

UNIVERSIDADE FEDERAL FLUMINENSE

ANAND SUBRAMANIAN

# Heuristic, Exact and Hybrid Approaches for Vehicle Routing Problems

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Thesis presented to the Computing Graduate Program of the Universidade Federal Fluminense in partial fulfillment of the requirements for the degree of Doctor of Science.  
Topic Area: Algorithms and Optimization.

Advisor:  
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Co-advisor:  
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Anand Subramanian

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Niterói, March 05, 2012

*“You can fight without ever winning  
but never ever win without a fight.”*

*Neil Peart (Rush)*

*To my parents.*

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# Abstract

The Vehicle Routing Problem (VRP) is a classical combinatorial optimization problem that was proposed in the late 1950's and it is still one of the most studied in the field of Operations Research. The great interest in the VRP is due to its practical importance, as well as the difficulty in solving it. This work deals with heuristic, exact and hybrid approaches for solving different variants of the VRP, namely: Capacitated VRP (CVRP), Open VRP (OVRP), Asymmetric CVRP (ACVRP), VRP with Simultaneous Pickup and Delivery (VRPSPD), VRP with Mixed Pickup and Delivery (VRPMPD), TSP with Mixed Pickup and Delivery (TSPMPD), Multi-Depot VRP (MDVRP), Multi-Depot Vehicle Routing Problem with Mixed Pickup and Delivery (MDVRPMPD) and Heterogeneous Fleet VRP (HFVRP). An extensive literature review is performed for all these variants, focusing on the main contributions of each work. Commodity flow formulations for the VRPSPD/VRPMPD are theoretically examined and their practical performance are measured by a Branch-and-cut (BC) algorithm. Another BC algorithm, based on a formulation defined only over the edge variables, is proposed for the VRPSPD, VRPMPD and MDVRPMPD, where the constraints that ensure that the capacity of the vehicle is not exceeded in the middle of the route and those that ensure that a route starts and ends at the same depot are treated in a lazy fashion. The third and last exact approach is a Branch-cut-and-price algorithm that is also designed to solve the VRPSPD/VRPMPD. These three exact approaches are tested in benchmark instances involving up to 200 customers and new optimal solutions are found for 67 open problems. A heuristic algorithm is proposed for solving the VRPs considered here. The algorithm, called ILS-RVND, is based on the Iterated Local Search (ILS) metaheuristic and it makes use of a Variable Neighborhood Descent with Random neighborhood ordering (RVND) in the local search phase. Finally, a hybrid algorithm that incorporates a Set Partitioning approach into the ILS-RVND heuristic is presented for solving 8 of the VRPs treated in this work. The developed heuristic and hybrid algorithms are tested in hundreds of benchmark instances and the results obtained are, on average, highly competitive.

**Keywords:** Vehicle Routing Problems. Exact approaches. Heuristic and hybrid algorithms. Iterated Local Search. Logistics.



# Resumo

O Problema de Roteamento de Veículos (PRV) é um problema clássico de otimização combinatória proposto no final da década de 1950, sendo ainda um dos mais estudados na área de Pesquisa Operacional. O elevado interesse no PRV é devido a sua importância prática, bem como a dificuldade em resolvê-lo. Este trabalho trata de abordagens heurísticas, exatas e híbridas para diferentes variantes do PRV, a saber: PRV Capacitado (PRVC), PRV com Rotas Abertas (PRVRA), PRV Capacitado e Assimétrico (PRVCA), PRV com Coleta e Entrega Simultânea (PRVCES), PRV com Coleta e Entrega Mista (PRVCEM), Problema do Caixeiro Viajante com Coleta e Entrega Mista (PCVCEM), PRV com Múltiplos Depósitos (PRVMD), PRV com Coleta e Entrega Mista e Múltiplos Depósitos (PRVCEMMD) e PRV com Frota Heterogênea (PRVFH). Uma extensa revisão da literatura é efetuada para todas estas variantes, dando destaque as principais contribuições de cada trabalho. Formulações baseadas em fluxo em rede para o PRVCES/PRVCEM são teoricamente examinadas e seus desempenhos práticos são medidos por meio de um algoritmo *Branch-and-Cut* (BC). Outro algoritmo BC, baseado em uma formulação definida apenas sobre as variáveis de aresta, é proposto para o PRVCES, PRVCEM e PRVCEMMD, onde as restrições que garantem que a capacidade do veículo não é excedida no meio da rota e aquelas que garantem que uma rota começa e termina no mesmo depósito são separadas de uma maneira *lazy*. A terceira e última abordagem exata é um algoritmo *Branch-cut-and-price* que também é desenvolvido para resolver o PRVCES/PRVCEM. Estas três abordagens exatas são testadas em instâncias da literatura envolvendo até 200 clientes e novas soluções ótimas são encontradas para 67 problemas em aberto. Um algoritmo heurístico é proposto para resolver os PRVs aqui considerados. O algoritmo, denominado, ILS-RVND, é baseado na metaheurística *Iterated Local Search* (ILS) que faz uso de procedimento inspirado no método *Variable Neighborhood Descent* com ordem aleatória na escolha das vizinhanças (RVND) na fase de busca local. Finalmente, um algoritmo híbrido, que incorpora uma abordagem baseada em Particionamento de Conjuntos na heurística ILS-RVND, é apresentado para resolver 8 dos PRVs tratados neste trabalho. Os algoritmos heurísticos e híbridos são testados em centenas de instâncias da literatura e os resultados obtidos são, em média, altamente competitivos.

**Palavras-chave:** Problemas de Roteamento de Veículos. Abordagens exatas. Algoritmos heurísticos e híbridos. *Iterated Local Search*. Logística.

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# Glossary

|        |  |
|--------|--|
| AC     | : Ant Colony;  |
| ACVRP  | : Asymmetric Capacited Vehicle Routing Problem;                          |
| ALNS   | : Adaptive Large Neighborhood Search;                                    |
| ADS    | : Auxiliary Data Structure;  |
| AMP    | : Adaptive Memory Procedure;   |
| ABHC   | : Attribute Based Hill Climber;  |
| BC     | : Branch-and-Cut;  |
| BCP    | : Branch-Cut-and-Price;  |
| BKS    | : Best Known Solution;   |
| BP     | : Branch-and-Price;  |
| CO     | : Combinatorial Optimization;  |
| CL     | : Candidate List;  |
| CVRP   | : Capacited Vehicle Routing Problem;                                     |
| ES     | : Evolutionary Strategies;   |
| FSM    | : Fleet Size and Mix;  |
| FSMD   | : Fleet Size and Mix with Dependent costs;                               |
| FSMF   | : Fleet Size and Mix with Fix costs;                                     |
| FSMFD  | : Fleet Size and Mix with Fix and Dependent costs;                       |
| GRASP  | : Greedy Randomized Adaptive Search Procedure;                           |
| GA     | : Genetic Algorithm;   |
| GLS    | : Guided Local Search;   |
| ILS    | : Iterated Local Search;   |
| ILP    | : Integer Linear Programming;  |
| HVRP   | : Heterogeneous Vehicle Routing Problem;                                 |
| HVRPFD | : Heterogeneous Vehicle Routing Problem with Fix and<br>Dependent costs; |
| HVRPD  | : Heterogeneous Vehicle Routing Problem with Dependent costs;            |
| HFVRP  | : Heterogeneous Fleet Vehicle Routing Problem;                           |
| LNS    | : Large Neighborhood Search;   |
| LB     | : Lower Bound;   |
| MA     | : Memetic Algorithm;   |

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|          |   |   |
|----------|---|---|
| MDVRP    | : | Multi-Depot Vehicle Routing Problem;                                |
| MDVRPMPD | : | Multi-Depot Vehicle Routing Problem with Mixed Pickup and Delivery; |
| MIP      | : | Mixed Integer Programming;  |
| MST      | : | Minimum Spanning Tree;  |
| NL       | : | Neighborhood List;  |
| MCFIC    | : | Modified Cheapest Feasible Insertion Criterion;                     |
| NFIC     | : | Nearest Feasible Insertion Criterion;                               |
| OR       | : | Operations Research;  |
| OVRP     | : | Open Vehicle Routing Problem;                                       |
| PSO      | : | Particle Swarm Optimization;  |
| PDP      | : | Pickup and Delivery Problem;  |
| RL       | : | Route List;   |
| RVND     | : | Variable Neighborhood Descent with Random neighborhood ordering;    |
| SA       | : | Simulated Annealing;  |
| SIS      | : | Sequential Insertion Strategy;                                      |
| PIS      | : | Parallel Insertion Strategy;  |
| SS       | : | Scatter Search;   |
| SP       | : | Set Partitioning;   |
| TSP      | : | Traveling Salesman Problem;   |
| TSPMPD   | : | Traveling Salesman Problem with Mixed Pickup and Delivery;          |
| TS       | : | Tabu Search;  |
| UB       | : | Upper Bound;  |
| VND      | : | Variable Neighborhood Descent;                                      |
| VNS      | : | Variable Neighborhood Search;                                       |
| VRP      | : | Vehicle Routing Problem;  |
| VRPMPD   | : | Vehicle Routing Problem with Mixed Pickup and Delivery;             |
| VRPSPD   | : | Vehicle Routing Problem with Simultaneous Pickup and Delivery;      |
| VRPTW    | : | Vehicle Routing Problem with Time Windows.                          |

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# Chapter 1

## Introduction

### 1.1 Definition of the theme

The Vehicle Routing Problem (VRP) is a classical Combinatorial Optimization (CO) problem that was proposed in the late 1950's and it is still one of the most studied in the field of Operations Research (OR). The great interest in the VRP is due to its practical importance, as well as the difficulty in solving it.

The dictionary defines the term *routing* as the act of sending someone or something along a particular course. According to Laporte [95], the name *Vehicle Routing Problem* was first employed in the mid 1970's by Christofides [30] and it had been widely adopted since then. In a general form, the VRP consists of designing a least-cost set of vehicle routes, subjected to some side constraints, in order to serve geographically dispersed customers.

The VRP is generally solved by exact or heuristic algorithms. The first aims to find strict optimal solutions while the latter seeks to obtain sub-optimal/approximate solutions. The term *heuristic* is derived from the greek word *heuriskein*, which means *to find*. In the area of computing, heuristic can be defined as a rule-based procedure developed for determining a good quality solution to a specific problem.

Hybrid approaches, i.e, those that combine heuristic and exact methods, are another alternative that can be employed to solve the VRP. Exact procedures can be integrated into heuristics and vice-versa.

This work presents heuristic, exact and hybrid approaches to solve a large class of VRPs. The next sections describe the motivation, objectives and outline of the thesis.

### 1.2 Motivation

The VRP plays an important role in the supply chain of several companies that are involved with the transportation of goods or people. This problem is regularly faced by the distribution systems of these corporations and its solution quality may have direct

implications on the logistic performance. In addition, the VRP can arise in different contexts within the same company, whether by transporting raw-materials and/or finished goods between the production unit and its subsidiaries, delivering (collecting) products to (from) the customers, or even transporting employees from their homes to the company and vice-versa.

Motivated by real-life situations, many VRP variants were proposed throughout the years. They may include additional constraints to satisfy customers' needs such as time windows and pickup and delivery services, or additional features such as route duration, multiple depots, mixed vehicle fleet, etc. In the end, regardless of the variant, the main goal is to optimize the use of the transportation resources and also to satisfy customers' demands.

The computing revolution as well as the industrial boom had impulsed the application of OR methods in real-life CO problems such as the VRP in the last 50 years. Golden et al. [77] described a number of case studies in which the application of computerized vehicle routing systems in the solid waste, beverage, food, dairy and newspaper industries led to substantial cost reductions. Toth and Vigo [175] state that the use of computerized procedures in distribution planning results in 5% to 10% savings in transportation costs.

However, solving the VRP is far from a simple task since the problem is  $\mathcal{NP}$ -hard [102]. Yet, there has been lot of advances in the development of exact algorithms for dealing with the VRP, particularly those based on mathematical programming techniques. Up to date, there is no exact algorithm that consistently solves VRP instances with more than 150 customers. Nonetheless, in those cases where the optimal solution could not be determined, one can still make use of the value of the dual bounds for evaluating the solution quality obtained by heuristic algorithms.

Due to their ability of obtaining good solutions in an acceptable time, heuristic procedures are the most common method employed to solve CO problems. A special attention must be given to metaheuristics, which can be defined as general master processes that guide a subordinate heuristic in order to efficiently find high quality solutions. Metaheuristics are the core of a huge number of successful heuristic algorithms for CO problems, including the VRP. Such popularity arises from the fact that metaheuristics are more flexible, easier to understand and, in general, require less implementation efforts than the exact approaches.

Combining (meta)heuristic and exact methods appears to be a very promising alternative in solving CO problems. The interest in hybrid approaches is rapidly growing especially due to several encouraging results obtained by the fusion of these two methods (see Maniezzo et al. [116]). The interaction between mathematical programming techniques and metaheuristics led to a new class of optimization algorithms called *matheuristics*. Nevertheless, the application of these kind of approaches have not received much

attention yet from the VRP literature. Some examples can be found in the works of De Franceschi et al. [43], Toth and Tramontani [173] and Alvarenga et al. [2].

Most VRP heuristics usually focus on a particular type of problem. A relatively small number of works have suggested unified heuristic procedures for dealing with several variants (see, for example, Røpke and Pisinger ([136], [148]), Cordeau et al. [38]). Seen from a practical point of view, these non-specific approaches are highly relevant. For instance, VRP commercial packages must be prepared to face real-life problems of different classes. Therefore, the development of efficient general algorithms are crucial for achieving a satisfactory performance. When talking about attributes for good heuristics, one should take into account not only the solution quality (accuracy) and computational time (speed), but also the simplicity and flexibility factors [36]. Hence, a general VRP heuristic that contains these four attributes, is more likely to be successfully employed in practice.

Given the above, one of the interests of this work is to propose general heuristic and hybrid algorithms for solving different VRPs. However, because of the huge number of existing variations it becomes virtually impossible to tackle all of them here. Therefore, it was thought advisable to turn attention only to a subset of variants, namely the following ones:

- (i) Capacitated VRP.
- (ii) Asymmetric CVRP.
- (iii) Open VRP.
- (iv) VRP with Simultaneous Pickup and Delivery.
- (v) VRP with Mixed Pickup and Delivery.
- (vi) Traveling Salesman Problem with Mixed Pickup and Delivery.
- (vii) Multi-depot VRP.
- (viii) Multi-depot VRP with Mixed Pickup and Delivery.
- (ix) Heterogeneous Fleet VRP.

Some versions of the problems listed above may also include route duration constraints. These cases were also treated in this work.

Although other variants were not explicitly considered, the heuristic and hybrid approaches suggested here can easily be adapted to solve other cases such as VRP with Time Windows, Site-dependent VRP, VRP with Backhauls, etc.

The proposed heuristic improves/extends the one suggested by Subramanian et al. [157] for the VRP with Simultaneous Pickup and Delivery. The algorithm consists of a

combination between the Iterated Local Search (ILS) [112] metaheuristic and the Variable Neighborhood Descent (VND) [120] procedure. The first is a stochastic local search procedure that focuses the search on a given subset of the solution space which is defined by local optimal solutions. The latter is characterized for systematically modifying the neighborhood operators during the local search.

According to Lourenço et al. [112], ILS contains several of the desirable features of a metaheuristic such as simplicity, robustness, effectiveness and the ease of implementing. These ingredients are highly important with regard to the development of general heuristics. The authors [112] also described a number of well-succeeded ILS implementations for different CO problems such as the Traveling Salesman Problem (TSP), Job Shop, Flow Shop, MAX-SAT, etc. Surprisingly, to date there are relatively few applications of this metaheuristic to VRPs (see, for example, [16], [28], [89], [140] and [157]). Yet, the computational results found by these researchers who have made use of an ILS approach to solve some VRP variant are quite encouraging.

One essential component of a good ILS algorithm is an efficient local search procedure. The VND is a simple and flexible strategy that can be embedded into any other neighborhood based heuristic. Hansen et al. [83] present various applications of the VND to many classes of CO problems, including VRPs. Differently from the usual VND approaches that employ a deterministic neighborhood ordering, the proposed heuristic adopted a random neighborhood ordering scheme (RVND). This not only avoids parameter tuning, but may also prevent premature convergence to poor local optimal solutions. In addition, the best order may be highly dependent on the instance.

The developed hybrid algorithm incorporates an exact procedure based on the Set Partitioning (SP) formulation into the ILS heuristic. This strategy is quite similar to the classical two-phase petal algorithm (see Laporte and Semet [100]). The idea is to store a pool of routes generated during the heuristic execution and then solve a SP problem in order to extract the best combination of routes. However, unlike traditional petal algorithms and other SP based approaches to VRPs [2, 91, 147], the proposed hybrid algorithm includes some enhanced features such as the cooperation between a Mixed Integer Programming (MIP) solver and the ILS heuristic (while solving the SP problem) and a reactive mechanism that dynamically controls the dimension of the SP models when dealing with large size instances.

Exact algorithms capable of solving instances with more than 50 customers can be found in the literature for problems (i)-(iii), (vi), (viii) and (ix). It was thus decided to develop exact approaches for problems (iv), (v) and (vi) in order to obtain new lower bounds and some optimal solutions that can be used as a reference for measuring the performance of the proposed heuristic and hybrid algorithms. In view of this, three mathematical formulations were implemented within a Branch-and-cut scheme for problems

(iv) and (v). These formulations are also theoretically compared. Furthermore, another Branch-and-cut algorithm, which is based on a mathematical formulation composed only by edge variables, was developed for problems (iv), (v) and (vii). Finally, a Branch-cut-and-price approach was put forward for problems (iv) and (v). Although there are more sophisticated exact algorithms for solving problems (i)-(iii) and (vi), the proposed exact strategies can still be used to solve these problems.

## 1.3 Objectives

The objectives of this work are as follows.

- Review the VRPs (i)-(ix), describing some practical applications and solution methods proposed in the literature.
- Develop exact approaches for problems (iv), (v) and (vii).
- Develop a general heuristic framework capable of solving a large class of VRPs with emphasis on problems (i)-(ix).
- Develop a general hybrid algorithm capable of solving a large class of VRPs with emphasis on problems (i)-(v) and (vii)-(ix).

## 1.4 Thesis outline

The remainder of this work is organized as follows.

- Chapter 2 describes the VRPs treated in this work, as well as their respective literature reviews.
- Chapter 3 presents a theoretical comparison between different commodity flow formulations and a Branch-and-cut approach for the VRP with Simultaneous/Mixed Pickup and Delivery.
- Chapter 4 presents a Branch-and-cut algorithm with a lazy separation scheme over a mathematical formulation composed only by the edge variables for the Single and Multi-depot VRP with Simultaneous/Mixed Pickup and delivery.
- Chapter 5 presents a Branch-cut-and-price algorithm for the VRP with Simultaneous/Mixed Pickup and Delivery.
- Chapter 6 presents the heuristic algorithm for general VRPs, describing the constructive procedures, neighborhood operators and perturbation mechanisms.

- 
- Chapter 7 presents the hybrid algorithm for general VRPs, describing how an exact procedure is integrated into the heuristic algorithm.
  - Chapter 8 contains the concluding remarks of this work.



# Chapter 2

## A Review of the Vehicle Routing Problems Considered in the Present Work

A huge number of works dealing with VRPs had been published since the seminal work of Dantzig and Ramser [42] entitled *The Truck Dispatching Problem* that was published in 1959. Different variants may include some specific characteristics such as time windows, pickup and delivery services, mixed fleet, stochastic components and so on. This chapter describes and reviews the VRP variants treated in this work, particularly the: (i) Capacitated VRP (CVRP); (ii) Open VRP (OVRP); (iii) Asymmetric CVRP (ACVRP); (iv) VRP with Simultaneous Pickup and Delivery (VRPSPD); (v) VRP with Mixed Pickup and Delivery (VRPMPD); (vi) TSP with Mixed Pickup and Delivery (TSPMPD); (vii) Multi-Depot VRP (MDVRP); (viii) Multi-Depot Vehicle Routing Problem with Mixed Pickup and Delivery (MDVRPMPD); and (ix) Heterogeneous Fleet VRP (HFVRP). A detailed and comprehensive survey of VRPs can be found in [39], [74], [75] and [175].

### 2.1 Capacitated Vehicle Routing Problem (CVRP)

The CVRP is considered to be the classical version of the VRP. A formal definition of the problem is as follows. Let  $G = (V, E)$  be a complete graph with a set of vertices  $V = \{0, \dots, n\}$ , where the vertex 0 represents the depot and the remaining ones the customers. Each edge  $\{i, j\} \in E$  has a non-negative cost  $c_{ij}$  and each customer  $i \in V' = V \setminus \{0\}$  has a demand  $d_i$ . Let  $C = \{1, \dots, m\}$  be the set of homogeneous vehicles with capacity  $Q$ . The CVRP consists in constructing a set up to  $m$  routes in such a way that: (i) every route starts and ends at the depot; (ii) all the demands are accomplished; (iii) the vehicle's capacity is not exceeded; (iv) a customer is visited by only a single vehicle; (v) the sum of costs is minimized.

Many exact algorithms had been proposed to solve the CVRP and the goal is not to review all of them here. There are some specific surveys in the literature that cover most of these approaches. A detailed review of algorithms based on Branch-and-bound, Branch-

and-cut and set-covering were respectively performed by Toth and Vigo [176], Naddef and Rinaldi [124] and Bramel and Simchi-Levi [19]. These three works, along with those of Laporte and Nobert [97] and Cordeau et al. [39], together survey the main CVRP exact algorithms developed in the past century and early 2000's. A review of the most latest advances regarding the exact approaches for solving the CVRP was performed by Baldacci et al. ([11], [12]). According to these authors, the best exact algorithms designed for the CVRP up to now are the BC of Lysgaard et al. [115], the robust Branch-cut-and-price (BCP) of Fukasawa et al. [63] and the Set Partitioning (SP) with additional cuts based algorithm of Baldacci et al. [7].

Recently, a robust BCP algorithm based on an extended formulation that makes use of capacity-indexed variables was proposed by Pessoa et al. [134]. In addition, new families of cuts over this formulation were also presented. Besides the CVRP, the authors applied their BCP for solving other VRP variants, namely: OVRP, ACVRP and Fleet Size and Mix VRP. The results obtained in all variants were highly competitive when compared to those found in the literature. Finally, they believe that the development of new cuts over the extended formulation appears to be promising and it can lead to novel research directions.

Even though there has been a lot of progress in the development of exact algorithms, the heuristic methods are still the most suitable approach, in terms of solution quality vs. computational time, when dealing with the CVRP. Some authors ([39], [95], [100]), divide them into two distinct classes: (i) classical heuristics, which consist on constructive, two-phase and improvement heuristics; and (ii) metaheuristics, which employ more sophisticated mechanisms such as memory structure, perturbation moves, route combination and so forth. A complete description of the classical heuristics can be found in [100], while a review of the application of metaheuristics to the CVRP is available in [34], [67] and [69]. In general, all of these solution methods can be extended to most VRP variants.

Due to the large number of (meta)heuristic based algorithms proposed for the CVRP, one will concentrate only on those that produced the best results for two of the most used sets of benchmark instances, specifically the one of Christofides et al. [31] and the one of Golden et al. [80].

There were several applications of the Tabu Search (TS) metaheuristic [72] to the CVRP during the 1990's and early 2000's (see [37]). Taillard's algorithm [162] was one of the first well succeeded implementations and it still remains as one of the best. In his algorithm, two decomposition methods are employed in which customers are randomly partitioned into subregions where each of these is treated as a subproblem that is solved separately using a TS strategy. A feasible solution is then generated by merging the routes produced by each subproblem. Rochat and Taillard [147] proposed a more sophisticated version of Taillard's algorithm by combining TS with an Adaptive Memory Procedure

(AMP). During the TS execution, the AMP stores a pool of promising routes which are periodically recombined/updated, with a view to obtain an improved incumbent solution. A post-optimization phase was also incorporated in which a SP formulation was applied over the pool of routes in order to build an improved solution. A similar AMP approach, called Solutions' Elite PArts Search (SEPAS), was suggested by Tarantilis [167]. Another version of the TS, known as Attribute Based Hill Climber (ABHC) [45], was implemented by Derigs and Kaiser [46]. This method extends the aspiration criterion concept by defining a set of attributes (e.g, in a boolean fashion) over a feasible solution.

Some CVRP algorithms based on other local search metaheuristics such as Large Neighborhood Search (LNS) [152], Variable Neighborhood Search (VNS) [120] and Guided Local Search (GLS) [179] started to appear in the mid 2000's. The Adaptive LNS (ALNS) of Pisinger and Røpke [136] seems to be one of the most robust in terms of flexibility. Their method consists of a set of removal (destroy) and insertion (repair) heuristics which are chosen, at each iteration, using roulette wheel selection based on the past performance of these heuristics. The authors also have applied their algorithm in the following VRP variants: VRP with Time Windows (VRPTW), Site-dependent VRP, OVRP and MD-VRP. Kytöjoki et al. [93] proposed an algorithm that uses VNS to guide an improvement heuristic and the GLS to help escaping from local optimal solutions. This heuristic was specially designed for very large scale VRPs. Zachariadis and Kiranoudis [187] developed a procedure based on TS, GLS and VNS which contains local search enhancements in terms of computational complexity. Chen et al. [28] developed a heuristic that integrates the Variable Neighborhood Descent (VND) procedure in an ILS scheme which in turn employs a Simulated Annealing (SA) approach for the acceptance criterion. Their scheme is quite similar to the one presented in this work (see Chapter 6).

In recent years, there emerged many successful CVRP heuristics inspired in Evolutionary Strategies (ES). Reimann et al. [143] proposed an Ant Colony (AC) [51] based heuristic using a divide and conquer decomposition. Both the complete and partial problems are solved as follows: (i) a solution is generated employing a nearest neighbor heuristic; (ii) local search is applied to the initial solution; (iii) the pheromones are updated; and (iv) information regarding the level of attractiveness between each pair of customers is augmented. Mester and Bräysy [118] proposed a two phase heuristic by combining GLS, LNS and ES. In the first phase, an initial solution is generated using a hybrid version of the cheapest insertion heuristic while the second one is divided into two stages: (i) a local search is applied using the GLS; and (ii) LNS is performed through removal and insertion heuristics in an evolutionary (genetic) fashion. Prins [139] put forward a Memetic Algorithm (MA) whose main characteristics are: (i) TSP representation of chromosomes (giant tour), without tour delimiters, which can be directly converted to a VRP solution using a splitting procedure; and (ii) first improvement local search as mutation operator.

The same metaheuristic was implemented by Nagata [125] and Nagata and Bräysy [127]. In both these works, the MA proposed combines the edge assembly crossover (EAX), that allows infeasible solutions, with an repairing/improvement (local search) procedure. An enhanced version of Nagata's MA was implemented by Nagata and Bräysy [126], in which efficient limitation strategies were incorporated into the local search neighborhoods. Prins [140] proposed some heuristics based on ILS, ES and Greedy Randomized Adaptive Search Procedure (GRASP) that use some of the features previously adopted in [139]. More recently, Vidal et al. [177] put forward a hybrid GA originally designed to solve the MDVRP, Period VRP and the Multi-depot Period VRP, which also turned out to be highly efficient for the CVRP.

Unlike pure exact and heuristic methods, hybrid versions combining these two approaches have not been much explored in the CVRP literature. De Franceschi et al. [43] proposed an LNS-like improvement heuristic based on Integer Linear Programming (ILP) that works as follows: (i) a set of nodes is removed, according to a predefined criterion, from a given solution which in turn is partially reconstructed by short-cutting all the deleted nodes; and (ii) a complete solution is generated by solving an ILP associated to a Reallocation Model which consists of re-inserting the extracted nodes into the partial solution. This solution method was also applied to the ACVRP. Toth and Tramontani [173] later extended this approach by generalizing the neighborhood structure which is evaluated by a two-phase method that: (i) applies a neighborhood reduction strategy; and (ii) explores this reduced neighborhood by solving the Column Generation problem associated with the linear programming relaxation of the ILP of the so-called Reduced Reallocation Model.

Table 2.1 presents a list of surveys regarding the CVRP, while Table 2.2 enumerates the most effective heuristics developed for the CVRP.

## 2.2 Asymmetric Capacitated Vehicle Routing Problem (ACVRP)

The ACVRP is a generalization of the CVRP where the cost between a pair of vertices is not necessarily symmetric, i.e.,  $c_{ij}$  need not be equal to  $c_{ji}$ ,  $\forall i, j \in V$ .

Examples of ACVRP applications appear in some cases where the travel costs between two destinations may differ due to certain road restrictions such as one-way directions or the existence of tolls.

There are very few works specially devoted to the ACVRP in the literature. In mid 1980's, Laporte et al. [96] suggested an exact Branch-and-bound algorithm in which the subproblems are formulated as modified assignment problems. Almost a decade later, Fischetti et al. [57] proposed a Branch-and-bound approach whose lower bounds are based

on: (i) assignment problems; (ii) disjunction on infeasible arcs subset; and (iii) min-cost flow relaxation.

Table 2.1: CVRP surveys

| <b>Work</b>                 | <b>Year</b> | <b>Topic of the survey</b>               |
|-----------------------------|-------------|--|
| Laporte and Nobert [97]     | 1987        | Exact algorithms                         |
| Toth and Vigo [176]         | 2002        | Branch-and-bound algorithms              |
| Naddef and Rinaldi [124]    | 2002        | Branch-and-cut algorithms                |
| Bramel and Simchi-Levi [19] | 2002        | Set-covering based algorithms            |
| Laporte and Semet [100]     | 2002        | Classical heuristics                     |
| Gendreau et al. [100]       | 2002        | Metaheuristics                           |
| Cordeau et al. [37]         | 2004        | TS heuristics                            |
| Cordeau et al. [34]         | 2005        | Metaheuristics                           |
| Cordeau et al. [39]         | 2007        | Exact and heuristic algorithms           |
| Laporte [94]                | 2007        | Exact and heuristic algorithms           |
| Baldacci et al. [11]        | 2007        | Latest exact approaches                  |
| Gendreau et al. [69]        | 2008        | Annotated bibliography on metaheuristics |
| Laporte [95]                | 2009        | Exact and heuristic algorithms           |
| Baldacci et al. [12]        | 2010        | Latest exact approaches (update of [11]) |

Table 2.2: CVRP effective heuristics

| <b>Work</b>                     | <b>Year</b> | <b>Approach</b>                 |
|---------------------------------|-------------|---------------------------------|
| Taillard [162]                  | 1993        | TS                              |
| Rochat and Taillard [147]       | 1995        | TS + AMP                        |
| Reinmann et al. [143]           | 2004        | AC                              |
| Prins [139]                     | 2004        | MA                              |
| Tarantilis [167]                | 2005        | TS + AMP                        |
| De Franceschi et al. [43]       | 2006        | ILP Local Search                |
| Pisinger and Røpke [136]        | 2007        | ALNS                            |
| Kytöjoki [93]                   | 2007        | VNS + GLS                       |
| Mester and Bräysy [118]         | 2007        | LNS + GLS + ES                  |
| Nagata [125]                    | 2007        | MA                              |
| Derigs and Kaiser [46]          | 2007        | ABHC                            |
| Nagata and Bräysy [126]         | 2008        | MA                              |
| Toth and Tramontani [173]       | 2008        | ILP Local Search                |
| Prins [140]                     | 2009        | GRASP + ILS + ES                |
| Nagata and Bräysy [127]         | 2009        | MA                              |
| Zacharidis and Kiranoudis [187] | 2010        | Enhanced Local Search algorithm |
| Chen et al. [28]                | 2010        | ILS + VND                       |
| Vidal et al. [177]              | 2011        | Hybrid GA                       |

Vigo [178] extended the classical CVRP heuristics of Clarke and Wright [33] and Fisher and Jaikumar [59] to the ACVRP and also proposed a refinement heuristic that repairs/improves an initial infeasible solution generated by the procedure developed by Fischetti et al. [57].

Table 2.3 present the works that dealt with the ACVRP including the ones of De Franceschi et al. [43] and Pessoa et al. [134] described in the previous section.

| Table 2.3: ACVRP related works |      |                                 |
|--------------------------------|------|---------------------------------|
| Work                           | Year | Approach                        |
| Laporte et al. [96]            | 1986 | Branch-and-bound Algorithm      |
| Fischetti et al. [57]          | 1994 | Branch-and-bound Algorithm      |
| Vigo [178]                     | 1996 | Improvement/repairing heuristic |
| De Franceschi et al. [43]      | 2006 | ILP Local Search                |
| Pessoa et al. [134]            | 2008 | Robust Branch-and-cut-and-price |

## 2.3 Open Vehicle Routing Problem (OVRP)

The OVRP is a special case of the ACVRP where the vehicles need not to return to the depot after visiting the last customer of a given route. Any OVRP instance can be converted to an ACVRP instance by simply setting  $c_{i0} = 0, \forall i \in V$ . Most authors also state that the number of vehicles must be minimized.

Applications of the OVRP may arise when a company chooses to hire a vehicle fleet to be in charge of the delivery services and, due to logistic reasons, these vehicles are not forced to return to the company's depot. In this case, the distribution costs are generally proportional to the length of the routes and/or to the number of vehicles used by the outsourced company.

Schrage [151] was the first to address the problem, in early 1980's, by describing certain characteristics of some real-life VRPs. One example mentioned by the author is the air express courier, in which aircrafts depart from a depot city, deliver their cargo to a set of customers geographically spread and then collect the cargo from the same set of customers by retracing their routes back to the depot. Bodin et al. [18] have presented a case study of this type of application at the FedEx Express company. Besides capacity constraints, other restrictions such as time windows and routes duration were also considered. The problem was solved by a procedure based on the Clarke and Wright savings heuristic [33].

The OVRP literature remained practically unchanged for nearly two decades until it was revisited by Sariklis and Powell [150] in 2000. The authors proposed a cluster-first, route second approach [17] where the first phase consists in grouping the customers according to the capacity constraints, while the second phase consists of a Minimum Spanning Tree (MST) heuristic that incorporates a penalty procedure.

Letchford et al. [103] presented an ILP formulation, a set of valid inequalities, as well as a BC algorithm that is mainly based on the one described in [115]. Their procedure is capable of solving to optimality several small and medium-sized instances. This work, along with the one of Pessoa et al. [134], are to date the only exact approaches that dealt

with the OVRP.

A considerable number of OVRP heuristic algorithms have been published since 2004. Some of these are based on the TS metaheuristic. Brandão [21] proposed a TS heuristic that makes use of a nearest neighborhood heuristic and a  $K$ -tree based procedure for generating initial solutions, whereas the local search is performed by shift and swap moves. Tarantilis et al. [23] suggested a heuristic that interactively combines the TS and AMP methods. Fu et al. ([61], [62]) developed a TS algorithm that employs a farthest-first heuristic for constructing an initial solution while shift, swap and 2-opt moves are used in the local search phase. Derigs and Reuter [47] proposed a ABHC procedure which is similar to the one presented in [46] for the CVRP.

OVRP algorithms based on other local search metaheuristics were also proposed. Tarantilis et al. [165] presented a threshold accepting approach [54] that consists of an adaptation of the SA procedure in which a worse solution is only accepted if it is within a given threshold. The same authors [166] also proposed another threshold accepting procedure that is integrated in a single-parameter metaheuristic. Li et al. [105] put forward a record-to-record [53] travel algorithm that, also as the threshold method, consists of a deterministic variant of the SA. Fleszar et al. [60] presented a VNS heuristic whose neighborhood operators are composed by exchanging segments between two routes and reversing segments of a single route. Zachariadis et al. [186] developed a local search metaheuristic that explores wide neighborhoods by only evaluating parts of a current solution that have been modified by a previous move.

Differently from pure local search approaches, there are few works containing applications of ES to the OVRP. Li and Tian [108] presented an AC algorithm combined with local search followed by a post-optimization procedure applied to the best solution obtained. A similar approach was later developed by Li et al. [109] where a TS procedure is incorporated into the AC framework. Repoussis et al. [146] suggested a heuristic based on ES in which offspring individuals (solutions) are generated through mutation operators and these intermediate solutions are improved by a procedure based on GLS and TS.

Table 2.4 summarizes the OVRP related works.

## 2.4 Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD)

The VRPSPD is a generalization of the CVRP in which a customer  $i \in V'$  have both a delivery demand  $d_i$  and also a pickup demand  $p_i$ . This problem can be considered as a Pickup and Delivery Problem (PDP). Berbeglia et al. [14] proposed a general scheme for classifying PDPs based on the characteristic of each problem. According to their framework, the VRPSPD was categorized as the multi-vehicle one-to-many-to-one PDP

with combined demands.

