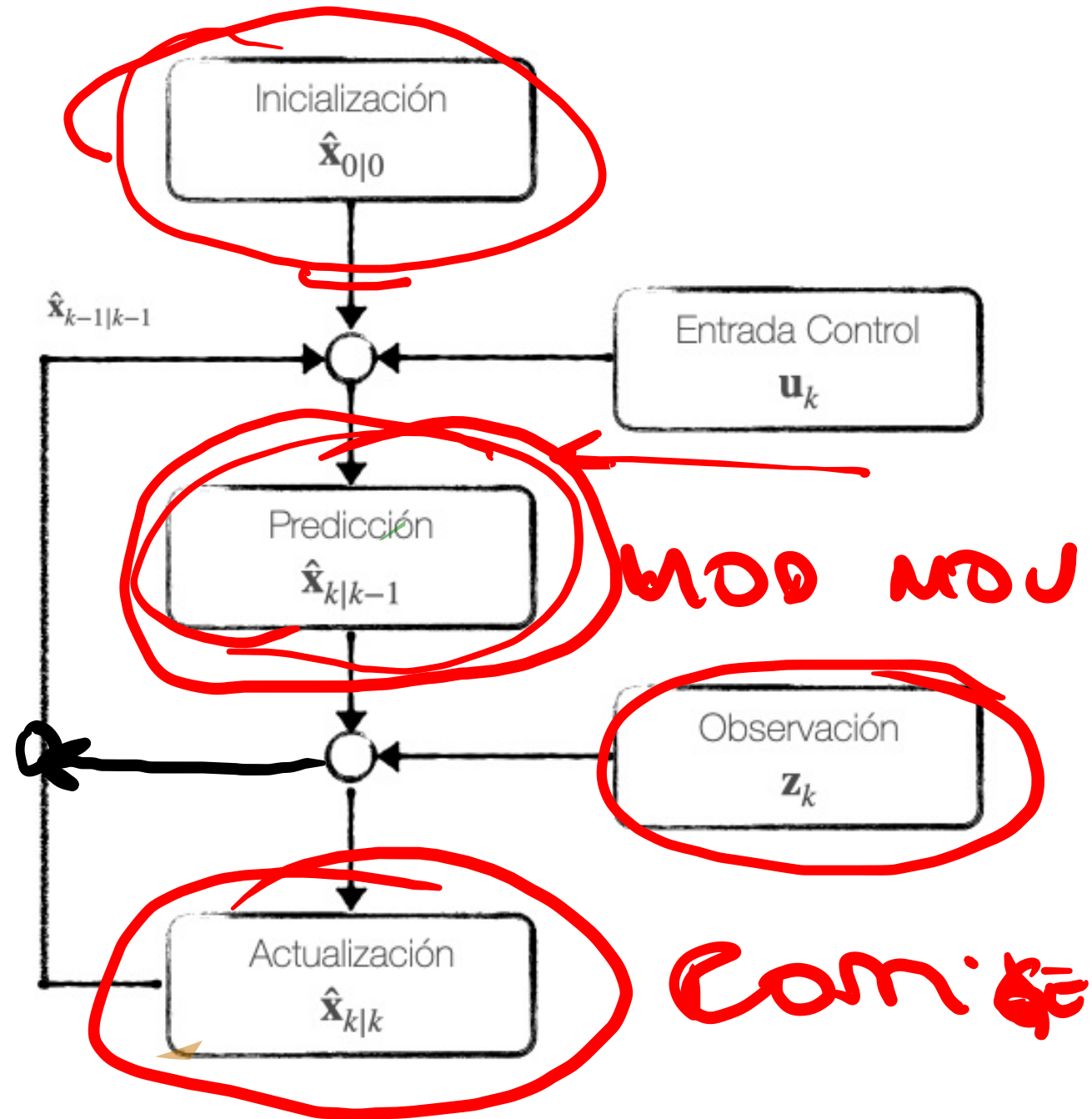


Estimación Probabilística - Sistema Lineal





Filtro de Kalman (lineal)



Filtro de Kalman (lineal)

