Communications Systems based on Software Defined Radio (SDR)

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Exponencial modulation

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- Starting from a continuous wave signal, with A(t) constant, but phase variable with time:

$$x_c(t) = A_c \cos[\varpi_c t + \phi(t)]$$

- Defining the argument or total angle:

$$\theta_c(t) = \varpi_c t + \phi(t)$$

- It is possible to write:

$$x_c(t) = A_c \cos\theta_c(t) = A_c \operatorname{Re}\left[e^{j\theta_c(t)}\right]$$

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- The information is in the phase or in the frequency of the signal.
- Therefore, the information of the message is in the zero crossings of the signal.
- The amplitude of the carrier does not contain information in exponential modulation.
- The described properties provide high noise immunity. The required bandwidth is usually greater (or even much greater) than that required in linear modulation.
- It is used in commercial broadcast service, analog TV, and two-way mobile radio systems.

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$$x_c(t) = A_c \cos\theta_c(t) = A_c \operatorname{Re}\left[e^{j\theta_c(t)}\right]$$

Then, if Ɵc(t) contains the message x(t), this process can be called exponential modulation. Which, Emphasizes the non-linear relationship between xc(t) and x(t).

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Phase Modulation:

- It is defined by the following dependency:

$$\phi(t) = \phi_{\Lambda} x(t) \qquad \phi_{\Lambda} \leq 180°$$

$$x_c(t) = A_c \cos[\varpi_c t + \phi_{\Lambda} x(t)]$$

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- The speed of rotation of the phasor in cycles per second will be:

$$f(t) = \frac{1}{2\pi}\frac{\theta_c(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = fc + \frac{1}{2\pi}\dot{\phi}(t)$$

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Frequency modulation:

•In the case of frequency modulation (FM), the instantaneous frequency is defined as:

$$f(t) = f_c + f_\Lambda x(t) \qquad f_c \geq f_\Lambda$$

•The frequency varies linearly with x(t) through a constant fΛ called the **frequency offset**.

•f_Λ represents the **maximum frequency offset** with respect to the carrier, this will occur when x(t)=1.

•In real systems $\mathbf{f_\Lambda \ll fc}$.

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- If we start from the instantaneous frequency expression for PM, it results:

$$X_c(t)= A_c.\cos(\phi_c(t)) = A_c \cos(2.\pi.f(t).t) = A_c.\cos[2.\pi(f_c+f_\Lambda x(t))t]$$

- Furthermore, using the instantaneous frequency expression in FM, it is possible to rewrite it as:

$$\boxed{f(t) = f_c + f_\Lambda x(t)} = f_c + \frac{1}{2\pi}\dot{\phi}(t) \qquad \dot{\phi}(t) = 2\pi f_\Lambda x(t)$$

$$\boxed{\phi(t) = 2\pi f_\Lambda \int_{t_0}^{t} x(\lambda)\,d\lambda \qquad t \geq t_0}$$

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- From the two previous expressions, the waveform for FM arises:

$$x_c(t) = A_c \cos[\varpi_c t + \phi(t)] = A_c \cos\left[\varpi_c t + \underbrace{2\pi f_\Delta \int^t x(\lambda)d\lambda}_{\phi(t)}\right]$$

- This shows that it is also possible by means of an integrator and a modulator of PM, to obtain FM.

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	Instantaneous phase	Instantaneous phase
PM	$\phi_\Delta x(t)$	$f_c + \frac{1}{2\pi}\phi_\Delta \frac{dx(t)}{dt}$
FM	$2\pi f_\Delta \int^t x(\lambda)d\lambda$	$f_c + f_\Delta x(t)$

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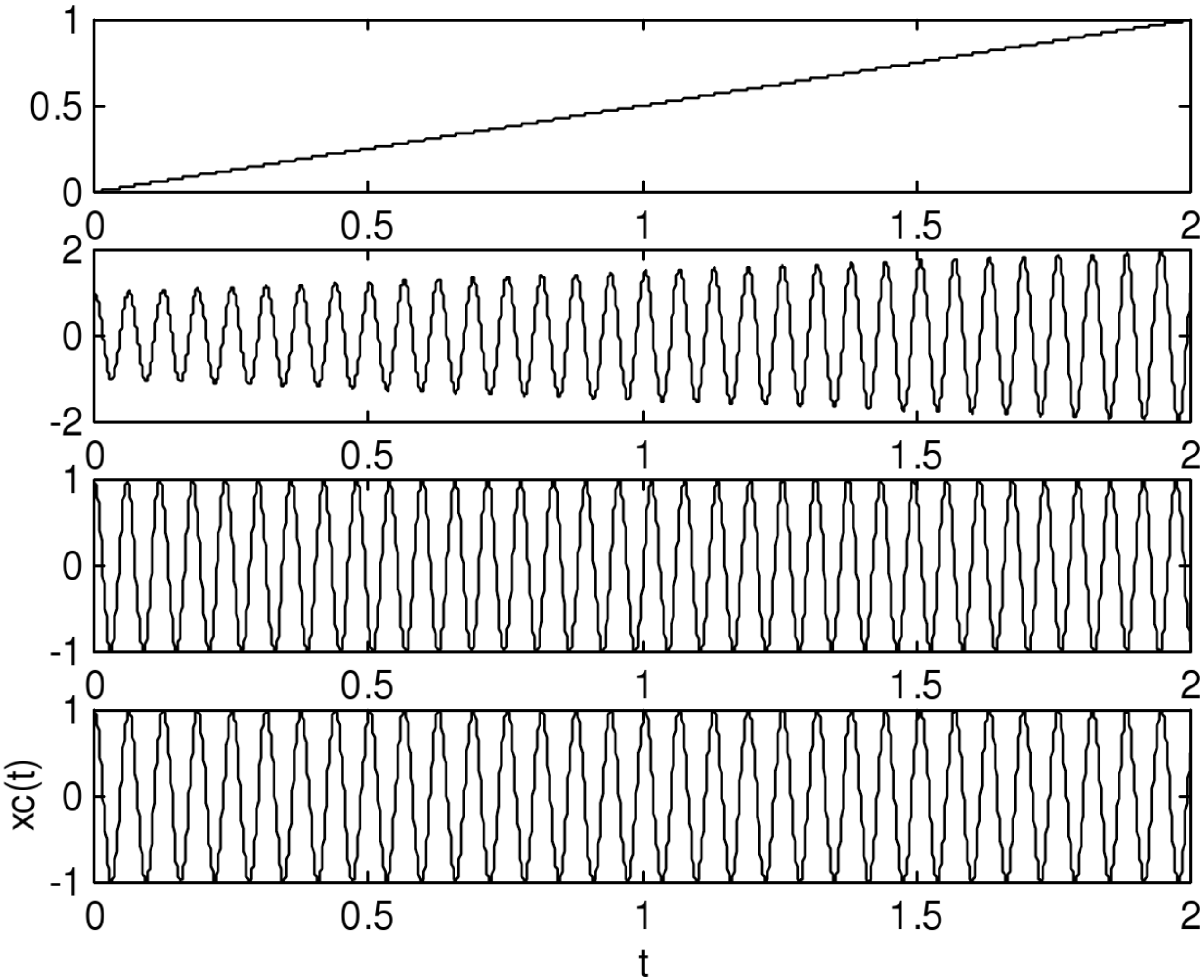
- Modulation of a linear signal through AM, PM and FM.



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- Since regardless of x(t), the amplitude of $x_c(t)$ does not vary in PM and FM.

$$S_T = \frac{1}{2} A_c^2$$

- The average transmitted power does not depend on x(t).

- In exponential modulation, zero crossings are not periodic.

- The modulated waveform is nothing like the message x(t).

- FM allows noise reduction without increasing S_T, but at the cost of increasing B_T.

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Continuous Wave Exponential Modulation

Distortion and Bandwidth

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Alternatively, a more compact expression can be written using the property:

$$J_{-n}(\beta) = (-1)^{-n} J_n(\beta)$$

So:

$$x_c(t) = A_c J_0(\beta)\cos\varpi_c t + \sum_{n\cdot impar}^{\infty} A_c J_n(\beta)\left[\cos(\varpi_c + \varpi_m)t - \cos(\varpi_c - \varpi_m)\right] +$$

$$+ \sum_{n\cdot par}^{\infty} A_c J_n(\beta)\left[\cos(\varpi_c + \varpi_m)t + \cos(\varpi_c - \varpi_m)\right]$$

It can be rewritten as:

$$\boxed{x_c(t) = A_c \sum_{n=-\infty}^{+\infty} J_n(\beta)\cos(\varpi_c + n\varpi_m)t}$$

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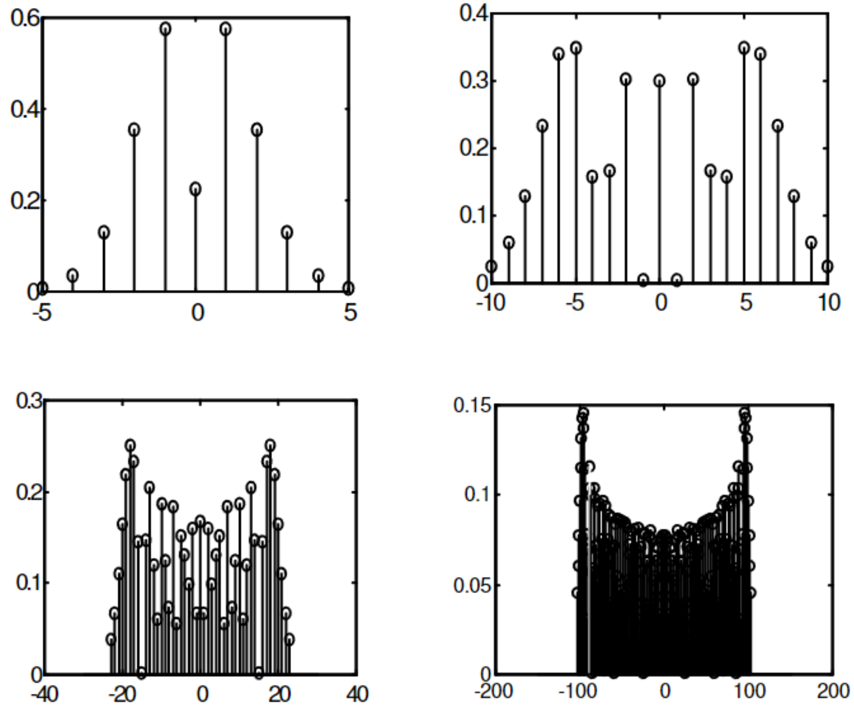
- Expanding, you get:

$$x_c(t) = A_c \sum_{n=-\infty}^{+\infty} \boxed{J_n(\beta)} \cos(\varpi_c + n\varpi_m)t$$

Bessel Coefficients

In either of the two forms (Eq. 7.33 or 7.34) it mathematically represents a wave of constant amplitude whose instantaneous frequency varies sinusoidally. We can see that the FM spectrum consists of a number infinity of lines at frequencies f_c±nf_m. All lines are equally spaced at the frequency of modulation, and the lines of odd order below f_c are inverted with respect to their similar one above of f_c.

