Communication Systems based on Software Defined Radio (SDR)

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Digital Communication Systems

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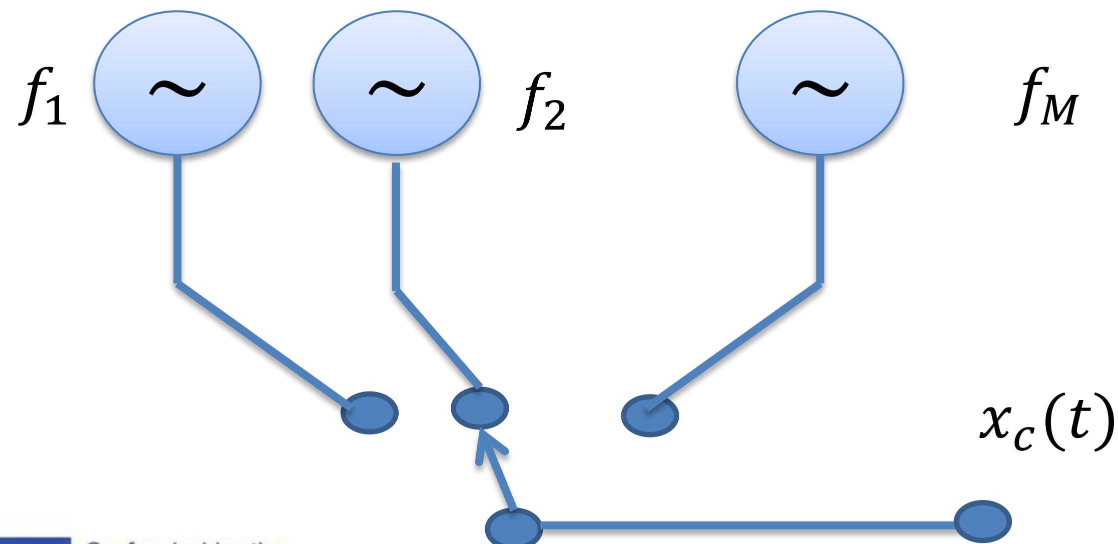
FSK. (Frequency Shift Keying)

Frequency modulation is basically done in two ways:

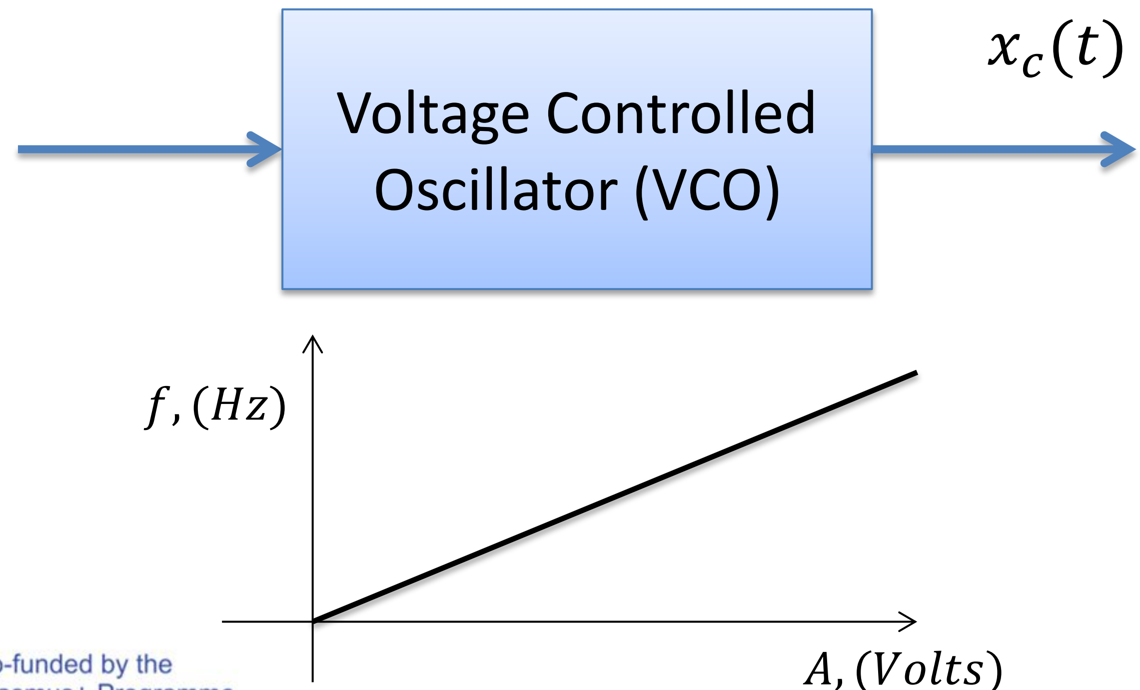
This binary type modulation consists of the transmission of two different frequencies that correspond to each of the bits to be transmitted. A first FSK generation procedure is the selection of a certain frequency of a set of oscillators through a key or switch.



CPFSK. (Continuous Phase Frequency Shift Keying)

The second method is based on the application of a digital format signal to a frequency modulator, that is, a voltage controlled oscillator (VCO)

$$x(t) = \sum_k a_k p(t - kD)$$



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FSK. (Frequency Shift Keying)

The first scheme, referred to simply as FSK, can present phase discontinuities when switching from one symbol to another, if the period of the modulating signal is not an integer number of carrier half-cycles.

If the modulating signal is an M-ary signal, an M-ary FSK signal will be generated. We have a set of oscillators with equal amplitude $A_c$and phase θ. The frequencies are related in the form

$$f_k = f_c \pm a_k f_d$$

$$a_k = \pm 1, \pm 3, \dots, \pm M - 1$$

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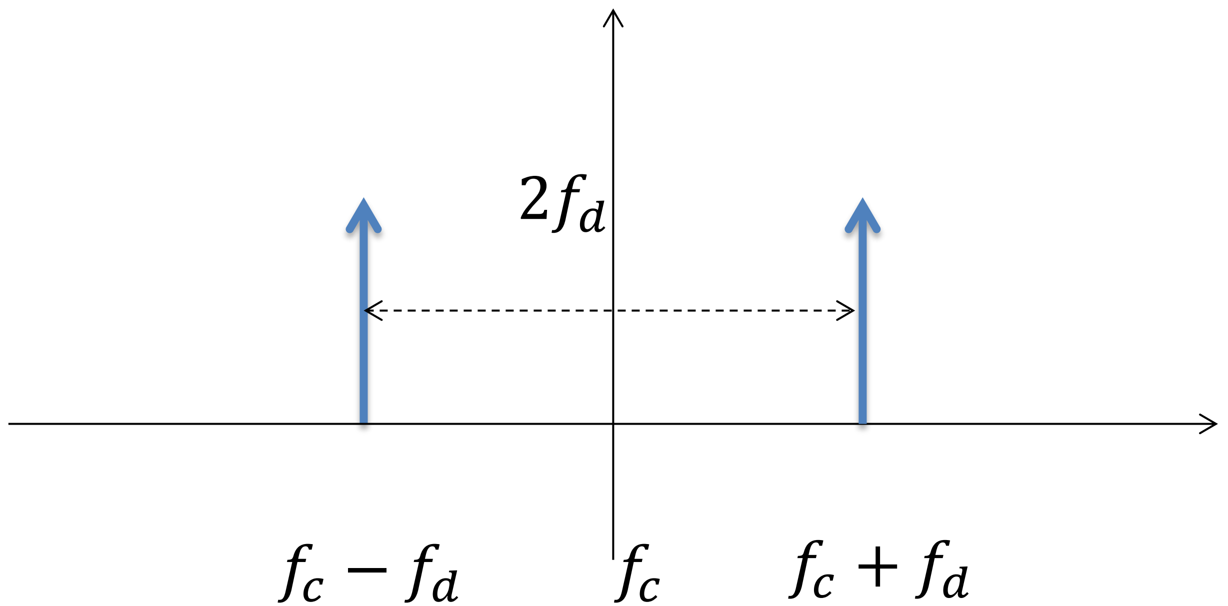
FSK. (Frequency Shift Keying)

$$x_c(t) = A_c \sum_k cos(\omega_c t + \theta + 2\pi f_d a_k t) p_D(t - kD)$$

The variable f_d is the frequency deviation to each side with respect to the central frequency.



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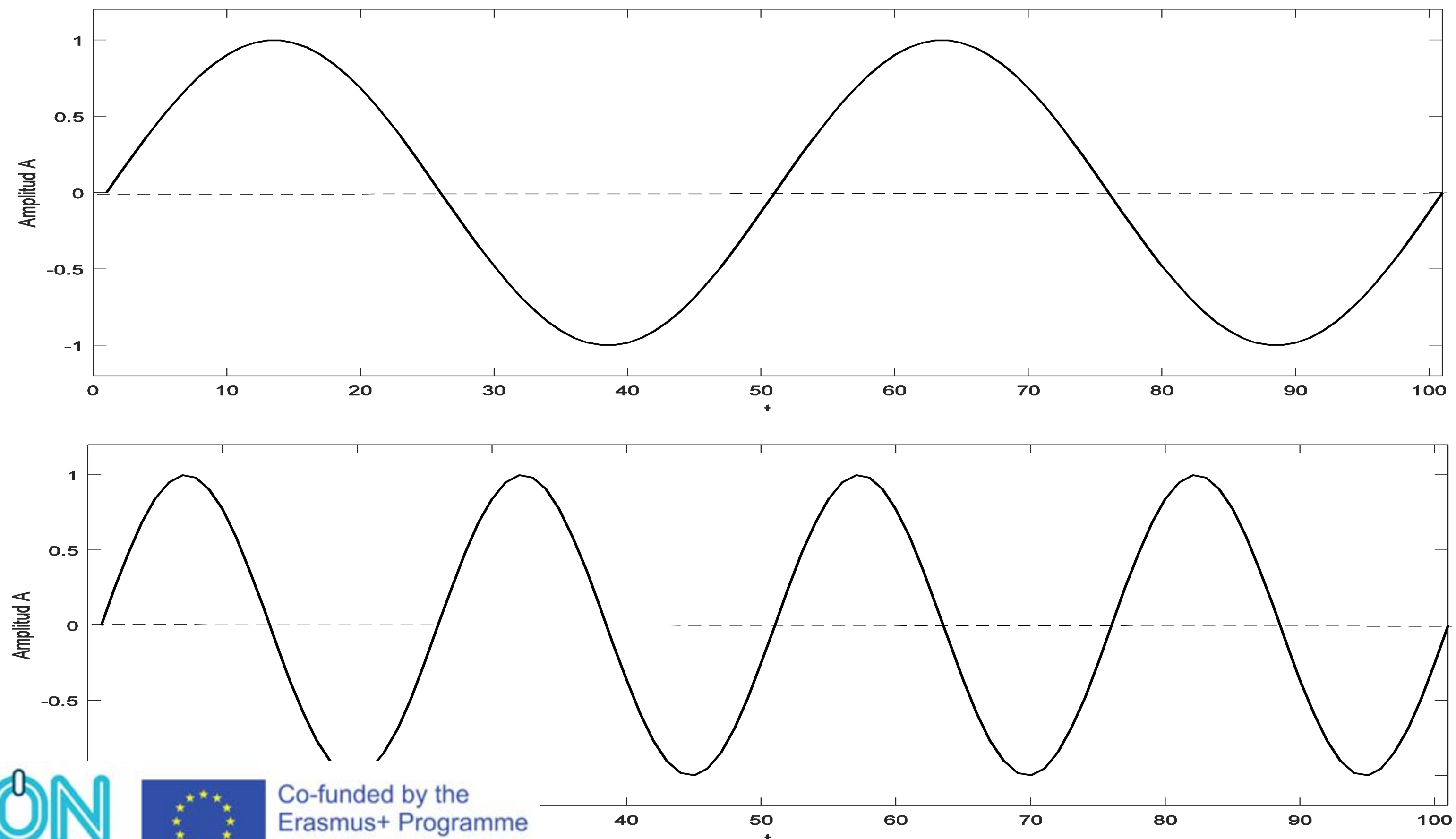
FSK. Phase continuity .

