# Communication Systems based on Software Defined Radio (SDR)

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## Digital Communication Systems

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# *Intersymbol Interference (ISI)*

As was previously described, the second problem to be solved in a baseband transmission system is the effect of the interference that one symbol (ISI)of the sequence can generate on the other. For a signal of the form:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kD)$$

Then, based on the concept of superposition, the effect of intersymbol interference is analyzed as if the noise did not exist. Having considered the separate and overlapping effects of noise and ISI, the two will be considered together.

From the contribution of the different effects on the transmitted signal after sampling the received signal:

# *Intersymbol Interference (ISI)*

$$y(t_K) = a_K + \sum_{k \neq K} a_k \tilde{p}(t - t_d - kD) + n(t_K)$$

If only taking into account the effect of ISI:

$$y(t_K) = a_K + \sum_{k \neq K} a_k \tilde{p}(t - t_d - kD)$$

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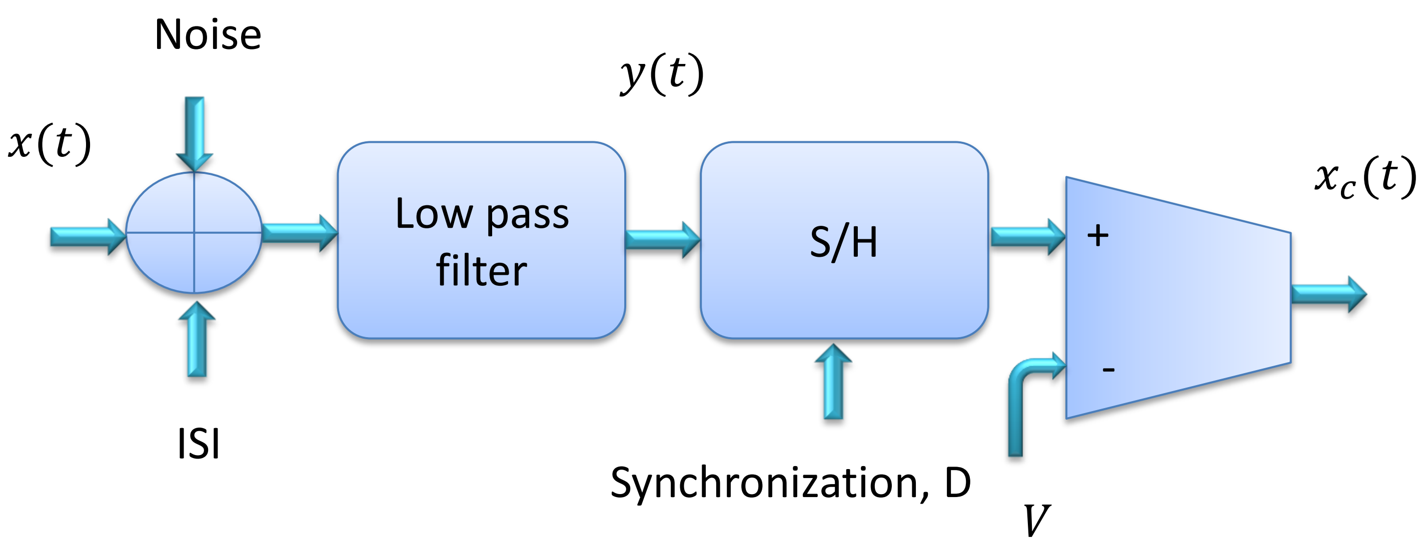
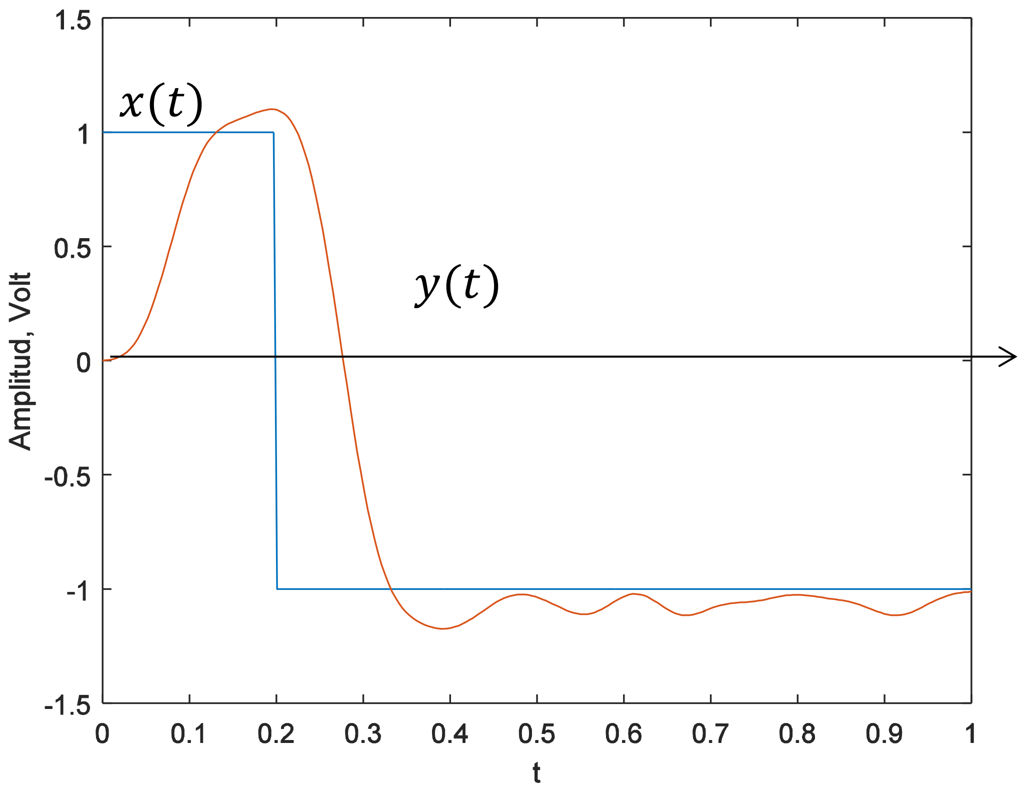
# *Intersymbol Interference (ISI)*

In input of the receiver appears the sum of the received signal, the noise and the possible ISI interference.