If the spectrum of lines is plotted in module only, without taking into account the sign changes, for values of β = 2, 7, 10 and 100, the line spectra shown in Fig (7.5) are obtained. In these graphs, the abscissa axis is not graduated in frequency, but in values of β . To pass to frequency, it must be multiplied by the frequency of the tone fm, and move the origin in fc. Note that as β increases, the number of lines increases, and the amplitude of the lines decreases, so that the average power remains constant and equal to $\frac{A_c^2}{2}$. Finally, for the case of β=100 it can be seen that the shape followed by the lines corresponds approximately to a bandwidth equal to 2β.



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Bessel Coefficients													
β	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12
0.00	1	0	0	0	0	0	0	0	0	0	0	0	0
0.10	0.998	0.050	0	0	0	0	0	0	0	0	0	0	0
0.25	0,984	0,124	0	0	0	0	0	0	0	0	0	0	0
0.50	0,938	0,242	0,031	0	0	0	0	0	0	0	0	0	0
0.75	0,864	0,349	0,067	0	0	0	0	0	0	0	0	0	0
1.00	0,765	0,44	**0,115**	0,02	0	0	0	0	0	0	0	0	0
1.25	0,646	0,511	0,171	0,037	0	0	0	0	0	0	0	0	0
1.50	0,512	0,558	0,232	0,061	0,012	0	0	0	0	0	0	0	0
1.75	0,369	0,58	0,294	0,092	0,021	0	0	0	0	0	0	0	0
2.00	0,224	0,577	0,353	**0,129**	0,034	0	0	0	0	0	0	0	0
2.50	-0,048	0,497	0,446	0,217	0,074	0,02	0	0	0	0	0	0	0
3.00	-0,26	0,339	0,486	0,309	**0,132**	0,043	0,011	0	0	0	0	0	0
3.50	-0,38	0,137	0,459	0,387	0,204	0,08	0,025	0	0	0	0	0	0
4.00	-0,397	-0,066	0,364	0,43	0,281	**0,132**	0,049	0,015	0	0	0	0	0
4.50	-0,321	-0,231	0,218	0,425	0,348	0,195	0,084	0,03	0	0	0	0	0
5.00	-0,178	-0,328	0,047	0,365	0,391	0,261	**0,131**	0,053	0,018	0	0	0	0
5.50	0	-0,341	-0,117	0,256	0,397	0,321	0,187	0,087	0,034	0,011	0	0	0
6.00	0,151	-0,277	-0,243	0,115	0,358	0,362	0,246	**0,13**	0,057	0,021	0	0	0
6.50	0,26	-0,154	-0,307	-0,035	0,275	0,374	0,3	0,18	0,088	0,037	0,013	0	0
7.00	0,3	0	-0,301	-0,168	0,158	0,348	0,339	0,234	**0,128**	0,059	0,024	0	0

- The question that arises is: How much of the spectrum is significant?

Practical rules establish that coefficients can be neglected from $\varepsilon<0.1$ or $\varepsilon<0.01$ depending on the application.

- Therefore, the table of coefficients is searched for a value of M such that:

$$\left|J_{M+1}(\beta)\right| < \varepsilon \qquad \text{y} \qquad \left|J_{M}(\beta)\right| > \varepsilon$$

For example: with b = 2, taking ε=0.1 results in |JM+1(β)| = 0.034 and |JM(β)| = 0.129. That is, J3(β) is the last significant line. Therefore, there will be 3 pairs of sidebands (in addition to the carrier).

Bessel Coefficients													
β	J0	J1	J2	J3	J4	J5	J6	J7	J8	J9	J10	J11	J12
0.00	1	0	0	0	0	0	0	0	0	0	0	0	0
0.10	0.998	0.050	0	0	0	0	0	0	0	0	0	0	0
0.25	0,984	0,124	0	0	0	0	0	0	0	0	0	0	0
0.50	0,938	0,242	0,031	0	0	0	0	0	0	0	0	0	0
0.75	0,864	0,349	0,067	0	0	0	0	0	0	0	0	0	0
1.00	0,765	0,44	**0,115**	0,02	0	0	0	0	0	0	0	0	0
1.25	0,646	0,511	0,171	0,037	0	0	0	0	0	0	0	0	0
1.50	0,512	0,558	0,232	0,061	0,012	0	0	0	0	0	0	0	0
1.75	0,369	0,58	0,294	0,092	0,021	0	0	0	0	0	0	0	0
2.00	0,224	0,577	0,353	**0,129**	0,034	0	0	0	0	0	0	0	0
2.50	-0,048	0,497	0,446	0,217	0,074	0,02	0	0	0	0	0	0	0
3.00	-0.26	0.339	0.486	0.309	**0.132**	0.043	0.011	0	0	0	0	0	0

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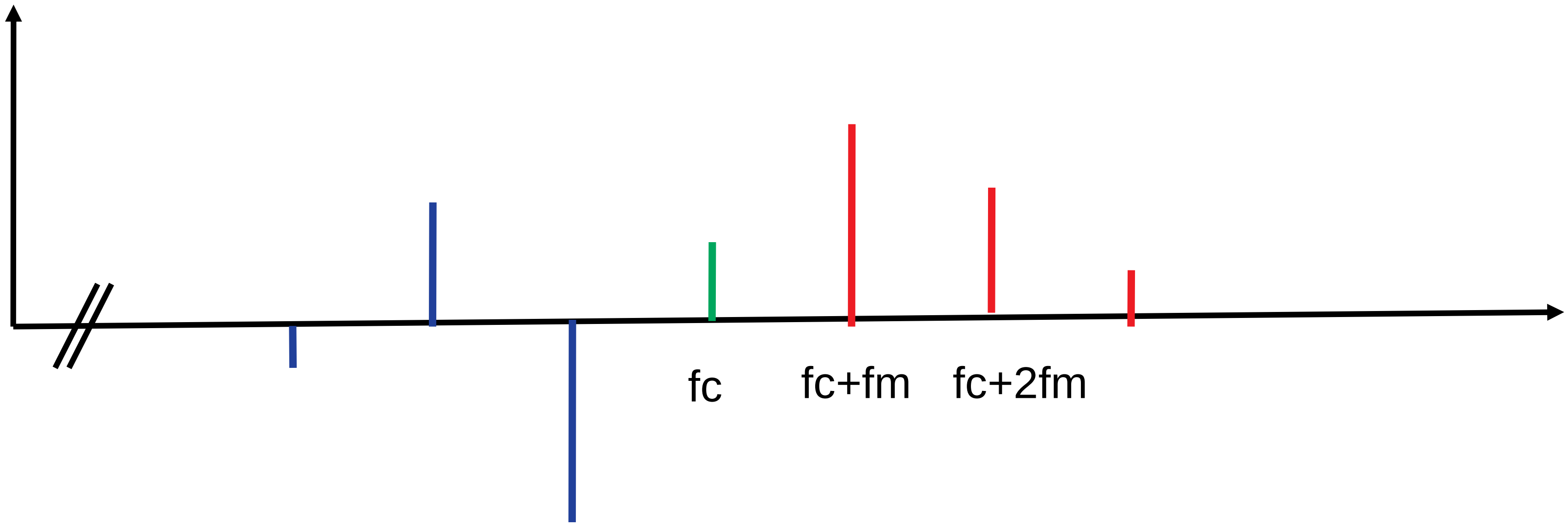
- Spectra (for f>0):



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Then the bandwidth is:

$$B = 2M(\beta) f_m \qquad\qquad M(\beta) > 1$$

- Where M is the number of significant lines.

Experimental studies show that ε<0.01 is very conservative. On the other hand, ε<0.1 can result in a small but distinguishable distortion.

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For signals that are not a tone, it must be fulfilled that: Am<1 and fm<W. Taking this into account, the values of M(β) approach M(β) = β+2 . It can be approximated:

$$B \approx 2(\beta + 2) f_m = 2\left(\frac{A_m f_\Delta}{f_m} + 2\right) f_m = 2\left(A_m f_\Delta + 2 f_m\right)$$

- And in the previous conditions, it can be said that:

$$B_T \approx 2\left(f_\Delta + 2W\right) \quad \text{if} \quad \beta > 2$$

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We can now write the bandwidth of an arbitrary signal x(t) as:

$$B_T = 2M(D)W$$

- It can be approximated as:

$$B_T = \begin{cases} 2DW = 2f\Delta & D >> 1 \\ 2W & D << 1 \end{cases}$$

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Both limits can be combined in a relationship called Carlson's rule.

$$B_T \approx 2(f_\Delta + W) = 2(D+1)W$$

•This is true if D>>1 and D<<1.

•But for values 2<D<10, Carlson's rule underestimates the bandwidth. Therefore, a better approximation in these cases is the following:

$$\boxed{B_T \approx 2(f_\Delta + 2W) = 2(D+2)W \qquad D > 2}$$

- Estimates based on power:

B_T = Bandwidth containing 90% of the total power.

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Distortion and Limiters

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Nonlinear distortion and limiters

The amplitude distortion of an FM wave produces a conversion from FM to AM. We will see that the modulation The resulting amplitude can be removed through the controlled use of non-linear distortion and filtering. For our analysis, suppose that the input signal in Fig. 7.11 is:

$$v_{in}(t) = A(t)\cos\theta_c(t) = A(t)\cos\left[\varpi_c t + \phi(t)\right] \qquad (7.76)$$

The nonlinear element is assumed without memory, that is, without elements that store energy, so the input and the output are related by an instantaneous transfer vout=T[vin]. Also, we will assume for convenience that T[0]=0. Although vin(t) is not necessarily periodic in time, it can be viewed as a periodic function of θ_C with period 2π. Since the output will be a periodic function of θ_C, it can be expanded into a trigonometric series of Fourier:

$$v_{out} = \sum_{n=1}^{\infty} \left|2a_n\right| \cos(n\theta_c + \arg a_n) \qquad (7.77)$$

where:

$$a_n = \frac{1}{2\pi} \int T\left[v_{in}\right] e^{-jn\theta_c}\, d\theta_c \qquad (7.78)$$

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The time variable does not appear here explicitly, but vout depends on t through the time variation of θc . Additionally, the coefficients an can be a function of time, when the amplitude of vin has variations in time. Initially we consider the case of an undistorted FM input, whereby A(t) is a constant Ac and all coefficients an are constant. Writing the Eq. 7.77 term by term, with t explicitly included, we have:

$$\boxed{v_{out}(t) = |2a_1| \cos[\varpi_c t + \phi(t) + \arg a_1] + |2a_2| \cos[2\varpi_c t + 2\phi(t) + \arg a_2] + \ldots\ldots \qquad (7.79)}$$

This expression reveals that nonlinear distortion produces FM modulated waves at the harmonics of the frequency carrier, the nth having a constant amplitude $|2a_n|$ and a phase modulation $n.\Phi(t)$ plus a constant phase shift $\arg(a_n)$. If these waves do not overlap in the frequency domain, the undistorted input can be recovered by applying the distorted output to a bandpass filter, which allows the carrier and its sidebands to pass through. That is, it changes A(t), which contains amplitude modulation by $|2a_1|$ which is a constant. We can conclude that FM has considerable immunity against the effects of non-linear distortion through the use of limiters. Returning to the unwanted amplitude variations A(t) of an FM wave, they can be eliminated by means of a limiter or trimmer whose transfer is shown in Fig. 7.12.