The binary type FSK modulation case is the so-called Sundee FSK. Phase continuity in an FSK scheme can be achieved if, for two signals of frequencies f_1 y f_2:



FSK. Phase continuity.

$$s_1(t) = sin(2\pi f_1 t)$$
$$s_2(t) = sin(2\pi f_2 t)$$

It can verified that the subtraction of both passes through zero at the instant of interruption of the data:

$$s_1(t) - s_2(t) = sen(2\pi f_1 t) - sen(2\pi f_2 t)$$

Using uso de $sin(\alpha) - sen(\beta) = 2cos\left(\frac{\alpha+\beta}{2}\right) sen\left(\frac{\alpha-\beta}{2}\right)$

$$sin(2\pi f_1 t) - sin(2\pi f_2 t) = 2cos\left(\frac{2\pi(f_1 + f_2)t}{2}\right) sin\left(\frac{2\pi(f_1 - f_2)t}{2}\right)$$

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FSK. Phase continuity.

In $t = T_b$

$$s_1(T_b) - s_2(T_b) = 2cos\left(\frac{2\pi(f_1 + f_2)T_b}{2}\right)sin\left(\frac{2\pi(f_1 - f_2)T_b}{2}\right) = 0$$

$$\frac{2\pi(f_1 - f_2)T_b}{2} = \frac{2\pi(2f_d)T_b}{2} = N\pi = \pi$$

$$f_d = \frac{1}{2T_b} = \frac{r_b}{2}$$

It is the minimum deviation between frequencies that allows phase continuity.

FSK. Power spectral density $G_{lp}(f)$.

Para obtener el espectro de esta señal será necesario conocer las formas de onda de las componentes $x_i(t)$ y $x_q(t)$. De la expresión de la señal modulada FSK:

$$x_c(t) = A_c \sum_k cos(\omega_c t + \omega_d a_k t + \theta) p_D(t - kD)$$

$$x_c(t) = A_c \left[\sum_k cos(\omega_c t + \theta) cos(\omega_d a_k t) p_D(t - kD) \right] - A_c \left[\sum_k sen(\omega_c t + \theta) sen(\omega_d a_k t) p_D(t - kD) \right]$$

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FSK. Power spectral density $G_{lp}(f)$..

$$x_c(t) = A_c\left[\sum_k cos(\omega_d a_k t)p_D(t-kD)\right]cos(\omega_c t+\theta)$$
$$-A_c\left[\sum_k sin(\omega_d a_k t)p_D(t-kD)\right]sin(\omega_c t+\theta)$$

The FSK modulation waveform is then presented as a waveform in its in-phase and quadrature components.

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FSK. Power spectral density $G_{lp}(f)$.

$$x_i(t) = \sum_k cos(\omega_d a_k t) p_D(t - kD)$$

$$x_q(t) = \sum_k sin(\omega_d a_k t) p_D(t - kD)$$

The a_k are binary polar form $a_k \pm 1$.

But:

$$cos(\omega_d a_k t) = cos(\omega_d t)$$
$$sin(\omega_d a_k t) = a_k sin(\omega_d t)$$

Then:

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FSK. Power spectral density $G_{lp}(f)$.

$$x_i(t) = \sum_k cos(\omega_d a_k t) p_D(t - kD) = \sum_k cos(\omega_d t) p_D(t - kD) = cos(\pi r_b t)$$

$$x_q(t) = \sum_k a_k sin(\pi r_b t) p_{T_b}(t - kT_b)$$

$$x_q(t) = \sum_k a_k sin(\pi r_b t)[u(t - kT_b) - u(t - kT_b - T_b)]$$

$$x_q(t) = \sum_k a_k sin(\pi r_b (t - kT_b) + k\pi)[u(t - kT_b) - u(t - kT_b - T_b)]$$

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FSK. Power spectral density $G_{lp}(f)$.

$$x_q(t) = \sum_k a_k sin(\pi r_b(t - kT_b) + k\pi)[u(t - kT_b) - u(t - kT_b - T_b)]$$

$$x_q(t) = \sum_k a_k (-1)^k sin\big(\pi r_b(t - kT_b)\big)[u(t - kT_b) - u(t - kT_b - T_b)]$$

$$x_q(t) = \sum_k Q_k p(t - kT_b)$$

$$p(t) = sin(\pi r_b t)[u(t) - u(t - T_b)]$$

$$Q_k = a_k(-1)^k$$

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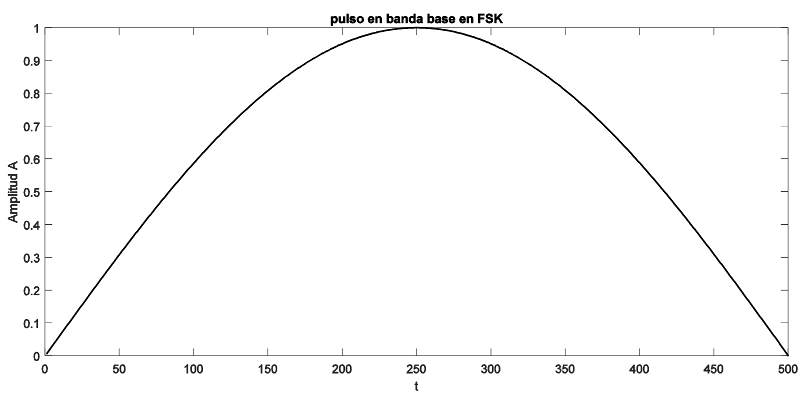
FSK. Power spectral density $G_{lp}(f)$.

The in-phase carrier contains no information. In fact it will only produce two $\delta(f \pm r_b/2)$ in the resultant spectra.

The quadrature component is transformed into a PAM signal where the pulse shape $p(t)$ is no longer square.



$$p(t) = sin(\pi r_b t)[u(t) - u(t - T_b)]$$

The statistical averages in $x_q(t)$ are:

$$\overline{Q_k} = \overline{a_k}(-1)^k = 0;\ \overline{Q_k{}^2} = \overline{a_k{}^2}(-1)^{2k} = 1$$

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FSK. Power spectral density $G_{lp}(f)$.

La expresión de la densidad espectral de potencia es función de la estadística de la información y de la forma espectral de pulso, que ha dejado de ser un pulso cuadrado, para convertirse en un pulso que es un semiciclo de una senoide.

El espectro asociado se calcula obteniendo la transformada de Fourier de la expresión de $p(t)$.

$$|P(f)|^2 = \frac{1}{4{r_b}^2}\left[sinc\left(\frac{f - r_b/2}{r_b}\right) + sinc\left(\frac{f + r_b/2}{r_b}\right)\right]^2$$

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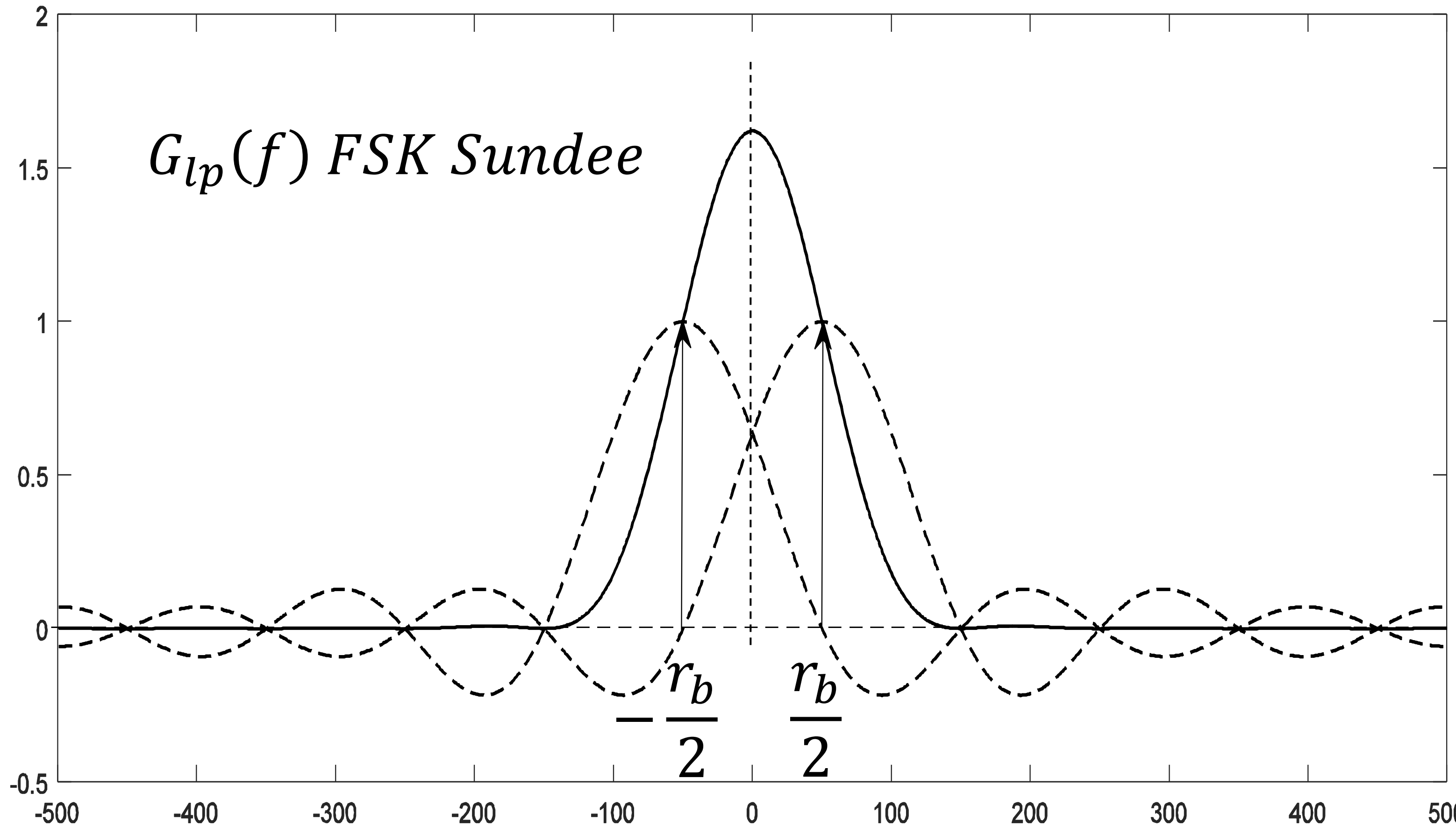
FSK. Power spectral density $G_{lp}(f)$.



