Logistic Map Part 2

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Abstract

In this experiment, we combined some of the previous topics we learned while learning more about the chaos and order within the natural world. In the video, we learned about how seemingly chaotic chemical reactions could be modelled fairly easily using mathematics and in the lab, we focused on different manipulations of the logistic mapping. These included Poincare plots and cobweb plots.

Introduction and Theory

The lab focused on the logistic map $x_{n+1} = rx(1-x)$ for some initial condition x_0 and a constant r often called the growth rate. Something we looked at somewhat was the Poincare plot of our function. This plotting allows us to examine some self-similarity of our function and helps us determine more so if there is just randomness or a desirable pattern we can examine. Next, we hearkened back to the previous lab and plotted the bifurcation diagram. This was very similar to the last lab where we could see the splitting branches. Finally, we examined the cobweb plots, which provide another method for examining the irregularity of a function.

Poincare Plot

The plot essentially plotted our our function as a sequence of points (x_i, x_{i+1}). The plot ends up looking like an upside parabola, which makes sense because our function, when expanded is $x_{n+1} = rx - rx^2$, which is an upside parabola. We then plotted a graph of a set of randomly generated values to compare. It is clear from the structure of the logistic vs. the randomness of the second plot that there is not just pure chaos in the logistic plot.

Bifurcation Diagram

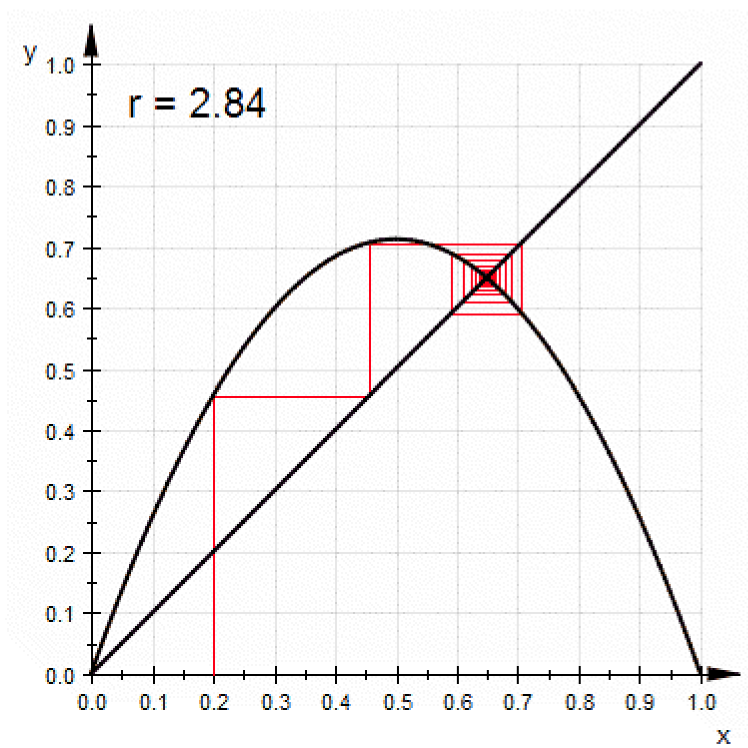
This is very similar to the previous lab. We were able to see splitting around 3 and then much more chaotic activity at 3.57.

Cobweb Plot

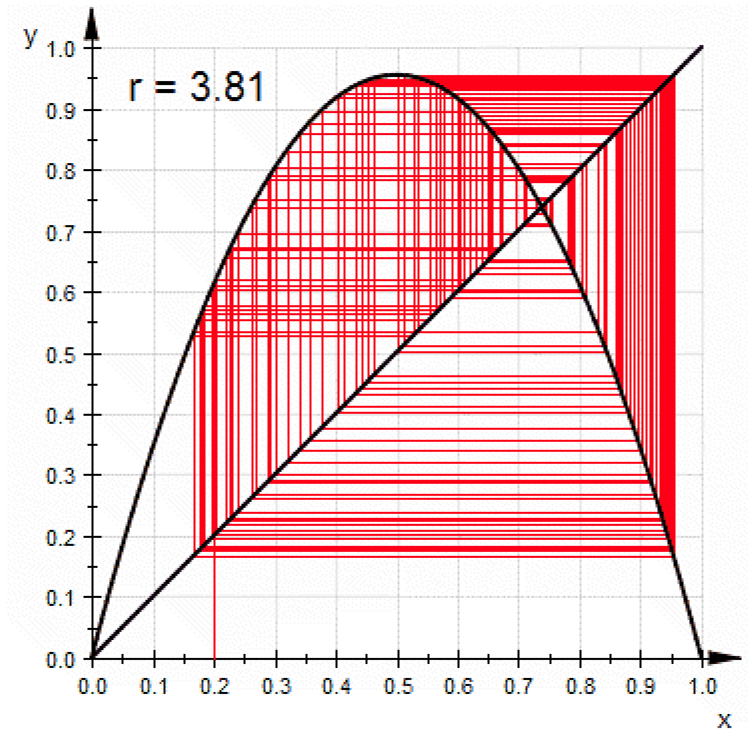
Finally, we examined the cobweb plot of our function $y = rx(1-x)$. The cobweb plot is a simple iterative process that goes like this.

1. Start at the point $(x_0, 0)$ and draw a vertical line up
2. Find the initial point $(x_0, f(x_0))$ and stop the line there
3. Draw a horizontal line until you hit the line $y = x$
4. Draw a vertical line from that point until you hit the function again
5. Repeat the process for the desired number of iterations

Below is a picture of the process for $r = 2.84$



As you can see, it stabilizes near a single point. This point helps us say that there is some sort of singular stable point that the function will tend to. However, if we change r to be some over 3.57, we see a filled in graph that is much more chaotic, which indicates a possibly infinite number of possible convergence values. Below is an image of such a plot.



Conclusions

In conclusion, this lab examined in detail the possible methods to examine the chaos of the logistic map. We saw how changing it's parameters affects the cobweb plot and bifurcation plot while also reiterating the fact that there is not some inherent sense of randomness but a methodical and predictable outcome.