Communication Systems based on Software Defined Radio (SDR)

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Digital Communication Systems

Network of Competence on Internet of Things

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Coherent binary systems.

Bandpass digital systems can be demodulated in two ways:

Demodulation or coherent detection:

Information of the frequency f_c and phase of the carrier θ is used in the detection of the signal. It is also necessary to determine the information bit frequency r or r_b.

Demodulation or non-coherent detection:

Information of the frequency f_c or the phase of the carrier θ is NOT used in the detection of the signal. Whether it is necessary to determine the information bit frequency r or r_b.

The performance of coherent systems is superior to non-coherent systems.

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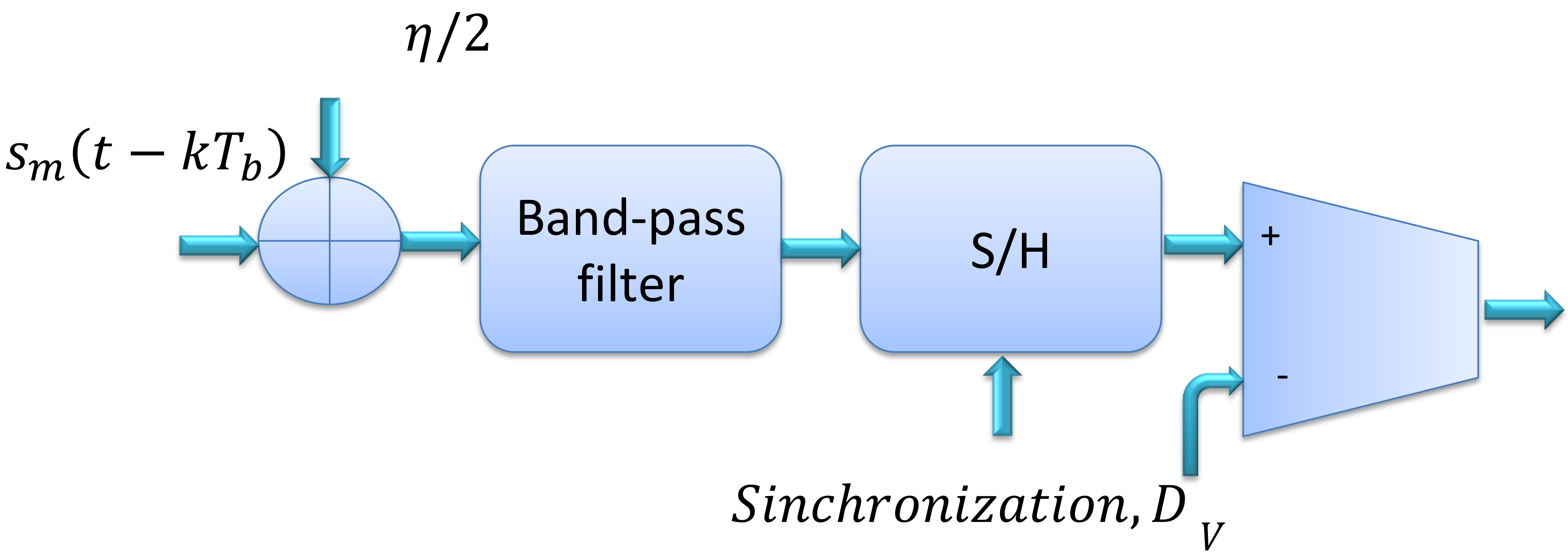
Optimal binary detection. block diagram.

The system is considered to be in the presence of additive and Gaussian white noise. The result of a baseband detection is that the optimal filter is the matched filter. A similar analysis is carried out considering that the waveforms to be detected are two.



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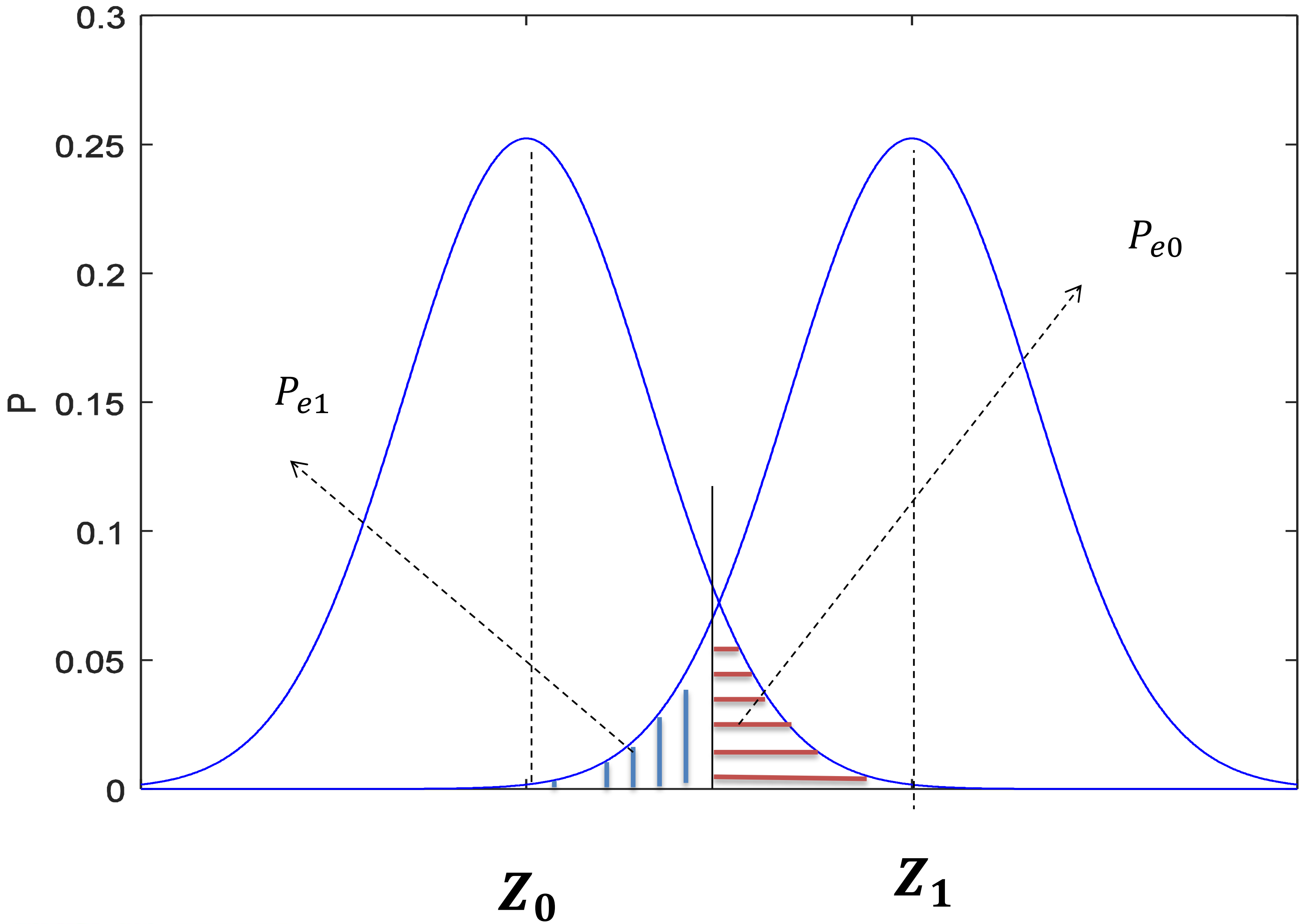
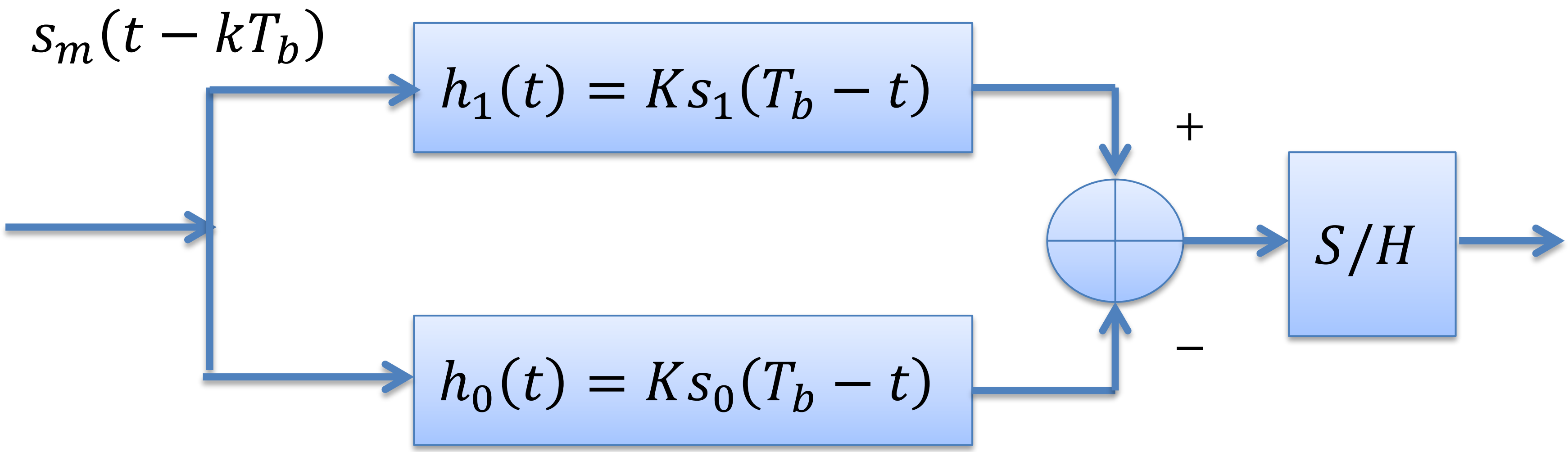
Optimal binary detection. Error probability.

