Communication Systems based on Software Defined Radio (SDR)

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Digital Communication Systems

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Bandpass digital transmission.

The transmission of signals through a radio frequency link or another type of transmission of greater complexity, **requires the use of modulation** as a transmission expansion technique, which basically consists of the application of information about some of the properties of a high-frequency signal, generally called a carrier.

$$A\cos(2\pi f_c t + \theta)$$

$$It\ is\ possible\ to\ modulate\ by\ modifying \begin{cases} A \\ \theta \\ f_c \end{cases}$$

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Bandpass digital transmission.

The information applied on the carrier is generally classified as binary or M-ary. The baseband format modifies the carrier digitally, sending it in a discrete set of possible signals.

$$\begin{cases} Binary\ modulation\ M = 2 \\ \\ M - ary\ modulation\ M = 2^n \end{cases}$$

If the signals sent are two different waveforms, the modulation is binary. If the set of possible waveforms is M<2 signals (Generally $M = 2^n$), the signaling is called M-ary.

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Bandpass digital transmission.

En este sentido la información parece ser presentada como un "switching" o selección discreta entre diferentes opciones, que son en definitiva formas de onda que se transmiten sobre el canal.

Esta selección entre diferente formas de onda suele denominarse en ingles " Shift keying".

$$\text{formas de modulación digital} = \begin{cases} \boldsymbol{ASK}, & \text{Amplitude Shift Keying} \\ \boldsymbol{PSK}, & \text{Phase Shift Keying} \\ \boldsymbol{FSK}, & \text{Frequency Shift Keying} \end{cases}$$

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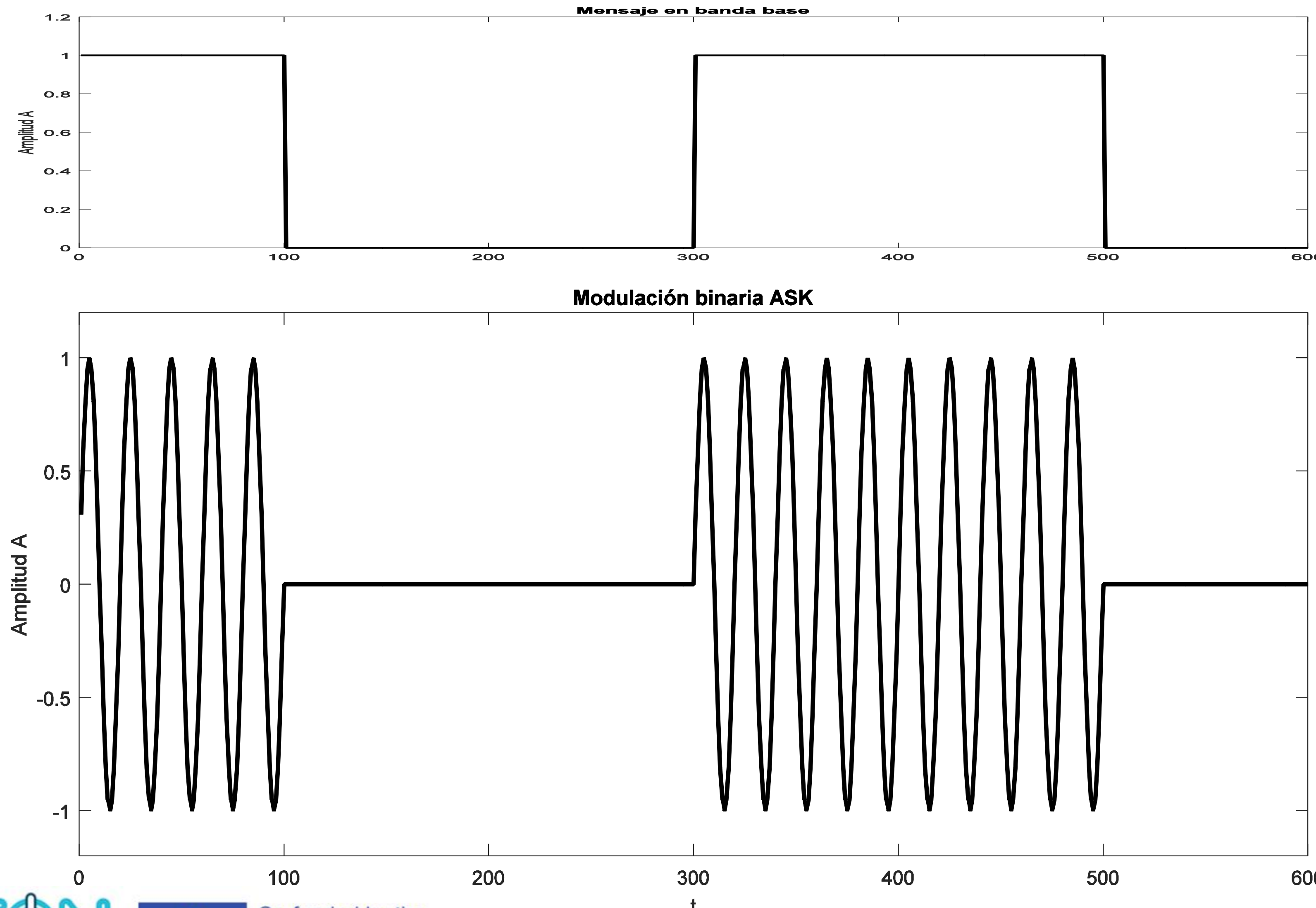
Binary ASK (Amplitud Shift Keying).



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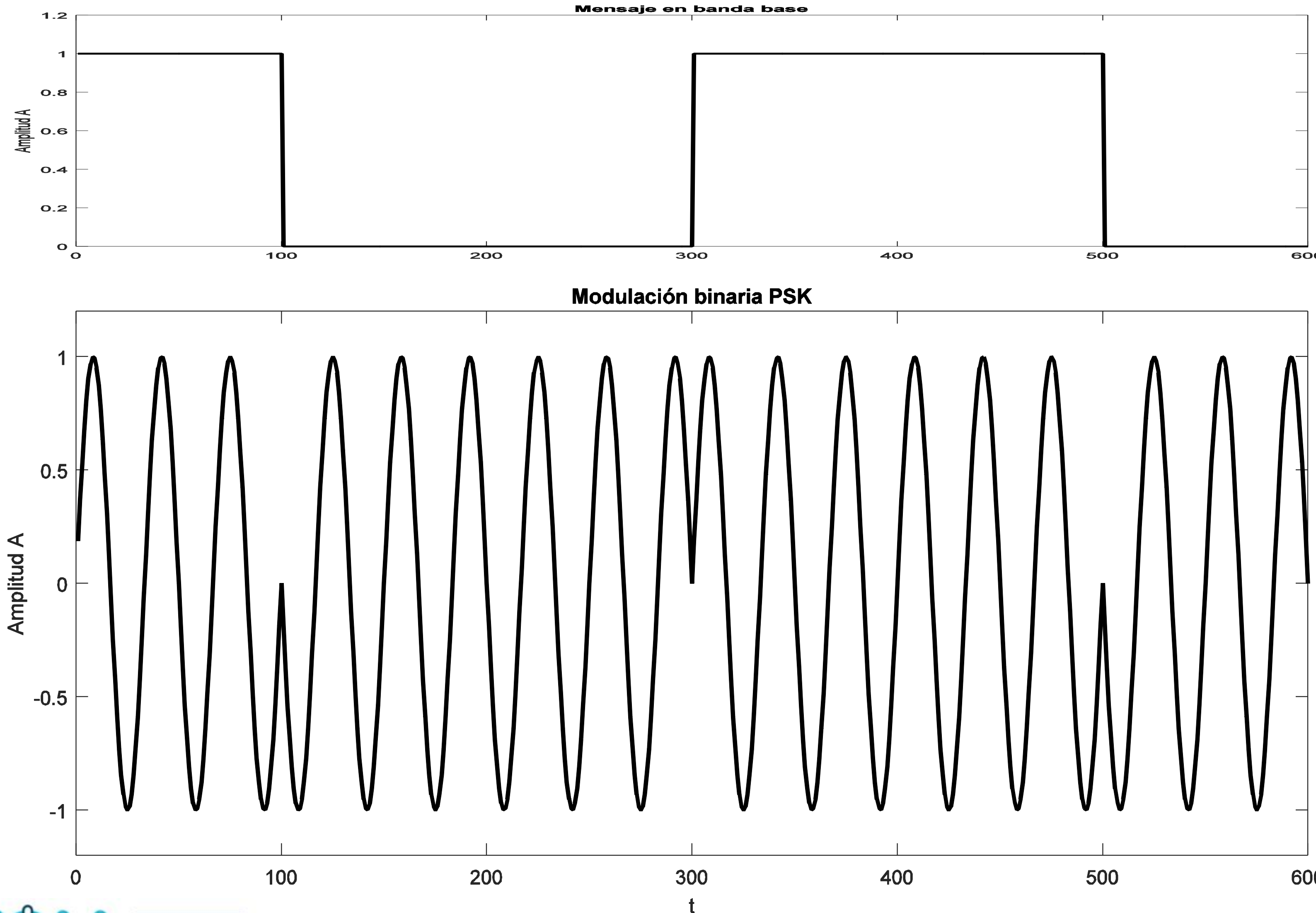
Binary PSK (Phase Shift Keying).