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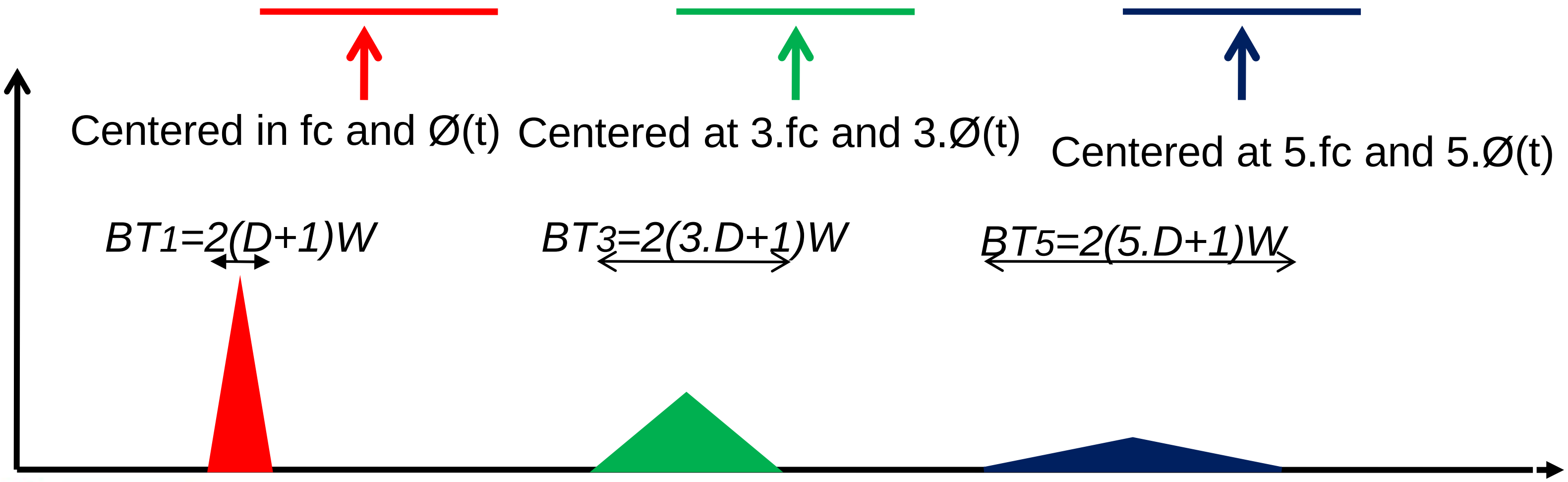
The coefficients obtained with Equation. 7.78 are:

$$a_n = \begin{cases} +\dfrac{2V_0}{n\pi} & n = 1,5,9,.... \\ -\dfrac{2V_0}{n\pi} & n = 3,7,11,.... \\ 0 & n = 2,4,6,... \end{cases} \tag{7.81}$$

Which are independent of time, since A(t) >0 does not affect the sign of vin . Therefore the output will be:

$$v_{out}(t) = \frac{4V_0}{\pi}\cos[w_c t + \phi(t)] - \frac{4V_0}{3\pi}\cos[3w_c t + 3\phi(t)] + \frac{4V_0}{5\pi}\cos[5w_c t + 5\phi(t)] + ... \tag{7.82}$$



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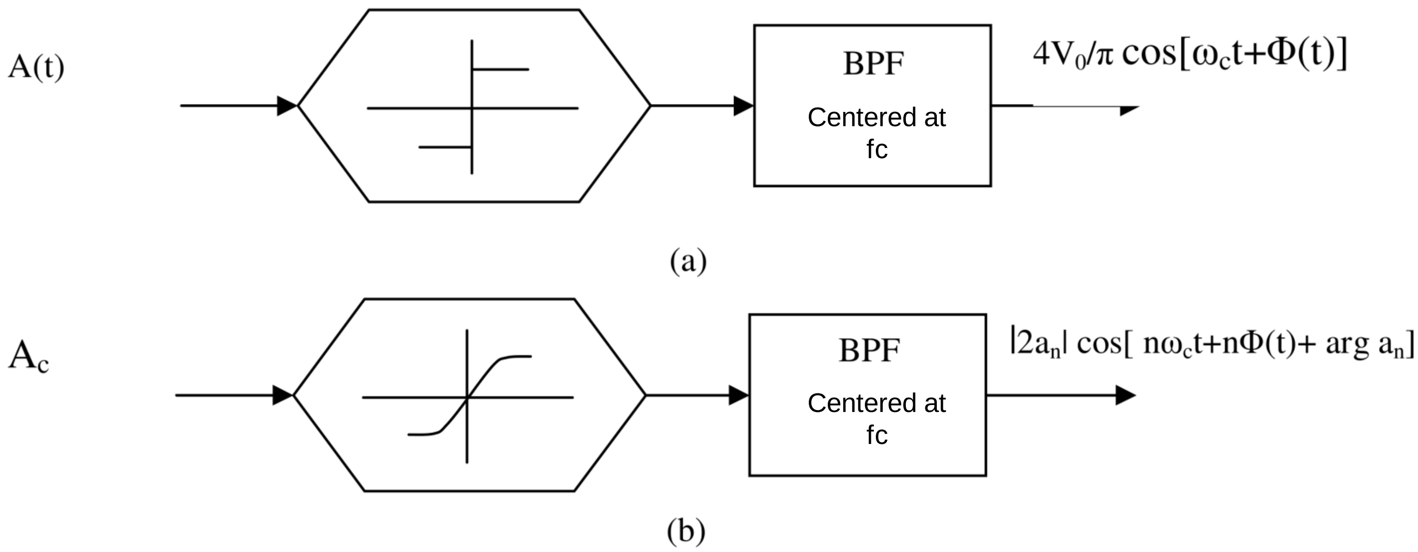

Fig. 7.13 summarizes the results. The limiter plus a bandpass filter centered on the carrier frequency, removes PM or FM amplitude variations, and is used in receivers, as a pre-switch operation. demodulation. In this way, we make sure that the demodulator output will only respond to the crossings by zero, where the information resides and not to the amplitude variations that the non-linear distortions have created. If the bandpass filter is centered on a harmonic of the carrier, a frequency multiplier is obtained. His primary use is in certain types of FM transmitters, to increase the rate of deviation of frequency, or simply to get a multiple of a frequency.

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FM and PM generation

VCO and direct FM
Indirect FM and phase modulators

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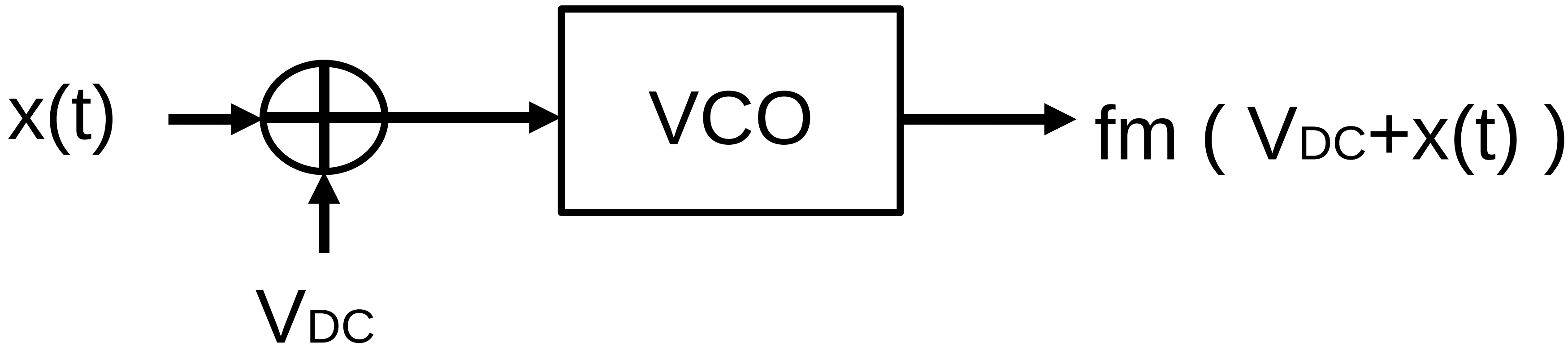
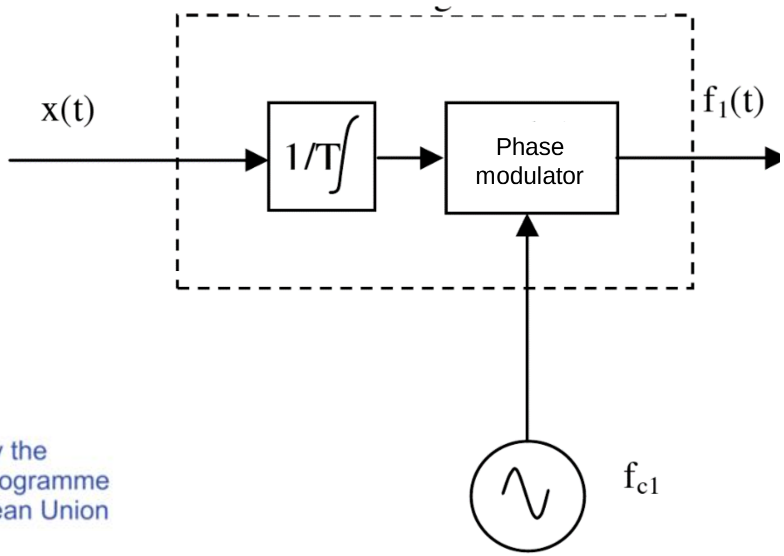
As previously seen, there are two alternatives for FM generation.

- Direct method.



- Indirect method.