Table 2.4: OVRP related works

| Work                             | Year | Approach                               |
|----------------------------------|------|--|
| Bodin et al. [18]                | 1983 | Clarke and Wright based heuristic      |
| Sariklis and Powell [150]        | 2000 | Cluster-first, route-second heuristic  |
| Brandão [21]                     | 2004 | TS                                     |
| Tarantilis et al. [23]           | 2004 | TS + AMP                               |
| Tarantilis et al. [165]          | 2004 | Threshold accepting approach           |
| Tarantilis et al. [166]          | 2005 | Single-parameter metaheuristic         |
| Fu et al. ([61], [62])           | 2005 | TS                                     |
| Li and Tian [108]                | 2006 | AC                                     |
| Letchford et al. [103]           | 2007 | Branch-and-cut algorithm               |
| Pisinger and Røpke [136]         | 2007 | ALNS                                   |
| Li et al. ([105])                | 2007 | Record-to-record travel algorithm      |
| Pessoa et al. [134]              | 2008 | Robust Branch-and-cut-and-price        |
| Derigs and Reuter [47]           | 2009 | ABHC                                   |
| Fleszar et al. [60]              | 2009 | VNS                                    |
| Li et al. [109]                  | 2009 | AC + TS                                |
| Repoussis et al. [146]           | 2010 | ES + GLS + TS                          |
| Zachariadis and Kiranoudis [186] | 2010 | Wide neighborhoods based metaheuristic |

A number of applications of the VRPSPD can be found in the beverage industry, where filled bottles are delivered while the empty ones are collected; in grocery stores, where pallets or containers are collected for re-use in merchandise transportation, etc. On the other hand, some customers can demand that the delivery and pickup services should be performed at the same time, since, if it is carried out separately, it may imply in additional costs and operational efforts for these customers.

The VRPSPD was first proposed by Min [119] in the late 1980's. The author presented a heuristic to solve a real-life problem concerning the distribution and collection of books of a public library.

Very few exact approaches were proposed in the VRPSPD literature. A Branch-and-price algorithm was developed by Dell'Amico et al. [44], in which two different strategies were used to solve the subpricing problem: (i) exact dynamic programming; and (ii) state space relaxation. The authors managed to find optimal solutions for instances with up to 40 customers. Angelelli and Manisini [3] also developed a Branch-and-price approach based on the set covering formulation, but for the VRPSPD with time-windows constraints. The subproblem was formulated as a shortest-path with resource constraints but without the elementary condition and it was solved by applying a permanent labeling algorithm. The authors were able to find optimal solutions for instances with up to 20 customers. Three-index formulations for the VRPSPD were proposed by Dethloff [48] and Montané and Galvão [121], however only the last authors had tested it in practice. They



ran their formulation in CPLEX 9.0 within a time limit of 2 hours and had reported the lower bounds produced for benchmark instances with 50-400 customers. Subramanian [154] presented a two-commodity flow formulation for the VRPSPD but no practical experiments were performed.

As a matter of fact, heuristic methods have been by far the most usual approach applied for solving the VRPSPD. However, only two works were published during the 1990's. Halse [82] proposed a two-phase heuristic based on the cluster-first, route-second concept for some VRPs including the VRPSPD and the VRPMPD, while some insertion heuristics also capable of solving the VRPMPD and the MDVRPMPD, were implemented by Salhi and Nagy [149].

A considerable number of VRPSPD works started to appear at the beginning of this century. Dethloff [48] proposed an insertion heuristic based on the cheapest feasible criterion, radial surcharge and residual capacity. The author also investigates the relation between the VRPSPD and other VRP variants. Røpke and Pisinger [148] developed a LNS heuristic associated with a procedure similar to the VNS metaheuristic to solve several variants of the VRP with Backhauls including the VRPSPD, VRPMPD and MDVRPMPD. Nagy and Salhi [128] developed a heuristic algorithm that considers different levels of feasibility. The authors also dealt with the VRPMPD and MDVRPMPD.

Most of the heuristics developed for the VRPSPD are based on the TS metaheuristic. Crispim and Brandão [40] presented a procedure where TS and the VND approach are combined. Montané and Galvão [121] proposed a TS algorithm involving traditional VRP neighborhood structures. Chen and Wu [27] proposed a local search procedure based on the record-to-record travel approximation and tabu lists. Chen [26] presented a heuristic based on SA and TS. Bianchessi and Righini [15] suggested some constructive and local search heuristics as well as a TS procedure that uses a variable neighborhood structure, in which the node-exchange-based and arc-exchange-based movements were combined. Wassan et al. [183] presented a reactive TS with the following neighborhood structures: relocation of a ADS, exchanging two customers between two different routes and reversing the route direction (reverse). Zachariadis et al. [189] developed an algorithm which combines the principles of the TS and GLS metaheuristics. The same authors [190] later suggested an AMP associated with a granular TS heuristic.

Some ES were also developed for the VRPSPD. Vural [180] proposed two Genetic Algorithms, where the first one is inspired on the random key representation [13] while the second one consists in an improvement heuristic that applies Or-opt movements. Gajpal and Abad [64] also developed an AC heuristic which has two main steps: (i) the trail intensities and parameters are initialized using an initial solution obtained by means of a nearest neighborhood constructive heuristic; and (ii) an ant-solution is generated for each ant using the trail intensities, followed by a local search in every ant-solution and updating

of the elitist ants and trail intensities. The authors also dealt with the VRPMPD. Ai and Kachitvichyanukul [1] suggested a Particle Swarm Optimization (PSO) [92] algorithm with multiple structures that employs a random key based representation.

Subramanian [154] and Subramanian et al. [156] put forward an ILS algorithm which uses a VND approach, with deterministic neighborhood ordering, in the local search phase. A parallel version of this algorithm, in which the VND procedure has a random neighborhood ordering (RVND), was later developed by Subramanian et al. [157]. Subramanian and Cabral [155] also applied the ILS-VND combination to solve the VRPSPD with route duration constraints. The unified VRP framework developed in the present work is an extension of these ILS approaches. Souza et al. [153] also implemented an ILS algorithm but combined with a GENIUS [66] approach. Finally, a local search based metaheuristic was recently proposed by Zachariadis and Kiranoudis [188].

Table 2.5 summarizes the VRPSPD related works mentioned in this section.

| Table 2.5: VRPSPD related works  |      |   |
|----------------------------------|------|---|
| Work                             | Year | Approach  |
| Min [119]                        | 1989 | Three-phase heuristic                               |
| Halse [82]                       | 1992 | Cluster-first, route-second strategy                |
| Salhi and Nagy [149]             | 1999 | Insertion based heuristics                          |
| Dethloff [48]                    | 2001 | Constructive heuristics                             |
| Angelelli and Mansini[3]         | 2001 | Branch-and-price for the VRPSPD with TW             |
| Vural [180]                      | 2003 | GA  |
| Røpke and Pisinger [148]         | 2004 | LNS   |
| Nagy and Salhi [128]             | 2005 | Constructive and Improvement/Feasibility Heuristics |
| Crispim and Brandão [40]         | 2005 | TS + VND  |
| Dell’amico et al. [44]           | 2006 | Branch-and-price                                    |
| Chen and Wu [27]                 | 2006 | Record-to-record travel + Tabu Lists                |
| Chen [26]                        | 2006 | TS + SA   |
| Montané and Galvão [121]         | 2006 | TS Algorithm  |
| Bianchessi and Righini [15]      | 2007 | Constructive, Local Search and TS + VNS Heuristics  |
| Wassan et al. [183]              | 2008 | Reactive TS   |
| Subramanian and Cabral [155]     | 2008 | ILS + VND for the VRPSPD with route durations       |
| Subramanian [154]                | 2008 | ILS + VND   |
| Subramanian et al. [156]         | 2008 | ILS + VND   |
| Zachariadis et al. [189]         | 2009 | TS + GLS  |
| Gajpal and Abad [64]             | 2009 | AC  |
| Zachariadis et al. [190]         | 2010 | TS + AMP  |
| Subramanian et al. [157]         | 2010 | Parallel ILS + RVND                                 |
| Souza et al. [157]               | 2011 | ILS + GENIUS  |
| Zachariadis and Kiranoudis [188] | 2011 | Local search metaheuristic                          |

## 2.5 Vehicle Routing Problem with Mixed Pickup and Delivery (VRPMPD)

The VRPMPD, also known as the VRP with Mixed Backhauls, is a particular case of the VRPSPD, in which customers either have a pickup or a delivery demand. More precisely, if  $d_i > 0$ , then  $p_i = 0$  and vice-versa. In principle, any VRPSPD solution method can be directly applied to solve the VRPMPD. According to the classification scheme of Berbeglia et al. [14], this problem is referred as the multi-vehicle one-to-many-to-one PDP with single demands and mixed solutions.

The first VRPMPD works appeared in the 1980's. Golden et al. [78] suggested a so-called stop-based backhaul insertion procedure, where routes are first generated in terms of the delivery customers, and then the pickup customers are evaluated for insertion in these routes. Casco et al. [24] proposed a so-called load-based backhaul insertion procedure, in which a penalty term, associated to the delivery load after pickup, is incorporated to the insertion cost.

A decade later, Mosheiov [123] developed two tour partitioning heuristics that consist of breaking a tour into disjoint segments which are assigned to different vehicles. Dethloff [49] applied the VRPSPD insertion approach developed in [48] to the VRPMPD. Wade and Salhi [181] proposed an AC heuristic that include some features such as the incorporation of a look ahead approach into the visibility function and enhanced trail updating rules. Recently, Wassan et al. [182] investigated the relationship between the VRPSPD and the VPMPD, besides presenting a Reactive TS heuristic based on the one developed for the VRPSPD [183].

Table 2.6 presents the works that dealt with the VRPMPD, including those of Salhi and Nagy [149], R pke and Pisinger [148], Crispim and Brand o [40] and Gajpal and Abad [64], that were cited in the previous subsection.

Table 2.6: VRPMPD related works

| Work                     | Year | Approach  |
|--------------------------|------|---|
| Golden et al. [78]       | 1985 | Stop-based backhaul insertion procedure             |
| Casco et al. [24]        | 1988 | Load-based backhaul insertion procedure             |
| Mosheiov [123]           | 1998 | Tour partitioning based heuristics                  |
| Salhi and Nagy [149]     | 1999 | Insertion based heuristics                          |
| Dethloff [49]            | 2002 | Insertion based heuristics                          |
| Wade and Salhi [181]     | 2003 | AC  |
| R pke and Pisinger [148] | 2004 | LNS   |
| Nagy and Salhi [128]     | 2005 | Constructive and Improvement/Feasibility Heuristics |
| Crispim and Brandao [40] | 2005 |   |
| Wassan et al. [182]      | 2008 | Reactive TS   |
| Gajpal and Abad [64]     | 2009 | AC  |

## 2.6 Traveling Salesmen Problem with Mixed Pickup and Delivery (TSPMPD)

The TSPMPD, sometimes referred as TSP with Pickup and Delivery or TSP with Delivery and Backhauls, is a special case of the VRPMPD in which only one vehicle is considered, i.e.,  $m = 1$ . Berbeglia et al. [14] classified this problem as the single-vehicle one-to-many-to-one PDP with single demands and mixed solutions.

Mosheiov [122] was one of the first to deal with the TSPMPD, in mid 1990's, when he described a real-life application concerning a welfare non-profit organization that arranges summer vacations for underprivileged children in out of town locations. The institution was faced with the problem of transporting the children between a central station and the vacation locations. The author also proposed a mathematical formulation as well as two heuristic procedures. The first one obtains a feasible solution for the TSPMPD by adapting an (sub-)optimal TSP solution, while the second one is based on the cheapest insertion criterion.

Some TSPMPD exact approaches have been proposed during the past decade. Baldacci et al. [8] developed a BC algorithm based on a two-commodity flow formulation that is capable of solving instances with up to 200 customers. Hernández-Pérez and Salazar-González [84] analyzed the relationship between the TSPMPD and a similar problem called *One-Commodity Pickup and Delivery TSP* (1-PDTSP) and they also proposed a BC algorithm that solves both problems. The same authors [86] later described a new set of valid inequalities and an improved BC algorithm that is capable of solving TSPMPD instances with up to 260 customers.

The number of heuristic works devoted to the TSPMPD are more or less equivalent to the exact ones. Anily and Mosheiov [4] suggested a two-phase MST heuristic where the first step consists of solving a relaxation of the problem and the second one consists of extending the relaxed solution into a feasible solution. Two heuristics were proposed by Gendreau et al. [68] in which the first one was designed to solve a special case of the TSPMPD that arises when the graph  $G$  is a cycle, whereas the second one consists of a TS algorithm that solves the general TSPMPD. Zhao et al. [191] presented a GA that makes use of a pheromone-based crossover operator as well as a local search procedure.

Hernández-Pérez and Salazar-González [85] presented a local search heuristic that uses 2-opt and 3-opt moves. They also proposed a hybrid algorithm, implemented using a Local Branching [56] scheme, that is based on their BC algorithm developed in [84]. This procedure also incorporates a primal heuristic that is periodically applied in order to obtain feasible solutions.

Table 2.7 compiles the TSPMPD works mentioned above.

Table 2.7: TSPMPD related works

| Work                                      | Year | Approach                                     |
|---|------|--|
| Mosheiov [122]                            | 1994 | TSP based and Cheapest Insertion heuristics  |
| Anily and Mosheiov [4]                    | 1994 | MST heuristics                               |
| Gendreau et al. [68]                      | 1999 | TS   |
| Baldacci et al. [8]                       | 2003 | BC based on a two-commodity flow formulation |
| Hernández-Pérez and Salazar-González [84] | 2004 | BC algorithm                                 |
| Hernández-Pérez and Salazar-González [85] | 2004 | Local Search and Hybrid algorithms           |
| Hernández-Pérez and Salazar-González [86] | 2007 | Improved BC algorithm                        |
| Zhao et al. [191]                         | 2009 | GA   |

## 2.7 Multi-depot Vehicle Routing Problem (MDVRP)

Let  $G$  be the set of depots. The MDVRP is a generalization of the CVRP where more than one depot may be considered, that is,  $|G| \geq 1$ . Also, the vehicle must start and end at the same depot. Typically, the number of vehicles per depot is given as an input data.

Applications of the MDVRP may arise when a company has multiple warehouses and/or subsidiaries, each of these with their own vehicle fleet, that together are capable of meeting customers' demands.

Tillman [170] was the first to address the MDVRP in late 1960's when he proposed an algorithm based on Clarke and Wright savings heuristic.

Exact Branch-and-bound algorithms were proposed in the 1980's by Laporte et al. [98] and Laporte et al. [99], where the first was capable of solving instances with up to 50 customers and 8 depots, while the latter managed to solve problems with up to 80 customers and 8 depots. Baldacci and Mingozzi [10] put forward a SP based algorithm that uses bounding procedures based on linear relaxation and lagrangean relaxation. Their method was designed to deal with several VRPs, including the HFVRP, Site-dependent VRP and the MDVRP. The authors found optimal solutions of MDVRP instances with up to 200 customers and 4 depots.

Some MDVRP algorithms based on classical VRP heuristics were suggested between early 1970's and early 1980's. Tillman and Hering [172] proposed an algorithm that incorporates a look ahead approach into Tillman's savings heuristic [170]. Tillman and Cain [171] also extended Tillman's algorithm [170] by including a partial enumerative procedure to compute the savings of joining customers on routes. Wren and Holliday [184] put forward an algorithm that employs a sweep procedure to generate routes which in turn are further refined by a set of improvement heuristics. Gillet and Johnson [70] applied the classical sweep algorithm of Gillet and Miller [71] to build an initial solution that is later refined by an improvement procedure that reassigns customers to different depots. Golden et al. [79] proposed two heuristic approaches where the first is a savings based method, while the second is a cluster-first route-second procedure. Raft [142] suggested

a modular algorithm that decomposes the problem into subproblems which are solved separately and then merged by means of an iterative procedure.

Like some other VRP variants, MDVRP algorithms inspired in metaheuristics started to appear in the 1990's. Chao et al. [25] developed an algorithm that makes use of a record-to-record procedure to improve an initial solution generated using the savings heuristic of Golden et al. [79]. Renaud et al. [145] implemented a TS heuristic composed of three phases: fast improvements, intensification and diversification. Cordeau et al. [35] suggested a TS algorithm that is embedded with a GENI mechanism, which consists of a general insertion procedure originally developed by Gendreau et al. [66] for the TSP.

Table 2.8 lists the works that dealt with the MDVRP, including those of Pisinger and R pke [136] and Vidal et al. [177] that were already described in Subsection 2.1.

Table 2.8: MDVRP related works

| Work                       | Year | Approach  |
|----------------------------|------|---|
| Tillman [170]              | 1969 | Savings heuristic                                 |
| Tillman and Hering [172]   | 1971 | Savings heuristic + look ahead approach           |
| Tillman and Cain [171]     | 1972 | Savings heuristic + partial enumerative procedure |
| Gillet and Johnson [70]    | 1976 | Sweep algorithm                                   |
| Golden et al. [79]         | 1977 | Savings and route-first cluster-second heuristics |
| Raft [142]                 | 1982 | Modular algorithm                                 |
| Chao et al. [25]           | 1993 | Record-to-record algorithm                        |
| Renaud et al. [145]        | 1996 | TS  |
| Cordeau et al. [35]        | 1997 | TS + GENI   |
| Pisinger and R pke [136]   | 2007 | ALNS  |
| Baldacci and Mingozzi [10] | 2009 | SP based exact algorithm                          |
| Vidal et al. [177]         | 2011 | Hybrid GA   |

## 2.8 Multi-depot Vehicle Routing Problem with Mixed Pickup and Delivery (MDVRPMPD)

The MDVRPMPD generalizes the VRPMPD by allowing  $|G| \geq 1$  depot(s). As in the MDVRP, the vehicle must start and end at the same depot.

Salhi and Nagy [149] introduced the MDVRPMPD in late 1990's and their solution method has been already described in Subsection 2.4.

To date, the problem has received very little attention in the literature. Only two other works dealt with it, particularly those of R pke and Pisinger [148] and Nagy and Salhi [128]. Both were mentioned in Subsection 2.4.

Table 2.9 contains the MDVRPMPD works cited above.

Table 2.9: MDVRPMPD related works

| Work                     | Year | Approach  |
|--------------------------|------|---|
| Salhi and Nagy [149]     | 1999 | Insertion based heuristics                          |
| R pke and Pisinger [148] | 2004 | LNS   |
| Nagy and Salhi [128]     | 2005 | Constructive and Improvement/Feasibility Heuristics |

## 2.9 Heterogeneous Fleet Vehicle Routing Problem (HFVRP)

The HFVRP is a generalization of the classical VRP because it allows vehicles with different capacities, instead of a homogeneous fleet. The fleet is composed by  $m$  different types of vehicles, with  $M = \{1, \dots, m\}$ . For each  $u \in M$ , there are  $m_u$  available vehicles, each with a capacity  $Q_u$ . Every vehicle is associated with a fixed cost denoted by  $f_u$  and a dependent (variable) cost denoted by per distance unit.

This situation can be often found in practice and the HFVRP might be a suitable model for dealing with this kind of applications. According to Hoff et al. [87], in industry, a fleet of vehicles is rarely homogeneous. Generally, either an acquired fleet is already heterogeneous or they become heterogeneous over the time when vehicles with different features are incorporated into the original fleet. In addition, insurance, maintenance and operating costs usually have distinct values according to the level of depreciation or usage time of the fleet. Moreover, from both tactical and operational point of view, a mixed vehicle fleet also increases the flexibility in terms of distribution planning.

There are several HFVRP variants often found in the literature. They are basically related to the fleet limitation (limited or unlimited) and the costs considered (dependent and/or fixed). The HFVRP with unlimited fleet, also known as the Fleet Size and Mix (FSM), was proposed by Golden et al. [76] and it consists of determining the best fleet composition and its optimal routing scheme. Another HFVRP version, called Heterogeneous VRP (HVRP), was proposed by Taillard [164] and it consists in optimizing the use of the available fixed fleet.

The present work deals with the five following variants:

- i. HVRPFD, limited fleet, with fixed and dependent costs;
- ii. HVRPD, limited fleet, with dependent costs but without fixed costs, i.e.,  $f_u = 0, \forall k \in M$ ;
- iii. FSMFD, unlimited fleet, i.e.,  $m_u = +\infty, \forall k \in M$ , with fixed and dependent costs;
- iv. FSMF, unlimited fleet, with fixed costs but without dependent costs;
- v. FSM, unlimited fleet, with dependent costs but without fixed costs.

The first HFVRP variant studied in the literature was the FSM, initially proposed by Golden et al. [76]. The authors developed two heuristics where the first one is based on the savings algorithm of [33] while the second one makes use of a giant tour scheme. They also proposed a mathematical formulation for the FSMF and presented some lower bounds.

Some exact approaches were developed for the FSM. Yaman [185] suggested valid inequalities and presented lower bounds for the FSMF. Baldacci et al. [6] proposed some valid inequalities as well as a two-commodity Mixed Integer Programming (MIP) formulation for the same variant. Choi and Tcha [29] obtained lower bounds for all FSM variants by means of a Column Generation algorithm based on a set covering formulation. Pessoa et al. [135] proposed a BCP algorithm also capable of solving all FSM variants. The general exact approach of Baldacci and Mingozzi [10], mentioned in Subsection 2.7, was also applied to the five HFVRP variants dealt in the present work.

Several TS heuristics were proposed to solve the FSMF and the FSMD. Gendreau et al. [68] suggested a TS algorithm that incorporates a GENIUS approach and an AMP. Lee et al. [101] developed an algorithm that combines TS with a SP approach. More recently, Brandão [22] proposed a deterministic TS that makes use of different procedures for generating initial solutions.

Some authors implemented heuristic procedures based on ES. Ochi et al. [129] developed a hybrid evolutionary heuristic that combines a Genetic Algorithm (GA) [88] with Scatter Search (SS) [73] to solve the FSMF. A parallel version, based on the island model, of the same algorithm was presented by Ochi et al. [130]. A hybrid GA that applies a local search as a mutation method was proposed by Liu et al. [111] to solve the FSMF and the FSMD. A MA was proposed by Lima et al. [110] for solving the FSMF. Two heuristic procedures based on the same metaheuristic were developed by Prins [141] to solve all FSM variants and the HVRPD.

Renaud and Boctor [144] proposed a sweep-based heuristic for the FSMF that integrates classical construction and improvement VRP approaches. Imran et al. [90] developed a VNS algorithm that makes use of a procedure based on Dijkstra's and sweep algorithms for generating an initial solution and several neighborhood structures in the local search phase. The authors considered all FSM variants.

The HVRP was proposed by Taillard [164]. The author developed an algorithm based on AMP, TS and Column Generation which was also applied to solve the FSM.

Prins [138] dealt with the HVRP by implementing a heuristic that extends a series of VRP classical heuristics followed by a local search procedure based on the Steepest Descent Local Search and TS. Tarantilis et al. [168] solved the HVRPD by means of a threshold accepting approach that consists of an adaptation of the SA procedure in which a worse solution is only accepted if it is within a given threshold. The same authors [169]



also proposed another threshold accepting procedure to solve the same variant. Li et al. [106] put forward a record-to-record travel algorithm that, also as the threshold method, consists of a deterministic variant of the SA. The authors considered both HVRPFD and HVRPD. Li et al. [107] proposed a multi-start adaptive memory procedure combined with Path Relinking and a modified TS to solve the HVRPFD. More recently, [20] proposed a TS algorithm for the HVRP which includes additional features such as strategic oscillation, shaking and frequency-based memory.

A HFVRP comprehensive survey containing all the five variants mentioned here can be found in [5]. The HFVRP works are summarized in Table 2.10.

Table 2.10: HFVRP related works

| Work                       | Year | Variant(s)         | Approach                               |
|----------------------------|------|--------------------|--|
| Golden et al. [76]         | 1984 | FSMF               | Heuristic algorithms                   |
| Ochi et al. [129]          | 1998 | FSMF               | GA + SS                                |
| Ochi et al. [130]          | 1998 | FSMF               | Parallel GA + SS                       |
| Taillard [164]             | 1999 | HVRPD, FSMF        | TS + AMP + Column Generation           |
| Gendreau et al. [68]       | 1999 | FSMF, FSMF         | TS + GENIUS + AMP                      |
| Renaud and Boctor [144]    | 2002 | FSMF               | Sweep algorithm                        |
| Prins [138]                | 2002 | HVRPD              | Heuristic algorithms                   |
| Tarantilis et al. [168]    | 2003 | HVRPD              | Threshold Accepting approach           |
| Tarantilis et al. [169]    | 2004 | HVRPD              | Threshold Accepting approach           |
| Lima et al. [110]          | 2004 | FSMF               | MA                                     |
| Yaman [185]                | 2006 | FSMF               | Valid inequalities                     |
| Choi and Tcha [29]         | 2007 | FSMF, FSMF, FSMF   | Column Generation Algorithm            |
| Li et al. [106]            | 2007 | HVRPFD, HVRPD      | Record-to-record                       |
| Lee et al. [101]           | 2008 | FSMF, FSMF         | TS + SP                                |
| Baldacci et al. [5]        | 2008 | All                | Survey                                 |
| Liu et al. [111]           | 2009 | FSMF, FSMF         | Hybrid GA                              |
| Pessoa et al. [135]        | 2009 | FSMF, FSMF, FSMF   | BCP algorithm                          |
| Imran et al. [90]          | 2009 | FSMF, FSMF, FSMF   | VNS                                    |
| Brandão [22]               | 2009 | FSMF, FSMF         | Deterministic TS                       |
| Prins [141]                | 2009 | All, except HVRPFD | Two MAs                                |
| Baldacci et al. [6]        | 2009 | FSMF               | MIP formulation and valid inequalities |
| Baldacci and Mingozzi [10] | 2009 | All                | SP based exact algorithm               |
| Li et al. [107]            | 2010 | HVRPFD             | SP AMP + Path Relinking + TS           |
| Brandão [20]               | 2011 | HVRPD              | TS algorithm                           |

## Chapter 3

# Branch-and-cut over Flow Formulations for the Vehicle Routing Problem with Simultaneous/Mixed Pickup and Delivery

This chapter presents commodity flow formulations for the VRPSPD/VRPMPD. Two-commodity flow formulations — an undirected, developed by Subramanian [154], and a directed, proposed in this work — are theoretically compared with the one-commodity flow formulation presented by Dell’Amico et al. [44]. In addition, these three formulations were implemented within a BC algorithm, including cuts from the CVRPSEP library [114], and they were tested in well-known VRPSPD and VRPMPD benchmark problems with up to 200 customers. The contents of the present chapter were partially published in [159].

### 3.1 One-commodity flow formulation

Reasonably simple and effective formulations for the CVRP can be defined only over the natural edge variables (arc variables in the asymmetric case), see [174]. Similar formulations are not available for the VRPSPD. The difference between these two problems can be explained as follows. In the CVRP, the feasibility of a route can be determined by only checking whether the sum of its customer demands does not exceed the vehicle’s capacity. In contrast, the feasibility of a VRPSPD route depends crucially on the sequence of visitation of the clients. In the example shown in Fig. 3.1, the route  $0 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 0$  is feasible, but the shorter routes  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 0$  or  $0 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0$  are not. This fact suggests the use of extended formulations, where auxiliary flow variables are used to enforce route feasibility.

The following directed one-commodity flow formulation for the VRPSPD was proposed by Dell’Amico et al. [44]. Define  $A$  as the set of arcs consisting of a pair of opposite arcs  $(i, j)$  and  $(j, i)$  for each edge  $\{i, j\} \in E$  and let  $D_{ij}$  and  $P_{ij}$  be the flow variables which

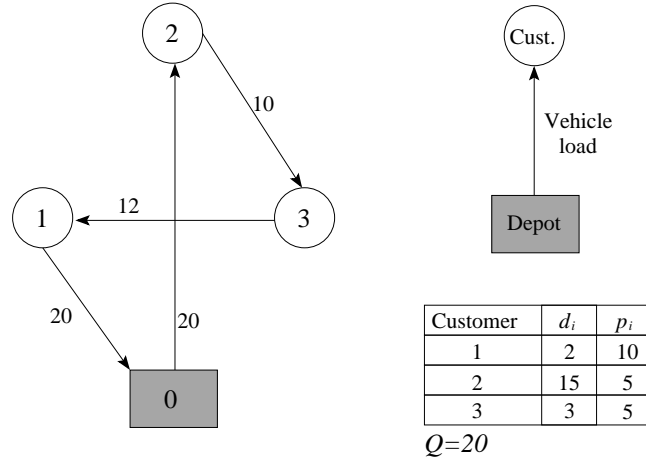


Figure 3.1: VRPSPD example

indicate, respectively, the delivery and pickup loads carried along the arc  $(i, j) \in A$ . Let  $x_{ij}$  be 1 if the arc  $(i, j) \in A$  is in the solution and 0 otherwise. The formulation F1C is described next.

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij} \quad (3.1)$$

$$\text{s.t.} \quad \sum_{j \in V} x_{ij} = 1 \quad \forall i \in V' \quad (3.2)$$

$$\sum_{j \in V} x_{ji} = 1 \quad \forall i \in V' \quad (3.3)$$

$$\sum_{j \in V'} x_{0j} \leq m \quad (3.4)$$

$$\sum_{j \in V} D_{ji} - \sum_{j \in V} D_{ij} = d_i \quad \forall i \in V' \quad (3.5)$$

$$\sum_{j \in V} P_{ij} - \sum_{j \in V} P_{ji} = p_i \quad \forall i \in V' \quad (3.6)$$

$$D_{ij} + P_{ij} \leq Q x_{ij} \quad \forall (i, j) \in A \quad (3.7)$$

$$D_{ij} \geq 0 \quad \forall (i, j) \in A \quad (3.8)$$

$$P_{ij} \geq 0 \quad \forall (i, j) \in A \quad (3.9)$$

$$x_{ij} \in \{0, 1\} \quad \forall (i, j) \in A \quad (3.10)$$

The objective function (3.1) minimizes the sum of the travel costs. Constraints (3.2)-(3.3) impose that each client should be visited exactly once. Constraints (3.4) refer to the number of vehicles available. Constraints (3.5)-(3.7) are the flow conservation equalities. Constraints (3.8)-(3.10) are related to the nature of the decision variables.

Dell'Amico et al. [44] basically extended the one-commodity flow formulation pro-

posed by Gavish and Graves [65] for the CVRP by adding constraints (3.6) and (3.9), and the term  $P_{ij}$  in (3.7). Gouveia [81] showed that it is possible to obtain stronger inequalities for  $D_{ij}$  by using the tighter bounds (3.11) instead of (3.8) in the Gavish and Graves formulation. Accordingly, the same idea can be applied to develop stronger inequalities for  $P_{ij}$  by replacing (3.9) with (3.12) and for  $D_{ij} + P_{ij}$  by replacing (3.7) with (3.13).

$$d_j x_{ij} \leq D_{ij} \leq (Q - d_i) x_{ij} \quad \forall (i, j) \in A \quad (3.11)$$

$$p_i x_{ij} \leq P_{ij} \leq (Q - p_j) x_{ij} \quad \forall (i, j) \in A \quad (3.12)$$

$$D_{ij} + P_{ij} \leq (Q - \max\{0, p_j - d_j, d_i - p_i\}) x_{ij} \quad \forall (i, j) \in A \quad (3.13)$$

It should be noticed that a lower bound for (3.13) is implicit in (3.11) and (3.12), i.e.,  $D_{ij} + P_{ij} \geq d_j x_{ij} + p_i x_{ij}$ . Another valid inequality for F1C, given by (3.14), is due to the fact that each edge not adjacent to the depot is traversed at most once.

$$x_{ij} + x_{ji} \leq 1 \quad \forall i, j, i < j, \in V' \quad (3.14)$$

## 3.2 Two-commodity flow formulations

This section presents both an undirected and a directed two-commodity flow formulations for the VRPSPD which are based on the one proposed by Baldacci et al. [9] for the CVRP. It is important to mention that the idea of employing two-commodities was originally developed by Finke et al. [55] for the TSP. Their formulation was later generalized by Lucena [113] for the CVRP.

### 3.2.1 Undirected two-commodity flow formulation

For the sake of convenience let vertex  $n+1$  be a copy of the depot,  $\bar{V} = V \cup \{n+1\}$  and  $\bar{E}$  be the complete set of edges  $\bar{E}$ , excepting  $\{0, n+1\}$ . Let  $x'_{ij}$  be 1 if the edge  $\{i, j\} \in \bar{E}$  is in the solution and 0 otherwise. Let the variables  $D'_{ij}$ ,  $P'_{ij}$  and  $SPD_{ij}$  denote, respectively, the delivery, pickup and simultaneous pickup and delivery flows when a vehicle goes from  $i \in \bar{V}$  to  $j \in \bar{V}$  and let the same variables denote, respectively, the associated residual capacities when a vehicle goes from  $j \in \bar{V}$  to  $i \in \bar{V}$ , in such a way that  $D'_{ij} + D'_{ji} = Qx'_{ij}$ ,  $P'_{ij} + P'_{ji} = Qx'_{ij}$  and  $SPD_{ij} + SPD_{ji} = Qx'_{ij}$ . Also, an integer variable  $v$ , which denotes the number of vehicles utilized, is included with an upper bound  $m$ . In the Baldacci et al. formulation [9] the precise number of vehicles  $m$  is assumed to be known in advance, since their formulation will produce feasible solutions with exact  $m$  vehicles.

Fig. 3.2 shows an example of the scheme used by the two-commodity flow formulation for the VRPSPD, where (i), (ii) and (iii) denote, respectively, the delivery, pickup and simultaneous pickup and delivery flows. In this case,  $Q = 20$  and two routes are considered where  $r_1$  is the one composed by  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow n+1$  and  $r_2$  is composed by

$0 \rightarrow 5 \rightarrow 6 \rightarrow 7 \rightarrow 8 \rightarrow n+1$ . Moreover, it can be observed that the flows when the vehicle is leaving the depot are equivalent in (i) and (iii), whereas the flows when the vehicle is returning to the depot are equivalent in (ii) and (iii). This fact can be generalized to any VRPSPD instance by means of the following relationships:  $SPD_{0j} = D'_{0j}$ ,  $SPD_{j0} = D'_{j0}$ ,  $SPD_{j,n+1} = P'_{j,n+1}$  and  $SPD_{n+1,j} = D'_{n+1,j}$ ,  $\forall j \in V'$ .