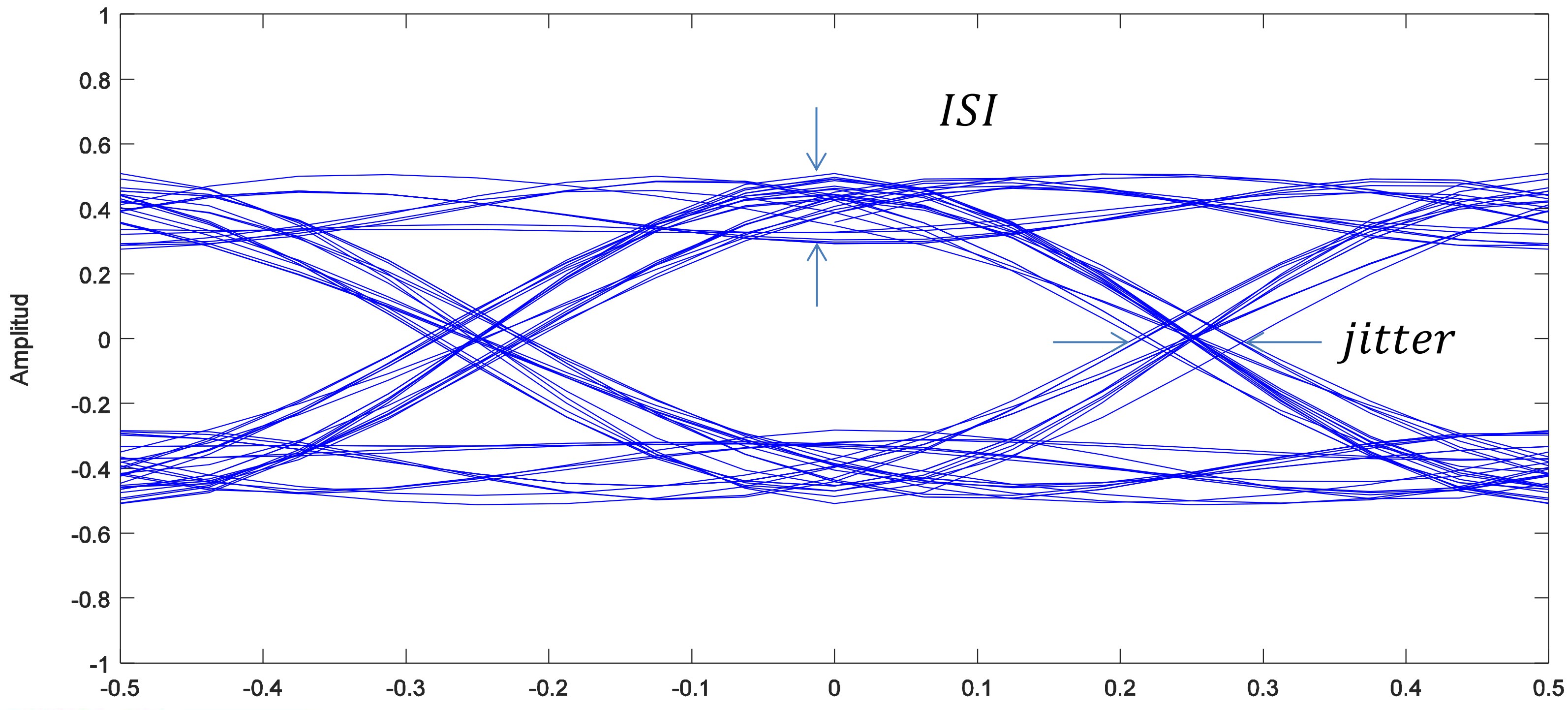
# *Intersymbol Interference (ISI)*

If a rectangular pulse that is unaffected by noise were transmitted, and passed through a low-pass filter, it would look like the figure:



# *ISI: Eye pattern*

The effect of ISI can be analyzed on an oscilloscope. When a sequence of digital information modulated in a PAM signal is observed, what is called the eye pattern is produced.

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# Nyquist shaped pulse.

Different alternatives are presented, and since a possible loss of signaling speed is accepted to shape a zero ISI pulse, but achievable, the symbol rate losses is described as percentage of shaping or "roll-off" that is identified with the letter $\beta$, then the available transmission bandwidth is divided between signaling and shaping:

$$B = \frac{r}{2} + \beta$$

Using as design criteria:

$$0 \leq \beta \leq \frac{r}{2}$$

This spectrum configuration allows signaling at rates between:

$$B \leq r \leq 2B$$

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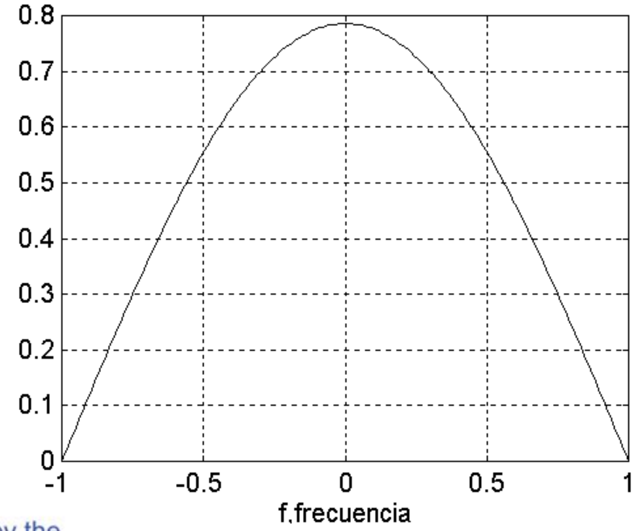
# Nyquist shaped pulse

It can be shown that if the pulse shape is:

$$p(t) = p_\beta(t) sinc(rt)$$

where:

$$P_\beta(f) = 0 \; si \; |f| > \beta$$



# Nyquist shaped pulse

The Figure presents the spectrum $P_\beta(f)$ for $\beta$=1. The waveform has a zero crossing at the sampling instants corresponding to the other values of the pulse sequence. This type of pulse will be zero ISI.

For the shaped pulse $p(t) = p_\beta(t) sinc(rt)$ studied in the frequency domain:

$$P(f) = P_\beta(f) * \frac{1}{r} \Pi\left(\frac{f}{r}\right)$$

And as a result of the convolution between spectra, the resulting bandwidth will be the sum of the corresponding bandwidths: $B = \frac{r}{2} + \beta$. This type of pulse is called a Nyquist shaped pulse

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# Nyquist shaped pulse

There are many functions like the one described that meet the established conditions. In general, $p_\beta(t)$ is adopted as an even function, such that $P_\beta(f)$ has even symmetry, and $P(f)$ has vestigial symmetry around $\pm r/2$.

It should be noted that the conclusions obtained about the Nyquist shaped pulse actually apply to the waveform resulting from the filtering produced by the transmitter system filter, plus the filtering caused by the channel, in the case that the channel response not flat over the bandwidth of interest, and filtering applied to the input of the receiver. When it is considered that the filter is flat, and that the transmitter and receiver apply a rectangular filter with a bandwidth greater than or equal to that of the signal, then all the properties correspond to the transmitted pulse.

# Nyquist shaped pulse

Due to the pulse $p(t)$ contains the function $sinc(rt)$, it has its zeros at the sampling instants $t\_k$=±$D$, ±2$D$, ±3$D$, … and fulfills the ISI zero condition.

On the other hand, it is at the same time a band-limited pulse, as was initially proposed.

There are several waveforms that meet these conditions. A typical waveform for $P_\beta(f)$ is the so-called cosine roll-off, whose Fourier transform has the form

$$P_\beta(f) = \frac{\pi}{4\beta} cos\left(\frac{\pi f}{2\beta}\right) \Pi\left(\frac{f}{2\beta}\right)$$