Predicción del estado y la varianza

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k$$

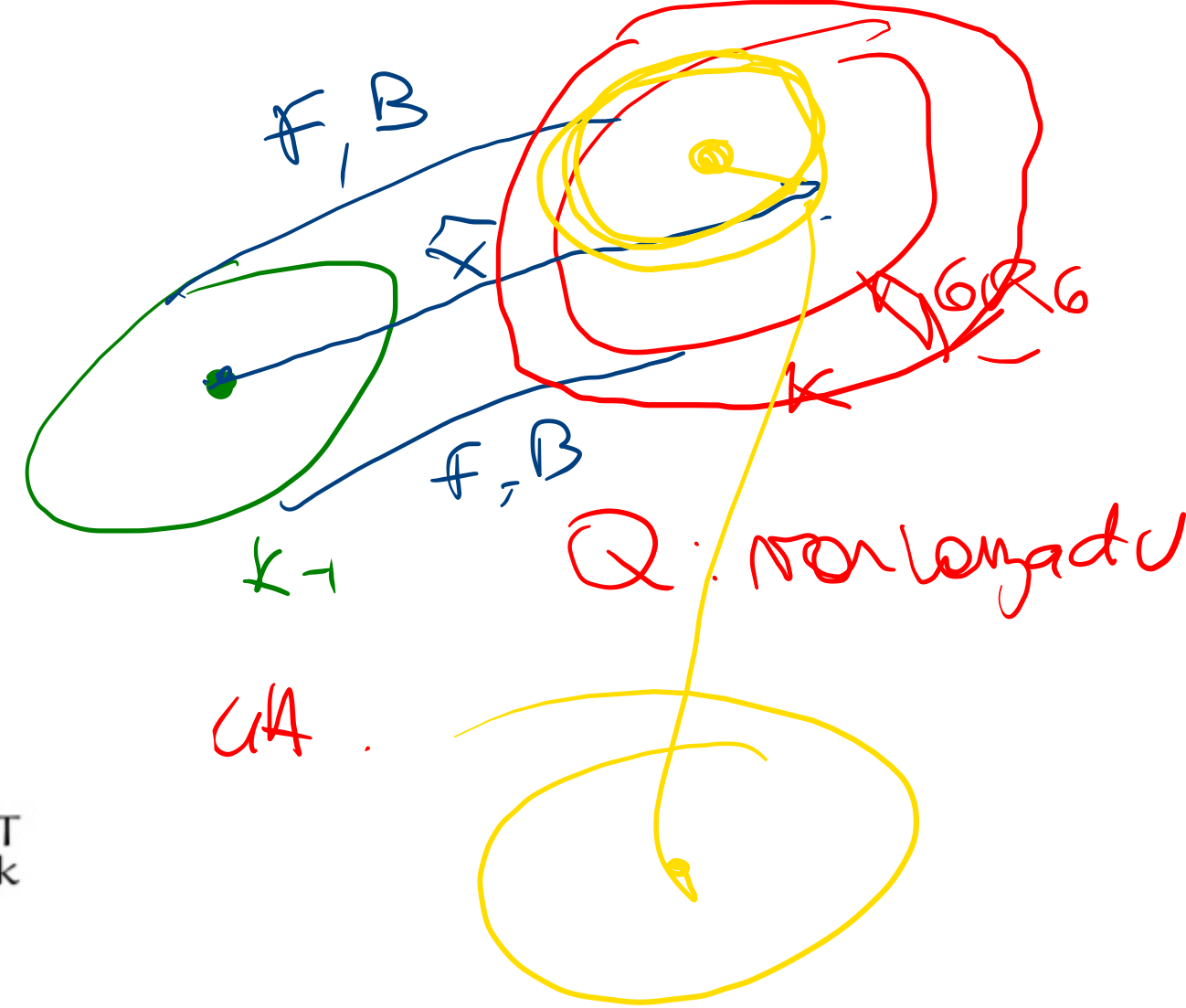
$$\mathbf{P}_{k|k-1} = \mathbf{F}_k \mathbf{P}_{k-1|k-1} \mathbf{F}_k^T + \mathbf{G}_k \mathbf{Q}_k \mathbf{G}_k^T$$