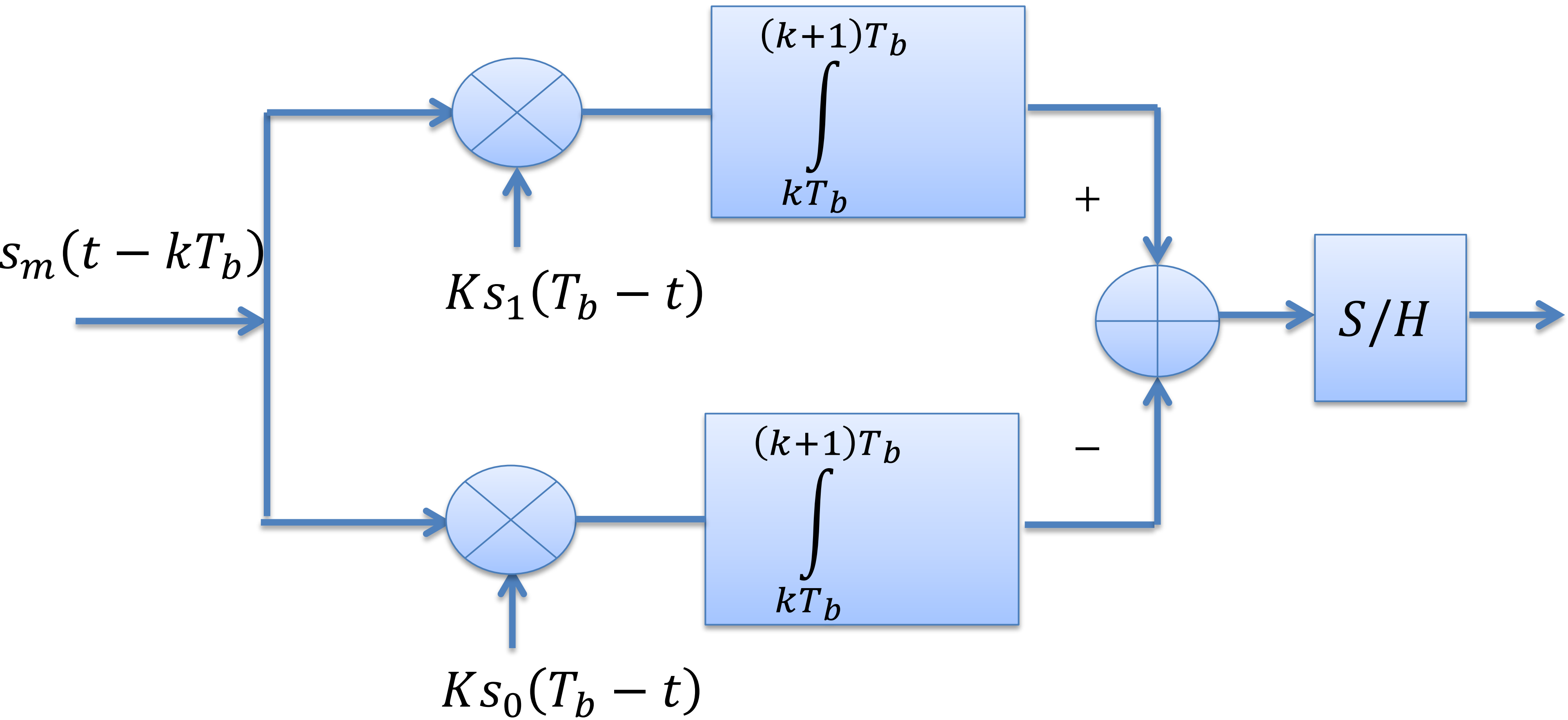


Optimal binary detection receiver.



Receiver by optimal binary detection correlation.

The block diagram of the correlation detector would then be:



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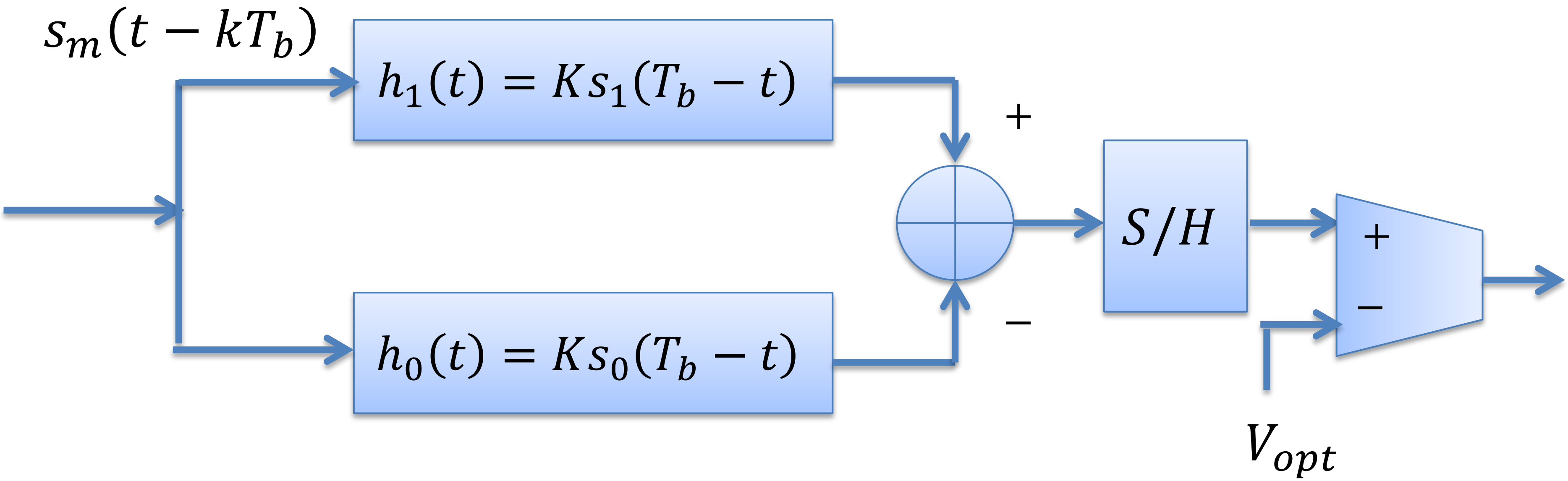
Optimal binary detection OOK matched filter.



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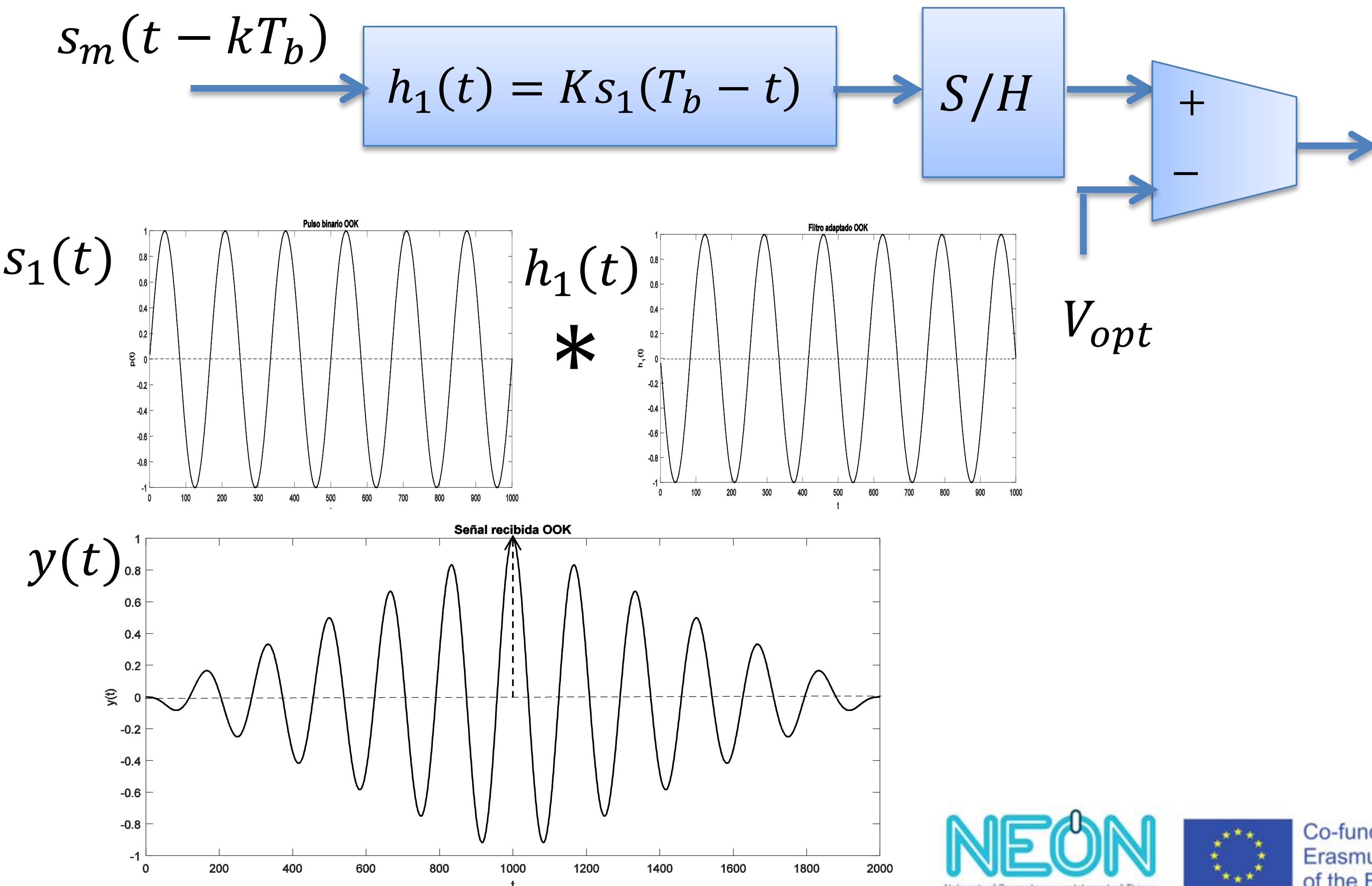
Optimal binary detection OOK matched filter.



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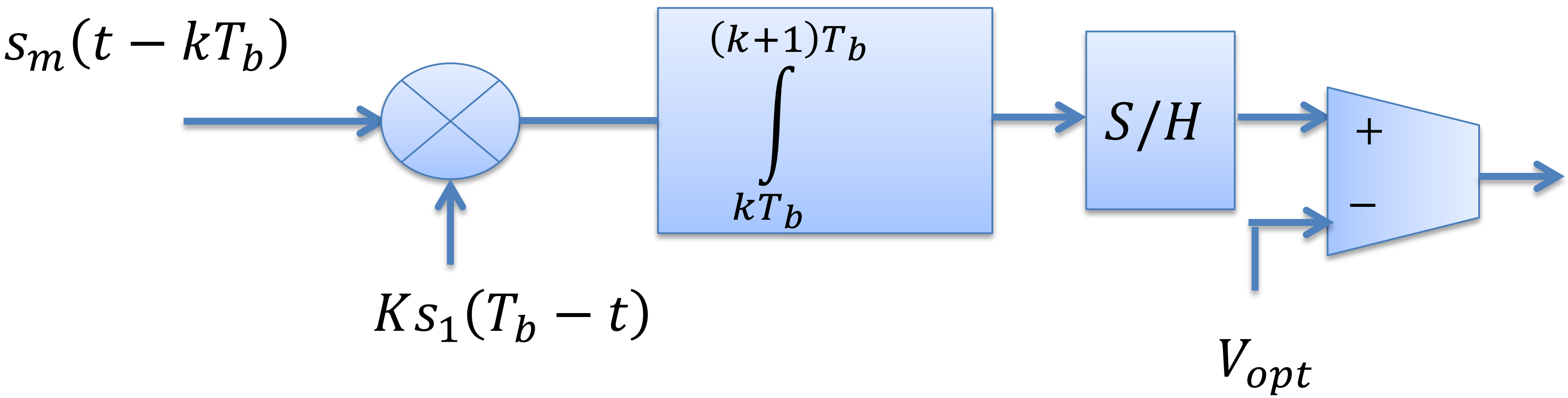
Optimal binary detection OOK correlation filter.

