Communication Systems based on Software Defined Radio (SDR)

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Digital Communication Systems

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Comparison between IQ constellations.

The graphical description of the IQ plane for a given modulation allows an approximate analysis of the behavior of that modulation against noise, and its probability of error.

One of the most important parameters is the minimum distance of the constellation, which we will consider raised to the square of ${d^2}_{min}$.

Another parameter of interest is the average energy per symbol E.

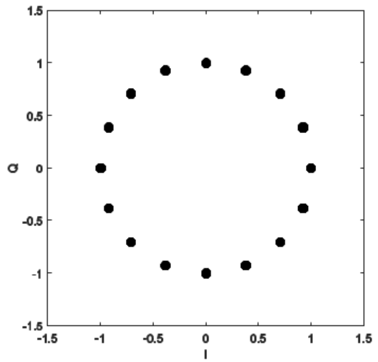
The combined parameter that characterizes a certain constellation in a modulation is the normalized minimum square distance:

$$ {d^2}_{N,min} = \frac{{d^2}_{min}}{E} $$

IQ constellations. Energy per symbol.

In the case of MPSK modulation the average energy per symbol is:

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \frac{{A_c}^2}{2} D\left(cos(\varphi_k)^2 + sin(\varphi_k)^2\right) = \frac{{A_c}^2 D}{2}$$



The average energy per symbol in MPSK, adopting a unitary module, would be associated with a geometric factor $FGN = \frac{M(1)}{M} = 1.$

Then $E = FGN \frac{{A_c}^2 D}{2} = \frac{{A_c}^2 D}{2}$

In the case of QAM (4QAM) modulation the average energy per symbol is:

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \frac{{A_c}^2}{2} D\left({I_k}^2 + {Q_k}^2\right) = {A_c}^2 D$$

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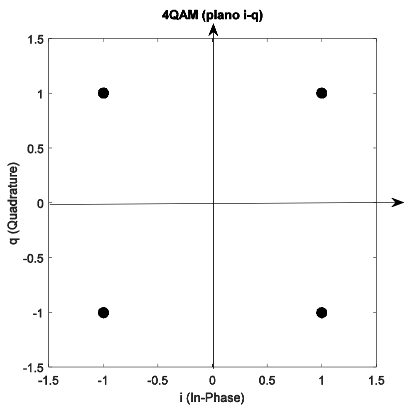
IQ constellations. Energy per symbol.

In the case of QAM modulation the average energy per symbol is :

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = 2\frac{{A_c}^2 D}{2} = {A_c}^2 D$$



The average energy per symbol in QAM, adopting IQ coordinates of the form ±1, would be associated with a geometric factor $FGN = \frac{4(1^2+1^2)}{4} = 2.$

Then $E = FGN\frac{{A_c}^2 D}{2}$=2$\frac{{A_c}^2 D}{2} = {A_c}^2 D$

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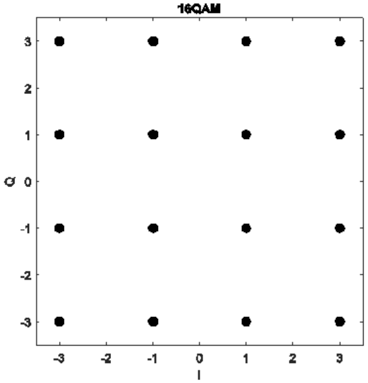
IQ constellations. Energy per symbol.

n the case of MQAM modulation the average energy per symbol is:

$$E = \frac{1}{2} {A_c}^2 ({I_k}^2 + {Q_k}^2) D = \frac{\mu^2 - 1}{3} {A_c}^2 D$$



The average energy per symbol in MQAM, adopting IQ coordinates of the form ±1, ±3, would be associated with a geometric factor $FGN = \frac{4(40)}{16}$ =10.

Then $E = FG \frac{{A_c}^2 D}{2}$=10$\frac{{A_c}^2 D}{2} = \frac{16-1}{3} {A_c}^2 D$

$$FGN = \frac{4}{16}(1^2 + 1^2 + 1^2 + 3^2 + 3^2 + 1^2 + 3^2 + 3^2) = 10$$

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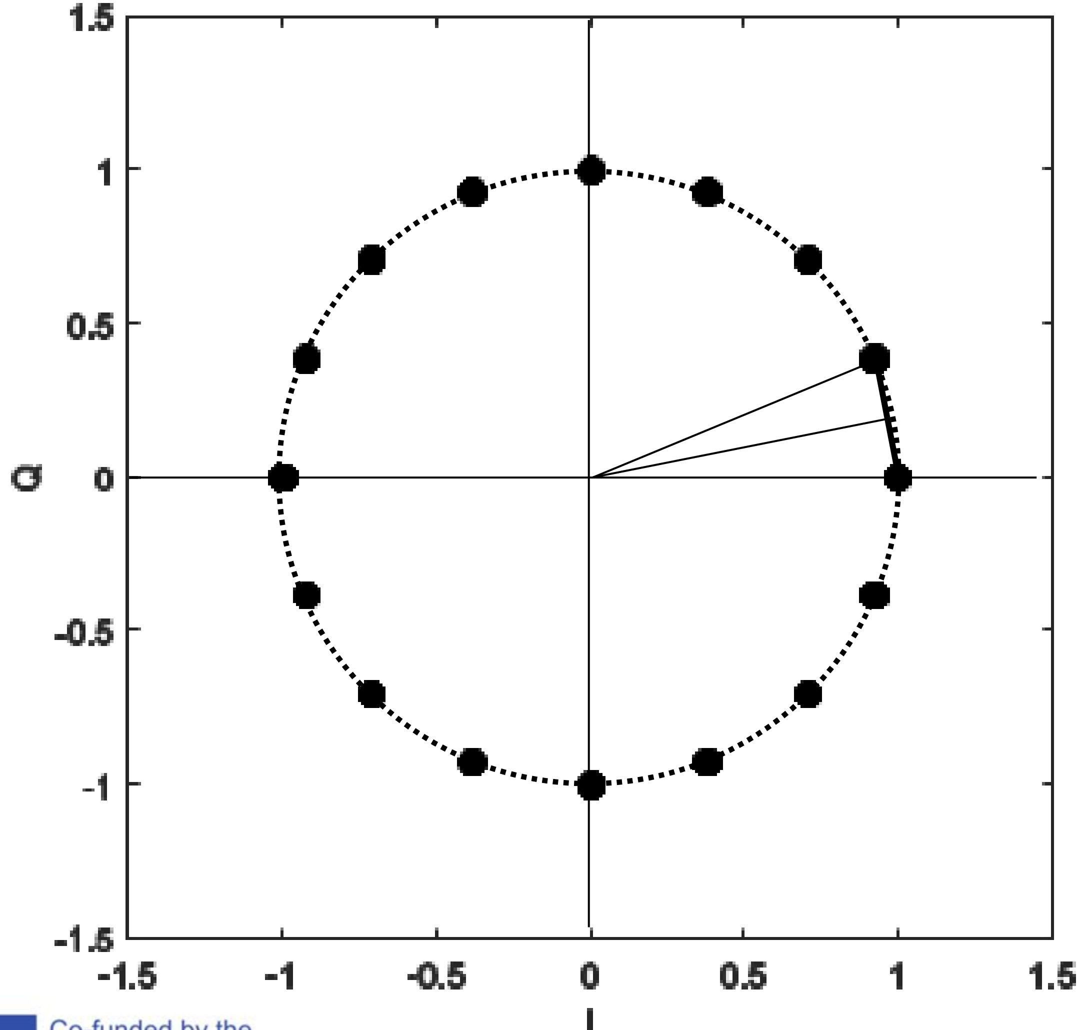
Normalized minimum squared distance. MPSK.



$$d^2{}_{min} = 4sin^2\left(\frac{\pi}{M}\right)$$

$$d^2{}_{min} = 4sin^2\left(\frac{\pi}{16}\right) = 0.1522$$

$$FGN = 16/16 = 1$$

$$d^2{}_{N,min} = \frac{d^2{}_{min}}{E} = 0.1522$$

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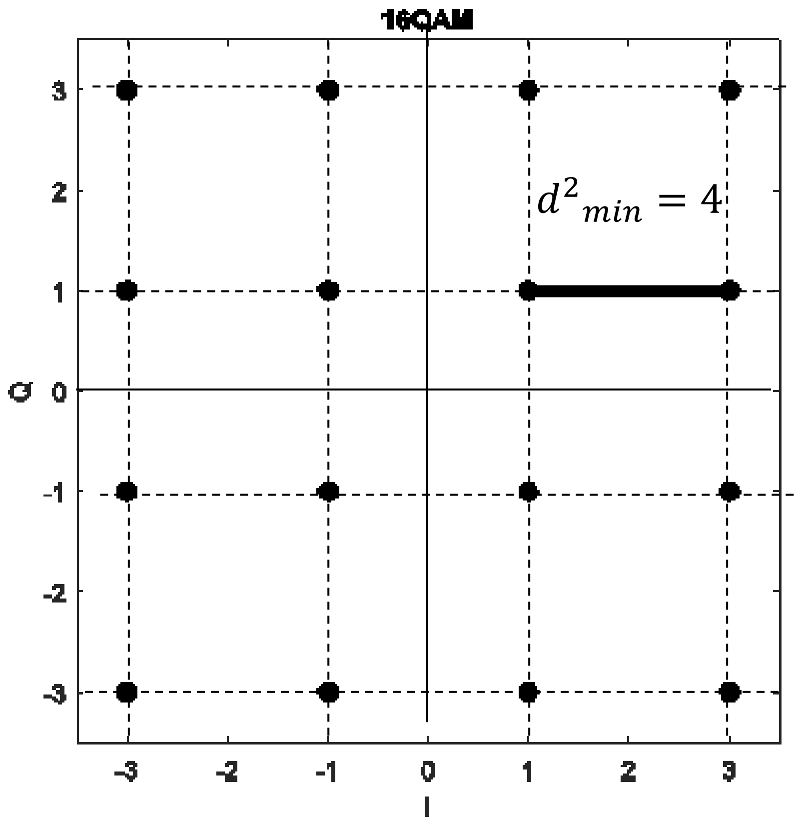
Normalized minimum squared distance. MQAM.



$$d^2{}_{min} = 4$$

$$d^2{}_{N,min} = \frac{d^2{}_{min}}{E} = 0.4$$

$$FGN = \frac{1}{16}4(1^2 + 1^2 + 1^2 + 3^2 + 3^2 + 1^2 + 3^2 + 3^2) = \frac{1}{16}x4x40 = 10$$

Comparison between IQ constellations.