FSK. Power spectral density $G_{lp}(f)$.

The final spectrum is componsed by two functions $\delta(f \pm r_b/2)$ and the calculated spectrum $|P(f)|^2$.

$$G_{lp}(f) = \frac{1}{4}\left[\delta\left(f - \frac{r_b}{2}\right) + \delta\left(f + \frac{r_b}{2}\right)\right] + r_b|P(f)|^2$$

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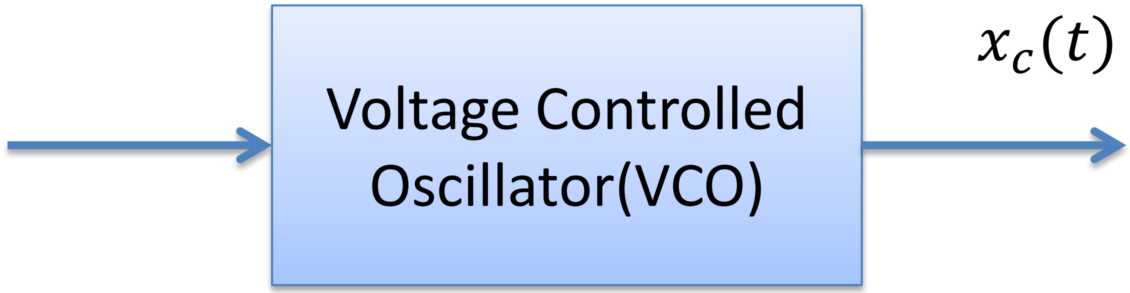
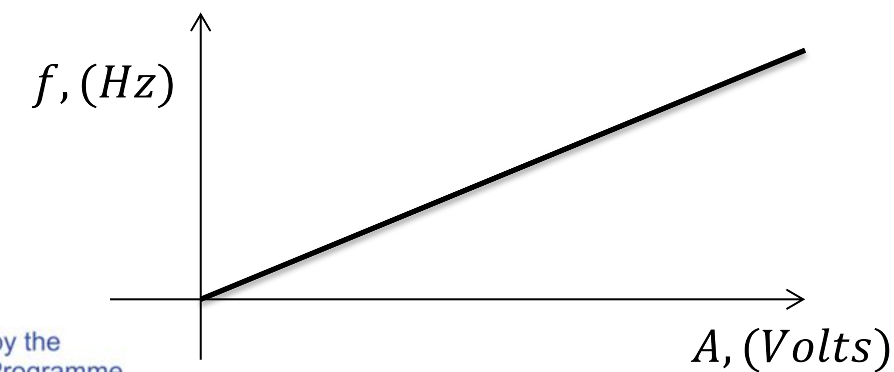
CPFSK. (Continuous Phase Frequency Shift Keying)

The second procedure is based on the application of a digital format signal to a frequency modulator, that is, a voltage controlled oscillator (VCO).

$$x(t) = \sum_k a_k p(t - kD)$$





CPFSK. (Continuous Phase Frequency Shift Keying)

Digital frequency modulation can also be accomplished by applying a digitally amplitude modulated (PAM) waveform to a device whose output is a frequency that depends on the amplitude applied to the input, called a voltage controlled oscillator.

In this sense the system diagram is identical to a typical analog signal FM modulator.

The input signal is:

$$x(t) = \sum_k a_k p_D(t - kD); \quad a_k = \pm 1, \pm 2, \dots, \pm(M-1)$$

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CPFSK. (Continuous Phase Frequency Shift Keying)

The modulated signal will then be:

$$x_c(t) = A_c \sum_k cos\left(\omega_c t + \theta + 2\pi f_d \int_0^t x(\lambda) d\,\lambda \right) p_D(t - kD)$$

$$\int_0^t x(\lambda) d\,\lambda = \sum_{k=0}^{\infty} \int_0^t a_k p_D(\lambda - kD)\, d\lambda$$

Where in the interval $kD < \lambda < (k+1)D$ it is verifieds that $p_D(\lambda - kD) = 1$.

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CPFSK. (Continuous Phase Frequency Shift Keying)

Then:

$$\int_0^t x(\lambda)d\,\lambda = a_0 t \qquad\qquad 0 < t < D$$

$$\int_0^t x(\lambda)d\,\lambda = a_0 D + a_1(t - D) \qquad\qquad D < t < 2D$$

$$\vdots$$

$$\int_0^t x(\lambda)d\,\lambda = \left(\sum_{j=0}^{k-1} a_j\right) D + a_k(t - kD) \qquad\qquad kD < t < (k+1)D$$

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CPFSK. (Continuous Phase Frequency Shift Keying)

$$x_c(t) = A_c \sum_k cos(\omega_c t + \theta + \varphi_k + 2\pi f_d(t - kD))p_D(t - kD)$$

$$\varphi_k = \left(\sum_{j=0}^{k-1} a_j\right)D \qquad \varphi_k = 0 \text{ para } k = 0$$

The most widely used case in practice of CPFSK modulation is the so-called MSK (Minimum Shift Keying).

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MSK. (Minimum Shift Keying)

MSK is the type of digital frequency modulation that has the minimum modulation index for orthogonal symbol modulation.

In this case the deviation is half of that applied in Sundee FSK, so that:

$$f_d = \frac{r_b}{4}$$

In the phase expression φ_k, where $a_k = \pm 1$:

$$\varphi_k = \left(\sum_{j=0}^{k-1} a_j \right) \frac{\pi}{2}$$

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MSK. (Minimum Shift Keying)

The phase is a continuous sequence of values that alternate by $\pm\frac{\pi}{2}$.

The modulation symbols are:

$$s_0(t) = A_c p(t) cos(2\pi(f_c - f_d)t)$$
$$s_1(t) = A_c p(t) cos(2\pi(f_c + f_d)t)$$

The correlation between the symbols defines what is called the orthogonality coefficient:

$$E_{10} = \int_0^{T_b} s_0(t)s_1(t)dt = \frac{T_b {A_c}^2}{2} sinc[2(2f_d)T_b] = \frac{T_b {A_c}^2}{2} sinc\left(\frac{f_d}{(r_b/4)}\right)$$

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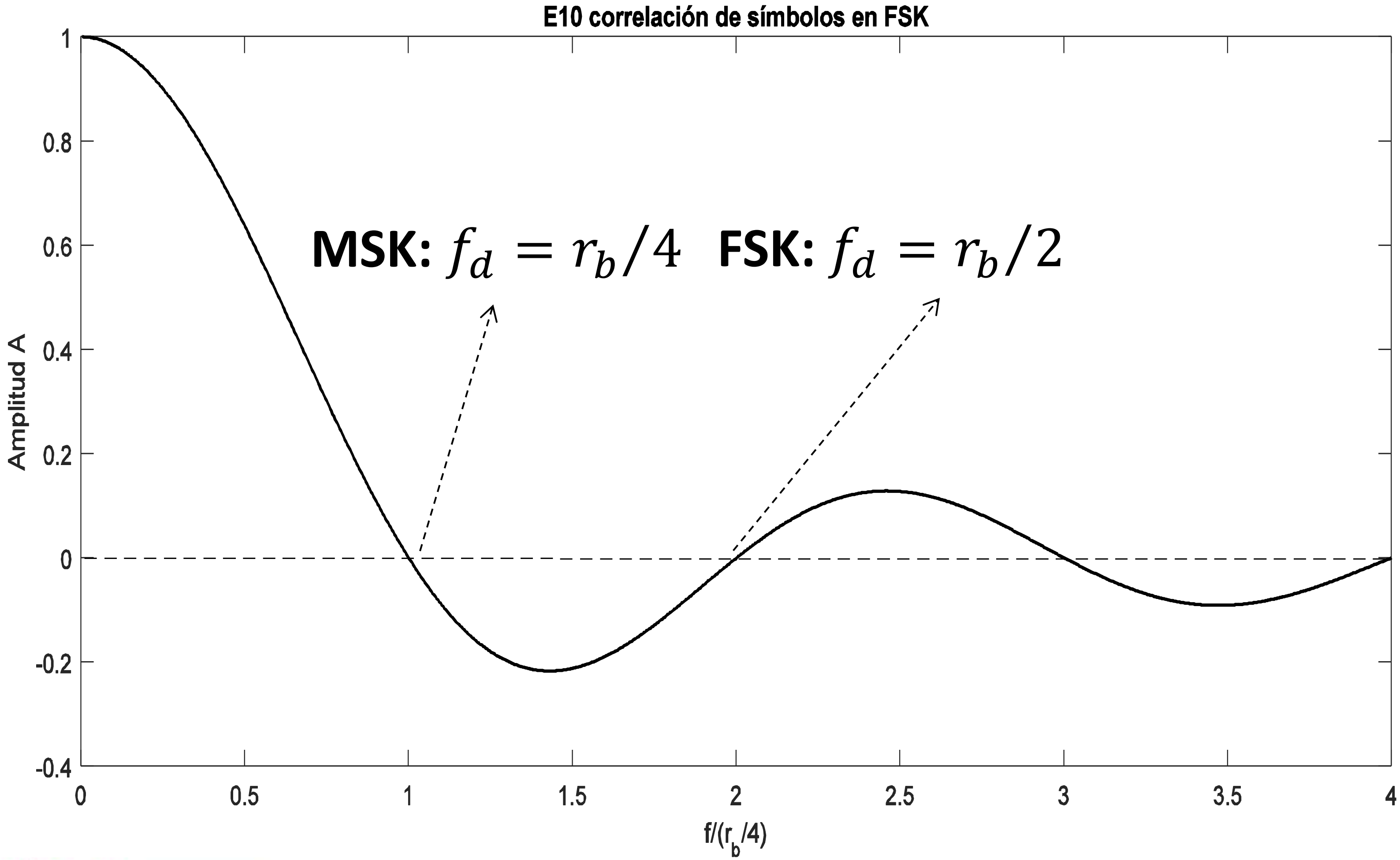
MSK. Correlation E_{10}.



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MSK. (Minimum Shift Keying)

In the expression of E_{10} there is a second term not shown, which has to do with the sum of the frequencies, and which is considered equal to zero based on two possibilities:

If $f_c = Nr_b$ the expression is exact, since that second term vanishes.

If $f_c \gg r_b$ the expression is approximate, and is based on the fact that the value of the second term is null because it is a number obtained in a sinc function with the argument $\gg$1).