Actualización del estado y la varianza

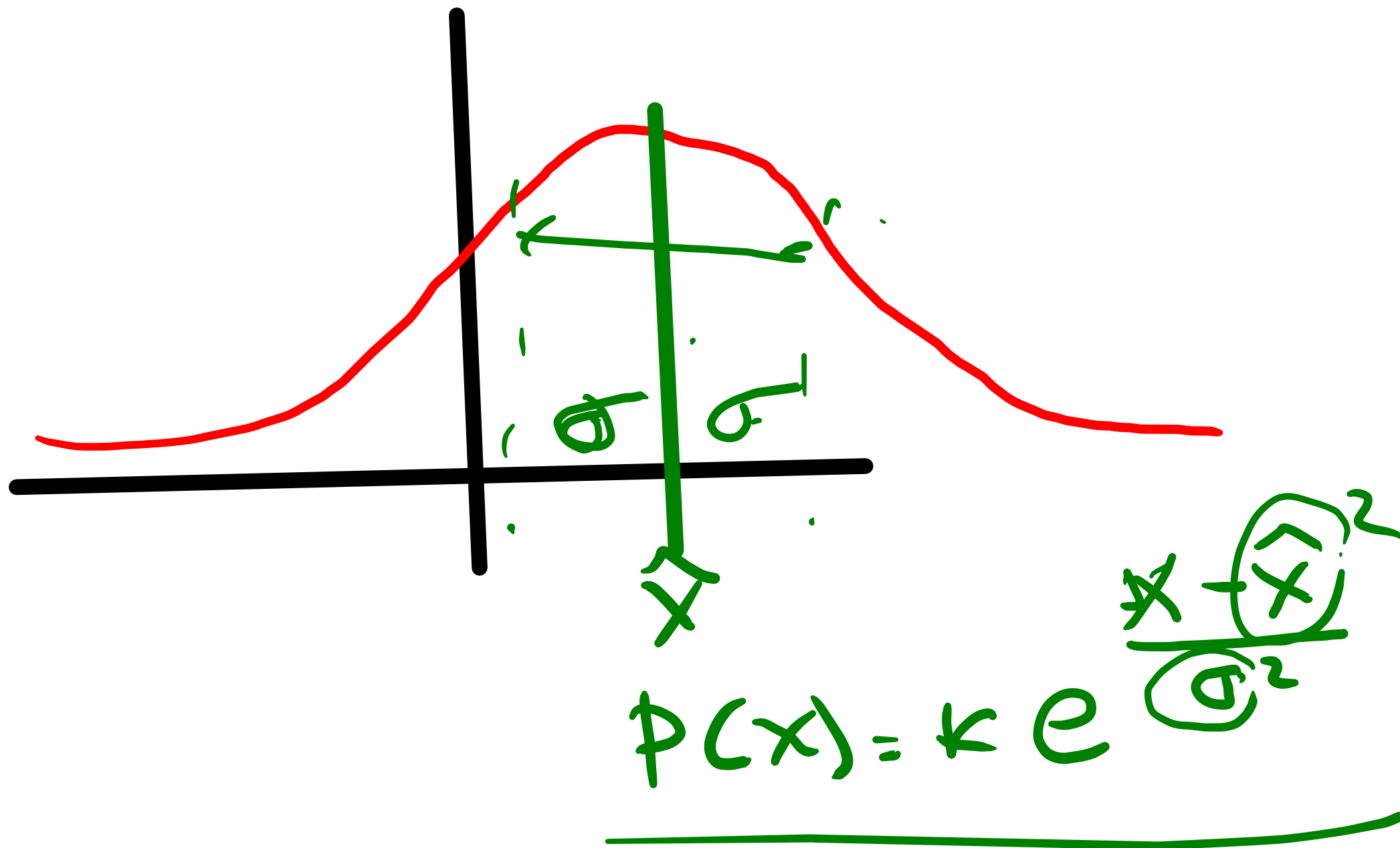
$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1})$$