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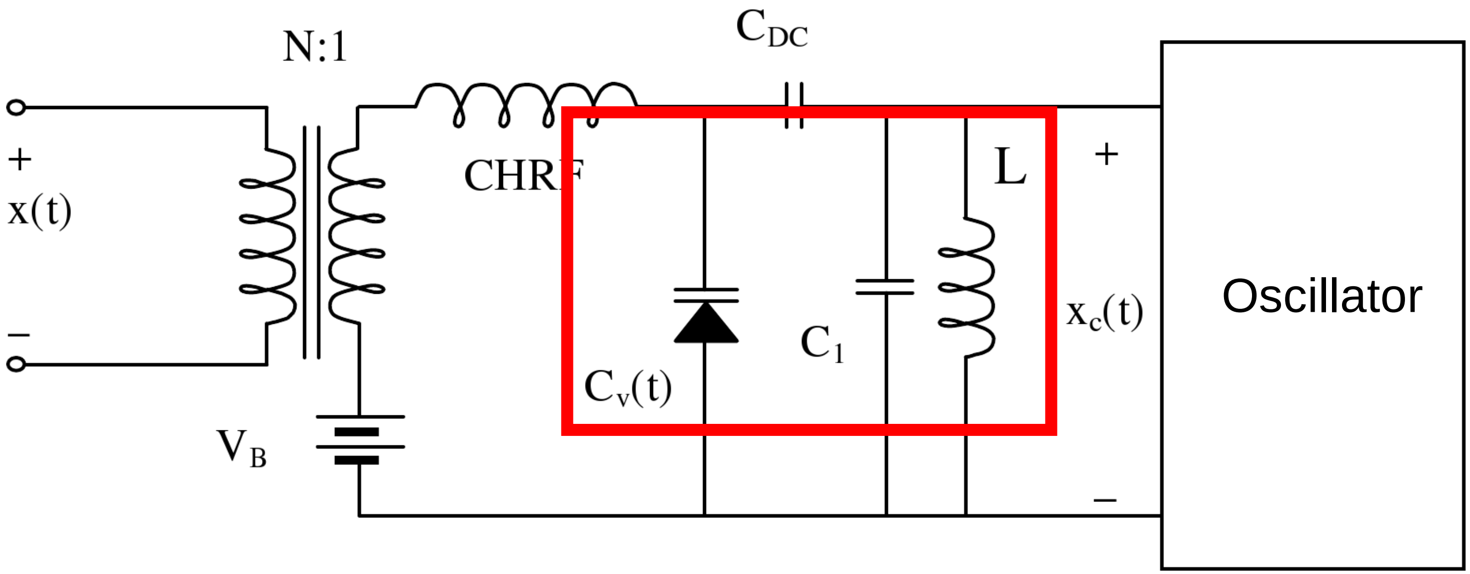
• Direct method of FM generation



This circuit consists of a VCO. Its resonant frequency depends on L and C. The total capacity of the system is controlled by x(t).

- But the biggest disadvantage of this circuit is that its carrier frequency tends to drift, so it must be stabilized with a more elaborate control circuit, known as a PLL. This is done with digital dividers and comparators.

- For this reason, until digital circuits appeared, modulators were of the indirect type.

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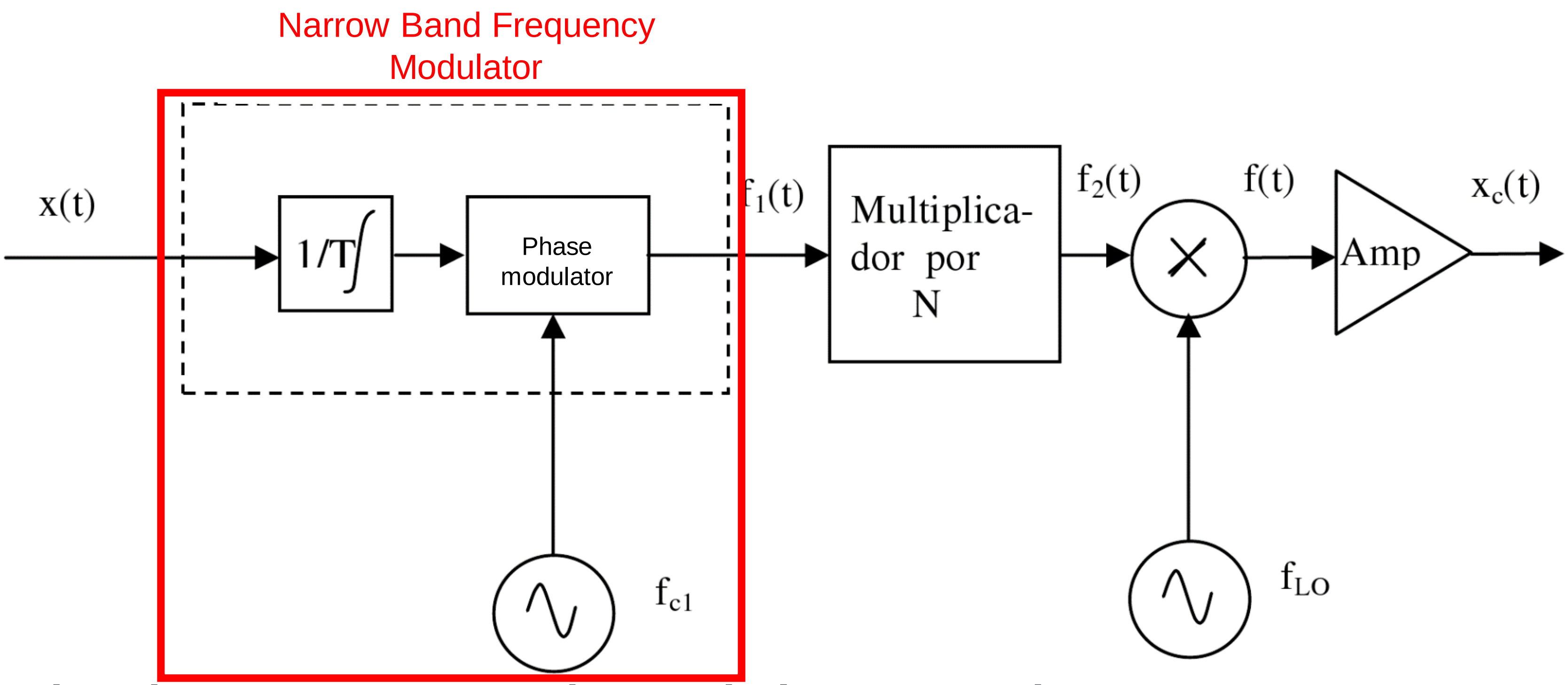
Indirect method of FM generation

- Although PM is rarely used, its modulators are interesting because:

- Its implementation is simple.

- The carrier can be obtained by means of a stable local oscillator (for example: a crystal). By integrating the message, it can be obtained at your FM output.



The integrator and modulator make up a narrowband FM modulator. This generates NBFM with :

$$f_1(t) = f_{c1} + \frac{\phi_\Delta}{2\pi T} x(t)$$

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Then, the instantaneous frequency at the output will be:

$$f(t) = f_c + f_\Delta x(t)$$

This conversion can even be done at any point in the multipliers in order to keep the frequency at each point in the system relatively low.

Likewise, only by changing the local oscillator, it is possible to relocate the carrier frequency.

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Frequency detection

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Frequency detection

A frequency detector, commonly called a discriminator, produces an output voltage that varies linearly with the instantaneous frequency of the input. There are many variations on detector circuits. frequency, but all can be classified into the following four categories:

1. FM to AM conversion.
2. Phase shift discriminators.
3. Zero crossing detection.
4. Frequency feedback.

Phase detectors will not be discussed, because they are rarely used, and if necessary they can be implement simply by integrating the output of an FM detector.

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A device or circuit that produces at the output, the time derivative of its input, produces FM to AM conversion. Specifically, if the input is an exponentially modulated wave:

$$x_c(t) = A_c \cos\theta_c(t) \tag{7.92}$$

$$\dot{\theta}_c(t) = 2\pi\left[f_c + f_\Delta x(t)\right] \tag{7.93}$$

$$\dot{x}_c(t) = -Ac\,\dot{\theta}_c(t) sen\theta_c(t) = \boxed{2\pi A_c\left[f_c + f_\Delta x(t)\right]} sen\left[\theta_c(t) \pm 180^\circ\right] \tag{7.94}$$

Expression 7.91 has a constant amplitude Ac that is not a function of time, but in Expression 7.94 it appears an amplitude as a function of time, which has the form of an amplitude modulation, with a modulation index f_Δ/fc . Therefore if we apply the waveform of Equation. 7.94 to an envelope detector, will give us an output proportional to f(t)=fc+f_Δx(t), that is, proportional to the message.

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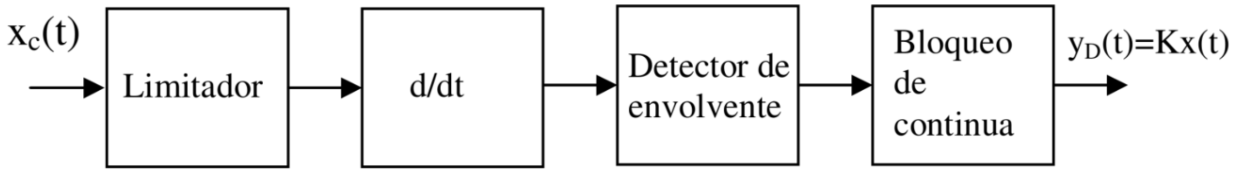
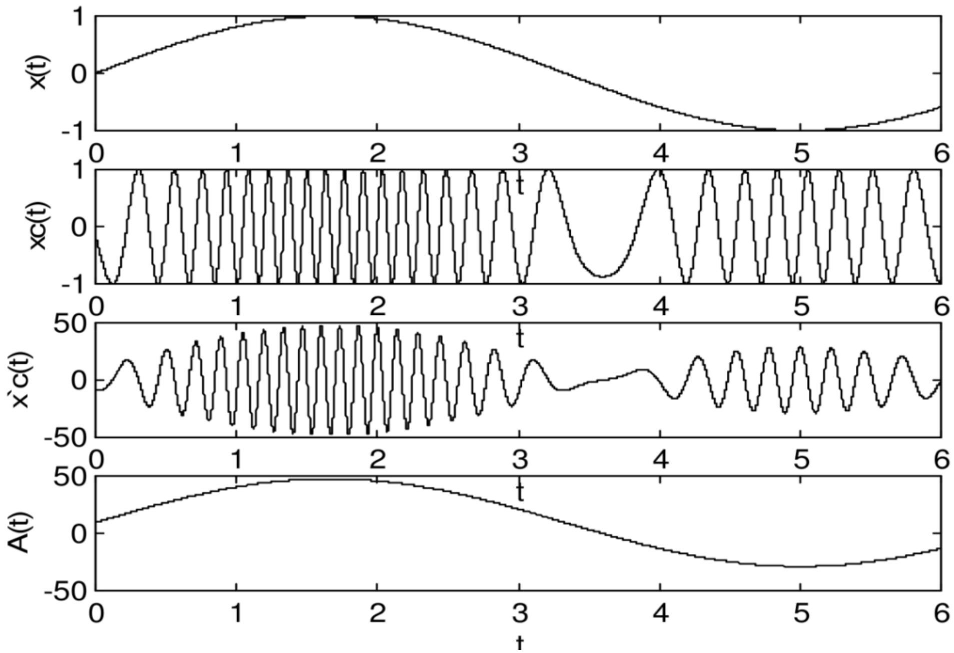


(a)



It is seen that the amplitude of the carrier Ac is a factor into the output in Equation 7.94. If Ac is constant, It will simply be a scale factor, but as a consequence of the nonlinearities that xc(t) surely went through in earlier stages, it will no longer be constant and it is necessary to use a limiter to eliminate the variations of amplitude.