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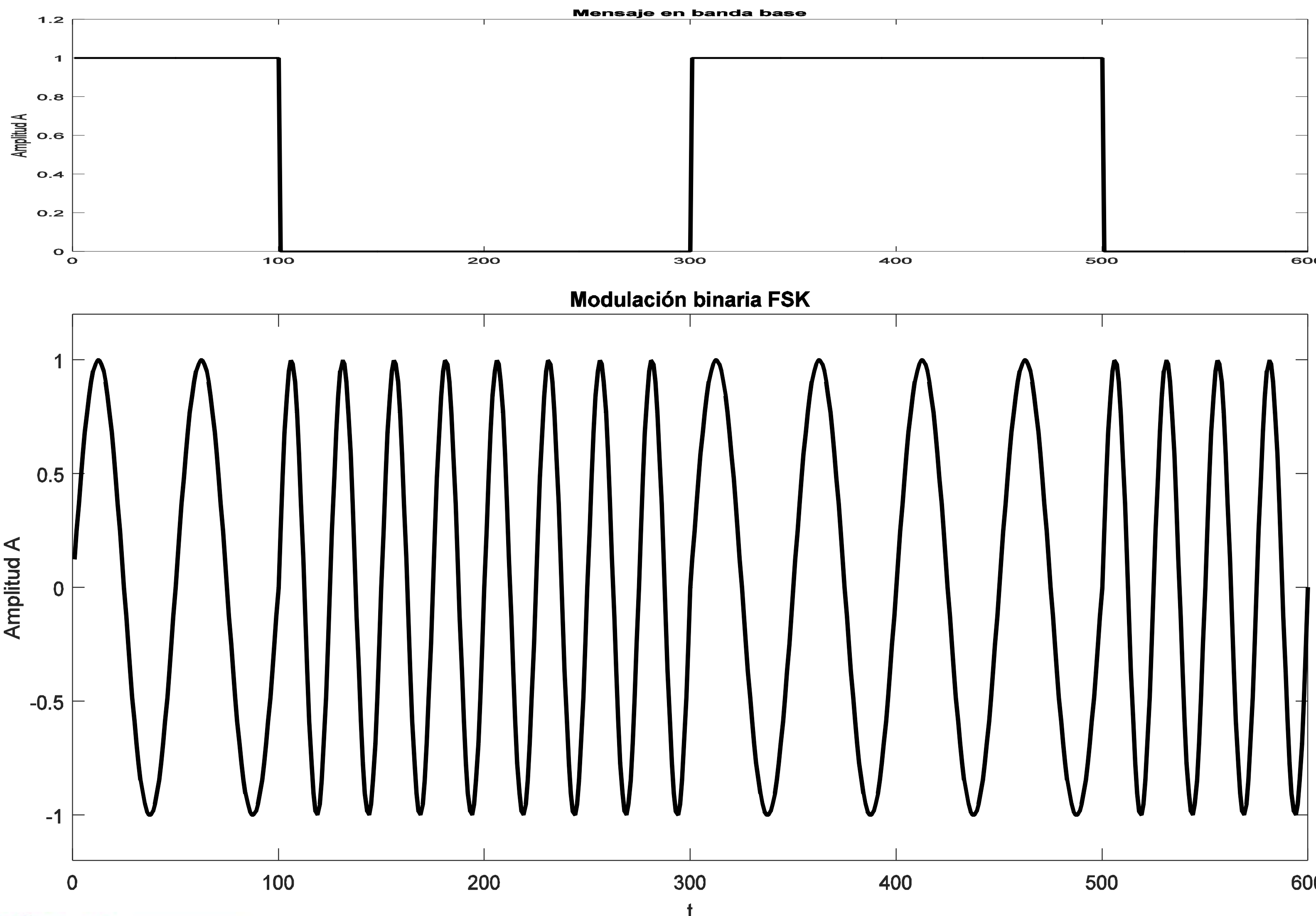
Binary FSK (Frequency Shift Keying).



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Spectral analysis of bandpass digital signals.

A modulated bandpass signal can be expressed in the form of component in phase and quadrature:

$$x_c(t) = A_c\left[x_i(t)\cos(\omega_c t + \theta) - x_q(t)\, sin(\omega_c t + \theta)\right]$$

The information is applied to the components in phase and quadrature, $x_i(t)$ and $x_q(t)$, respectively.

In general, the in-phase and quadrature components are independent random variables, which means that the cross-correlation of these variables is equal to zero:

$$R_{x_i,x_q} = E\left[x_i(t)x_q(t)\right] = E[x_i(t)]\, E\left[x_q(t)\right]$$

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Spectral analysis of bandpass digital signals.

As a consequence of this property:

$$\overline{\left(x_i + x_q\right)^2} = \overline{x_i{}^2} + \overline{x_q{}^2}$$

The modulated signal can be modeled as the product of the signal by a cosine function:

$$z(t) = x(t)\cos(\omega_c t + \theta)$$

And, the autocorrelation of a function modulated by a random cosine function is:

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Spectral analysis of bandpass digital signals.

$$G_z(f) = \frac{1}{4}[G_x(f - f_c) + G_x(f + f_c)]$$

And, the signal

$$x_c(t) = A_c\left[x_i(t)\cos(\omega_c t + \theta) - x_q(t)\, sin(\omega_c t + \theta)\right]$$

$$G_c(f) = \frac{{A_c}^2}{4}\left[G_i(f - f_c) + G_i(f + f_c) + G_q(f - f_c) + G_q(f + f_c)\right]$$

The spectral densities $G_i(f)$ and $G_q(f)$ are the spectral components of $x_i(t)$ and $x_q(t)$.

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Spectral analysis of bandpass digital signals.

The equivalent lowpass spectrum can be defined as:

$$G_{lp}(f) = G_i(f) + G_q(f)$$

Then:

$$G_c(f) = \frac{{A_c}^2}{4}\left[G_{lp}(f - f_c) + G_{lp}(f + f_c)\right]$$

The bandpass spectrum is expressed as a function of the lowpass equivalent.

Spectral analysis of bandpass digital signals.

For the signal:

$$x(t) = \sum_k a_k p(t - kD)$$

Which has the power spectral density presented below:

$$G_x(f) = \frac{{\sigma_a}^2}{D} |P(f)|^2 + \frac{m_a}{D} \sum_{n=-\infty}^{\infty} \left|P\left(\frac{n}{D}\right)\right|^2 \delta\left(f - \frac{n}{D}\right)$$

For the analysis of the modulation schemes, it is considered that the pulse shape $p(t)$ is rectangular and begins at the sampling instant kD. Then:

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Amplitude modulation M-ASK.

The binary form of digital amplitude modulation, 2ASK, consists of turning a carrier on and off, a modulation that is also called OOK (On-Off Keying).

When the baseband waveform is an M-ary signal, $M-1$ amplitudes and the zero level are generated. In this case there is no phase modulation, so all the modulation component is represented in the in-phase component, the quadrature component being equal to zero.

$$x_i(t) = \sum_k a_k p(t - kD)$$

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Amplitude modulation M-ASK.

In order not to incorporate phase modulation, the modulation amplitudes are all positive (unipolar format):

$$a_k = 0, 1, 2, 3, \dots, M-1$$

It can be seen that the mean value and root mean square can be obtained as:

$$m_a = \overline{a_k} = \frac{1}{M}\sum_{i=0}^{M-1} i = \frac{M-1}{2}$$

$$\overline{{a_k}^2} = \frac{1}{M}\sum_{i=0}^{M-1} i^2 = \frac{2M^2 - 3M + 1}{6}$$

$$\sigma_a^{\,2} = \overline{{a_k}^2} - {m_a}^2 = \frac{M^2 - 1}{12}$$

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Amplitude modulation M-ASK.