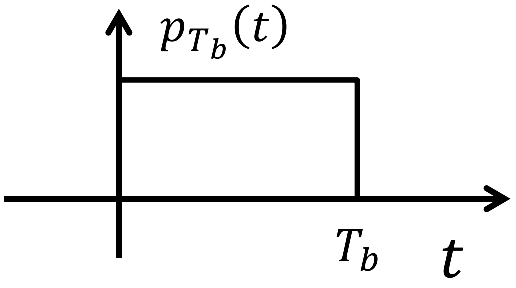
The block diagram of the correlation detector would then be:



Consistent PRK detection.

The signal corresponding to the PRK modulation has two symbols, $s_1(t)$, y $s_0(t) = -s_1(t)$:

$$s_1(t) = A_c p_{T_b} \cos(\omega_c t + \theta)$$
$$s_0(t) = -A_c p_{T_b} \cos(\omega_c t + \theta)$$



As was defined $f_c = N r_b$ then

$$s_1(t) = A_c \cos(\omega_c t) = -s_0(t)$$

The detector for this type of modulation becomes the schematic in the figure, which is the same as that used in OOK. The difference lies in the determination of the comparison threshold value V_{opt}.

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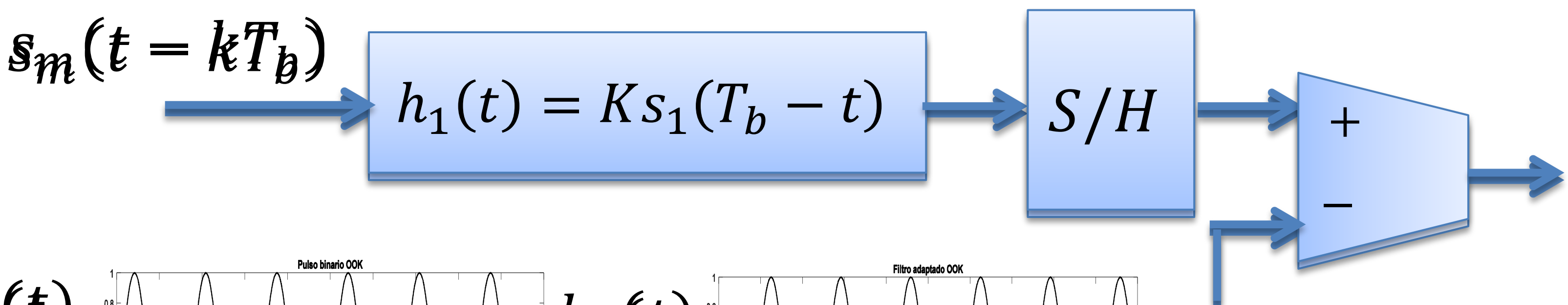
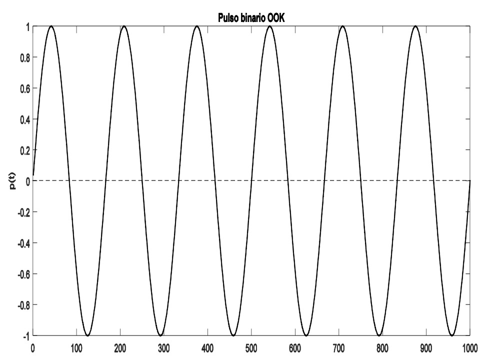
Optimal binary detection PRK matched filter.



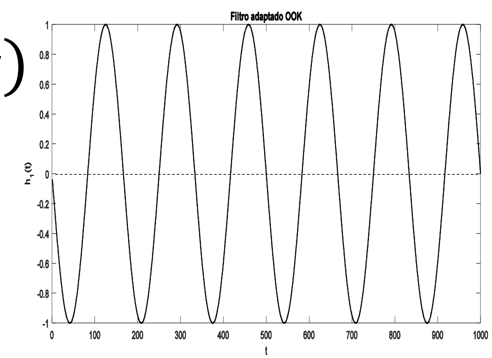
$V_{opt} \equiv 0$

$s_1(t)$

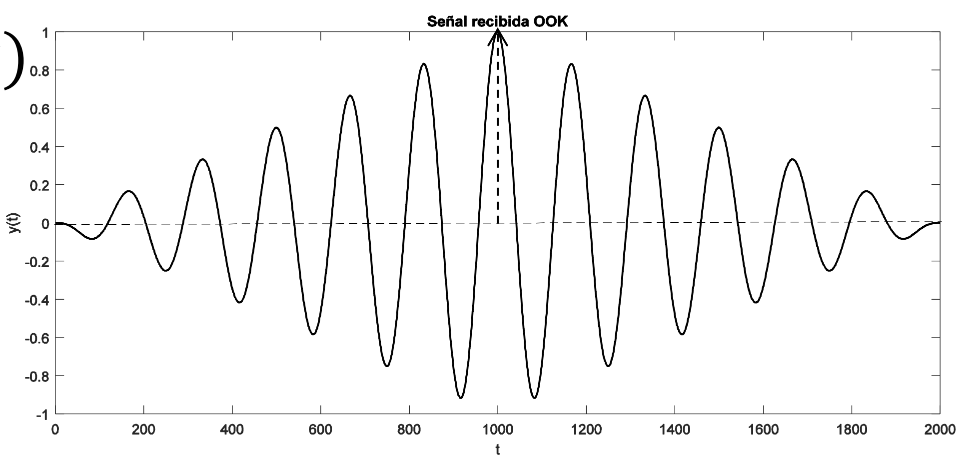


$h_1(t)$

$*$



$y(t)$



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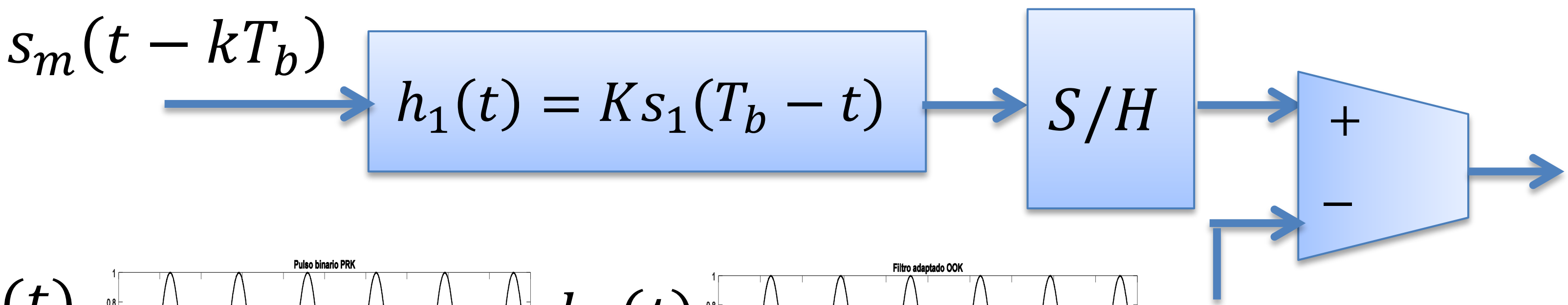
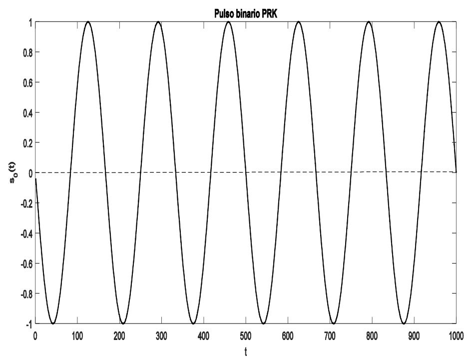
Optimal binary detection PRK matched filter