The expression of $x_c(t)$ can be decomposed in the form:

MSK. (Minimum Shift Keying)

$$x_c(t) = A_c \sum_k cos(\omega_c t + \theta + \varphi_k + a_k \omega_d (t - kD))p_D(t - kD)$$

$$x_c(t) = A_c \left[\sum_k cos(\omega_c t + \theta) cos(\varphi_k + a_k \omega_d (t - kD))p_D(t - kD) \right]$$
$$- A_c \left[\sum_k sin(\omega_c t + \theta) sen(\varphi_k + a_k \omega_d (t - kD))p_D(t - kD) \right]$$

$$x_c(t) = A_c \left[\sum_k cos(\varphi_k + a_k \omega_d (t - kD))p_D(t - kD) \right] cos(\omega_c t + \theta)$$
$$- A_c \left[\sum_k sin(\varphi_k + a_k \omega_d (t - kD))p_D(t - kD) \right] sin(\omega_c t + \theta)$$

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MSK. (Minimum Shift Keying)

Taking into account that $D = T_b$, the components in phase and quadrature are then :

$$x_i(t) = \sum_k cos(\varphi_k + a_k \omega_d (t - kD)) p_D(t - kD)$$

$$x_q(t) = \sum_k sin(\varphi_k + a_k \omega_d (t - kD)) p_D(t - kD)$$

With an algebraic work based on trigonometry it is possible to take the previous expressions to the form:

MSK. (Minimum Shift Keying)

$$x_i(t) = \sum_{k\ even} I_k p\ (t - kT_b)$$

$$x_q(t) = \sum_{k\ odd} Q_k p(t - kT_b)$$

Where:

$$I_k = cos(\varphi_k)$$
$$Q_k = sin(\varphi_k)$$

$$p(t) = cos\left(\frac{\pi r_b t}{2}\right)[u(t + T_b) - u(t - T_b)]$$

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MSK. (Minimum Shift Keying)

If the input value is assigned such that:

$a_k = +1$ the bit is a one $'1'$,

$a_k = -1$ the bit is a zero $'0'$,

For a given sequence of input the output can be seen as a sequence of phases that change by $\pm\pi/2$.

For example, if the input sequence were $\mathbf{1\,0\,0\,1\,1\,0\,1\,0\,1}$, the output would be a sequence of phases like the one seen in the figure:

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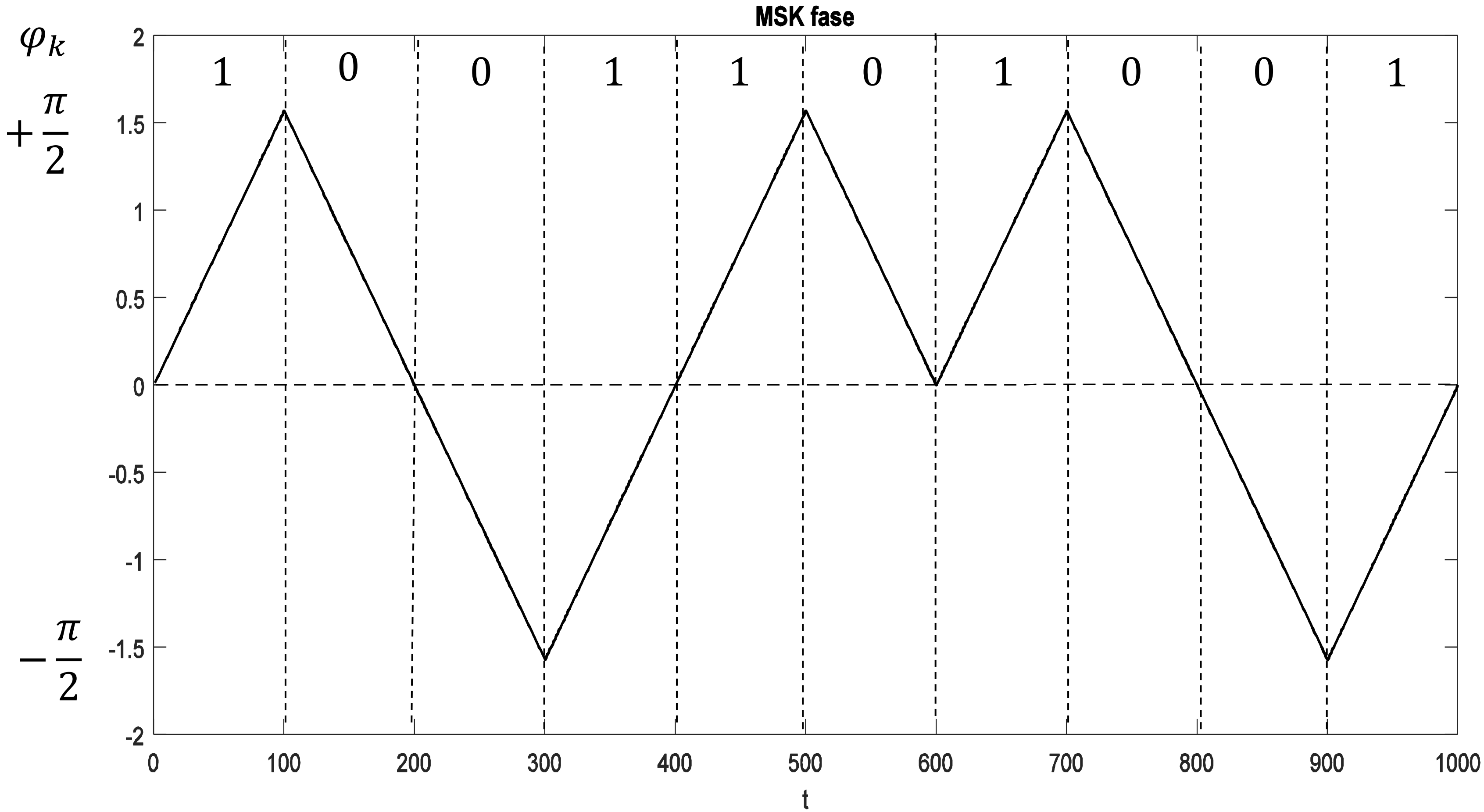
MSK. Phase.



The in-phase and quadrature components also represent the phase information.

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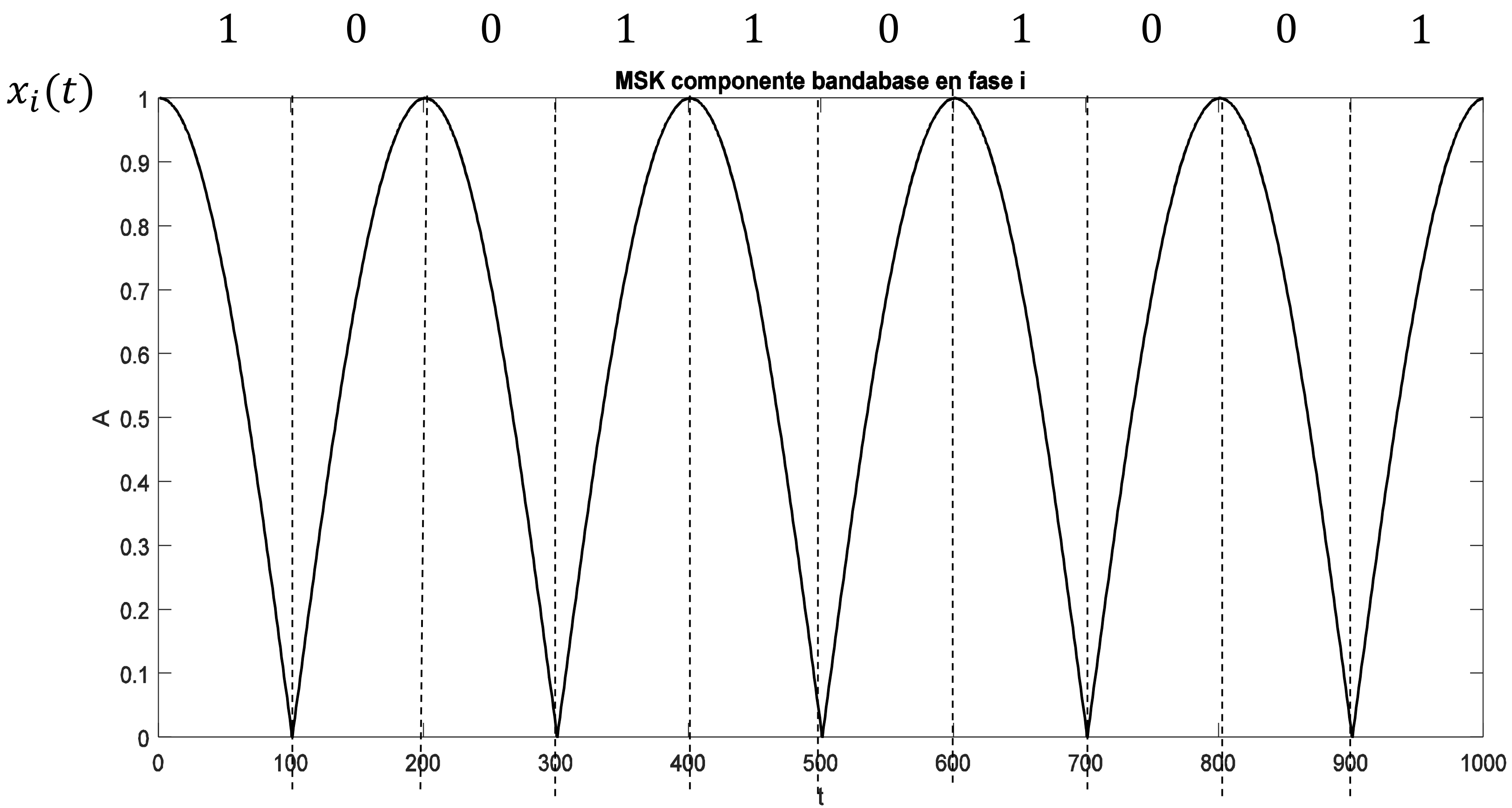
MSK. Component I.