The equivalent lowpass spectrum is that of the PAM signal with the power spectral density $G_x(f)$, which is shifted to the carrier f_c:

$$G_{lp}(f) = G_i(f) = \frac{M^2 - 1}{12r} sinc^2\left(\frac{f}{r}\right) + \frac{M^2 - 1}{4}\delta(f)$$

$$G_c(f) = \frac{{A_c}^2}{4}\left[G_{lp}(f - f_c) + G_{lp}(f + f_c)\right]$$

For example a sequence of binary data modulates a signal in the form ASK (OOK) was shown at the beginning. A sequence of M-ary modulates a signal in the form 8ASK shown in the figure

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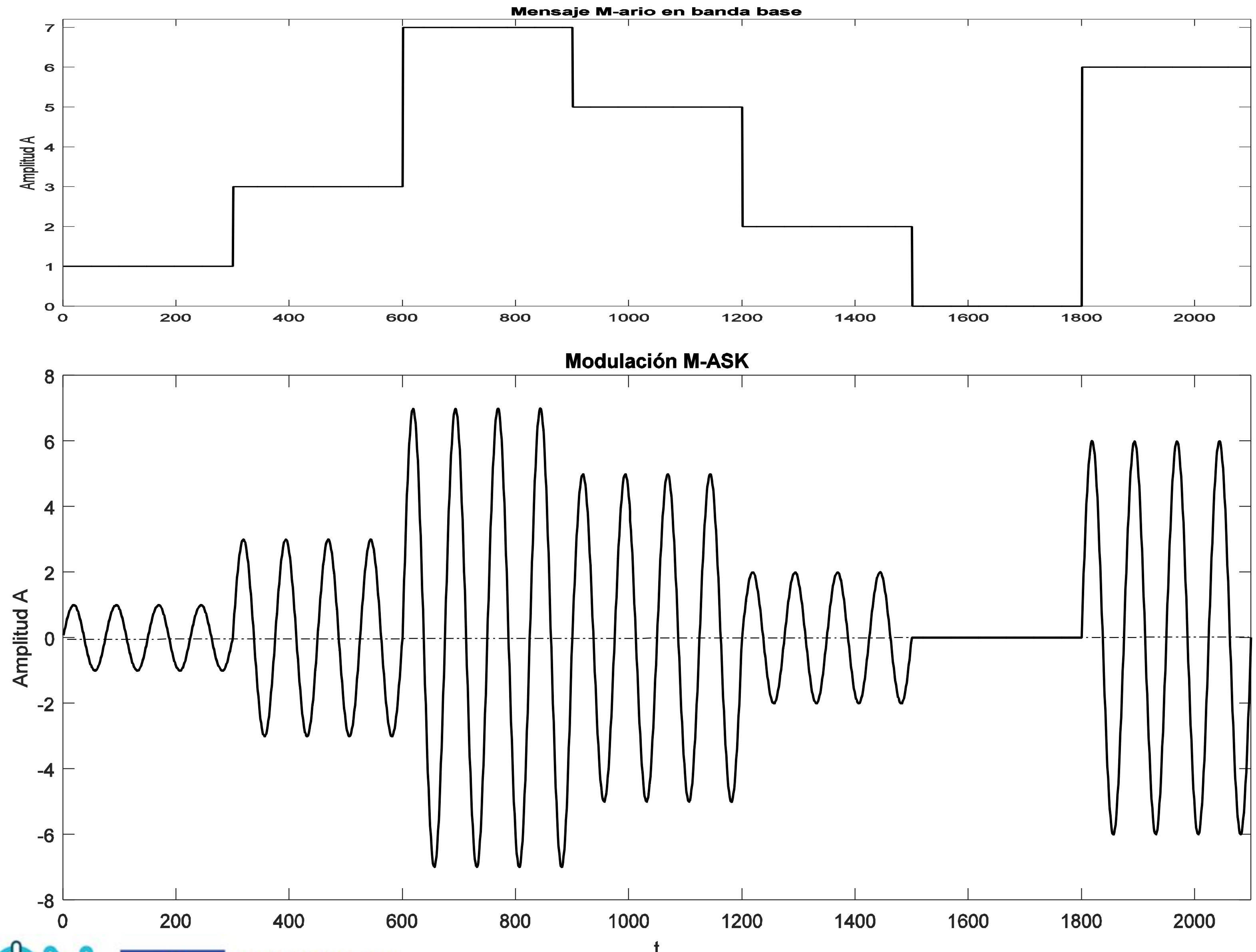
Amplitude modulation M-ASK.



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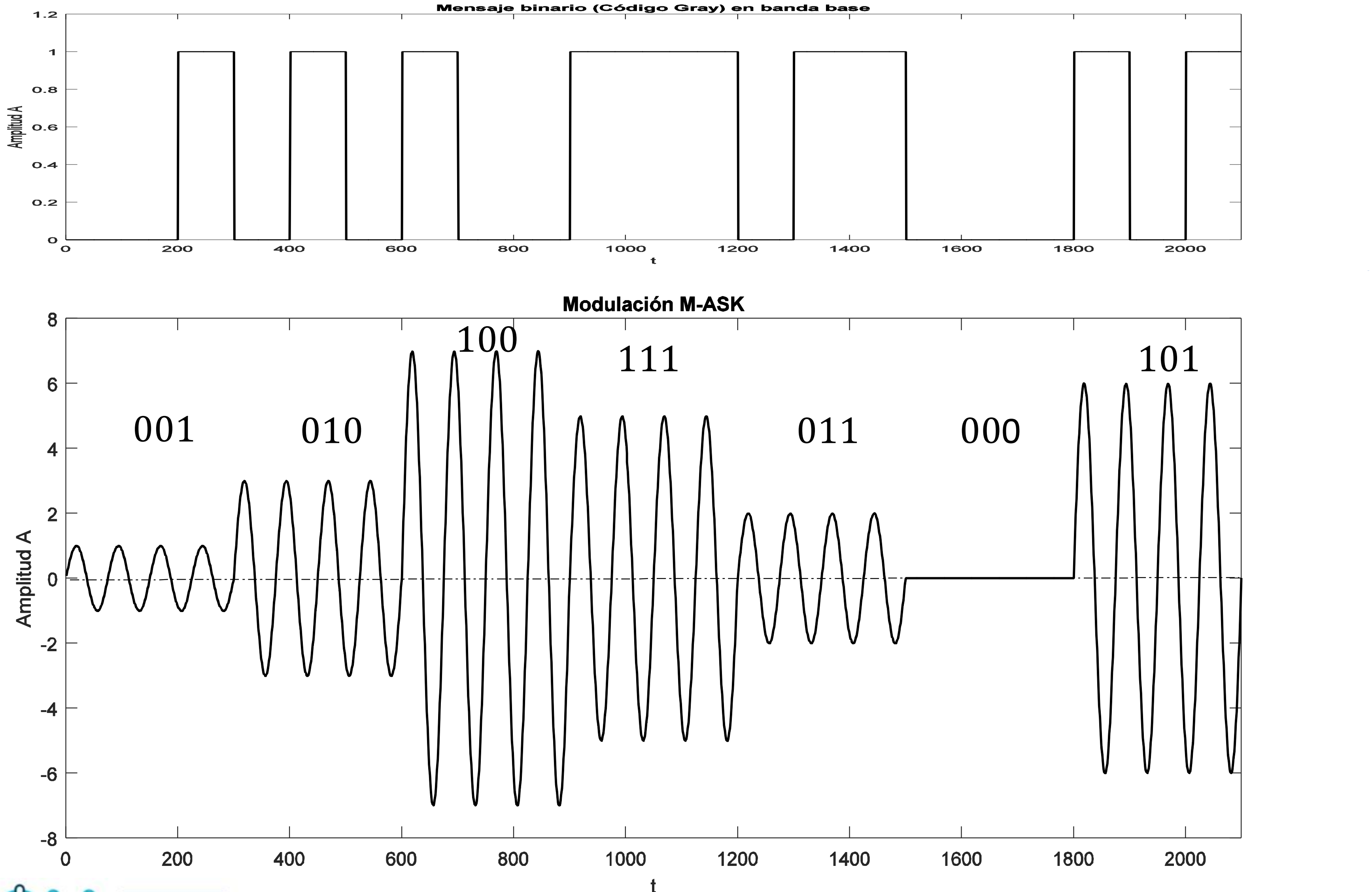
Amplitude modulation M-ASK. Gray code.