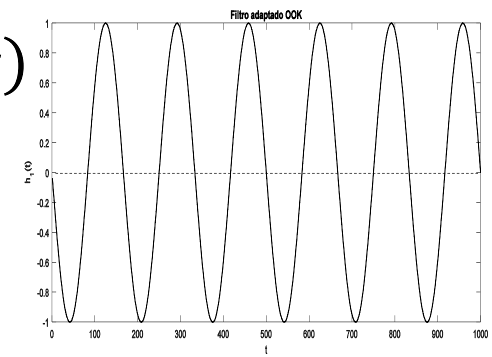
$V_{opt} = 0$

$s_0(t)$

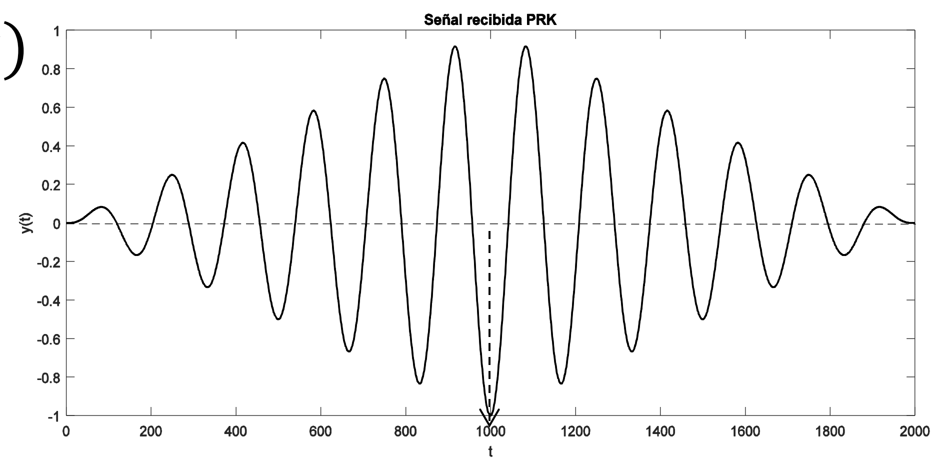


$h_1(t)$

$*$



$y(t)$



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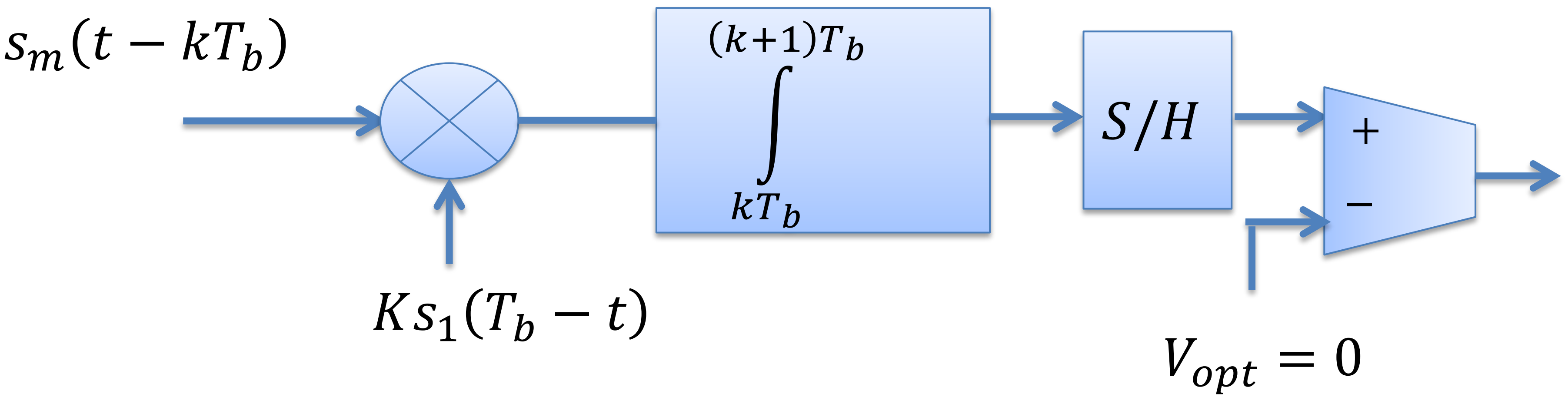
Optimal binary detection PRK filter correlation.

The block diagram of the correlation detector would then be:



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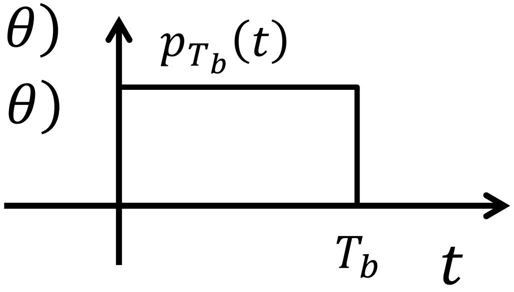
FSK coherent detection.

The signal corresponding to the FSK modulation presents two symbols, $s_1(t)$, and $s_0(t)$ of different frequencies:

$$s_1(t) = A_c p_{T_b} \cos(2\pi(f_c + f_d)t + \theta)$$
$$s_0(t) = A_c p_{T_b} \cos(2\pi(f_c - f_d)t + \theta)$$

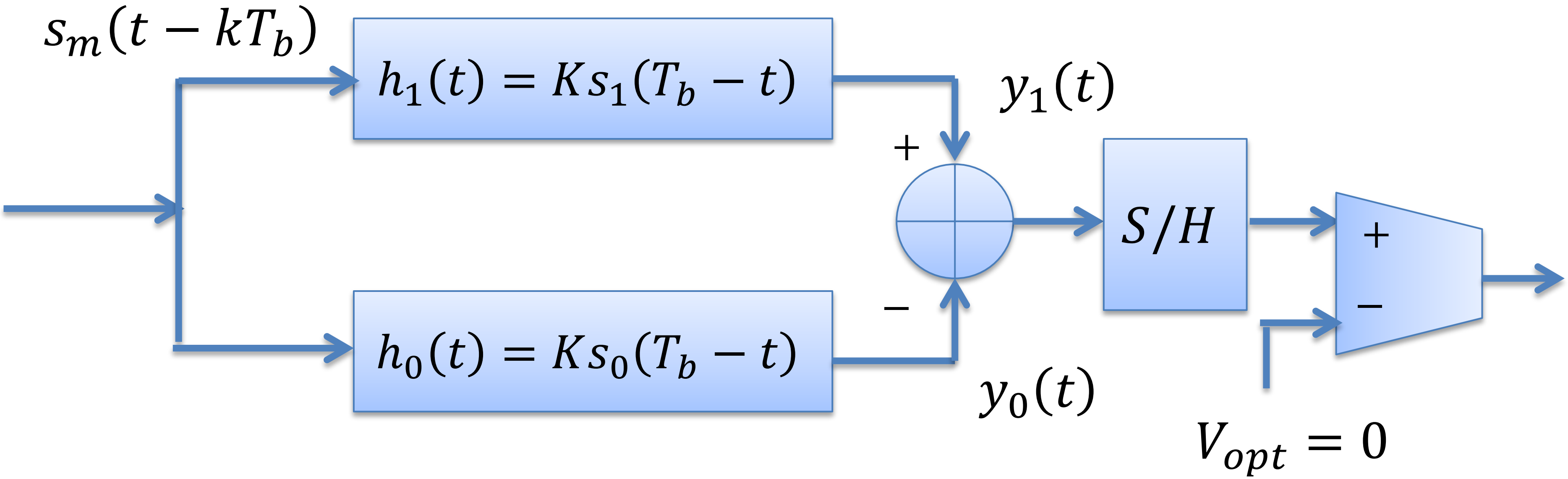


As was defined $f_c = N r_b$ then

$$s_1(t) = A_c \cos(2\pi(f_c + f_d)t)$$
$$s_0(t) = A_c \cos(2\pi(f_c - f_d)t)$$

The detector for this type of modulation becomes the schematic in the figure, which is no longer the same as that used in OOK. The difference is that both branches of the optimal receiver have to be considered

Optimal binary detection FSK matched filter.



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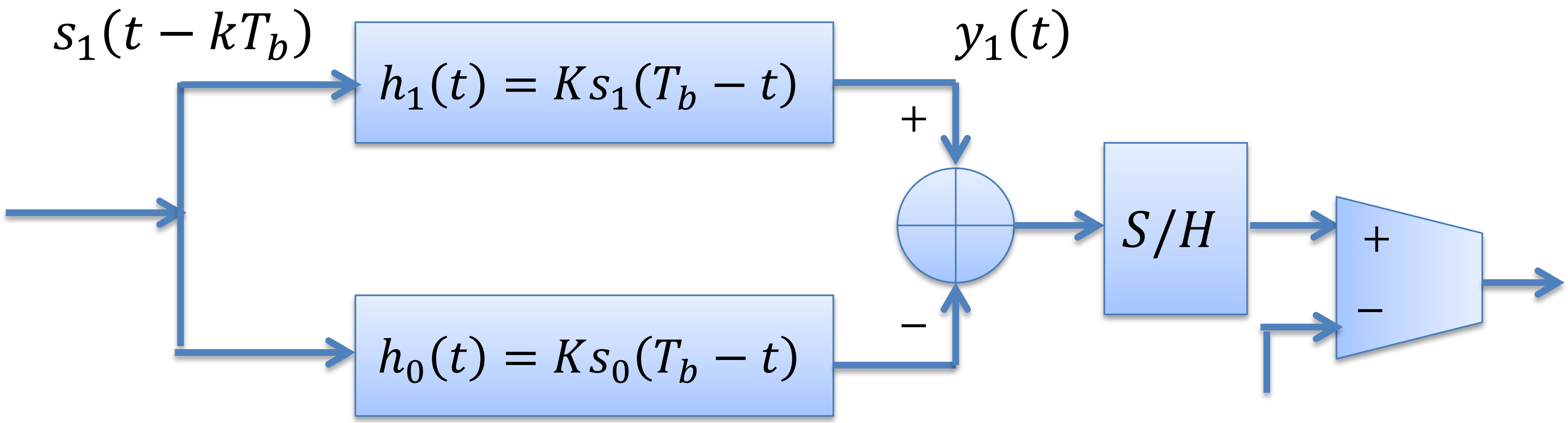
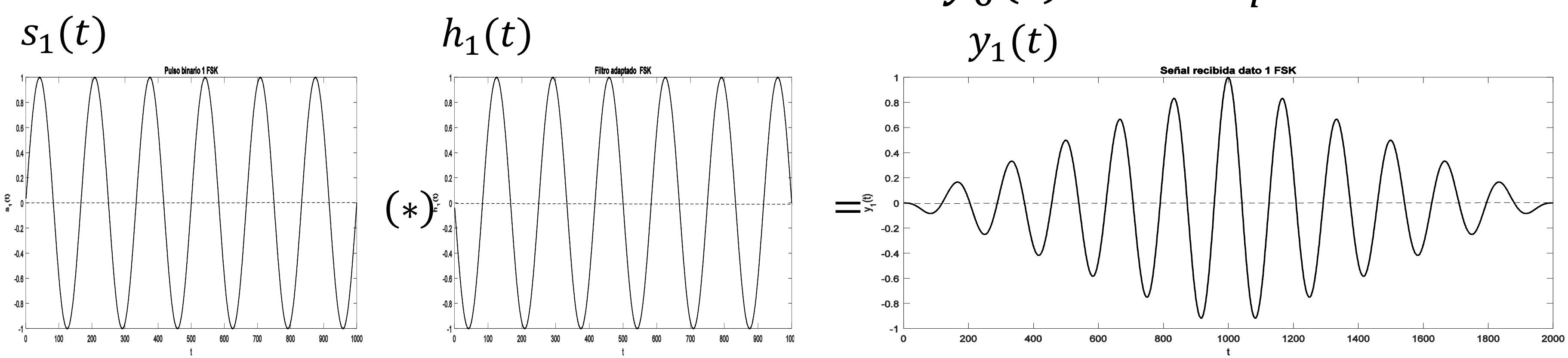
Optimal binary detection FSK matched filter.