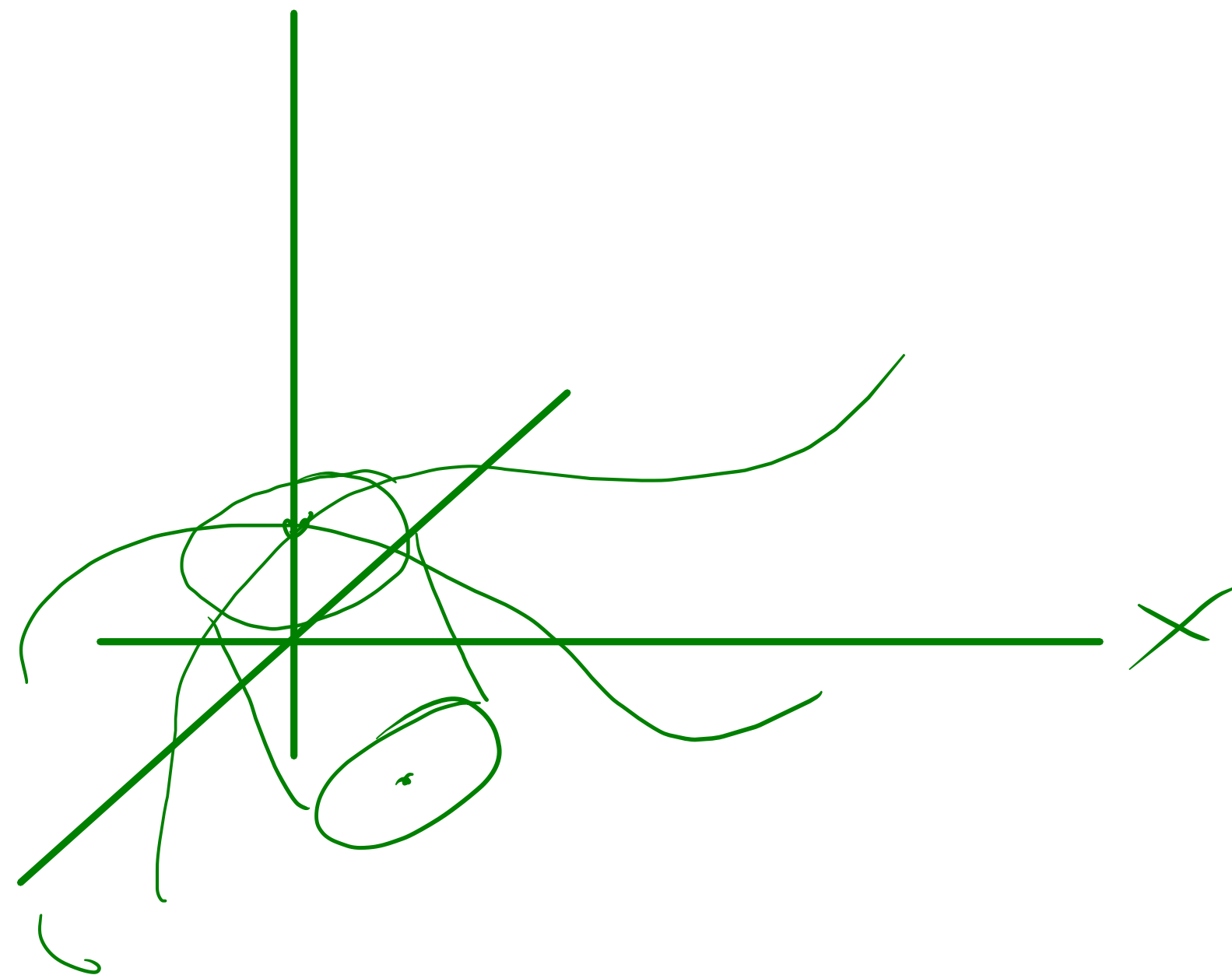
$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{W}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{W}_k \mathbf{H}_k)^T + \mathbf{W}_k \mathbf{R}_k \mathbf{W}_k^T$$

$$\mathbf{W}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k]^{-1}$$



1 medida / seg
1000 medidas / seg.





Innovación

\hat{z}

$$v_k = z_k - H_k \hat{x}_{k|k-1}$$

$$E\{v_k | Z^{k-1}\} = 0 \quad E\{v_i v_j^T\} = S_i \delta_{ij}$$

$$S_k = R_k + H_k P_{k|k-1} H_k^T$$

varianza
de w_k

varianza
de lo pred.

Filtro de Kalman (lineal)

Actualización del estado y la varianza

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_k (\mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1})$$