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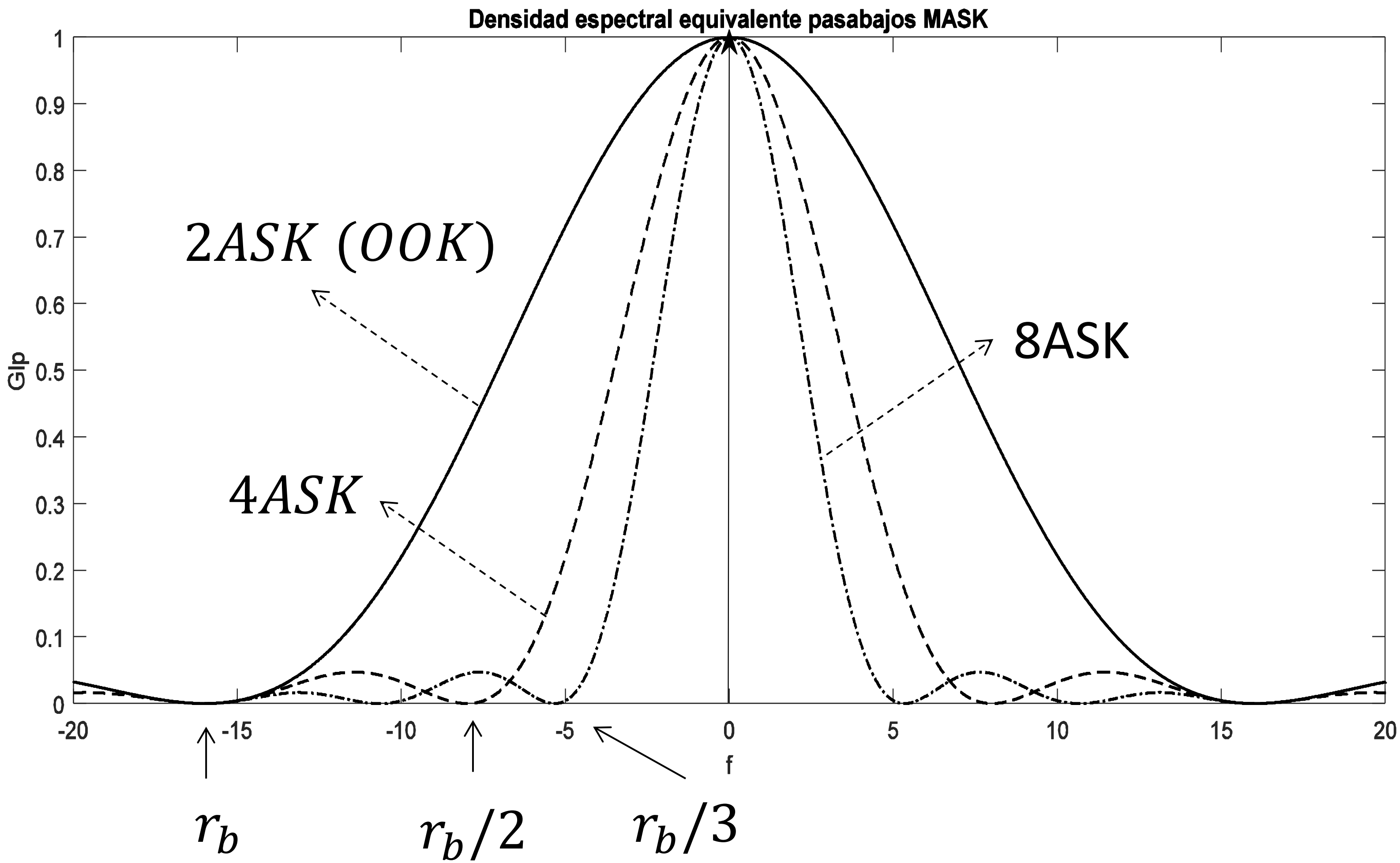
M-ASK. Spectral Density $G_{lp}(f)$.



Quadrature amplitude modulation QAM.

As seen in the case of ASK modulation, the information is applied to the component in phase. Then, the modulated vector is a vector that only changes its amplitude between discrete values of the phase component of the message.

$$x_c(t) = A_c[x_i(t)\cos(\omega_c t + \theta)]$$

However, it can be seen in the expression of a bandpass digital signal:

$$x_c(t) = A_c\left[x_i(t)\cos(\omega_c t + \theta) - x_q(t)\,sin(\omega_c t + \theta)\right]$$

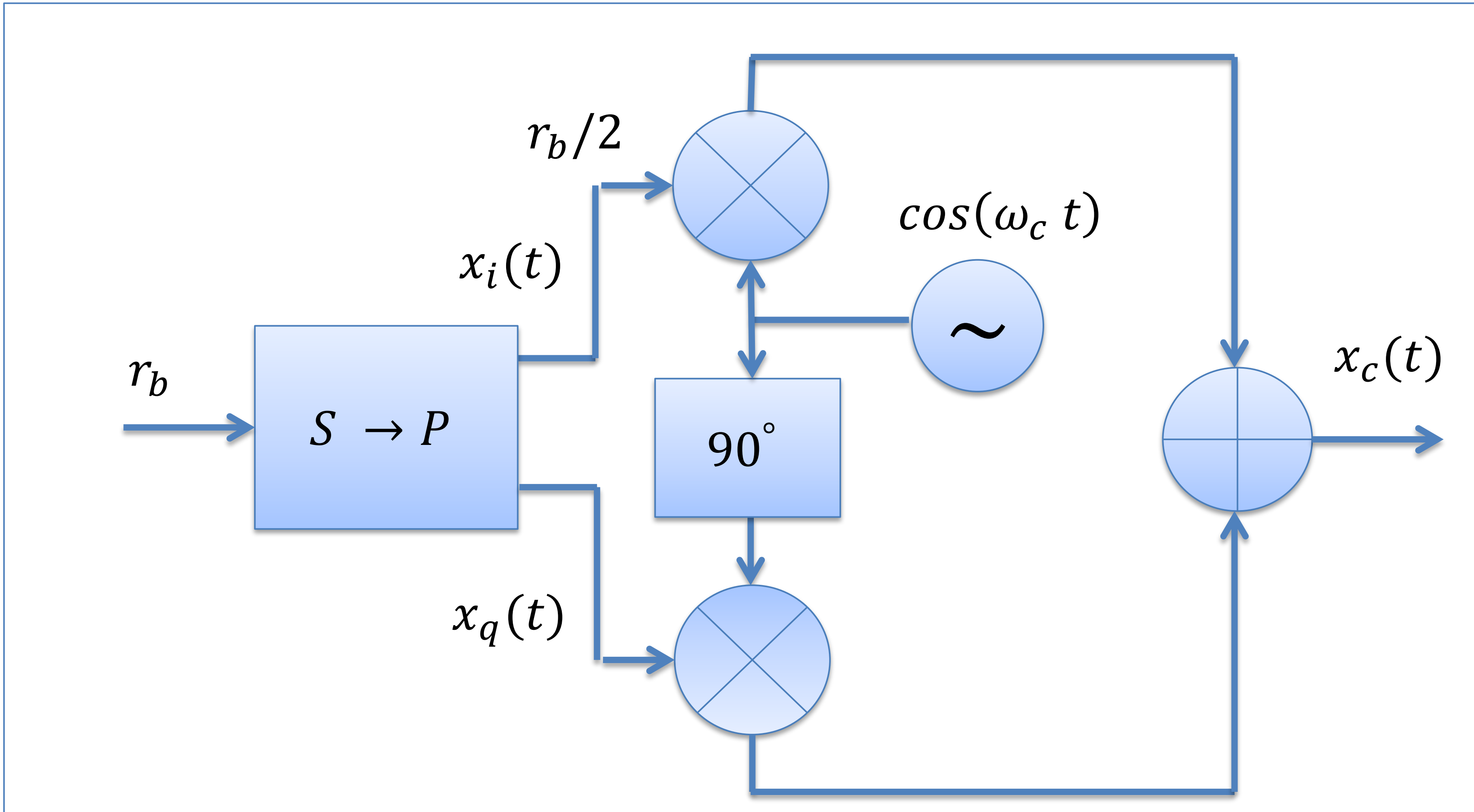
Then, the information could also be applied at the same time on the component in quadrature, or rather multiplexed and distributed on both components

Quadrature Amplitude Modulation QAM.