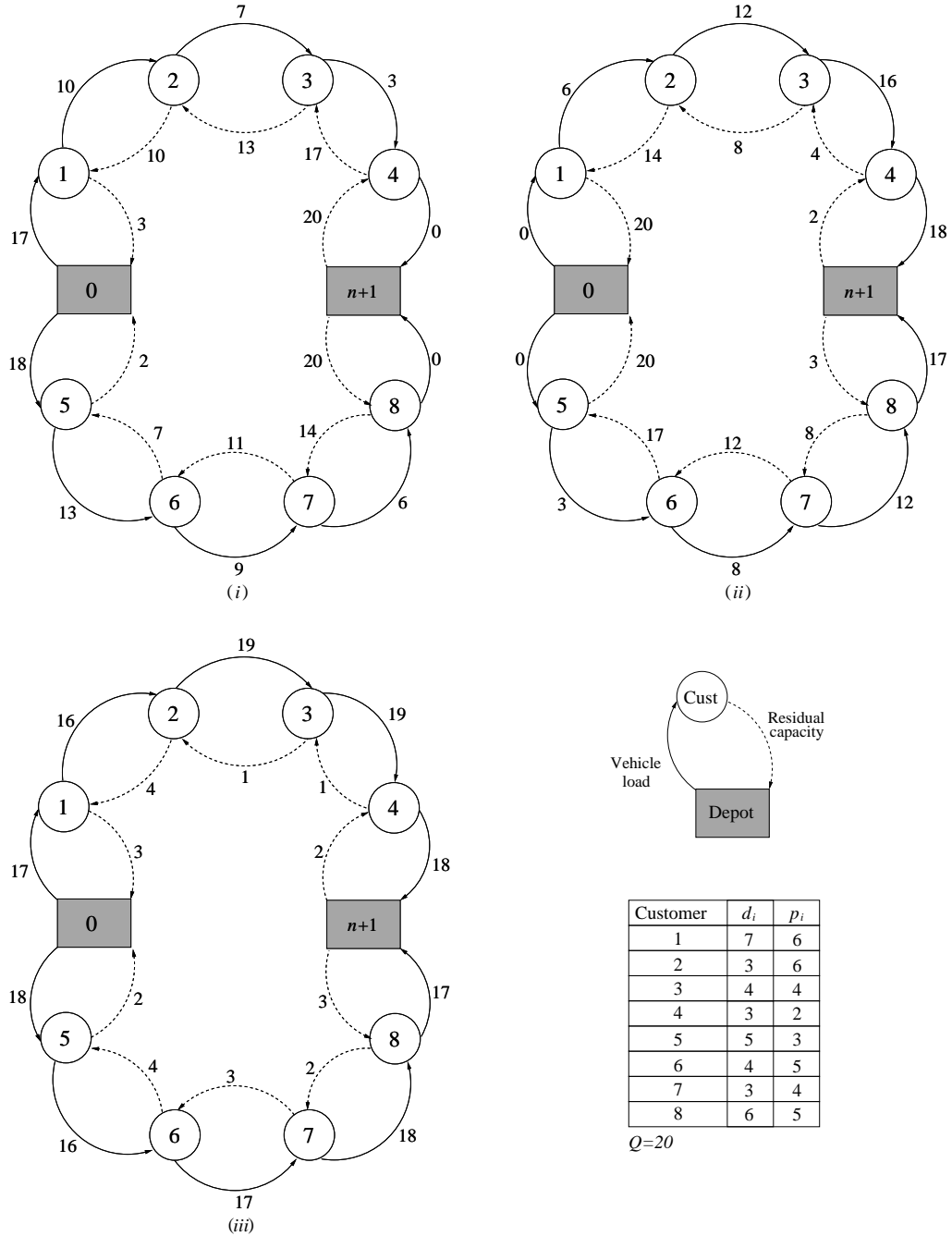


Figure 3.2: The two-commodity formulation scheme for the VRPSPD

The undirected formulation F2C-U is as follows.

$$\min \sum_{\{i,j\} \in \bar{E}} c_{ij} x'_{ij} \quad (3.15)$$

$$\text{s.t.} \quad \sum_{i \in \bar{V}, i < k} x'_{ik} + \sum_{j \in \bar{V}, j > k} x'_{kj} = 2 \quad \forall k \in V' \quad (3.16)$$

$$\sum_{j \in \bar{V}} (D'_{ji} - D'_{ij}) = 2d_i \quad \forall i \in V' \quad (3.17)$$

$$\sum_{j \in V'} D'_{0j} = \sum_{i \in V'} d_i \quad (3.18)$$

$$\sum_{j \in V'} D'_{j0} = vQ - \sum_{i \in V'} d_i \quad (3.19)$$

$$\sum_{j \in \bar{V}} (P'_{ij} - P'_{ji}) = 2p_i \quad \forall i \in V' \quad (3.20)$$

$$\sum_{j \in V'} P'_{j,n+1} = \sum_{i \in V'} p_i \quad (3.21)$$

$$\sum_{j \in V'} P'_{n+1,j} = vQ - \sum_{i \in V'} p_i \quad (3.22)$$

$$\sum_{j \in \bar{V}} (SPD_{ji} - SPD_{ij}) = 2(d_i - p_i) \quad \forall i \in V' \quad (3.23)$$

$$SPD_{0j} = D'_{0j} \quad \forall j \in V' \quad (3.24)$$

$$SPD_{j0} = D'_{j0} \quad \forall j \in V' \quad (3.25)$$

$$SPD_{j,n+1} = P'_{j,n+1} \quad \forall j \in V' \quad (3.26)$$

$$SPD_{n+1,j} = D'_{n+1,j} \quad \forall j \in V' \quad (3.27)$$

$$D'_{ij} + D'_{ji} = Qx'_{ij} \quad \forall \{i, j\} \in \bar{E} \quad (3.28)$$

$$P'_{ij} + P'_{ji} = Qx'_{ij} \quad \forall \{i, j\} \in \bar{E} \quad (3.29)$$

$$SPD_{ij} + SPD_{ji} = Qx'_{ij} \quad \forall \{i, j\} \in \bar{E} \quad (3.30)$$

$$D'_{j,n+1} = P'_{0j} = 0 \quad \forall j \in V' \quad (3.31)$$

$$\sum_{j \in V'} D'_{n+1,j} = \sum_{j \in V'} P'_{j0} = vQ \quad (3.32)$$

$$\sum_{j \in V'} x'_{0j} = \sum_{j \in V'} x'_{j,n+1} = v \quad (3.33)$$

$$0 \leq v \leq m \quad (3.34)$$

$$D'_{ij} \geq 0, D'_{ji} \geq 0 \quad \forall \{i, j\} \in \bar{E} \quad (3.35)$$

$$P'_{ij} \geq 0, P'_{ji} \geq 0 \quad \forall \{i, j\} \in \bar{E} \quad (3.36)$$

$$SPD_{ij} \geq 0, SPD_{ji} \geq 0 \quad \forall \{i, j\} \in \bar{E} \quad (3.37)$$

$$x'_{ij} \in \{0, 1\} \quad \forall (i, j) \in \bar{E} \quad (3.38)$$

The objective function (3.15) minimizes the sum of the travel costs. Constraints (3.16) are the degree equations. Constraints (3.17) ensure that the delivery demands are satisfied. Constraints (3.18) state that the sum of the vehicle loads leaving the vertex 0 must be equal to the sum of the demand of all costumers. Constraints (3.19) enforce that the sum of the vehicle loads arriving at the vertex 0 must be equal to the sum of the residual capacity of all vehicles. Constraints (3.20)-(3.22) are related to the pickup flow and their meaning are, respectively, analogous to (3.17)-(3.19). Constraints (3.23) guarantee that the pickup and delivery demands are simultaneously satisfied. Constraints (3.24)-(3.27) are self-explanatory. Constraints (3.28)-(3.30) state, respectively, that the sum of the delivery, pickup and combined loads arriving and leaving each customer must be equal to the vehicle capacity. Constraints (3.31)-(3.32) are self-explanatory. Constraint (3.33) is related to the number of vehicles. Constraints (3.34)-(3.38) define the domain of the decision variables.

The formulation F2C-U was obtained by simply adding constraints (3.20)-(3.27), (3.29)-(3.34) and (3.36)-(3.37) to the formulation presented in [9]. As in F1C, stronger inequalities can be developed by tightening the bounds of the flow variables, i.e, replacing (3.35)-(3.36) with (3.39)-(3.40) and (3.37) with (3.41).

$$D'_{ij} \geq d_j x'_{ij} \quad \forall (i, j) \in \bar{E} \quad (3.39)$$

$$P'_{ij} \geq p_i x'_{ij} \quad \forall (i, j) \in \bar{E} \quad (3.40)$$

$$SPD_{ij} \geq \max\{0, d_j - p_j, p_i - d_i\} x'_{ij} \quad \forall (i, j) \in \bar{E} \quad (3.41)$$

Although the lower bounds of the flow variables are not explicit in (3.39)-(3.41) it can be easily verified that they become inherent to the formulation when these upper bound inequalities are combined with (3.28)-(3.30), resulting in  $D'_{ij} \leq (Q - d_i) x'_{ij}$ ,  $P'_{ij} \leq (Q - d_j) x'_{ij}$  and  $SPD_{ij} \leq (Q - \max\{0, d_i - p_i, p_j - d_j\}) x'_{ij}$ .

### 3.2.2 Directed two-commodity flow formulation

Let  $\bar{A}$  be the set of arcs  $(i, j)$ ,  $\forall i, j \in \bar{V}$  and  $\bar{x}_{ij}$  be 1 if the arc  $(i, j) \in \bar{A}$  is in the solution and 0 otherwise. A directed version of the two-commodity flow formulation (F2C-D) is as follows.

$$\min \sum_{i \in \bar{V}} \sum_{j \in \bar{V}} c_{ij} \bar{x}_{ij} \quad (3.42)$$

$$\text{s.t.} \quad \sum_{j \in \bar{V}} \bar{x}_{ij} = 1 \quad \forall i \in V' \quad (3.43)$$

$$\sum_{j \in \bar{V}} \bar{x}_{ji} = 1 \quad \forall i \in V' \quad (3.44)$$

$$\bar{x}_{j0} = \bar{x}_{n+1,j} = 0 \quad \forall j \in V' \quad (3.45)$$

$$D'_{ij} + D'_{ji} = Q(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall (i, j), i < j, i \in A \quad (3.46)$$

$$P'_{ij} + P'_{ji} = Q(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall (i, j), i < j, i \neq 0 \in \bar{A} \quad (3.47)$$

$$SPD_{ij} + SPD_{ji} = Q(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall i, j, i < j, i \in V' \quad (3.48)$$

$$\sum_{j \in V'} \bar{x}_{0j} = \sum_{j \in V'} \bar{x}_{j,n+1} = v \quad (3.49)$$

$$\bar{x}_{ij} \in \{0, 1\} \quad \forall (i, j) \in \bar{A} \quad (3.50)$$

(3.17)-(3.27), (3.31)-(3.32) and (3.34)-(3.37)

Constraints (3.43)-(3.44) are the degree equations. The meaning of constraint (3.45) is self-explanatory. Constraints (3.46)-(3.48) are the capacity equalities. Constraints (3.49)-(3.50) have already been defined.

The stronger flow inequalities defined for F2C-U also hold for F2C-D as can be observed in (3.51)-(3.53). Also, the arc inequalities (3.14) used in F1C can be directly converted to F2C-D as shown in (3.54).

$$D'_{ij} \geq d_j(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall (i, j) \in \bar{A} \quad (3.51)$$

$$P'_{ij} \geq p_i(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall (i, j) \in \bar{A} \quad (3.52)$$

$$SPD_{ij} \geq (\max\{0, d_j - p_j, p_i - d_i\})(\bar{x}_{ij} + \bar{x}_{ji}) \quad \forall (i, j) \in \bar{A} \quad (3.53)$$

$$\bar{x}_{ij} + \bar{x}_{ji} \leq 1 \quad \forall i, j, i < j, i \in V' \quad (3.54)$$

F2C-D is clearly at least as strong as F2C-U since the degree constraints (3.43)-(3.44) along with (3.54) dominate (3.16) and the linear relaxation of (3.38), whereas the remaining constraints are equivalent in both formulations.

Letchford and Salazar-Gonzalez [104] have shown that the one-commodity formulation and the directed two-commodity flow formulation with their respective stronger inequalities are equivalent for the CVRP. However, this fact is not verified for the VRPSD as stated by Proposition 1.

**Proposition 1.** *The linear relaxation of F1C with (3.11)-(3.14) is stronger than the one obtained by F2C-D with (3.51)-(3.54).*

*Proof.* First, one shall prove that given the solution vector  $(x^*, D^*, P^*)$  with cost  $z^*$  of the linear programming relaxation of the one-commodity flow formulation, it is possible to build a feasible solution of the linear program of F2C-D (in terms of  $(\bar{x}, D', P', SPD, v)$ ) with the same cost.



The values of the variables of F2C-D can be directly obtained by means of (3.55)-(3.68). For the sake of simplicity let  $P_{j,n+1} = P_{j0}$  and  $P_{n+1,j} = P_{0j}$ ,  $\forall j \in V'$ .

$$\bar{x}_{ij} = x_{ij} \quad \forall i, j \in V' \quad (3.55)$$

$$\bar{x}_{0j} = x_{0j} \quad \forall j \in V' \quad (3.56)$$

$$\bar{x}_{j,n+1} = x_{j0} \quad \forall j \in V' \quad (3.57)$$

$$D'_{ij} = D_{ij} + (Q\bar{x}_{ji} - D_{ji}) \quad \forall (i, j), i < j, \in A \quad (3.58)$$

$$D'_{ji} = D_{ji} + (Q\bar{x}_{ij} - D_{ij}) \quad \forall (i, j), i < j, \in A \quad (3.59)$$

$$P'_{ij} = P_{ij} + (Q\bar{x}_{ji} - P_{ji}) \quad \forall (i, j), i < j, i \neq 0 \in \bar{A} \quad (3.60)$$

$$P'_{ji} = P_{ji} + (Q\bar{x}_{ij} - P_{ij}) \quad \forall (i, j), i < j, i \neq 0 \in \bar{A} \quad (3.61)$$

$$SPD_{ij} = D_{ij} + P_{ij} + (Q\bar{x}_{ji} - D_{ji} - P_{ji}) \quad \forall i, j, i < j, \in V' \quad (3.62)$$

$$SPD_{ji} = D_{ji} + P_{ji} + (Q\bar{x}_{ij} - D_{ij} - P_{ij}) \quad \forall i, j, i < j, \in V' \quad (3.63)$$

$$D'_{j,n+1} = P'_{0j} = \bar{x}_{j0} = \bar{x}_{n+1,j} = 0 \quad \forall j \in V' \quad (3.64)$$

$$D'_{n+1,j} = Q\bar{x}_{n+1,j}, P'_{j0} = Q\bar{x}_{j0} \quad \forall j \in V' \quad (3.65)$$

$$SPD_{0j} = D'_{0j}, SPD_{j0} = D'_{j0} \quad \forall j \in V' \quad (3.66)$$

$$SPD_{j,n+1} = P'_{j,n+1}, SPD_{n+1,j} = P'_{n+1,j} \quad \forall j \in V' \quad (3.67)$$

$$v = \sum_{j \in V'} \bar{x}_{0j} \quad (3.68)$$

Note that constraints (3.46)-(3.48) are automatically satisfied since they can be easily obtained from (3.58)-(3.63). Constraints (3.51) are satisfied since, according to (3.11),  $D_{ij} \geq d_j \bar{x}_{ij}$  and  $Q\bar{x}_{ji} - D_{ji} \geq d_j \bar{x}_{ji}$ , which implies in  $D'_{ij} \geq d_j(\bar{x}_{ij} + \bar{x}_{ji})$ . The same idea can be employed, using (3.12), to show that constraints (3.52) are also satisfied.

To verify if constraints (3.53) are not violated the following statement must be proven:  $D_{ij} + P_{ij} + (Q\bar{x}_{ij} - D_{ji} - P_{ji}) \geq (\max\{0, d_j - p_j, p_i - d_i\})(\bar{x}_{ij} + \bar{x}_{ji})$ . Using the fact that  $D_{ij} + P_{ij} \geq d_j \bar{x}_{ij} + p_i \bar{x}_{ij}$  (see (3.11)-(3.12)) and after some algebraic manipulation one can obtain:  $d_j \bar{x}_{ij} + p_i \bar{x}_{ij} + (Q - \max\{0, d_j - p_j, p_i - d_i\})\bar{x}_{ji} \geq D_{ji} + P_{ji} + (\max\{0, d_j - p_j, p_i - d_i\})\bar{x}_{ji}$ . From (3.13) it can be observed that  $(Q - \max\{0, d_j - p_j, p_i - d_i\})\bar{x}_{ji} \geq D_{ji} + P_{ji}$  and it is clear that  $d_j \bar{x}_{ij} + p_i \bar{x}_{ij} \geq (\max\{0, d_j - p_j, p_i - d_i\})\bar{x}_{ij}$ , which proves that (3.53) is satisfied.

Thus one can conclude that the vector  $(\bar{x}, D', P', SPD, v)$  is indeed a feasible solution of the linear program of F2C-D.

On the other hand, given the solution vector  $(\bar{x}^*, D'^*, P'^*, SPD^*, v^*)$  with cost  $\bar{z}^*$  of the linear programming relaxation of F2C-D it is not always possible to build a feasible solution in terms of  $(x, D, P)$  with the same cost. Tables 3.1 and 3.3; 3.4 and 3.6; and 3.7 and 3.9; all presented in Section 5, show that the value of the linear relaxation obtained by the F1C is always greater or equal than the one found by F2C-D.  $\square$

### 3.3 A Branch-and-cut approach

A simple BC algorithm was employed to evaluate the formulations presented in this work. Traditional CVRP inequalities were used, namely the rounded capacity, multistar and comb inequalities. They can be directly applied to the VRPSPD. The cuts were separated using the CVRPSEP package [114]. The reader is referred to [115] for details concerning the separation routines.

At first, the delivery demands are used to separate the cuts. When no valid inequalities are found then the pickup demands are used. All of the three kinds of cuts are generated at the root node, but just the rounded capacity cuts are used throughout the tree up to the 5th level. Preliminary tests have shown that the overhead of separating comb and multistar inequalities outside the root node was not worthwhile. For each separation routine of the CVRPSEP package a limit of 50 violated cuts per iteration was established.

In the case of the VRPSPD instances, the best upper bound (UB) solutions pointed out in the literature were given as initial primal bound for the BC, namely those reported in [157]. This definitely helps the algorithm to find optimal solutions in much less computational time. As for the VRPMPD instances, the UBs found by Gajpal and Abad [64] were provided as a cutoff value for the BC.

### 3.4 Computational experiments

The BC procedures were implemented using the CPLEX 11.2 callable library and executed in an Intel Core 2 Quad with 2.4 GHz and 4 GB of RAM running under Linux 64 bits (kernel 2.6.27-16). Only a single thread was used in the experiments. Each BC is respectively associated with the formulations F1C, F2C-U and F2C-D. A time limit of 2 hours of execution was imposed for the BC algorithms. In some very particular cases, the CPLEX have slightly exceeded this time limit, namely on few instances involving more than 100 customers.

#### 3.4.1 VRPSPD

Three set of test-problems are available in the VRPSPD literature. These benchmark instances were proposed by Dethloff [48], Salhi and Nagy [149] and Montané and Galvão [121]. The first group contains 40 instances with 50 customers, the second contains 14 instances with 50-199 customers, while the third contains 12 instances with 100-200 customers. The number of vehicles is not explicitly specified in these 66 instances. The barrier algorithm was used to solve the initial linear relaxation of the last two group of instances. It is noteworthy to mention that the Montané and Galvão's instances involving 400 customers were not considered.

In the tables presented hereafter,  $\#v$  represents the number of vehicles in the best

known solution, **LP** is the linear relaxation, **Root LB** indicates the root lower bound, after CVRPSEP cuts are added, **Root Time** is the CPU time in seconds spent at the root node, **Tree size** corresponds to the the number of nodes opened, **Total time** is the total CPU time in seconds of the BC procedure, **Prev. LB** is the lower bound obtained in [121], **New LB** is the best lower bound determined among the three flow formulations, **F-LB** is the lower bound found by the respective formulation, **UB** is the upper bound reported in the literature, and **Gap** corresponds to the gap between the LB and the UB. Proven optimal solutions are highlighted in boldface. If the F-LB is the one associated with the New LB ( $F-LB = \text{New LB}$ ), then its value is underlined only if New LB is not an optimal solution.

Tables 3.1, 3.2 and 3.3 contain, respectively, the results obtained by F1C, F2C-U and F2C-D on the set of instances of Dethloff. It can be seen that the three formulations were able to prove the optimality of almost all instances of 4 vehicles. F2C-U appears to be the most effective under this aspect, being capable of proving the optimality of 17 instances. The performance of the three formulations on the instances of 9 vehicles were inferior in terms of optimality proof, but their LBs are significantly better than the previous values reported in [121]. F2C-U also seems to be the most effective in terms of LBs, with an average gap of 0.94%, against 1.34% and 1.26% of F1C and F2C-D, respectively.

In order to check if the values of the UB of the instances SCA3-0, SCA3-6, SCA8-3, SCA8-6, CON3-2, CON8-1, CON8-4 and CON8-7 are optimal the F2C-U was executed with a time limit of 48 hours. The formulation was successful to prove the optimality of each of these instances within up to 36 hours of execution.

The results found by F1C, F2C-U and F2C-D on the set of instances of Salhi and Nagy are presented, respectively, in Tables 3.4, 3.5 and 3.6. The optimality of the instances CMT1X and CMT1Y has been proven by all the three formulations. Nevertheless, these are the first LBs presented for this set of instances. Montané and Galvão [121] had reported LBs for the case where the demands were rounded to the nearest integer. When comparing the LBs obtained by each of the three formulations it can be verified that F2C-U produced superior results, with an average gap of 4,27 %, against 4,57% and 4,31% of F2C-D and F1C, respectively.

The results obtained by the three formulations on the set of instances of Montané and Galvão are shown in Tables 3.7, 3.8 and 3.9. Three optimal solutions were proven by all formulations, namely in the instances r201, c201 and rc201. The main characteristic of these three instances is the fact of having relatively very few vehicles. When comparing the LBs of the different formulations, it can be verified that F2C-U found the best results, with an average gap of 2.94%, whereas for F2C-U and F1C the average gap was 3.57% and 3.62%, respectively.

Tables 3.10-3.12 present the statistics of the root node of each formulation over a

set of representative instances. In these tables, **Sep. Rounds** represent the number of calls to the separation routines, **LP Time** is the time in seconds spent solving the linear relaxations, **Sep. Time** is the time in seconds spent separating the cuts, **Root Time** is the sum of the LP Time and Sep. Time, and **Gap** is the gap between the root relaxation and the UB.

Table 3.1: Results obtained by F1C on Dethloff’s instances

| Instance/<br>Customers | #v | LP     | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | Prev.<br>LB | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|----|--------|------------|------------------|--------------|-------------------|-------------|-----------|---------------|---------------|------------|
| SCA3-0/50              | 4  | 551.14 | 613.38     | 41               | 73473        | 7200              | 583.77      | 627.66    | 622.73        | 635.62        | 2.03       |
| SCA3-1/50              | 4  | 645.95 | 682.40     | 107              | 1712         | 1230              | 655.63      | 697.84    | <b>697.84</b> | <b>697.84</b> | 0.00       |
| SCA3-2/50              | 4  | 592.56 | 658.35     | 19               | 1            | 19                | 627.12      | 659.34    | 659.34        | 659.34        | 0.00       |
| SCA3-3/50              | 4  | 586.30 | 667.37     | 70               | 1083         | 415               | 633.56      | 680.04    | <b>680.04</b> | <b>680.04</b> | 0.00       |
| SCA3-4/50              | 4  | 627.29 | 672.92     | 80               | 8718         | 1599              | 642.89      | 690.50    | <b>690.50</b> | <b>690.50</b> | 0.00       |
| SCA3-5/50              | 4  | 604.31 | 646.14     | 83               | 15560        | 1901              | 603.06      | 659.90    | <b>659.90</b> | <b>659.90</b> | 0.00       |
| SCA3-6/50              | 4  | 587.97 | 624.92     | 47               | 37655        | 7200              | 607.53      | 645.56    | 639.97        | 651.09        | 1.71       |
| SCA3-7/50              | 4  | 584.69 | 654.30     | 82               | 26           | 103               | 616.40      | 659.17    | <b>659.17</b> | <b>659.17</b> | 0.00       |
| SCA3-8/50              | 4  | 638.75 | 688.77     | 94               | 72785        | 7200              | 668.04      | 719.48    | 703.12        | 719.48        | 2.27       |
| SCA3-9/50              | 4  | 597.02 | 668.09     | 82               | 1674         | 417               | 619.03      | 681.00    | <b>681.00</b> | <b>681.00</b> | 0.00       |
| SCA8-0/50              | 9  | 849.35 | 922.36     | 96               | 8854         | 7200              | 877.55      | 936.89    | 933.12        | 961.50        | 2.95       |
| SCA8-1/50              | 9  | 937.71 | 998.04     | 75               | 9948         | 7200              | 954.29      | 1020.28   | 1015.05       | 1049.65       | 3.30       |
| SCA8-2/50              | 9  | 931.93 | 1008.83    | 78               | 10334        | 7200              | 950.74      | 1024.24   | 1019.99       | 1039.64       | 1.89       |
| SCA8-3/50              | 9  | 874.31 | 954.55     | 74               | 11375        | 7200              | 905.29      | 975.87    | 970.88        | 983.34        | 1.27       |
| SCA8-4/50              | 9  | 958.58 | 1022.44    | 72               | 12054        | 7200              | 972.62      | 1041.65   | 1036.45       | 1065.49       | 2.73       |
| SCA8-5/50              | 9  | 923.50 | 996.01     | 79               | 9207         | 7200              | 940.60      | 1015.19   | 1011.57       | 1027.08       | 1.51       |
| SCA8-6/50              | 9  | 870.58 | 933.57     | 133              | 6219         | 7200              | 885.34      | 959.91    | 944.53        | 971.82        | 2.81       |
| SCA8-7/50              | 9  | 937.30 | 1013.86    | 65               | 12533        | 7200              | 955.86      | 1031.56   | 1029.97       | 1051.28       | 2.03       |
| SCA8-8/50              | 9  | 962.50 | 1023.86    | 102              | 8510         | 7200              | 986.52      | 1048.93   | 1036.90       | 1071.18       | 3.20       |
| SCA8-9/50              | 9  | 953.36 | 1012.73    | 89               | 9031         | 7200              | 978.90      | 1034.28   | 1031.51       | 1060.50       | 2.73       |
| CON3-0/50              | 4  | 577.74 | 606.00     | 91               | 40753        | 4836              | 592.38      | 616.52    | 616.46        | 616.52        | 0.01       |
| CON3-1/50              | 4  | 506.41 | 543.71     | 73               | 52033        | 6498              | 532.55      | 554.47    | <b>554.47</b> | <b>554.47</b> | 0.00       |
| CON3-2/50              | 4  | 468.40 | 503.14     | 61               | 13874        | 7200              | 491.04      | 517.26    | 514.11        | 518.00        | 0.75       |
| CON3-3/50              | 4  | 541.46 | 581.45     | 55               | 20044        | 1941              | 557.99      | 591.19    | <b>591.19</b> | <b>591.19</b> | 0.00       |
| CON3-4/50              | 4  | 537.90 | 577.61     | 63               | 78398        | 7200              | 558.26      | 588.79    | 588.47        | 588.79        | 0.06       |
| CON3-5/50              | 4  | 511.88 | 553.87     | 107              | 32652        | 5975              | 531.33      | 563.70    | <b>563.70</b> | <b>563.70</b> | 0.00       |
| CON3-6/50              | 4  | 468.90 | 486.59     | 128              | 14248        | 7200              | 475.33      | 499.05    | 493.01        | 499.05        | 1.21       |
| CON3-7/50              | 4  | 533.86 | 562.10     | 38               | 53629        | 5522              | 550.73      | 576.48    | <b>576.48</b> | <b>576.48</b> | 0.00       |
| CON3-8/50              | 4  | 477.81 | 513.90     | 87               | 15317        | 1923              | 492.69      | 523.05    | <b>523.05</b> | <b>523.05</b> | 0.00       |
| CON3-9/50              | 4  | 528.34 | 564.87     | 63               | 15461        | 5602              | 547.31      | 578.25    | <b>578.25</b> | <b>578.25</b> | 0.00       |
| CON8-0/50              | 9  | 774.69 | 829.80     | 47               | 16498        | 7200              | 795.45      | 845.19    | 842.62        | 857.17        | 1.70       |
| CON8-1/50              | 9  | 680.24 | 719.03     | 80               | 7552         | 7200              | 693.22      | 734.71    | 732.44        | 740.85        | 1.14       |
| CON8-2/50              | 9  | 636.18 | 682.76     | 128              | 9856         | 7200              | 650.81      | 695.70    | 693.07        | 712.89        | 2.78       |
| CON8-3/50              | 10 | 732.55 | 784.93     | 71               | 7536         | 7200              | 754.41      | 797.57    | 796.31        | 811.07        | 1.82       |
| CON8-4/50              | 9  | 710.36 | 749.83     | 122              | 6374         | 7200              | 729.09      | 767.63    | 759.11        | 772.25        | 1.70       |
| CON8-5/50              | 9  | 696.85 | 728.10     | 78               | 7901         | 7200              | 709.76      | 741.51    | 736.79        | 754.88        | 2.40       |
| CON8-6/50              | 9  | 611.16 | 647.04     | 61               | 10400        | 7200              | 631.41      | 662.14    | <b>662.14</b> | 678.92        | 2.47       |
| CON8-7/50              | 9  | 729.28 | 787.89     | 64               | 11861        | 7200              | 762.03      | 810.08    | 800.22        | 811.96        | 1.44       |
| CON8-8/50              | 9  | 689.23 | 741.02     | 74               | 10324        | 7200              | 705.08      | 757.45    | 753.42        | 767.53        | 1.84       |
| CON8-9/50              | 9  | 716.21 | 770.66     | 101              | 5435         | 7200              | 729.10      | 786.40    | 778.65        | 809.00        | 3.75       |
| Avg. Gap (%)           |    |        |            |                  |              |                   |             |           |               | 1.34          |            |

From the results of Tables 3.10-3.12 it can be seen that in most cases the Sep. Rounds increases with the number of vehicles, given a fixed number of customers. Also, it is possible to verify that the LP Time is considerably higher than the Sep. Time and, as expected, this difference tends to increase with the size of the instance as well as the number of vehicles. It appears that all the three formulations became very “heavy” in the instances involving 200 customers, since in almost all cases, they took about 2 hours to solve less than 13 linear programs. An attempt has been made to use the barrier algorithm to solve all the linear programs, but unfortunately the results were not satisfactory.

Table 3.2: Results obtained by the F2C-U on Dethloff's instances

| Instance/<br>Customers | # <i>v</i> | LP     | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | Prev.<br>LB | New<br>LB | F-LB           | UB            | Gap<br>(%) |
|------------------------|------------|--------|------------|------------------|--------------|-------------------|-------------|-----------|----------------|---------------|------------|
| SCA3-0/50              | 4          | 550.85 | 613.36     | 28               | 211456       | 7200              | 583.77      | 627.66    | 625.00         | 635.62        | 1.67       |
| SCA3-1/50              | 4          | 645.59 | 682.33     | 42               | 1467         | 142               | 655.63      | 697.84    | <b>697.84</b>  | <b>697.84</b> | 0.00       |
| SCA3-2/50              | 4          | 592.44 | 658.89     | 12               | 1            | 12                | 627.12      | 659.34    | <b>659.34</b>  | <b>659.34</b> | 0.00       |
| SCA3-3/50              | 4          | 586.02 | 667.37     | 25               | 847          | 81                | 633.56      | 680.04    | <b>680.04</b>  | <b>680.04</b> | 0.00       |
| SCA3-4/50              | 4          | 626.93 | 673.28     | 50               | 2866         | 252               | 642.89      | 690.50    | <b>690.50</b>  | <b>690.50</b> | 0.00       |
| SCA3-5/50              | 4          | 603.95 | 646.29     | 38               | 19724        | 731               | 603.06      | 659.90    | <b>659.90</b>  | <b>659.90</b> | 0.00       |
| SCA3-6/50              | 4          | 587.85 | 624.89     | 27               | 176561       | 7200              | 607.53      | 645.56    | 644.37         | 651.09        | 1.03       |
| SCA3-7/50              | 4          | 584.59 | 653.76     | 34               | 30           | 43                | 616.40      | 659.17    | <b>659.17</b>  | <b>659.17</b> | 0.00       |
| SCA3-8/50              | 4          | 638.41 | 693.71     | 42               | 108017       | 4899              | 668.04      | 719.48    | <b>719.48</b>  | <b>719.48</b> | 0.00       |
| SCA3-9/50              | 4          | 596.79 | 668.09     | 36               | 1452         | 96                | 619.03      | 681.00    | <b>681.00</b>  | <b>681.00</b> | 0.00       |
| SCA8-0/50              | 9          | 847.39 | 922.58     | 79               | 17695        | 7200              | 877.55      | 936.89    | 936.89         | 961.50        | 2.56       |
| SCA8-1/50              | 9          | 933.44 | 997.07     | 56               | 18086        | 7200              | 954.29      | 1020.28   | <u>1020.28</u> | 1049.65       | 2.80       |
| SCA8-2/50              | 9          | 931.34 | 1008.27    | 75               | 12789        | 7200              | 950.74      | 1024.24   | <u>1024.24</u> | 1039.64       | 1.48       |
| SCA8-3/50              | 9          | 872.37 | 953.67     | 49               | 18487        | 7200              | 905.29      | 975.87    | <u>975.87</u>  | 983.34        | 0.76       |
| SCA8-4/50              | 9          | 955.74 | 1021.35    | 44               | 25853        | 7200              | 972.62      | 1041.65   | <u>1041.65</u> | 1065.49       | 2.24       |
| SCA8-5/50              | 9          | 922.25 | 995.93     | 58               | 19464        | 7200              | 940.60      | 1015.19   | 1013.87        | 1027.08       | 1.29       |
| SCA8-6/50              | 9          | 868.00 | 933.76     | 74               | 10467        | 7200              | 885.34      | 959.91    | <u>959.91</u>  | 971.82        | 1.23       |
| SCA8-7/50              | 9          | 935.55 | 1015.11    | 69               | 15193        | 7200              | 955.86      | 1031.56   | <u>1031.56</u> | 1051.28       | 1.88       |
| SCA8-8/50              | 9          | 960.17 | 1023.60    | 87               | 8262         | 7200              | 986.52      | 1048.93   | <u>1048.93</u> | 1071.18       | 2.08       |
| SCA8-9/50              | 9          | 952.34 | 1014.89    | 64               | 16262        | 7200              | 978.90      | 1034.28   | <u>1034.28</u> | 1060.50       | 2.47       |
| CON3-0/50              | 4          | 577.52 | 606.82     | 46               | 3048         | 247               | 592.38      | 616.52    | <b>616.52</b>  | <b>616.52</b> | 0.00       |
| CON3-1/50              | 4          | 506.23 | 545.53     | 54               | 16039        | 823               | 532.55      | 554.47    | <b>554.47</b>  | <b>554.47</b> | 0.00       |
| CON3-2/50              | 4          | 468.22 | 504.44     | 59               | 22107        | 7200              | 491.04      | 517.26    | 516.23         | 518.00        | 0.34       |
| CON3-3/50              | 4          | 541.40 | 582.83     | 35               | 6608         | 330               | 557.99      | 591.19    | <b>591.19</b>  | <b>591.19</b> | 0.00       |
| CON3-4/50              | 4          | 537.73 | 577.57     | 42               | 50663        | 3198              | 558.26      | 588.79    | <b>588.79</b>  | <b>588.79</b> | 0.00       |
| CON3-5/50              | 4          | 511.59 | 554.35     | 65               | 10191        | 729               | 531.33      | 563.70    | <b>563.70</b>  | <b>563.70</b> | 0.00       |
| CON3-6/50              | 4          | 468.75 | 486.61     | 100              | 48466        | 5230              | 475.33      | 499.05    | <b>499.05</b>  | <b>499.05</b> | 0.00       |
| CON3-7/50              | 4          | 533.73 | 561.87     | 37               | 9822         | 1141              | 550.73      | 576.48    | <b>576.48</b>  | <b>576.48</b> | 0.00       |
| CON3-8/50              | 4          | 477.45 | 514.13     | 71               | 5541         | 450               | 492.69      | 523.05    | <b>523.05</b>  | <b>523.05</b> | 0.00       |
| CON3-9/50              | 4          | 527.94 | 564.78     | 53               | 6372         | 790               | 547.31      | 578.25    | <b>578.25</b>  | <b>578.25</b> | 0.00       |
| CON8-0/50              | 9          | 773.46 | 827.14     | 74               | 13038        | 7200              | 795.45      | 845.19    | <u>845.19</u>  | 857.17        | 1.40       |
| CON8-1/50              | 9          | 678.95 | 719.09     | 67               | 13302        | 7200              | 693.22      | 734.71    | <u>734.71</u>  | 740.85        | 0.83       |
| CON8-2/50              | 9          | 635.23 | 682.37     | 127              | 9409         | 7200              | 650.81      | 695.70    | <u>695.70</u>  | 712.89        | 2.41       |
| CON8-3/50              | 10         | 731.55 | 785.00     | 71               | 18680        | 7200              | 754.41      | 797.57    | <u>797.57</u>  | 811.07        | 1.66       |
| CON8-4/50              | 9          | 708.64 | 751.32     | 60               | 15700        | 7200              | 729.09      | 767.63    | <u>767.63</u>  | 772.25        | 0.60       |
| CON8-5/50              | 9          | 696.08 | 727.26     | 66               | 9765         | 7200              | 709.76      | 741.51    | <u>741.51</u>  | 754.88        | 1.77       |
| CON8-6/50              | 9          | 610.20 | 646.78     | 94               | 11947        | 7200              | 631.41      | 662.14    | 661.36         | 678.92        | 2.59       |
| CON8-7/50              | 9          | 726.55 | 788.64     | 74               | 5520         | 7200              | 762.03      | 810.08    | <u>810.08</u>  | 811.96        | 0.23       |
| CON8-8/50              | 9          | 688.25 | 741.76     | 81               | 13325        | 7200              | 705.08      | 757.45    | <u>757.45</u>  | 767.53        | 1.31       |
| CON8-9/50              | 9          | 713.85 | 770.85     | 109              | 12833        | 7200              | 729.10      | 786.40    | <u>786.40</u>  | 809.00        | 2.79       |
| Avg. Gap (%)           |            |        |            |                  |              |                   |             |           |                |               | 0.94       |

Table 3.13 shows a summary of the results obtained by the three formulations in all set of instances. In this table, **G1** is the average gap between the linear relaxation and the UB, **G2** is the average gap with respect to the root LB, including the CVRPSEP cuts, and **G3** is average gap for the LB, possibly after branching, found within the time limit established. Those results can be explained as follows. The linear relaxation of F1C is indeed a little better than the linear relaxations of F2C-D and F2C-U. However, after the cuts, there is no significant difference in the LB quality. This can be clearly seen in the column **G2** under Dethloff instances. For those smaller instances, the cut separation in the root node could always be completed within the time limit. In those cases, the small gap differences (2.96%, 2.94% and 2.92%) are not significant and can be attributed to the heuristic nature of the routines in the CVRPSEP library. The consistent advantage of formulation F2C-U shown in columns **G3** is explained by the fact that CPLEX has a significantly better performance when reoptimizing its LPs. This means that more cuts