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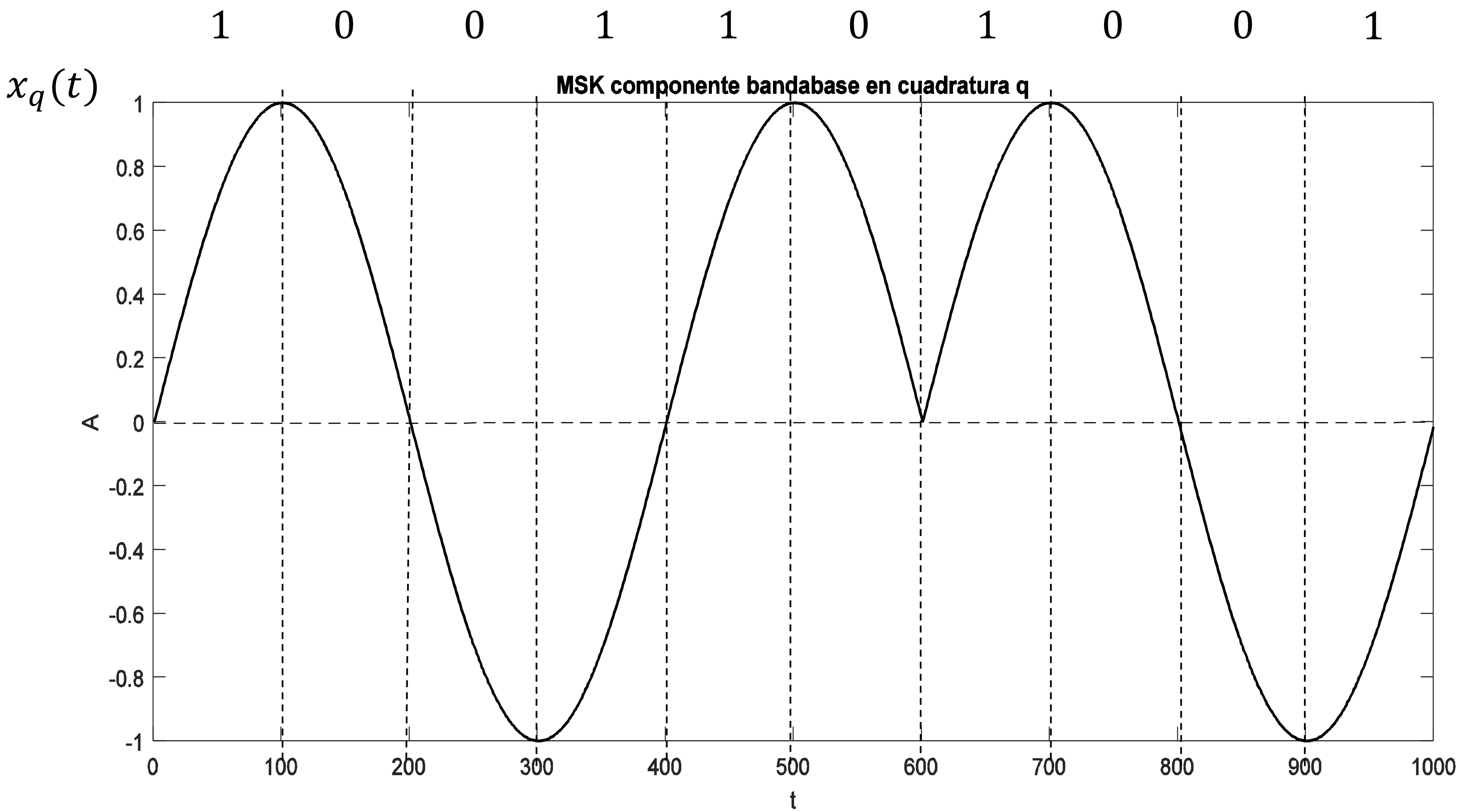
MSK. Component Q.



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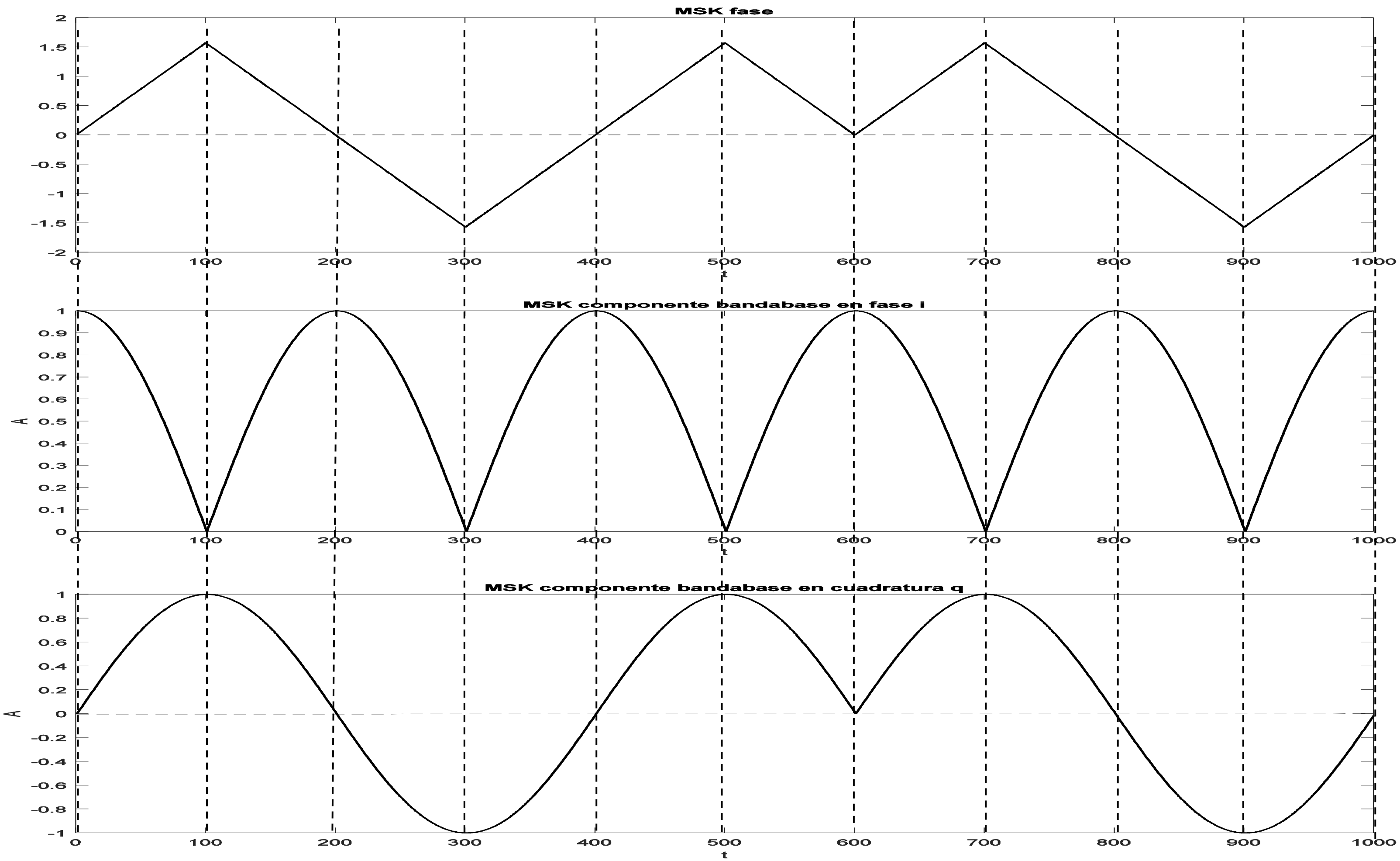
MSK. I and Q Components.

1 0 0 1 1 0 1 0 0 1



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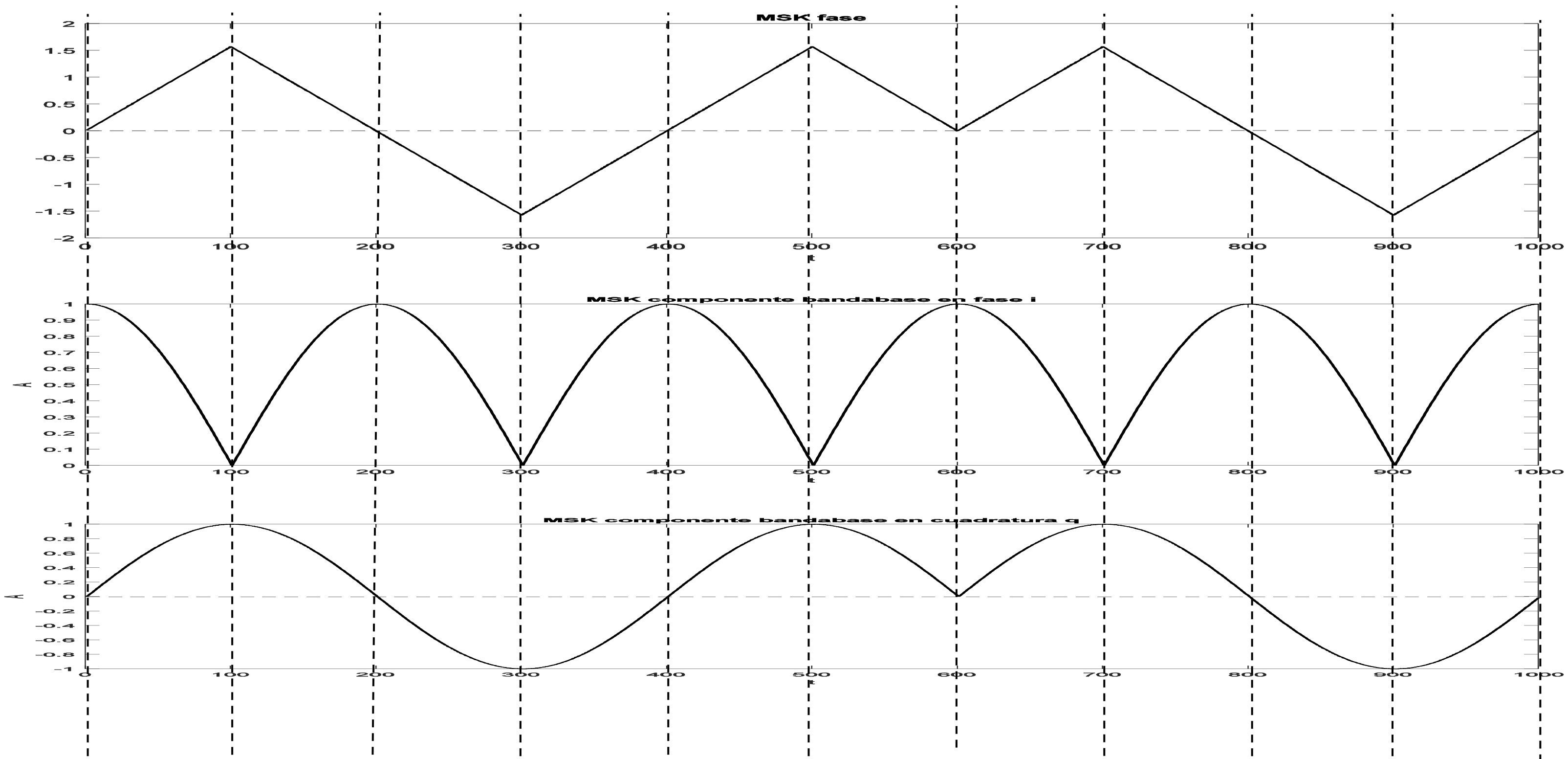
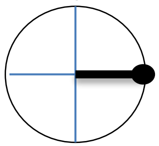
MSK. I and Q Components.