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# Nyquist shaped pulse

Performing convolution in the spectral domain

$$P(f) = P_\beta(f) * \frac{1}{r}\Pi\left(\frac{f}{r}\right)$$

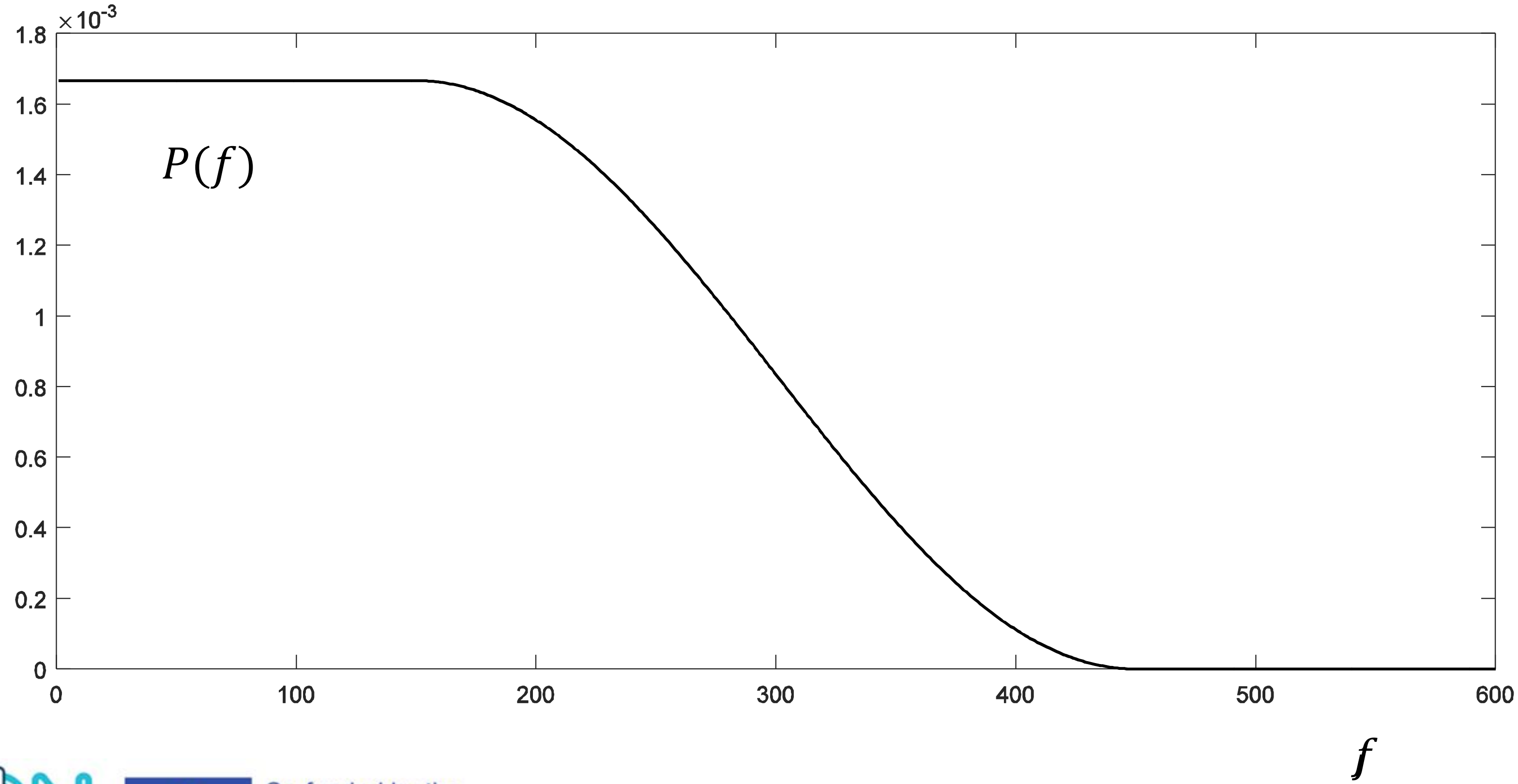
which can be written in the time domain as:

$$p(t) = \frac{cos(2\pi\beta t)}{1-(4\beta t)^2} sinc(rt)$$

Which decays as $1/|t|^2$ when $t$ is large enough. This amplitude reduction of the side lobes of the sync gives a factible shape to the pulse.

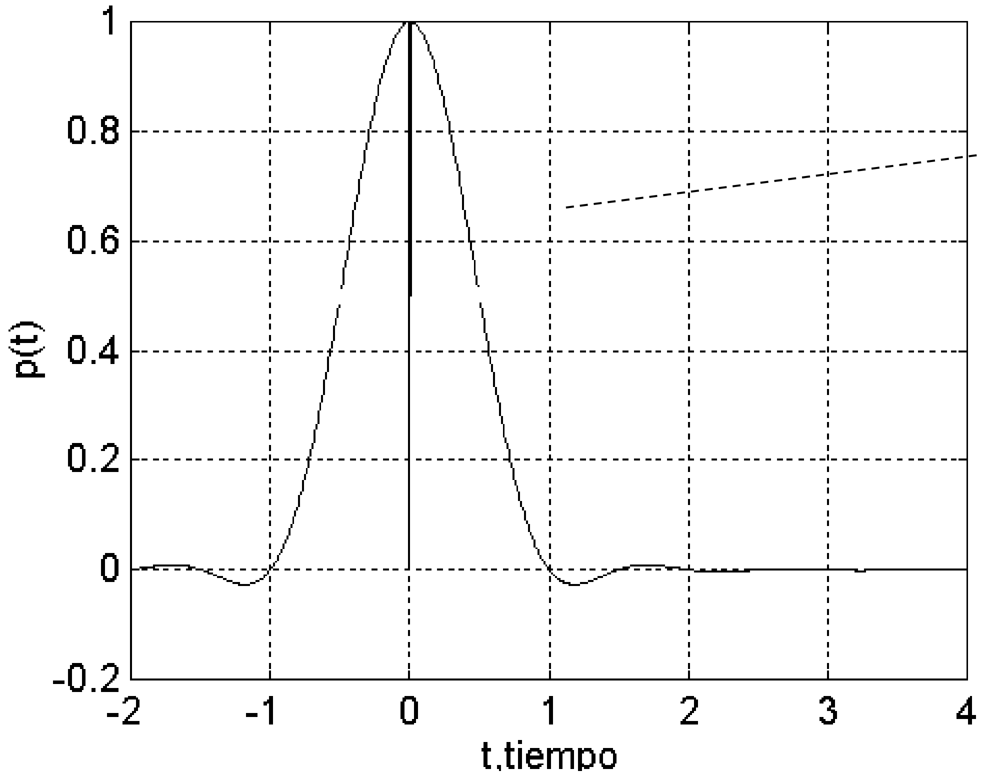
# Nyquist shaped pulse

The spectrum of the Nyquist pulse with a roll-off percentage of 50% is observed in the figure:



# Nyquist shaped pulse

The expression and shape of the Nyquist pulse $p(t)$ with a roll-off percentage of 100% is observed in the figure:




$$p(t) = \frac{sinc(2rt)}{1-(2rt)^2}$$

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# Nyquist shaped pulse

A polar signal that uses this type of pulse will present zero crossings at the midpoints of the intervals. This pulse provides a synchronizing signal but it does so at the expense of reducing the signaling rate to $r$ = $B$.

The Nyquist shaped pulse satisfies the zero ISI condition and is a type of pulse that can be implemented in practice. When digitally programmed devices are used, such as DSPs (Digital Signal Processors), the waveform is stored in sampled values and then generated at the output by digital-analog conversion. In this case the values to be stored include the pulse tails, which quickly cancel out, making the corresponding file of samples quite limited. According to the adopted roll-off value, the signaling speed may be between $B$ and $2B$.

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# Part 3

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# *Pulse-Coded Modulation: PCM*

In order to transmit the signal in digital mode, in the form of a train of rectangular pulses for example, it is necessary in the case of analog signals to first carry out the conversion from the analog to the digital form.

The most common analog signals are audio (with a bandwidth of approximately 4 $khz$), sound and music (with a bandwidth of approximately 20 $khz$) and television video (with a bandwidth of approximately 6 $Mhz$). .

In general, the signals use a transducer to get to have an electrical version, a measurable signal in volts, which can be digitized by electronic means.