Table 3.5: Results obtained by the F2C-U on Salhi and Nagy’s instances (VRPSPD)

| Instance/<br>Customers | # <i>v</i> | LP     | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|------------|--------|------------|------------------|--------------|-------------------|-----------|---------------|---------------|------------|
| CMT1X/50               | 3          | 449.00 | 459.98     | 102              | 2282         | 300               | 466.77    | <b>466.77</b> | <b>466.77</b> | 0.00       |
| CMT1Y/50               | 3          | 449.00 | 460.02     | 70               | 3205         | 213               | 466.77    | <b>466.77</b> | <b>466.77</b> | 0.00       |
| CMT2X/75               | 6          | 632.11 | 652.85     | 346              | 2073         | 7200              | 655.88    | 655.21        | 684.21        | 4.24       |
| CMT2Y/75               | 6          | 632.11 | 653.13     | 449              | 2610         | 7200              | 655.41    | <u>655.41</u> | 684.21        | 4.21       |
| CMT3X/100              | 5          | 682.18 | 701.10     | 504              | 13820        | 7200              | 705.54    | <u>704.35</u> | 721.27        | 2.35       |
| CMT3Y/100              | 5          | 682.18 | 701.12     | 612              | 19865        | 7200              | 705.62    | 705.28        | 721.27        | 2.22       |
| CMT12X/100             | 5          | 564.08 | 628.59     | 813              | 991          | 7201              | 629.39    | <u>629.39</u> | 662.22        | 4.96       |
| CMT12Y/100             | 5          | 564.08 | 628.58     | 923              | 118          | 7201              | 629.18    | 629.09        | 662.22        | 5.00       |
| CMT11X/120             | 4          | 687.42 | 775.51     | 4835             | 42           | 7201              | 776.35    | <u>776.35</u> | 833.92        | 6.90       |
| CMT11Y/120             | 4          | 687.42 | 775.40     | 6138             | 22           | 7200              | 775.74    | <u>775.74</u> | 833.92        | 6.98       |
| CMT4X/150              | 7          | 796.48 | 817.11     | 7288             | 1            | 7292              | 817.11    | <u>817.11</u> | 852.46        | 4.15       |
| CMT4Y/150              | 7          | 796.48 | 816.99     | 5747             | 1            | 7201              | 816.99    | <u>816.99</u> | 852.46        | 4.16       |
| CMT5X/200              | 10         | 933.21 | 954.87     | 6939             | 1            | 7201              | 954.87    | <u>954.87</u> | 1029.25       | 7.23       |
| CMT5Y/200              | 10         | 933.21 | 953.56     | 6600             | 1            | 7202              | 953.56    | <u>953.56</u> | 1029.25       | 7.35       |
| Avg. Gap (%)           |            |        |            |                  |              |                   |           |               |               | 4.27       |

Table 3.6: Results obtained by F2C-D on Salhi and Nagy’s instances (VRPSPD)

| Instance/<br>Customers | # <i>v</i> | LP     | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|------------|--------|------------|------------------|--------------|-------------------|-----------|---------------|---------------|------------|
| CMT1X/50               | 3          | 449.00 | 459.89     | 56               | 1971         | 204               | 466.77    | <b>466.77</b> | <b>466.77</b> | 0.00       |
| CMT1Y/50               | 3          | 449.00 | 460.02     | 71               | 3204         | 239               | 466.77    | <b>466.77</b> | <b>466.77</b> | 0.00       |
| CMT2X/75               | 6          | 632.11 | 653.05     | 788              | 5719         | 7200              | 655.88    | <u>655.88</u> | 684.21        | 4.14       |
| CMT2Y/75               | 6          | 632.11 | 652.95     | 625              | 1800         | 7200              | 655.41    | 654.96        | 684.21        | 4.28       |
| CMT3X/100              | 5          | 682.18 | 701.77     | 610              | 14470        | 7200              | 705.54    | <u>705.54</u> | 721.27        | 2.18       |
| CMT3Y/100              | 5          | 682.18 | 701.74     | 460              | 13533        | 7200              | 705.62    | <u>705.62</u> | 721.27        | 2.17       |
| CMT12X/100             | 5          | 564.18 | 628.58     | 1804             | 88           | 7200              | 629.39    | 628.81        | 662.22        | 5.05       |
| CMT12Y/100             | 5          | 564.18 | 628.53     | 1564             | 88           | 7200              | 629.18    | 629.02        | 662.22        | 5.01       |
| CMT11X/120             | 4          | 687.42 | 774.36     | 7222             | 1            | 7224              | 776.35    | 774.36        | 833.92        | 7.14       |
| CMT11Y/120             | 4          | 687.42 | 774.56     | 7237             | 1            | 7239              | 775.74    | 774.56        | 833.92        | 7.12       |
| CMT4X/150              | 7          | 796.48 | 816.90     | 7154             | 1            | 7201              | 817.11    | 816.90        | 852.46        | 4.17       |
| CMT4Y/150              | 7          | 796.48 | 816.90     | 7185             | 1            | 7201              | 816.99    | 816.91        | 852.46        | 4.17       |
| CMT5X/200              | 10         | 933.21 | 952.38     | 7419             | 1            | 7422              | 954.87    | 952.38        | 1029.25       | 7.47       |
| CMT5Y/200              | 10         | 933.21 | 952.62     | 6798             | 1            | 7201              | 953.56    | 952.62        | 1029.25       | 7.45       |
| Avg. Gap (%)           |            |        |            |                  |              |                   |           |               |               | 4.31       |

Table 3.7: Results obtained by the F1C on Montané and Galvão’s instances

| Instance/<br>Customers | # <i>v</i> | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | Prev.<br>LB | New<br>LB | F-LB           | UB            | Gap<br>(%) |
|------------------------|------------|---------|------------|------------------|--------------|-------------------|-------------|-----------|----------------|---------------|------------|
| r101/100               | 12         | 939.75  | 972.57     | 3084             | 104          | 7201              | 934.97      | 973.91    | <u>973.17</u>  | 1009.95       | 3.64       |
| r201/100               | 3          | 643.08  | 664.87     | 562              | 17           | 575               | 643.65      | 666.20    | <b>666.20</b>  | <b>666.20</b> | 0.00       |
| c101/100               | 16         | 1070.82 | 1195.47    | 1788             | 909          | 7200              | 1066.19     | 1196.70   | <u>1196.70</u> | 1220.18       | 1.92       |
| c201/100               | 5          | 598.51  | 657.97     | 260              | 88           | 325               | 278.05      | 662.07    | <b>662.07</b>  | <b>662.07</b> | 0.00       |
| rc101/100              | 10         | 946.99  | 1028.72    | 4006             | 82           | 7201              | 937.41      | 1029.38   | 1029.08        | 1059.32       | 2.85       |
| rc201/100              | 3          | 600.31  | 672.31     | 323              | 1            | 324               | 602.70      | 672.92    | <b>672.92</b>  | <b>672.92</b> | 0.00       |
| r1.2.1/200             | 23         | 3023.35 | 3078.76    | 7015             | 1            | 7202              | 2951.12     | 3084.97   | 3078.76        | 3360.02       | 8.37       |
| r2.2.1/200             | 5          | 1549.88 | 1607.89    | 6824             | 1            | 7201              | 1501.82     | 1618.76   | 1607.89        | 1665.58       | 3.46       |
| c1.2.1/200             | 28         | 3326.05 | 3396.32    | 5377             | 1            | 7201              | 3299.07     | 3475.03   | 3396.32        | 3629.89       | 6.43       |
| c2.2.1/200             | 9          | 1560.54 | 1611.74    | 5601             | 1            | 7201              | 1542.96     | 1647.83   | 1611.74        | 1726.59       | 6.65       |
| rc1.2.1/200            | 23         | 3020.18 | 3092.57    | 7124             | 1            | 7202              | 2939.98     | 3093.30   | 3092.57        | 3306.00       | 6.46       |
| rc2.2.1/200            | 5          | 1439.13 | 1513.14    | 6757             | 1            | 7200              | 1396.95     | 1551.07   | 1513.14        | 1560.00       | 3.00       |
| Avg. Gap (%)           |            |         |            |                  |              |                   |             |           |                | 3.57          |            |

### 3.4.2 VRPMPD

A set of 21 VRPMPD instances involving 50-199 customers was proposed by Salhi and Nagy [149]. As in the VRPSPD, the number of vehicles is not specified. Also, Gajpal and Abad [64] did not report the number of vehicles associated with their UBs. The barrier

algorithm was employed to solve the initial linear relaxation.

Table 3.8: Results obtained by F2C-U on Montané and Galvão's instances

| Instance/<br>Customers | #v | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | Prev.<br>LB | New<br>LB | F-LB           | UB            | Gap<br>(%) |
|------------------------|----|---------|------------|------------------|--------------|-------------------|-------------|-----------|----------------|---------------|------------|
| r101/100               | 12 | 939.19  | 972.88     | 2910             | 122          | 7201              | 934.97      | 973.91    | 973.10         | 1009.95       | 3.65       |
| r201/100               | 3  | 643.07  | 664.80     | 292              | 21           | 307               | 643.65      | 666.20    | <b>666.20</b>  | <b>666.20</b> | 0.00       |
| c101/100               | 16 | 1070.40 | 1195.53    | 1396             | 302          | 7201              | 1066.19     | 1196.70   | 1195.89        | 1220.18       | 1.99       |
| c201/100               | 5  | 598.47  | 657.97     | 197              | 17           | 241               | 596.85      | 662.07    | <b>662.07</b>  | <b>662.07</b> | 0.00       |
| rc101/100              | 10 | 944.21  | 1028.15    | 2940             | 138          | 7201              | 937.41      | 1029.38   | <u>1029.38</u> | 1059.32       | 2.83       |
| rc201/100              | 3  | 600.24  | 671.84     | 134              | 4            | 134               | 602.70      | 672.92    | <b>672.92</b>  | <b>672.92</b> | 0.00       |
| r1.2.1/200             | 23 | 3013.16 | 3084.97    | 6971             | 1            | 7200              | 2951.12     | 3084.97   | <u>3084.97</u> | 3360.02       | 8.19       |
| r2.2.1/200             | 5  | 1549.60 | 1618.76    | 7869             | 1            | 7874              | 1501.82     | 1618.76   | <u>1618.76</u> | 1665.58       | 2.81       |
| c1.2.1/200             | 28 | 3325.20 | 3475.03    | 7041             | 1            | 7202              | 3299.07     | 3475.03   | <u>3475.03</u> | 3629.89       | 4.27       |
| c2.2.1/200             | 9  | 1560.22 | 1647.83    | 7370             | 1            | 7374              | 1542.96     | 1647.83   | <u>1647.83</u> | 1726.59       | 4.56       |
| rc1.2.1/200            | 23 | 3015.44 | 3093.30    | 7064             | 1            | 7201              | 2939.98     | 3093.30   | <u>3093.30</u> | 3306.00       | 6.43       |
| rc2.2.1/200            | 5  | 1438.91 | 1551.07    | 7301             | 1            | 7308              | 1396.95     | 1551.07   | <u>1551.07</u> | 1560.00       | 0.57       |
|                        |    |         |            |                  |              |                   |             |           |                | Avg. Gap (%)  | 2.94       |

Table 3.9: Results obtained by F2C-D on Montané and Galvão's instances

| Instance/<br>Customers | #v | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | Prev.<br>LB | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|----|---------|------------|------------------|--------------|-------------------|-------------|-----------|---------------|---------------|------------|
| r101/100               | 12 | 939.26  | 972.85     | 5867             | 12           | 7200              | 934.97      | 973.91    | <u>973.91</u> | 1009.95       | 3.57       |
| r201/100               | 3  | 643.07  | 665.05     | 490              | 14           | 524               | 643.65      | 666.20    | <b>666.20</b> | <b>666.20</b> | 0.00       |
| c101/100               | 16 | 1070.40 | 1196.16    | 2752             | 666          | 7200              | 1066.19     | 1196.70   | 1196.65       | 1220.18       | 1.93       |
| c201/100               | 5  | 598.47  | 657.97     | 317              | 61           | 404               | 596.85      | 662.07    | <b>662.07</b> | <b>662.07</b> | 0.00       |
| rc101/100              | 10 | 944.39  | 1028.49    | 6570             | 6            | 7200              | 937.41      | 1029.38   | 1028.52       | 1059.32       | 2.91       |
| rc201/100              | 3  | 600.26  | 671.84     | 200              | 6            | 201               | 602.70      | 672.92    | <b>672.92</b> | <b>672.92</b> | 0.00       |
| r1.2.1/200             | 23 | 3013.21 | 3074.77    | 7088             | 1            | 7201              | 2951.12     | 3084.97   | 3074.77       | 3360.02       | 8.49       |
| r2.2.1/200             | 5  | 1549.62 | 1615.14    | 7383             | 1            | 7386              | 1501.82     | 1618.76   | 1615.14       | 1665.58       | 3.03       |
| c1.2.1/200             | 28 | 3325.20 | 3389.36    | 7517             | 1            | 7521              | 3299.07     | 3475.03   | 3389.36       | 3629.89       | 6.63       |
| c2.2.1/200             | 9  | 1560.39 | 1596.24    | 6028             | 1            | 7201              | 1542.96     | 1647.83   | 1596.24       | 1726.59       | 7.55       |
| rc1.2.1/200            | 23 | 3015.98 | 3067.65    | 7280             | 1            | 7283              | 2939.98     | 3093.30   | 3067.65       | 3306.00       | 7.21       |
| rc2.2.1/200            | 5  | 1439.01 | 1526.68    | 6896             | 1            | 7202              | 1396.95     | 1551.07   | 1526.68       | 1560.00       | 2.14       |
|                        |    |         |            |                  |              |                   |             |           |               | Avg. Gap (%)  | 3.62       |

Table 3.10: Root node statistics of F1C over a set of VRPSD representative instances

| Instance/<br>Customers | #v | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|----|----------------|----------------|------------------|------------------|------------|
| SCA3-1/50              | 4  | 19             | 106.1          | 0.8              | 106.9            | 2.21       |
| SCA8-1/50              | 9  | 17             | 72.8           | 1.8              | 74.6             | 4.92       |
| CON3-1/50              | 4  | 26             | 71.5           | 1.5              | 73.1             | 1.94       |
| CON8-1/50              | 9  | 37             | 73.9           | 6.5              | 80.4             | 2.95       |
| CMT1X/50               | 3  | 38             | 59.0           | 3.7              | 62.7             | 1.46       |
| CMT2X/75               | 6  | 65             | 1005.0         | 19.9             | 1025.0           | 4.58       |
| CMT3X/100              | 5  | 38             | 1369.0         | 13.4             | 1382.3           | 3.70       |
| CMT12X/100             | 5  | 47             | 3006.4         | 10.2             | 3016.5           | 5.07       |
| CMT11X/120             | 4  | 87             | 7152.3         | 48.1             | 7200.4           | 7.09       |
| CMT4X/150              | 7  | 21             | 7030.6         | 2.6              | 7033.2           | 4.23       |
| CMT5X/200              | 10 | 7              | 7312.7         | 2.2              | 7314.9           | 7.78       |
| r101/100               | 12 | 82             | 2946.5         | 137.5            | 3084.0           | 3.70       |
| r201/100               | 3  | 50             | 551.7          | 10.5             | 562.2            | 0.20       |
| r1.2.1/200             | 23 | 12             | 7008.9         | 6.5              | 7015.4           | 8.37       |
| r2.2.1/200             | 5  | 4              | 6823.0         | 0.5              | 6823.5           | 3.46       |

Tables 3.14, 3.15 and 3.16 present the results obtained, respectively, by F1C, F2C-U and F2C-D. It can be observed that the optimality of the instances CMT1H, CMT1Q, CMT1T, CMT3Q and CMT12T was proven by all formulations. When comparing the



LBs, one can verify that they are very similar, but F1C slightly outperformed the other formulations, with an average gap of 2.37% against 2.42% of F2C-U and 2.40% of F2C-D. This little difference in favor of F1C is mostly due to the significant better LBs found in all the three instances involving 200 customers.

Table 3.11: Root node statistics of F2C-U over a set of VRPSPD representative instances

| Instance/<br>Customers | # $v$ | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|-------|----------------|----------------|------------------|------------------|------------|
| SCA3-1/50              | 4     | 22             | 40.8           | 1.2              | 42.0             | 2.22       |
| SCA8-1/50              | 9     | 29             | 50.9           | 5.1              | 56.0             | 5.01       |
| CON3-1/50              | 4     | 33             | 52.4           | 1.6              | 54.1             | 1.61       |
| CON8-1/50              | 9     | 33             | 62.3           | 4.4              | 66.7             | 2.94       |
| CMT1X/50               | 3     | 45             | 96.4           | 5.2              | 101.5            | 1.46       |
| CMT2X/75               | 6     | 44             | 330.8          | 15.0             | 345.8            | 4.58       |
| CMT3X/100              | 5     | 39             | 493.0          | 10.6             | 503.6            | 2.80       |
| CMT12X/100             | 5     | 41             | 805.5          | 7.1              | 812.6            | 5.08       |
| CMT11X/120             | 4     | 88             | 4770.0         | 64.5             | 4834.5           | 7.00       |
| CMT4X/150              | 7     | 55             | 7173.3         | 114.6            | 7287.9           | 4.15       |
| CMT5X/200              | 10    | 8              | 6936.7         | 2.6              | 6939.3           | 7.23       |
| r101/100               | 12    | 76             | 2802.9         | 106.8            | 2909.7           | 3.67       |
| r201/100               | 3     | 28             | 288.7          | 3.6              | 292.4            | 0.21       |
| r1_2_1/200             | 23    | 13             | 6966.9         | 3.6              | 6970.5           | 8.19       |
| r2_2_1/200             | 5     | 8              | 7868.2         | 1.0              | 7869.2           | 2.81       |

Table 3.12: Root node statistics of F2C-D over a set of VRPSPD representative instances

| Instance/<br>Customers | # $v$ | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|-------|----------------|----------------|------------------|------------------|------------|
| SCA3-1/50              | 4     | 18             | 40.0           | 0.7              | 40.8             | 2.29       |
| SCA8-1/50              | 9     | 26             | 78.2           | 2.8              | 81.0             | 5.35       |
| CON3-1/50              | 4     | 25             | 39.8           | 1.4              | 41.2             | 1.98       |
| CON8-1/50              | 9     | 55             | 126.0          | 6.1              | 132.1            | 3.04       |
| CMT1X/50               | 3     | 33             | 53.1           | 2.7              | 55.8             | 1.50       |
| CMT2X/75               | 6     | 49             | 778.3          | 9.9              | 788.2            | 4.77       |
| CMT3X/100              | 5     | 45             | 586.9          | 22.7             | 609.6            | 2.78       |
| CMT12X/100             | 5     | 44             | 1798.3         | 5.8              | 1804.1           | 5.35       |
| CMT11X/120             | 4     | 80             | 7173.9         | 47.7             | 7221.6           | 7.69       |
| CMT4X/150              | 7     | 29             | 7135.6         | 18.1             | 7153.8           | 4.35       |
| CMT5X/200              | 10    | 6              | 7417.3         | 1.5              | 7418.7           | 8.07       |
| r101/100               | 12    | 80             | 5776.4         | 90.8             | 5867.3           | 3.81       |
| r201/100               | 3     | 56             | 472.4          | 17.2             | 489.5            | 0.17       |
| r1_2_1/200             | 23    | 11             | 7084.8         | 2.9              | 7087.6           | 9.28       |
| r2_2_1/200             | 5     | 8              | 7381.9         | 0.8              | 7382.7           | 3.12       |

Table 3.13: Summary of the results obtained by the three formulations

| Formulation | Dethloff |        |        | Salhi and Nagy |        |        | Montané and Galvão |        |        |
|-------------|----------|--------|--------|----------------|--------|--------|--------------------|--------|--------|
|             | G1 (%)   | G2 (%) | G3 (%) | G1 (%)         | G2 (%) | G3 (%) | G1 (%)             | G2 (%) | G3 (%) |
| F1C         | 9.74     | 2.96   | 1.34   | 9.21           | 4.85   | 4.57   | 8.75               | 3.66   | 3.57   |
| F2C-U       | 9.85     | 2.92   | 0.94   | 9.30           | 4.62   | 4.27   | 8.82               | 3.04   | 2.94   |
| F2C-D       | 9.85     | 2.94   | 1.26   | 9.30           | 4.66   | 4.31   | 8.82               | 3.57   | 3.62   |

Since the Gaps of the instances CMT12H and CMT12Q were relatively small for all formulations, it was thought advisable, to verify if the UBs found by Gajpal and Abad [64] for these instances are indeed optimal solutions by running F1C with a time limit of 48 hours. The BC algorithm was capable of proving the optimality of the instance

CMT12H after 7.1 hours of execution. On the other hand, the optimal solution found by the BC algorithm (**729.25**) for the instance CMT12Q, after 22.7 hours of execution, was better than the UB found by Gajpal and Abad (729.46). Moreover, the number of vehicles associated with optimal solutions of the instances CMT12H and CMT12Q are, respectively, 5 and 7.

Table 3.14: Results obtained by F1C on Salhi and Nagy’s instances (VRPMPD)

| Instance/<br>Customers | # <i>v</i> | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | New<br>LB | F-LB           | UB            | Gap<br>(%) |
|------------------------|------------|---------|------------|------------------|--------------|-------------------|-----------|----------------|---------------|------------|
| CMT1H/50               | 3          | 442.77  | 460.11     | 39               | 794          | 87                | 465.02    | <b>465.02</b>  | <b>465.02</b> | 0.00       |
| CMT1Q/50               | 4          | 468.98  | 488.10     | 36               | 15           | 40                | 489.74    | <b>489.74</b>  | <b>489.74</b> | 0.00       |
| CMT1T/50               | 5          | 488.08  | 513.07     | 47               | 908          | 193               | 520.06    | <b>520.06</b>  | <b>520.06</b> | 0.00       |
| CMT2H/75               | -          | 622.00  | 643.52     | 292              | 6174         | 7200              | 647.84    | <u>647.84</u>  | 662.63        | 2.23       |
| CMT2Q/75               | -          | 682.68  | 707.50     | 698              | 5827         | 7200              | 711.30    | 710.83         | 732.76        | 2.99       |
| CMT2T/75               | -          | 733.20  | 760.18     | 574              | 4996         | 7200              | 764.99    | 764.19         | 782.77        | 2.37       |
| CMT3H/100              | -          | 675.40  | 691.44     | 621              | 4735         | 7200              | 694.92    | 694.40         | 701.31        | 0.98       |
| CMT3Q/100              | 6          | 719.22  | 744.42     | 901              | 309          | 1112              | 747.15    | <b>747.15</b>  | <b>747.15</b> | 0.00       |
| CMT3T/100              | -          | 757.25  | 784.23     | 3602             | 1016         | 7200              | 787.12    | 786.14         | 798.07        | 1.50       |
| CMT12H/100             | -          | 542.24  | 623.03     | 526              | 20280        | 7200              | 627.32    | <u>627.32</u>  | 629.37        | 0.33       |
| CMT12Q/100             | -          | 641.08  | 721.71     | 863              | 2880         | 7200              | 726.71    | 724.63         | 729.46        | 0.66       |
| CMT12T/100             | 9          | 706.28  | 787.52     | 457              | 1            | 457               | 787.52    | <b>787.52</b>  | <b>787.52</b> | 0.00       |
| CMT11H/120             | -          | 671.59  | 800.99     | 7200             | 1            | 7204              | 801.05    | 800.99         | 820.35        | 2.36       |
| CMT11Q/120             | -          | 816.16  | 926.87     | 7182             | 1            | 7200              | 928.74    | 926.87         | 939.36        | 1.33       |
| CMT11T/120             | -          | 904.02  | 984.35     | 7224             | 1            | 7228              | 985.03    | 984.35         | 998.80        | 1.45       |
| CMT4H/150              | -          | 778.93  | 798.15     | 7240             | 1            | 7242              | 798.38    | 798.15         | 831.39        | 4.00       |
| CMT4Q/150              | -          | 857.79  | 889.58     | 7133             | 1            | 7201              | 890.12    | 889.58         | 913.93        | 2.66       |
| CMT4T/150              | -          | 920.63  | 949.50     | 7101             | 1            | 7200              | 950.59    | 949.50         | 990.39        | 4.13       |
| CMT5H/200              | -          | 905.32  | 922.88     | 6913             | 1            | 7201              | 922.88    | <u>922.88</u>  | 992.37        | 7.00       |
| CMT5Q/200              | -          | 1023.95 | 1040.25    | 6541             | 1            | 7201              | 1040.25   | <u>1040.25</u> | 1134.72       | 8.33       |
| CMT5T/200              | -          | 1118.60 | 1139.93    | 7448             | 1            | 7449              | 1139.93   | <u>1139.93</u> | 1232.08       | 7.48       |
| Avg. Gap (%)           |            |         |            |                  |              |                   |           |                |               | 2.37       |

Table 3.15: Results obtained by F2C-U on Salhi and Nagy’s instances (VRPMPD)

| Instance/<br>Customers | # <i>v</i> | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|------------|---------|------------|------------------|--------------|-------------------|-----------|---------------|---------------|------------|
| CMT1H/50               | 3          | 442.09  | 460.12     | 52               | 2893         | 128               | 465.02    | <b>465.02</b> | <b>465.02</b> | 0.00       |
| CMT1Q/50               | 4          | 468.58  | 488.13     | 49               | 15           | 54                | 489.74    | <b>489.74</b> | <b>489.74</b> | 0.00       |
| CMT1T/50               | 5          | 488.08  | 512.83     | 71               | 1050         | 193               | 520.06    | <b>520.06</b> | <b>520.06</b> | 0.00       |
| CMT2H/75               | -          | 620.06  | 643.20     | 365              | 1642         | 7200              | 647.84    | 646.11        | 662.63        | 2.49       |
| CMT2Q/75               | -          | 681.52  | 707.91     | 765              | 1596         | 7200              | 711.30    | 709.96        | 732.76        | 3.11       |
| CMT2T/75               | -          | 733.01  | 760.81     | 511              | 2763         | 7200              | 764.99    | 763.47        | 782.77        | 2.47       |
| CMT3H/100              | -          | 674.46  | 691.53     | 1499             | 10735        | 7200              | 694.92    | 694.15        | 701.31        | 1.02       |
| CMT3Q/100              | 6          | 718.88  | 744.50     | 1583             | 271          | 1788              | 747.15    | <b>747.15</b> | <b>747.15</b> | 0.00       |
| CMT3T/100              | -          | 757.23  | 784.43     | 4414             | 1267         | 7200              | 787.12    | 785.86        | 798.07        | 1.53       |
| CMT12H/100             | -          | 537.95  | 623.32     | 674              | 57192        | 7200              | 627.32    | 626.70        | 629.37        | 0.42       |
| CMT12Q/100             | -          | 639.80  | 721.95     | 1160             | 5675         | 7200              | 726.71    | 726.49        | 729.46        | 0.41       |
| CMT12T/100             | 9          | 706.08  | 787.52     | 508              | 7            | 509               | 787.52    | <b>787.52</b> | <b>787.52</b> | 0.00       |
| CMT11H/120             | -          | 665.43  | 800.91     | 7057             | 4            | 7214              | 801.05    | 800.91        | 820.35        | 2.37       |
| CMT11Q/120             | -          | 811.84  | 928.74     | 7272             | 1            | 7275              | 928.74    | <u>928.74</u> | 939.36        | 1.13       |
| CMT11T/120             | -          | 903.15  | 984.67     | 5670             | 12           | 7201              | 985.03    | <u>985.03</u> | 998.80        | 1.38       |
| CMT4H/150              | -          | 778.19  | 798.38     | 7199             | 1            | 7202              | 798.38    | <u>798.38</u> | 831.39        | 3.97       |
| CMT4Q/150              | -          | 857.54  | 890.12     | 7280             | 1            | 7286              | 890.12    | <u>890.12</u> | 913.93        | 2.61       |
| CMT4T/150              | -          | 920.63  | 950.59     | 7255             | 1            | 7259              | 950.59    | <u>950.59</u> | 990.39        | 4.02       |
| CMT5H/200              | -          | 902.33  | 917.99     | 6756             | 1            | 7201              | 922.88    | 917.99        | 992.37        | 7.50       |
| CMT5Q/200              | -          | 1022.01 | 1039.20    | 7837             | 1            | 7840              | 1040.25   | 1039.20       | 1134.72       | 8.42       |
| CMT5T/200              | -          | 1118.44 | 1133.52    | 7538             | 1            | 7540              | 1139.93   | 1133.52       | 1232.08       | 8.00       |
| Avg. Gap (%)           |            |         |            |                  |              |                   |           |               |               | 2.42       |

The statistics of the root node of each formulation over a set of representative instances is presented in Tables 3.17-3.19. The interpretation of the results contained in these tables

are quite similar to those reported for the VRPSPD instances (see Tables 3.10-3.12). The amount of time spent by all formulations to solve the LPs considerably increases with the size of the instances. Since the estimated number of vehicles of most instances is unknown, a further analysis regarding their influence in the statistics of the root node could not be performed.

Table 3.16: Results obtained by F2C-D on Salhi and Nagy's instances (VRPMPD)

| Instance/<br>Customers | #v | LP      | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | New<br>LB | F-LB          | UB            | Gap<br>(%) |
|------------------------|----|---------|------------|------------------|--------------|-------------------|-----------|---------------|---------------|------------|
| CMT1H/50               | 3  | 442.09  | 460.07     | 71               | 1748         | 148               | 465.02    | <b>465.02</b> | <b>465.02</b> | 0.00       |
| CMT1Q/50               | 4  | 468.58  | 488.21     | 61               | 11           | 64                | 489.74    | <b>489.74</b> | <b>489.74</b> | 0.00       |
| CMT1T/50               | 5  | 488.08  | 512.92     | 85               | 813          | 208               | 520.06    | <b>520.06</b> | <b>520.06</b> | 0.00       |
| CMT2H/75               | -  | 620.06  | 643.42     | 730              | 5735         | 7200              | 647.84    | 647.45        | 662.63        | 2.29       |
| CMT2Q/75               | -  | 681.52  | 707.13     | 953              | 6287         | 7200              | 711.30    | <u>711.30</u> | 732.76        | 2.93       |
| CMT2T/75               | -  | 733.01  | 760.91     | 918              | 6183         | 7200              | 764.99    | <u>764.99</u> | 782.77        | 2.27       |
| CMT3H/100              | -  | 674.46  | 691.53     | 1158             | 19031        | 7200              | 694.92    | <u>694.92</u> | 701.31        | 0.91       |
| CMT3Q/100              | 6  | 718.88  | 744.47     | 1366             | 411          | 1608              | 747.15    | <b>747.15</b> | <b>747.15</b> | 0.00       |
| CMT3T/100              | -  | 757.23  | 784.13     | 3957             | 1904         | 7200              | 787.12    | <u>787.12</u> | 798.07        | 1.37       |
| CMT12H/100             | -  | 538.07  | 623.32     | 773              | 42656        | 7200              | 627.32    | 626.45        | 629.37        | 0.46       |
| CMT12Q/100             | -  | 639.80  | 721.90     | 1309             | 3676         | 7200              | 726.71    | <u>726.71</u> | 729.46        | 0.38       |
| CMT12T/100             | 9  | 706.08  | 787.25     | 750              | 6            | 752               | 787.52    | <b>787.52</b> | <b>787.52</b> | 0.00       |
| CMT11H/120             | -  | 665.46  | 801.05     | 7273             | 1            | 7277              | 801.05    | <u>801.05</u> | 820.35        | 2.35       |
| CMT11Q/120             | -  | 811.92  | 926.91     | 7197             | 1            | 7200              | 928.74    | 926.91        | 939.36        | 1.33       |
| CMT11T/120             | -  | 903.19  | 984.48     | 7196             | 1            | 7200              | 985.03    | 984.48        | 998.80        | 1.43       |
| CMT4H/150              | -  | 778.19  | 798.15     | 7295             | 1            | 7298              | 798.38    | 798.15        | 831.39        | 4.00       |
| CMT4Q/150              | -  | 857.54  | 890.11     | 7168             | 1            | 7200              | 890.12    | 890.11        | 913.93        | 2.61       |
| CMT4T/150              | -  | 920.63  | 950.12     | 7152             | 1            | 7200              | 950.59    | 950.12        | 990.39        | 4.07       |
| CMT5H/200              | -  | 902.34  | 922.85     | 7262             | 1            | 7265              | 922.88    | 922.85        | 992.37        | 7.01       |
| CMT5Q/200              | -  | 1022.08 | 1037.29    | 7249             | 1            | 7251              | 1040.25   | 1037.29       | 1134.72       | 8.59       |
| CMT5T/200              | -  | 1118.44 | 1129.15    | 6383             | 1            | 7201              | 1139.93   | 1129.15       | 1232.08       | 8.35       |
| Avg. Gap (%)           |    |         |            |                  |              |                   |           |               |               | 2.40       |

Table 3.17: Root node statistics of the F1C over a set of VRPMPD representative instances

| Instance/<br>Customers | #v | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|----|----------------|----------------|------------------|------------------|------------|
| CMT1H/50               | 3  | 27             | 37.0           | 1.8              | 38.8             | 1.06       |
| CMT1T/50               | 5  | 34             | 44.5           | 2.1              | 46.6             | 1.34       |
| CMT2H/75               | -  | 45             | 280.8          | 11.1             | 291.9            | 2.88       |
| CMT2T/75               | -  | 62             | 553.4          | 20.5             | 573.8            | 2.89       |
| CMT3H/100              | -  | 33             | 604.0          | 16.8             | 620.8            | 1.41       |
| CMT3T/100              | -  | 91             | 3520.0         | 81.9             | 3601.9           | 1.73       |
| CMT12H/100             | -  | 67             | 7168.0         | 32.2             | 7200.2           | 1.01       |
| CMT12T/100             | 9  | 72             | 7206.0         | 17.5             | 7223.5           | 0.00       |
| CMT11H/120             | -  | 32             | 522.4          | 4.0              | 526.4            | 2.36       |
| CMT11T/120             | -  | 39             | 455.0          | 1.7              | 456.7            | 1.45       |
| CMT4H/150              | -  | 22             | 7236.6         | 3.4              | 7240.1           | 4.00       |
| CMT4T/150              | -  | 30             | 7096.0         | 5.3              | 7101.3           | 4.13       |
| CMT5H/200              | -  | 13             | 6908.8         | 4.1              | 6912.9           | 7.00       |
| CMT5T/200              | -  | 8              | 7445.6         | 2.1              | 7447.8           | 7.48       |

Table 3.20 shows the summary of the results obtained in the set of VRPMPD instances of Salhi and Nagy. The value of the average gaps G1, G2 and G3 suggest that there are practically no difference between the three formulations. Nonetheless, differently from the VRPSPD results, where F2C-U consistently found better results, it was F1C that produce, on average, slightly better LBs.

Table 3.18: Root node statistics of F2C-U over a set of VRPMPD representative instances

| Instance/<br>Customers | # $v$ | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|-------|----------------|----------------|------------------|------------------|------------|
| CMT1H/50               | 3     | 30             | 49.6           | 2.0              | 51.7             | 1.05       |
| CMT1T/50               | 5     | 41             | 68.3           | 3.2              | 71.4             | 1.39       |
| CMT2H/75               | -     | 53             | 355.2          | 9.4              | 364.7            | 2.93       |
| CMT2T/75               | -     | 58             | 492.1          | 19.2             | 511.3            | 2.81       |
| CMT3H/100              | -     | 52             | 1477.7         | 21.5             | 1499.2           | 1.39       |
| CMT3T/100              | -     | 110            | 4340.3         | 73.6             | 4413.9           | 1.71       |
| CMT12H/100             | -     | 45             | 671.4          | 2.9              | 674.3            | 0.96       |
| CMT12T/100             | 9     | 39             | 505.1          | 2.9              | 508.0            | 0.00       |
| CMT11H/120             | -     | 83             | 6994.5         | 62.1             | 7056.6           | 2.37       |
| CMT11T/120             | -     | 118            | 5589.4         | 80.9             | 5670.3           | 1.41       |
| CMT4H/150              | -     | 31             | 7194.2         | 4.3              | 7198.5           | 3.97       |
| CMT4T/150              | -     | 30             | 7249.4         | 5.9              | 7255.2           | 4.02       |
| CMT5H/200              | -     | 4              | 6755.2         | 1.1              | 6756.4           | 7.50       |
| CMT5T/200              | -     | 4              | 7537.2         | 0.8              | 7538.0           | 8.00       |

Table 3.19: Root node statistics of F2C-D over a set of VRPMPD representative instances

| Instance/<br>Customers | # $v$ | Sep.<br>Rounds | LP<br>Time (s) | Sep.<br>Time (s) | Root<br>Time (s) | Gap<br>(%) |
|------------------------|-------|----------------|----------------|------------------|------------------|------------|
| CMT1H/50               | 3     | 32             | 68.2           | 2.9              | 71.1             | 1.06       |
| CMT1T/50               | 5     | 39             | 82.8           | 2.4              | 85.3             | 1.37       |
| CMT2H/75               | -     | 75             | 704.4          | 25.3             | 729.7            | 2.90       |
| CMT2T/75               | -     | 82             | 883.9          | 33.9             | 917.8            | 2.79       |
| CMT3H/100              | -     | 44             | 1142.7         | 15.5             | 1158.2           | 1.39       |
| CMT3T/100              | -     | 90             | 3882.4         | 74.2             | 3956.7           | 1.75       |
| CMT12H/100             | -     | 49             | 769.0          | 3.8              | 772.8            | 0.96       |
| CMT12T/100             | 9     | 37             | 748.0          | 2.2              | 750.2            | 0.03       |
| CMT11H/120             | -     | 65             | 7246.7         | 26.2             | 7272.9           | 2.35       |
| CMT11T/120             | -     | 74             | 7178.3         | 17.6             | 7195.9           | 1.43       |
| CMT4H/150              | -     | 30             | 7274.2         | 21.1             | 7295.2           | 4.00       |
| CMT4T/150              | -     | 21             | 7148.7         | 3.0              | 7151.7           | 4.07       |
| CMT5H/200              | -     | 8              | 7259.8         | 2.2              | 7261.9           | 7.01       |
| CMT5T/200              | -     | 3              | 6382.6         | 0.6              | 6383.2           | 8.35       |

Table 3.20: Summary of the results obtained by the three formulations (VRPMPD)

| Formulation | Salhi and Nagy |        |        |
|-------------|----------------|--------|--------|
|             | G1 (%)         | G2 (%) | G3 (%) |
| F1C         | 8.16           | 2.68   | 2.37   |
| F2C-U       | 8.33           | 2.70   | 2.42   |
| F2C-D       | 8.33           | 2.72   | 2.40   |

## 3.5 Concluding remarks

This chapter dealt with Mixed Integer Programming formulations for the the Vehicle Routing Problem with Simultaneous Pickup and Delivery (VRPSPD). An undirected and a directed two-commodity flow formulations were proposed. They were tested within a branch-and-cut scheme and their results were compared with the one-commodity flow formulation of Dell’Amico et al. [44]. The optimal solutions of 30 VRPSPD open problems were proved, as can be seen in Table 3.21. The three formulations were also tested in benchmark instances of the Vehicle Routing Problem with Mixed Pickup and Delivery (VRPMPD), which is a particular case of the VRPSPD, and were able to prove the optimality of 7 open problems (see Table 3.21). Furthermore, new lower bounds were

produced for both VRPSPD and VRPMPD instances with up to 200 customers. In addition, although it has been shown that the one-commodity flow formulation produces a stronger linear relaxation, the two-commodity flow formulations have found, on average, better lower bounds in the VRPSPD instances. As for the VRPMPD, the lower bounds were, on average, quite similar, but with a slight superiority of F1C.