The normalized square distance parameter allows us to make a comparative analysis between the modulations, being able to determine the relative gain of one with respect to the other, calculating the quantity:

$$G_{dB} = 10\log_{10}\left(\frac{{d^2}_{N,min\,1}}{{d^2}_{N,min\,2}}\right) = 10\log_{10}\left(\frac{{d^2}_{min\,1}/E_1}{{d^2}_{min\,2}/E_2}\right) = 10\log_{10}\left(\frac{{d^2}_{min\,1}/FGN_1}{{d^2}_{min\,2}/FGN_2}\right)$$

where $E = FGN\frac{{A_c}^2}{2}D$

Then:

$$G_{dB} = 10\log_{10}\left(\frac{{d^2}_{N,min\,1}}{{d^2}_{N,min\,2}}\right) = 10\log_{10}\left(\frac{0.4}{0.1522}\right) = 4.2\ dB$$

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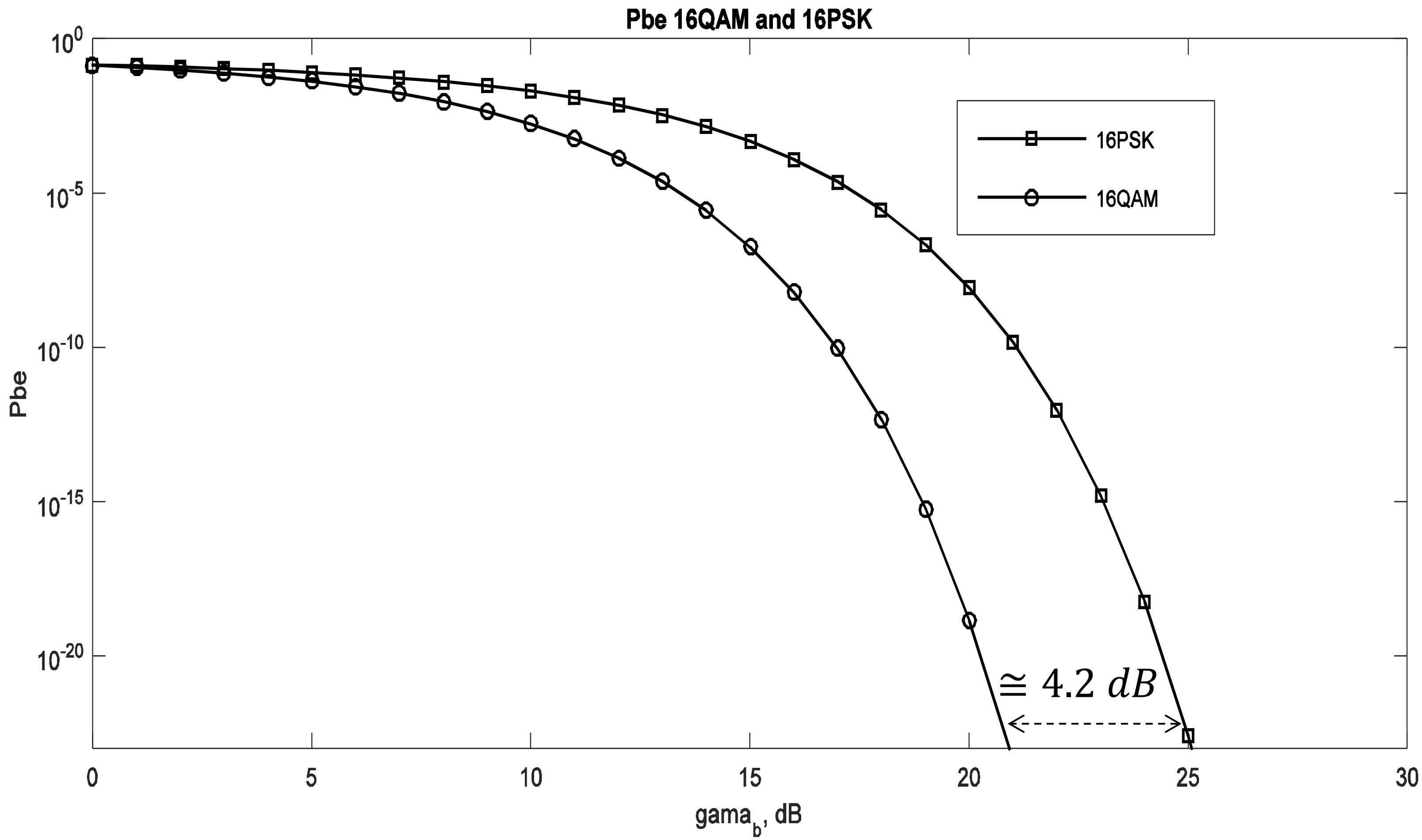
Comparison between 16PSK and 16QAM constellations.



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M-ary and quadrature carrier systems.

Error probability based on the IQ constellation

M-ary modulation systems that are received by coherent detection or by phase comparison are analyzed.

The M-ary QAM, PSK and the so-called APK (Amplitude Phase Keying) modulation systems are the most appropriate for modulation on telephone channels, or in general limited band, to expand the signaling speed.

They are also used in cable signal communications systems, due to the incorporation of data transmission by this means to offer Internet service.

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M-ary and quadrature carrier systems.

Relationship between the IQ representation and the probability of error in digital modulation systems.

2PSK or PRK binary phase modulation

PRK binary modulation can be represented in the IQ plane and appears as having two signals that differ by 180° in phase:

The signal corresponding to the PRK modulation has two symbols, $s_0(t) = -s_1(t)$:

$$s_1(t) = A_c p_{T_b}(t) cos(\omega_c t + \theta)$$

$$s_0(t) = -A_c p_{T_b}(t) cos(\omega_c t + \theta)$$

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M-ary and quadrature carrier systems.

since $f_c = Nr_b$ is normally considered, then:

$$s_1(t) = A_c \cos(\omega_c t) = -s_0(t)$$

Consequently:

$$E_1 = \int_0^{T_b} [s_1(t)]^2 \, dt = E_1 = \int_0^{T_b} [A_c \cos(\omega_c t)]^2 \, dt = \frac{{A_c}^2 T_b}{2}$$

$$E_0 = \int_0^{T_b} [s_0(t)]^2 \, dt = E_1 = \int_0^{T_b} [-A_c \cos(\omega_c t)]^2 \, dt = \frac{{A_c}^2 T_b}{2}$$

$$E_b = \frac{E_1 + E_0}{2} = \frac{{A_c}^2 T_b}{2}$$

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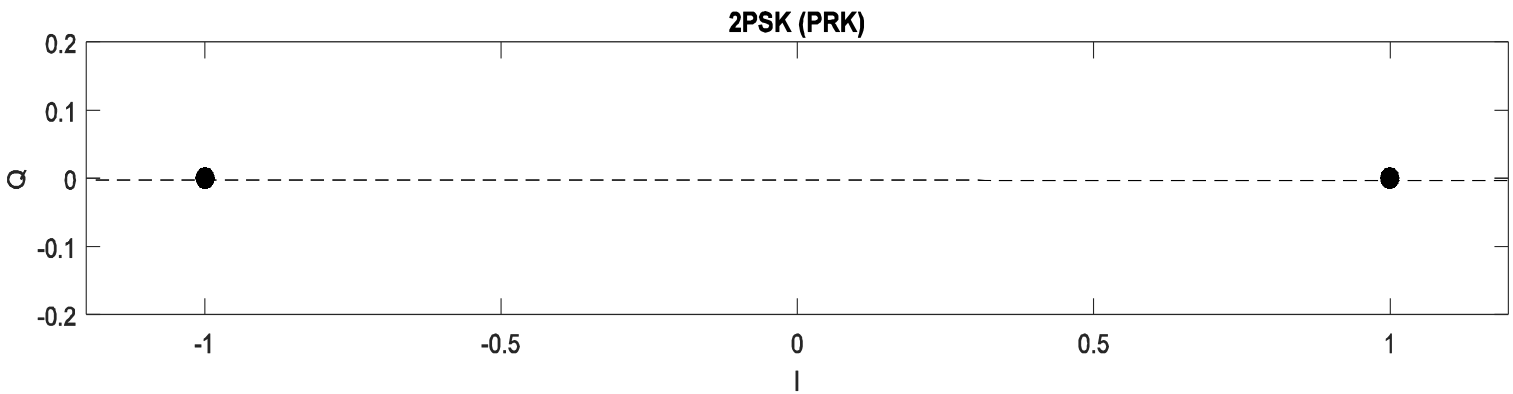
M-ary and quadrature carrier systems.

