Logistic Map

Neo Nguyen

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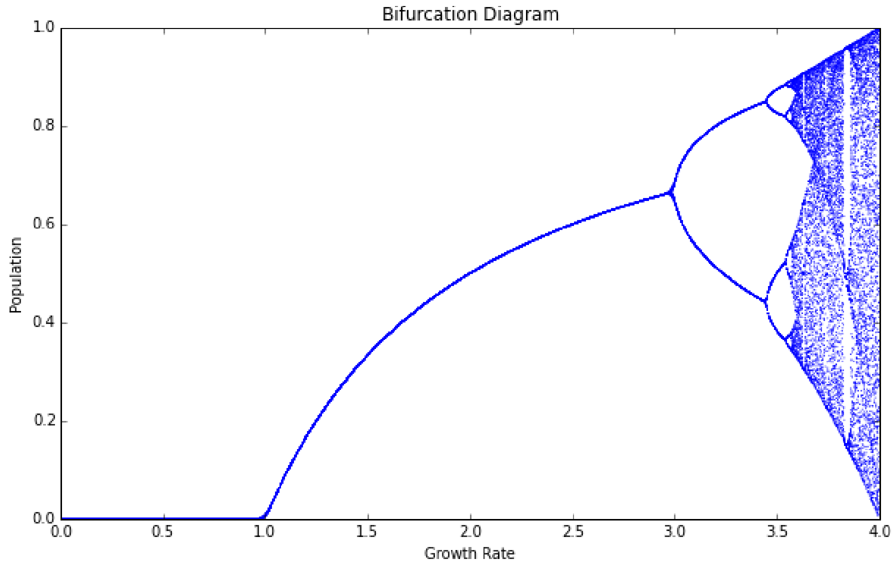
Abstract

In this lab we studied the logistic map, modelled by the equation $x_{n+1} = rx_n(1-x_n)$ where r is a growth rate and x_n is a real number between 0 and 1, normally called the population. This recursive equation can be used in real life to model many different things, from faucet dripping to rabbit reproduction, and has even been shown to exist in things like eye flickering rates. Just like the last lab report, this simple structure can output a multitude of things, predictable and not. In this lab specifically, we tried to understand how changing initial conditions and the growth rate affects the output of the map. In a broad sense, modifying the initial population x doesn't affect the end behavior much but changing the growth rate can change it very much.

1 Introduction and Theory

As stated in the abstract, the equation we will be studying is $x_{n+1} = rx_n(1 - x_n)$. For r values below 1, the end behavior of the map goes to 0. This makes sense as with a growth rate lower than 1 would cut down on the total population as time goes on no matter what the starting value is. For $0 \leq r \leq 3$ the value of the map converges to a number. This can be seen in the graph of the function where the y value is a consistent curve. Then around 3, the function is periodic, leading to the graph splitting. That means there are two values the function jumps between. This sort of 2-period bifurcation can be seen in real life when flashing lights into your eyes. The rate at which your eyes flicker will begin to jump between two values after certain light flashing's. After some more time, the graph once again splits, leading to 4 possible values. It ends up splitting again and again until about 3.57. Here the function is totally chaotic and unpredictable. This can be seen in the graph but in essence, we cannot know what is going to happen in the function just based on the input. Sometime after the chaotic split, there are some moments of calmness where the function goes through a 3-period bifurcation but that quickly ends and we return to randomness. This habit of not having a singular or even periodic output results in was even used as some of the first pseudo-random number generators.

The ratio of the lengths of any two distinct succesive splits are equivalent to about 4.6. This constant is called the Feigenbaum constant.



The above figure plots the r values along the x-axis and the resulting convergence value as the y axis. As you can see, after about 3.5, the function is chaotic and there is no predictable value for the function.

2 Methods

In the lab, we began with computing the end behavior of a specific r and x_0 value after 30 iterations. We then plotted that result. We were able to see how the output changes based on the changes in r and x_0. The next section takes what we did in the first section and applies it to a whole set of different r values. It loops through a vector of 10 different r values from 2.5 to 4. Then it graphs the function at each of these r values. Looking through the graph, we can see the singular values, then the splitting values, and then the chaotic values.

3 Conclusions

In conclusion, we see from a simple equation can come very complex outputs. Knowing the equations that model the natural world is important but understand where and why they converge and diverge to specific points is more important. Our lab gives us a closer look into how changing the parameters of the logistic map affects the maps output.