Lab 6 Report

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Abstract

This experiment introduces dynamical systems as flows and continuous systems. Instead of examining iterative maps like before, we observe a simple continuous trajectory calculation in response to initial conditions.

Book Notes

Simplest ODE

$$\frac{dx}{dt} = ax$$

for $x = x(t)$ being a real valued function of a real variable t and dx/dt being its derivative. This means

$$x'(t) = ax(t)$$

Using calculus we obtain the unique solution

$$f'(t) = aKe^{at} = af(t)$$

Here k is any constant and can only be specified if given an initial condition of the form $x(o) = K$

If $a \neq 0$ then the equation is stable, meaning it does not resort to chaos within a small perturbation of a. 0 can be called the bifurcation point.

For the system

$$x_1'(t) = a_1 x_1$$

$$x_2'(t) = a_2 x_2$$

we can immediately write down the solution following the rule above since each equation is uncoupled.

$$x_1(t) = K_1 e^{a_1 t}$$

$$x_2(t) = K_2 e^{a_2 t}$$

Each of these can be observed geometrically forming a vector field.

We focus on the study of dynamical systems, where the independent variable is taken as time and the solution as some sort of physical movement, such as a particle moving in space.Thus we can graph the position of the particle by

$$\phi_t(u) = (u_1 e^{a_1 t}, u_2 e^{a_2 t})$$

This map is a linear transformation, meaning $\phi_t(u + r) = \phi_t(u) + phi_t(r)$ where $\phi_t(\lambda u) = \lambda\phi_t(u)$ for real numbers λ

For coupled systems, one must find the diagonal form of the system (uncoupled), which can be found by applying a change of coordinates and substituting.

Generalizing, instead of working with two coordinates we work with n differential equations each with n many real number constants. Here we are working in the R^n plane, with n-tuples as coordinates. Addition, scalar product, and size are as defined in a general vector sense. A differential equation in this sense can be defined as

$$x' = Ax$$

where x is the vertical n tuple of all xs and A is the matrix of coefficients.

Working in the gravitational field of the sun, the acceleration vector can be modeled as

$$a(t) = x(t)$$

Newton's second law is

$$F(x(t)) = mx(t)x = F(x)/m$$

Where $F(x)$ will be the force function.

For one dimensional harmonic oscillation we get

$$x + p^2 x = 0$$

, with solution

$$x(t) = A\cos pt + B\sin pt = a\cos(pt + t_0)$$

With a being the amplitude, which is the size of vector (A, B). The nonhomogenous version of the system with constant K will have solution

$$x(t) = a\cos(pt + t_0) + \frac{K}{p^2}$$

A two dimensional version expands similarly as we have seen in the two dimensional system mentioned previously.

When the force field is given by

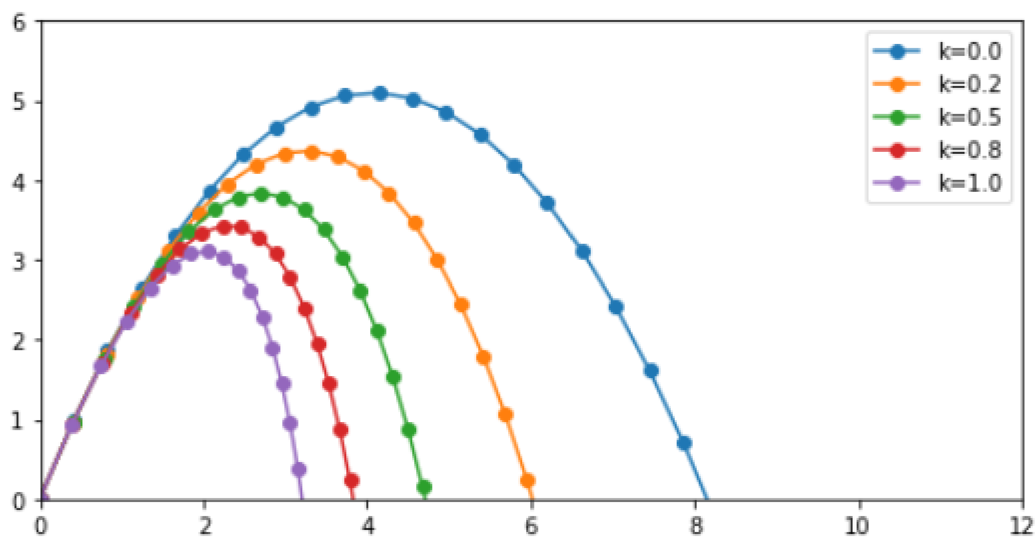
$$F(x) = -(\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \frac{\partial V}{\partial x_3} = -\text{grad}V(x)$$

for $V : R^3 \to R^3$ It is called conservative.

The force field $F(x) = -mkx$ gives rise to the planar harmonic oscillation, which is conservative. Our V in this case is the potential energy $V(x) = 1/2mk|x|^2$ The kinetic energy is defined by $T = 1/2m|\dot{x}(t)|^2$. The total energy is defined as $E = T + V$

For a particle in a conservative field, the total energy is independent of time.

Lab



The above is the trajectory of a particle in two dimensions, modeled by the solutions of the differential equations from newton's laws. Here we see how the initial condition does not induce chaos.