$y_0(t) = 0 \quad V_{opt} = 0$



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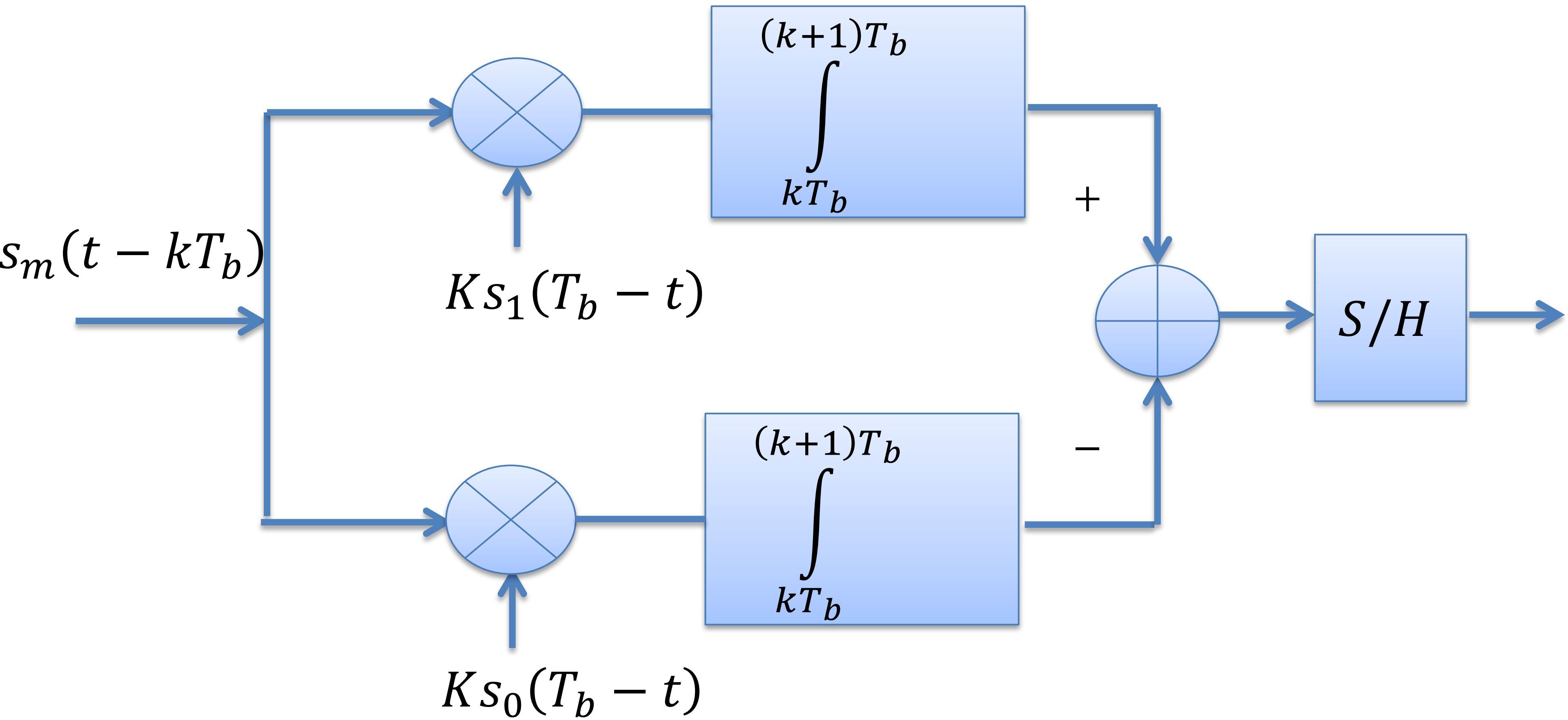
Receiver using FSK correlation .

The block diagram of the correlation detector would then be:



PART 2

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Non-coherent binary systems

The binary receiver can be simplified by avoiding having to know the frequency and phase of the carrier.

Detection of this type is called non-coherent. In the non-coherent demodulation or detection, neither the information of the frequency f_c nor of the phase of the carrier θ is used in the detection of the signal.

Whether it is necessary to determine the information bit frequency r or r_b.

The performance of non-coherent systems is slightly inferior to that of coherent systems.

Envelope of a sinusoid with bandpass noise

Since the detection will be carried out without taking into account the phase, but rather the envelope of a signal, it will be convenient to use the models of envelope and phase signals, rather than the model of components in phase and quadrature.

This will lead to a modification in the approach to the probability of error in the system, due to the change in the type of probability density function that appears as a consequence of performing the detection by amplitude.

The signal that is studied is of the form:

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Envelope of a sinusoid with bandpass noise

$$A_c cos(\omega_c t + \theta) + n(t)$$

The noise signal $n(t)$ corresponds to the case of bandpass noise with mean value zero and dispersion σ^2.

Bandpass noise has a signal pattern that can be expressed in the form of an in-phase and quadrature component.

$$n(t) = n_i(t) cos(\omega_c t + \theta) - n_q(t) sin(\omega_c t + \theta)$$

Adding this expression to the previous equation is obtained:

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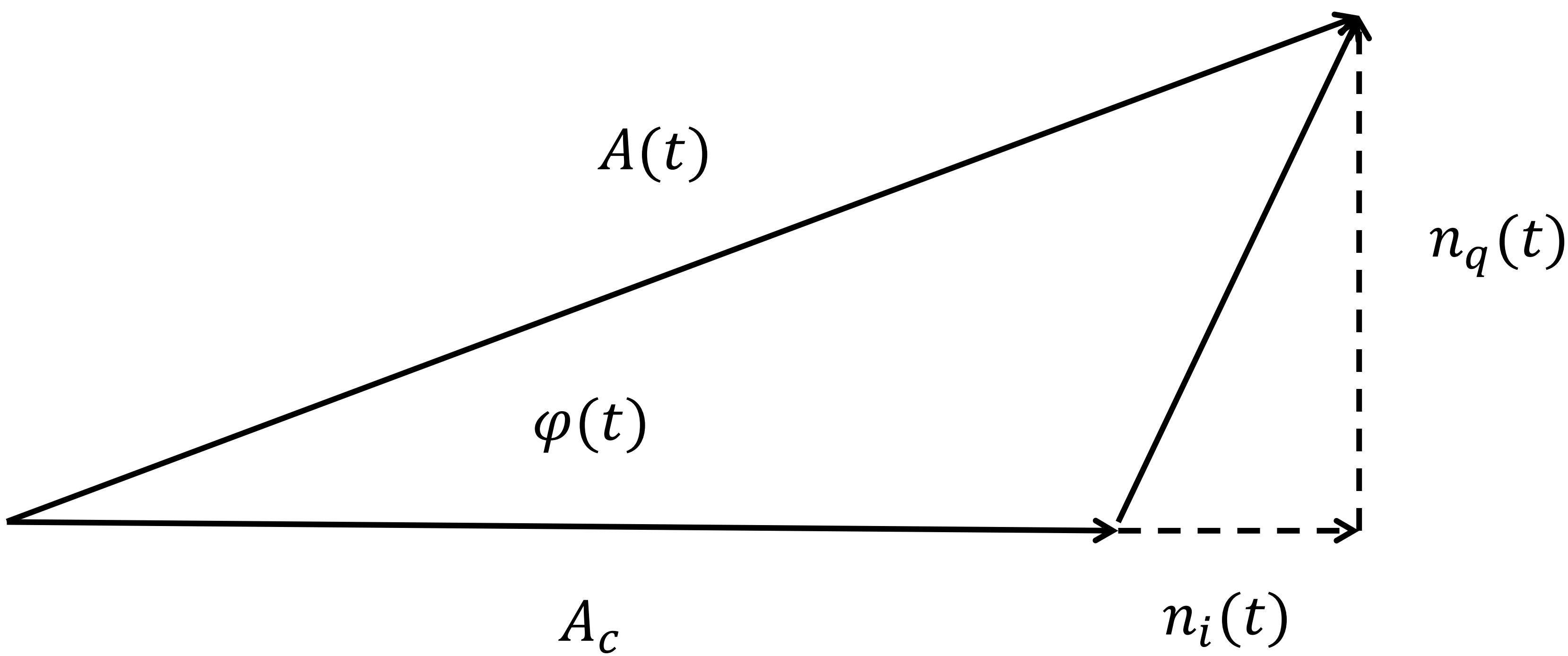
Envelope of a sinusoid with bandpass noise



$$A_c cos(\omega_c t + \theta) + n_i(t) cos(\omega_c t + \theta) - n_q(t) sin(\omega_c t + \theta)$$

Envelope of a sinusoid with bandpass noise

$n_i(t)$ y $n_q(t)$ son variables aleatorias independientes que tienen igual distribución que . Si el ruido fuera de valor cero, la señal $A(t)$ es igual a A_c. Si la señal es nula, la distribución será la de la envolvente del ruido.

Esto significa que la función densidad de probabilidad (pdf) que caracteriza a la señal, que sería ruido Gaussiano y blanco rectificado, es la pdf de Rayleigh:

$$P_A = \frac{A}{\sigma^2} e^{-\frac{A^2}{2\sigma^2}}$$

En la notación A representa a $A(t)$ que es una amplitud $A(t) \geq 0$.

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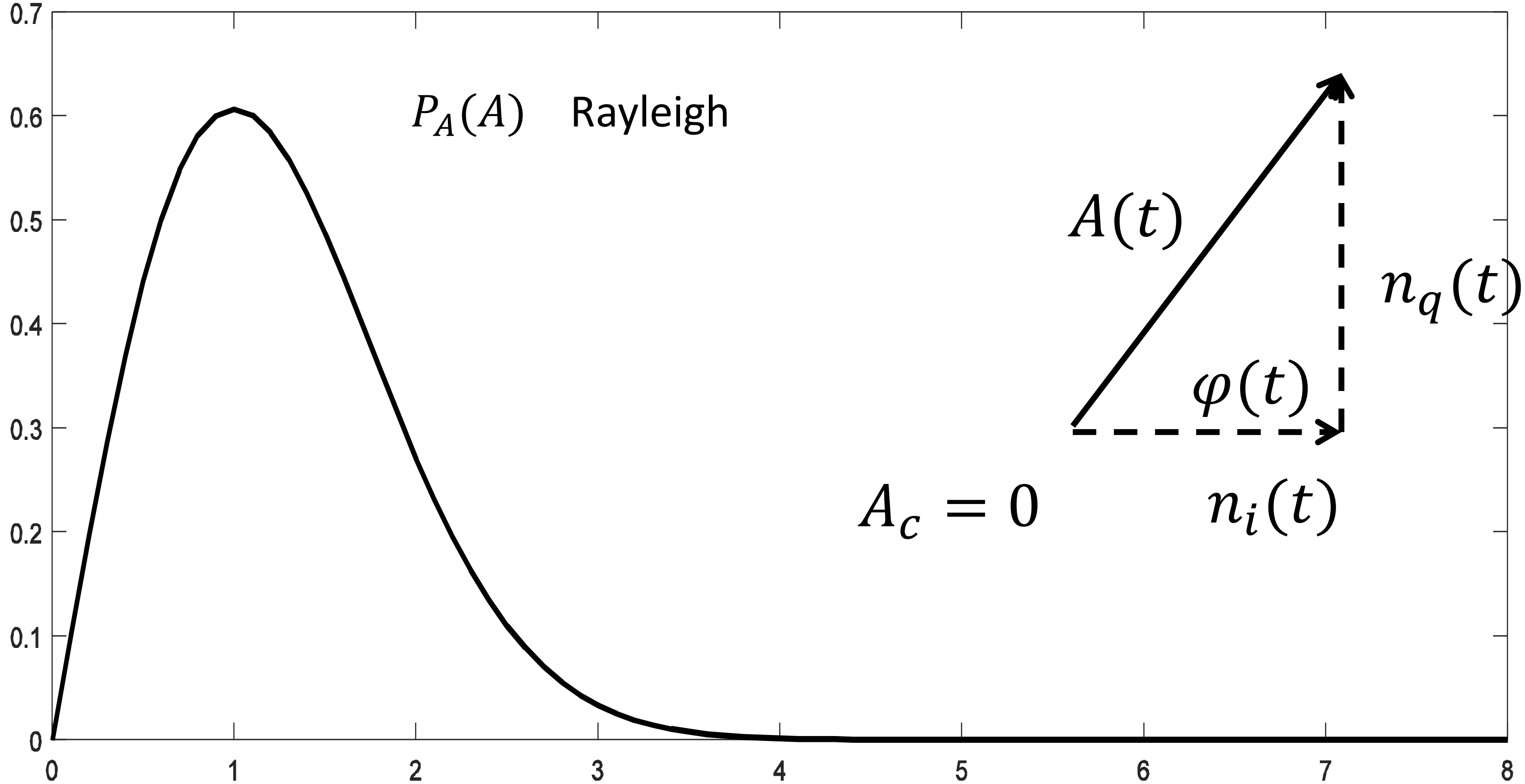
Envelope of a sinusoid with bandpass noise