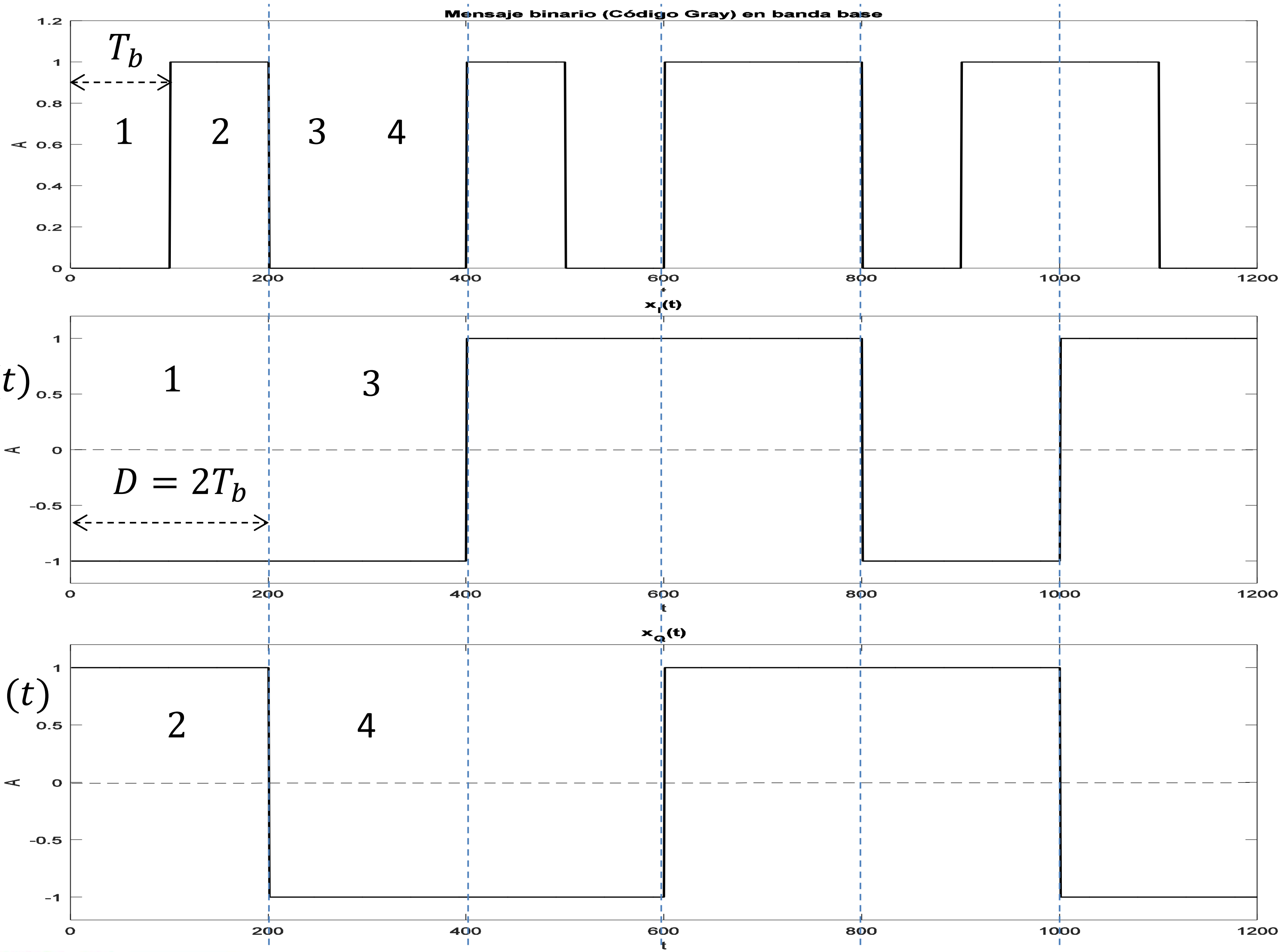
This type of modulation creates the appearance of four symbols, and is generally referred to as QAM (Quadrature Amplitude Modulation).

Serial-parallel conversion distributes the input bits into two groups, reducing the signaling rate for each branch of the modulator.

QAM modulation. Block diagram.



QAM Modulation. Multiplexation process.



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QAM Modulation. Multiplexation process.

The waveforms of each component can be written as a pair of rectangular pulse trains:

$$x_i(t) = \sum_k a_{2k+1} p(t - kD)$$

$$x_q(t) = \sum_k a_{2k} p(t - kD)$$

That alternate the parity of the position of the data in the original message. In these expressions $a_k = \pm 1$ y $D = 2T_b$.

For any interval $kD < t < (k+1)D$ the possible values of the i and q components are ±1. It can be plotted in a plane of i and q components, called the constellation of the signal.

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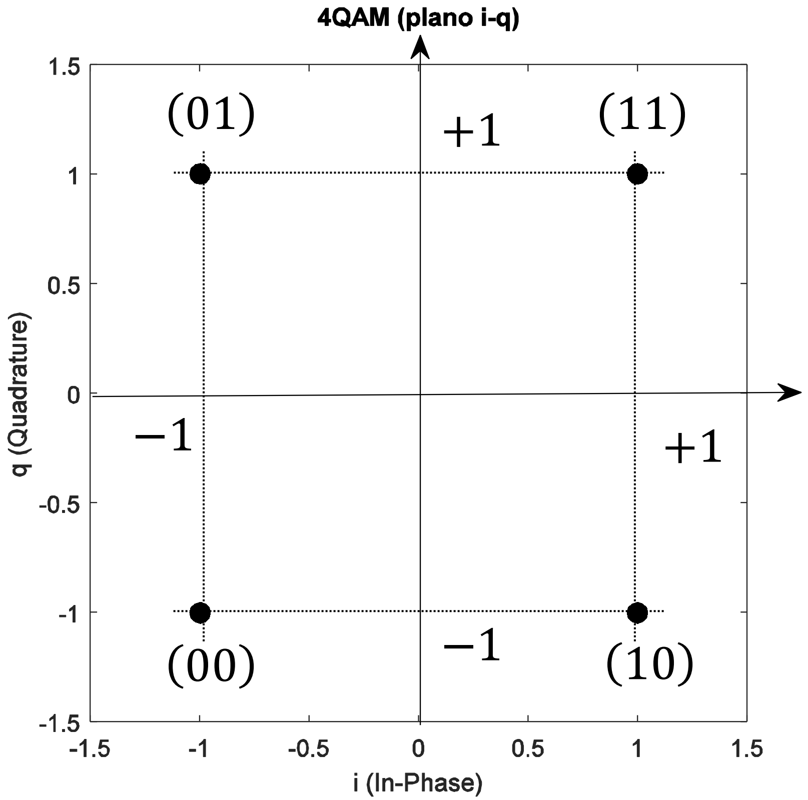
QAM modulation. Constelation.



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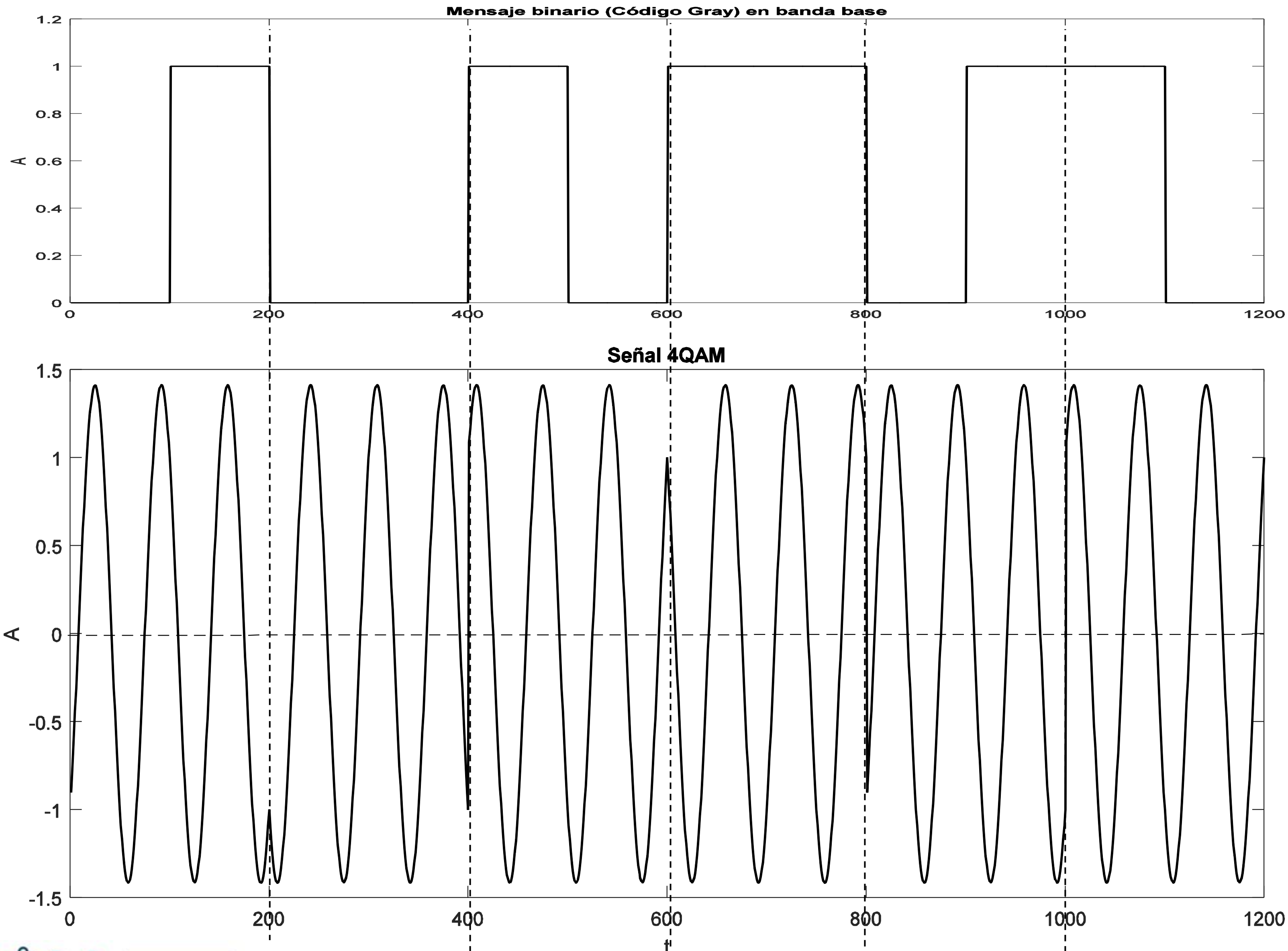
QAM modulation. Waveforms.