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The quadrature or phase shift demodulator, in contrast to the previous ones that use a Amplitude linear transfer uses linear phase transfer circuitry. Use as a beginning approximation for the derivative with respect to time, given by:

$$\dot{v}(t) \approx \frac{1}{t_1}\left[v(t) - v(t - t_1)\right] \qquad (795)$$

That is, the quotient between the increment of the function and the increment of the variable, will give us the value of the slope at that point, provided the increment is very small, thus obtaining the derivative with respect to time.
An FM modulated wave has:

$$f(t) = \frac{1}{2\pi}\frac{\theta_c(t)}{dt} = f_c + \frac{1}{2\pi}\frac{d\phi(t)}{dt} = fc + f_\Delta x(t) \qquad (7.96)$$

$$\dot{\phi}(t) = 2\pi f_\Delta x(t) \qquad (7.97)$$

Using the approximation:

$$\boxed{\phi(t) - \phi(t - t_1) \approx t_1\,\dot{\phi}(t) = 2\pi f_\Delta t_1 x(t)} \qquad (7.98)$$

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Interference in linear and exponential modulation

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INTERFERENCE

With interference is meant the contamination of the signal with another signal, which generally originates from a system or device used by any user. An example may be when a receiving antenna takes two signals in the same frequency band, and that correspond to different stations. Another example is the multipath propagation, whereby two or more signals are received from the same antenna, but with amplitude and phase differences between them. Unlike noise, control of the source of an interference, it depends on human activity, while noise depends on natural causes, and cannot be governed. Of however, whatever its cause, severe interference can make message recovery impossible. We will begin our study with simple examples of interference caused by sinusoids, which represent unmodulated carriers, to see the differences between the effects of interference on AM, PM and FM.

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Interfering sinusoids.

Let us consider a receiver tuned to a carrier frequency fc, and whose received signal is formed by the wanted signal, in the form of an unmodulated carrier, and another term is the interfering carrier, with amplitude *Ai*, frequency *fc+fi* and a relative angle *Φi*

$$v(t) = A_c \cos\varpi_c t + A_i \cos[(\varpi_c + \varpi_i)t + \phi_i] = A_v(t)\cos[\varpi_c t + \phi_v(t)] \qquad (7.104)$$

We will introduce the following relationships:

$$\rho \equiv \frac{A_i}{A_c} \qquad \theta_i(t) \equiv \varpi_i t + \phi_i \qquad (7.105)$$

The sum of the terms of Eq. 7.104 can be expressed as the module and phase of a bandpass signal, as is shown in the phasor diagram of Fig. 7.20:

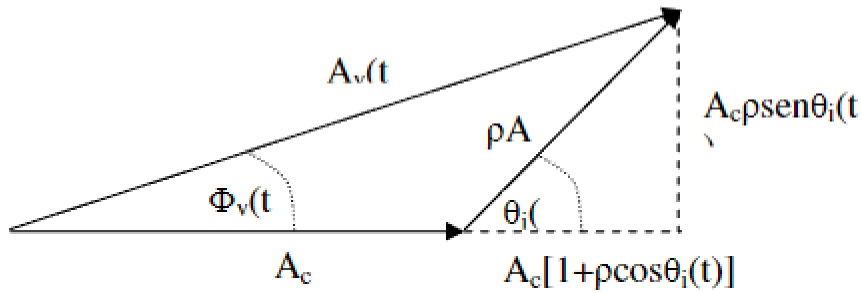

Fig. 7.20 Interference phasor diagram

$$\boxed{A_v(t) = A_c\sqrt{1+\rho^2+2\rho\cos\theta_i(t)} \qquad \phi_v(t) = \arctan\frac{\rho sen\,\theta_i(t)}{1+\rho\cos\theta_i(t)}}$$

These expressions show us that interference produces both amplitude modulation and frequency modulation. phase. In the case that *ρ <<1*, then:

$$A_v(t) \approx A_c\left[1 + \rho\cos(\varpi_i t + \phi_i)\right] \tag{7.109}$$

$$\phi_v(t) \approx \rho sen(\varpi_i t + \phi_i) \tag{7.110}$$

Therefore, the envelope contains amplitude modulation, which corresponds to a tone of frequency fi, that is, to the difference between the frequency of the carrier and that of the interference, with modulation index ρ, and in addition phase or frequency modulation, with index *β=ρ*.
At the other extreme, if ρ >> 1, then:

$$A_v(t) \approx A_i\left[1 + \rho^{-1}\cos(\varpi_i t + \phi_i)\right] \tag{7.111}$$

$$\phi_v(t) \approx (\varpi_i t + \phi_i) \tag{7.112}$$

The envelope still contains amplitude modulation, but the phase corresponds to a carrier shifted to *fc+fi*, plus a constant *Φi*.

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We will see what happens if this signal enters an ideal detector, which can be AM, FM or PM, with a constant K.D. We will take the case of weak interference (*ρ <<1*) and use the approximations of Eq. 7,109 and 7,110, with *Φi=0*. The detector output will be in each case:

$$y_D(t) \approx \begin{cases} A_v(t) = K_D(1+\rho\cos\varpi_i t) & AM \\ \phi_v(t) = K_D \rho sen \varpi_i t & PM \\ \dfrac{1}{2\pi}\dot{\phi}_v(t) = K_D \rho \underline{f_i} \cos\varpi_i t & FM \end{cases} \qquad (7.113)$$

As long as |*fi*| < *W*, otherwise the lowpass filter at the output of the detector will reject the frequencies at which |*fi*| > *W*. The constant term in the AM detector will be removed if there is continuous blocking at its output. Since $\Phi_i=0$ was assumed, the results for the envelope detector will hold for synchronous detection, for DSB and SSB.

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The Equation 7.113 shows that weak interference in a linear modulation system, or in a phase modulation, produces a spurious tone at the output, with amplitude proportional to *ρ=Ai/Ac*, independent of *fi*. But in FM the amplitude of the tone is proportional to ρfi so, if the interfering signal is in the same frequency than the carrier of the desired signal, with which fi=0, the spurious tone will not be produced at the output. By Therefore, FM is more vulnerable to adjacent channel interference, where fi is not zero. Fig. 7.21 shows this difference by plotting the amplitude of the demodulated interference versus | *fi*|. The crossing point corresponds to a frequency of 1 Hz. if all detectors have the same constant *KD*.

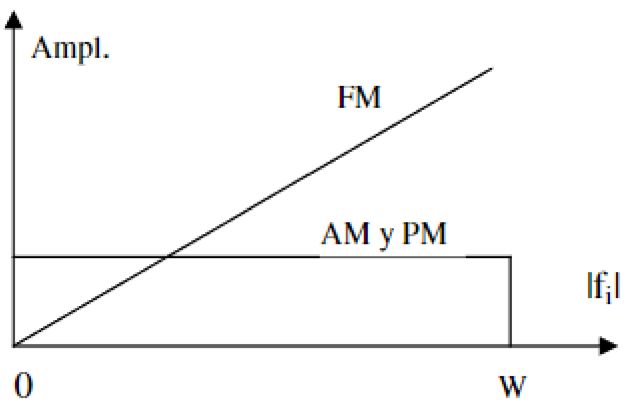

Fig. 7.21 Amplitude of demodulated interference from a carrier at fc+fi

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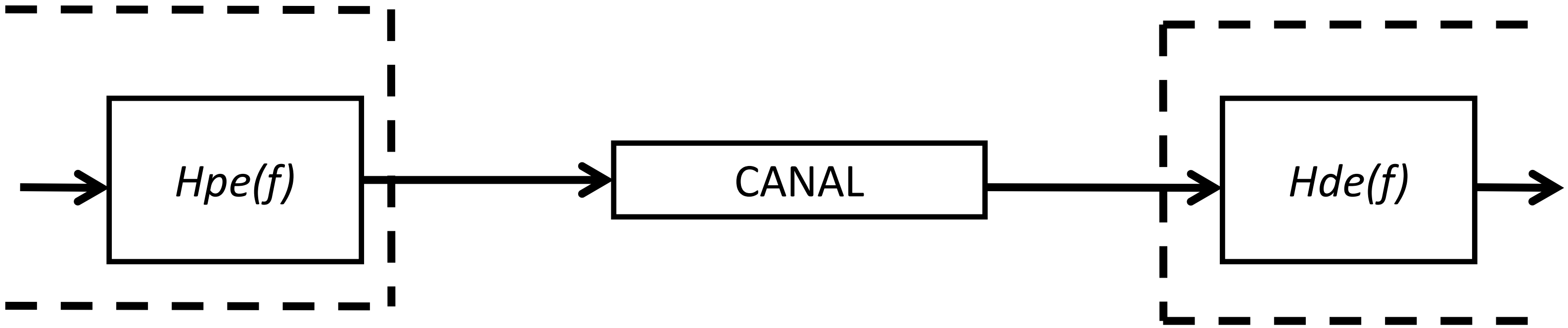
De-emphasis pre-emphasis filtering

The fact that the interference detected in FM is more severe for large values of | fi |, suggests a method to improve system performance, through selective post-detection filtering, called de-emphasis. Suppose that after the detector we place a low-pass filter, whose ratio of amplitudes begins to gradually decrease at frequencies below W; this will reduce the amplitudes of the frequencies higher, and therefore the interference. Obviously, it will also attenuate the high-frequency portion of the message, causing distortion to the output of the detected message. But this distortion can be compensated by means of a pre- distortion or pre-emphasis, in the signal that is sent to the transmitter, prior to its modulation. so that compensation between both operations, the de-emphasis and the pre-emphasis, either in a way that does not modify the message, must be fulfilled that:

$$H_{pe}(f) = \frac{1}{H_{de}(f)} \qquad |f| \leq W \qquad (7.114)$$



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Exponential modulation with noise

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8.4 Exponential modulation with noise

In this section we will see how FM and PM systems behave in the presence of noise. given nature nonlinear exponential modulation, which makes analysis difficult, we will start under conditions of large ratio signal-to-noise $(S/N)_R$ >>1, to determine the post-detection noise, and the signal-to-noise ratios of post detection

Generalized model:

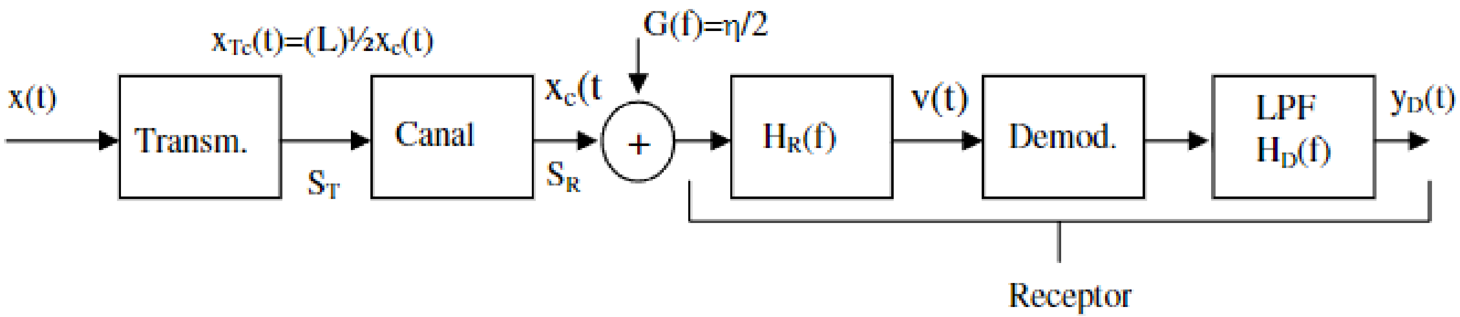

Figure 8.1 Model of the linear modulation system

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Post detection noise

The predetection portion of an exponentially modulated receiver has the structure shown previously in Fig. 8.4. The received signal is:

$$x_c(t) = A_c \cos[\varpi_c t + \phi(t)] \qquad (8.68)$$

Where $\phi(t) = \phi_\Delta x(t)$ for a PM wave, or $\dot{\phi}(t) = 2\pi f_\Delta x(t)$ for an FM wave. Anyways, the carrier amplitude remains constant, so:

$$S_R = \frac{A_c^2}{2} \qquad \left(\frac{S}{N}\right)_R = \frac{A_c^2}{2\eta B_T} \qquad (8.69)$$

The predetection signal to noise ratio of Equation. 8.69 is commonly known as the carrier to noise relation (CNR) for exponential modulation systems. It is assumed that the predetection filter $H_R(f)$ is pretty much ideal, it has B_T bandwidth and is fc centric.