Then, if we consider it as an MPSK modulation constellation, with $M = 2$,

$$\varphi_k = \frac{2\pi a_k}{M} = \pi a_k$$
$$a_k = 0, 1$$



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M-ary and quadrature carrier systems.

$$P_e = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{2E}{\eta}}\right) = Q\left(\sqrt{\frac{2E_b}{\eta}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$

4PSK or QPSK ($\mathbf{M} = 4$) M-ary phase modulation

4PSK or QPSK ($\mathbf{M=4}$) M-ary phase modulation

In the following analysis it is assumed that the carrier signal has synchronization with the modulation so that $f_c = Nr_b$. In a signaling interval the waveform is equal to:

$$x_c(t) = s_i(t - kD) - s_q(t - kD)$$

$$s_i(t - kD) = A_c cos(\varphi_k) p_D(t) cos(\omega_c t)$$

$$s_q(t - kD) = A_c \sin(\varphi_k) p_D(t) \sin(\omega_c t)$$

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4PSK or QPSK (M=4) M-ary phase modulation

Donde

$$\varphi_k = \frac{2\pi a_k}{4}$$

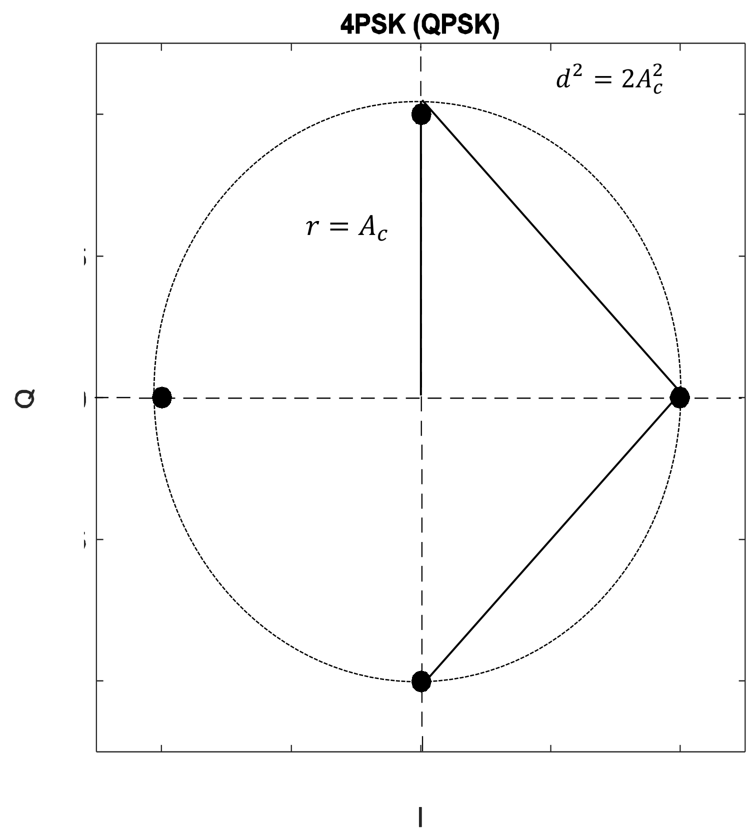
$$a_k = 0, 1, 2, 3$$

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \int_{kD}^{(k+1)D} (s_i(t))^2 dt + \int_{kD}^{(k+1)D} \left(s_q(t)\right)^2 dt) =$$

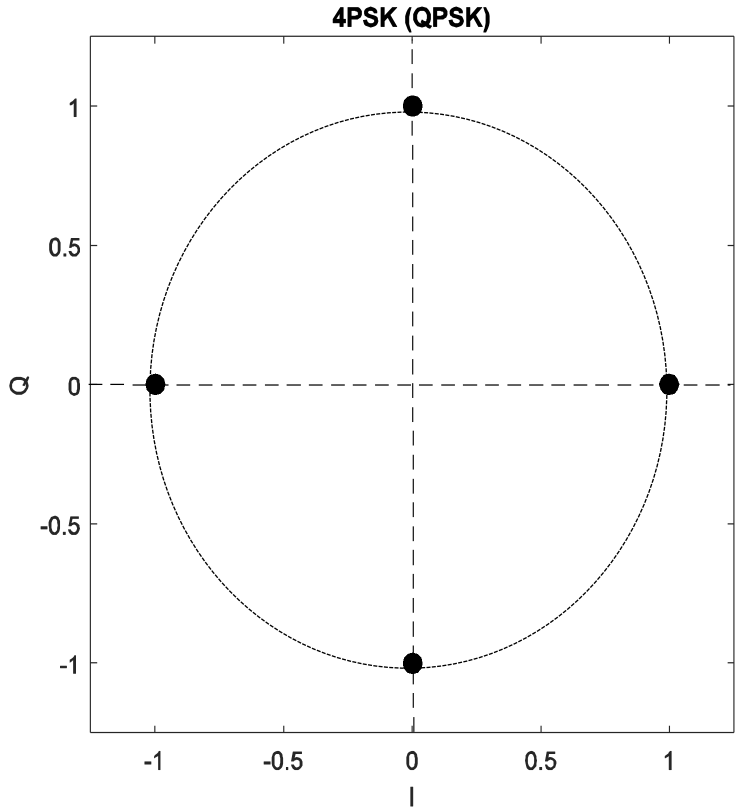
$$= \frac{{A_c}^2}{2} D\left(cos(\varphi_k)^2 + sin(\varphi_k)^2\right) = \frac{{A_c}^2 D}{2}$$

The constellation corresponding to 4PSK is shown in the following figures, in dimensional and normalized mode:

Modulación M-aria de fase 4PSK o QPSK $(\mathbf{M = 4})$



Modulación M-aria de fase 4PSK o QPSK $(\mathbf{M} = 4)$



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Modulación M-aria de fase 4PSK o QPSK $(\mathbf{M = 4})$

On the normalized constellation a normalized geometric factor can be defined:

$$FGN = \frac{1}{4}(1^2 + 1^2 + 1^2 + 1^2) = 1$$

In this case:

$$E = FGN\frac{{A_c}^2 D}{2} = \frac{{A_c}^2 D}{2}$$

Then:

$${A_c}^2 = \frac{2E}{D} = 2Er$$

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M-ary and quadrature carrier systems.

Now the distance is$= \sqrt{2}A_c$, then $d^2 = 2A_c^2$

$$\frac{d}{2} = \frac{\sqrt{2}}{2} A_c$$

$$\frac{(d/2)^2}{\eta r} = \frac{{A_c}^2}{2\eta r}$$

Each symbol s_i is associated with a probability $P_{ei} = 2Q\left(\sqrt{\frac{(d/2)^2}{\eta r}}\right)$, since each symbol can be confused with two neighbors on its sides that are at a distance:

$$d = \sqrt{2}A_c$$

M-ary and quadrature carrier systems.

$$P_e = \frac{1}{M}(P_{e0} + P_{e2} + P_{e2} + P_{e3}) =$$

$$\frac{1}{4}4x2Q\left(\sqrt{\frac{(d/2)^2}{\eta r}}\right) = 2Q\left(\sqrt{\frac{2Er}{2\eta r}}\right) = 2Q\left(\sqrt{\frac{E}{\eta}}\right) =$$

$$2Q\left(\sqrt{\frac{2E_b}{\eta}}\right) = 2Q\left(\sqrt{2\gamma_b}\right)$$

$$E = 2E_b$$

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4QAM or QAM ($\mathbf{M = 4}$) M-ary modulation

4QAM or QAM ($\mathbf{M}$=$\mathbf{4}$) M-ary modulation

Coherent quadrature carrier synchronization requires demodulation synchronization. In the interval $k-th$ interval $kD < t < (k+1)D$:

$$x_c(t) = s_i(t-kD) - s_q(t-kD)$$

$$s_i(t-kD) = A_c I_k p_D(t) cos(\omega_c t) \quad I_k = \pm 1$$

$$s_q(t-kD) = A_c Q_k p_D(t) \sin(\omega_c t) \quad Q_k = \pm 1$$

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4QAM or QAM (M=4) M-ary modulation

If we consider $f_c = Nr_b = 2Nr$

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \int_{kD}^{(k+1)D} (s_i(t))^2 dt + \int_{kD}^{(k+1)D} \left(s_q(t)\right)^2 dt = \frac{{A_c}^2}{2} D\left({I_k}^2 + {Q_k}^2\right) = {A_c}^2 D$$

The energy per symbol is twice the energy per bit $E = 2E_b$.

The constellation corresponding to 4QAM is shown in the following figures, in dimensional and normalized mode:

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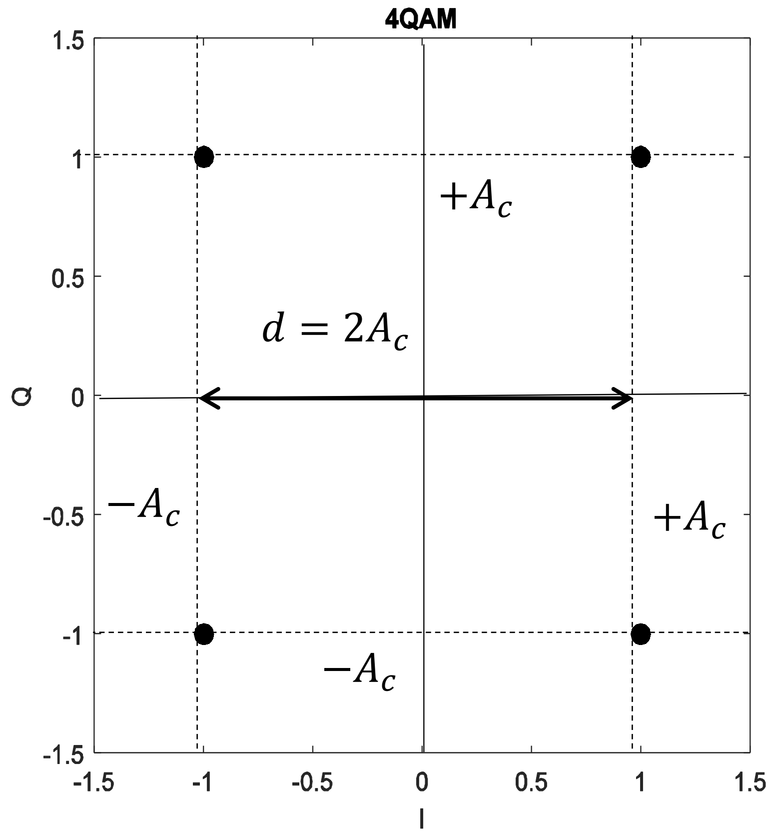
4QAM or QAM ($\mathbf{M=4}$) M-ary modulation