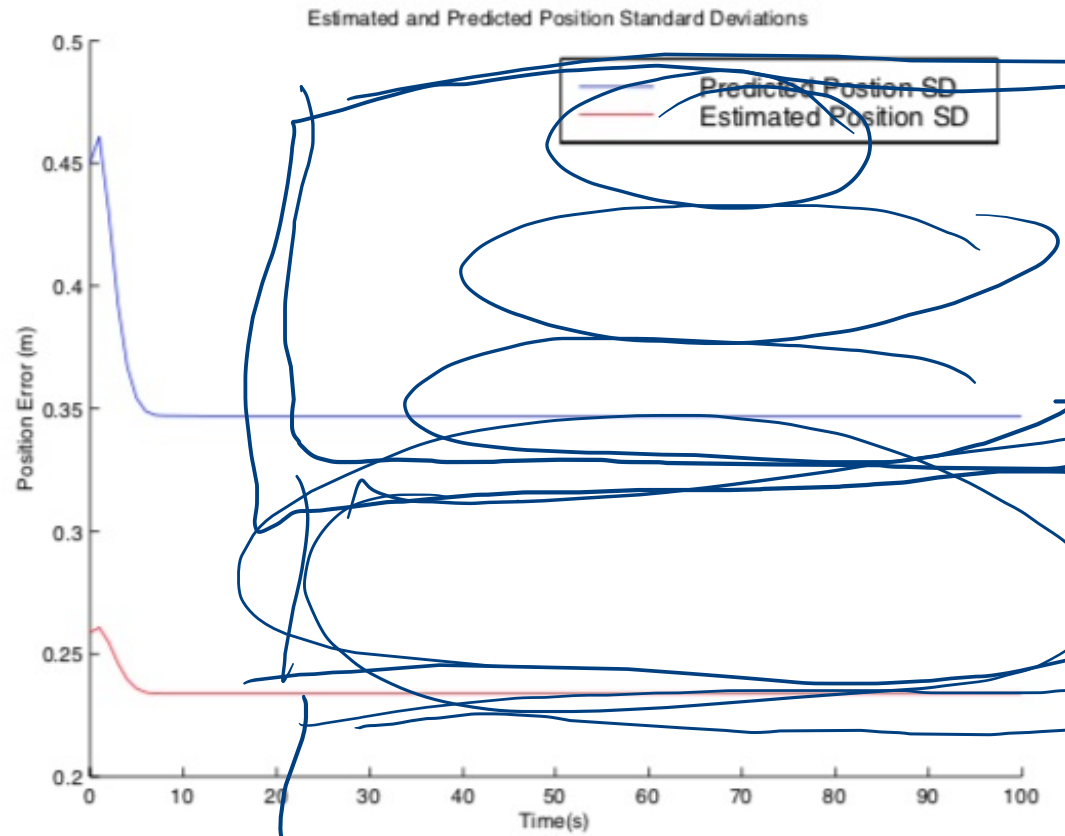
$$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{W}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} (\mathbf{I} - \mathbf{W}_k \mathbf{H}_k)^T + \mathbf{W}_k \mathbf{R}_k \mathbf{W}_k^T$$

$$\mathbf{W}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T [\mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^T + \mathbf{R}_k]^{-1}$$

Handwritten notes in blue ink summarizing the Kalman filter equations:

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_k \mathbf{v}_k$$
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^T$$
$$\mathbf{W}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^T \mathbf{S}_k^{-1}$$