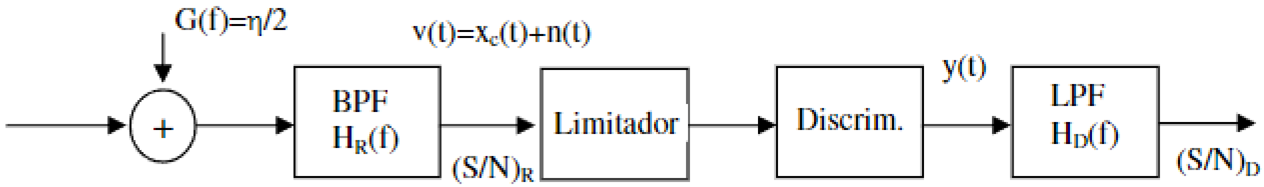

Fig. 8.9 Detection model for exponential modulation with noise.

In Fig. 8.9 you can see our detection model for exponential modulation with noise, highlighting the presence of the limiter block, which always precedes detection in modulation receivers exponential, and also, the denomination of the detector is changed by the discriminator, as it is known habitually. The input voltage to the discriminator is:

$$v(t) = x_c(t) + n(t) = A_v(t)\cos[\varpi_c t + \phi_v(t)] \qquad (8.70)$$

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The limiter suppresses any amplitude variation in $A_v(t)$ to find the signal and noise contained in $\Phi_v(t)$, we will express n(t) in the form of envelope and phase, and we will write:

$$v(t) = A_c(t)\cos[\varpi_c t + \phi(t)] + A_n(t)\cos[\varpi_c t + \phi_n(t)] \qquad (8.71)$$

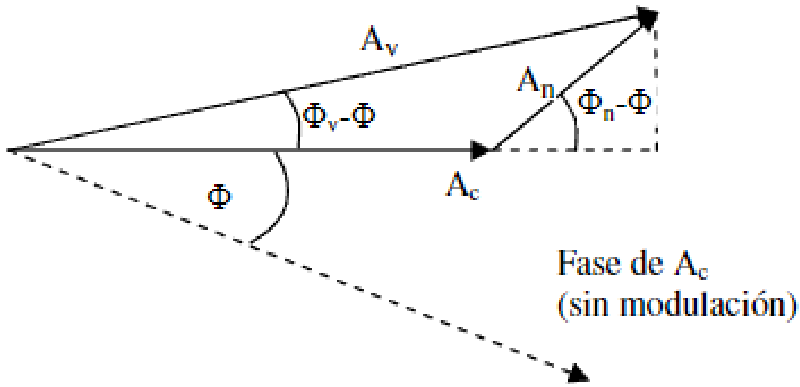

Fig. 8.10 Phasor diagram of exponential modulation with noise.

The construction of the phasor in Fig. 8.10 shows us that:

$$\phi_v(t) = \phi(t) + \arctan \frac{A_n(t) sen[\phi_n(t) - \phi(t)]}{A_c + A_n(t)\cos[\phi_n(t) - \phi(t)]} \quad (8.72)$$

The first term of Φv(t) is the phase of the signal itself, but the second term involves both the signal as to noise, and it is the one that contaminates the total phase, but we will not be able to advance without making some simplifications to the same. A logical simplification is that $(S/N)_R$ >> 1, so Ac >> An most of the time. We can use the small argument approximation for the arctangent, so we will plug in the arctangent function by its argument. A less obvious simplification ignores Φ(t) in the second term of the Equation 8.72, replacing (Φn(t)- Φ(t)) by Φ(t). We will justify this step for the purposes of noise analysis, remembering that Φn(t) has a uniform distribution over 2π radians. Therefore, the average in the ensemble (Φn(t)- Φ(t)) differs from Φn(t) only by a shift in the mean value. With these simplifications, Equation. 8.72 becomes:

$$\phi_v(t) \approx \phi(t) + \psi(t) \quad (8.73)$$

where we have decomposed the phase into two terms, one that depends only on the signal, and another that depends only from noise. We can then say that in large signal conditions, Ac >> An, signal and noise are additives. We then define the instantaneous phase of noise as:

$$\psi(t) \equiv \frac{An(t) sen \phi n(t)}{A_c} = \frac{1}{\sqrt{2S_R}} n_q(t) \quad (8.74)$$

$$\psi(t) \equiv \frac{An(t)sen\phi n(t)}{A_c} = \frac{1}{\sqrt{2S_R}} n_q(t) \tag{8.74}$$

With the replacement suggested by Equation. 8.69, we can see that the instantaneous phase of noise depends on the quadrature component of noise, and that decreases when the received signal increases. We will use these results to obtain the post-detection noise power in PM, for which we will assume that the input signal is not modulated, so the output of the detector is just noise. For it, we will make Φ(t)=0, and we will consider the resulting noise ψ(t) at the output of a phase discriminator, which in PM will be y(t)=Φv(t). The average power is then:

$$\overline{\psi^2} = \frac{\overline{n_q^2}}{2S_R} \tag{8.75}$$

The noise power spectrum has the form of $G_{nq}(f)$ in Fig 8.3(b), i.e. it corresponds to the symmetric case, but like what we have in Equation. 8.75 is the power of the quadrature component of noise, but divided by $2S_R$, the noise power spectrum must also be divided by $2S_R$. Then:

$$G_\psi(f) \approx \frac{\eta}{2S_R} \prod\left(\frac{f}{B_T}\right) \tag{8.76}$$

which is essentially flat over the frequency range $-B_T/2$ to $+B_T/2$, as shown in Fig. 8.11.

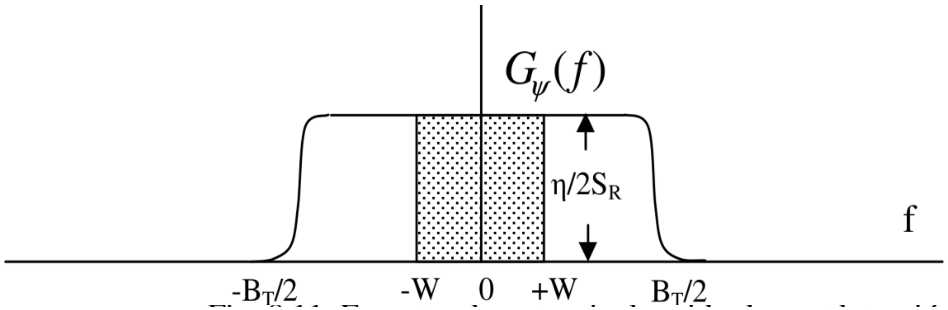

Fig.8.11. Post-detection noise power spectrum in PM

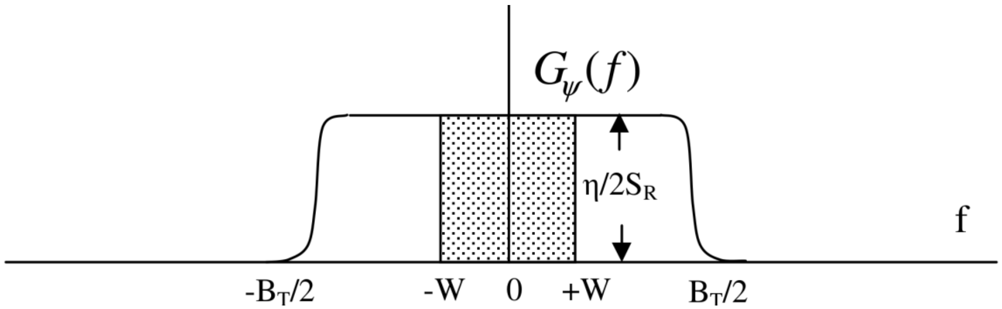

Fig.8.11. Post-detection noise power spectrum in PM

Since $B_T/2$ exceeds the bandwidth of the W message, except for the special case of NBPM, the receiver must include a post-detection filter $H_D(f)$ to remove the noise power that is out of band, i.e. by above W. If $H_D(f)$ is an ideal lowpass filter, with unity gain and width W, the noise power in the destination will be:

$$N_D = \int_{-W}^{+W} G_\psi(f)\,df = \frac{\eta W}{S_R} \tag{8.77}$$

The PM post-detection noise power increases with W and is inversely proportional to S_R. In Fig. 8.11, the shaded area is equal to N_D, the remainder is the excess noise power, which will grow as B_T is elderly.

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To obtain the FM post-detection noise power, we now consider the output of a frequency discriminator FM, whose input is:

$$\phi_v(t) = \psi(t) \tag{8.78}$$

That is to say that we continue considering only the noise, since we assume that there is no modulation. The discriminator of FM delivers a voltage proportional to the instantaneous frequency deviation of the input, so the output will be an instantaneous noise frequency, which we will define as:

$$\xi(t) \equiv \frac{1}{2\pi}\dot{\psi}(t) = \frac{1}{2\pi\sqrt{2S_R}}\dot{n}_q(t) \tag{8.79}$$

To obtain the FM noise power spectrum at the output of the discriminator, we will use the property that Relates the derivative of a function to its power spectrum:

$$y(t) = \frac{dx(t)}{dt} \Rightarrow G_y(f) = (2\pi f)^2 G_x(f) \tag{8.80}$$

This is the case of a transfer *H(f)=j2πf* in the frequency domain, which is equivalent to the derivation in the time domain.

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Using this property in Equation. 8.79, we can obtain the power spectrum of post-detection noise in FM

$$G_{\xi}(f)=(2\pi f)^2\left(\frac{1}{2\pi\sqrt{2S_R}}\right)^2 G_{n_q}(f)=\frac{f^2}{2S_R}G_{n_q}(f)=\frac{\eta f^2}{2S_R}\prod\left(\frac{f}{B_T}\right) \qquad (8.81)$$

This parabolic function is shown in Fig. 8.12, where it is seen to have components above W as in PM, but they are incremented with f_2.