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4QAM or QAM (M=4) M-ary modulation

On the normalized constellation a normalized geometric factor can be defined:

$$FGN = \frac{1}{4}\left((1^2 + 1^2) + (1^2 + 1^2) + (1^2 + 1^2) + (1^2 + 1^2)\right) = 2$$

In this case:

$$E = 2\frac{{A_c}^2 D}{2} = {A_c}^2 D$$

Then:

$${A_c}^2 = \frac{E}{D} = Er$$

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4QAM or QAM (M=4) M-ary modulation

Now the distance is $d = 2A_c$, then $d^2 = 4A_c^2$ y $d/2 = A_c$

$$\frac{d}{2} = A_c$$

$$\frac{(d/2)^2}{\eta r} = \frac{{A_c}^2}{\eta r}$$

Each symbol s_i is associated with a probability $P_{ei} = 2Q\left(\sqrt{\frac{(d/2)^2}{\eta r}}\right)$, since each symbol can be confused with two neighbors on its sides that are at a distance: $d = 2A_c$

4QAM or QAM (M=4) M-ary modulation

$$P_e = \frac{1}{M}(P_{e0} + P_{e2} + P_{e2} + P_{e3}) = \frac{1}{4} 4x2xQ\left(\sqrt{\frac{A_c^2}{\eta r}}\right) =$$

$$2Q\left(\sqrt{\frac{E}{\eta}}\right) = 2Q\left(\sqrt{\frac{2E_b}{\eta}}\right) = 2Q(\sqrt{2\gamma_b})$$

$$E = 2E_b$$

La tasa de error es igual a 4PSK.

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MPSK M-ary phase modulation

MPSK M-ary phase modulation

In the following analysis it is assumed that the carrier signal has synchronization with the modulation so that $f_c = Nr_b$. In a signaling interval the waveform is equal to:

$$x_c(t) = s_i(t - kD) - s_q(t - kD)$$

$$s_i(t - kD) = A_c cos(\varphi_k) p_D(t) cos(\omega_c t)$$
$$s_q(t - kD) = A_c sin(\varphi_k) p_D(t) sin(\omega_c t)$$

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MPSK M-ary phase modulation

Where $$\varphi_k = \frac{2\pi a_k}{M}$$

$$a_k = 0, 1, 2, 3, \dots, M-1$$

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \int_{kD}^{(k+1)D} (s_i(t))^2 dt + \int_{kD}^{(k+1)D} \left(s_q(t)\right)^2 dt) =$$
$$= \frac{{A_c}^2}{2} D\left(cos(\varphi_k)^2 + sin(\varphi_k)^2\right) = \frac{{A_c}^2 D}{2}$$

The constellation corresponding to 8PSK is shown in the following figures, in dimensional and normalized mode:

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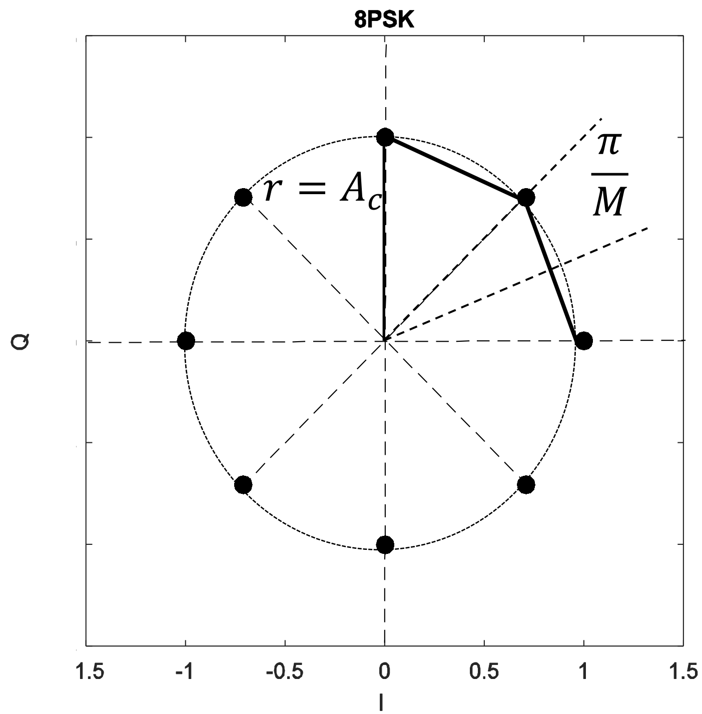
MPSK M-ary phase modulation



$$d = 2A_c sin\left(\frac{\pi}{M}\right)$$
$$= 2A_c sin\left(\frac{\pi}{8}\right)$$

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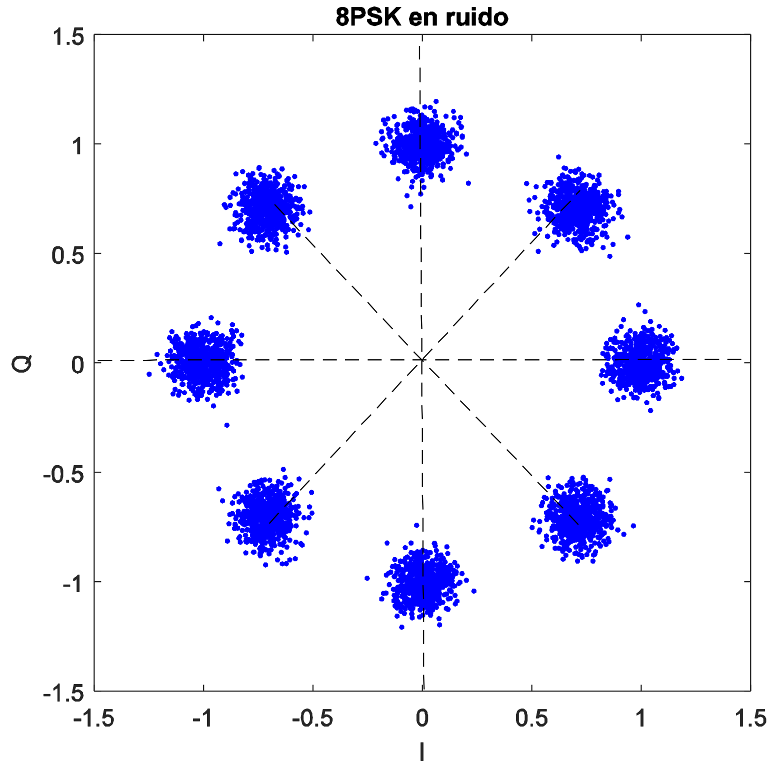
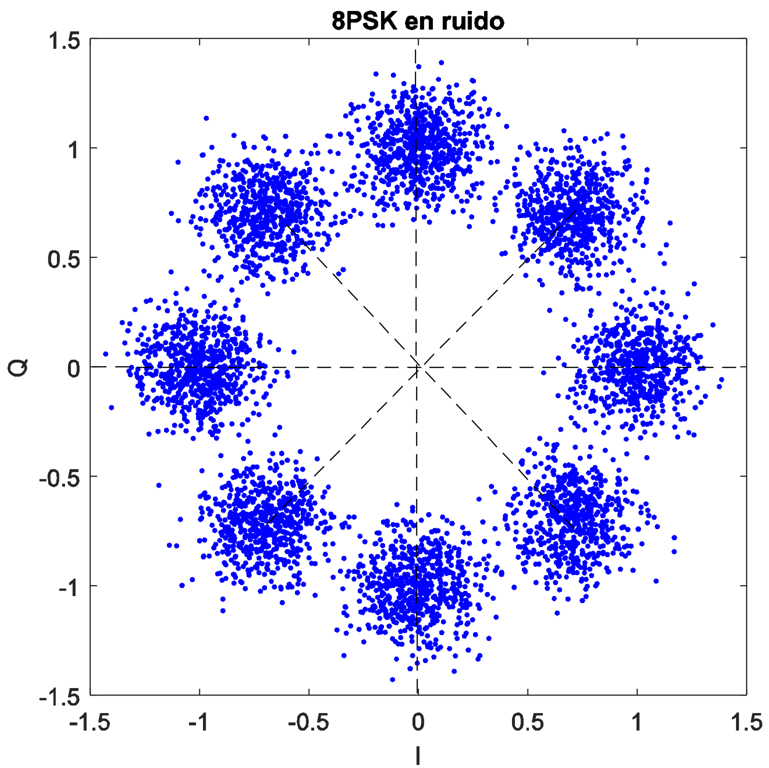
8PSK with noise. Error Probability.



8PSK with noise. Error Probability.