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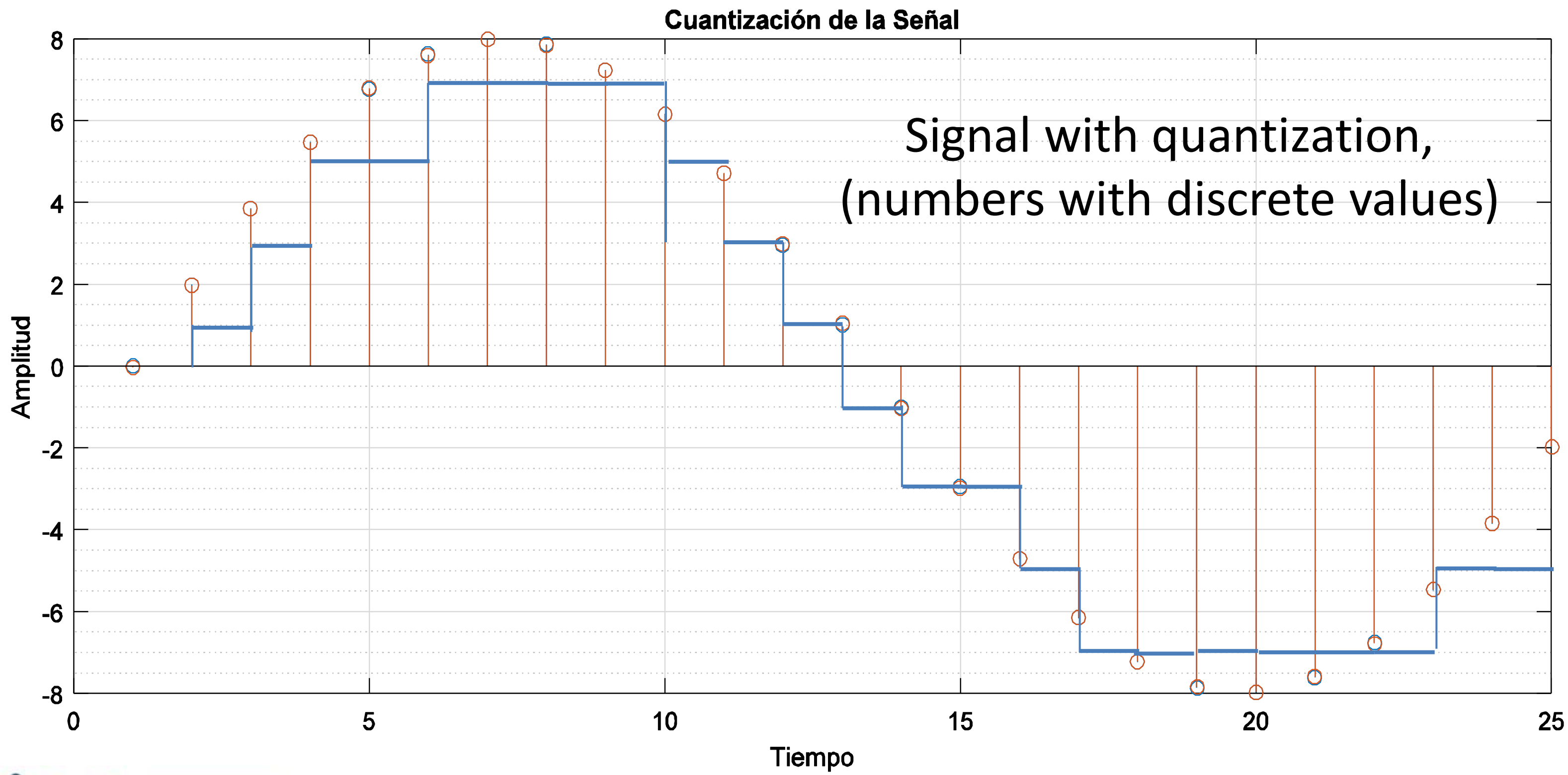
# *Pulse-Coded Modulation: PCM*

The samples acquired are real numbers, but to send the signal in digitized form, a discrete alphabet of amplitudes must be adopted, which forces the process that we will call quantization.





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# *PCM. Features*

Advantages of the digital transmission of signals:

- It can be encoded and compressed, thereby allowing error correction and bandwidth reduction respectively.
- Techniques are more easily implemented to provide a transmission signal with privacy.
- The use of regenerative repeaters allows transmission over long distances.
- Some limitations of this digitization process are the expansion of the bandwidth and the quantization noise of the system.

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# *Pulse-Coded Modulation: PCM*

The input signal is filtered to be able to be sampled at a speed $f_s \geq 2W$ without causing aliasing.

The output of the sample and hold block is a ladder waveform with values $x(kT_s)$.

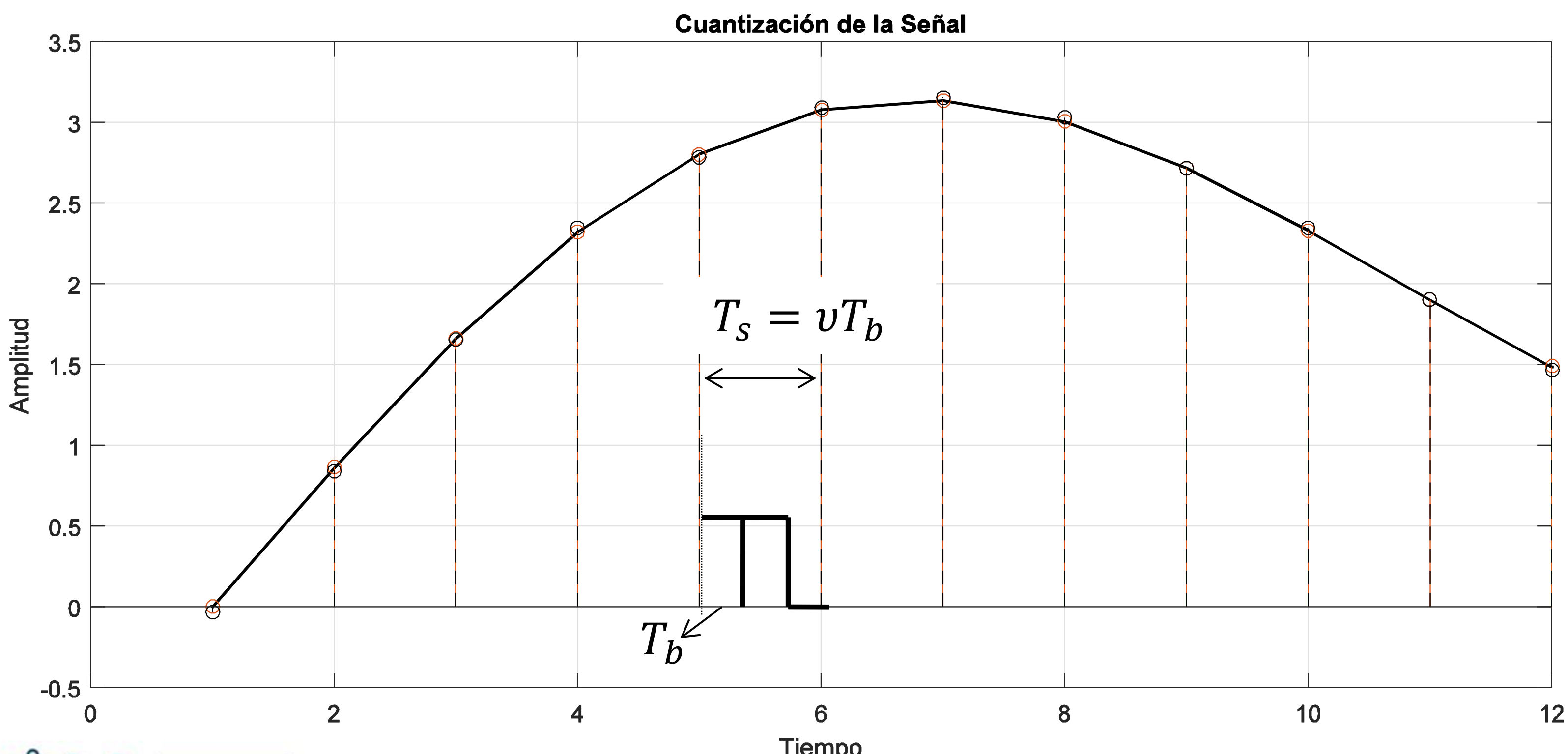
A quantifier adopts the closest value within a set of $q$ levels, adopting the shape of a discretized level $x_q(kT_s)$.