Table 3.21: Optimal Solutions

| Instance/<br>Customers | #v | OPT           | Instance/<br>Customers | #v | OPT           |
|------------------------|----|---------------|------------------------|----|---------------|
| SCA3-0/50              | 4  | <b>635.62</b> | CON3-7/50              | 4  | <b>576.48</b> |
| SCA3-1/50              | 4  | <b>697.84</b> | CON3-8/50              | 4  | <b>523.05</b> |
| SCA3-2/50              | 4  | <b>659.34</b> | CON3-9/50              | 4  | <b>578.25</b> |
| SCA3-3/50              | 4  | <b>680.04</b> | CON8-1/50              | 9  | <b>740.85</b> |
| SCA3-4/50              | 4  | <b>690.50</b> | CON8-4/50              | 9  | <b>772.25</b> |
| SCA3-5/50              | 4  | <b>659.90</b> | CON8-7/50              | 9  | <b>811.96</b> |
| SCA3-6/50              | 4  | <b>651.09</b> | CMT1X/50               | 3  | <b>466.77</b> |
| SCA3-7/50              | 4  | <b>659.17</b> | CMT1Y/50               | 3  | <b>466.77</b> |
| SCA3-8/50              | 4  | <b>719.48</b> | r201/100               | 3  | <b>666.20</b> |
| SCA3-9/50              | 4  | <b>681.00</b> | c201/100               | 5  | <b>662.07</b> |
| SCA8-3/50              | 9  | <b>983.34</b> | rc201/100              | 3  | <b>672.92</b> |
| SCA8-6/50              | 9  | <b>971.82</b> | CMT1H/50               | 3  | <b>465.02</b> |
| CON3-0/50              | 4  | <b>616.52</b> | CMT1Q/50               | 4  | <b>489.74</b> |
| CON3-1/50              | 4  | <b>554.47</b> | CMT1T/50               | 5  | <b>520.06</b> |
| CON3-2/50              | 4  | <b>518.00</b> | CMT3Q/100              | 6  | <b>747.15</b> |
| CON3-3/50              | 4  | <b>591.19</b> | CMT12H/100             | 5  | <b>629.37</b> |
| CON3-4/50              | 4  | <b>588.79</b> | CMT12Q/100             | 7  | <b>729.25</b> |
| CON3-5/50              | 4  | <b>563.70</b> | CMT12T/100             | 9  | <b>787.52</b> |
| CON3-6/50              | 4  | <b>499.05</b> |                        |    |               |

## Chapter 4

# Branch-and-cut with Lazy Separation for the Single and Multi-depot Vehicle Routing Problem with Simultaneous/Mixed Pickup and Delivery

This chapter presents a BC algorithm that is capable of dealing with the VRPSPD, VRPMPD and MDVRPMPD. This BC includes cuts from the CVRPSEP library [114] and it is based on a mathematical formulation composed only by edge variables. The constraints that ensure that the capacities are not exceeded in the middle of a route and those that ensure that a route starts and ends at the same depot are applied in a lazy fashion. The developed solution approach was tested in well-known VRPSPD/VRPMPD instances with up to 200 customers and it was capable of improving most of the previously known lower bounds. The contents of the present chapter were partially published in [161].

### 4.1 Mathematical formulation

Let  $x_{ij}$  be an integer variable counting the number of times that an edge  $\{i, j\} \in E$  appears in a route, this number can only be 2 for an edge used in a route with a single customer. Given a set  $S \subseteq V'$ , let  $d(S)$  and  $p(S)$  be the sum of the delivery and pickup demands, respectively, of all customers in  $S$ . Let  $e(S) = \lceil d(S)/Q \rceil$  and  $q(S) = \lceil p(S)/Q \rceil$ . Finally, let  $v_k$  be integer variables representing the number of vehicles used at each depot  $k \in G$  in the solution. Finally, define  $\bar{S}$  as the complementary set of  $S$ , plus the depot  $\{0\}$ . Define the following IP:

$$\min \sum_{i \in V'} \sum_{j \in V, j > i} c_{ij} x_{ij} \tag{4.1}$$

$$\text{s.t. } \sum_{i \in V, i < k} x_{ik} + \sum_{j \in V, j > k} x_{kj} = 2 \quad \forall k \in V' \quad (4.2)$$

$$\sum_{j \in V'} x_{0j} = 2v_k \quad \forall k \in G \quad (4.3)$$

$$\sum_{i \in S} \sum_{j \in \bar{S}, i < j} x_{ij} + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} x_{ij} \geq 2e(S) \quad \forall S \subseteq V' \quad (4.4)$$

$$\sum_{i \in S} \sum_{j \in \bar{S}, i < j} x_{ij} + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} x_{ij} \geq 2q(S) \quad \forall S \subseteq V' \quad (4.5)$$

$$0 \leq v_k \leq m_k \quad \forall k \in G \quad (4.6)$$

$$x_{ij} \in \{0, 1\} \quad \forall \{i, j\} \in E, i > 0 \quad (4.7)$$

$$x_{ij} \in \{0, 1, 2\} \quad \forall \{0, j\} \in E. \quad (4.8)$$

This is not a complete formulation for the VRPSPD. While Constraints (4.4) and (4.5) are enough to ensure that all vehicles leave and return to the depot with load at most  $Q$ , it is possible that a vehicle capacity may be exceeded in the middle of the route. In fact, previous VRPSPD formulations use auxiliary flows for controlling vehicle load ([44, 159, 160]). Such additional variables and constraints have the drawback of increasing the solution time of the associated LPs. The new formulation proposed in this work eliminates those unfeasible routes with constraints over the edge variables. Let  $\mathcal{R}$  be the set of all subsets of edges, not adjacent to the depot, representing routes (in both directions) that are feasible with respect to both pickup and delivery alone, but not with respect to the simultaneous pickup and delivery. The following constraints are added to (4.1-4.8) in order to obtain a complete formulation for the VRPSPD, called F1:

$$\sum_{\{i,j\} \in R} x_{ij} \leq |R| - 1 \quad \forall R \in \mathcal{R} \quad (4.9)$$

Constraints similar to (4.9) are sometimes called “no-good cuts” since they only remove a single infeasible integral point, which is usually weak in a polyhedral sense, because these kind of cuts are seldom violated by fractional solutions. In this case, a constraint associated to an unfeasible route  $R$  actually eliminates all integral solutions that contain such route. Nevertheless, they are still weak and only worthy to be applied in a lazy fashion in a BC algorithm, with the purpose of checking the feasibility of the integral solutions found along the tree.

Formulation F1 is a complete formulation for the VRPSPD/VRPMPD, but not for the MDVRPMPD, because there are no constraints preventing a route from starting and ending in different depots. Let  $\mathcal{R}'$  be the set of all feasible VRPSPD routes, represented by their edges (including those adjacent to the depot), that start and end in different depots. With a view of obtaining a complete formulation for the MDVRPMPD, called F1-MD, one can add the following constraints to F1:

$$\sum_{\{0,j\} \in R} x_{ij} + 2 \sum_{\{i,j\} \in R, i>0} x_{ij} \leq 2(|R| - 2) + 1 \quad \forall R \in \mathcal{R}' \quad (4.10)$$

Constraints (4.10) follow the same idea of constraints (4.9) and therefore they are only worthy to be applied in a lazy fashion within a BC scheme.

## 4.2 Computational experiments with a Branch-and-cut approach

A BC algorithm over F1 and F1-MD was implemented. Besides capacity inequalities (4.4-4.5), two other families of valid CVRP inequalities, namely the multistar and comb inequalities, are also separated using the CVRPSEP package [114, 115]. Firstly, the separation is performed considering only the delivery demands (like in Constraints (4.4)). When no violated inequalities are found one then starts separating the pickup demands (like in (4.5)). For each separation routine of the CVRPSEP package a limit of 50 violated cuts per iteration was established. The multistar and comb inequalities are generated only at the root node. The rounded capacity cuts (4.4-4.5) are generated throughout the tree up to the 7th level and whenever an integer solution is found. Moreover, every time a feasible integer CVRP solution is found, it is necessary to check whether it is also feasible for the VRPSPD. If it is not the case, inequality (4.9) is added for each unfeasible route  $R$ . For the MDVRPMPD, if a route is feasible with respect to the VRPMPD but not feasible for the MDVRPMPD, i.e., a route that starts and ends in different depots, constraints (4.10) are added.

The BC was implemented using the CPLEX 11.2 callable library and executed in an Intel Core 2 Quad with 2.4 GHz and 4 GB of RAM running under Linux 64 bits. Only a single thread was used.

In the tables presented hereafter,  $\mathbf{v}$  is the number of vehicles in the best known solution of VRPSPD/VRPMPD instances and the number of available vehicles per depot for MDVRPMPD instances,  $\mathbf{m}$  is the number of depots, **Root LB** indicates the root lower bound, after CVRPSEP cuts are added, **Root Time** is the CPU time in seconds spent at the root node, **Tree size** is the number of nodes opened, **Total time** is the total CPU time in seconds of the BC procedure, **#Lazy Cuts 1** and **#Lazy Cuts 2** denote the number of lazy cuts (4.9) and (4.10), respectively, added, **Prev. LB** is the best lower bound obtained in Chapter 3, **LB** is the lower bound (LB) determined by the proposed BC, **UB** is the upper bound reported in [157], [64] and Chapter 6 for the VRPSPD, VRPMPD and MDVRPMPD, respectively, and **Gap** corresponds to the gap between the LB and the UB. Proven optimal solutions are highlighted in boldface and new optimal



solutions are underlined.

### 4.2.1 VRPSPD

Tables 4.1-4.3 present the results obtained by the BC on the set of instances of Dethloff [48], Salhi and Nagy [149] and Montané and Galvão [121], respectively. For the first set of instances, the BC was ran until the optimal solution was found, whereas a time limit of 2 hours of execution was imposed for the remaining sets.

From Table 4.1, it is possible to observe that, except for the instance CON8-9, the optimality of all instances were proved within up to 5000 seconds. In the instances that require less vehicles the BC spent at most 33 seconds to find the optimal solution. Furthermore, 15 new optimal solutions were proved. From Table 4.2 it can be seen that, except for the instance CMT5Y, all the LBs were equaled or improved. Also, the optimality of the instances CMT3X and CMT3Y, involving 100 customers, were proved. Lastly, from Table 4.3, one can verify that the BC equaled or improved the LBs of the instances involving 100 customers, regardless of the number of vehicles. However, for the instances involving 200 customers the results were rather inconsistent. On one hand, the BC found the optimal solutions for the instances requiring few vehicles. On the other hand, the BC obtained poor LBs for the instances requiring a large number of vehicles.

### 4.2.2 VRPMPD

Table 4.4 shows the results obtained in the instances of Salhi and Nagy. A time limit of 2 hours was imposed for the BC (which was sometimes exceeded by CPLEX), except for the instances CMT3H, CMT3T and CMT11Q, where we the BC algorithm was ran until the optimal solution was found. It can be verified that all LBs were either equaled or improved and one new optimal solution was found (CMT11Q).

### 4.2.3 MDVRPMPD

The proposed algorithm was tested in the set of MDVRPMPD instances developed by Salhi and Nagy [149]. This set consists of 33 test-problems with 50-249 customers. Only those without route duration constraints (50-100 customers) were considered. Table 4.5 presents the results found by the BC algorithm in these instances. It can be observed that BC managed to find the optimal solutions of 4 open-problems and to obtain the first lower bounds for this set of instances. Furthermore, it is possible to verify that the quality of the linear relaxation decreases when the number of depots and vehicles per depot increases. Although the overall results are reasonably satisfactory, the BC algorithm was not as effective for the MDVRPMPD as it was for the VRPSPD/VRPMPD.

Table 4.1: Results obtained on the instances of Dethloff [48]

| Instance/<br>Customers | $v$ | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | #Lazy<br>Cuts 1 | Prev.<br>LB | LB      | UB             | Gap<br>(%) |
|------------------------|-----|------------|------------------|--------------|-------------------|-----------------|-------------|---------|----------------|------------|
| SCA3-0/50              | 4   | 619.21     | 5.18             | 543          | 16.8              | 5               | 635.62      | 635.62  | <b>635.62</b>  | 0.00       |
| SCA3-1/50              | 4   | 682.34     | 1.90             | 129          | 6.75              | 0               | 697.84      | 697.84  | <b>697.84</b>  | 0.00       |
| SCA3-2/50              | 4   | 659.34     | 0.60             | 1            | 0.61              | 0               | 659.34      | 659.34  | <b>659.34</b>  | 0.00       |
| SCA3-3/50              | 4   | 667.56     | 1.76             | 35           | 2.75              | 0               | 680.04      | 680.04  | <b>680.04</b>  | 0.00       |
| SCA3-4/50              | 4   | 676.56     | 6.75             | 98           | 10.4              | 0               | 690.50      | 690.50  | <b>690.50</b>  | 0.00       |
| SCA3-5/50              | 4   | 647.90     | 1.16             | 623          | 5.71              | 26              | 659.90      | 659.91  | <b>659.91</b>  | 0.00       |
| SCA3-6/50              | 4   | 625.60     | 1.29             | 2655         | 239               | 0               | 651.09      | 651.09  | <b>651.09</b>  | 0.00       |
| SCA3-7/50              | 4   | 654.97     | 3.70             | 5            | 3.87              | 0               | 659.17      | 659.17  | <b>659.17</b>  | 0.00       |
| SCA3-8/50              | 4   | 688.21     | 1.97             | 5880         | 710               | 0               | 719.48      | 719.48  | <b>719.48</b>  | 0.00       |
| SCA3-9/50              | 4   | 671.07     | 2.04             | 27           | 2.88              | 0               | 681.00      | 681.00  | <b>681.00</b>  | 0.00       |
| SCA8-0/50              | 9   | 926.03     | 18.8             | 2836         | 557               | 0               | 936.89      | 961.50  | <b>961.50</b>  | 0.00       |
| SCA8-1/50              | 9   | 1002.29    | 15.5             | 14292        | 3288              | 2               | 1020.28     | 1049.65 | <b>1049.65</b> | 0.00       |
| SCA8-2/50              | 9   | 1013.37    | 19.8             | 4321         | 553               | 0               | 1024.24     | 1039.64 | <b>1039.64</b> | 0.00       |
| SCA8-3/50              | 9   | 956.10     | 11.9             | 350          | 66.5              | 0               | 983.34      | 983.34  | <b>983.34</b>  | 0.00       |
| SCA8-4/50              | 9   | 1027.24    | 25.9             | 9136         | 931               | 1               | 1041.65     | 1065.49 | <b>1065.49</b> | 0.00       |
| SCA8-5/50              | 9   | 998.05     | 13.7             | 948          | 149               | 3               | 1015.19     | 1027.08 | <b>1027.08</b> | 0.00       |
| SCA8-6/50              | 9   | 948.26     | 12.8             | 750          | 95.3              | 12              | 971.82      | 971.82  | <b>971.82</b>  | 0.00       |
| SCA8-7/50              | 9   | 1018.20    | 16.4             | 6303         | 600               | 4               | 1031.56     | 1051.28 | <b>1051.28</b> | 0.00       |
| SCA8-8/50              | 9   | 1038.86    | 6.15             | 957          | 100               | 1               | 1048.93     | 1071.18 | <b>1071.18</b> | 0.00       |
| SCA8-9/50              | 9   | 1019.25    | 12.0             | 9068         | 1381              | 1               | 1034.28     | 1060.50 | <b>1060.50</b> | 0.00       |
| CON3-0/50              | 4   | 611.30     | 2.58             | 21           | 3.39              | 0               | 616.52      | 616.52  | <b>616.52</b>  | 0.00       |
| CON3-1/50              | 4   | 546.65     | 8.26             | 142          | 13.7              | 0               | 554.47      | 554.47  | <b>554.47</b>  | 0.00       |
| CON3-2/50              | 4   | 505.06     | 3.94             | 293          | 22.4              | 6               | 518.01      | 518.01  | <b>518.01</b>  | 0.00       |
| CON3-3/50              | 4   | 584.28     | 2.05             | 22           | 2.62              | 0               | 591.19      | 591.19  | <b>591.19</b>  | 0.00       |
| CON3-4/50              | 4   | 577.52     | 1.04             | 322          | 10.4              | 5               | 588.79      | 588.79  | <b>588.79</b>  | 0.00       |
| CON3-5/50              | 4   | 552.91     | 3.22             | 350          | 20.2              | 2               | 563.70      | 563.70  | <b>563.70</b>  | 0.00       |
| CON3-6/50              | 4   | 486.98     | 5.22             | 352          | 32.5              | 0               | 499.05      | 499.05  | <b>499.05</b>  | 0.00       |
| CON3-7/50              | 4   | 561.62     | 0.97             | 494          | 29.1              | 0               | 576.48      | 576.48  | <b>576.48</b>  | 0.00       |
| CON3-8/50              | 4   | 514.84     | 5.03             | 97           | 8.55              | 3               | 523.05      | 523.05  | <b>523.05</b>  | 0.00       |
| CON3-9/50              | 4   | 564.78     | 2.51             | 175          | 19.4              | 0               | 578.25      | 578.25  | <b>578.25</b>  | 0.00       |
| CON8-0/50              | 9   | 830.03     | 41.5             | 1760         | 272               | 0               | 845.19      | 857.17  | <b>857.17</b>  | 0.00       |
| CON8-1/50              | 9   | 722.38     | 27.7             | 1179         | 200               | 1               | 740.85      | 740.85  | <b>740.85</b>  | 0.00       |
| CON8-2/50              | 9   | 685.66     | 39.2             | 14315        | 4842              | 1               | 695.70      | 712.89  | <b>712.89</b>  | 0.00       |
| CON8-3/50              | 10  | 787.84     | 39.3             | 12377        | 1599              | 2               | 797.57      | 811.07  | <b>811.07</b>  | 0.00       |
| CON8-4/50              | 9   | 751.95     | 21.2             | 806          | 159               | 3               | 772.25      | 772.25  | <b>772.25</b>  | 0.00       |
| CON8-5/50              | 9   | 729.92     | 15.1             | 6369         | 1212              | 0               | 741.51      | 754.88  | <b>754.88</b>  | 0.00       |
| CON8-6/50              | 9   | 648.64     | 18.7             | 10738        | 2865              | 0               | 662.14      | 678.92  | <b>678.92</b>  | 0.00       |
| CON8-7/50              | 9   | 793.50     | 15.5             | 228          | 38.4              | 2               | 811.96      | 811.96  | <b>811.96</b>  | 0.00       |
| CON8-8/50              | 9   | 744.52     | 26.8             | 3520         | 606               | 0               | 757.45      | 767.53  | <b>767.53</b>  | 0.00       |
| CON8-9/50              | 9   | 773.65     | 45.0             | 90738        | 38137             | 4               | 786.40      | 809.00  | <b>809.00</b>  | 0.00       |

Table 4.2: Results obtained on the VRPSPD instances of Salhi and Nagy [149]

| Instance/<br>Customers | $v$ | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | #Lazy<br>Cuts 1 | Prev.<br>LB | LB     | UB            | Gap<br>(%) |
|------------------------|-----|------------|------------------|--------------|-------------------|-----------------|-------------|--------|---------------|------------|
| CMT1X/50               | 3   | 460.73     | 5.56             | 78           | 10.3              | 0               | 466.77      | 466.77 | <b>466.77</b> | 0.00       |
| CMT1Y/50               | 3   | 460.69     | 15.5             | 60           | 19.3              | 0               | 466.77      | 466.77 | <b>466.77</b> | 0.00       |
| CMT2X/75               | 6   | 658.38     | 100              | 17718        | 7200              | 0               | 655.98      | 666.57 | 684.21        | 2.65       |
| CMT2Y/75               | 6   | 658.12     | 200              | 19875        | 7200              | 0               | 655.41      | 666.69 | 684.21        | 2.63       |
| CMT3X/100              | 5   | 711.77     | 79.7             | 8521         | 2461              | 36              | 705.54      | 721.27 | <b>721.27</b> | 0.00       |
| CMT3Y/100              | 5   | 711.89     | 140              | 9313         | 3226              | 36              | 705.62      | 721.27 | <b>721.27</b> | 0.00       |
| CMT12X/100             | 5   | 635.52     | 50.3             | 10460        | 7200              | 1               | 629.39      | 643.76 | 662.22        | 2.87       |
| CMT12Y/100             | 5   | 635.45     | 65.1             | 11621        | 7200              | 0               | 629.18      | 644.10 | 662.22        | 2.81       |
| CMT11X/120             | 4   | 793.11     | 892              | 5988         | 7200              | 0               | 776.35      | 799.67 | 833.92        | 4.28       |
| CMT11Y/120             | 4   | 793.54     | 1098             | 4063         | 7200              | 0               | 775.74      | 799.02 | 833.92        | 4.37       |
| CMT4X/150              | 7   | 826.74     | 1950             | 4012         | 7200              | 0               | 817.11      | 831.18 | 852.46        | 2.56       |
| CMT4Y/150              | 7   | 826.34     | 2568             | 4380         | 7200              | 0               | 816.99      | 831.65 | 852.46        | 2.50       |
| CMT5X/200              | 10  | 971.07     | 7202             | 1            | 7206              | 0               | 954.87      | 971.07 | 1029.25       | 5.99       |
| CMT5Y/200              | 10  | 937.66     | 7248             | 1            | 7252              | 0               | 953.56      | 937.66 | 1029.25       | 9.77       |

Table 4.3: Results obtained on the instances of Montané and Galvão [121]

| Instance/<br>Customers | $v$ | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | #Lazy<br>Cuts 1 | Prev.<br>LB | LB      | UB             | Gap<br>(%) |
|------------------------|-----|------------|------------------|--------------|-------------------|-----------------|-------------|---------|----------------|------------|
| r101/100               | 12  | 972.58     | 1282             | 5528         | 7200              | 0               | 973.91      | 978.28  | 1009.95        | 3.24       |
| r201/100               | 3   | 664.80     | 42.9             | 10           | 43.7              | 0               | 666.20      | 666.20  | <b>666.20</b>  | 0.00       |
| c101/100               | 16  | 1194.69    | 722              | 10480        | 7200              | 0               | 1196.70     | 1202.59 | 1220.18        | 1.46       |
| c201/100               | 5   | 657.97     | 24.6             | 12           | 28.3              | 0               | 662.07      | 662.07  | <b>662.07</b>  | 0.00       |
| rc101/100              | 10  | 1028.06    | 1067             | 8208         | 7200              | 0               | 1029.38     | 1041.06 | 1059.32        | 1.75       |
| rc201/100              | 3   | 671.84     | 6.37             | 3            | 6.42              | 0               | 672.92      | 672.92  | <b>672.92</b>  | 0.00       |
| r1.2.1/200             | 23  | 2681.05    | 7198             | 1            | 7202              | 0               | 3084.97     | 2681.05 | 3360.02        | 25.32      |
| r2.2.1/200             | 5   | 1656.62    | 4477             | 974          | 5551              | 0               | 1618.76     | 1665.58 | <b>1665.58</b> | 0.00       |
| c1.2.1/200             | 27  | 2857.45    | 7196             | 1 the        | 7201              | 0               | 3475.03     | 2857.45 | 3629.89        | 27.03      |
| c2.2.1/200             | 9   | 1638.99    | 7197             | 1            | 7203              | 0               | 1647.83     | 1638.99 | 1726.59        | 5.34       |
| rc1.2.1/200            | 23  | 2656.78    | 7196             | 1            | 7201              | 0               | 3093.30     | 2656.78 | 3306.00        | 24.44      |
| rc2.2.1/200            | 5   | 1551.54    | 1932             | 483          | 2323              | 0               | 1551.07     | 1560.00 | <b>1560.00</b> | 0.00       |

Table 4.4: Results obtained on the VRPMPD instances of Salhi and Nagy [149]

| Instance/<br>Customers | $v$ | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | #Lazy<br>Cuts 1 | Prev.<br>LB | LB      | UB            | Gap<br>(%) |
|------------------------|-----|------------|------------------|--------------|-------------------|-----------------|-------------|---------|---------------|------------|
| CMT1H/50               | 3   | 460.10     | 7.14             | 103          | 9.67              | 10              | 465.02      | 465.02  | <b>465.02</b> | 0.00       |
| CMT1Q/50               | 4   | 488.15     | 5.65             | 5            | 5.87              | 0               | 489.74      | 489.74  | <b>489.74</b> | 0.00       |
| CMT1T/50               | 5   | 512.61     | 4.98             | 126          | 11.7              | 0               | 520.06      | 520.06  | <b>520.06</b> | 0.00       |
| CMT2H/75               | -   | 643.28     | 101              | 15003        | 7200              | 0               | 647.84      | 653.79  | 662.63        | 1.33       |
| CMT2Q/75               | -   | 707.66     | 100              | 12256        | 7200              | 0               | 711.30      | 717.36  | 732.76        | 2.10       |
| CMT2T/75               | -   | 759.92     | 151              | 15585        | 7200              | 0               | 764.99      | 770.35  | 782.77        | 1.59       |
| CMT3H/100              | 3   | 691.32     | 58.0             | 58219        | 42606             | 481             | 694.92      | 700.94  | <b>700.94</b> | 0.00       |
| CMT3Q/100              | 6   | 744.50     | 95.5             | 39           | 113               | 0               | 747.15      | 747.15  | <b>747.15</b> | 0.00       |
| CMT3T/100              | 5   | 784.13     | 342              | 92974        | 164046            | 0               | 787.12      | 798.07  | <b>798.07</b> | 0.00       |
| CMT12H/100             | -   | 623.00     | 40.3             | 17791        | 3018              | 180             | 629.37      | 629.37  | <b>629.37</b> | 0.00       |
| CMT12Q/100             | -   | 721.81     | 83.9             | 1577         | 686               | 4               | 729.25      | 729.25  | <b>729.25</b> | 0.00       |
| CMT12T/100             | 9   | 787.27     | 37.7             | 3            | 37.9              | 0               | 787.52      | 787.52  | <b>787.52</b> | 0.00       |
| CMT11H/120             | -   | 801.24     | 353              | 4296         | 7200              | 0               | 801.05      | 806.46  | 820.35        | 1.69       |
| CMT11Q/120             | 6   | 928.28     | 653              | 95065        | 209428            | 9               | 928.74      | 939.36  | <b>939.36</b> | 0.00       |
| CMT11T/120             | -   | 984.81     | 359              | 7232         | 7200              | 0               | 985.03      | 989.32  | 998.80        | 0.95       |
| CMT4H/150              | -   | 799.19     | 1122             | 3544         | 7200              | 0               | 798.38      | 804.10  | 831.39        | 3.28       |
| CMT4Q/150              | -   | 892.99     | 2920             | 3243         | 7200              | 0               | 890.12      | 898.28  | 913.93        | 1.71       |
| CMT4T/150              | -   | 953.41     | 4246             | 1873         | 7200              | 0               | 950.59      | 956.54  | 990.39        | 3.42       |
| CMT5H/199              | -   | 931.02     | 7209             | 1            | 7215              | 0               | 922.88      | 931.02  | 992.37        | 6.18       |
| CMT5Q/199              | -   | 1056.74    | 7206             | 1            | 7213              | 0               | 1040.25     | 1056.74 | 1134.72       | 6.87       |
| CMT5T/199              | -   | 1145.61    | 7192             | 1            | 7200              | 0               | 1139.93     | 1145.63 | 1232.08       | 7.02       |

### 4.3 Concluding remarks

The success of the proposed BC when compared to previous approaches can be attributed to the use of a formulation where the load of a vehicle in the middle of a route is only controlled by weak constraints, that are only separated in a lazy way over integral solutions, avoiding complicated and larger extended formulations. In practice (at least on the instances from the literature), it seems that there are relatively few routes where the load capacity is respected in both ends but not in the middle, as can be observed by the small number of lazy cuts added throughout the BC (see Tables 4.1-2.8). However, the BC becomes less effective when the number of vehicles and/or depots increases.

Table 4.5: Results obtained on the MDVRPMPD instances of Salhi and Nagy [149]

| Instance/<br>Customers | $v$ | $ G $ | Root<br>LB | Root<br>Time (s) | Tree<br>size | Total<br>Time (s) | #Lazy<br>Cuts 1 | #Lazy<br>Cuts 2 | LB     | UB            | Gap<br>(%) |
|------------------------|-----|-------|------------|------------------|--------------|-------------------|-----------------|-----------------|--------|---------------|------------|
| GJ01H/50               | 4   | 4     | 459.594    | 5.28             | 34210        | 7200              | 0               | 263             | 480.43 | 499.12        | 3.74       |
| GJ01Q/50               | 4   | 4     | 492.635    | 3.34             | 44224        | 7200              | 2               | 132             | 515.09 | 528.30        | 2.50       |
| GJ01T/50               | 4   | 4     | 532.425    | 11.0             | 33046        | 7200              | 0               | 45              | 558.12 | 569.43        | 1.99       |
| GJ02H/50               | 2   | 4     | 428.195    | 0.46             | 2554         | 93.7              | 2               | 132             | 440.00 | <b>440.00</b> | 0.00       |
| GJ02Q/50               | 2   | 4     | 437.079    | 1.00             | 4065         | 224               | 0               | 127             | 449.72 | <b>449.72</b> | 0.00       |
| GJ02T/50               | 2   | 4     | 447.174    | 3.90             | 5321         | 517               | 0               | 151             | 464.13 | <b>464.13</b> | 0.00       |
| GJ03H/75               | 3   | 5     | 559.289    | 15.1             | 27805        | 7200              | 5               | 71              | 572.75 | 579.45        | 1.16       |
| GJ03Q/75               | 3   | 5     | 580.047    | 14.3             | 22271        | 7200              | 0               | 56              | 591.35 | 605.25        | 2.30       |
| GJ03T/75               | 3   | 5     | 594.704    | 60.9             | 20880        | 7200              | 0               | 10              | 606.68 | 624.44        | 2.84       |
| GJ04H/100              | 8   | 2     | 745.99     | 250              | 6362         | 7200              | 0               | 0               | 754.33 | 789.19        | 4.42       |
| GJ04Q/100              | 8   | 2     | 831.759    | 364              | 8446         | 7200              | 0               | 0               | 841.74 | 874.78        | 3.78       |
| GJ04T/100              | 8   | 2     | 915.006    | 589              | 8595         | 7200              | 0               | 0               | 923.90 | 962.25        | 3.99       |
| GJ05H/100              | 5   | 2     | 659.855    | 13.6             | 16762        | 7200              | 9               | 12              | 668.72 | 676.81        | 1.20       |
| GJ05Q/100              | 5   | 2     | 692.707    | 36.3             | 493          | 278               | 0               | 12              | 700.15 | <b>700.15</b> | 0.00       |
| GJ05T/100              | 5   | 2     | 715.925    | 97.8             | 10080        | 7200              | 0               | 2               | 723.35 | 733.17        | 1.34       |
| GJ06H/100              | 6   | 3     | 702.486    | 167              | 8408         | 7200              | 0               | 0               | 709.71 | 742.18        | 4.38       |
| GJ06Q/100              | 6   | 3     | 756.244    | 358              | 9531         | 7200              | 0               | 0               | 765.70 | 793.85        | 3.55       |
| GJ06T/100              | 6   | 3     | 805.96     | 735              | 8561         | 7200              | 0               | 0               | 815.61 | 850.82        | 4.14       |
| GJ07H/100              | 4   | 4     | 699.282    | 64.3             | 10448        | 7200              | 0               | 0               | 706.86 | 732.73        | 3.53       |
| GJ07Q/100              | 4   | 4     | 758.286    | 166              | 10011        | 7200              | 0               | 0               | 768.21 | 802.20        | 4.24       |
| GJ07T/100              | 4   | 4     | 806.337    | 315              | 11053        | 7200              | 0               | 0               | 816.31 | 853.54        | 4.36       |

# Chapter 5

## Branch-cut-and-price for the Vehicle Routing Problem with Simultaneous/Mixed Pickup and Delivery

This chapter presents a Branch-cut-and-price (BCP) algorithm for the VRPSPD that is also capable of solving the VRPMPD. To the knowledge of the author this is the first attempt to solve this problem using a BCP approach. The developed algorithm consists of an extension of the one proposed by Fukasawa et al. [63] for the CVRP. Computational experiments were carried out on instances with up to 200 customers. Four instances were solved for the first time and some LBs were improved.

BCP is a generalization of both Branch-and-cut (BC) and Branch-and-price (BP) algorithms. It can be used to solve formulations with a very large number of constraints and variables. In the case of VRPs, such variables (columns) are usually associated to routes. The columns are generated by solving a so-called pricing subproblem which aims at finding the single vehicle route with most negative reduced cost. The cuts (constraints) are expressed in terms of edge variables and then translated to route variables. The reader is referred to [137] for further details on how this approach can be generally applied to CO problems.

### 5.1 Introducing the *pd*-route

Let a  $q$ -route be a walk that starts at the depot vertex, traverses a sequence of customer vertices with total delivery (or pickup) demand at most  $Q$ , and returns to the depot. Some customers may be visited more than once, so the set of valid CVRP routes is strictly contained in the set of  $q$ -routes. For the VRPSPD, the direct use of  $q$ -routes does not yield strong relaxations because the pickup (or delivery) demands are ignored. Therefore, the concept of  $pd$ -route, which extends the definition of  $q$ -route by considering both pickup and delivery demands, is introduced, as given by Definition 1.

**Definition 1.** A *pd-route* is a sequence of vertices  $v_1, v_2, \dots, v_k$  with  $v_i \neq v_{i+1}$ ,  $i = 1, 2, \dots, k-1$ ,  $v_1 = v_k = 0$ ,  $v_i \in V'$ ,  $i = 2, \dots, k-1$ , and  $\sum_{j=2}^i p_{v_j} + \sum_{j=i+1}^{k-1} d_{v_j} \leq Q$ ,  $i = 1, \dots, k-1$ . The cost of this *pd-route* is given by the sum of its edge costs, i.e.,  $\sum_{i=1}^{k-1} c_{v_i, v_{i+1}}$ .

Fig. 5.1 illustrates an example of a *pd-route*. In this case, the *pd-route* is defined by  $0 \rightarrow 1 \rightarrow 2 \rightarrow 3 \rightarrow 1 \rightarrow 0$ . The vehicle leaves the depot with a load of 22 and returns to depot with a load of 30. Notice that the edge  $\{0,1\}$  is traversed twice, which shows that the existence of cycles is valid on *pd-routes*.