MPSK M-ary phase modulation. P_{be}

On the normalized constellation a normalized geometric factor can be defined:

$$FGN = \frac{1}{8}(1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2 + 1^2) = 1$$

In this case:

$$E = FGN\frac{{A_c}^2 D}{2} = \frac{{A_c}^2 D}{2}$$

Then:

$${A_c}^2 = \frac{2E}{D} = 2Er$$

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MPSK M-ary phase modulation. P_{be}

Now the distance is$d = 2A_c sin\left(\frac{\pi}{M}\right)$

then $d^2 = \left(2A_c sin\left(\frac{\pi}{M}\right)\right)^2 = 4A_c^2 \sin^2\left(\frac{\pi}{M}\right)$

and

$$\frac{d}{2} = A_c sin\left(\frac{\pi}{M}\right)$$

$$\frac{(d/2)^2}{\eta r} = \frac{\left(A_c sin\left(\frac{\pi}{M}\right)\right)^2}{\eta r} = \frac{A_c^2 \sin^2\left(\frac{\pi}{M}\right)}{\eta r}$$

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MPSK M-ary phase modulation. P_{be}

Each symbol s_i is associated with a probability $P_{ei} = 2\,Q\left(\sqrt{\frac{(d/2)^2}{\eta r}}\right)$, since each symbol can be confused with two neighbors on its sides that are at a distance : $d = 2A_c sin\left(\frac{\pi}{M}\right)$

$$P_e = \frac{1}{M}(P_{e0} + P_{e2} + P_{e2} + P_{e3} + P_{e4} + P_{e6} + P_{e6} + P_{e7})$$

$$= \frac{1}{8} 8x2Q\left(\sqrt{\frac{A_c^2 \sin^2\left(\frac{\pi}{M}\right)}{\eta r}}\right) = 2Q\left(\sqrt{\frac{2Er \sin^2\left(\frac{\pi}{M}\right)}{\eta r}}\right)$$

$$= 2Q\left(\sqrt{\frac{2E \sin^2\left(\frac{\pi}{M}\right)}{\eta}}\right)$$

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MPSK M-ary phase modulation. P_{be}

$$= 2Q\left(\sqrt{\frac{2E\sin^2\left(\frac{\pi}{M}\right)}{\eta}}\right)$$

$$E = (\log_2 M)E_b = KE_b\,, \qquad K = \log_2 M$$

$$P_e = 2Q\left(\sqrt{\frac{2K\sin^2\left(\frac{\pi}{M}\right)E_b}{\eta}}\right) = 2Q\left(\sqrt{2K\sin^2\left(\frac{\pi}{M}\right)\gamma_b}\right)$$

$$P_{\mathrm{be}} = \frac{2}{K}Q\left(\sqrt{\frac{2K\sin^2\left(\frac{\pi}{M}\right)E_b}{\eta}}\right) = \frac{2}{K}Q\left(\sqrt{2K\sin^2\left(\frac{\pi}{M}\right)\gamma_b}\right)$$

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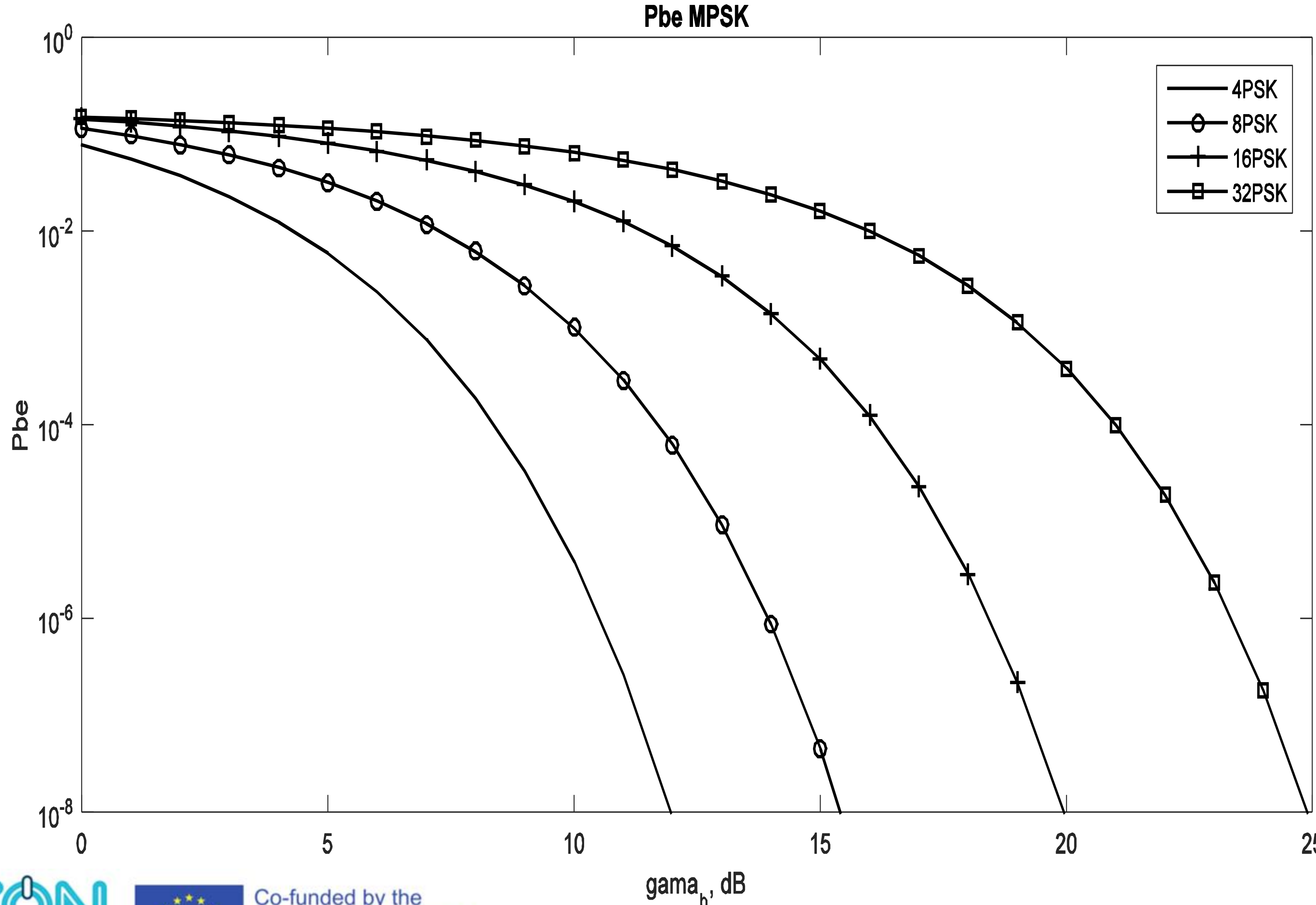
MPSK M-ary phase modulation. P_{be}



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APK or MQAM. Modulation in Amplitude and Phase.

MQAM. Modulation in Amplitude and Phase.

In the following analysis it is assumed that the carrier signal has synchronization with the modulation so that $f_c = Nr_b$.

At the el $k-th$ interval $kD < t < (k+1)D$:

$$x_c(t) = s_i(t-kD) - s_q(t-kD)$$

$$s_i(t-kD) = A_c I_k p_D(t) cos(\omega_c t) I_k = \pm 1, \pm 3, .., \pm \pm \mu - 1$$
$$s_q(t-kD) = A_c Q_k p_D(t) \sin(\omega_c t) Q_k = \pm 1, \pm 3, .., \pm \pm \mu - 1$$

$$\mu = \sqrt{M}$$

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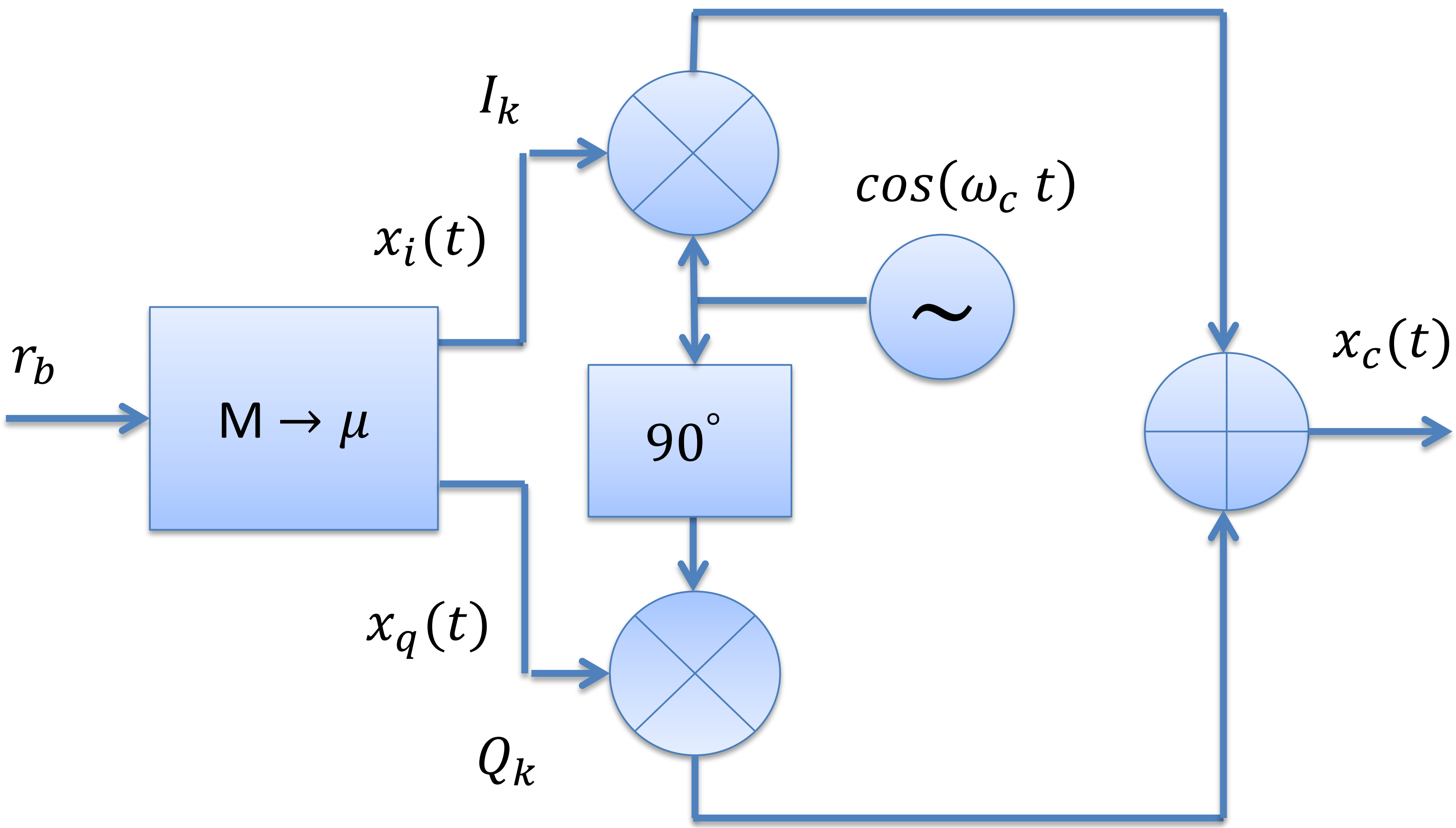
MQAM Modulation. Block diagram.