In this quantization process, a signal distortion occurs which is characterized by so-called quantization noise. The M-ary encoder takes the described signal and converts it into a digital word of digits that is formed into a serial output to be finally transmitted over the channel.

The signal $x(t)$ entering the system is considered to be normalized in such a way that $|x(t)| \leq 1\ Volt$.

# *PCM. Signal rate*

The encoder converts each quantized sample into a digital word. The conversion must be done before the next sample has been acquired.

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# *PCM. Signal rate*

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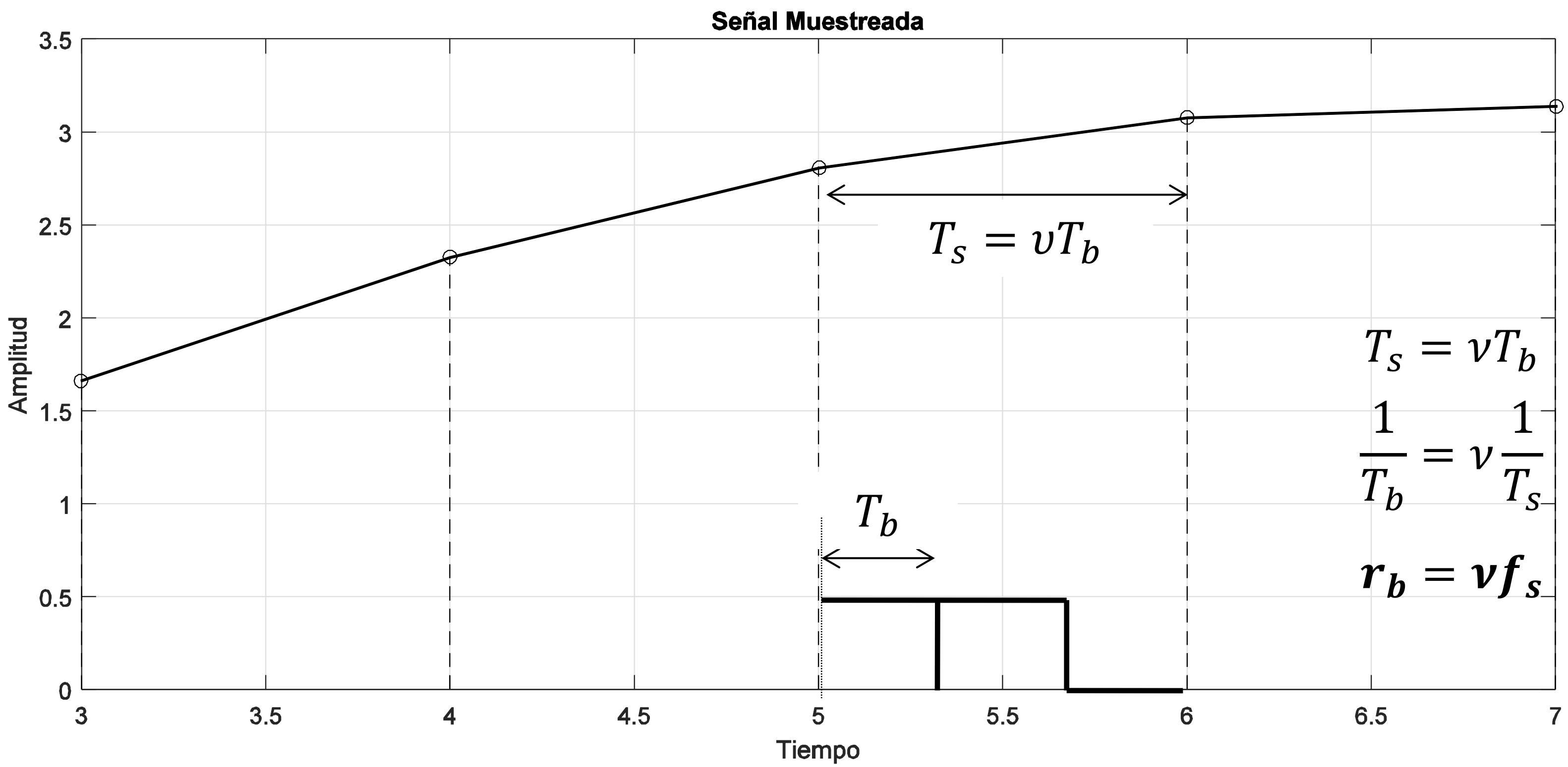


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# *PCM. Signal rate*

The previous figure shows the case in which a sample of the quantized signal $x_q(kT_s)$ is converted into a three-bit serial digital word, with which it follows that there would be 8 different levels that would correspond to this set of digital words. It is generally required that for the transmission of $\nu$ digits there exist at least:

$$2^{\nu} = q, \qquad \nu = \log_2 q, \qquad (2^{\nu} \geq q)$$

If the transmission of digitized data and typically interpreted as a sequence of '1's and '0's is carried out as M-ary signalling, then:

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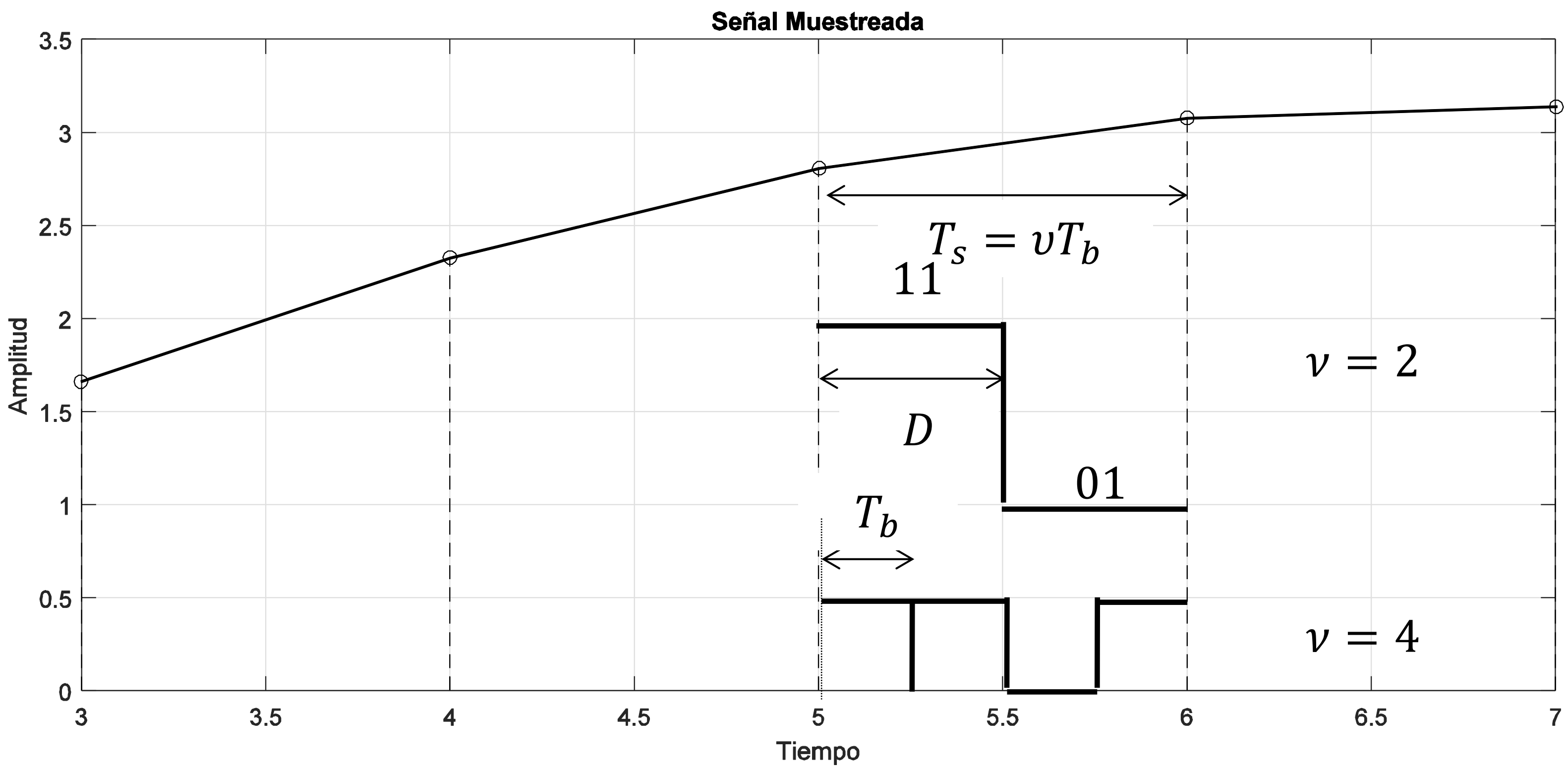
# PCM. Signal rate