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Envelope of a sinusoid with bandpass noise

Las probabilidades de A y φ son conjuntas y no es posible separarlas en el producto de las funciones de densidad independientes.

Si la relación señal-ruido fuera alta, las funciones podría ser independientes (Sería el caso en que puede hacerse $A = A_c$). Para señal cero la función se transforma en la función de Rayleigh, como se indicó anteriormente.

En el caso general, para conocer la probabilidad de error $P_A(A)$ que caracteriza la amplitud o la fase es necesario realizar la integración de la ecuación de $P_{A,\varphi}(A, \varphi)$ respecto de la variable φ. Así:

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Envelope of a sinusoid with bandpass noise

$$P_A(A) = \int_{-\pi}^{\pi} \frac{A}{2\pi\sigma^2} e^{-\left(\frac{A^2+A_c{}^2-2AA_c cos(\varphi)}{2\sigma^2}\right)} d\varphi$$

$$P_A(A) = \frac{A}{2\pi\sigma^2} e^{-\left(\frac{A^2+A_c{}^2}{2\sigma^2}\right)} \int_{-\pi}^{\pi} e^{\left(\frac{AA_c cos(\varphi)}{\sigma^2}\right)} d\varphi$$

Bessel functions are used:

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Envelope of a sinusoid with bandpass noise

$$P_A(A) = \frac{A}{2\pi\sigma^2} e^{-\left(\frac{A^2+A_c{}^2}{2\sigma^2}\right)} I_0\left(\frac{AA_c}{\sigma^2}\right)$$

The function described in the previous equation is known as Rice function. When the signal to noise ratio is large, $A_c \gg \sigma$:

$$P_A(A) = \sqrt{\frac{A}{2\pi A_c \sigma^2}} e^{-\left(\frac{A^2+A_c{}^2}{2\sigma^2}\right)} e^{\frac{AA_c}{\sigma^2}}$$

$$P_A(A) = \frac{1}{\sqrt{2\pi}\sigma} \sqrt{\frac{A}{A_c}} e^{-\frac{(A-A_c)^2}{2\sigma^2}}$$

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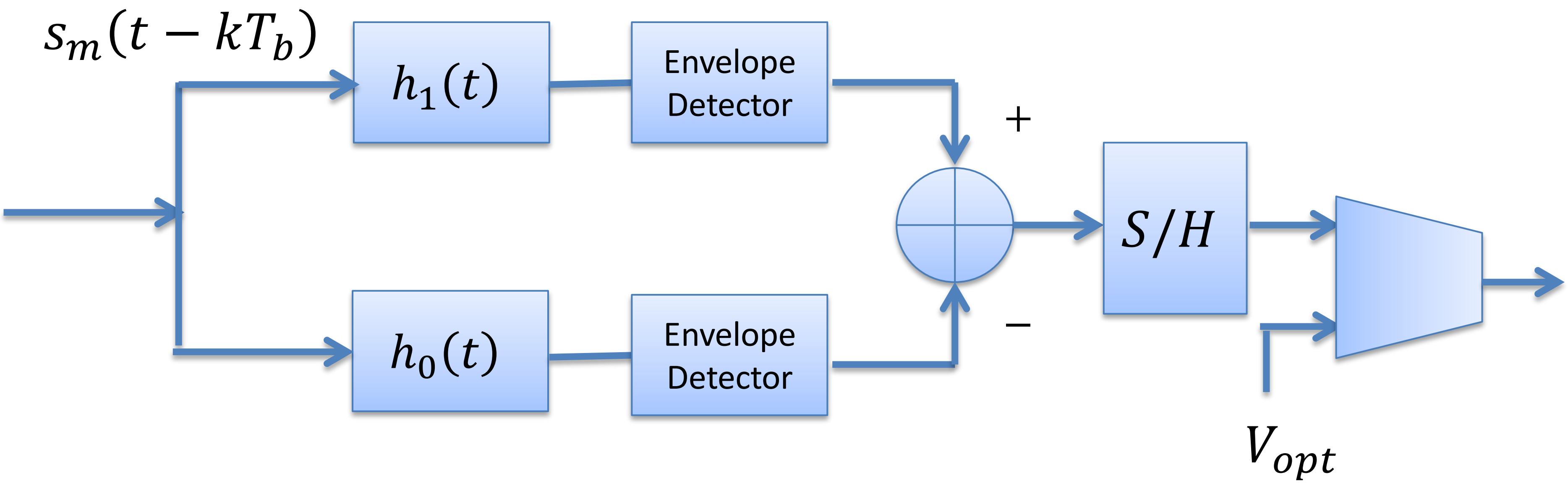
Non-coherent OOK



Non-coherent OOK

The reception filter is still a matched filter, which is followed by an envelope detection block and a sensed value shaper. The bandpass filter has the following form as a response:

$$h(t) = KA_c p_{T_b}(t) cos(\omega_c t)$$

The carrier phase is unknown. A normalization of the transfer function is performed such that the detected amplitude is A_c .

$$y(T_b) = KE_1 = A_c$$

Then $$K = \frac{A_c}{E_1}$$

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Non-coherent OOK

The value of the signal-to-noise ratio is calculated:

$$\sigma^2 = \frac{\eta}{2} \int_{-\infty}^{\infty} h^2(t)dt$$

$$\sigma^2 = \frac{\eta}{2} \int_{-\infty}^{\infty} \left(KA_c p_{T_b}(t) cos(\omega_c t)\right)^2 dt$$

$$\sigma^2 = \frac{K^2 {A_c}^2 T_b \eta}{4}$$

Then

$$\frac{{A_c}^2}{\sigma^2} = \frac{4}{K^2 T_b \eta} = \frac{4x4{E_b}^2}{{A_c}^2 T_b \eta} = 4\frac{E_b}{\eta} = 4\gamma_b$$

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Non-coherent OOK

The random variable Y results from sampling the detected signal $y(t)$ at instants t_k.

$$y(t_k) = A_c + n(t_k)$$

The transmitted symbols are zero or carrier. When $a_k = 0$ the resulting random variable is just noise. The distribution function for this case is $p_Y(Y/H_0)$, and it is of Rayleigh type.

When the signal sent is a one '1' , $a_k = A_c$ and the associated pdf $p_Y(Y/H_1)$ has the form of a Rice function.

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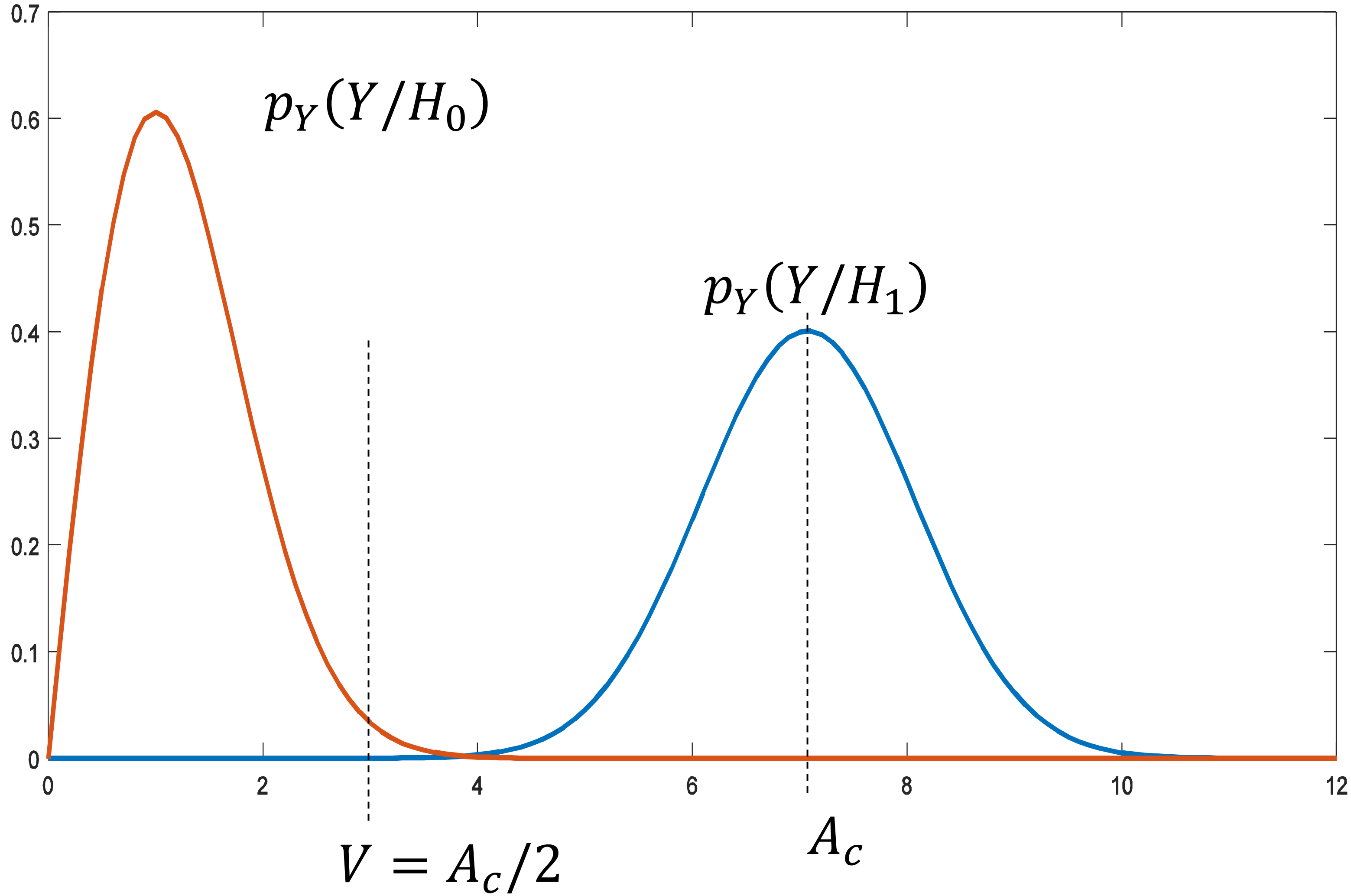
Non-coherent OOK