1 0 0 1 1 0 1 0 0 1





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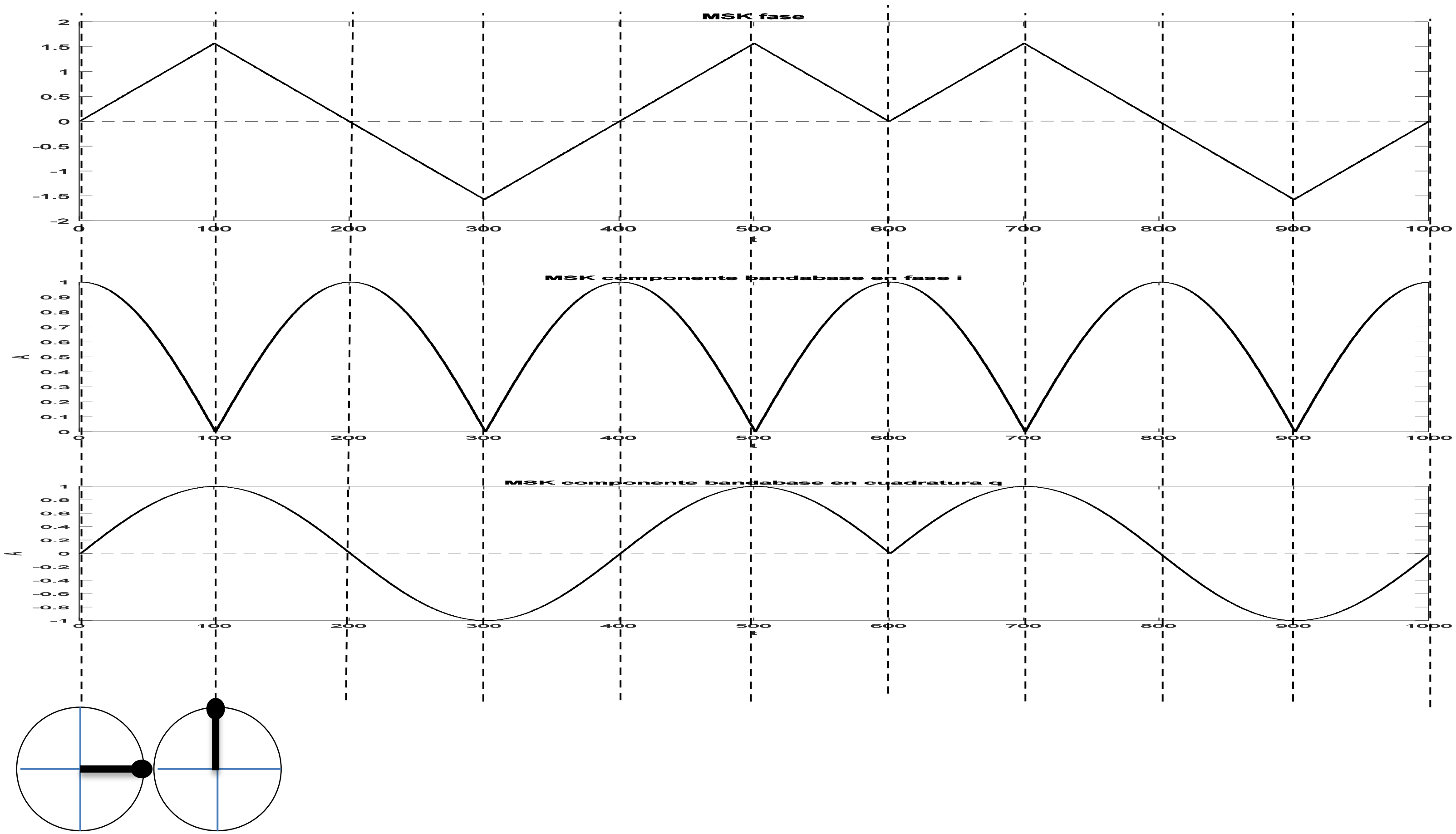
MSK. I and Q Components.

1 0 0 1 1 0 1 0 0 1



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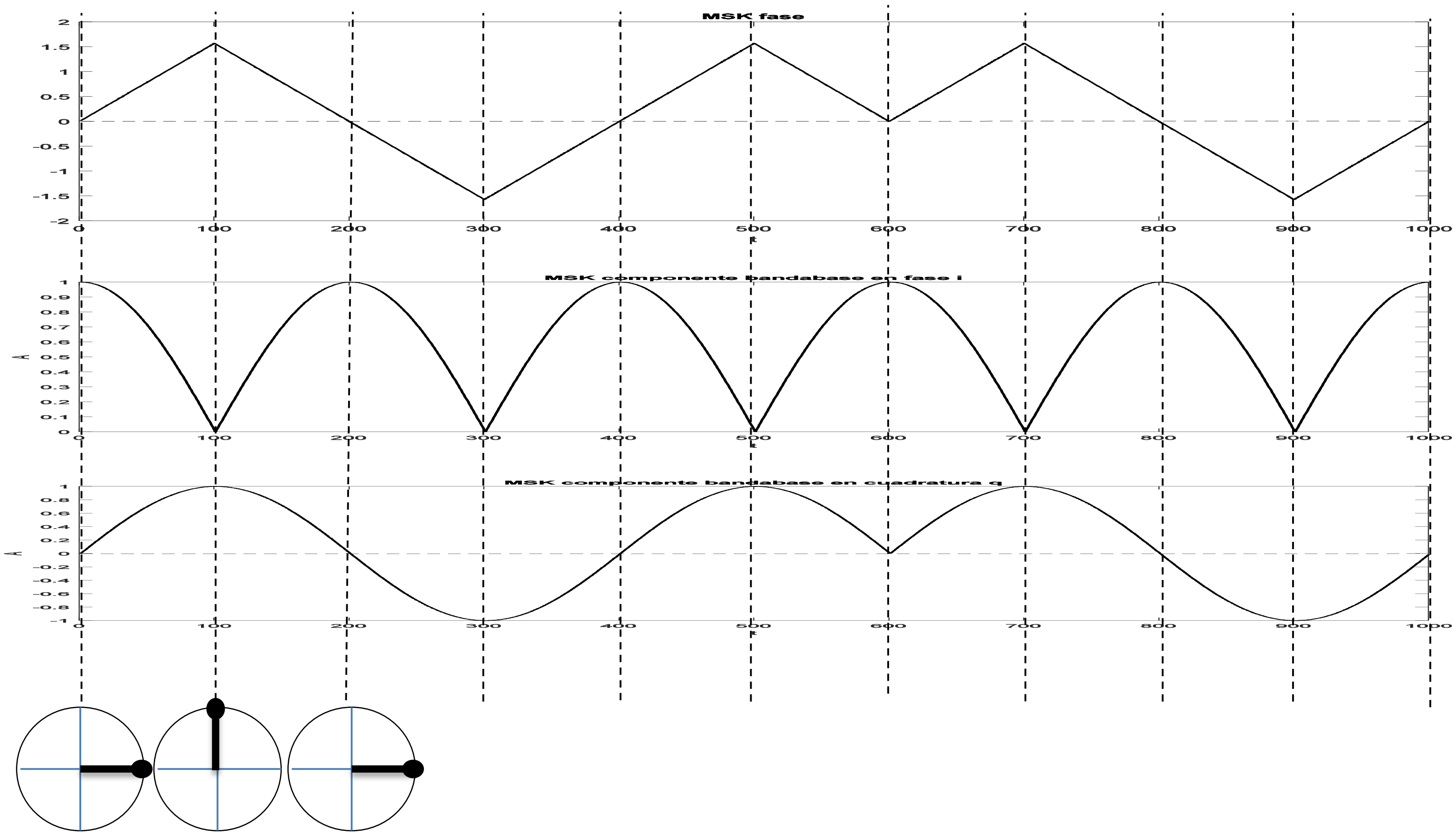
MSK. I and Q Components.

1 0 0 1 1 0 1 0 0 1



MSK. I and Q Components.

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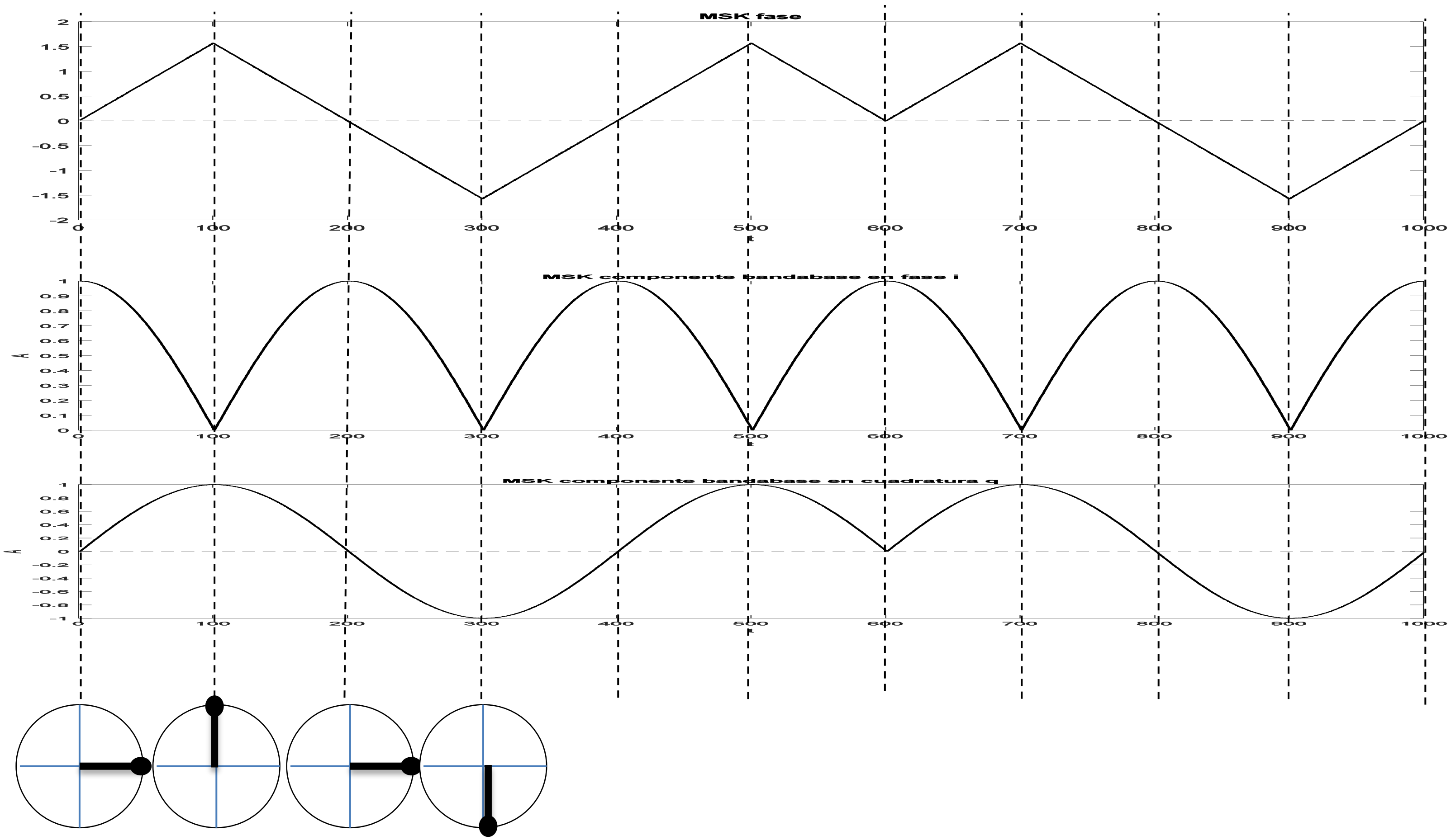

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MSK. I and Q Components.