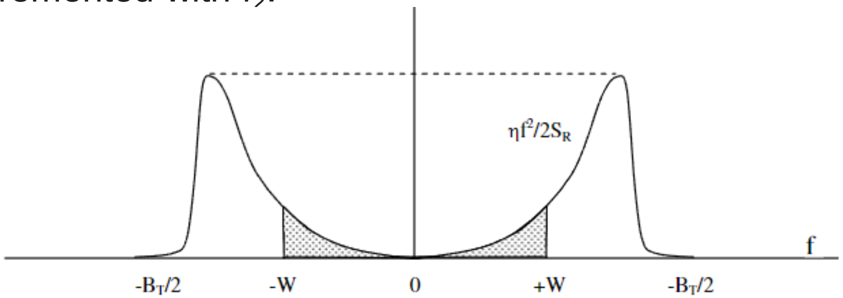

Fig.8.12. FM post-detection noise power spectrum

We are interested in allowing only the shaded part of Fig. 8.12 to pass through, which is the area where the message is contained. To do this, we use a low-pass filter that we assume to be ideal, that is, with unit transfer in the band of pitch, and rectangular width W. The noise power at the destination for FM will be:

$$N_D=\int_{-W}^{+W} G_{\xi}(f)df=\frac{\eta W^3}{3S_R} \qquad (8.82)$$

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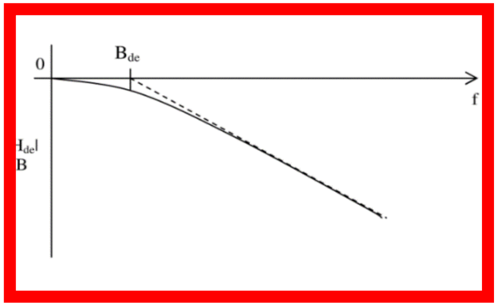
However, pre-emphasis and de-emphasis networks are commonly used in FM, since they allow improving the behavior of the system against interference and noise. In this case, we incorporate a filter with a transfer such that:

$$|H_D(f)| = |H_{de}(f)| \prod\left(\frac{f}{2W}\right) \tag{8.83}$$



The frequency of de-emphasis Bde is the one that gives the efficiency of the filtering, the lower, the more efficient the suppression, and the transfer is:

$$|H_{de}(f)| = \left[1 + \left(\frac{f}{B_{de}}\right)^2\right]^{-1/2} \tag{8.84}$$

Applying this transfer to obtain the post-detection noise power in FM with de-emphasis, we obtain:

$$N_D = \int_{-W}^{+W} |H_{de}(f)|^2 G_\xi(f)\,df = 2\int_0^{+W} \frac{f^2}{1+\left(\frac{f}{B_{de}}\right)^2} \frac{\eta}{2S_R}\,df \tag{8.85}$$

$$N_D = \frac{\eta B_{de}^{\ 3}}{S_R}\left[\left(\frac{W}{B_{de}}\right) - \arctan\left(\frac{W}{B_{de}}\right)\right] \tag{8.86}$$

In the usual case that $W/B_{de} \gg 1$ a simplification can be made, since from $\tan^{-1}\left(\frac{W}{B_{de}}\right) \approx \frac{\pi}{2} \ll \frac{W}{B_{de}}$ with which:

$$N_D \approx \frac{\eta B_{de}^2 W}{S_R} \tag{8.87}$$

Which is the post-detection noise power in FM with de-emphasis, when W >> Bde.

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Summarizing the results obtained so far, we have:

1) Post-detection noise in FM and PM have components outside the message band, which make post-detection filtering is necessary. In linear modulation, on the other hand, the $H_R(f)$ filter made it unnecessary post-detection filtering.

2) The FM post-detection noise power spectrum has a parabolic shape, while the spectrum post-detection noise power in PM is flat. Consequently, the higher frequencies of the message suffer more contamination in FM than the lower frequencies. De-emphasis filtering in FM, compensates for this effect, provided pre-emphasis filtering has been used prior to the FM transmitter.

3) The average power of post-sensing noise N_D, both in PM and FM, decreases when S_R increases. As S_R is increased, the detected noise is squelched. Theoretically, if S_R were zero, the noise power N_D according to Equation 8.82 would be infinite, but since we started from the Assuming that $S_R >> N_R$, these expressions are not valid for the limit of this analysis. In fact, the output of the limiter will set the maximum input level to the discriminator, and this will be the maximum available power of detected noise. Being this value so high, it is necessary to silence this noise when there is not enough signal for it. A circuit called squelch is usually used, which compares the detected level of noise power above W, with an adjustable value of continuous, and decides, according to whichever is greater, if the detector output should be cut off.

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Signal to Noise Ratio at the destination

Let's calculate the signal-to-noise ratio at the destination, for PM, FM, and FM with de-emphasis. We continue with the condition that $S_R >> N_R$, so that previously obtained results are still valid. The presence of a phase that corresponds to the signal Φ(t) at the input does not invalidate it, since if instead of only Φn(t) we would have considered Φn(t) - Φ(t) in Eq. 8.72, a more complicated analysis would have shown additional components in the noise power spectrum, which fall beyond W, so they will be rejected by $H_D(f)$.
In a PM system, the demodulated signal plus noise will be:

$$y(t) = \phi_v(t) = \phi(t) + \psi(t) = \phi_\Delta x(t) + \psi(t) \qquad (8.88)$$

The post-detection filter will pass the term that corresponds to the signal in its entirety, so the root mean square value of it will give us S_D:

$$S_D = \overline{\phi_\Delta^2 x^2(t)} = \phi_\Delta^2 S_x$$

Note that unlike linear modulation, the increase in transmitted signal power S_T does not increase the output of the discriminator, but decreases the noise power, and by this means, improves the signal to signal ratio. noise at the destination.

$$\left(\frac{S}{N}\right)_D = \frac{S_D}{N_D} = \frac{S_D}{\overline{\psi^2(t)}} = \frac{\phi_\Delta^2 S_x}{\left(\frac{\eta W}{S_R}\right)} = \phi_\Delta^2 S_x \frac{S_R}{\eta W} = \phi_\Delta^2 S_x \gamma$$

Since the parameter γ is equal to the output S/N of a baseband system that has the same bandwidth W, equal signal power S_R, and equal noise spectral density η, we see that a PM system provides an improvement of Φ_Δ2Sx with respect to base band. But since Φ_Δ must be less than π, to avoid ambiguity, the maximum value of Φ_Δ2Sx will be π2, which represents approx. 10 dB. at most. This is an extreme limit, usually Φ_Δ2Sx < 1, with which the performance is lower than the base band, while the B_T bandwidth ≥ 2W.

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Moving to an FM system, the signal plus the demodulated noise will be proportional to the instantaneous frequency:

$$y(t) = \frac{1}{2\pi}\dot{\phi}_v(t) = \frac{1}{2\pi}\dot{\phi}(t) + \frac{1}{2\pi}\dot{\psi}(t) = f_\Delta x(t) + \xi(t) \tag{8.91}$$

We obtain the average power of the output, taking into account that the post-detection filter will be of width W:

$$\overline{y^2(t)} = \overline{[f_\Delta x(t)]^2} + \overline{\xi^2(t)} = S_D + N_D = f_\Delta^2 S_x + N_D \tag{8.92}$$

Replacing ND by Equation. 8.82, we finally obtain the FM signal-to-noise ratio:

$$\left(\frac{S}{N}\right)_D = \frac{f_\Delta^2 S_x}{\left(\frac{\eta W^3}{3S_R}\right)} = 3\left(\frac{f_\Delta}{W}\right) S_x \frac{S_R}{\eta W} = 3D^2 S_x \gamma \tag{8.93}$$

We have used the deviation ratio, $D=f_\Delta/W$, whose value increases the S/N squared, without the need for increase S_T. According to this, we could make the S/N as big as we want, without increasing S_T, with just increase D. We will see later that this is not exact. Meanwhile, let us remember that the required transmission bandwidth B_T increases with the deviation D.

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With small deviation ratios, the breakpoint compared to baseband occurs when in the Equation 8.93 the factor 3D2Sx is equal to one, which implies D ≈ 0.577 with Sx=1. For this reason, it is often called this value as the boundary between NBFM and WBFM.
If de-emphasis filtering is now included, we can obtain the de-emphasized FM signal-to-noise ratio, which can be simplified in its derivation, if we adopt the case where Bde << W, with which the noise output is reduces according to Eq. 8.78. Substituting, we get:

$$\left(\frac{S}{N}\right)_D = \frac{f_\Delta^2 S_x}{\frac{\eta B_{de}^2 W}{S_R}} = \frac{f_\Delta^2}{B_{de}^2} S_x \gamma \qquad (8.95)$$

Which tells us that the signal-to-noise ratio with de-emphasis provides an improvement factor with respect to FM without de-emphasis, of approx. 1/3(W/Bde)2 as can be verified:

$$\frac{\left|\frac{S}{N}\right|_{FM\ con\,de-énfasis}}{\left|\frac{S}{N}\right|_{FM\ \sin de-énfasis}} = \frac{\frac{f_\Delta^2}{B_{de}^2} S_x \gamma}{3D^2 S_x \gamma} = \frac{W^2}{3B_{de}^2} \qquad (8.96)$$

This improvement with de-emphasis can be obtained with a filtering prior to transmission (pre-emphasis) and another in the detection, (de-emphasis) but can bring a hidden penalty. If the spectrum widths of the message do not have a drop of at least 1/f from Bde, the pre-emphasis will increase the deflection ratio and with it, the transmission bandwidth.

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Threshold effect

The small signal condition $(S/N)_R << 1$ can be represented by a diagram like the one in Fig. 8.10, where the signal and noise phasors have been reversed. Under these conditions, as An(t) >> Ac during the largest part of the time, the phase of the resultant at the detector input will be:

$$\phi_v(t) \approx \phi_n(t) + \frac{A_c}{A_n(t)} sen[\phi(t) - \phi_n(t)] \qquad (8.97)$$

The dominant now is noise, and the message, contained in Φ(t), has been mutilated, unable to recover the message. Actually, a major mutilation starts when $(S/N)_R \approx 1$ ie An ≈ Ac modulo. With phasors of practically equal length, we have a situation similar to that of co-channel interference, when ρ=Ai/Ac ≈ 1. Small noise variations can produce spikes in the output of the FM discriminator. He The phasor diagram in Fig. 8.13(a) illustrates this point, taking Φ(t)=0 and Φn(t) ≈ -π, so that Φv(t) ≈ -π. If the variations of An(t) and Φn(t) follow the dashed line between t_1 and t_2, then $\Phi v(t_2) \approx +\pi$. Correspondingly, the phase waveform, in Fig. 8.13(b), has a step height 2π and the output of the discriminator (which corresponds to the step derivative), has a peak in the interval t_1-t_2, during the phase transition from –π up to +π. The area under the curve of the peak has a value of one, so if the interval t_1-t_2 is small, the amplitude of the peak will be very high Since the noise voltage is random, the peaks will have random duration and amplitudes, and when An(t) ≈ Ac, they will be heard as crackles that mask the message.