APK or MQAM. Modulation in Amplitude and Phase.

Statistical averages can be obtained:

$$\overline{I_k} = \overline{Q_k} = 0$$

$$\overline{{I_k}^2} = \overline{{Q_k}^2} = \frac{\mu^2 - 1}{3}$$

Example where $M = 16, \mu = 4$:

$$x_c(t) = s_i(t - kD) - s_q(t - kD)$$

$$s_i(t - kD) = A_c I_k p_D(t) cos(\omega_c t) \quad I_k = \pm 1, \pm 3$$

$$s_q(t - kD) = A_c Q_k p_D(t) \sin(\omega_c t) \quad Q_k = \pm 1, \pm 3$$

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APK or MQAM. Modulation in Amplitude and Phase..

$$E = \int_{kD}^{(k+1)D} (x_c(t))^2 dt = \int_{kD}^{(k+1)D} (s_i(t))^2 dt + \int_{kD}^{(k+1)D} \left(s_q(t)\right)^2 dt) =$$

$$= \frac{{A_c}^2}{2} D\left(I_k^2 + Q_k^2\right) = \frac{\mu^2 - 1}{3} {A_c}^2 D$$

The constellation corresponding to 16QAM is shown in the following figures, in dimensional and normalized mode. The values on the I and Q axes take the form M-ary polar:

$$a_k = \pm 1, \pm 3, \pm 5, \dots, \pm (M - 1)$$

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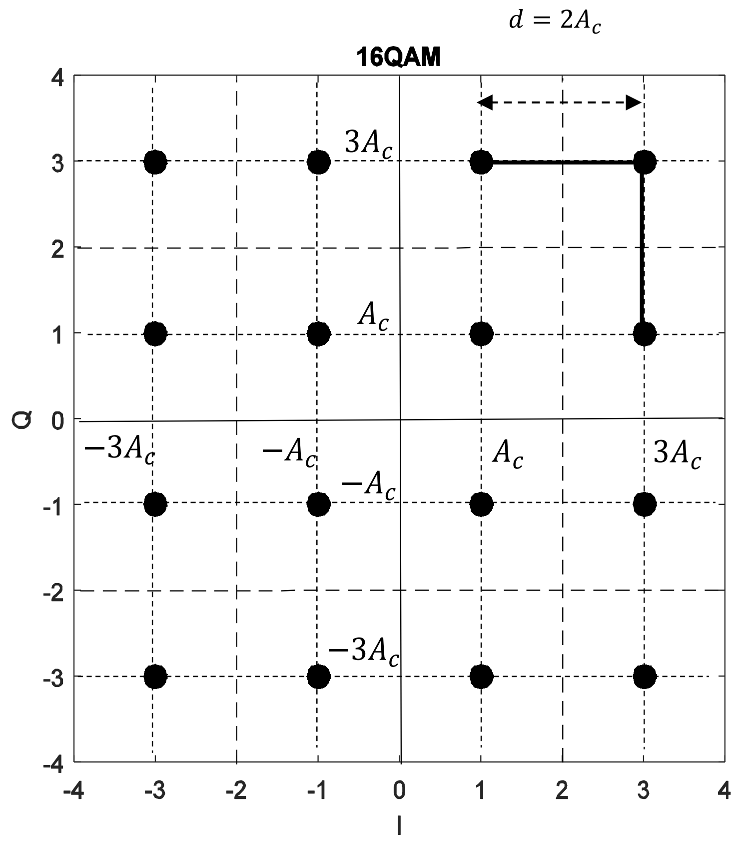
MQAM Modulation. 16QAM



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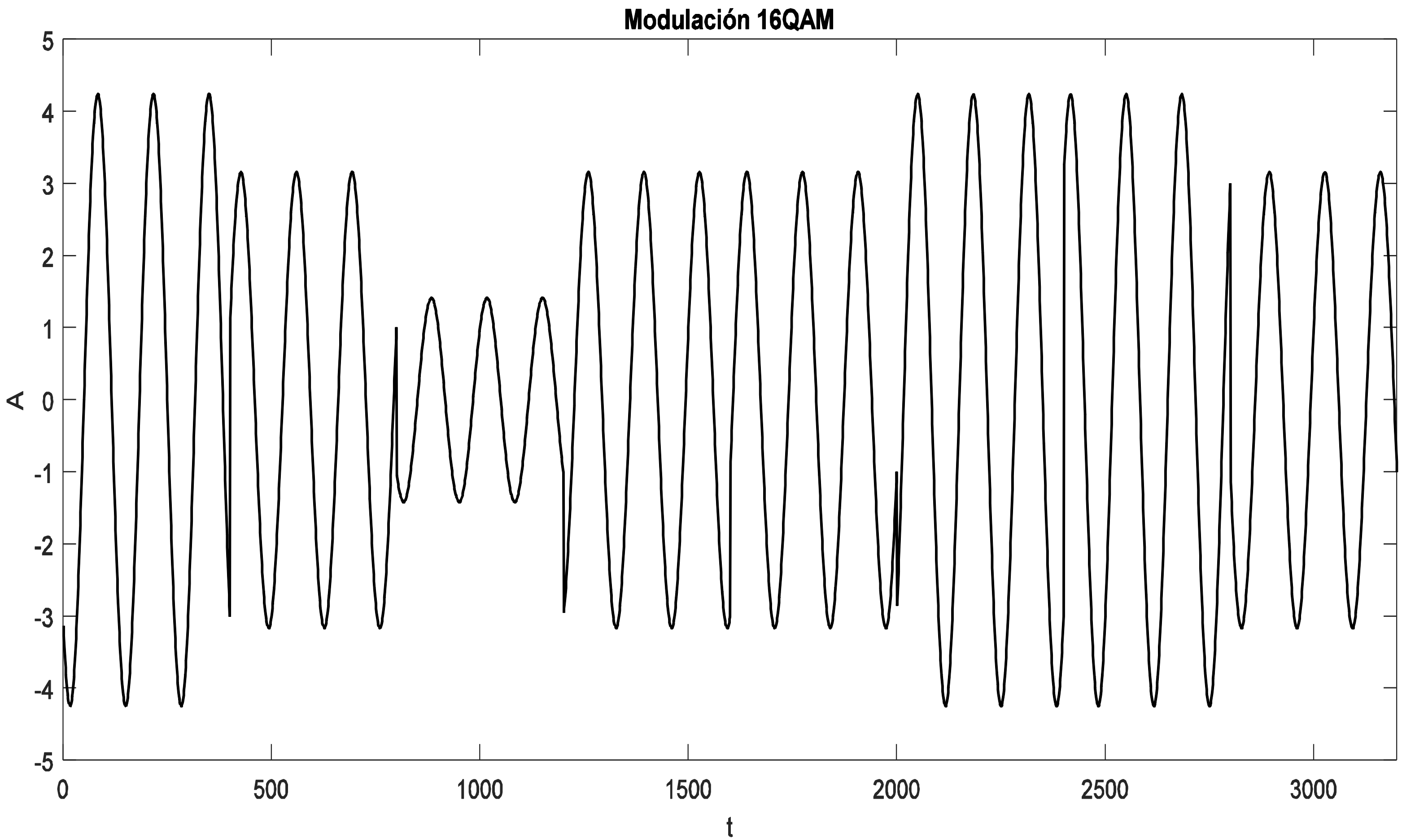
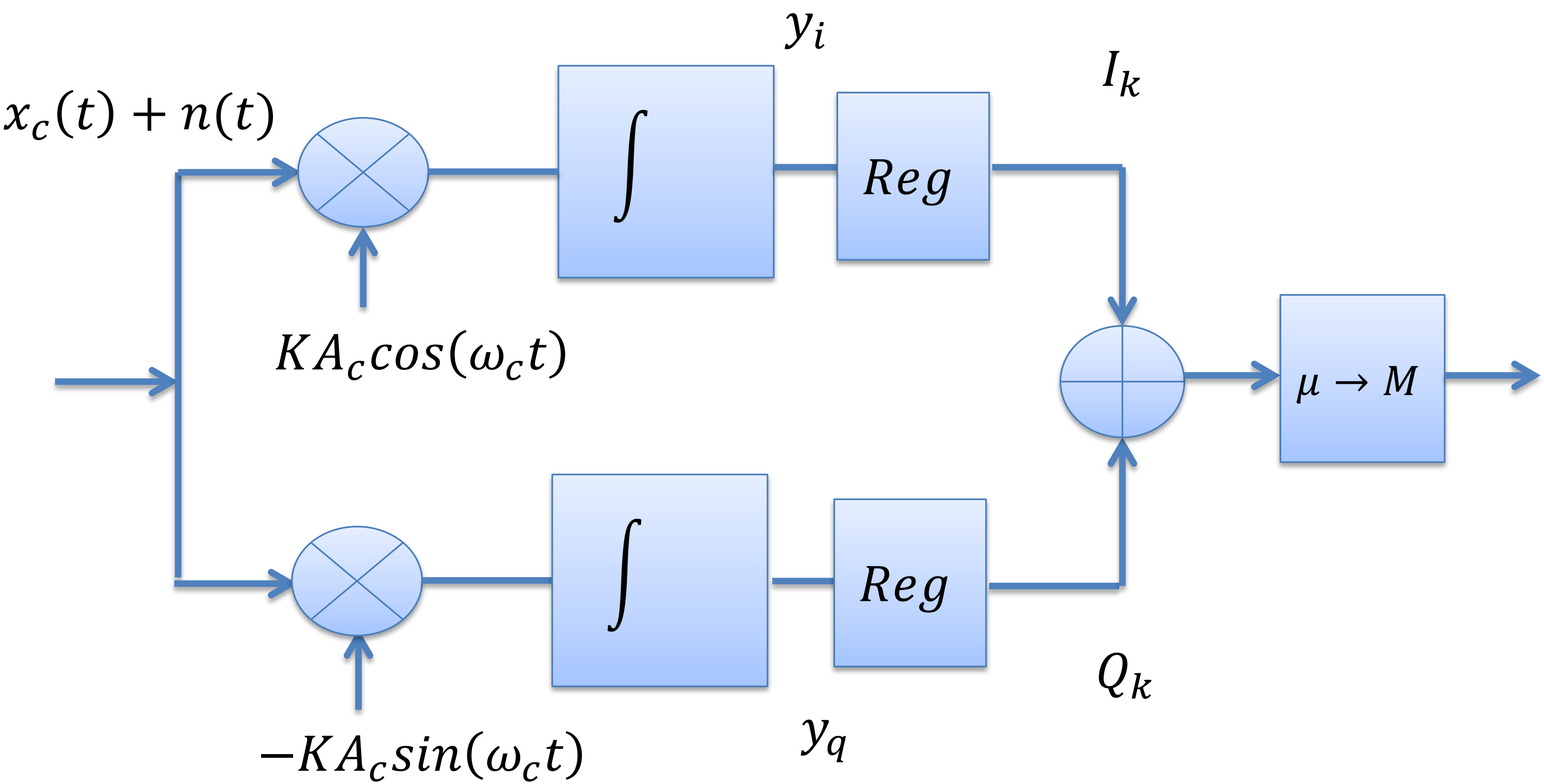
16QAM. Waveforms.