$$M^{\nu} = q, \qquad \nu = \log_M q, \qquad (M^{\nu} \geq q)$$





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# *PCM. Transmission bandwidth.*

Each sample is a word of $\nu$ digits. These digits must be transmitted serially at a speed $r_b = \nu f_s$ where $f_s \geq 2W$ . The bandwidth required to transmit a PCM signal is then converted into

$$B_T \geq \frac{r_b}{2} = \frac{1}{2}\nu f_s \geq \nu \mathrm{W}$$

The minimum value of the transmission bandwidth is equal to $B_T = \nu W$ and this situation occurs if:

- The sampling frequency is chosen at its minimum possible value $f_s = 2W$
- Transmission is performed using an ideal Nyquist pulseo posible $p(t) = sinc(rt)$

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# Quantization noise

The original signal $x(t)$ is affected by the quantization noise, represented by the effect of the values of that noise passing through an ideal lowpass filter.

The quantization noise $\epsilon_k$ is actually a random variable that, according to the previous expression, depends on $x(t)$.

If the number of quantization levels is high, then the quantization error becomes independent of the signal. It will then be assumed that the quantization error $\epsilon_k$ is independent of the signal from the statistical point of view.

The quantization noise $\epsilon_k$ is a random variable that is assumed to have a uniform probability density function and root mean square value ${\epsilon_k}^2$ that has a range of variation:

$$-1/q \leq \epsilon_k \leq 1/q$$

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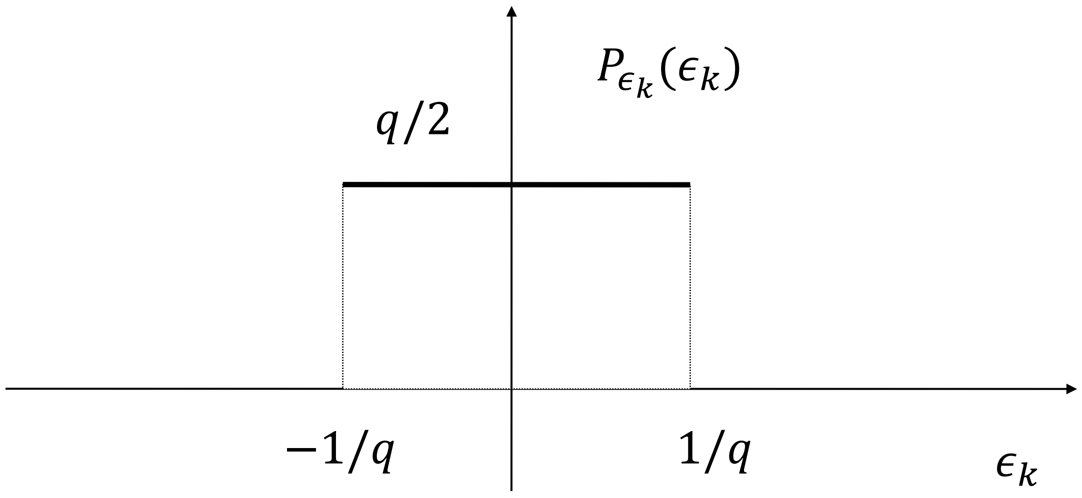
# Quantization noise. PDF function

The assumption of the quantization noise distribution is proper when the signal is uniformly distributed, since the quantization noise is a linear function of the signal.

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# Quantization noise. Noise power.

Then if an uniform pdf for the quantization noise is assumed, the noise power of this quantity can be determined:

$$\overline{\sigma_q{}^2} = \overline{\epsilon_k{}^2} = \frac{q}{2}\int_{-1/q}^{1/q} \epsilon_k{}^2 d\epsilon_k = \frac{q}{2}\frac{2x(1/q)^3}{3} = \frac{1}{3q^2}$$

The quantization noise power decreases as the number of levels $q$ increases.

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# Signal-to-Noise radio of quantification

The receiving power at the destination is given by:

$$S_D = \overline{x^2} = S_x \leq 1$$

Assuming a normalized signal such that: $|x(t)|$≤1.

Then the signal noise ratio at the destination can be expressed as:

$$\left(\frac{S}{N}\right)_D = \frac{S_x}{\overline{\overline{\sigma_q{}^2}}} = 3q^2 S_x$$

If the number of levels is expressed as a power of 2, $q = 2^\nu$, and applying logarithms to use decibels:

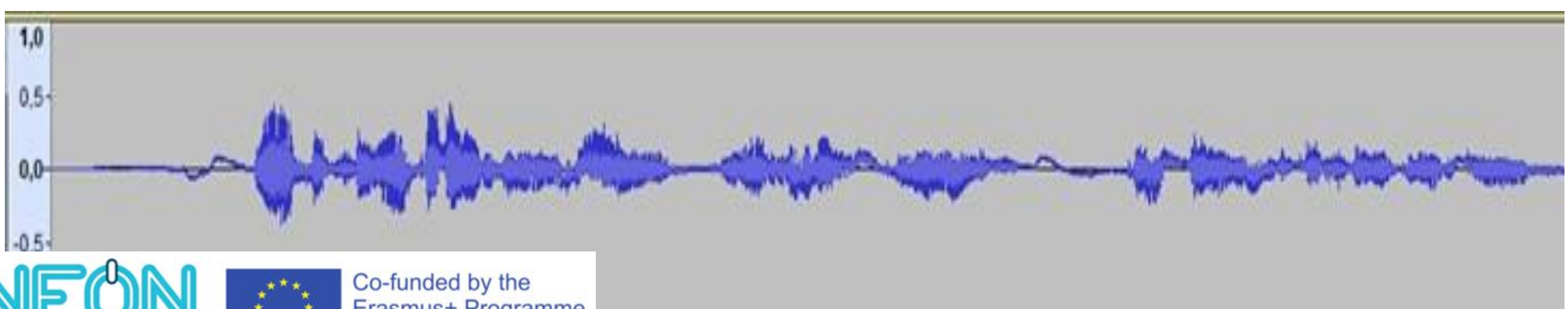
$$\left(\frac{S}{N}\right)_{D,dB} = 10\log_{10}(3x2^{2\nu}S_x) \leq 4.8 + 6\nu\ [dB]\ ,(S_x \leq 1)$$

Where " $=$ " is in the case when $S_x = 1$.