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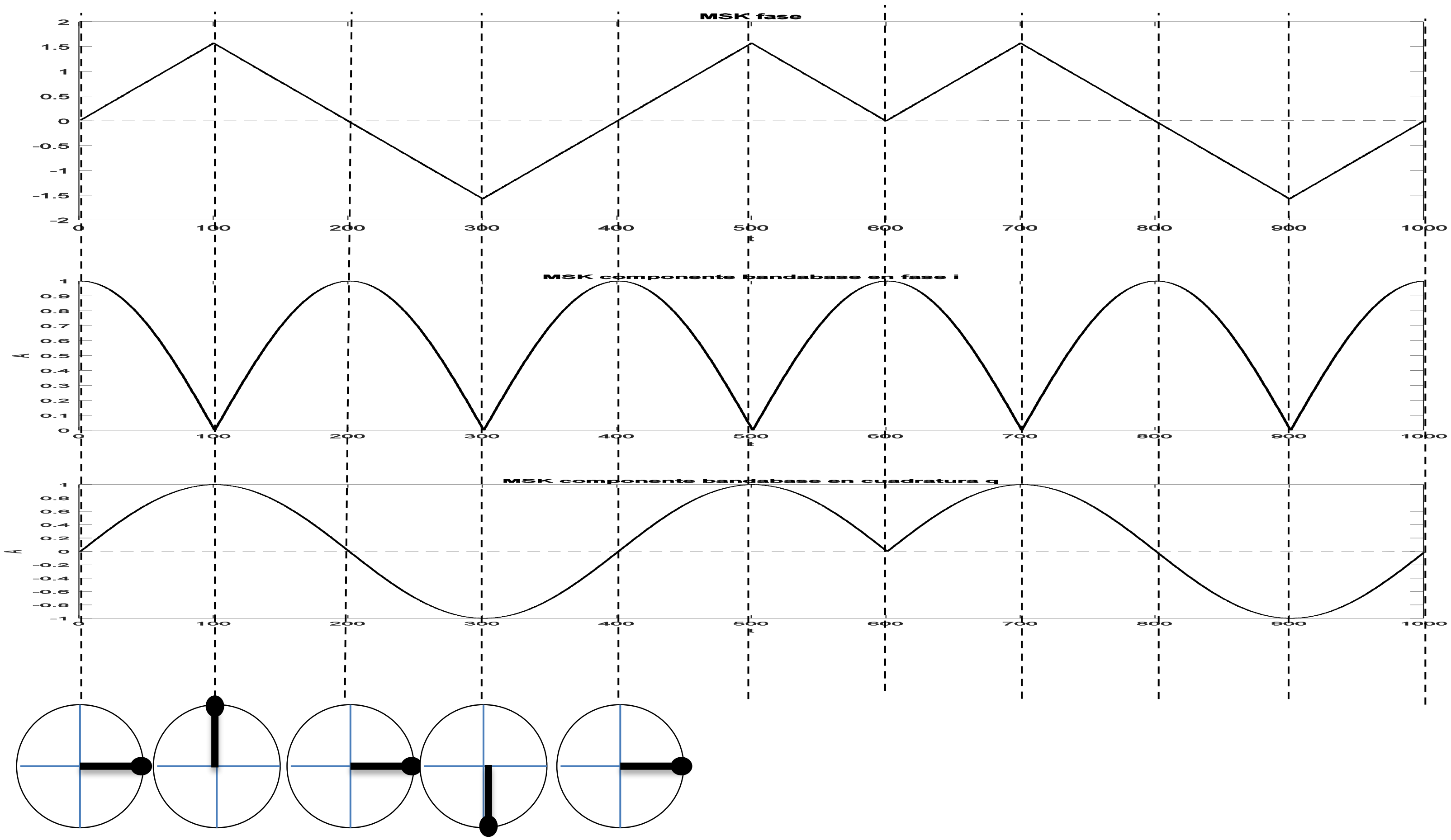

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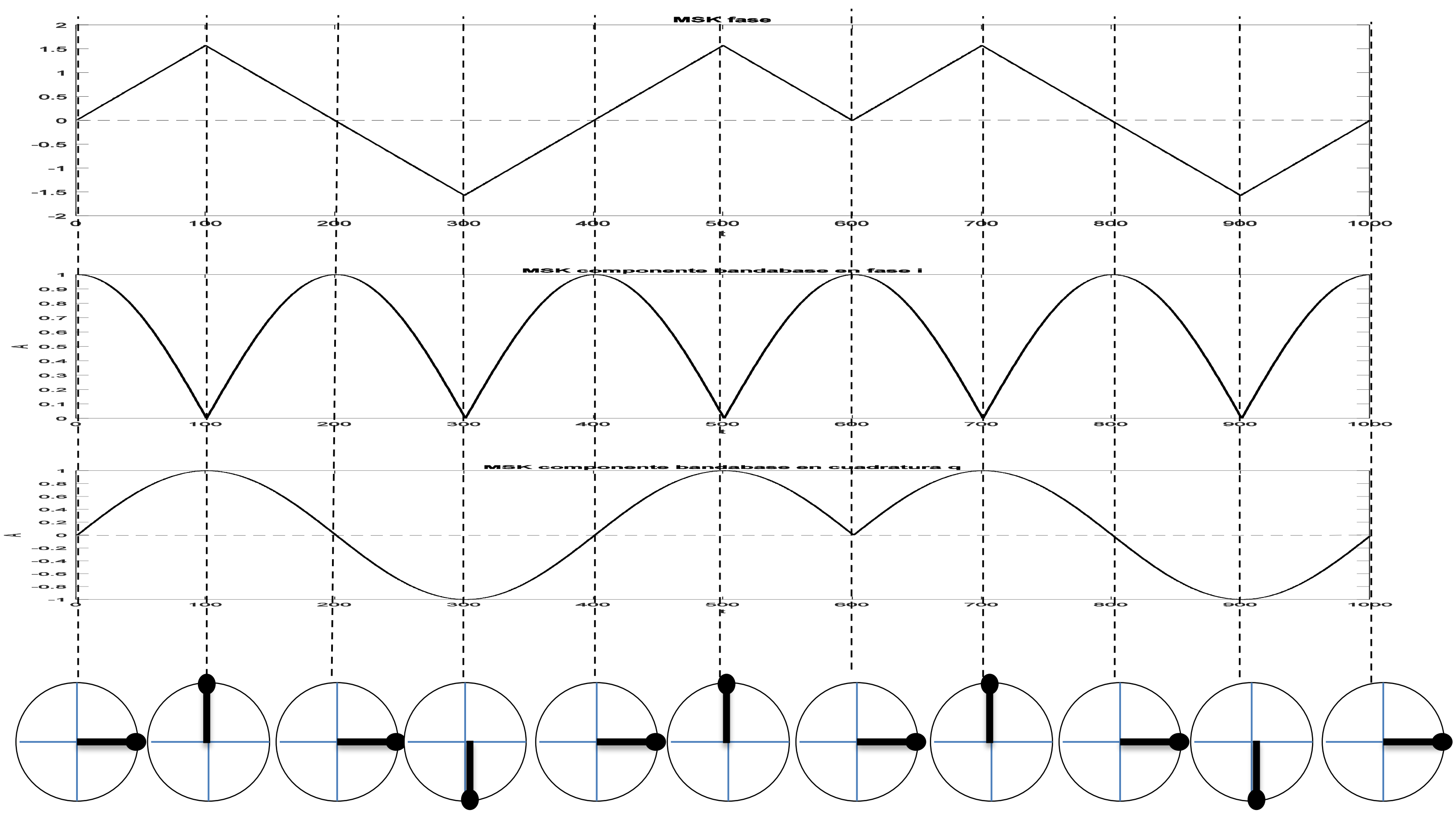
MSK. I and Q Components.

1 0 0 1 1 0 1 0 0 1



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MSK. Spectral density $G_{lp}(f)$.

Where

$$I_k = cos(\varphi_k)$$
$$Q_k = sen(\varphi_k)$$

The average statistical values are:

$$\overline{I_k} = \overline{Q_k} = 0$$
$$\overline{I_k}^2 = \overline{Q_k}^2 = 1$$

And being the shape of the modified pulse:

$$p(t) = cos\left(\frac{\pi r_b t}{2}\right)[u(t + T_b) - u(t - T_b)]$$

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MSK. Spectral Density $G_{lp}(f)$.

$$|P(f)|^2 = \frac{1}{{r_b}^2}\left[sinc\left(\frac{f - r_b/4}{r_b/2}\right) + sinc\left(\frac{f + r_b/4}{r_b/2}\right)\right]^2$$

Then:

$$G_{lp}(f) = {\sigma_a}^2 r_b |P(f)|^2 = \frac{1}{\pi^2 r_b}\left[\frac{cos(2\pi f/r_b)}{\left(\frac{4f}{r_b}\right)^2 - 1}\right]$$

The spectrum of the MSK modulation is very efficient due to two factors, one of them is that it efficiently uses the two components in phase and quadrature to naturally distribute and multiplex the information, doubling the duration in time of the baseband waveforms, the other is the conformation of the pulse to another softer than the rectangular one, in the form of a half-cycle of cosine.