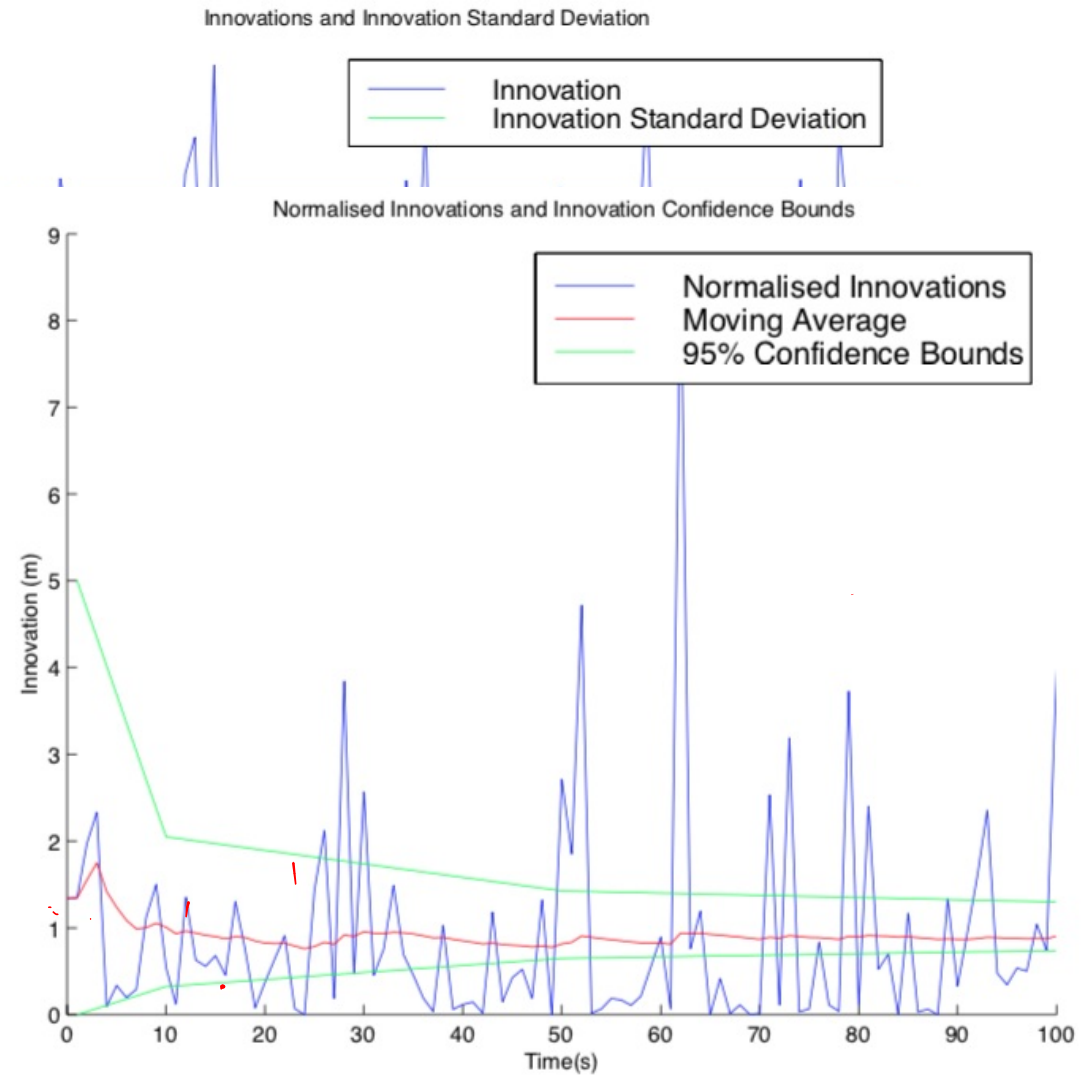
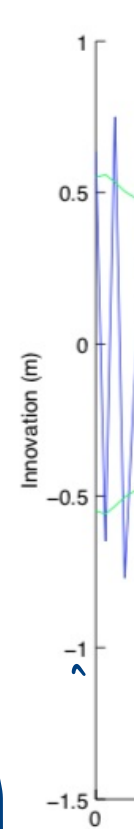
Kalman



Secuencia Innovación

Desempeño de estado estac.

Condiciones del Error



$$\frac{V_0}{S_0}$$

$$\frac{V_0 + U_1}{S_0 + S_1}$$

$$\frac{U_2}{S_0} + \frac{1}{S_1} + \frac{U_2}{S_2}$$

Sistema No lineal

$$\mathbf{x}_k = \mathbf{f}[\mathbf{x}_{k-1}, \mathbf{u}_k, k] + \mathbf{v}_k$$

$$\mathbf{z}_k = \mathbf{h}[\mathbf{x}_k, k] + \mathbf{w}_k$$

$$r = \sqrt{x^2 + y^2}$$

$$\varphi = \tan^{-1} \left(\frac{y}{x} \right)$$



$$F = \begin{bmatrix} A & 0 & b & 0 \\ 0 & 1 & c & 0 \\ 0 & \dots & \dots & \dots \end{bmatrix}$$

$$F = \begin{bmatrix} f_1(x, u) \\ f_2(x, u) \\ \vdots \end{bmatrix}$$

EKF: Filtro extendido de Kalman (no lineal)

Predicción del estado y la varianza

$$\hat{\mathbf{x}}_{k|k-1} = \mathbf{f}[\hat{\mathbf{x}}_{k-1|k-1}, \mathbf{u}_k]$$

$$\mathbf{P}_{k|k-1} = \nabla \mathbf{f}_x \mathbf{P}_{k-1|k-1} \nabla \mathbf{f}_x^T + \mathbf{Q}_k$$