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Non-coherent OOK

If the signal-to-noise ratio is good, $\gamma_b \gg 1$, the Rice function becomes a Gaussian-like function. The optimum detection point is approximately defined according to the following expression

$$V_{opt} \cong \frac{A_c}{2}\sqrt{1+\frac{2}{\gamma_b}} \cong \frac{A_c}{2}$$

And, the threshold is adopted at $\frac{A_c}{2}$ accepting the approximation.

The error probabilities corresponding to the two symbols are calculated by integrating the corresponding probability density functions:

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Non-coherent OOK

$$P_{e1} = Q\left(\frac{A}{2\sigma}\right) = Q(\sqrt{\gamma_b})$$

Due to:

$$\frac{{A_c}^2}{\sigma^2} = 4\gamma_b; \qquad \frac{{A_c}^2}{4\sigma^2} = \gamma_b$$

For $k > 3$

$$Q(k) \cong \frac{1}{\sqrt{2\pi}k} e^{-\frac{k^2}{2}}$$

$$P_{e1} = Q(\sqrt{\gamma_b}) \cong \frac{1}{\sqrt{2\pi\gamma_b}} e^{-\frac{\gamma_b}{2}}$$

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Non-coherent OOK

Then the error probability is:

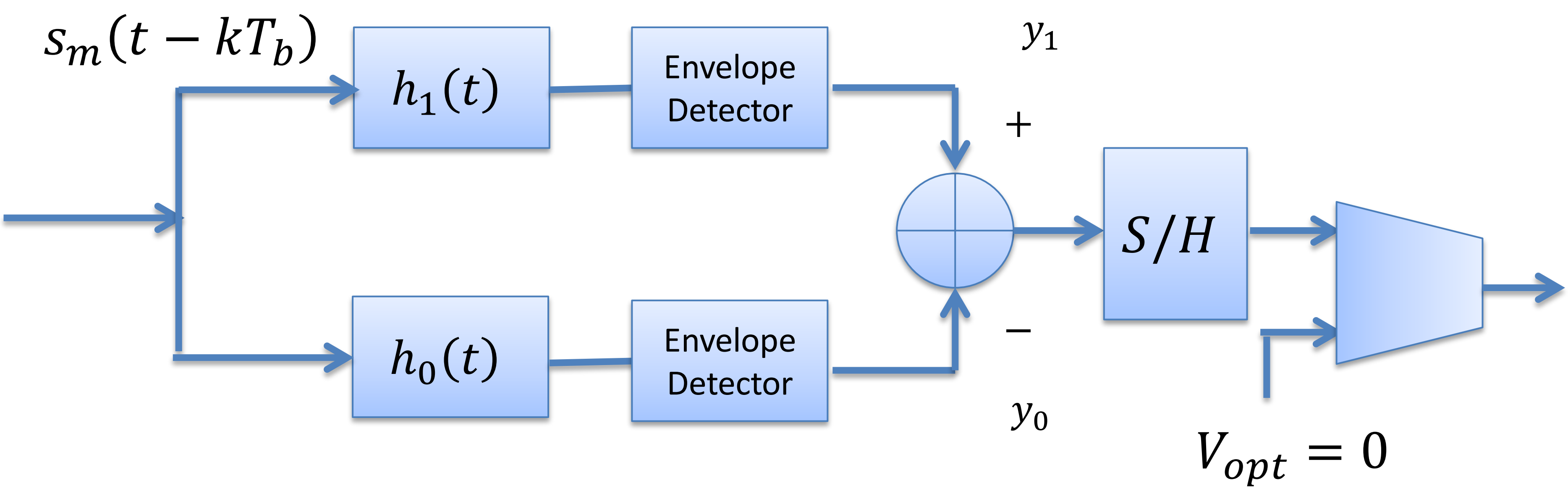
$$P_e = \frac{P_{e0} + P_{e1}}{2} = \frac{1}{2}\left(e^{-\frac{\gamma_b}{2}} + \frac{1}{\sqrt{2\pi\gamma_b}} e^{-\frac{\gamma_b}{2}}\right) \cong \frac{1}{2} e^{-\frac{\gamma_b}{2}}$$

when $\gamma_b \gg 1$

The error probability strongly depends of the error of the zero symbol.

Non-coherent OOK

The binary FSK signal can be interpreted as two interleaved OOK signals of different carriers with frequencies $f_c - f_d$ and $f_c + f_d$ and equal amplitude. The non-coherent FSK receiver can be configured as follows.



Non-coherent OOK

In this case there are two different waveforms and each one is associated with a corresponding matched filter.

The transfers of the bandpass filters are those corresponding to filters adapted to each transmitted waveform.

$$h_1(t) = KA_c p_{T_b}(t) cos(\omega_1 t)$$
$$h_0(t) = KA_c p_{T_b}(t) cos(\omega_0 t)$$

The filter ratio constant, K, is adopted such that the received signal is normalized to be of amplitude A_c. Then, $K = A_c / E_b$.

Also: $$E_b = E_1 = E_0 = \frac{{A_c}^2 T_b}{2}$$

Non-coherent OOK

$$\sigma^2 = \frac{\eta}{2}\int_{-\infty}^{\infty} h^2(t)dt$$

$$\sigma^2 = \frac{\eta}{2}\int_{-\infty}^{\infty} \left(KA_c p_{T_b}(t)cos(\omega_1 t)\right)^2 dt$$

$$\sigma^2 = \frac{K^2 {A_c}^2 T_b \eta}{4}$$

Then

$$\frac{{A_c}^2}{\sigma^2} = \frac{4}{K^2 T_b \eta} = \frac{4{E_b}^2}{{A_c}^2 T_b \eta} = 2\frac{E_b}{\eta} = 2\gamma_b$$

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Non-coherent OOK

Orthogonal type FSK signals are considered. In this case, the detection occurs in such a way that if the signal received in one of the branches of the receiver in the figure has a certain value, in the other branch the detection produces a null signal.

If a '1' was transmitted, the emitted frequency was f_1, so in the receiver, the upper branch has a certain detected value, while the lower branch detects a null value.

The reverse situation occurs when a zero was transmitted. The distribution functions alternate depending on the case. When the data transmitted is a '1', the random variable $y_1 = y_1(t_K)$ presents a Rice function in the neighborhood of A_c (If $K = A_c/E_b$) while the variable has a Rayleigh-type probability density function . The opposite situation happens if the data sent is a '0' .

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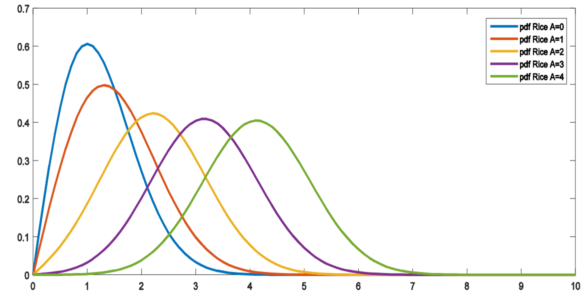
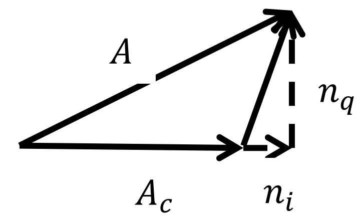
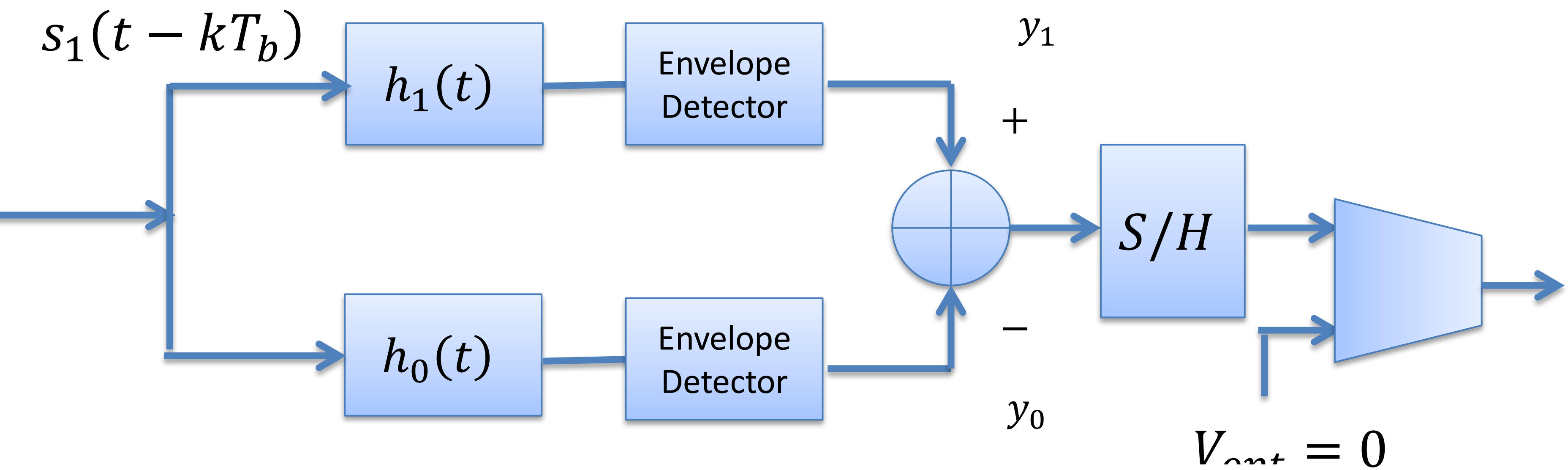
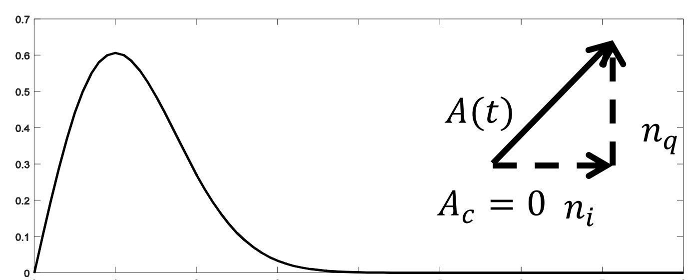
Non-coherent OOK

