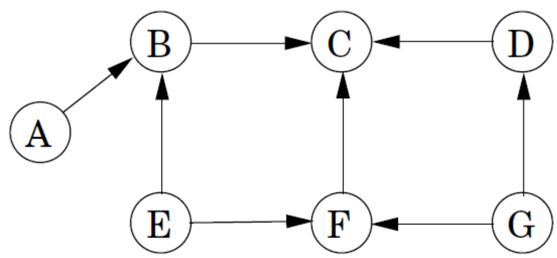
COMS 4771 HW5

for practice only; no need to submit

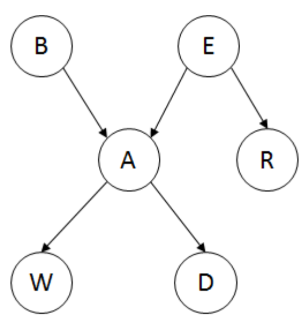
1 **[Directed graphical models]**

(a) Consider the following directed graphical model:



(i) Which variables are independent of A?
(ii) Which variables are independent of D?
(iii) Which variables are independent of D given F?
(iv) Which variables are independent of D given C?
(v) Define random variables $X = (A, B, E)$, $Y = (C, F)$, $Z = (D, G)$. Draw a directed model which correctly represents the dependencies between these variables. (It should have as few edges as possible and three nodes)?

(b) Consider the following network over six binary variables.



The semantics of this network are as follows. The alarm (A) in your house can be triggered by two possible events: a burglary (B), or an earthquake (E). If there is a strong enough earthquake, there may be a news report (R). If the alarm is ringing, your neighbor Watson calls (W) or your daughter calls (D) you (if they happen to hear the alarm), they may call you even if the alarm is not ringing just to say ‘hi’.

(i) Give a simple expression for the joint distribution $P\big[(A, B, D, E, R, W)\big]$.

Given the probability functions: $P(E) = 0.01$, $P(b) = 0.0001$, and

E	$P(R = 1 \mid E)$
0	0.0
1	0.4

A	$P(D = 1 \mid A)$
0	0.0
1	0.7

A	$P(W = 1 \mid A)$
0	0.1
1	1.0

B	E	$P(A = 1 \mid B, E)$
0	0	0.01
0	1	0.2
1	0	0.95
1	1	0.96

(ii) What is the probability that Watson will call?

(iii) What is the probability of a burglary, given that Watson called but the daughter didn’t?

(iv) What is the probability of an earthquake, given that there was no news report, but both Watson and the daughter called?

(v) What is the most likely explanation of the following scenario: Watson doesn’t call, daughter calls, and there is no news report?

(c) The notation “$A \perp B \mid C$” means “A and B are independent given C”. Show that:

$$X \perp Y \mid W, Z \text{ and } X \perp W \mid Z \implies X \perp W, Y \mid Z.$$