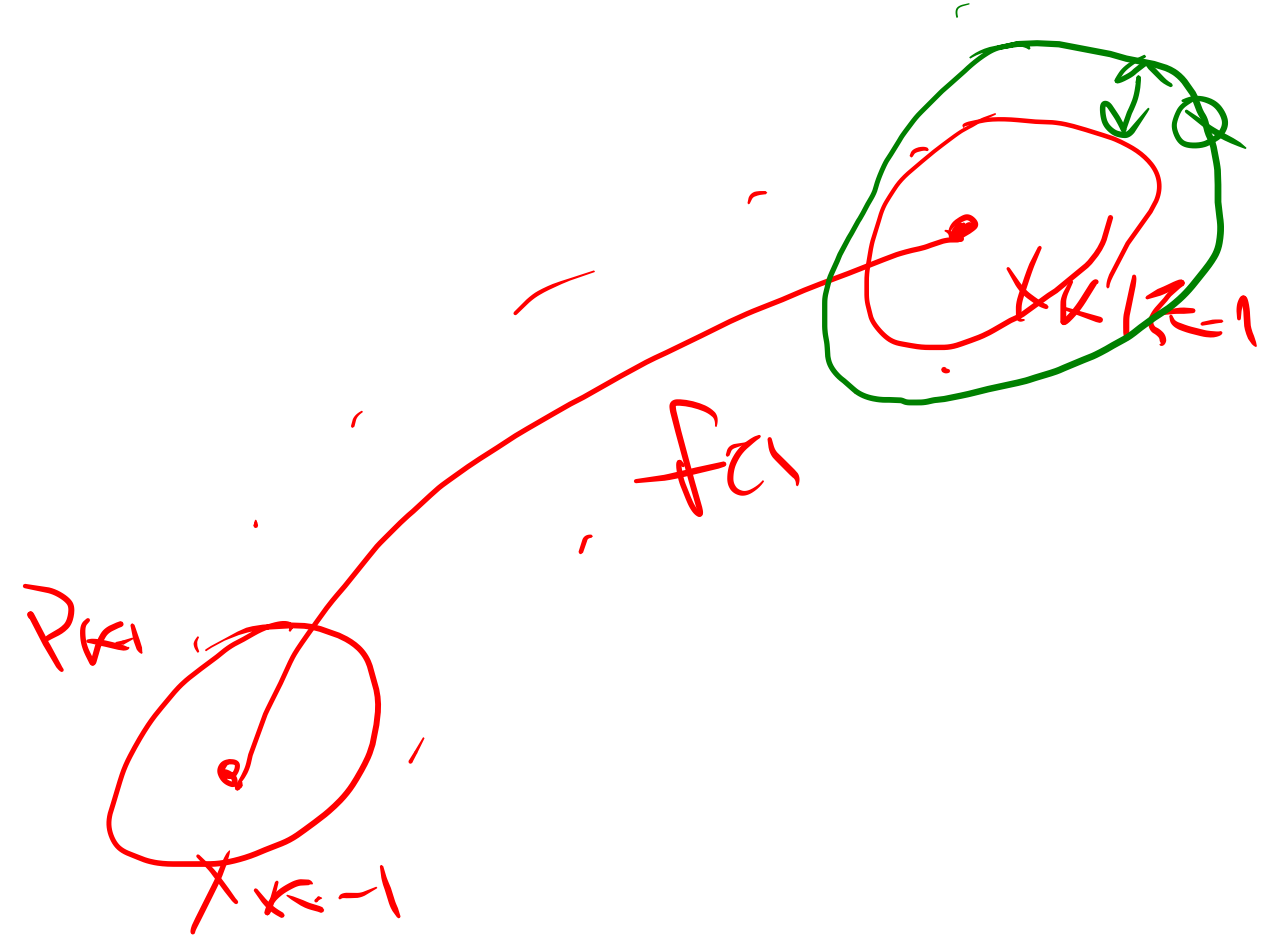
Actualización del estado y la varianza

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_k (\mathbf{z}_k - \mathbf{h}[\hat{\mathbf{x}}_{k|k-1}])$$