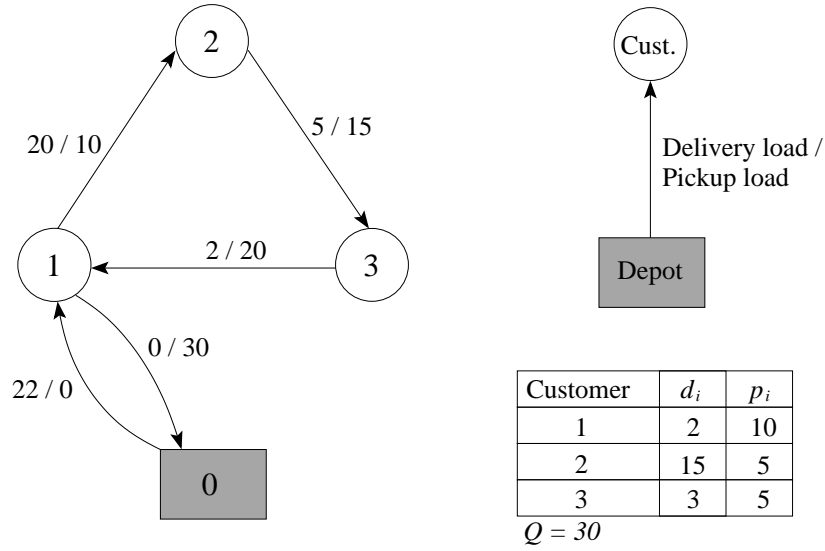


Figure 5.1: Example of a *pd-route*

## 5.2 Mathematical formulation

Consider the following Formulation F1 described in the previous chapter.

$$(F1) \quad \min \sum_{i \in V} \sum_{j \in V, j > i} c_{ij} x_{ij} \quad (5.1)$$

$$\text{s.t.} \quad \sum_{i \in V, i < k} x_{ik} + \sum_{j \in V, j > k} x_{kj} = 2 \quad \forall k \in V' \quad (5.2)$$

$$\sum_{j \in V'} x_{0j} = 2v \quad (5.3)$$

$$\sum_{i \in S} \sum_{j \in \bar{S}, i < j} x_{ij} + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} x_{ij} \geq 2e(S) \quad \forall S \subseteq V' \quad (5.4)$$

$$\sum_{i \in S} \sum_{j \in \bar{S}, i < j} x_{ij} + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} x_{ij} \geq 2q(S) \quad \forall S \subseteq V' \quad (5.5)$$

$$\sum_{\{i,j\} \in R} x_{ij} \leq |R| - 1 \quad \forall R \in \mathcal{R} \quad (5.6)$$

$$v \in \mathbb{Z}_+ \quad (5.7)$$

$$x_{ij} \in \{0, 1\} \quad \forall \{i, j\} \in E, i > 0 \quad (5.8)$$

$$x_{ij} \in \{0, 1, 2\} \quad \forall \{0, j\} \in E. \quad (5.9)$$

Formulation F1 can be strengthened by adding an exponential number of variables corresponding to the  $pd$ -routes. Enumerate all possible  $pd$ -routes from 1 to  $P$  and let  $a_{ij}^t$  be the number of times an edge  $\{i, j\}$  appears in the  $t$ -th  $pd$ -route. Define  $\lambda_t$  as a positive variable associated to the  $t$ -th  $pd$ -route. In order to improve F1 the following constraints are added:

$$\sum_{t=1}^P a_{ij}^t \lambda_t - x_{ij} = 0 \quad \forall \{i, j\} \in E \quad (5.10)$$

$$\lambda_t \geq 0 \quad (t = 1, \dots, P) \quad (5.11)$$

Constraints (5.10) state that  $x$  is as a weighted sum of edge-incidence vectors of  $pd$ -routes. Notice that constraints (5.6) can be dropped from F1 since they are implied by (5.10). Hence, define F2 as the IP formulation composed by (5.1)-(5.5) and (5.7)-(5.11).

When solving the linear relaxation of F2 by column and row generation, a more compact Master LP is obtained if every occurrence  $x_{ij}$  in (5.2)-(5.5) is replaced by its equivalent given by (5.10). The resulting LP will be referred to as the Dantzig-Wolfe Master (DWM):

$$(\text{DWM}) \quad \min \sum_{t=1}^P \left( \sum_{i \in V} \sum_{j \in V, j > i} c_{ij} a_{ij}^t \right) \lambda_t \quad (5.12)$$

$$\text{s.t.} \quad \sum_{t=1}^P \left( \sum_{i \in V, i < k} a_{ik}^t + \sum_{j \in V, j > k} a_{kj}^t \right) \lambda_t = 2 \quad \forall k \in V' \quad (5.13)$$

$$\sum_{t=1}^P \left( \sum_{j \in V'} a_{0j}^t \right) \lambda_t = 2v \quad (5.14)$$

$$\sum_{t=1}^P \left( \sum_{i \in S} \sum_{j \in \bar{S}, i < j} a_{ij}^t + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} a_{ij}^t \right) \lambda_t \geq 2e(S) \quad \forall S \subseteq V' \quad (5.15)$$

$$\sum_{t=1}^P \left( \sum_{i \in S} \sum_{j \in \bar{S}, i < j} a_{ij}^t + \sum_{i \in \bar{S}} \sum_{j \in S, i < j} a_{ij}^t \right) \lambda_t \geq 2q(S) \quad \forall S \subseteq V' \quad (5.16)$$

$$\sum_{t=1}^P a_{ij}^t \lambda_t \leq 1 \quad \forall \{i, j\} \in E, i > 0 \quad (5.17)$$

$$\lambda_t \geq 0 \quad (t = 1, \dots, P) \quad (5.18)$$

The reduced cost of a given  $\lambda$  variable is the sum of the reduced costs of the edges in the corresponding  $pd$ -route. Let  $\mu$ ,  $\nu$ ,  $\pi$ ,  $\omega$  and  $\sigma$  be the dual variables associated with constraints (5.13), (5.14), (5.15), (5.16) and (5.17) respectively. Define  $S : \bar{S}$  as the set composed by the edges with one end in  $S$  and the other end in  $\bar{S}$ . The reduced cost  $\bar{c}_{ij}$  of an edge is given by:

$$\bar{c}_{ij} = \begin{cases} c_{ij} - \mu_i - \mu_j - \sum_{S|S:\bar{S} \ni \{i,j\}} \pi_S - \sum_{S|S:\bar{S} \ni \{i,j\}} \omega_S - \sigma_{ij} & \forall \{i, j\} \in E, i > 0 \\ c_{ij} - \nu - \sum_{S|j \in S} \pi_S - \sum_{S|j \in S} \omega_S & \forall \{0, j\} \in E \end{cases}$$

In general, a cut of the form  $\sum_{t=1}^P (\sum_{\{i,j\} \in E} h_{ij} a_{ij}^t) \lambda_t \geq b$ , with dual variable  $\alpha$ , contributes with  $-h_{ij}\alpha$  to the value of  $\bar{c}_{ij}$ .

## 5.3 The Branch-cut-and-price algorithm

As already mentioned, the BCP algorithm presented in this work is mainly based on the one developed in [63] for the CVRP. Some elements such as branching rules, node selection strategy and strong branching scheme remained practically unchanged with respect to the original version. On the other hand, the column generation procedure had to be completely redesigned in order to deal with both pickup and delivery demands.

### 5.3.1 Pricing subproblem

The pricing subproblem can be seen as a shortest path problem with resource constraints, which is known to be strongly  $\mathcal{NP}$ -hard when cycles are forbidden (see Dror [52]). However, it can be solved in pseudo-polynomial time when  $pd$ -routes are considered. Therefore, the pricing subproblem consists of finding  $pd$ -routes with minimum reduced costs.

In this work a dynamic programming based algorithm is used to solve the pricing subproblem. The load resources are denoted by  $\mathcal{D}$  and  $\mathcal{P}$ , where the first indicates the total amount of load to be delivered, while the second indicates the cumulative amount of load that was collected. Resource  $\mathcal{D}$  is initialized with a value less or equal to  $Q$  and



resource  $\mathcal{P}$  is initialized with 0. Every time a customer  $j$  is visited the value of  $\mathcal{D}$  decreases by  $d_j$  and the value of  $\mathcal{P}$  increases by  $p_j$ . A state is here denoted by a triple  $(j, \mathcal{D}, \mathcal{P})$  and it represents that customer  $j$  has just been visited, with  $\mathcal{D}$  demand units to be still delivered and  $\mathcal{P}$  demand units already picked up.

Let  $\bar{c}(i, \mathcal{D}, \mathcal{P})$  be the least costly walk from customer  $i$  to the depot with respect to the edge reduced costs. The recursive expression of the dynamic programming can be expressed as follows.

$$\bar{c}(i, \mathcal{D}, \mathcal{P}) = \begin{cases} \min_j \{ \bar{c}_{ij} + \bar{c}(j, \mathcal{D} - d_j, \mathcal{P} + p_j) \mid \mathcal{D} - d_j \geq 0, (\mathcal{D} - d_j) + (\mathcal{P} + p_j) \leq Q \}, & \text{if } \mathcal{D} > 0 \\ \bar{c}_{i0}, & \text{if } \mathcal{D} = 0 \end{cases}$$

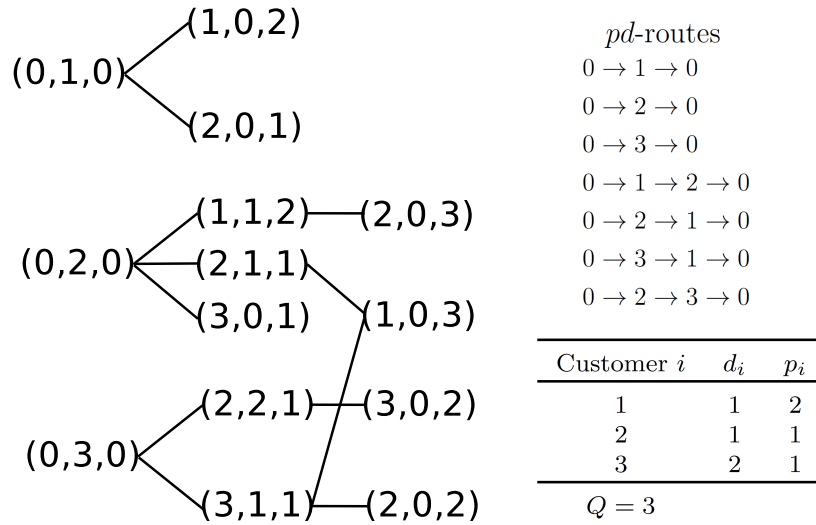
The minimum reduced cost of a  $pd$ -route is given by  $\min_{i, \mathcal{D}} \{ \bar{c}_{0i} + \bar{c}(i, \mathcal{D}, p_i) \}$ .

In classical dynamic programming recursion, the optimal cost of a given state is written as a function of other known optimal state costs. Conversely, in label correcting approaches, a state with known optimal cost is extended to others with unknown optimal cost. In view of the latter, for each state there is an associated label containing three elements, namely: the customer identifier  $j$ ; the minimum reduced cost, given by  $\underline{c}(j, \mathcal{D}, \mathcal{P})$ , of a walk that starts at the depot and ends at state  $(j, \mathcal{D}, \mathcal{P})$ ; and a pointer to a label representing the walk up to the previous customer.

The developed label correcting based dynamic programming algorithm starts by considering all feasible expansions from the depot given by states  $(0, \mathcal{D}, 0)$ ,  $\mathcal{D} \in \{1, 2, \dots, Q\}$ , to each customer  $j \in V'$ , thus generating the states  $(j, \mathcal{D} - d_j, p_j)$ . This procedure is repeated for every new generated state and a label is updated when  $\underline{c}(i, \mathcal{D}, \mathcal{P}) + \underline{c}_{ij} < \underline{c}(j, \mathcal{D} - d_j, \mathcal{P} + p_j)$ . Of course, a state can be only expanded when its optimal walk had been already computed. The algorithm terminates when feasible expansions are no longer possible. The optimal walk of each customer is then extended to the depot in order to obtain a  $pd$ -route. The columns associated to  $pd$ -routes with negative reduced costs are added to the master problem. The computational complexity of the dynamic programming approach is  $O(n^2 Q^2)$ .

Fig. 5.2 shows an example of all feasible states and  $pd$ -routes generated during the dynamic programming. It can be seen that the initial states are  $(0, 1, 0)$ ,  $(0, 2, 0)$  and  $(0, 3, 0)$ . A total of 7 possible  $pd$ -routes can be obtained but only those with negative reduced costs are considered as mentioned above. Notice that  $(1, 0, 3)$  is the only state that is generated from two other states, namely  $(3, 1, 1)$  and  $(2, 1, 1)$ . In this case, the optimal walk for this state is the one with minimum cost between walks  $0 \rightarrow 2 \rightarrow 1$  and  $0 \rightarrow 3 \rightarrow 1$ . The example assumes that the latter walk is the optimal one.

A crucial aspect regarding the performance of the proposed dynamic programming

Figure 5.2: Example of states and *pd*-routes generated during the dynamic programming

algorithm is the state elimination. A particular expansion can be avoided if it is known in advance that the optimal walk of the state generated from such expansion will never be in a *pd*-route with negative reduced cost. In order to apply this type of strategy one must compute a valid LB for every possible state. In view of this, a relaxed version of the pricing subproblem was formulated by only considering the delivery demands. This relaxed problem is also solved using dynamic programming to fill a matrix where each entry  $M(j, \mathcal{D})$  corresponds to the least costly walk given by  $\underline{c}(M(j, \mathcal{D}))$  starting from the depot and ending at customer  $j$  using total delivery demand exactly  $\mathcal{D}$ . Therefore one can avoid all expansions from a state  $(i, \mathcal{D}, \mathcal{P})$  if  $\underline{c}(i, \mathcal{D}, \mathcal{P}) + \underline{c}(M(i, \mathcal{D} + d_i)) \geq 0$ . Furthermore, one can also avoid an expansion from state  $(i, \mathcal{D}, \mathcal{P})$  to state  $(j, \mathcal{D} - d_j, \mathcal{P} + p_j)$  if  $\underline{c}(i, \mathcal{D}, \mathcal{P}) + \underline{c}_{ij} + \underline{c}(M(j, \mathcal{D})) \geq 0$ .

The full execution of the dynamic programming algorithm can be very time consuming. A common speed-up strategy is to employ heuristic acceleration with a view of finding columns (*pd*-routes) with negative reduced costs more faster. The full dynamic programming procedure is then only applied when the heuristics fail to obtain *pd*-routes with negative reduced costs. The proposed heuristic strategy consists of a combination between two techniques: scaling and sparsification. Let  $g > 1$  be the scaling factor. The scaling consists of modifying the customers original delivery and pickup demands to  $d'_i(g) = \lceil d_i/g \rceil, i \in V'$  and  $p'_i(g) = \lceil p_i/g \rceil, i \in V'$  respectively, and the original vehicle capacity to  $Q' = \lfloor Q/g \rfloor$ . Hence, the computational complexity will be in function of the capacity  $Q' \leq Q$ . The sparsification limits the number of possible expansions from a state  $(i, \mathcal{D}, \mathcal{P})$  to those states associated with the vertices that are in the neighborhood of  $i$ . These neighbors are computed only once in the beginning of the algorithm using the Minimum Spanning Tree based approach presented in [63].

Finally, in order to strengthen the formulation, *pd*-routes with 2-cycle are forbidden. This is done by simply keeping the label of the second least costly walk of each state [32].

### 5.3.2 Cut generation

The rounded capacity cuts (5.15)-(5.16) and the bound cuts (5.17) are separated in every node of the branch-and-bound tree. In addition to these cuts, the strengthened comb cuts are also applied, but only at the root node. The capacity and comb cuts are separated using the CVRPSEP package [114].

## 5.4 Computational experiments

The BCP algorithm was coded in C++ and executed in an Intel Core 2 Quad with 2.4 GHz and 4 GB of RAM running under Linux 64 bits. Linear Programs were solved using CPLEX 11.2. Only a single thread was used in the experiments. The same set of VRPSPD/VRPMPD instances used in the previous two chapters were considered here.

### 5.4.1 Data preprocessing

The data of the VRPSPD instances proposed in [48, 149] contain fractional values with 6 decimal places. This becomes an issue to the dynamic programming algorithm presented in Section 5.3.1, since the conversion of demands and capacities to integer values results in very large values of  $Q$ . In order to avoid this, the capacity is divided by a scaling factor  $r$  such that the new capacity  $Q/r$  is integral and not greater than 250. The demands are also divided by  $r$  and rounded down. In addition, for each delivery demand that is zero, its value is set one and the vehicle capacity is incremented by one unit. Notice that one must also ensure that the total increase on the vehicle capacity does not exceed the maximum number of customers with incremented delivery demands that could fit into a single route (considering the pickup demands). These adaptations correspond to a (usually mild) relaxation of the pricing subproblem, allowing some routes that violate the true capacities. Since the rounded capacity cuts are applied over the original instance, one can ensure that the original vehicle capacity will be never violated in the depot. Nevertheless, these cuts are still not sufficient to prevent the algorithm of generating infeasible integer solutions, i.e., those containing at least one vehicle whose capacity is violated in the middle of the route. The cuts that ensure that the vehicle capacity is not exceeded in the middle of a route, i.e. those given by (5.6) are separated in a lazy fashion. Very few such cuts are needed in practice.

### 5.4.2 Results

In the following tables, **Instance** is the name of the instance, **Customers** is the number of customers, **#vehicles** is the number of vehicles in the best known solution or a LB on the number of vehicles, **Root LB** indicates the root LB, **Root Time** is the CPU time in seconds spent at the root node, **Tree size** is the number of nodes opened, **Total time** is the total CPU time in seconds of the BCP procedure, **Prev. LB** is the best known LB obtained considering Chapters 3 and 4, **LB** is the LB determined by BCP, **UB** is the upper bound reported in [157] and [64, 161] for the VRPSPD and VRPMPD, respectively. Previously known optimal solutions and improved LBs are highlighted in boldface and new optimal solutions are underlined.

The LBs obtained at the root node are reported for all VRPSPD/VRPMPD. For those instances with a relatively large number of customers per vehicle a time limit of 2 hours was imposed (CMT11X, CMT11Y, r201, rc101, r2\_2\_1, rc2\_2\_1 and CMT11H and CMT4T). Nevertheless, this limit was exceeded in some instances because it was decided not to interrupt the execution of the algorithm during a column generation procedure. A comparison is performed with the root relaxations obtained by the BC presented in Chapter 4. The full BCP algorithm was only executed in the instances where the BC has an inferior performance.

#### 5.4.2.1 VRPSPD

Table 4.1 shows the results found by the BCP algorithm on the set of instances proposed in [48]. It is noteworthy to mention that the optimal solutions of all instances of this set were found by the BC presented in Chapter 4. From Table 5.1, it can be observed that the BCP algorithm had a considerable better performance in terms of both root LB and computational time in those cases where the average number of customers per vehicle (given by  $n/v$ ) is smaller, i.e. instances SCA8-0, SCA8-1,...,SCA8-9, CON8-0, CON8-1,...,CON8-9. The BC and BCP average gaps between the root LBs and the optimal solutions in such subset of 20 instances were 3.24% and 1.28%, respectively. Moreover, the average computational time of the complete execution of BCP in these instances was 197 seconds, whereas for BC this value was 2883 seconds.

Table 5.2 illustrates the results found in the instances suggested in [149]. In these set of instances, BC appears to be more suitable, in terms of overall performance, when compared to the BCP. Yet, the best known LBs of the instances CMT5X and CMT5Y were improved at the root node when solving such problems using the BCP algorithm. For the remaining the instances there is no advantage in applying the BCP instead of a pure BC. Moreover, closing the gap between the UB and the LB of instances CMT2X and CMT2Y, both with 75 customers and 6 vehicles, still remains a challenge.

Table 5.3 presents the results obtained in the set of benchmark instances proposed

in [121]. As it happened in the set of instances of Dethloff [48], the BCP algorithm outperformed the BC on those instances where the average number of customers per vehicle is relatively small. Three instances containing 100 customers were solved to optimality for the first time, namely r101, c101 and rc101. As a result, all the six 100-customer instances of this set are now solved. In addition, the LB of the instances r1\_2\_1, c1\_2\_1, c2\_2\_1 and rc1\_2\_1 were dramatically improved. On the other hand, the BCP algorithm failed completely to solve the linear relaxation of the instances r2\_2\_1 and rc2\_2\_1 (where they are about 40 customers per vehicle) in the time limit specified. These instances were solved to optimality by the pure BC.

Table 5.1: Results obtained on the instances of Dethloff [48]

| Instance/<br>Customers/<br>#vehicles | BC         |                  | BCP        |                  |              |                   |         | Prev.<br>LB | UB             |
|--------------------------------------|------------|------------------|------------|------------------|--------------|-------------------|---------|-------------|----------------|
|                                      | Root<br>LB | Root<br>Time (s) | Root<br>LB | Root<br>Time (s) | Tree<br>Size | Total<br>Time (s) | LB      |             |                |
| SCA3-0/50/4                          | 619.21     | 5.18             | 620.87     | 149              | -            | -                 | -       | 635.62      | <b>635.62</b>  |
| SCA3-1/50/4                          | 682.34     | 1.90             | 682.75     | 27.2             | -            | -                 | -       | 697.84      | <b>697.84</b>  |
| SCA3-2/50/4                          | 659.34     | 0.60             | 659.34     | 64.8             | -            | -                 | -       | 659.34      | <b>659.34</b>  |
| SCA3-3/50/4                          | 667.56     | 1.76             | 667.56     | 102              | -            | -                 | -       | 680.04      | <b>680.04</b>  |
| SCA3-4/50/4                          | 676.56     | 6.75             | 675.97     | 51.6             | -            | -                 | -       | 690.50      | <b>690.50</b>  |
| SCA3-5/50/4                          | 647.90     | 1.16             | 649.56     | 48.5             | -            | -                 | -       | 659.91      | <b>659.91</b>  |
| SCA3-6/50/4                          | 625.60     | 1.29             | 625.16     | 67.0             | -            | -                 | -       | 651.09      | <b>651.09</b>  |
| SCA3-7/50/4                          | 654.97     | 3.70             | 655.99     | 72.8             | -            | -                 | -       | 659.17      | <b>659.17</b>  |
| SCA3-8/50/4                          | 688.21     | 1.97             | 690.29     | 161              | -            | -                 | -       | 719.48      | <b>719.48</b>  |
| SCA3-9/50/4                          | 671.07     | 2.04             | 670.56     | 85.2             | -            | -                 | -       | 681.00      | <b>681.00</b>  |
| SCA8-0/50/9                          | 926.03     | 18.8             | 946.50     | 2.91             | 63           | 76.2              | 961.50  | 961.50      | <b>961.50</b>  |
| SCA8-1/50/9                          | 1002.29    | 15.5             | 1028.22    | 2.71             | 369          | 437               | 1049.65 | 1049.65     | <b>1049.65</b> |
| SCA8-2/50/9                          | 1013.37    | 19.8             | 1026.07    | 2.90             | 307          | 270               | 1039.64 | 1039.64     | <b>1039.64</b> |
| SCA8-3/50/9                          | 956.10     | 11.9             | 975.45     | 1.93             | 11           | 10.6              | 983.34  | 983.34      | <b>983.34</b>  |
| SCA8-4/50/9                          | 1027.24    | 25.9             | 1052.79    | 1.11             | 51           | 34.4              | 1065.49 | 1065.49     | <b>1065.49</b> |
| SCA8-5/50/9                          | 998.05     | 13.7             | 1017.77    | 2.75             | 107          | 115               | 1027.08 | 1027.08     | <b>1027.08</b> |
| SCA8-6/50/9                          | 948.26     | 12.8             | 964.03     | 3.56             | 19           | 20.8              | 971.82  | 971.82      | <b>971.82</b>  |
| SCA8-7/50/9                          | 1018.20    | 16.4             | 1028.30    | 2.01             | 547          | 601               | 1051.28 | 1051.28     | <b>1051.28</b> |
| SCA8-8/50/9                          | 1038.86    | 6.15             | 1057.44    | 1.85             | 35           | 53.7              | 1071.18 | 1071.18     | <b>1071.18</b> |
| SCA8-9/50/9                          | 1019.25    | 12.0             | 1037.29    | 1.51             | 245          | 341               | 1060.50 | 1060.50     | <b>1060.50</b> |
| CON3-0/50/4                          | 611.30     | 2.58             | 612.16     | 66.9             | -            | -                 | -       | 616.52      | <b>616.52</b>  |
| CON3-1/50/4                          | 546.65     | 8.26             | 546.38     | 104              | -            | -                 | -       | 554.47      | <b>554.47</b>  |
| CON3-2/50/4                          | 505.06     | 3.94             | 507.43     | 71.5             | -            | -                 | -       | 518.01      | <b>518.01</b>  |
| CON3-3/50/4                          | 584.28     | 2.05             | 585.16     | 167              | -            | -                 | -       | 591.19      | <b>591.19</b>  |
| CON3-4/50/4                          | 577.52     | 1.04             | 578.09     | 48.4             | -            | -                 | -       | 588.79      | <b>588.79</b>  |
| CON3-5/50/4                          | 552.91     | 3.22             | 555.76     | 114              | -            | -                 | -       | 563.70      | <b>563.70</b>  |
| CON3-6/50/4                          | 486.98     | 5.22             | 487.83     | 67.9             | -            | -                 | -       | 499.05      | <b>499.05</b>  |
| CON3-7/50/4                          | 561.62     | 0.97             | 563.49     | 58.8             | -            | -                 | -       | 576.48      | <b>576.48</b>  |
| CON3-8/50/4                          | 514.84     | 5.03             | 516.45     | 50.6             | -            | -                 | -       | 523.05      | <b>523.05</b>  |
| CON3-9/50/4                          | 564.78     | 2.51             | 567.69     | 59.0             | -            | -                 | -       | 578.25      | <b>578.25</b>  |
| CON8-0/50/9                          | 830.03     | 41.5             | 850.16     | 2.82             | 53           | 45.2              | 857.17  | 857.17      | <b>857.17</b>  |
| CON8-1/50/9                          | 722.38     | 27.7             | 732.24     | 0.98             | 151          | 85.9              | 740.85  | 740.85      | <b>740.85</b>  |
| CON8-2/50/9                          | 685.66     | 39.2             | 700.78     | 2.94             | 249          | 576               | 712.89  | 712.89      | <b>712.89</b>  |
| CON8-3/50/10                         | 787.84     | 39.3             | 806.23     | 1.73             | 95           | 85.2              | 811.07  | 811.07      | <b>811.07</b>  |
| CON8-4/50/9                          | 751.95     | 21.2             | 764.50     | 1.16             | 121          | 107               | 772.25  | 772.25      | <b>772.25</b>  |
| CON8-5/50/9                          | 729.92     | 15.1             | 746.25     | 1.69             | 273          | 358               | 754.88  | 754.88      | <b>754.88</b>  |
| CON8-6/50/9                          | 648.64     | 18.7             | 666.09     | 1.42             | 151          | 202               | 678.92  | 678.92      | <b>678.92</b>  |
| CON8-7/50/9                          | 793.50     | 15.5             | 804.51     | 1.35             | 25           | 35.7              | 811.96  | 811.96      | <b>811.96</b>  |
| CON8-8/50/9                          | 744.52     | 26.8             | 761.79     | 2.93             | 31           | 66.2              | 767.53  | 767.53      | <b>767.53</b>  |
| CON8-9/50/9                          | 773.65     | 45.0             | 798.12     | 1.61             | 419          | 422               | 809.00  | 809.00      | <b>809.00</b>  |

#### 5.4.2.2 VRPMPD

The results found in the VRPMPD instances generated in [149] are depicted in Table 5.4. The best known upper bounds for VRPMPD instances that are not proven optimal were

taken from [64]. However, the number of vehicles was not specified in the referred work. The LB on the number of vehicles is reported in such cases. From Table 5.4 it is possible to verify that the BCP algorithm had an overall performance similar to the one observed in the VRPSPD instances of Salhi and Nagy [149] (see Table 4.2). The optimality of the instance CMT2T was proved for the first time and LBs of the instances CMT4T, CMT5H, CMT5Q and CMT5T were improved.

Table 5.2: Results obtained on the VRPSPD instances of Salhi and Nagy [149]

| Instance/<br>Customers/<br>#vehicles | BC         |                  | BCP           |                  |              |                   |    | Prev.<br>LB | UB            |
|--------------------------------------|------------|------------------|---------------|------------------|--------------|-------------------|----|-------------|---------------|
|                                      | Root<br>LB | Root<br>Time (s) | Root<br>LB    | Root<br>Time (s) | Tree<br>Size | Total<br>Time (s) | LB |             |               |
| CMT1X/50/3                           | 460.73     | 5.56             | 461.90        | 96.0             | -            | -                 | -  | 466.77      | <b>466.77</b> |
| CMT1Y/50/3                           | 460.69     | 15.5             | 460.86        | 238              | -            | -                 | -  | 466.77      | <b>466.77</b> |
| CMT2X/75/6                           | 658.38     | 100              | 661.11        | 94.2             | -            | -                 | -  | 666.57      | 684.21        |
| CMT2Y/75/6                           | 658.12     | 200              | 659.76        | 308              | -            | -                 | -  | 666.69      | 684.21        |
| CMT3X/100/5                          | 711.77     | 79.7             | 711.70        | 1404             | -            | -                 | -  | 721.27      | <b>721.27</b> |
| CMT3Y/100/5                          | 711.89     | 140              | 711.49        | 4519             | -            | -                 | -  | 721.27      | <b>721.27</b> |
| CMT12X/100/5                         | 635.52     | 50.3             | 638.72        | 3130             | -            | -                 | -  | 643.76      | 662.22        |
| CMT12Y/100/5                         | 635.45     | 65.1             | 636.29        | 3608             | -            | -                 | -  | 644.10      | 662.22        |
| CMT11X/120/4                         | 793.11     | 892              | 766.22        | 9570             | -            | -                 | -  | 799.67      | 833.92        |
| CMT11Y/120/4                         | 793.54     | 1098             | 765.47        | 9421             | -            | -                 | -  | 799.02      | 833.92        |
| CMT4X/150/7                          | 826.74     | 1950             | 829.58        | 11433            | -            | -                 | -  | 831.18      | 852.46        |
| CMT4Y/150/7                          | 826.34     | 2568             | 827.50        | 791912           | -            | -                 | -  | 831.65      | 852.46        |
| CMT5X/199/10                         | 971.07     | 7202             | <b>988.28</b> | 66063            | -            | -                 | -  | 971.07      | 1029.25       |
| CMT5Y/199/10                         | 937.66     | 7248             | <b>980.63</b> | 260737           | -            | -                 | -  | 953.56      | 1029.25       |

Table 5.3: Results obtained on the instances of Montané and Galvão [121]

| Instance/<br>Customers/<br>#vehicles | BC         |                  | BCP            |                  |              |                    |                | Prev.<br>LB | UB             |
|--------------------------------------|------------|------------------|----------------|------------------|--------------|--------------------|----------------|-------------|----------------|
|                                      | Root<br>LB | Root<br>Time (s) | Root<br>LB     | Root<br>Time (s) | Tree<br>Size | Total<br>Time (s)  | LB             |             |                |
| r101/100/12                          | 972.58     | 1282             | 996.18         | 208              | 23465        | $1.06 \times 10^6$ | <b>1009.95</b> | 978.28      | <b>1009.95</b> |
| r201/100/3                           | 664.80     | 42.9             | 661.95         | 9968             | -            | -                  | -              | 666.20      | <b>666.20</b>  |
| c101/100/16                          | 1194.69    | 722              | 1208.99        | 3.99             | 31631        | 31042              | <b>1220.18</b> | 1202.59     | <b>1220.18</b> |
| c201/100/5                           | 657.97     | 24.6             | 659.11         | 17086            | -            | -                  | -              | 662.07      | <b>662.07</b>  |
| rc101/100/10                         | 1028.06    | 1067             | 1049.84        | 78.5             | 653          | 17491              | <b>1059.32</b> | 1041.06     | <b>1059.32</b> |
| rc201/100/3                          | 671.84     | 6.37             | 668.84         | 15021            | -            | -                  | -              | 672.92      | <b>672.92</b>  |
| r1_2_1/200/23                        | 2681.05    | 7198             | <b>3273.27</b> | 1094             | -            | -                  | -              | 3084.97     | 3360.02        |
| r2_2_1/200/5                         | 1656.62    | 4477             | -              | -                | -            | -                  | -              | 1665.58     | <b>1665.58</b> |
| c1_2_1/200/28                        | 2857.45    | 7196             | <b>3609.16</b> | 78.4             | -            | -                  | -              | 3475.03     | 3629.89        |
| c2_2_1/200/9                         | 1638.99    | 7197             | <b>1704.00</b> | 97487            | -            | -                  | -              | 1647.83     | 1726.59        |
| rc1_2_1/200/23                       | 2656.78    | 7196             | <b>3241.98</b> | 888              | -            | -                  | -              | 3093.30     | 3306.00        |
| rc2_2_1/200/5                        | 1551.54    | 1932             | -              | -                | -            | -                  | -              | 1560.00     | <b>1560.00</b> |

## 5.5 Concluding remarks

This chapter presented a BCP approach for the VRPSPD that is also capable of solving the VRPMPD. The proposed algorithm is an adaptation of the one developed in [63] for solving the CVRP. The fundamental difference of this BCP relies on the column generation procedure that was redesigned to deal with the delivery and pickup demands simultaneously. The BCP algorithm was tested in well-known benchmark instances and it was found capable of proving the optimality of 4 problems for the first time, where three

of them are VRPSPD instances containing 100 customers and one of them corresponds to a VRPMPD instance containing 75 customers. Also, the best known LBs of other 10 instances involving 150-200 customers were improved. Moreover, the BCP clearly outperformed previous methods in a subset of instances proposed in [48] and [121]. The main characteristic of such instances is the fact of having a small average number of customers per vehicle, which is known to be favorable to column generation based techniques.

Table 5.4: Results obtained on the VRPMPD instances of Salhi and Nagy [149]

| Instance/<br>Customers/<br>#vehicles | BC         |                  | BCP            |                  |              |                   |               | Prev.<br>LB | UB            |
|--------------------------------------|------------|------------------|----------------|------------------|--------------|-------------------|---------------|-------------|---------------|
|                                      | Root<br>LB | Root<br>Time (s) | Root<br>LB     | Root<br>Time (s) | Tree<br>Size | Total<br>Time (s) | LB            |             |               |
| CMT1H/50/3                           | 460.10     | 7.14             | 461.32         | 64.7             | -            | -                 | -             | 465.02      | <b>465.02</b> |
| CMT1Q/50/4                           | 488.15     | 5.65             | 488.26         | 31.2             | -            | -                 | -             | 489.74      | <b>489.74</b> |
| CMT1T/50/5                           | 512.61     | 4.98             | 515.20         | 5.81             | -            | -                 | -             | 520.06      | <b>520.06</b> |
| CMT2H/75/6                           | 643.28     | 101              | 648.63         | 174              | -            | -                 | -             | 653.79      | 662.63        |
| CMT2Q/75/8                           | 707.66     | 100              | 710.00         | 14.0             | -            | -                 | -             | 717.36      | 732.76        |
| CMT2T/75/9                           | 759.92     | 151              | 767.77         | 7.31             | 14627        | 109348            | <b>782.77</b> | 770.35      | <b>782.77</b> |
| CMT3H/100/3                          | 691.32     | 58.0             | 691.42         | 3376             | -            | -                 | -             | 700.94      | <b>700.94</b> |
| CMT3Q/100/6                          | 744.50     | 95.5             | 744.84         | 854              | -            | -                 | -             | 747.15      | <b>747.15</b> |
| CMT3T/100/5                          | 784.13     | 342              | 784.75         | 346              | -            | -                 | -             | 798.07      | <b>798.07</b> |
| CMT12H/100/5                         | 623.00     | 40.3             | 623.33         | 881              | -            | -                 | -             | 629.37      | <b>629.37</b> |
| CMT12Q/100/7                         | 721.81     | 83.9             | 722.35         | 948              | -            | -                 | -             | 729.25      | <b>729.25</b> |
| CMT12T/100/9                         | 787.27     | 37.7             | 787.52         | 52.7             | -            | -                 | -             | 787.52      | <b>787.52</b> |
| CMT11H/120/4                         | 801.24     | 353              | 776.36         | 7680             | -            | -                 | -             | 806.46      | 820.35        |
| CMT11Q/120/6                         | 928.28     | 653              | 899.45         | 7831             | -            | -                 | -             | 939.36      | <b>939.36</b> |
| CMT11T/120/7                         | 984.81     | 359              | 985.75         | 2884             | -            | -                 | -             | 989.32      | 998.80        |
| CMT4H/150/6                          | 799.19     | 1122             | 795.22         | 8465             | -            | -                 | -             | 804.10      | 831.39        |
| CMT4Q/150/9                          | 892.99     | 2920             | 895.15         | 13090            | -            | -                 | -             | 898.28      | 913.93        |
| CMT4T/150/11                         | 953.41     | 4246             | <b>960.59</b>  | 228              | -            | -                 | -             | 956.54      | 990.39        |
| CMT5H/199/9                          | 931.02     | 7209             | <b>948.46</b>  | 276318           | -            | -                 | -             | 931.02      | 992.37        |
| CMT5Q/199/12                         | 1056.74    | 7206             | <b>1069.91</b> | 97259            | -            | -                 | -             | 1056.74     | 1134.72       |
| CMT5T/199/15                         | 1145.61    | 7192             | <b>1175.62</b> | 755              | -            | -                 | -             | 1145.63     | 1232.08       |

# Chapter 6

## An ILS Heuristic for General Vehicle Routing Problems

This chapter presents a general heuristic algorithm for solving different VRPs. The proposed approach is mostly based on the ILS framework. Before describing the solution method, a brief outline of this metaheuristic is provided. The contents of this chapter were partially published in [133].