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Modulación QAM. Densidad espectral.

The i and q components are independent but have the same pulse shape and the same statistical averages.

$$m_a = \frac{1}{2}(1) + \frac{1}{2}(-1) = 0$$

$$\sigma_a{}^2 = \frac{1}{2}(1)^2 + \frac{1}{2}(-1)^2 = 1$$

$$|P_D(f)|^2 = \frac{4}{r_b{}^2} sinc^2\left(\frac{2f}{r_b}\right)$$

$$G_{lp}(f) = G_i(f) + G_q(f) = 2G_i(f) = 2r|P_D(f)|^2$$

$$G_{lp}(f) = r_b|P_D(f)|^2 = \frac{4}{r_b} sinc^2\left(\frac{2f}{r_b}\right)$$

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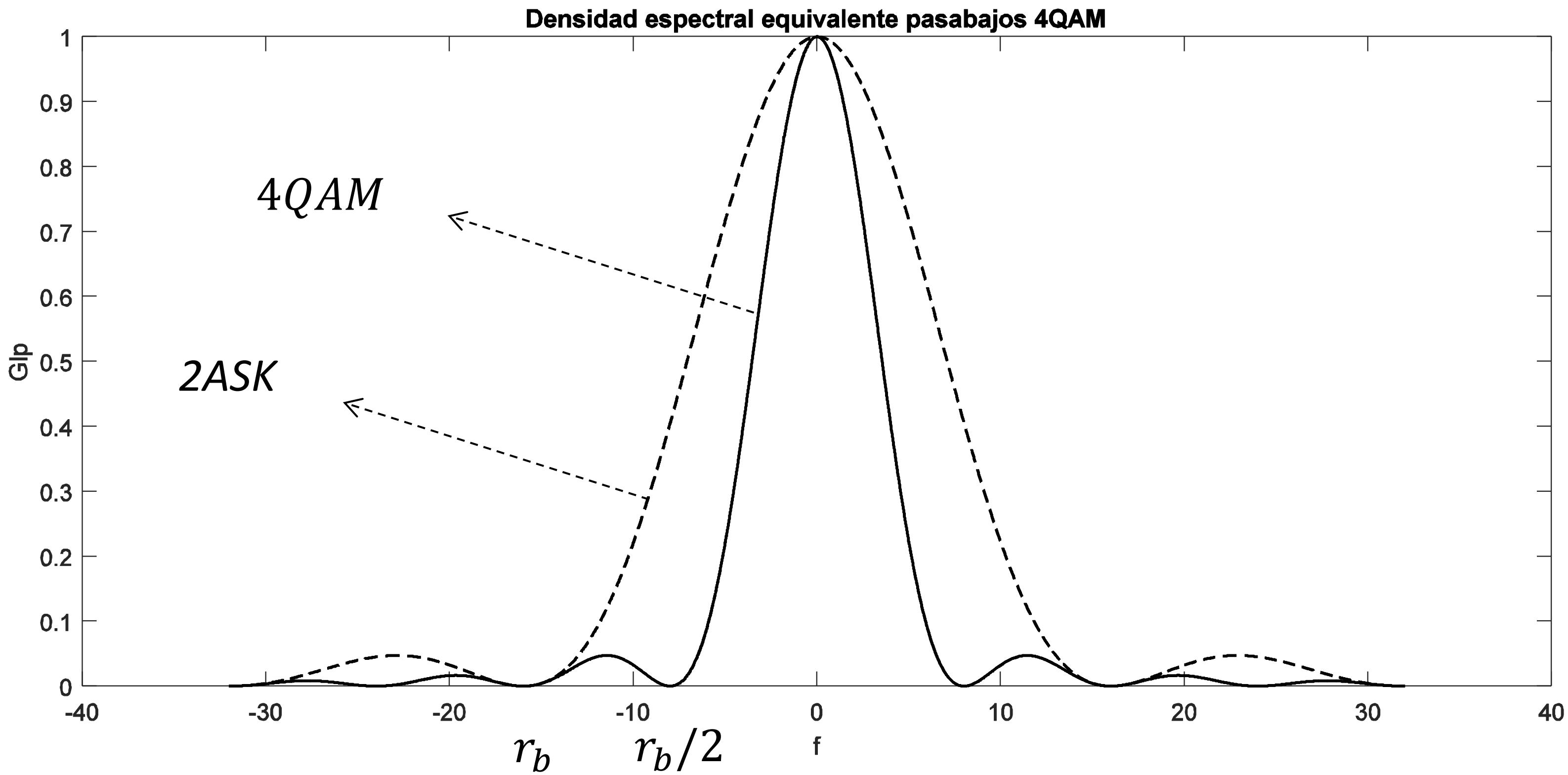
QAM modulation. Spectral density.

The spectral eficiency can be defined as:

$$\frac{r_b}{B_T} = \frac{2\ bps}{Hz}$$



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MPSK modulation.

The PSK binary waveform (2PSK BPSK or PRK, “Phase Reversal Keying”) represented in the figure, is basically a binary transmission where a signal is transmitted with the same carrier frequency and with a phase shift of $\pm\pi$.

If the baseband information is a $M\!-\!ary$ waveform, the modulation is performed by univocally relating each amplitude level to a given carrier phase. In the interval $kD<t<(k+1)D$ the carrier has a phase shift φ_k.

The PSK modulated signal can be expressed as:

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MPSK modulation.

$$x_c(t) = A_c \sum_k cos(\omega_c t + \varphi_k + \theta) p_D(t - kD)$$

Using $cos(\alpha + \beta) = cos(\alpha)cos(\beta) - sen(\alpha)sen(\beta)$

$$x_c(t) = A_c \left[\sum_k cos(\omega_c t + \theta) cos(\varphi_k) p_D(t - kD) \right] - A_c \left[\sum_k sen(\omega_c t + \theta) sen(\varphi_k) p_D(t - kD) \right]$$

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MPSK modulation.

$$x_c(t) = A_c\left[\sum_k cos(\varphi_k)p_D(t-kD)\right]cos(\omega_c t+\theta) - A_c\left[\sum_k sen(\varphi_k)p_D(t-kD)\right]sen(\omega_c t+\theta)$$

La forma de onda de modulación PSK esta entonces presentada como una forma de onda en sus componentes en fase y cuadratura.

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MPSK modulation.

$$x_i(t) = \sum_k cos(\varphi_k) p_D(t - kD) = \sum_k I_k p_D(t - kD)$$

$$x_q(t) = \sum_k sen(\varphi_k) p_D(t - kD) = \sum_k Q_k p_D(t - kD)$$

$$I_k = cos(\varphi_k)$$

$$Q_k = sen(\varphi_k)$$

The a_k values of the M-ary waveform in unipolar, modulating mode correspond to the different phases φ_k.

$$\varphi_k = \frac{2\pi a_k}{M}; \qquad a_k = 0, 1, 2, \ldots, M - 1$$

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4PSK or QPSK modulations.