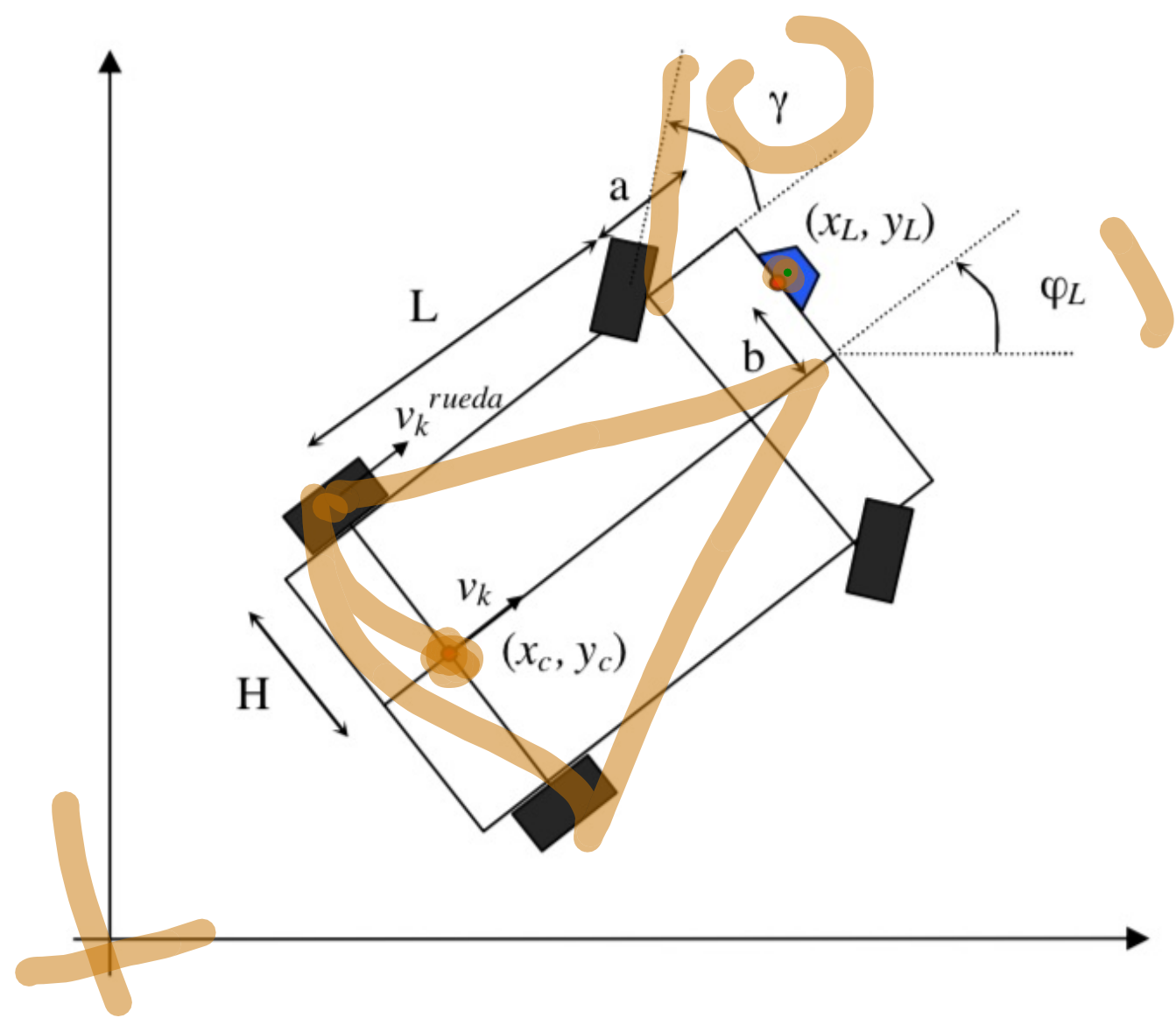
$$\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1} - \mathbf{W}_k \mathbf{S}_k \mathbf{W}_k^T$$

$$\mathbf{W}_k = \mathbf{P}_{k|k-1} \nabla \mathbf{h}_x^T \mathbf{S}_k^{-1}$$

$$\mathbf{S}_k = \nabla \mathbf{h}_x \mathbf{P}_{k|k-1} \nabla \mathbf{h}_x^T + \mathbf{R}_k$$



Modelo de un Vehículo Terrestre



Modelo de un Vehículo Terrestre

$$\begin{bmatrix} x_{Lk} \\ y_{Lk} \\ \varphi_{Lk} \end{bmatrix} = \begin{bmatrix} x_{Lk-1} + \Delta t \cdot v_k \cdot [\mathbb{G} \cos(\varphi_{Lk-1}) - \mathbb{H} \sin(\varphi_{Lk-1})] \\ y_{Lk-1} + \Delta t \cdot v_k \cdot [\mathbb{G} \sin(\varphi_{Lk-1}) + \mathbb{H} \cos(\varphi_{Lk-1})] \\ \varphi_{Lk-1} + \Delta t \cdot \frac{v_k}{L} \cdot \tan(\gamma_k) \end{bmatrix}$$

- $L = 2,38\text{m}$

- $a = 0,95\text{m}$

- $b = 0,5\text{m}$

- $H = 0,75\text{m}$

$$\mathbb{G} = \left(1 - \frac{b \tan(\gamma_k)}{L}\right)$$

$$\mathbb{H} = \frac{(L + a) \tan(\gamma_k)}{L}$$

$$v = \frac{v_r}{1 - \frac{H}{L} \tan(\alpha)}$$