### 6.1 A brief overview of the ILS metaheuristic

Consider a local optimum solution that has been found by a local search algorithm. Instead of restarting the same procedure from a completely new solution, the ILS metaheuristic applies a local search repeatedly to a set of solutions obtained by perturbing previously visited local optimal solutions. As stated in Chapter 1, the essential idea of ILS resides in the fact that it focuses on a smaller subset, instead of considering the total space of solutions. This subset is defined by the local optimums of a given optimization procedure [112]. To implement an ILS algorithm, four procedures should be specified: (i) **GenerateInitialSolution**, where an initial solution is constructed; (ii) **LocalSearch**, which improves the solution initially obtained; (iii) **Perturb**, where a new starter point is generated through a perturbation of a solution found in the **LocalSearch**; (iv) **AcceptanceCriterion**, that determines from which solution the search should continue. Alg. 6.1 describes how these components are combined to build the ILS framework.

The modification performed in the perturbation phase is used in order to try escape from a current locally optimal solution. Frequently, the move is randomly chosen within a larger neighborhood than the case utilized in the local search, or a move that the local search cannot undo in just one step. In principle, any local search method can be used. However, its performance, in terms of the solution quality and computational effort, strongly depends on the chosen procedure. The acceptance criterion is used to decide the next solution that should be perturbed. The choice of this criterion is important



**Algorithm 6.1** ILS

---

```

1: Procedure ILS
2:  $s_0 \leftarrow \text{GenerateInitialSolution}$ ;
3:  $s^* \leftarrow \text{LocalSearch}(s_0)$ ;
4: while Stopping criterion is not met do
5:    $s' \leftarrow \text{Perturb}(s^*, \text{history})$ ;
6:    $s^{*'} \leftarrow \text{LocalSearch}(s')$ ;
7:    $s^* \leftarrow \text{AcceptanceCriterion}(s^*, s^{*'}, \text{history})$ ;
8: end ILS;

```

---

because it controls the balance between intensification and diversification. The search history is employed for deciding if some found previously local optimum solution should be chosen. The ILS procedure can lead to good samples of the search space as long as the perturbations are not too large or too small. If it is small, not many new solutions will be explored, while if it is too large, it will adopt almost randomly starting points.

## 6.2 The ILS-RVND heuristic

This section describes the proposed heuristic algorithm, called ILS-RVND, whose main steps are summarized in Alg. 6.2. Let  $v$  be the number of vehicles (or routes). If its value is not specified in advance then an estimation is performed (lines 3-5). The multi-start heuristic executes *MaxIter* iterations (lines 6-19), where at each iteration a solution is generated by means of a constructive procedure (line 7). The main ILS loop (lines 10-16) seeks to improve this initial solution using a RVND procedure (line 11) in the local search phase combined with a set of perturbation mechanisms (line 15). Notice that the perturbation is always performed on the best current solution ( $s'$ ) of a given iteration (acceptance criterion). The parameter *MaxIterILS* represents the maximum number of consecutive perturbations allowed without improvements.

The next subsections provide a detailed explanation of the main components of the ILS-RVND heuristic.

### 6.2.1 Estimating the number of vehicles

Sometimes the number of available vehicles is not specified by the instance. Since the constructive method depends on this information, a procedure to estimate the initial number of vehicles had to be developed, as can be observed in Alg. 6.3. Let CL denote the Candidate List composed by customers that have not been added to the partial solution. At first, just one vehicle is considered (line 2). A customer  $k \in \text{CL}$  is chosen at random and inserted into the single route (lines 3-5). Next, while the CL is not empty, one of the insertion criteria described in Section 6.2.2 is selected at random to evaluate the inclusion of a customer  $k \in \text{CL}$  in each position of the route  $v$  and the least cost insertion is considered (lines 6-14). If the vehicle  $v$  is full then a new vehicle is added (lines 11-12).

**Algorithm 6.2** ILS-RVND

---

```

1: Procedure ILS-RVND(MaxIter, MaxIterILS, v)
2: LoadData( );
3: if v was not defined then
4:   v  $\leftarrow$  EstimateTheNumberOfVehicles(seed);
5:  $f^* \leftarrow \infty$ ;
6: for  $i:=1, \dots, \text{MaxIter}$  do
7:   s  $\leftarrow$  GenerateInitialSolution(v, MaxIter, seed);
8:    $s' \leftarrow s$ ;
9:   iterILS  $\leftarrow 0$ ;
10:  while iterILS  $\leq$  MaxIterILS do
11:    s  $\leftarrow$  RVND(s);
12:    if  $f(s) < f(s')$  {or v of s < v of s' (considered only when v must be minimized)} then
13:       $s' \leftarrow s$ ;
14:      iterILS  $\leftarrow 0$ ;
15:      s  $\leftarrow$  Perturb(s', seed);
16:      iterILS  $\leftarrow$  iterILS + 1;
17:      if  $f(s') < f^*$  then
18:         $s^* \leftarrow s'$ ;
19:         $f^* \leftarrow f(s')$ ;
20:  return  $s^*$ ;
21: end ILS-RVND.

```

---

For the HFVRP with limited fleet and for the VRPs with multiple depots, the maximum number of vehicles of each type/depot is initially considered. In the case of HFVRP, these values are computed considering the sum of the customers demands and the capacity of each vehicle. For the HFVRP with unlimited fleet, one vehicle of each type is first considered. As for the OVRP, use is made of a lower bound on the number of vehicles ( $v_{min}$ ) which is computed dividing the sum of the customers demands by the capacity of the vehicle.

### 6.2.2 Constructive procedure

In essence any classical VRP construction method can be used to generate initial solutions. However, the main interest is to build diversified initial solutions using relatively simple schemes. The developed procedure makes use of two insertion criteria, namely the Modified Cheapest Feasible Insertion Criterion (MCFIC) and the Nearest Feasible Insertion Criterion (NFIC). Also, two insertion strategies were employed, specifically the Sequential Insertion Strategy (SIS) and the Parallel Insertion Strategy (PIS).

The pseudocode of the constructive procedure is presented in Alg. 6.4. Each route is filled with a seed customer *k*, randomly selected from the CL (lines 5-7). An insertion criterion and an insertion strategy is chosen at random (lines 8-9). An initial solution is generated using the selected combination of criterion and strategy (lines 10-13). If an infeasible solution has been found the procedure restarts from line 3. In this case, an infeasible solution necessarily corresponds to an incomplete solution, which means that the selected insertion procedure was not capable of including all customers using *v* vehicles.

**Algorithm 6.3** EstimateTheNumberOfVehicles

---

```

1: Procedure EstimateTheNumberOfVehicles(seed)
2:  $v \leftarrow 1$ ;
3: Initialize the CL
4:  $s^v \leftarrow k \in \text{CL}$  selected at random;
5: Update CL;
6: while CL  $\neq \emptyset$  do
7:   Evaluate the cost  $g(k)$  for each  $k \in \text{CL}$  using an insertion criterion selected at random;
8:    $g^{\min} \leftarrow \min\{g(k) | k \in \text{CL}\}$ ;
9:    $n \leftarrow$  client  $e$  associated to  $g^{\min}$ ;
10:   $s^v \leftarrow s^v \cup \{n\}$ ;
11:  if vehicle  $v$  is full then
12:     $v \leftarrow v + 1$ ;
13:  else
14:    Update CL;
15: return  $v$ ;
16: end EstimateTheNumberOfVehicles.

```

---

Moreover, if the number of vehicles was not specified by the instance and the number of consecutive trials for generating a feasible initial solution (*ConsecutiveTrials*) is equal to *MaxIter* then an extra vehicle is added (lines 14-20). If a feasible solution is found then the procedure ends (lines 21-25). Notice that if the fleet is heterogeneous and unlimited then an empty route associated to each type of vehicle is added to the constructed solution  $s$  (lines 22-23). These empty routes are necessary to allow a possible fleet resizing during the local search phase.

**6.2.2.1 Insertion criteria**

The cost of inserting an unrouted customer  $k \in \text{CL}$  in a given route using MCFIC is expressed in Eq. 6.1, where function  $g(k)$  represents the insertion cost. The value of  $g(k)$  is computed by the sum of two parcels. The first computes the insertion cost of the client  $k$  between every pair of adjacent customers  $i$  and  $j$  while the second corresponds to a surcharge used to avoid late insertions of customers located far away from the depot. The cost back and forth from the depot is weighted by a factor  $\gamma$ . This greedy insertion criterion was inspired in [48].

$$g(k) = (c_{ik} + c_{kj} - c_{ij}) - \gamma(c_{0k} + c_{k0}) \quad (6.1)$$

NFIC directly computes the distance between a customer  $k \in \text{CL}$  and every customer  $i$  that has been already included into the partial solution, as can be observed by function  $g$  in Eq. 6.2. It is assumed that the insertion of  $k$  is always performed after  $i$ .

$$g(k) = c_{ik} \quad (6.2)$$

In both criteria, the insertion associated with the least-cost is done, i.e.  $\min\{g(k) | k \in \text{CL}\}$ .

**Algorithm 6.4** GenerateInitialSolution

---

```

1: Procedure GenerateInitialSolution( $v$ ,  $MaxIter$ , seed)
2:  $ConsecutiveTrials \leftarrow 0$ ;
3: Initialize the CL;
4: Let  $s = \{s^1, \dots, s^{v-1}\}$  be the set composed by  $v - 1$  empty routes;
5: for  $v' = 1 \dots v - 1$  do
6:    $s^{v'} \leftarrow k \in CL$  selected at random;
7:   Update CL;  $\{CL \leftarrow CL - \{k\}\}$ 
8:   InsertionCriterion  $\leftarrow$  MCFIC or NFIC (chosen at random);
9:   InsertionStrategy  $\leftarrow$  SIS or PIS (chosen at random);
10: if InsertionStrategy = SIS then
11:    $s \leftarrow$  SequentialInsertion( $s$ ,  $v$ , CL, InsertionCriterion);
12: else
13:    $s \leftarrow$  ParallelInsertion( $s$ ,  $v$ , CL, InsertionCriterion);
14: if  $s$  is infeasible then
15:   if  $v$  was not defined then
16:      $ConsecutiveTrials \leftarrow ConsecutiveTrials + 1$ ;
17:     if  $ConsecutiveTrials = MaxIter$  then
18:        $v \leftarrow v + 1$ ;
19:        $ConsecutiveTrials \leftarrow 0$ ;
20:   Go to line 3;
21: else
22:   if the vehicle fleet is heterogeneous and unlimited then
23:     Add an empty route associated to each type of vehicle to  $s$ 
24:   return  $s$ ;
25: end GenerateInitialSolution.

```

---

**6.2.2.2 Insertion strategies**

In SIS, only a single route is considered for insertion at each iteration. The pseudocode of SIS is presented in Alg. 6.5. If the insertion criterion corresponds to MCFIC then a value of  $\gamma$  is chosen at random within the set  $\{0.00, 0.05, 0.10, \dots, 1.65, 1.70\}$  (lines 3-4). This set was defined in [157] after some preliminary experiments. While the CL is not empty and there is at least one customer  $k \in CL$  that can be added to the current partial solution without violating any constraint (lines 7-16), each route is filled with a customer selected using the correspondent insertion criterion (lines 8-15). If the solution  $s$  is incomplete and there are multiple depots and route duration constraints, a local search is performed with a view of reducing the duration of the routes, thus may allowing further insertions (lines 17-20). The procedure than restarts from line 7 (line 21). Moreover, if the vehicle fleet is heterogeneous and unlimited and the solution  $s$  is incomplete then a new vehicle, chosen at random from the available types, is added and the procedure restarts from line 7 (lines 21-24).

PIS differs from SIS because all routes are considered while evaluating the least-cost insertion. Alg. 6.6 illustrates the pseudocode of PIS. While the CL is not empty and there is at least one customer  $k \in CL$  that can be included in  $s$  (lines 6-12), the insertions are evaluated using the selected insertion criterion and the customer associated with the least-cost insertion is then included in the correspondent route  $v$  (lines 7-11). The rest of

the code is likewise SIS.

---

**Algorithm 6.5** SequentialInsertion
 

---

```

1: Procedure SequentialInsertion( $s, v, CL, \text{InsertionCriterion}$ )
2:  $\gamma \leftarrow 0$ ;
3: customerInserted  $\leftarrow$  false;
4: if InsertionCriterion = MCFIC then
5:    $\gamma \leftarrow$  random value within a given set;
6:  $v_0 \leftarrow 1$ ;
7: while  $CL \neq \emptyset$  and at least one customer  $k \in CL$  can be added to  $s$  do
8:   for  $v' = v_0 \dots v$  and  $CL \neq \emptyset$  do
9:     if at least one customer  $k \in CL$  can be inserted into the vehicle  $v'$  then
10:      Evaluate the value of each cost  $g(k)$  for  $k \in CL$ ;
11:       $g^{\min} \leftarrow \min\{g(k) | k \in CL\}$ ;
12:       $k' \leftarrow$  customer  $k$  associated to  $g^{\min}$ ;
13:       $s^{v'} \leftarrow s^{v'} \cup \{k'\}$ ;
14:      Update  $CL$ ;
15:      customerInserted  $\leftarrow$  true;
16:      Update  $v_0$ ;
17: if  $CL > 0$  and there are multiple depots + route duration and customerInserted = true then
18:    $s \leftarrow \text{RVND}(s)$ ;
19:   customerInserted  $\leftarrow$  false;
20:   Go to line 7;
21: if  $CL > 0$  and the vehicle fleet is heterogeneous and unlimited then
22:   Add a new vehicle chosen at random;  $\{v \leftarrow v + 1\}$ 
23:   Update  $v_0$ ;  $\{v_0 \leftarrow v\}$ 
24:   Go to line 7;
25: return  $s$ ;
26: end SequentialInsertion.

```

---



---

**Algorithm 6.6** ParallelInsertion
 

---

```

1: Procedure ParallelInsertion( $s, v, CL, \text{InsertionCriterion}$ )
2:  $\gamma \leftarrow 0$ ;
3: customerInserted  $\leftarrow$  false;
4: if InsertionCriterion = MCFIC then
5:    $\gamma \leftarrow$  random value within a given set;
6: while  $CL \neq \emptyset$  and at least one customer  $k \in CL$  can be added to  $s$  do
7:   Evaluate the value of each cost  $g(k)$  for  $k \in CL$ ;
8:    $g^{\min} \leftarrow \min\{g(k) | k \in CL\}$ ;
9:    $k' \leftarrow$  customer  $k$  associated to  $g^{\min}$ ;
10:   $v' \leftarrow$  route associated to  $g^{\min}$ ;
11:   $s^{v'} \leftarrow s^{v'} \cup \{k'\}$ ;
12:  Update  $CL$ ;
13: if  $CL > 0$  and there are multiple depots + route duration and customerInserted = true then
14:    $s \leftarrow \text{RVND}(s)$ ;
15:   customerInserted  $\leftarrow$  false;
16:   Go to line 7;
17: if  $CL > 0$  and the vehicle fleet is heterogeneous and unlimited then
18:   Add a new vehicle chosen at random;  $\{v \leftarrow v + 1\}$ 
19:   Go to line 6;
20: return  $s$ ;
21: end ParallelInsertion.

```

---

### 6.2.3 Local search

The local search is performed by a VND procedure which utilizes a random neighborhood ordering (RVND). Let  $N = \{N^{(1)}, N^{(2)}, N^{(3)}, \dots, N^{(r)}\}$  be the set of neighborhood structures. Whenever a given neighborhood of the set  $N$  fails to improve the incumbent solution, the RVND randomly chooses another neighborhood from the same set to continue the search throughout the solution space. In this case,  $N$  is composed only by inter-route neighborhood structures.

The pseudocode of the RVND procedure is presented in Alg. 6.7. Firstly, a Neighborhood List (NL) containing a predefined number of inter-route moves is initialized (line 3). In the main loop (lines 4-13), a neighborhood  $N^{(n)} \in NL$  is chosen at random (line 5) and then the best admissible move is determined (line 6). In case of improvement, an intra-route local search is performed, the fleet is updated (only when the vehicle fleet is unlimited) and the NL is populated with all the neighborhoods (lines 7-12). Otherwise,  $N^{(n)}$  is removed from the NL (line 14). A set of Auxiliary Data Structures (ADSs) is updated (see Subsection 6.2.3.1) at the beginning of the process (line 2) and whenever a neighborhood search is performed (line 15). If the problem has a primary objective of minimizing the sum of the vehicles, which is the case of the OVRP, then a procedure that tries to empty a route is applied (line 16).

---

**Algorithm 6.7** RVND

---

```

1: Procedure RVND( $s$ )
2:   Update ADSs;
3:   Initialize the inter-route Neighborhood List (NL);
4:   while NL  $\neq$  0 do
5:     Choose a neighborhood  $N^{(n)} \in NL$  at random;
6:     Find the best neighbor  $s'$  of  $s \in N^{(n)}$ ;
7:     if  $f(s') < f(s)$  then
8:        $s \leftarrow s'$ ;
9:        $s \leftarrow \text{IntraRouteSearch}(s)$ ;
10:      if the vehicle fleet is heterogeneous and unlimited then
11:        Update Fleet in such a way that there is one empty vehicle of each type
12:        Update NL; {NL is populated with all inter-route neighborhood structures}
13:      else
14:        Remove  $N^{(n)}$  from the NL;
15:      Update ADSs;
16: TryToEmptyRoute( $s$ ); {considered only when vehicle fleet must be necessarily minimized}
17: return  $s$ ;
18: end RVND.

```

---

Let  $N'$  be a set composed by  $r'$  intra-route neighborhood structures. Alg. 6.8 describes how the intra-route search procedure was implemented. At first, a neighborhood list  $NL'$  is initialized with all the intra-route neighborhood structures (line 2). Next, while  $NL'$  is not empty a neighborhood  $N'^{(n)} \in NL'$  is randomly selected and a local search is exhaustively performed until no more improvements are found (lines 3-9).

In the case of single-vehicle routing problems such as the TSPMPD, it does not make

**Algorithm 6.8** IntraRouteSearch

---

```

1: Procedure IntraRouteSearch( $s$ )
2: Initialize the Intra-Route Neighborhood List ( $NL'$ );
3: while  $NL' \neq \emptyset$  do
4:   Choose a neighborhood  $N'^{(\eta)} \in NL'$  at random;
5:   Find the best neighbor  $s'$  of  $s \in N'^{(\eta)}$ ;
6:   if  $f(s') < f(s)$  then
7:      $s \leftarrow s'$ ;
8:   else
9:     Remove  $N'^{(\eta)}$  from the  $NL'$ ;
10: return  $s$ ;
11: end IntraRouteSearch.

```

---

sense to use inter-route neighborhoods in the local search. Hence, in these situations the set  $N$  utilized in the RVND procedure is replaced by the set  $N'$  composed only by intra-route neighborhood structures and the call to the function `IntraRouteSearch` (line 9 in Alg. 6.7) is ignored.

**6.2.3.1 Auxiliary Data Structures (ADSs)**

In order to enhance the neighborhood search, some ADSs were adopted. The following arrays store useful informations regarding each route.

- `SumDelivery[ ]` — sum of the delivery demands. For example, if `SumDelivery[2] = 100`, it means that the sum of the delivery demands of all customers of route 2 corresponds to 100.
- `SumPickup[ ]` — sum of the pickup demands.
- `MinDelivery[ ]` — minimum delivery demand. For example, if `MinDelivery[3] = 5`, it means that 5 is the least delivery demand among all customers of route 3.
- `MinPickup[ ]` — minimum pickup demand.
- `MaxDelivery[ ]` — maximum delivery demand.
- `MaxPickup[ ]` — maximum pickup demand.
- `MinPairDelivery[ ]` — minimum sum of delivery demands of two adjacent customers. For example, if `MinPairDelivery[1] = 10`, it means that the least sum of the delivery demands of two adjacent customers of route 1 corresponds to 10.
- `MinPairPickup[ ]` — minimum sum of pickup demands of two adjacent customers.
- `MaxPairDelivery[ ]` — maximum sum of delivery demands of two adjacent customers.
- `MaxPairPickup[ ]` — maximum sum of pickup demands of two adjacent customers.

- **CumulativeDelivery**[ ][ ] — cumulative delivery load at each point of the route. For example, if **CumulativeDelivery**[2][4] = 78, it means that the sum of the delivery demands of the first four customers of route 2 corresponds to 78.
- **CumulativePickup**[ ][ ] — cumulative pickup load at each point of the route.
- **NeighbStatus**[ ][ ] — informs if the route has been modified after the neighborhood has failed to find an improvement move involving the same route. For example, if **NeighbStatus**[1][3] = **true**, it means that the last time the neighborhood  $N^{(1)}$  was applied, no improvement move involving route 3 was found, but this route was later modified by another neighborhood structure or by a perturbation move. If **NeighbStatus**[1][3] = **false**, it means that the route 3 did not suffer any change after the last time  $N^{(1)}$  was unsuccessful to find an improvement move involving this route.

To update the information of the ADSs one should take into account only the routes that were modified. Let  $\bar{n}$  be the total number of customers in the modified routes. Except for the **NeighbStatus**, the ADSs updating is as follows. For each modified route, a verification is performed along the whole tour to update the corresponding values of the ADSs. Hence, the computational complexity is of the order of  $\mathcal{O}(\bar{n})$ . As for the **NeighbStatus**, for each route, the information regarding all inter-routes neighborhoods are updated which results in a computational complexity of  $\mathcal{O}(\bar{v}|N|)$ , where  $\bar{v}$  is the number of modified routes.

### 6.2.3.2 Inter-Route neighborhood structures

Six VRP neighborhood structures involving inter-route moves were employed for the VRPs with homogeneous fleet and for the HVRP. Five of them are based on the  $\lambda$ -interchanges scheme [132], which consists of exchanging up to  $\lambda$  customers between two routes, while one is based on the Cross-exchange operator [163], which consists of exchanging two segments of different routes. For the FSM, besides these six neighborhood structures, another one, called  $K$ -Shift, was also implemented. As for the MDVRP and MDVRPMPD, two new neighborhood structures called ShiftDepot and SwapDepot were also employed.

To limit the number of possibilities one considered  $\lambda = 2$ . According to Cordeau and Laporte [37], these exchanges are better described as couples  $(\lambda_1, \lambda_2)$  (with  $\lambda_1 \leq \lambda$  and  $\lambda_2 \leq \lambda$ ) that represent an operation where  $\lambda_1$  customers are transferred from route 1 to route 2 and  $\lambda_2$  customers are removed from route 2 to route 1. Therefore, disregarding symmetries, the following combinations are possible in 2-exchanges: (1,0), (1,1), (2,0), (2,1), (2,2). Remark that such combinations include swap moves ((1,1), (2,1), (2,2)) and shift moves ((1,0), (2,0)).



The solutions associated to all neighborhoods structures are explored exhaustively, that is, all possible combinations are examined, and the best improvement strategy is considered. The computational complexity of each one of these moves is  $\mathcal{O}(n^2)$ , with the exception of the neighborhoods structures ShiftDepot and SwapDepot whose complexity is of the order of  $\mathcal{O}(v|G|)$ .

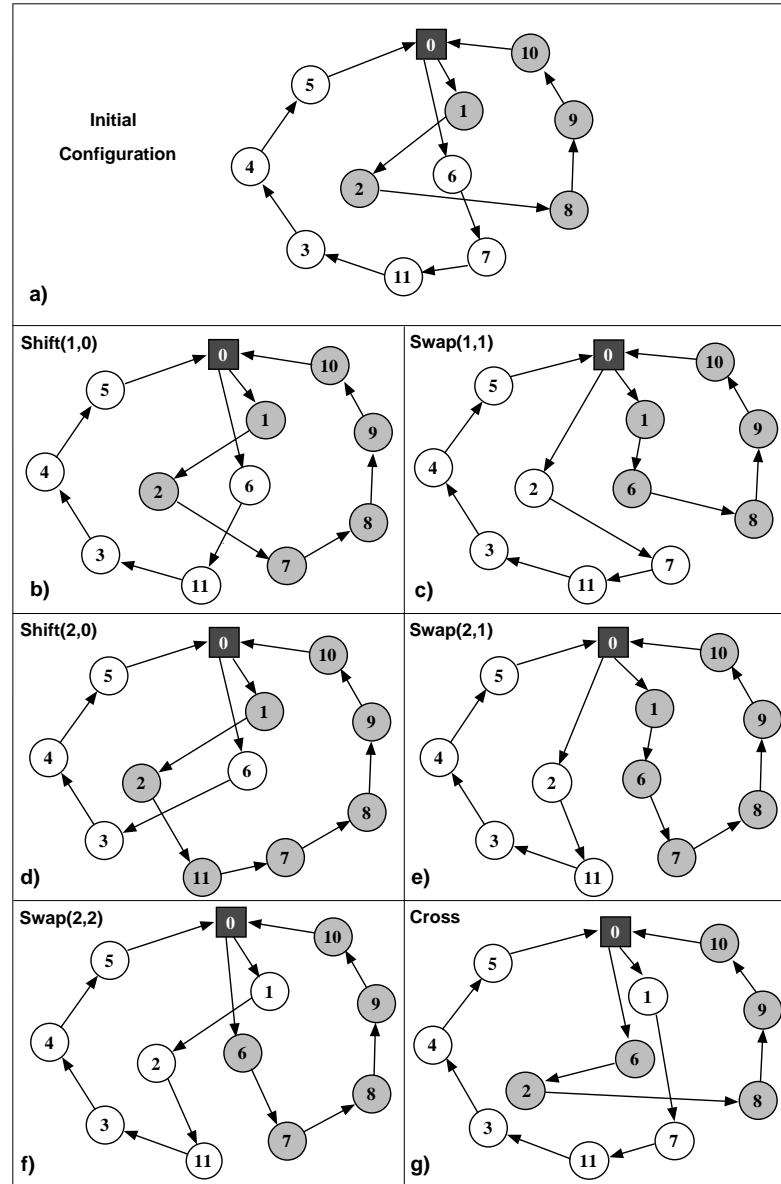
In order to accelerate the local search, a set of conditions can be specified to avoid the examination of moves that are known to be infeasible. It is in this context that the ADSs play an important role. The information stored in these data structures are used in the process of identifying unnecessary moves during the local search. Each neighborhood structure has its own conditions but the idea is essentially the same. Moreover, all of them share the condition that, given a inter-route movement involving the routes  $r_1$  and  $r_2$ , it is worth examining a move using neighborhood  $N^{(\eta)}$  only if `NeighbStatus` $[\eta][r_1] = \text{true}$  or `NeighbStatus` $[\eta][r_2] = \text{true}$ .

Only feasible moves are admitted, i.e., those that do not violate the vehicle load and route duration constraints. Therefore, every time an improvement occurs, the algorithm checks whether this new solution is feasible or not. In some VRP variants such as the CVRP, ACVRP, OVRP, MDVRP and the HFVRP checking the vehicle load is trivial and it can be performed in a constant time by just verifying if the sum of the customers demands of a given route does not exceed the vehicle's capacity when the same is leaving (or arriving at) the depot. In other variants, specially those that include both pickup and delivery services such as the VRPSPD, VRPMPD, TSPMPD and the MDVRPMPD, a more elaborated procedure must be designed for checking the feasibility of a move with respect to the vehicle capacity. As for the route duration, it is possible to check if a route exceeds the maximum limit in constant time. For that, one should keep track of the duration of each route throughout the local search procedure.

The inter-route neighborhood structures are described next. The feasibility checking procedures for those problems that contain both pickup and delivery (VRPSPD, VRPMPD and MDVRPMPD) are also presented and, in all these cases, the computational complexity is  $\mathcal{O}(n)$ . In addition, the particular conditions of each neighborhood that must be satisfied to avoid evaluating some infeasible moves are presented as well. Implementation steps of the neighborhoods Shift(1,0), Swap(1,1), Shift(2,0), Swap(2,1), Swap(2,2), Cross and  $K$ -Shift are respectively provided in Algs. 6.9-6.17. Fig. 6.1 illustrates an example of the effect of the  $\lambda$ -interchanges and Cross based neighborhoods over a given solution, while Figs. 6.2 and 6.3 show, respectively, the effect of the  $K$ -Shift and multi-depot neighborhoods.

**Shift(1,0)** —  $N^{(1)}$  — A customer  $k$  is transferred from a route  $r_1$  to a route  $r_2$ . In Fig. 6.1.b the customer 7 was moved from one route to the other one. If `MinDelivery` $[r_1]$

+  $\text{SumDelivery}[r_2] > Q$  or  $\text{MinPickup}[r_1] + \text{SumPickup}[r_2] > Q$  it means that transferring any customer from  $r_1$  to  $r_2$  implies an infeasible solution. This fact is easy to verify because if even the customer with the least delivery (or pickup) demand cannot be transferred to the other route, it is clear that the remaining customers also cannot. In addition, if  $d_k + \text{SumDelivery}[r_2] > Q$  or  $p_k + \text{SumPickup}[r_2] > Q$  then there is no point in evaluating the transfer of  $k \in r_1$  to any position in  $r_2$ , since the vehicle load will always be violated. Thus a verification should be performed to avoid the evaluation of these infeasible moves. The vehicle load is checked as follows. All customers located before the insertion's position have their loads added by  $d_k$ , while the ones located after have their loads added by  $p_k$ .

Figure 6.1:  $\lambda$ -interchanges and Cross based neighborhoods

**Algorithm 6.9** Shift(1,0)

---

```

1: Procedure Shift-1-0( $s$ )
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and ( $\text{NeighbStatus}[1][r_1] = \text{true}$  or  $\text{NeighbStatus}[1][r_2] = \text{true}$ ) and
        $\text{MinDelivery}[r_1] + \text{SumDelivery}[r_2] \leq Q$  and  $\text{MinPickup}[r_1] + \text{SumPickup}[r_2] \leq Q$  then
5:       for every customer  $k \in r_1$  do
6:         if  $d_k + \text{SumDelivery}[r_2] \leq Q$  and  $p_k + \text{SumPickup}[r_2] \leq Q$  then
7:           for each position  $z$  in  $r_2$  do
8:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
               transferring  $k \in r_1$  to the position  $z$  in  $r_2$ ;
9:             if  $f(s') < f^*$  and  $s'$  is feasible then
10:               $f^* \leftarrow f(s')$ ;
11:               $s^* \leftarrow s'$ ;
12: if  $f^* < f(s)$  then
13:    $s \leftarrow s^*$ ;
14: else
15:   Update NeighbStatus;
16: return  $s$ ;
17: end Shift-1-0.

```

---

**Swap(1,1)** —  $N^{(2)}$  — Permutation between a customer  $k$  from a route  $r_1$  and a customer  $l$ , from a route  $r_2$ . In Fig. 6.1.c the customers 2 and 6 were swapped. To avoid evaluating infeasible moves one should verify if  $\text{MinDelivery}[r_1] - \text{MaxDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$ ,  $\text{MinPickup}[r_1] - \text{MaxPickup}[r_2] + \text{SumPickup}[r_2] \leq Q$ ,  $d_k + \text{SumDelivery}[r_2] - \text{MaxDelivery}[r_2] \leq Q$  and  $p_k + \text{SumPickup}[r_2] - \text{MaxPickup}[r_2] \leq Q$ . The loads of the vehicles of both routes are examined in the same way. For example, in case of  $r_2$ , all customers situated before the position that  $l$  was found (now replaced by  $k$ ), have their values added by  $d_k$  and subtracted by  $d_l$  while the load of the customers positioned after  $k$  increases by  $p_k$  and decreases by  $p_l$ .

**Shift(2,0)** —  $N^{(3)}$  — Two adjacent customers,  $k$  and  $l$ , are transferred from a route  $r_1$  to a route  $r_2$ . This move can also be seen as an arc transfer. In this case, the move examines the transfer of both arcs  $(k, l)$  and  $(l, k)$ . In Fig. 6.1.d the adjacent customers 7 and 11 were moved from one route to the other one. Before starting to evaluate the customers transferring from  $r_1$  to  $r_2$  one should verify following conditions are met:  $\text{MinPairDelivery}[r_1] + \text{SumDelivery}[r_2] \leq Q$  and  $\text{MinPairPickup}[r_1] + \text{SumPickup}[r_2] \leq Q$ . The vehicle load of  $r_2$  is checked using an approach similar to the one adopted in Shift(1,0). All customers located before the insertion's position in  $r_2$  have their loads added by  $d_k + d_l$ , while the ones located after have their loads added by  $p_k + p_l$ . Also, the new load carried out in the arc  $(k, l)$  must be verified.

**Swap(2,1)** —  $N^{(4)}$  — Permutation of two adjacent customers,  $k$  and  $l$ , from a route  $r_1$  by a customer  $k'$  from a route  $r_2$ . As in Shift(2,0), both arcs  $(k, l)$  and  $(l, k)$  are considered. In Fig. 6.1.e the adjacent customers 6 and 7 were exchanged with customer 2. The evaluation of some infeasible moves are avoided by checking if  $\text{MinPairDelivery}[r_1] - \text{MaxDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$  and  $\text{MinPairPickup}[r_1] - \text{MaxPickup}[r_2]$

+  $\text{SumPickup}[r_2] \leq Q$ . The load is verified by means of an extension of the approach used in the neighborhoods  $\text{Shift}(2,0)$  and  $\text{Swap}(1,1)$ .

---

**Algorithm 6.10**  $\text{Swap}(1,1)$ 


---

```

1: Procedure  $\text{Swap-1-1}(s)$ 
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = r_1 + 1 \dots v$  do
4:     if ( $\text{NeighbStatus}[2][r_1] = \text{true}$  or  $\text{NeighbStatus}[2][r_2] = \text{true}$ ) and
        $\text{MinDelivery}[r_1] - \text{MaxDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$  and
        $\text{MinPickup}[r_1] - \text{MaxPickup}[r_2] + \text{SumPickup}[r_2] \leq Q$  then
5:       for every customer  $k \in r_1$  do
6:         if  $d_k + \text{SumDelivery}[r_2] - \text{MaxDelivery}[r_2] \leq Q$  and
            $p_k + \text{SumPickup}[r_2] - \text{MaxPickup}[r_2] \leq Q$  then
7:           for every customer  $l \in r_2$  do
8:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
               exchanging  $k \in r_1$  with  $l$  in  $r_2$ ;
9:             if  $f(s') < f^*$  and  $s'$  is feasible then
10:                $f^* \leftarrow f(s')$ ;
11:                $s^* \leftarrow s'$ ;
12: if  $f^* < f(s)$  then
13:    $s \leftarrow s^*$ ;
14: else
15:   Update  $\text{NeighbStatus}$ ;
16: return  $s$ ;
17: end  $\text{Swap-1-1}$ .

```

---



---

**Algorithm 6.11**  $\text{Shift}(2,0)$ 


---

```

1: Procedure  $\text{Shift-2-0}(s)$ 
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and ( $\text{NeighbStatus}[3][r_1] = \text{true}$  or  $\text{NeighbStatus}[3][r_2] = \text{true}$ ) and
        $\text{MinPairDelivery}[r_1] + \text{SumDelivery}[r_2] \leq Q$  and  $\text{MinPairPickup}[r_1] + \text{SumPickup}[r_2] \leq Q$  then
5:       for every pair of adjacent customers  $k$  and  $l \in r_1$  do
6:         if  $d_k + d_l + \text{SumDelivery}[r_2] \leq Q$  and  $p_k + p_l + \text{SumPickup}[r_2] \leq Q$  then
7:           for each position  $z$  in  $r_2$  do
8:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
               transferring  $k$  and  $l \in r_1$  to the position  $z$  in  $r_2$ ;
9:             if  $f(s') < f^*$  and  $s'$  is feasible then
10:                $f^* \leftarrow f(s')$ ;
11:                $s^* \leftarrow s'$ ;
12: if  $f^* < f(s)$  then
13:    $s \leftarrow s^*$ ;
14: else
15:   Update  $\text{NeighbStatus}$ ;
16: return  $s$ ;
17: end  $\text{Shift-2-0}$ .

```

---

**Swap(2,2)** —  $N^{(5)}$  — Permutation between two adjacent customers,  $k$  and  $l$ , from a route  $r_1$  by another two adjacent customers  $k'$  and  $l'$ , belonging to a route  $r_2$ . All the four possible combinations of exchanging arcs  $(k, l)$  and  $(k', l')$  are considered. In Fig. 6.1.f the adjacent customers 6 and 7 were exchanged with the adjacent customers 1

and 2. In order to avoid evaluating some infeasible moves the following conditions must be satisfied:  $\text{MinPairDelivery}[r_1] - \text{MaxDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$  and  $\text{MinPairPickup}[r_1] - \text{MaxPickup}[r_2] + \text{SumPickup}[r_2] \leq Q$ . The load is checked just as  $\text{Swap}(2,1)$ .

---

**Algorithm 6.12**  $\text{Swap}(2,1)$ 


---

```

1: Procedure  $\text{Swap-2-1}(s)$ 
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and  $(\text{NeighbStatus}[4][r_1] = \text{true or } \text{NeighbStatus}[4][r_2] = \text{true})$  and
        $\text{MinPairDelivery}[r_1] - \text{MaxDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$  and
        $\text{MinPairPickup}[r_1] - \text{MaxPickup}[r_2] + \text{SumPickup}[r_2] \leq Q$  then
5:       for every pair of adjacent customers  $k$  and  $l \in r_1$  do
6:         if  $d_k + d_l + \text{SumDelivery}[r_2] - \text{MaxDelivery}[r_2] \leq Q$  and
            $p_k + p_l + \text{SumPickup}[r_2] - \text{MaxPickup}[r_2] \leq Q$  then
7:           for every customer  $k'$  in  $r_2$  do
8:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
               exchanging adjacent customers  $k$  and  $l \in r_1$  with  $k' \in r_2$ ;
9:             if  $f(s') < f^*$  and  $s'$  is feasible then
10:                $f^* \leftarrow f(s')$ ;
11:                $s^* \leftarrow s'$ ;
12: if  $f^* < f(s)$  then
13:    $s \leftarrow s^*$ ;
14: else
15:   Update  $\text{NeighbStatus}$ ;
16: return  $s$ ;
17: end  $\text{Swap-2-1}$ .