For example, for the case M=4 we have the constellation of the figure .

This modulation is called QPSK.

$$\varphi_k = \frac{2\pi a_k}{4}; \qquad a_k = 0, 1, 2, 3$$

$$\varphi_k = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

$$I_k = 1, 0, -1, 0$$

$$Q_k = 0, 1, 0, -1$$

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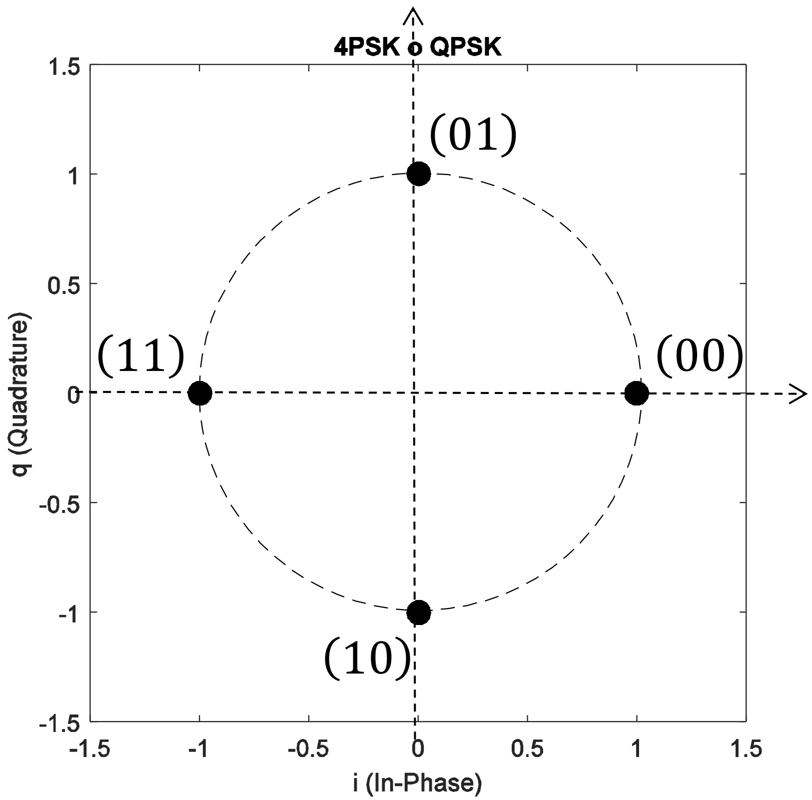
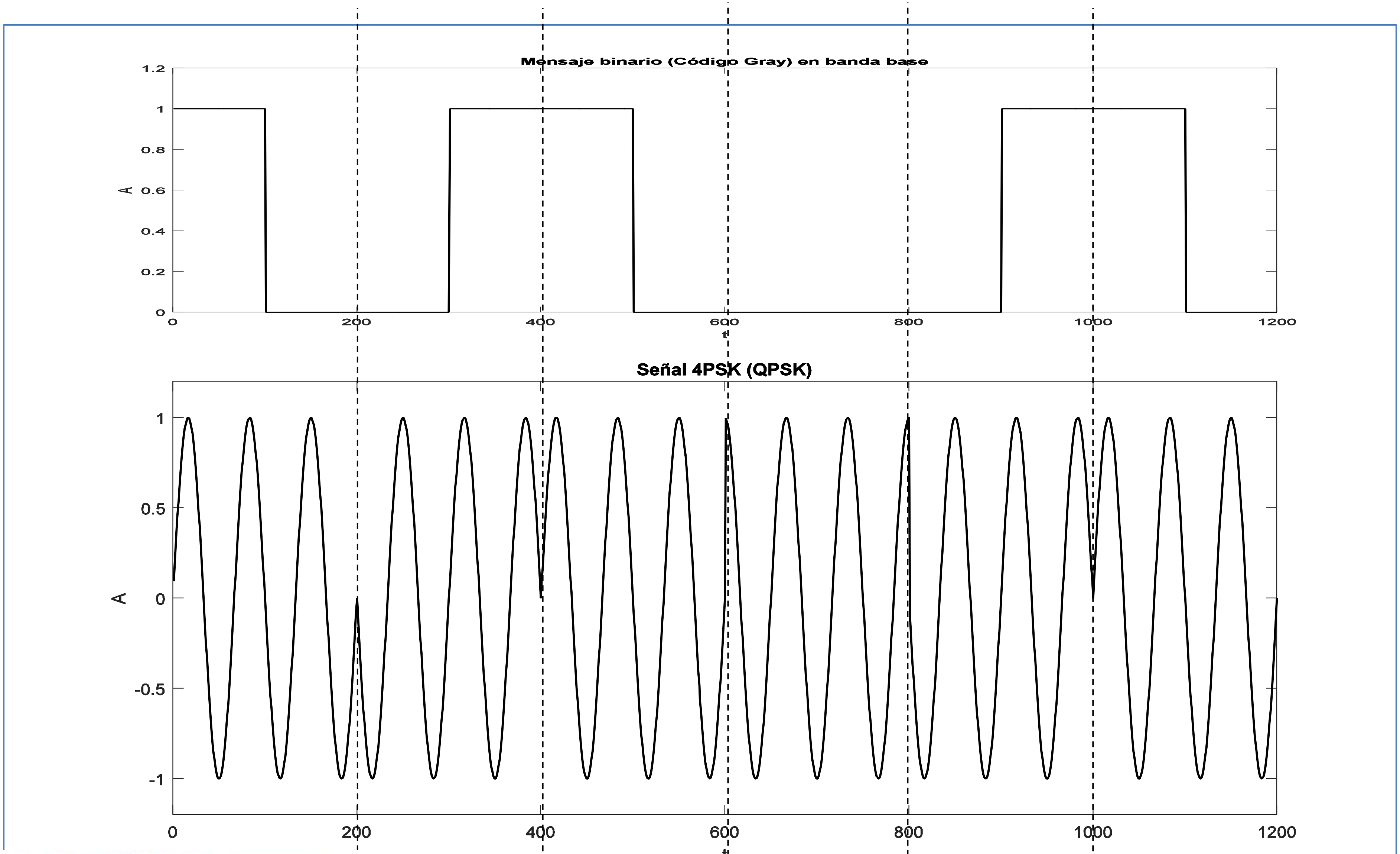
4PSK or QPSK modulations.



QAM modulation. Waveforms.



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4PSK or QPSK modulations.

For example, for the case M=4, it can be interpreted that this modulation is equivalent to 4QAM with the constellation rotated in $\pi/4$.

$$\varphi_k = \frac{2\pi a_k + 1}{4}; \qquad a_k = 0, 1, 2, 3$$

$$\varphi_k = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$I_k = \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$$

$$Q_k = \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}$$

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MPSK Spectral density.

To determine the power spectral density in MPSK modulation, the expressions obtained for the component in phase and quadrature of the modulated waveform can be used.

Such components are similar to the general waveform of a PAM signal, only that the value of a_k is now I_k or Q_k, for the in-phase and quadrature components respectively.

The expression of the power spectral density requires knowing the mean values of the signal statistic.

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MPSK Spectral density.

$$\overline{I_k} = \sum_k cos(\varphi_k)P(I_k) = \frac{1}{M}\sum_k cos(\varphi_k) = \frac{1}{M}\sum_k cos\left(\frac{2\pi a_k}{M}\right) = 0$$

Similarly, for the component Q_k it is verified that:

$$\overline{Q_k} = 0$$

And, it is possible to obtain the squared averages values:

$$\overline{{I_k}^2} = \sum_k cos^2(\varphi_k)P(I_k) = \frac{1}{M}\sum_k cos^2\left(\frac{2\pi a_k}{M}\right) = 1/2$$

$$\overline{{Q_k}^2} = 1/2$$

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MPSK Spectral density.