MQAM. In phase and quadrature receiver

The block diagram of the MQAM detector by correlation would then be:



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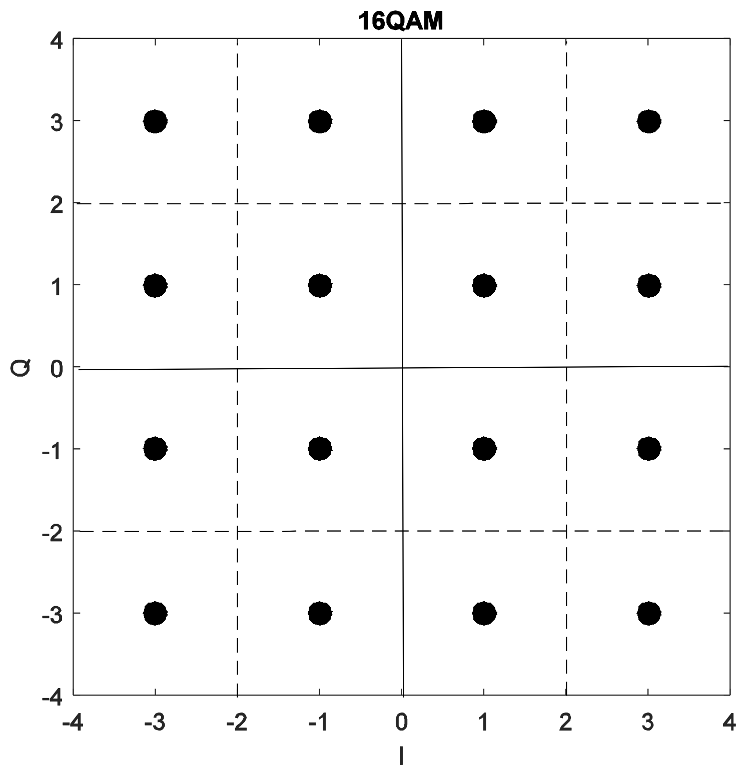
MQAM. Error probability.



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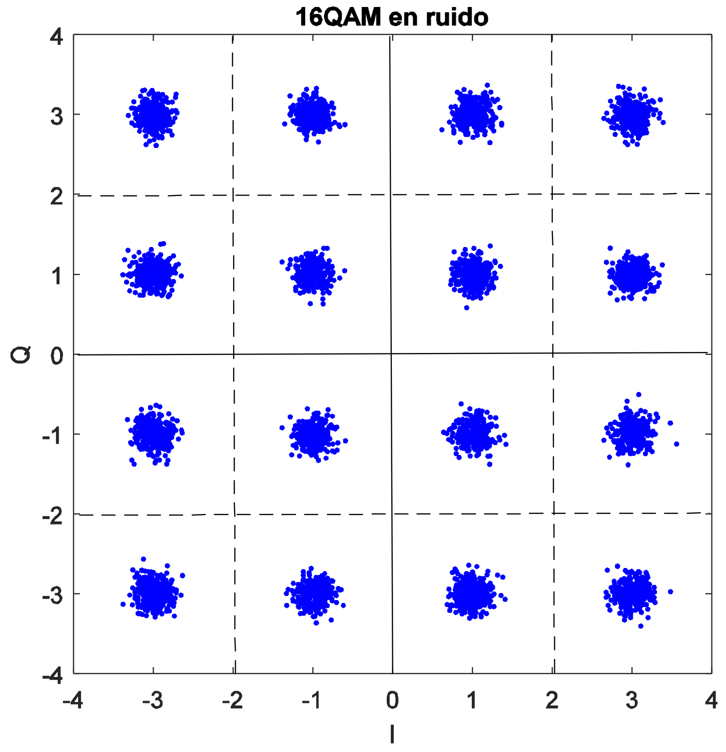
MQAM. Error probability. .



M-ary systems. Error probability.

On the normalized constellation a normalized geometric factor can be defined:

$$FGN = \frac{1}{16}4\left((1^2+1^2)+(1^2+3^2)+(3^2+1^2)+(3^2+3^2)\right)$$
$$= \frac{1}{16}4x40 = 10$$

In this case:

$$E = FGN\frac{{A_c}^2 D}{2} = 10\frac{{A_c}^2 D}{2}$$

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M-ary systems. Error probability.

$$A_c{}^2 = \frac{2E}{10D} = \frac{Er}{5}$$

Now, the distance $d = 2A_c$, then $d^2 = 4A_c^2$

And

$$\frac{d}{2} = A_c$$

$$\frac{(d/2)^2}{\eta r} = \frac{(A_c)^2}{\eta r} = \frac{A_c^2}{\eta r}$$

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M-ary systems. Error probability.

Each symbol s_i is associated with a probability $P_{ei} = Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)$, but the neighborhood is not symmetric, and some symbols have different error probabilities depending on how close the neighboring symbols are, and how many neighboring symbols there are.

The distance to define the neighborhood is always the same, and equal to $d = 2A_c$.

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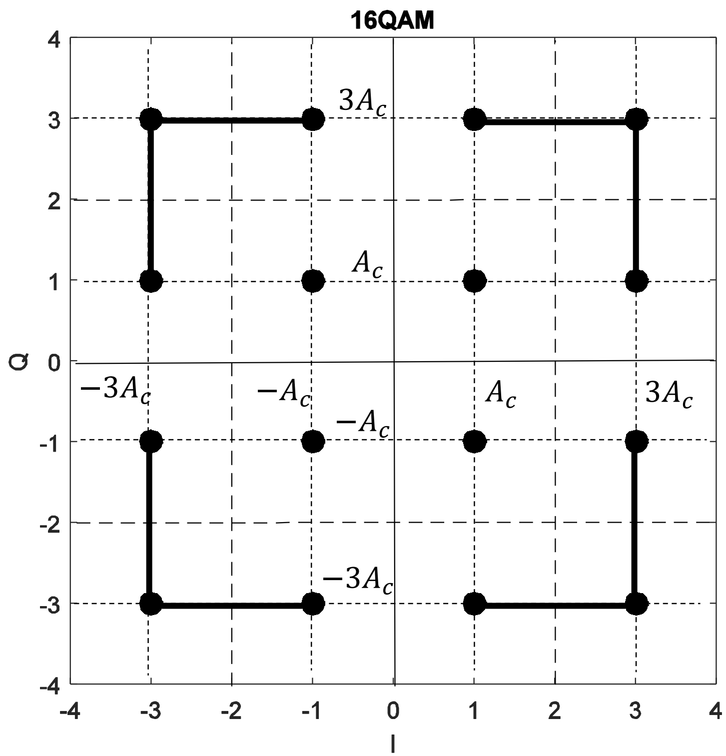
M-ary systems. Error probability.



The 4 symbols found on the corners are associated with the probability of having 2 neighbors at a distance d.