When a ‘1’ is transmitted



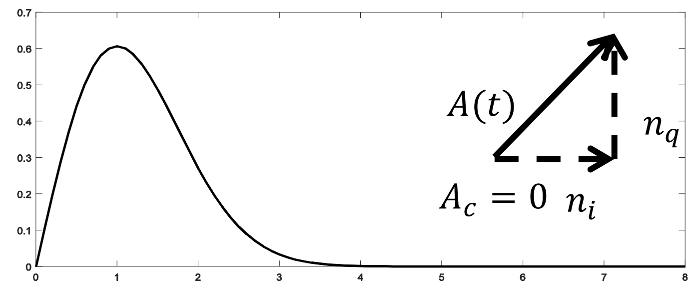


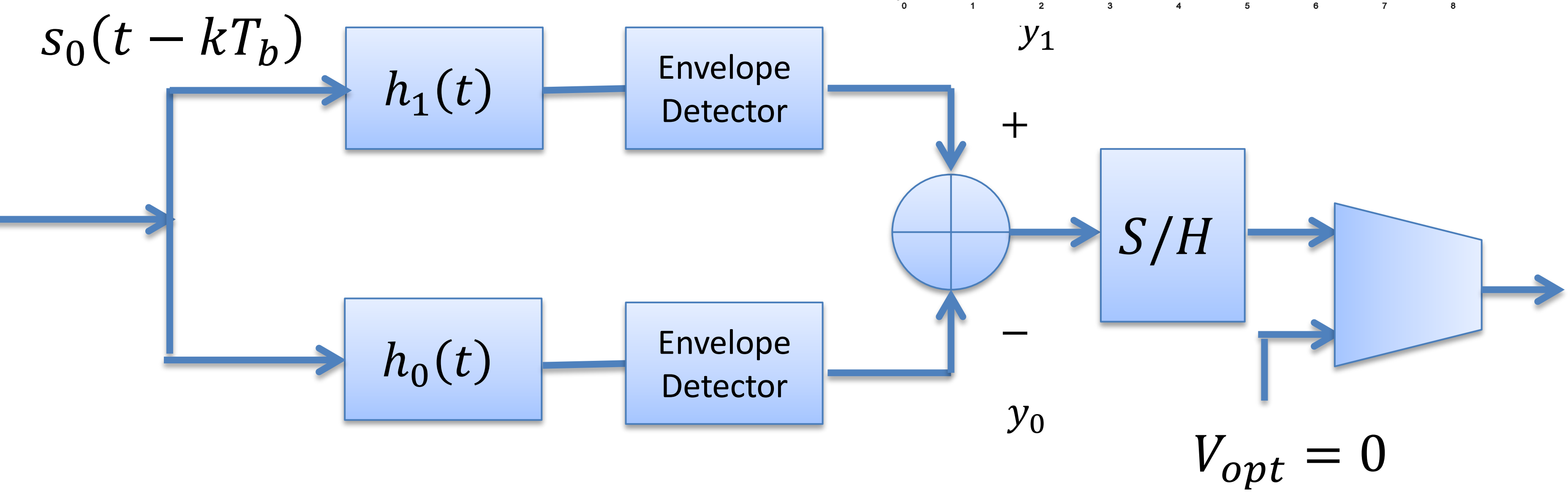


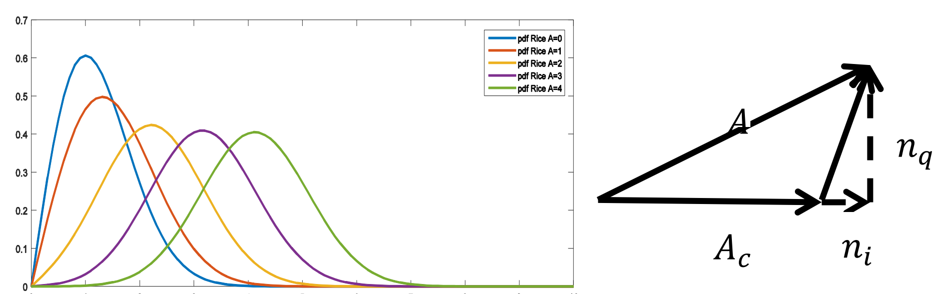


Non-coherent OOK

When a '0' is transmitted







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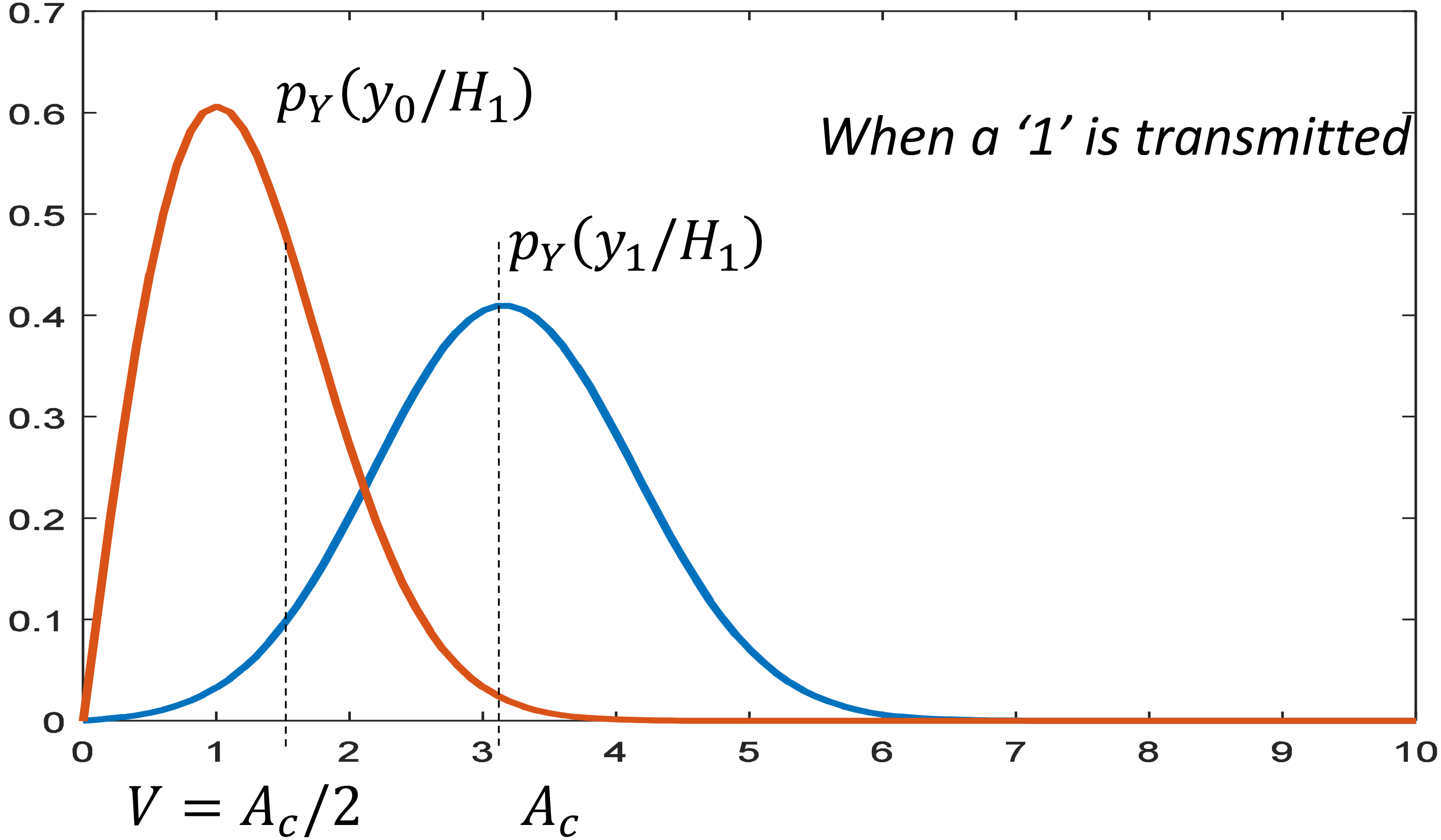
Non-coherent OOK. P_{be}



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Non-coherent OOK. P_{be}

The FSK detector recovers the difference signal of the detected symbols $y_1 - y_0$. The threshold is set to V=0. Therefore:

$$P_{e1} = P\big(((y_1 - y_0) < 0/H_1\big)$$

$$P_{e0} = P\big(((y_1 - y_0) > 0/H_0\big)$$

The error probability:

$$P_e = \frac{P_{e0} + P_{e1}}{2}$$

The error probabilities P_{e1} and P_{e0} are equal since the distributions are equal for each case. The probability of error is calculated taking into account that:

Non-coherent OOK. P_{be}

$$P_e = \int_0^{\infty} \frac{\lambda}{2\sigma^2} e^{-\left(\frac{\lambda^2+2\alpha^2}{2\sigma^2}\right)} I_0\left(\frac{\lambda\alpha}{\sigma^2}\right) d\lambda$$

$$P_e = \frac{1}{2} e^{-\left(\frac{\alpha^2}{2\sigma^2}\right)} \int_0^{\infty} \frac{\lambda}{\sigma^2} e^{-\left(\frac{\lambda^2+\alpha^2}{2\sigma^2}\right)} I_0\left(\frac{\lambda\alpha}{\sigma^2}\right) d\lambda$$

The second expression contains a term that is the integral of the Rice function over its full range. This integral is equal to one. Then:

$$P_e = \frac{1}{2} e^{-\left(\frac{{A_C}^2}{4\sigma^2}\right)} = \frac{1}{2} e^{-\frac{\gamma_b}{2}}$$

(The same value as Non-coherent)

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