In this way, the equivalent lowpass spectral density can be witten as:

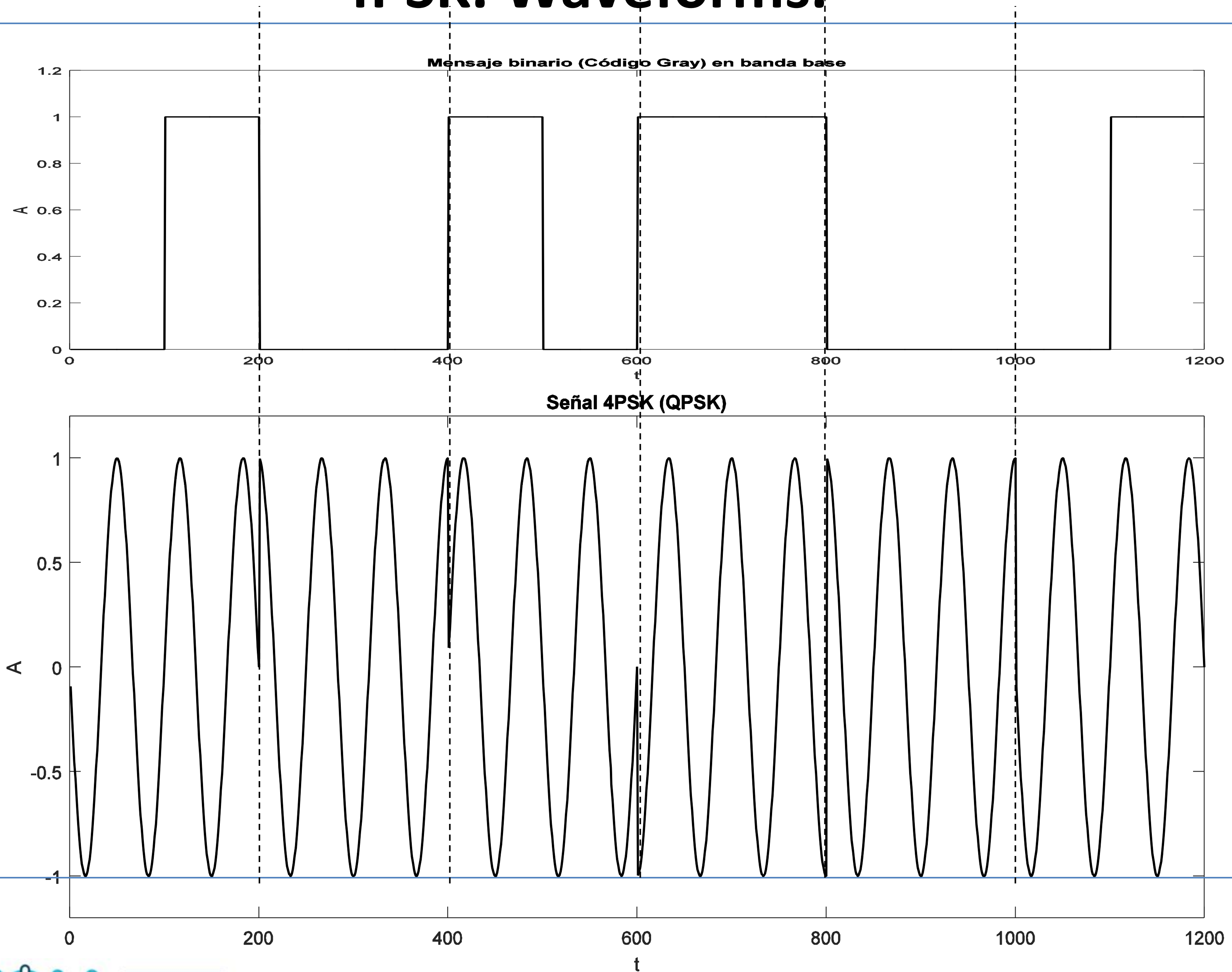
$$|P_D(f)|^2 = \frac{1}{r^2} sinc^2\left(\frac{f}{r}\right)$$

$$G_i(f) = G_q(f) = {\sigma_a}^2 r |P_D(f)|^2$$

$$G_{lp}(f) = G_i(f) + G_q(f) = 2G_i(f) = r|P_D(f)|^2$$

$$G_{lp}(f) = \frac{1}{r} sinc^2\left(\frac{f}{r}\right)$$

4PSK. Waveforms.



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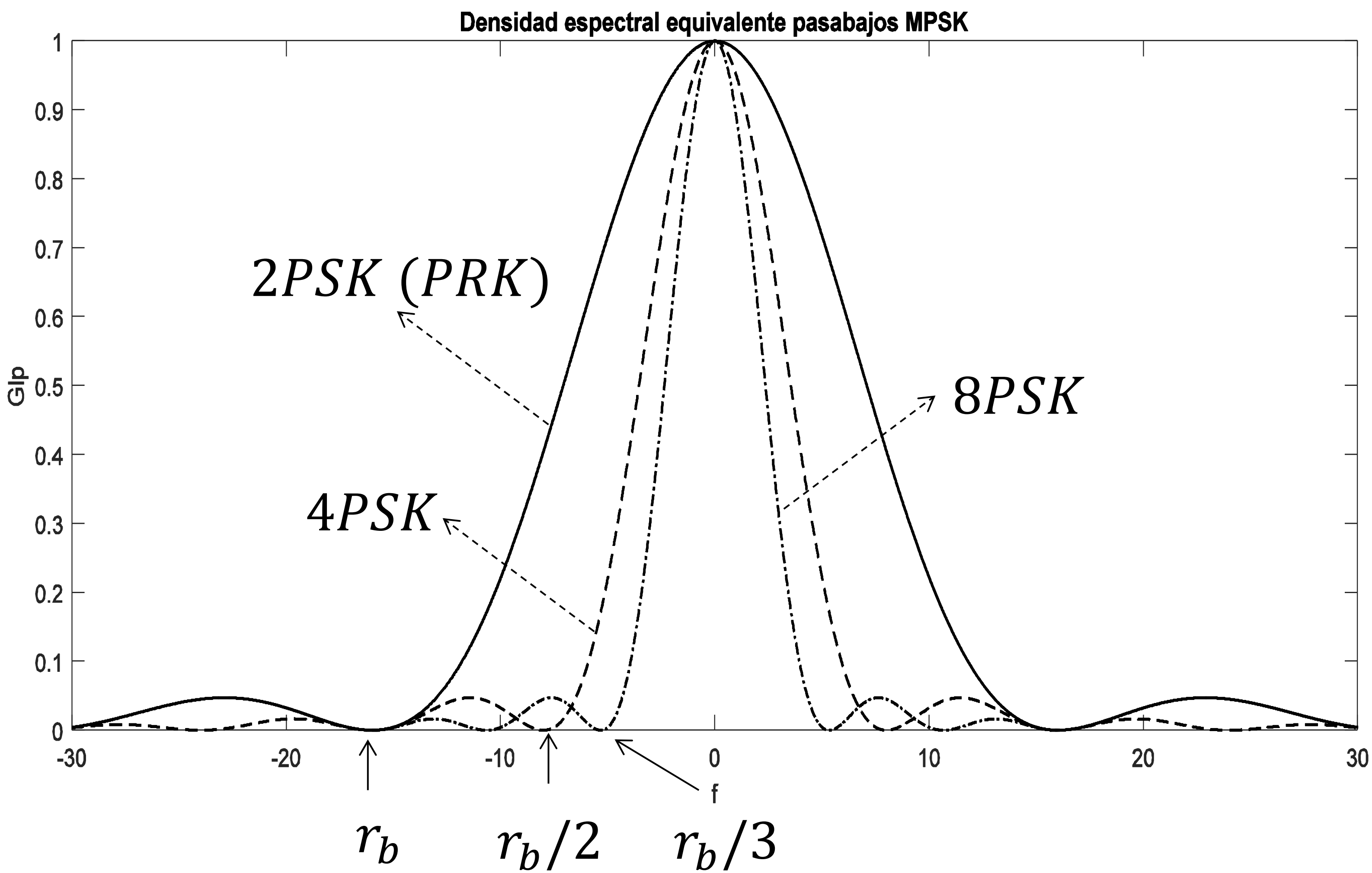
MPSK. Spectral Density $G_{lp}(f)$.



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MASK and MPSK Spectral efficiency.

The spectral density of MPSK is similar to that of MASK, but it does not contain a function $\delta(f \pm f_c)$ at the carrier frequency. MPSK has better power efficiency, and similar spectral efficiency.

The spectral efficiency for ASK and PSK is basically given by the expression of the speed r as a function of the bit rate r_b. If *M-ary* modulation is used, the signaling rate is given by:

$$r = \frac{r_b}{\log_2 M}$$

MASK and MPSK Spectral efficiency.

The spectral efficiency, or number of bits per second that is transmitted per unit of bandwidth is evaluated knowing that approximately the transmission bandwidth is

$$B_T \cong r = \frac{r_b}{\log_2 M}$$

for MASK and MPSK modulations. The spectral efficiency is then:

$$\frac{r_b}{B_T} \cong \log_2 M \quad bps/Hz$$