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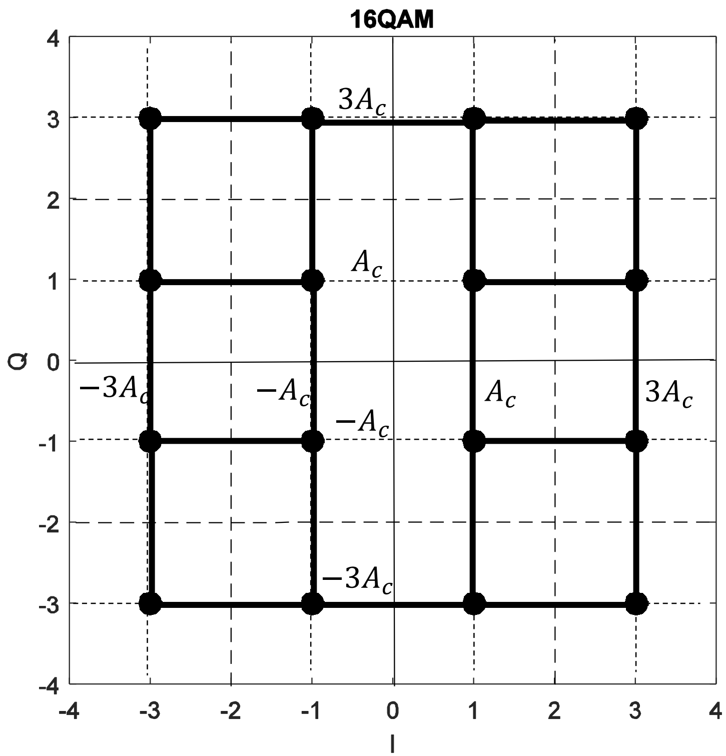
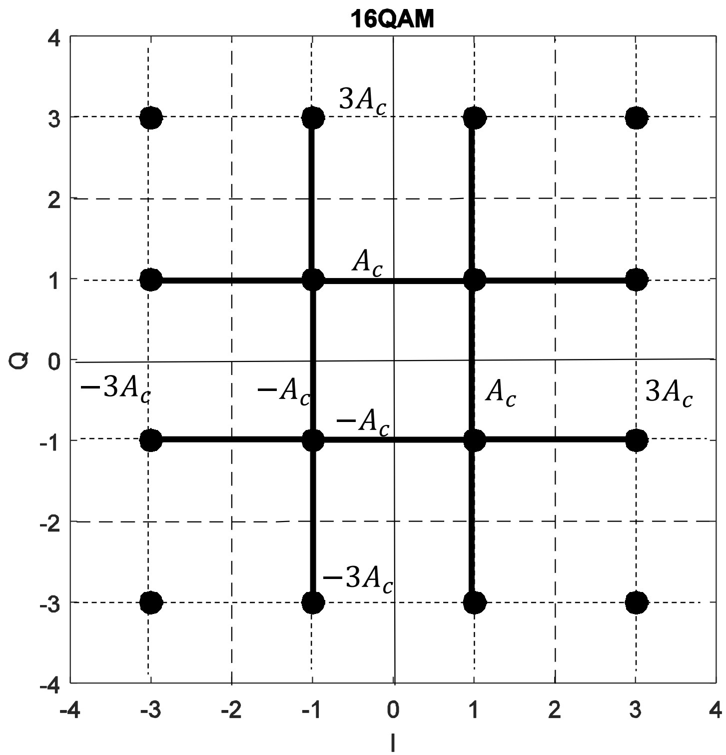
M-ary systems. Error probability.



The 8 symbols at the lateral ends excluding those at the corners are in an error event with 3 other neighbors in each case.

M-ary systems. Error probability.



The 4 inner symbols have 4 nearby neighbors with which they can be confused in an error event.

M-ary systems. Error probability.

The error probability per symbol P_e of the system is then :

$$P_e = \frac{1}{M}\left(\sum_{i=0}^{M-1} P_{ei}\right)$$

$$= \frac{1}{16}\left[4\left(2Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + 8\left(3Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + 4\left(4Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right)\right]$$

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M-ary systems. Error probability.

The error probability per symbol P_e of the system is then :

$$P_e = \frac{1}{M}\left(\sum_{i=0}^{M-1} P_{ei}\right)$$

$$= \frac{1}{M}\left[\frac{M}{4}\left(2Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + \frac{M}{2}\left(3Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + \frac{M}{4}\left(4Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right)\right]$$

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M-ary systems. Error probability.

$$P_e = \frac{1}{M}\left(\sum_{i=0}^{M-1} P_{ei}\right)$$

$$= \frac{1}{M}\left[\frac{M}{2}\left(Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + \frac{3M}{2}\left(Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right) + M\left(Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right)\right]$$

$$P_e = \frac{1}{M}\left(\sum_{i=0}^{M-1} P_{ei}\right) = \frac{1}{M}\left(\frac{M}{2} + \frac{3M}{2} + M\right)\left[Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)\right] = 3Q\left(\sqrt{\frac{A_c^2}{\eta r}}\right)$$

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M-ary systems. Error probability.

While:

$$A_c^2 = \frac{Er}{5} \quad \text{y} \quad \frac{d}{2} = A_c:$$

$$\frac{(d/2)^2}{\eta r} = \frac{A_c^2}{\eta r} = \frac{Er}{5\eta r} = \frac{E}{5\eta}$$

$$P_e = 3Q\left(\sqrt{\frac{E}{5\eta}}\right)$$

For squared 16QAM.

M-ary systems. Error probability.

Expression that matches the following if $M = 16$:

$$P_e = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E}{(M-1)\eta}}\right) = 4\left(1 - \frac{1}{4}\right) Q\left(\sqrt{\frac{3E}{(15)\eta}}\right) = 3Q\left(\sqrt{\frac{E}{5\eta}}\right)$$

If:

$$E = (\log_2 M)E_b = KE_b, \qquad K = \log_2 M$$

$$P_e = 3Q\left(\sqrt{\frac{KE_b}{5\eta}}\right) = 3Q\left(\sqrt{\frac{K}{5}\gamma_b}\right) = 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3K}{(M-1)}\gamma_b}\right)$$

M-ary systems. Error probability.

Calculation on the constellation

We start from the expression:

$$E = FGN \frac{{A_c}^2 D}{2}$$

$$r = \frac{1}{D} = \frac{FGN\ {A_c}^2}{2E}$$

The noise power is:

$$\sigma^2 = \eta r = \eta \frac{FGN\ {A_c}^2}{2E}$$

Then, for example:

For MQAM constellations the distance is $d = 2A_c$,

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M-ary systems. Error probability.

$\frac{d}{2} = A_c,$ $\qquad P_e = Q\left(\frac{A_c}{\sigma}\right)$

$$\left(\frac{A_c}{\sigma}\right)^2 = \frac{(d/2)^2}{\eta r} = \frac{{A_c}^2}{\eta r} = \frac{{A_c}^2}{\eta \frac{FGN\, {A_c}^2}{2E}} = \frac{2}{FGN}\frac{E}{\eta}$$

$$P_e = Q\left(\frac{A}{2\sigma}\right) = Q\left(\frac{d}{2\sigma}\right) = Q\left(\sqrt{\frac{(d/2)^2}{\eta r}}\right) Q\left(\sqrt{\frac{2}{FGN}\frac{E}{\eta}}\right)$$

the binary probability of error is:

$$P_{\mathrm{be}} = \frac{4}{K}\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3K}{(M-1)}\gamma_b}\right)$$

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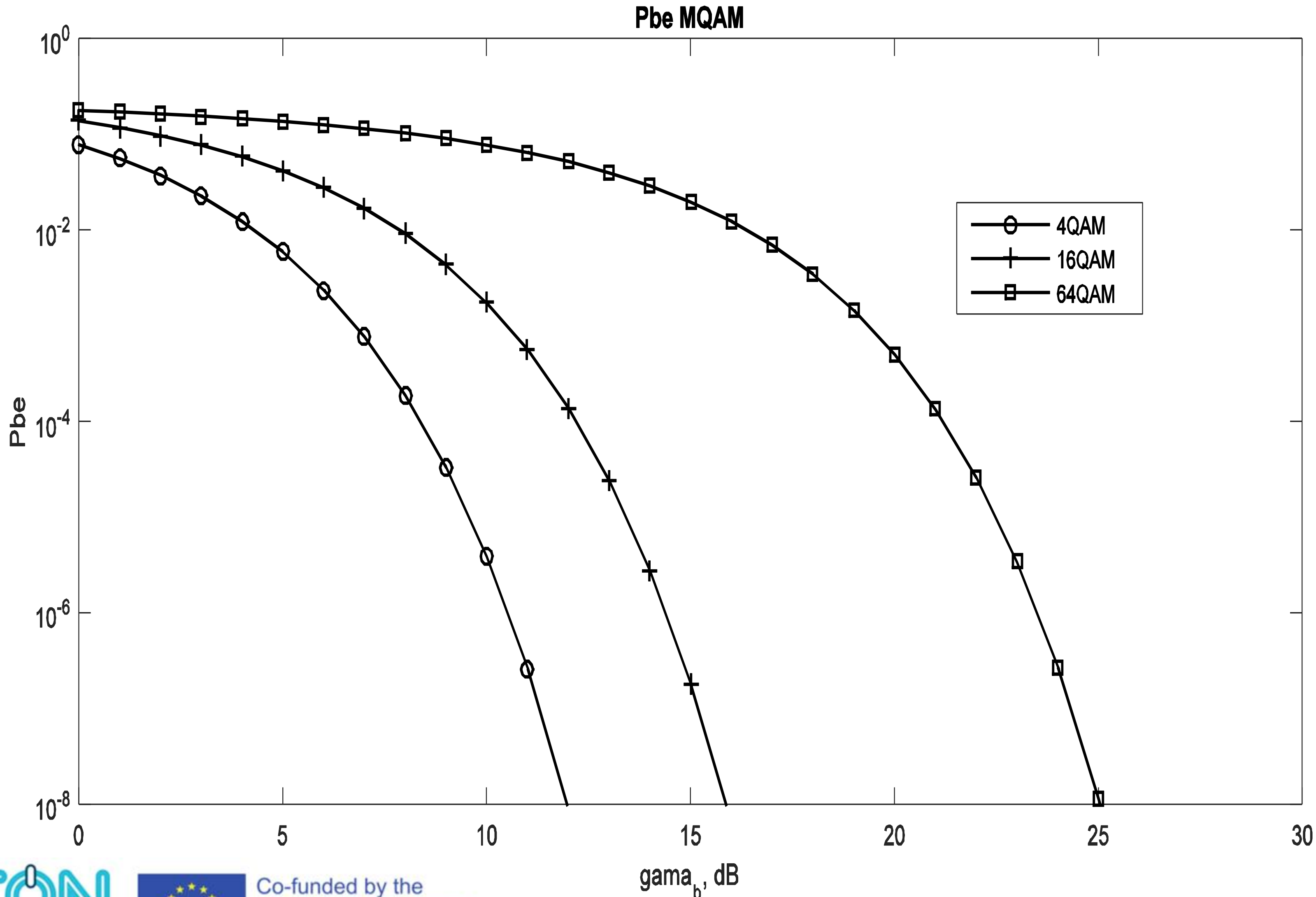
MQAM. Binary error probability.



MQAM-MPSK. Binary error probability.