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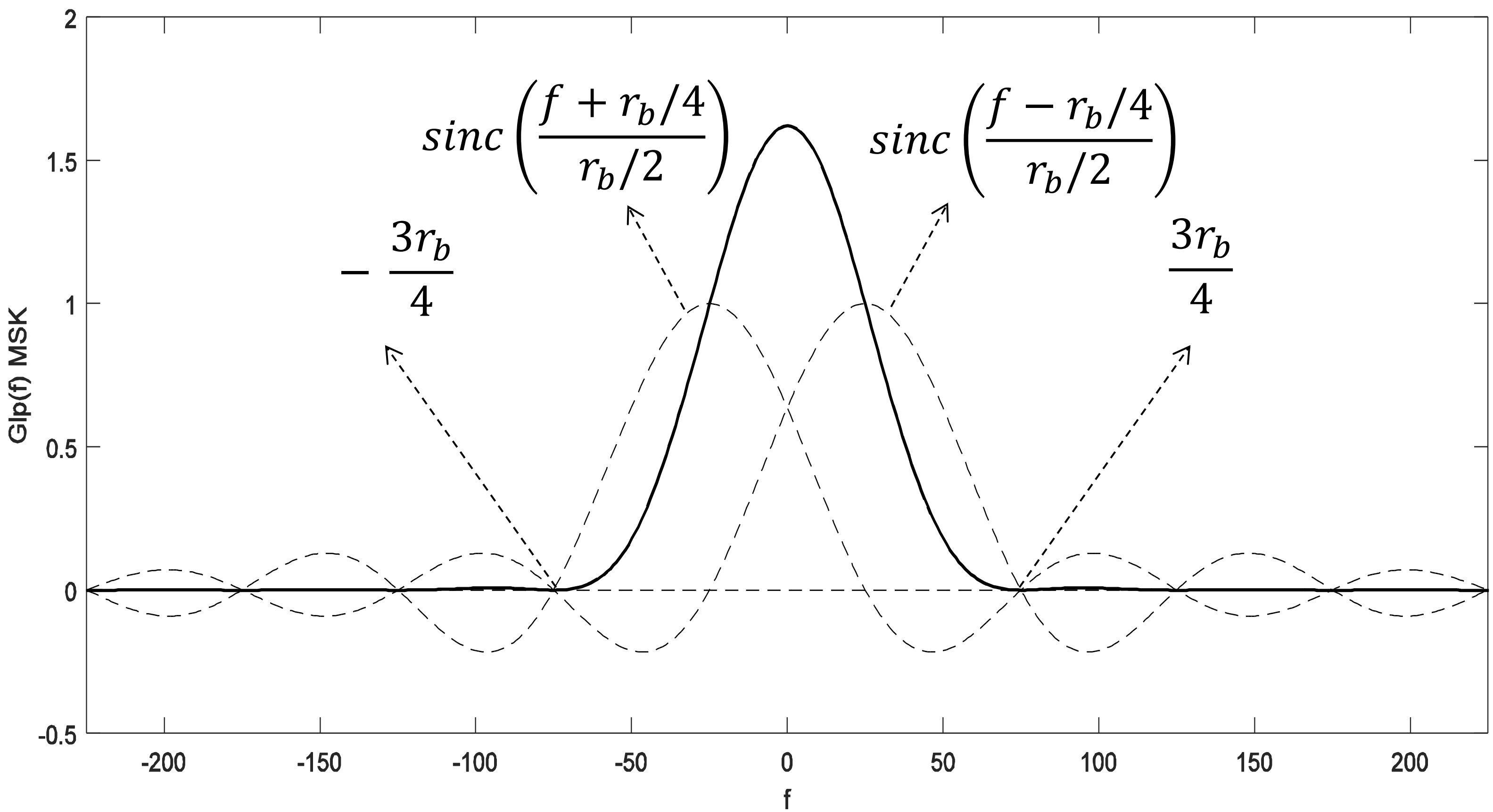
MSK. Spectral density $G_{lp}(f)$.



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Binary modulation. Spectral Density $G_{lp}(f)$.

Summarizing, the spectra $G_{lp}(f)$ for the digital modulations 2ASK (OOK), 2PSK (PRK), 2FSK (Sundee FSK) and MSK are represented in the following figure.

The spectra are normalized in amplitude and frequency, in the latter case as factors of the bit rate r_b.

The spectra of the 2ASK and 2PSK modulations are shown in a single graph, remembering that in the case of 2ASK the existence of a delta in the frequency origin must be considered, not represented in the comparison.

The FSK spectrum has an abrupt fall on the sides, and fewer lobes on either side of the center of the spectrum. MSK shows the same effect and lower bandwidth for the same case, having a spectral efficiency equal to $r_b/B_T = 2$.

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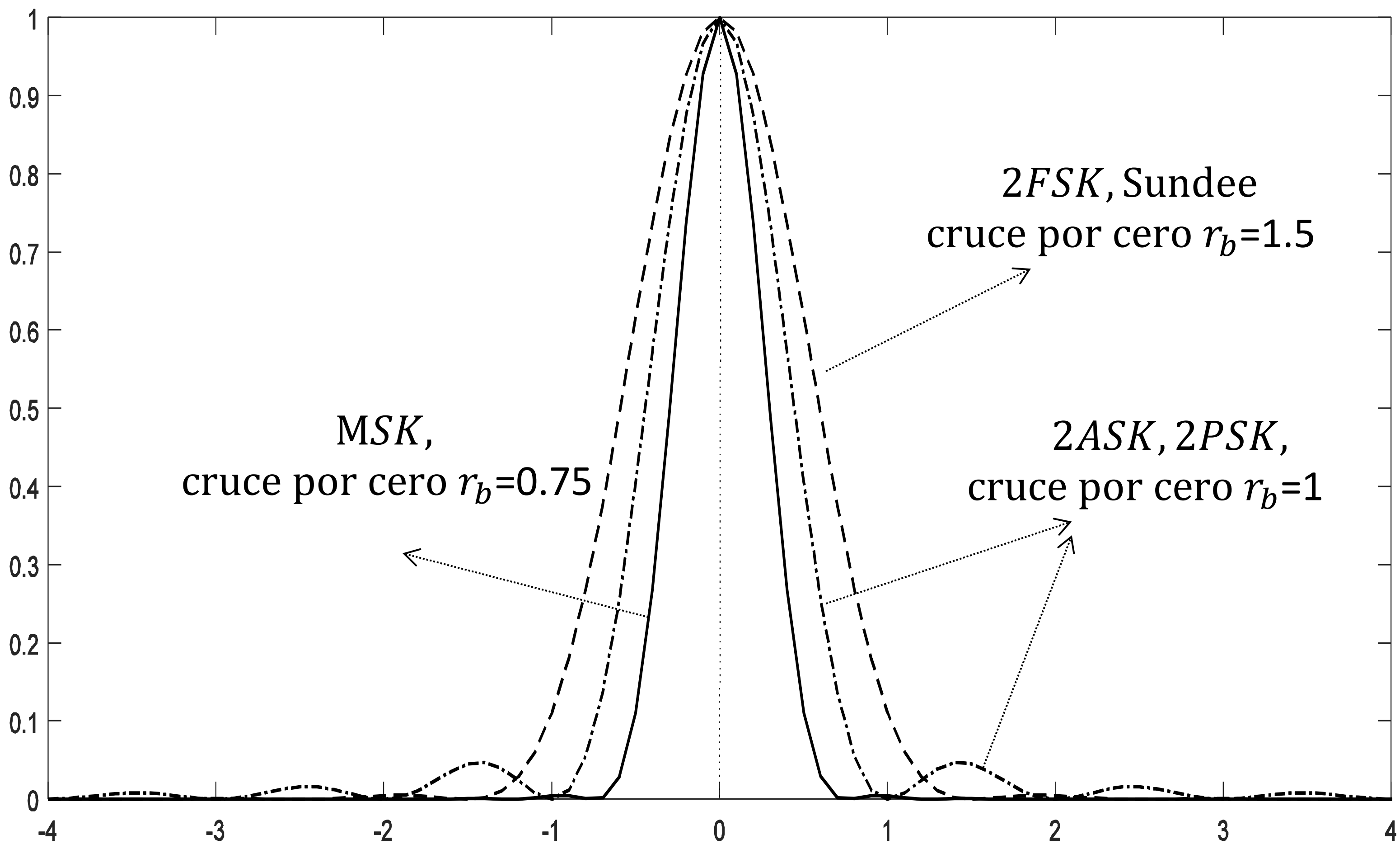
Binary Modulation. Spectral density $G_{lp}(f)$.



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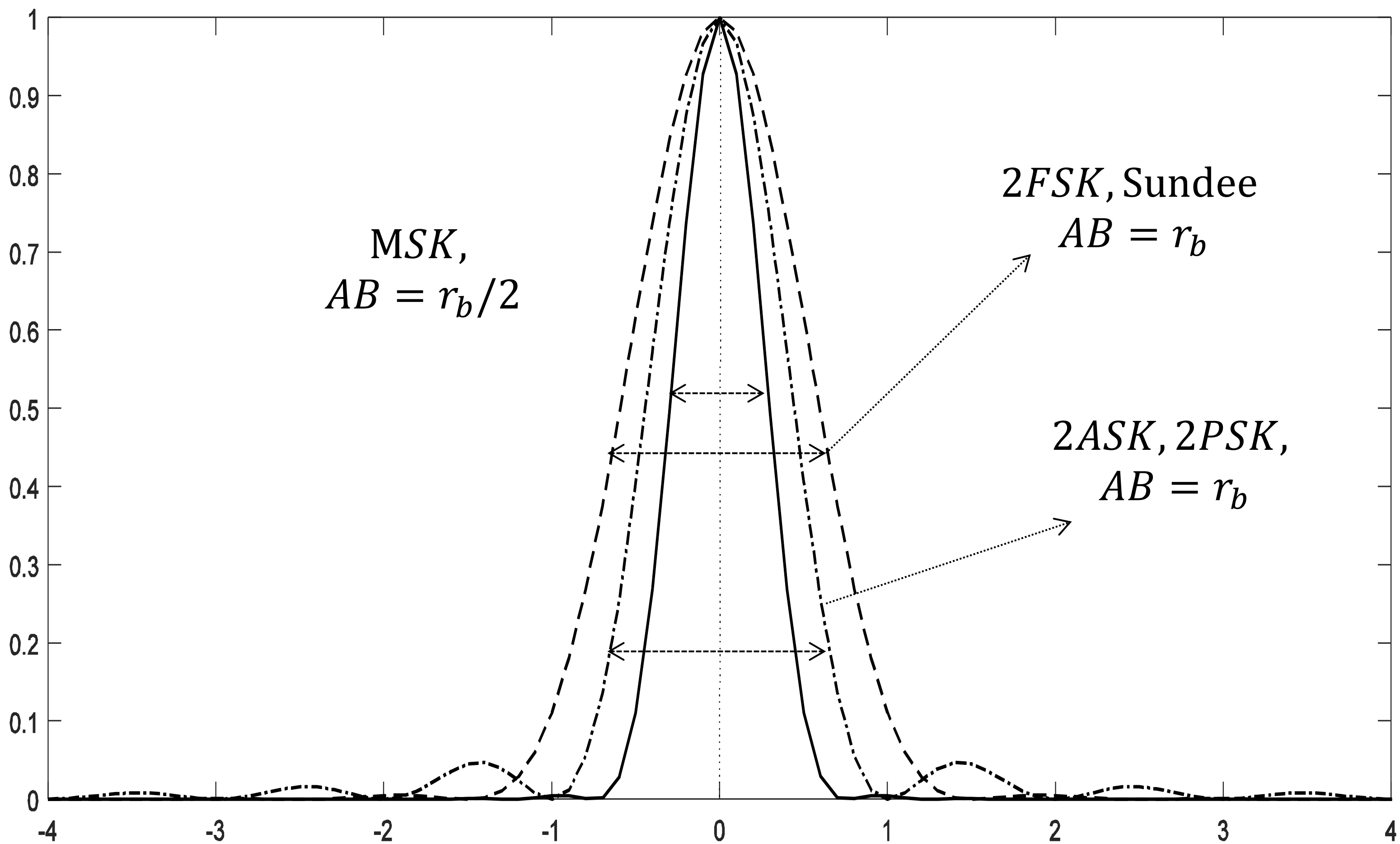
Binary Modulation. Spectral density $G_{lp}(f)$.



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