# Non-Uniform Quantization and Compansion

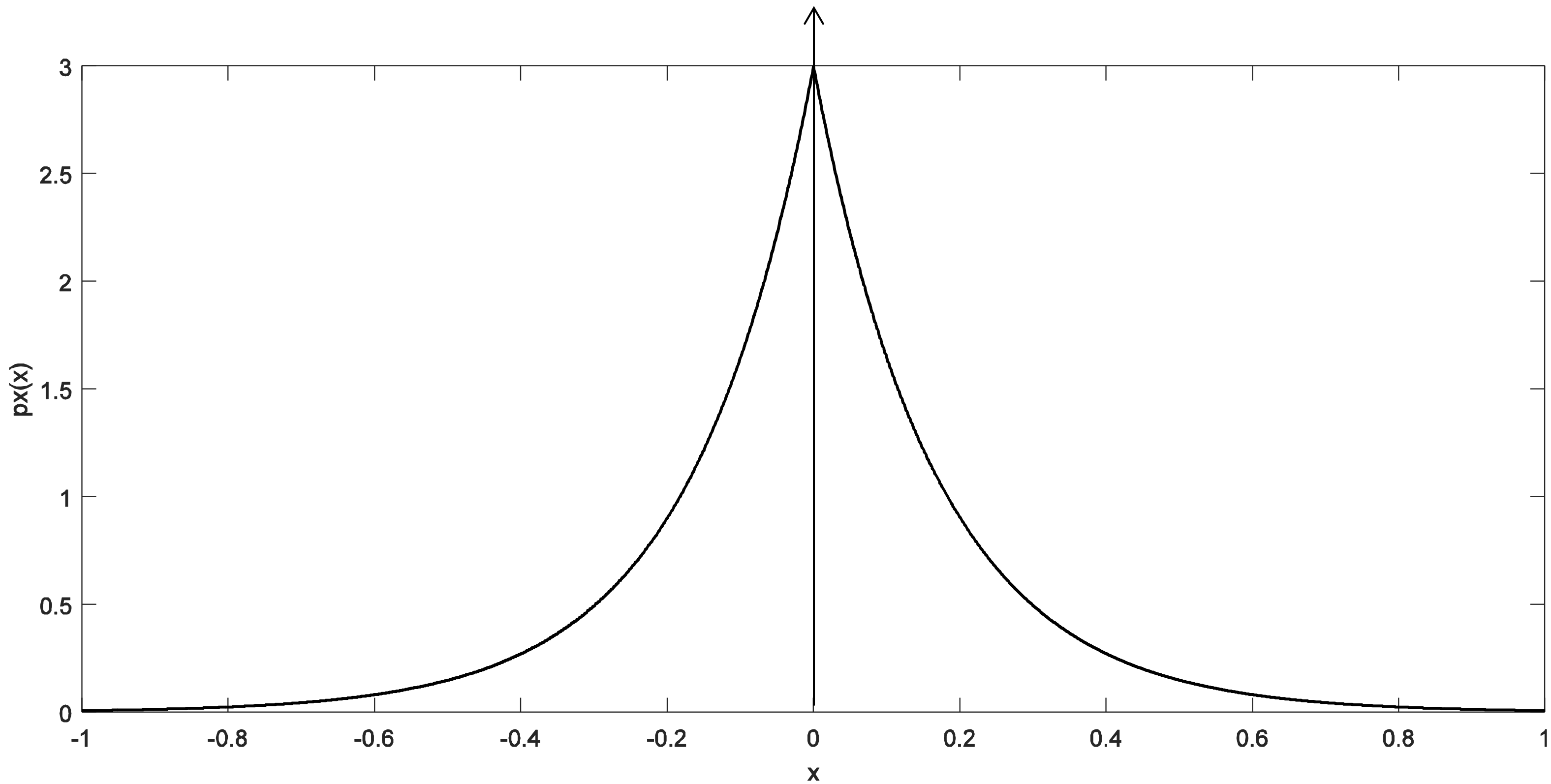
If $S_x = 1$, for example for a PCM signal of $\nu$=8 digits, the theoretical signal to noise ratio would be 52.8 $dB$.

However, real signals, particularly audio signals (See figure), have a fairly high crest factor. This means that the signal is most of the time at low voltage values, and reaches the maximum peak only a few times. As a consequence $S_x \ll 1$, which produces a significant reduction in the signal noise ratio. Thus, for example, and if you want to have a signal-to-noise ratio of 60 dB as calculated for 8 digits, you will actually need 14 digits in a practical case.



# Non-Uniform Quantization and Compansion.

A voice signal has a typical probability density function like the one seen in the figure, whose associated normalized power $S_x \ll 1$:



# Non-Uniform Quantization and Compansion.

Uniform quantization produces a worsening of the signal noise ratio because low voltage values are more likely to occur than high ones.

It is possible to standardize the probability of these levels if, for example, a greater number of them is quantified on the $x$ axis of the figure so that the voltage values can be grouped, making those that are low correspond to more digital words, which are finally discriminated for small intervals of $x$, while high stress values are clustered using fewer digital words for wider intervals of the $x$-axis. This is called non-uniform quantization.

# Non-Uniform Quantization and Compansion.

La cuantificación no uniforme puede realizarse como se describió anteriormente de forma digital, es decir cuantificando con mayor cantidad de niveles, y agrupando por programa dichos niveles de forma que lo valores bajos de tensión se dividen en pequeños intervalos del eje, y los mas altos con incrementos de mas anchos. Así la probabilidad de las palabras digitales a transmitir tiende a uniformarse.

Otra forma que ha tenido amplio uso es el proceso de la compresión y expansión analógica de una señal, proceso que abreviadamente se denomina compansión.

Se supone que la señal a transmitir tiene una función densidad de probabilidad como la anteriormente mostrada, con valor medio cero $\bar{x} = 0$, y dispersión $\sigma^2$.