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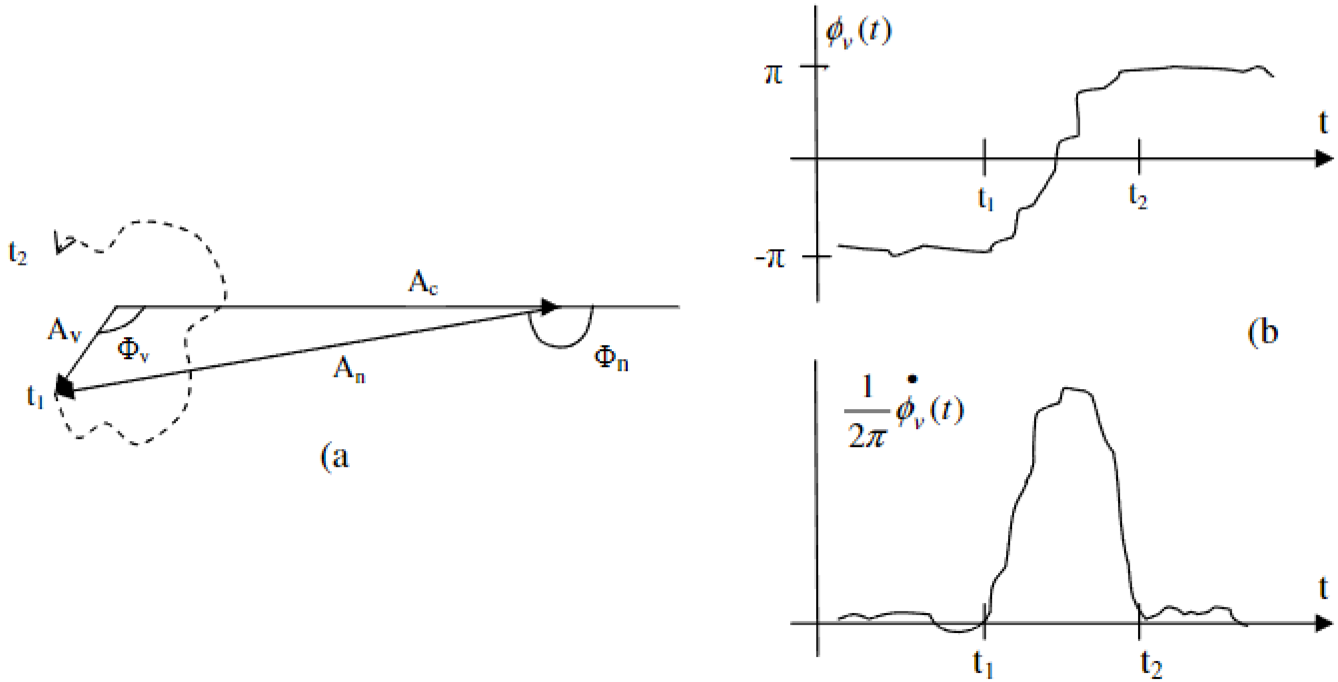


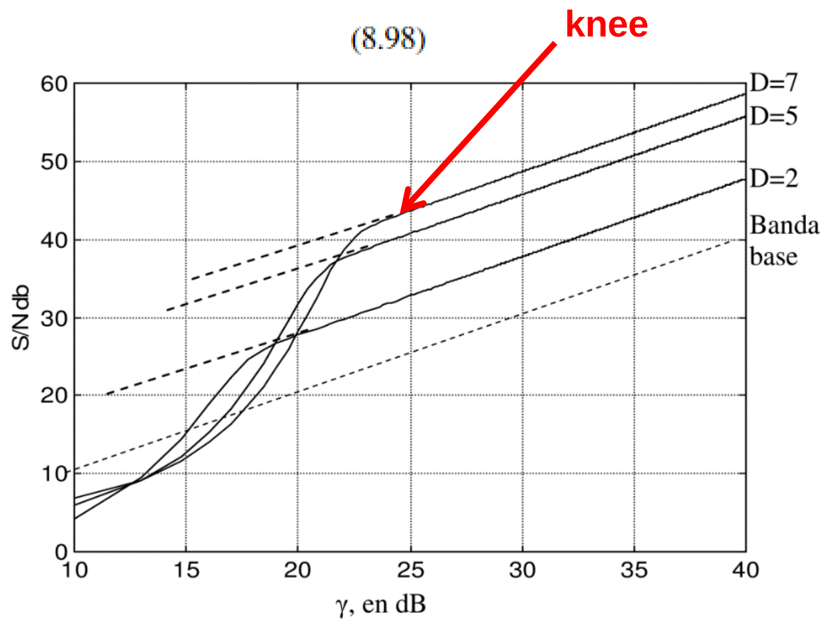
Fig.8.13. FM close to threshold. (a) Phasor diagram (b) Phase and instantaneous frequency.

We can infer with the previous qualitative analysis that the spectral distribution of noise is no longer parabolic, but it tends to fill out the lower frequencies. Without deducting it, we will say that if we take into account the effect of the peaks, for a modulation with pitch, the total noise power is:

$$N_D = \frac{\eta W^3}{3S_R}\left[1 + \frac{12D}{\pi}\gamma e^{-(W/B_T)\gamma}\right] \tag{8.97}$$

Where the second term is the contribution of the peaks. Fig. 8.14 shows the destination S/N ratio, for FM without pre-emphasis, versus γ, both in decibels. Three deviation values D=2.5 and 7 are shown, and the baseband curve as a reference. The three S/N curves were plotted using the noise power expression given by Eq. 8.97. It can be seen that each of them presents two well-differentiated zones: a zone linear, where the increase in γ (for example by means of an increasing of S_R) corresponds to an increase of the same magnitude in S/N; and another area where the S/N grows rapidly with γ, with a steeper slope. Each zone corresponds to the two terms of the square brackets of Equation. 8.97; the unit, is the linear zone, where this is the one with the greatest weight, for values of γ makes the exponential much less than one. For lower values of γ, the term of the exponential is the one with the most weight, and the variation is of the exponential type. Between both areas there is an “knee" or transition, in which small changes in the amplitude of the received signal cause large changes in the S/N and therefore, in the signal output. Above this knee, signal dominates detection, below noise captures output, as in the case of heavy co-channel interference. We will call threshold the value that locates the system above the elbow, so that the expressions derived under the condition $S_R >> N_R$ are valid. Experimental studies showed that, for the cases of interest, if $S_R /N_R>10$, mutilation of the noise message. Accordingly, we will define the threshold point as:

$$\left(\frac{S}{N}\right)_{R_{UMB}} = 10 \tag{8.98}$$



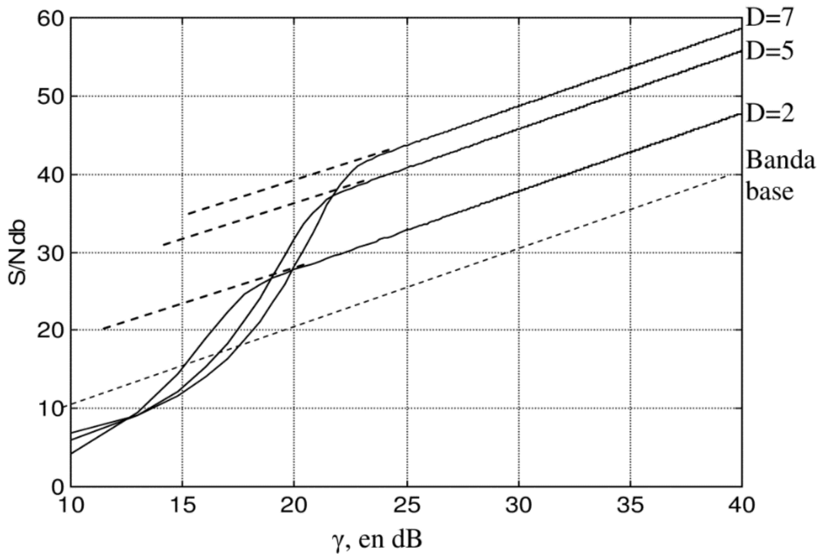


Fig. 8.14 Noisy FM performance (no pre-emphasis)

As an example, let us see in the case of commercial FM broadcasting what value γ has in the threshold. As we saw, the transmission bandwidth is 14 times greater than the audio bandwidth (BT=14W), so the N_R predetection noise power will be 14 times larger than that of baseband, for the same density noise spectral. As we define that $S_R=10N_R$ in the limit will be $\gamma_{THR}=140$, or in dB $\gamma_{THR}=21.4$dB. This point defines in Fig. 8.14 the limit of validity of the S/N expression of Equation 8.93 in FM. For values of γ older, Eq. 8.93 will give a valid result, while for smaller values (corresponding to the line of lines of the curves of Fig. 8.14), will not be valid. Furthermore, due to the mutilation of the message, no it makes sense to speak of the S/N ratio already, since the message cannot be recovered. We see that the exchange of bandwidth for transmission power has a condition, and that it cannot unrestrictedly increase the S/N by increasing the deviation D. If, for example, we locate ourselves in the value of γ of 21.4dB given above, and we intend to increase S/N as predicted by Eq. 8.93 increasing D, for example from 5 to 7, keeping γ constant, places us in the area of the dashed line of the D=7 curve. Then, it cannot in this case increase S/N without increasing γ, that is to say the received power S_R.

Starting from the definition of Eq. 8.98, we can make the following substitutions:

$$\left(\frac{S}{N}\right)_R = \frac{S_R}{\eta B_T} = \frac{S_R}{\eta B_T}\frac{W}{W} = \left(\frac{W}{B_T}\right)\gamma \qquad (8.99)$$

When (S/N)R=10, γ will correspond to the threshold:

$$\gamma_{UMB} = 10\frac{B_T}{W} \approx 20(D+2) \qquad D>2 \qquad (8.100)$$

In which we have used the expression for the FM bandwidth for a tone, when D > 2. If D is replaced by Φ_Δ, Eqs. 8.99 and 8.100 will be valid for PM. In view of the considerations made about the threshold point, it is useful to calculate the $(S/N)_D$ at that point.

$$\left(\frac{S}{N}\right)_{D_{UMB}} = 3D^2 S_x \gamma_{UMB} \approx 60D^2(D+2)S_x. \quad D>2 \qquad (8.101)$$

Which is equal to the minimum value of $(S/N)_D$ as a function of D. Given a value of $(S/N)_D$, if there are no constraints of bandwidth, you can solve Eq. 8.101 to find the value of the deviation D that allows obtain the most efficient performance in terms of signal power, for an FM system without pre-emphasis. Us It allows us to locate ourselves at the threshold, and therefore, use the minimum signal power to obtain a given $(S/N)_D$. Of course, for a reliable operation of the system, a certain margin must be allowed for possible signal fades.

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