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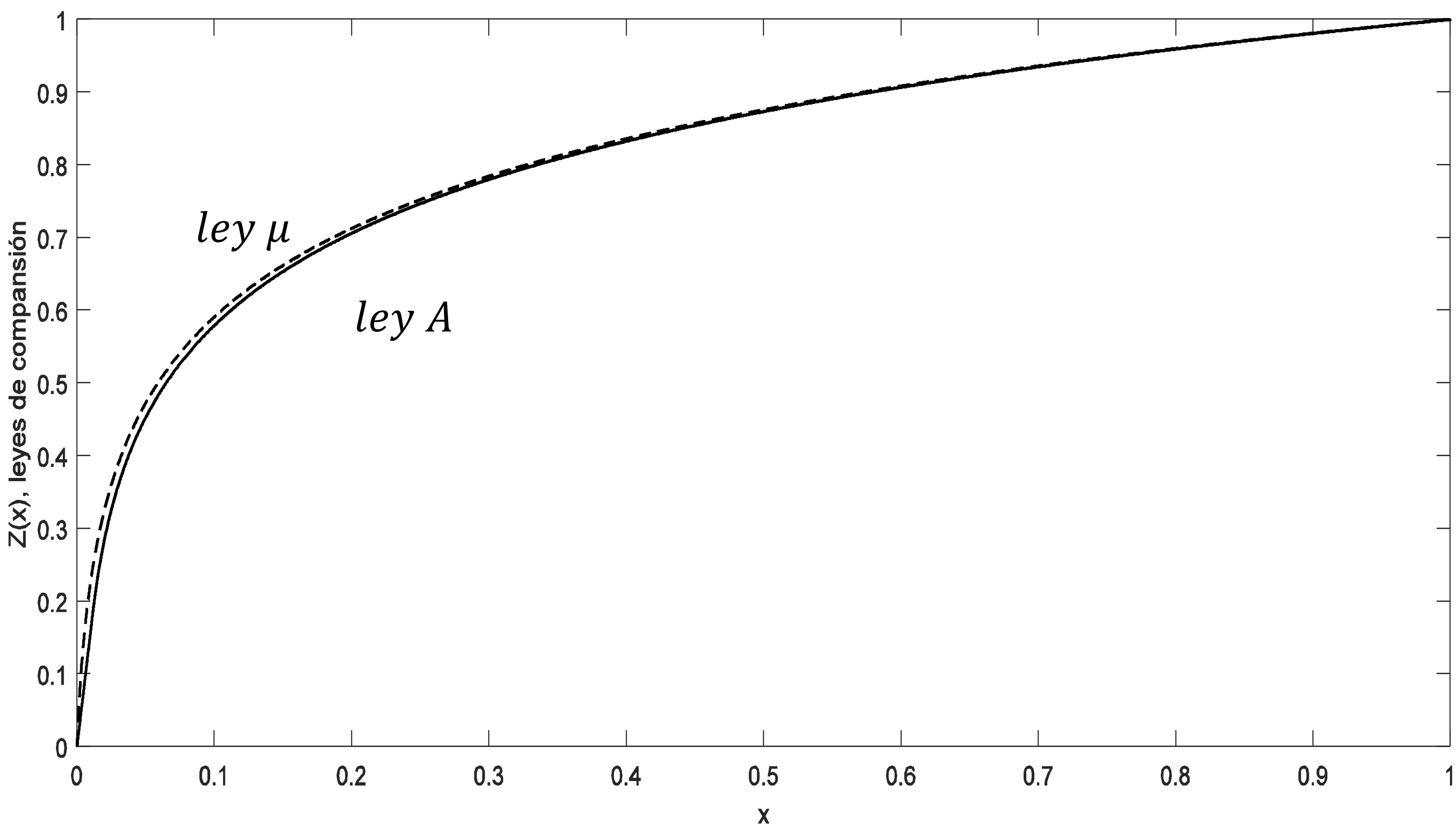
# Law $\mu$ and Law $A$.



# Part 4

# PCM with noise. Noise of decoding.

Up to now, only the so-called quantization noise has been considered, is that, a distortion that we add to the system if digitization is not implemented properly.

But once the signal is digitized, and it is decided to transmit it through a channel, transmission noise appears, which can alter the digital values that later, in the digital-to-analog conversion, can result in noise magnified by the process.

The signal generated by a PCM transmitter is input to the channel, where noise can affect the sequence values and cause errors. The effect of noise on the transmitted signal is then analyzed, noise known as decoding.

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# Noise of decoding.

The transmitted signal will then be affected by the quantization effect and by channel noise. The case of a binary modulated PCM signal with uniform quantization and small binary error probability is considered. The codeword is made up of $\nu$ digits and the number of bits in error in it is a binomial function:

$$P_{ew} = P_{1\,dig} + P_{2\,dig} + \cdots + P_{\nu\,dig} =$$
$$= \binom{\nu}{1} P_e (1 - P_e\,)^{\nu-1} + \binom{\nu}{2} P_e{}^2 (1 - P_e\,)^{\nu-2} + \cdots \cong \nu P_e$$

It is considered that the probability of error is approximate to that when there is only one error in the word, since the probability of two or more errors is small. The effect of the error has more or less influence depending on the place where it occurs .

# Noise of decoding.

If the use of natural code is considered and being the binary word:

$$b_{\nu-1} \quad b_{\nu-2} \quad \cdots \quad b_1 \quad b_0$$

Being organized in a natural code, the *mth* bit represents an amplitude of the digitized signal of value $2^m(2/q)$ Volts. According to this it is said that an error in the *mth* bit causes an error in the amplitude waveform

$$e_m = \pm 2^m(2/q)\, V$$

Then, in order to average the effect of the error in the different bit positions we calculate:

$$\overline{{e_m}^2} = \frac{1}{\nu}\sum_{m=1}^{\nu-1}(2^m(2/q))^2 = \frac{4}{\nu q^2}\sum_{m=1}^{\nu-1}4^m = \frac{4}{\nu q^2}\frac{4^\nu - 1}{3} \cong \frac{4}{3\nu}$$

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# Noise of decoding.

The decoding noise power is given by the product between the number of bits with error and this average power per bit:

$$\sigma_d{}^2 = (\nu P_e)\overline{e_m{}^2} = \frac{4}{3}P_e$$

The total noise power will be the sum of the decoding noise power plus the quantization noise power:

$$N_D = \sigma_q{}^2 + \sigma_d{}^2 = \frac{1 + 4q^2 P_e}{3q^2}$$

Noise powers are added because they are generated by de-correlated sources.

# Signal-to-Noise ratio

$$\left(\frac{S}{N}\right)_D = \frac{3q^2 S_x}{1 + 4q^2 P_e}$$

If the probability of error (*Pe)* is small, the signal-to-noise ratio is determined by the quantization noise:

$$\left(\frac{S}{N}\right)_D \cong 3q^2 S_x \quad \text{si } P_e \ll \frac{1}{q^2}$$

If the probability of error is high:

$$\left(\frac{S}{N}\right)_D \cong \frac{3S_x}{4P_e} \quad \text{si} \quad P_e \gg \frac{1}{q^2}$$

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# Threshold effect.

Cuando el ruido de decodificación es muy alto la relación señal ruido se deteriora fuertemente, creando el efecto de umbral.

El efecto de umbral en PCM se define como el punto en el que la relación señal ruido en destino se reduce en 1 dB.

La zona de umbral no aceptable es la gobernada por el ruido de decodificación. En la zona aceptable rige el ruido de cuantificación solamente.

Para que eso suceda, $P_e \ll \frac{1}{q^2}$. Asumiendo un valor clásico de cantidad de niveles $q = 2^8 = 256$, entonces $P_e \leq 10^{-5}$.

La expresión de la relación señal ruido en el caso de usar una señal M-aria con formato polar es:

# Threshold effect.

$$P_e = 2\left(1 - \frac{1}{M}\right) Q\left(\sqrt{\frac{3}{(M^2 - 1)}\left(\frac{S}{N}\right)_R}\right) \leq 10^{-5}$$

The function $Q(k)$ is $Q(k) = 10^{-5}$ if $k = 4.3$ . Then:

$$\left(\frac{S}{N}\right)_{R,umbral} \cong 6(M^2 - 1)$$

If the signal-to-noise ratio is below this value, the efect of signal mutilation due to the presence of noise in notorious. If related to analog transmission parameters:

# Threshold effect.

$$\gamma = \frac{S_R}{\eta W} = \left(\frac{B_T}{W}\right)\left(\frac{S}{N}\right)_R$$

It is possible to define

$$\gamma_{umbral} = \left(\frac{B_T}{W}\right)\left(\frac{S}{N}\right)_{R,umbral}$$

Where $B_T \geq \nu W$

$$\gamma_{umbral} \cong 6\frac{B_T}{W}(M^2 - 1) \geq 6\nu(M^2 - 1)$$

Which is the minimum value of $\gamma$ that can be used in PCM to be above the threshold

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