```

---



---

**Algorithm 6.13**  $\text{Swap}(2,2)$ 


---

```

1: Procedure  $\text{Swap-2-2}(s)$ 
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = r_1 + 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and  $(\text{NeighbStatus}[5][r_1] = \text{true or } \text{NeighbStatus}[5][r_2] = \text{true})$  and
        $\text{MinPairDelivery}[r_1] - \text{MaxPairDelivery}[r_2] + \text{SumDelivery}[r_2] \leq Q$  and
        $\text{MinPairPickup}[r_1] - \text{MaxPairPickup}[r_2] + \text{SumPickup}[r_2] \leq Q$  then
5:       for every pair of adjacent customers  $k$  and  $l \in r_1$  do
6:         if  $d_k + d_l + \text{SumDelivery}[r_2] - \text{MaxPairDelivery}[r_2] \leq Q$  and
            $p_k + p_l + \text{SumPickup}[r_2] - \text{MaxPairPickup}[r_2] \leq Q$  then
7:           for every pair of adjacent customers  $k'$  and  $l' \in r_2$  do
8:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
               exchanging adjacent customers  $k$  and  $l \in r_1$  with adjacent customers  $k'$  and  $l' \in r_2$ ;
9:             if  $f(s') < f^*$  and  $s'$  is feasible then
10:                $f^* \leftarrow f(s')$ ;
11:                $s^* \leftarrow s'$ ;
12: if  $f^* < f(s)$  then
13:    $s \leftarrow s^*$ ;
14: else
15:   Update  $\text{NeighbStatus}$ ;
16: return  $s$ ;
17: end  $\text{Swap-2-2}$ .

```

---

**Cross** —  $N^{(6)}$  — The arc between adjacent customers  $k$  and  $l$ , belonging to a route  $r_1$ , and the one between  $k'$  and  $l'$ , from a route  $r_2$ , are both removed. Next, an arc is

inserted connecting  $k$  and  $l'$  and another is inserted linking  $k'$  and  $l$ . In Fig. 6.1.g the arcs (1,2) and (6,7) were removed and the arcs (6,2) and (1,7) were inserted. The procedure for testing the vehicle load of each route is the same and is as follows. At first, the initial ( $\mathcal{L}_0$ ) and final ( $\mathcal{L}_f$ ) vehicle loads of both routes are computed in constant time using the ADSs `SumDelivery`, `SumPickup`, `CumulativeDelivery` and `CumulativePickup`. If the values of ( $\mathcal{L}_0$ ) and ( $\mathcal{L}_f$ ) do not exceed the vehicle capacity  $Q$  then the remaining loads are verified through the following expression:  $\mathcal{L}_i = \mathcal{L}_{i-1} + p_i - d_i$ . Hence, if  $\mathcal{L}_i$  surpasses  $Q$ , the move is infeasible.

---

**Algorithm 6.14** Cross
 

---

```

1: Procedure Cross( $s$ )
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and (NeighbStatus[6][ $r_1$ ] = true or NeighbStatus[6][ $r_2$ ] = true) then
5:       for every position  $i$  in  $r_1$  do
6:         {Let  $f$  be the last customer of  $r_2$ }
7:         if CumulativeDelivery[ $r_1$ ][ $i$ ] +  $d_f \leq Q$  and CumulativePickup[ $r_1$ ][ $i$ ] +  $p_f \leq Q$ 
           then
8:           for every position  $j$  in  $r_2$  do
9:             Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost
             of removing the arc  $(k, l) \in r_1$ , associated with positions  $i$  and  $i + 1$ , and the arc
              $(k', l') \in r_2$ , associated with positions  $j - 1$  and  $j$  and inserting arcs  $(k, l')$  and  $(k', l)$ ;
10:            if  $f(s') < f^*$  and  $s'$  is feasible then
11:               $f^* \leftarrow f(s')$ ;
12:               $s^* \leftarrow s'$ ;
13: if  $f^* < f(s)$  then
14:    $s \leftarrow s^*$ ;
15: else
16:   Update NeighbStatus;
17: return  $s$ ;
18: end Cross.

```

---

**K-Shift** —  $N^{(7)}$  — A subset of consecutive customers  $K$  is transferred from a route  $r_1$  to the end of a route  $r_2$ . In this case, it is assumed that the dependent and fixed costs of  $r_2$  is smaller than those of  $r_1$ . It should be pointed out that the move is also applied if  $r_2$  is an empty route. In Fig. 6.2, the consecutive customers 1, 2 and 7 are transferred from one route to the end of the other one.

**ShiftDepot** —  $N^{(8)}$  — A route  $r$  is transferred from a  $depot_1$  to a  $depot_2$ , as long as there is a vehicle available on the latter. In principle, any move is considered to be feasible. In Fig 6.3.b, the route composed by customers 14, 18 and 15 is transferred from one depot to the other one.

**SwapDepot** —  $N^{(9)}$  — Permutation between a route  $r_1$  from a  $depot_1$  and a route  $r_2$  from a  $depot_2$ . As in the previous case, any move is admitted to be feasible. In Fig 6.3.c, the route composed by customers 14, 18 and 15, belonging to depot D1 is exchanged with the route composed by customers 4, 5, 6 and 8, belonging to the depot D2.

**Algorithm 6.15** *K*-Shift

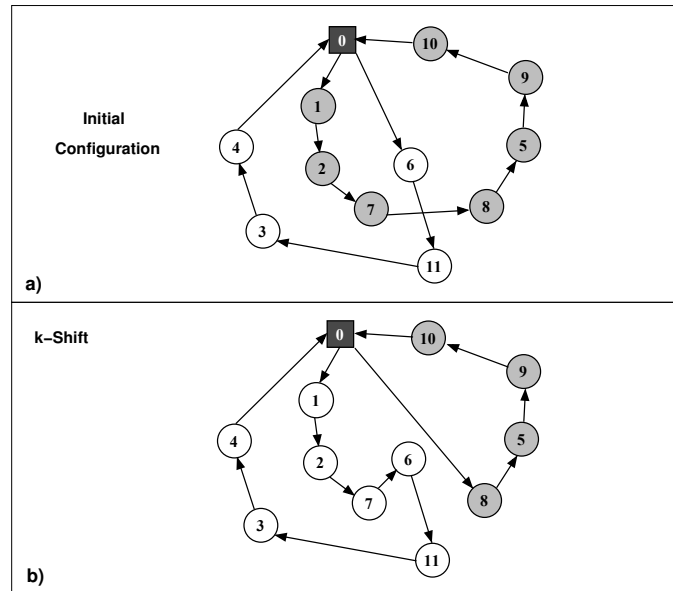
---

```

1: Procedure K-Shift(s)
2: for  $r_1 = 1 \dots v$  do
3:   for  $r_2 = 1 \dots v$  do
4:     if  $r_1 \neq r_2$  and  $Q_{r_1} < Q_{r_2}$  and
       (NeighbStatus[1][ $r_1$ ] = true or NeighbStatus[1][ $r_2$ ] = true) and MinDelivery[ $r_1$ ] +
       SumDelivery[ $r_2$ ]  $\leq Q_{r_2}$  then
5:       for every customer  $k \in r_1$  do
6:         if  $d_k + \text{SumDelivery}[r_2] \leq Q_{r_2}$  then
7:           SumDeliveryTemp  $\leftarrow d_k + \text{SumDelivery}[r_2]$ ;
8:            $z \leftarrow$  position of  $k$ ;
9:            $l \leftarrow k$ ;
10:          while SumDeliveryTemp  $\leq Q_{r_2}$  and  $z$  is a valid position of  $r_1$  do
11:            Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
            transferring the consecutive customers ranging from  $k \in r_1$  to  $l \in r_1$  to the end of  $r_2$ ;
12:            if  $f(s') < f^*$  and  $s'$  is feasible then
13:               $f^* \leftarrow f(s')$ ;
14:               $s^* \leftarrow s'$ ;
15:               $z \leftarrow z + 1$ ;
16:               $l \leftarrow$  customer associated with the position  $z \in r_1$ 
17:              SumDeliveryTemp  $\leftarrow \text{SumDeliveryTemp} + d_l$ 
18: if  $f^* < f(s)$  then
19:    $s \leftarrow s^*$ ;
20: else
21:   Update NeighbStatus;
22: return  $s$ ;
23: end K-Shift.

```

---

Figure 6.2: *K*-Shift neighborhood

It is important to mention that other conditions to avoid the examination of moves that are known to be infeasible were tested but the results were disappointing in terms of computational time. For example, in the case of Swap(1,1), it is clear that the following conditions must be met:  $\text{MinDelivery}[r_2] - \text{MaxDelivery}[r_1] + \text{SumDelivery}[r_1] \leq$

$Q, \text{MinPickup}[r_2] - \text{MaxPickup}[r_1] + \text{SumPickup}[r_1] \leq Q, d_i + \text{SumDelivery}[r_1] - \text{MaxDelivery}[r_1] \leq Q$ . However, preliminary experiments showed that the overhead of including these verifications in addition to the existing ones was not worth it.

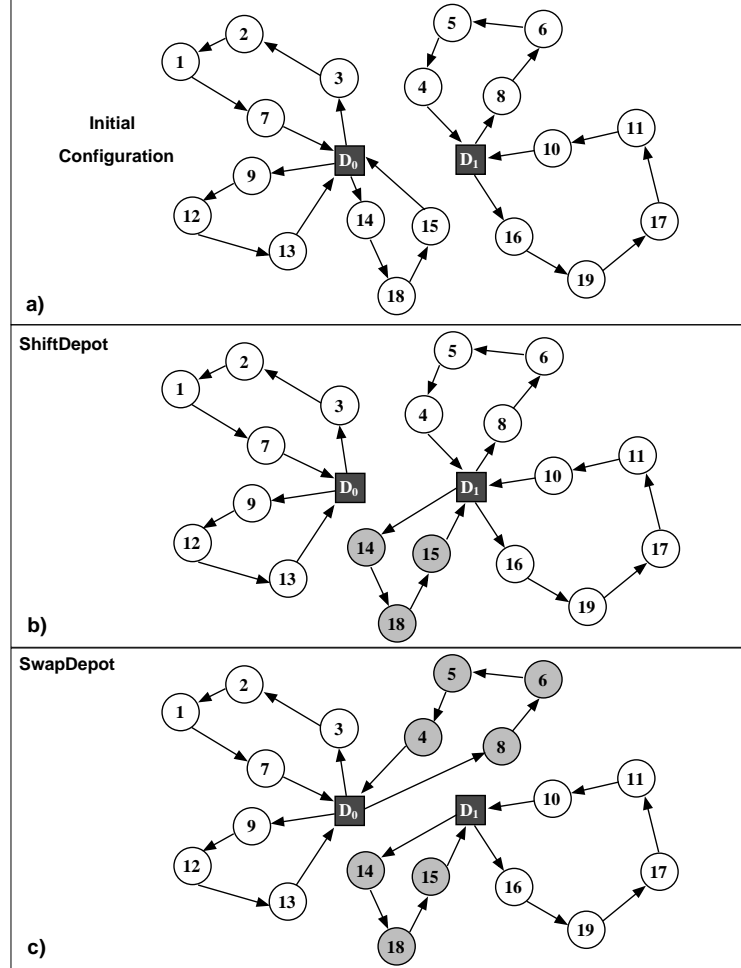


Figure 6.3: Multi-depot neighborhoods

**Algorithm 6.16** ShiftDepot

---

```

1: Procedure ShiftDepot( $s$ )
2:   for  $depot_1 = 1 \dots |G|$  do
3:     for each non-empty route  $r \in depot_1$  do
4:       for  $depot_2 = 1 \dots |G|$  do
5:         if  $depot_1 \neq depot_2$  and  $depot_2$  has an empty route then
6:           Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
             transferring route  $r \in depot_1$  to the  $depot_2$ ;
7:           if  $f(s') < f^*$  then
8:              $f^* \leftarrow f(s')$ ;
9:              $s^* \leftarrow s'$ ;
10:  if  $f^* < f(s)$  then
11:     $s \leftarrow s^*$ ;
12:  return  $s$ ;
13: end ShiftDepot.

```

---



**Algorithm 6.17** SwapDepot

---

```

1: Procedure SwapDepot( $s$ )
2: for  $depot_1 = 1 \dots |G|$  do
3:   for each non-empty route  $r_1 \in depot_1$  do
4:     for  $depot_2 = 1 \dots |G|$  do
5:       if  $depot_1 \neq depot_2$  then
6:         for each non-empty route  $r_2 \in depot_2$  do
7:           Evaluate the solution cost  $f(s')$  of the neighborhood solution  $s'$  of  $s$ , i.e., the cost of
             exchanging route  $r_1 \in depot_1$  with  $r_2 \in depot_2$ ;
8:           if  $f(s') < f^*$  then
9:              $f^* \leftarrow f(s')$ ;
10:             $s^* \leftarrow s'$ ;
11: if  $f^* < f(s)$  then
12:    $s \leftarrow s^*$ ;
13: return  $s$ ;
14: end SwapDepot.

```

---

**6.2.3.3 Intra-Route neighborhood structures**

The six intra-route neighborhood structures are described next. The set  $N'$  is composed by Reinsertion, Or-opt [131], 2-opt [41], exchange and reverse [128] moves. In all cases, the vehicle load of the PDP problems (VRPSPD, VRPMPD and MDVRPMPD) is verified utilizing the same approach of the neighborhood Cross, but it is known in advance that  $\mathcal{L}_0$  and  $\mathcal{L}_f$  never violate the maximum load allowed. Fig 6.4 shows an example of each one of these neighborhood operators.

**Reinsertion** — One, customer is removed and inserted in another position of the route. In Fig. 6.4.b the customer 3 was re-inserted in another position.

**Or-opt2** — Two adjacent customers are removed and inserted in another position of the route. In Fig. 6.4.c the adjacent customers 2 and 3 were re-inserted in another position.

**Or-opt3** — Three adjacent customers are removed and inserted in another position of the route. In Fig. 6.4.d the adjacent customers 1, 2 and 3 were re-inserted in another position.

**2-opt** — Two nonadjacent arcs are deleted and another two are added in such a way that a new route is generated. In Fig. 6.4.e The arcs (2,3) and (5,6) were deleted while the arcs (2,5) and (3,6) were inserted. This neighborhood structure was not employed in the ACVRP.

**Exchange** — Permutation between two customers. In Fig. 6.4.f the customers 2 and 6 were swapped.

**Reverse** — This move reverses the route direction if the value of the maximum load of the corresponding route is reduced. In Fig. 6.4.g all the arcs had their direction reversed. This neighborhood structure is only employed in PDP problems.

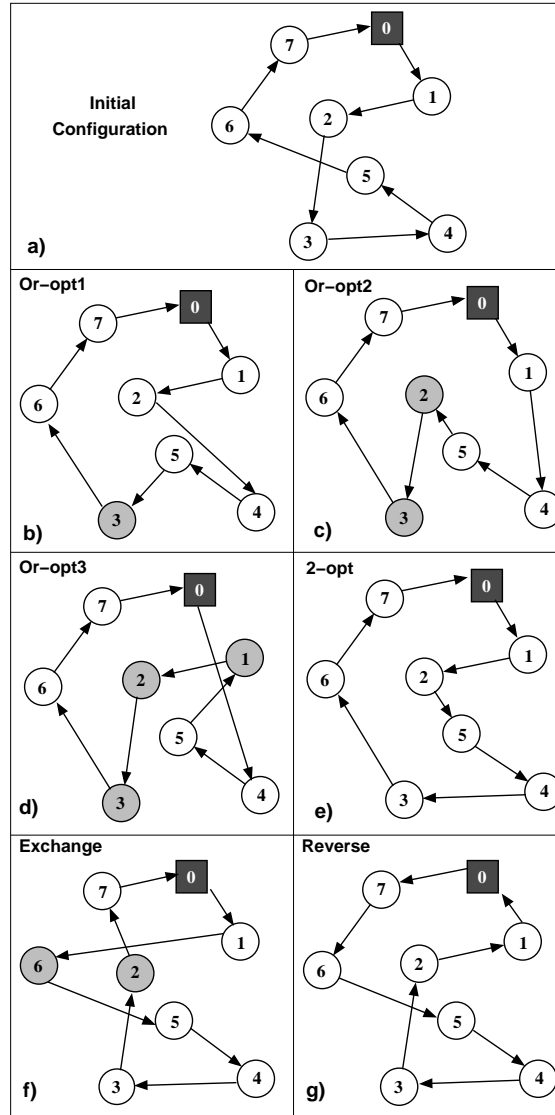


Figure 6.4: Intra-Route neighborhoods

The computational complexity of evaluating the neighborhoods Reinsertion, Or-opt2, Or-opt3, 2-opt and Exchange is  $\mathcal{O}(\bar{n}^2)$  while the complexity of evaluating the neighborhood Reverse is  $\mathcal{O}(\bar{n})$ .

#### 6.2.3.4 Trying to empty a route

In some VRP variants, minimizing the number of vehicles is the primary goal. Hence a greedy randomized procedure was developed for dealing with this issue, as can be observed in Alg. 6.18. The idea is to make use of the residual capacity and residual duration of the routes of a given solution  $s$  by means of local search, with a view of decreasing the number of routes of  $s$ . The procedure starts by storing a backup of the solution  $s$  in  $s'$  (line 2). Let Route List (RL) be the list composed by the routes of  $s$  (line 3). While  $|\text{RL}|$  is greater than 1 (lines 4-11), an attempt to empty a route is performed. A route  $r$  is selected to

be removed from RL (lines 6-7) according to one of the following criteria: (i) route with maximum load; (ii) route with maximum duration; (iii) random selection. The route selection criterion is chosen at random (line 5). Next, while it is still possible to move a customer from any route  $r' \in \text{RL}$  to  $r$  or it is still possible to exchange a customer from any route  $r' \in \text{RL}$  with another one in  $r$  in such a way that the load of  $r$  is increased, a local search is performed between the route  $r$  and those in RL by the neighborhood structures Shift(1,0), Shift(2,0) and Swap(1,1) (lines 9-11). The best admissible move is considered for each of these three neighborhoods. Moreover, in the case of Shift(1,0) and Shift(2,0), a move is immediately accepted if a route  $r' \in \text{RL}$  becomes empty, whereas in the case of Swap(1,1), a move is only accepted if the vehicle load of  $r$  is increased. An intra-route local search is performed in every modified route using 2-opt and exchange neighborhood structures. If the procedure is not capable of emptying a route then the current solution is restored (lines 12-13).

---

**Algorithm 6.18** TryEmptyRoute
 

---

```

1: Procedure TryEmptyRoute( $s$ )
2:  $s' \leftarrow s$ 
3: Initialize Route List (RL) with the routes of  $s$ 
4: while |RL| > 1 do
5:   Choose a route selection criterion at random;
6:   Choose a route  $r \in \text{RL}$  according to the selected criterion;
7:   Remove  $r$  from RL;
8:   while it is still possible to move a customer from any route  $r' \in \text{RL}$  to  $r$  or it is still possible to
       exchange a customer from any route  $r' \in \text{RL}$  with another one in  $r$  in such a way that the load of
        $r$  is increased do
9:      $s \leftarrow \text{Shift}(1,0)$ ;
10:     $s \leftarrow \text{Shift}(2,0)$ ;
11:     $s \leftarrow \text{Swap}(1,1)$ ;
12: if number of routes of  $s$  is equal to the number of routes of  $s'$  then
13:    $s \leftarrow s'$ ;
14: return  $s$ ;
15: end TryEmptyRoute.

```

---

### 6.2.4 Perturbation mechanisms

A set  $P$  of four perturbation mechanisms were adopted. Only feasible perturbation moves are accepted. Whenever the `Perturb()` function is called, one of the following moves is randomly selected.

**Multiple-Swap(1,1)** –  $P^{(1)}$  – Multiple random Swap(1,1) moves are performed in sequence.

**Multiple-Shift(1,1)** –  $P^{(2)}$  – Multiple random Shift(1,1) moves are performed in sequence. The Shift(1,1) consists in transferring a customer  $k$  from a route  $r_1$  to a route  $r_2$ , whereas a customer  $l$  from  $r_2$  is transferred to  $r_1$ .

**Double-Bridge** –  $P^{(3)}$  – Consists in cutting four arcs of a given route and inserting another four to form a new tour [117] (see Fig. 6.5). This perturbation can also be seen as a permutation of segments of routes. For those instances with less than 75 customers, an intra-route version of the neighborhood structure Swap(2,2) is employed. For the remaining cases the length of the segment of routes is chosen at random from the set  $[2, \dots, \lfloor n/25 \rfloor]$ . This perturbation is only considered for single-vehicle routing problems.

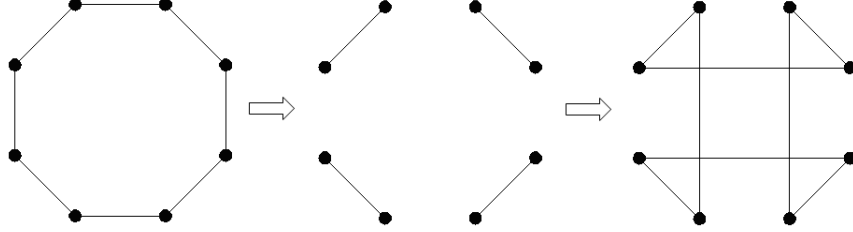


Figure 6.5: Double-Bridge

**Split**,  $P^{(4)}$ , a route  $r$  is divided into smaller routes. Let  $M' = \{2, \dots, m\}$  be a subset of  $M$  composed by all vehicle types, except the one with the smallest capacity. Firstly, a route  $r \in s$  (let  $s = s'$ ) associated with a vehicle  $u \in M'$  is selected at random. Next, while  $r$  is not empty, the remaining customers of  $r$  are sequentially transferred to a new randomly selected route  $r' \notin s$  associated with a vehicle  $u' \in \{1, \dots, u-1\}$  in such a way that the capacity of  $u'$  is not violated. The new generated routes are added to the solution  $s$  while the route  $r$  is removed from  $s$ . The procedure described is repeated multiple times where the number of repetitions is chosen at random from the set  $\{1, 2, \dots, v\}$ . This perturbation was applied only for the FSM, since it does not make sense for the HVRP.

## 6.3 Computational results

The algorithm ILS-RVND was coded in C++ (g++ 4.4.3). For the VRPs with homogeneous fleet and TSPMPD the tests were executed in an Intel® Core™ 2 Quad with 2.4 GHz and 4 GB of RAM running under Linux 64 bits (kernel 2.6.27-16). As for the HFVRP the tests were executed in an Intel® Core™ i7 with 2.93 GHz and 8 GB of RAM running under Linux 64 bits (kernel 2.6.32-22). Only a single thread was used.

The current section is divided into three parts: (i) VRPs with homogeneous fleet; (ii) HFVRP and (iii) TSPMPD. For each of these parts, a parameter tuning was performed.

In the tables presented hereafter, **Instance** denotes the number of the test-problem,  **$n$**  is the number of customers, **BKS** represents the best known solution reported in the literature, **Best Sol.**, **Avg. Sol.** and **Time(s)** indicate, respectively, the best solution, the average solution and the average computational time in seconds associated to the correspondent work, **Gap** denotes the gap between the best solution found by ILS-RVND and

the best known solution, **Avg. Gap** corresponds to the gap between the average solution found by ILS-RVND and the best known solution. The best solutions are highlighted in boldface and the solutions improved by ILS-RVND are underlined. The approximate speed, in Mflop/s, of the machines used by other authors is also reported considering the factors suggested by the benchmarks of Dongarra [50], when solving solving a system of equations of order 1000,

### 6.3.1 VRPs with Homogeneous Fleet

The seven VRP variants tackled in this subsection are: CVRP, ACVRP, OVRP, VRPSPD, VRPMPD, MDVRP and MDVRPMPD. The benchmark instances used in every type of problem were those most adopted in the literature. For each group of instances, the performance of the ILS-RVND, in terms of solution quality, was compared with the best known heuristic algorithms or simply with the optimal/BKS. The number of executions of the ILS-RVND on each instance of every variant was 50.

#### 6.3.1.1 Parameter tuning

A set of representative instances of different VRPs with homogeneous fleet was selected for tuning the two main parameters of the ILS-RVND heuristic, that is, *MaxIter* and *MaxIterILS*. It has been observed that the last one varies with the size of the instances, more precisely, with the number of customers and vehicles. For the sake of simplicity, it was decided to use an intuitive and straightforward linear expression for computing the value of *MaxIterILS* according to  $n$  and  $v$ , as shown in Eq. 6.3.

$$MaxIterILS = n + \beta \times v \quad (6.3)$$

The parameter  $\beta$  in Eq. 6.3 corresponds to a non-negative integer constant that indicates the level of influence of the number of vehicles  $v$  in the value of *MaxIterILS*.

Twenty three instances with varying number of customers (50 - 480) and vehicles were chosen as a sample for tuning the values of the parameters. For each of these test-problems the ILS-RVND was executed 20 times. Two values of *MaxIter* were tested, specifically 50 and 75. For each of these, five values of  $\beta$  were evaluated.

In order to select an attractive parameters configuration one took into account the quality of the solutions obtained in each variant, measured by the average gap between the solutions obtained using a given  $\beta$  value and the respective best solution found in the literature, and the computational effort, measured by the average computational time. The gap was calculated using Eq. 6.4. Negative values indicate an improvement.

$$gap = \frac{ILS\_RVND\_solution - literature\_solution}{literature\_solution} \times 100 \quad (6.4)$$

Table 6.1 contains the results of the average gap and the average CPU time for the tests involving  $MaxIter = 50$ , while those obtained for  $MaxIter = 75$  are presented in Table 6.2. It can be observed that the quality of the solutions and the computational time tend to increase with the value of  $\beta$  and  $MaxIter$ . This behavior was obviously expected since more trials are given to the algorithm when the values of these two parameters increase. The selected configuration was  $\beta = 5$  and  $MaxIter = 50$  since it was capable of producing, on average, satisfactory solutions in much less computational time when compared to the configuration that obtained the best results in terms of solution quality ( $\beta = 7$  and  $MaxIter = 75$ ).

Table 6.1: Results of the parameter tuning for  $MaxIter = 50$ 

| Instance               | $N$ | $v$ | $\beta$      |          |              |          |              |          |              |          |              |          |
|------------------------|-----|-----|--------------|----------|--------------|----------|--------------|----------|--------------|----------|--------------|----------|
|                        |     |     | 3            |          | 4            |          | 5            |          | 6            |          | 7            |          |
|                        |     |     | Avg. Gap (%) | Time (s) | Avg. Gap (%) | Time (s) | Avg. Gap (%) | Time (s) | Avg. Gap (%) | Time (s) | Avg. Gap (%) | Time (s) |
| CMT1X <sup>3</sup>     | 50  | 3   | 0.00         | 2.08     | 0.00         | 2.18     | 0.00         | 2.35     | 0.00         | 2.43     | 0.00         | 2.57     |
| CMT1Q <sup>4</sup>     | 50  | 4   | 0.00         | 1.76     | 0.00         | 1.87     | 0.00         | 1.93     | 0.00         | 2.05     | 0.00         | 2.12     |
| SCA3-0 <sup>3</sup>    | 50  | 4   | 0.07         | 1.51     | 0.07         | 1.60     | 0.07         | 1.68     | 0.07         | 1.74     | 0.07         | 1.82     |
| SCA8-0 <sup>3</sup>    | 50  | 9   | 0.00         | 1.86     | 0.00         | 1.95     | 0.00         | 2.14     | 0.00         | 2.31     | 0.00         | 2.54     |
| CMT2X <sup>2</sup>     | 75  | 6   | 0.22         | 6.15     | 0.15         | 6.41     | 0.19         | 6.62     | 0.15         | 7.00     | 0.12         | 7.29     |
| CMT2Q <sup>4</sup>     | 75  | 8   | -0.01        | 5.46     | -0.01        | 5.85     | -0.10        | 6.30     | 0.01         | 6.71     | -0.02        | 6.82     |
| CMT3X <sup>3</sup>     | 100 | 5   | 0.08         | 15.07    | 0.15         | 16.17    | 0.08         | 16.18    | 0.12         | 17.33    | 0.06         | 17.47    |
| CMT3Q <sup>4</sup>     | 100 | 6   | 0.00         | 14.32    | 0.00         | 14.86    | 0.00         | 15.43    | 0.00         | 16.18    | 0.00         | 16.95    |
| r101 <sup>3</sup>      | 100 | 11  | 0.46         | 13.07    | 0.41         | 14.37    | 0.29         | 15.30    | 0.32         | 16.31    | 0.34         | 16.86    |
| CMT11X <sup>3</sup>    | 120 | 4   | 2.02         | 40.52    | 1.77         | 42.90    | 1.77         | 42.78    | 1.89         | 43.86    | 1.88         | 45.75    |
| F-n135-k7 <sup>2</sup> | 134 | 7   | 0.09         | 38.91    | 0.10         | 40.12    | 0.11         | 42.10    | 0.11         | 42.86    | 0.09         | 45.60    |
| CMT4X <sup>3</sup>     | 150 | 7   | 0.19         | 52.00    | 0.18         | 54.15    | 0.17         | 55.85    | 0.12         | 56.96    | 0.12         | 59.26    |
| CMT4Q <sup>4</sup>     | 150 | 9   | 0.18         | 43.87    | 0.18         | 46.29    | 0.20         | 48.10    | 0.19         | 49.98    | 0.16         | 50.66    |
| CMT5X <sup>3</sup>     | 199 | 10  | 0.42         | 122.08   | 0.35         | 127.07   | 0.43         | 134.27   | 0.40         | 139.51   | 0.35         | 143.12   |
| CMT5Q <sup>4</sup>     | 199 | 12  | -1.85        | 124.03   | -1.90        | 132.49   | -1.87        | 136.82   | -1.90        | 139.11   | -1.87        | 146.94   |
| rc1_2_1 <sup>3</sup>   | 200 | 23  | 0.59         | 100.43   | 0.62         | 110.16   | 0.68         | 117.22   | 0.60         | 122.95   | 0.55         | 132.74   |
| c1_2_1 <sup>3</sup>    | 200 | 28  | 0.33         | 91.29    | 0.33         | 100.38   | 0.33         | 106.44   | 0.34         | 113.59   | 0.29         | 118.14   |
| G9 <sup>1</sup>        | 255 | 14  | 1.58         | 259.25   | 1.55         | 267.96   | 1.54         | 270.60   | 1.50         | 289.21   | 1.47         | 296.71   |
| G18 <sup>1</sup>       | 300 | 27  | 0.90         | 472.27   | 0.82         | 501.70   | 0.77         | 523.43   | 0.79         | 542.48   | 0.73         | 568.91   |
| c2_4_1 <sup>3</sup>    | 400 | 15  | 0.70         | 1135.35  | 0.55         | 1152.96  | 0.69         | 1142.65  | 0.61         | 1154.56  | 0.70         | 1187.65  |
| c1_4_1 <sup>3</sup>    | 400 | 63  | 0.15         | 857.68   | 0.10         | 924.39   | 0.07         | 988.83   | 0.06         | 1026.24  | 0.09         | 1117.27  |
| G20 <sup>1</sup>       | 420 | 38  | 0.98         | 1300.91  | 0.93         | 1375.48  | 0.85         | 1460.54  | 0.87         | 1497.13  | 0.91         | 1540.54  |
| G16 <sup>1</sup>       | 480 | 37  | 1.23         | 1285.28  | 1.27         | 1365.71  | 1.21         | 1379.75  | 1.14         | 1464.85  | 1.18         | 1521.31  |
| Average                |     |     | 0.36         | 260.22   | 0.33         | 274.22   | 0.32         | 283.36   | 0.32         | 293.71   | 0.31         | 306.48   |

(1) CVRP, (2) OVRP, (3) VRPSPD, (4) VRPMPD

### 6.3.1.2 CVRP

Different sets of well-known CVRP instances were used to verify the behavior of the proposed heuristic. The first group is composed by the so-called A, B, E, F, M, and P instances (available at [www.branchandcut.org](http://www.branchandcut.org), accessed 11 August 2010). These instances are generally used for testing exact algorithms and most of them had been solved to optimality (see [63, 115]). Also, it is worth mentioning that the distance matrix is rounded up to the nearest integer. The second group of instances was suggested by Christofides et al. [31] and it is composed of 14 instances with 50-199 customers. Half of these includes route duration constraints (C6-C10, C13-C14). The third group contains large-sized instances

Table 6.2: Results of the parameter tuning for  $MaxIter = 75$ 

| Instance               | $N$ | $v$ | $\beta$            |             |                    |             |                    |             |                    |             |                    |             |
|------------------------|-----|-----|--------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|-------------|--------------------|-------------|
|                        |     |     | 3                  |             | 4                  |             | 5                  |             | 6                  |             | 7                  |             |
|                        |     |     | Avg.<br>Gap<br>(%) | Time<br>(s) | Avg.<br>Gap<br>(%) | Time<br>(s) | Avg.<br>Gap<br>(%) | Time<br>(s) | Avg.<br>Gap<br>(%) | Time<br>(s) | Avg.<br>Gap<br>(%) | Time<br>(s) |
| CMT1X <sup>3</sup>     | 50  | 3   | 0.00               | 3.11        | 0.00               | 3.31        | 0.00               | 3.42        | 0.00               | 3.52        | 0.00               | 3.75        |
| CMT1Q <sup>4</sup>     | 50  | 4   | 0.00               | 2.65        | 0.00               | 2.75        | 0.00               | 2.94        | 0.00               | 2.99        | 0.00               | 3.18        |
| SCA3-0 <sup>3</sup>    | 50  | 4   | 0.07               | 2.24        | 0.07               | 2.36        | 0.07               | 2.48        | 0.07               | 2.59        | 0.07               | 2.72        |
| SCA8-0 <sup>3</sup>    | 50  | 9   | 0.00               | 2.76        | 0.00               | 3.04        | 0.00               | 3.35        | 0.00               | 3.53        | 0.00               | 3.73        |
| CMT2X <sup>3</sup>     | 75  | 6   | 0.09               | 8.87        | 0.16               | 9.70        | 0.13               | 9.99        | 0.08               | 10.43       | 0.10               | 11.03       |
| CMT2Q <sup>4</sup>     | 75  | 6   | -0.10              | 8.25        | -0.04              | 8.76        | -0.10              | 9.47        | -0.08              | 9.84        | -0.10              | 10.36       |
| CMT3X <sup>3</sup>     | 100 | 5   | 0.06               | 23.36       | 0.03               | 23.69       | 0.07               | 25.12       | 0.10               | 25.76       | 0.07               | 25.88       |
| CMT3Q <sup>4</sup>     | 100 | 6   | 0.00               | 21.82       | 0.00               | 22.42       | 0.00               | 23.29       | 0.00               | 23.89       | 0.00               | 24.75       |
| r101 <sup>3</sup>      | 100 | 11  | 0.39               | 20.32       | 0.31               | 22.17       | 0.24               | 22.87       | 0.34               | 24.21       | 0.28               | 25.41       |
| CMT11X <sup>3</sup>    | 120 | 4   | 1.61               | 60.28       | 1.60               | 62.48       | 1.44               | 64.19       | 1.54               | 66.65       | 1.58               | 69.25       |
| F-n135-k7 <sup>2</sup> | 134 | 7   | 0.08               | 58.60       | 0.09               | 59.94       | 0.07               | 62.54       | 0.08               | 64.11       | 0.06               | 66.86       |
| CMT4X <sup>3</sup>     | 150 | 7   | 0.12               | 78.72       | 0.15               | 81.28       | 0.12               | 86.12       | 0.14               | 88.48       | 0.11               | 89.70       |
| CMT4Q <sup>4</sup>     | 150 | 9   | 0.18               | 66.66       | 0.17               | 70.16       | 0.16               | 72.50       | 0.16               | 74.04       | 0.17               | 77.12       |
| CMT5X <sup>3</sup>     | 199 | 10  | 0.33               | 192.94      | 0.37               | 198.06      | 0.31               | 208.29      | 0.36               | 208.96      | 0.27               | 219.95      |
| CMT5Q <sup>4</sup>     | 199 | 12  | -1.94              | 187.50      | -1.87              | 199.37      | -1.92              | 202.91      | -1.91              | 214.19      | -1.95              | 218.94      |
| rc1_2_1 <sup>3</sup>   | 200 | 23  | 0.56               | 153.68      | 0.56               | 165.48      | 0.52               | 175.59      | 0.56               | 187.16      | 0.57               | 193.25      |
| c1_2_1 <sup>3</sup>    | 200 | 28  | 0.33               | 137.58      | 0.28               | 148.23      | 0.30               | 158.81      | 0.29               | 169.54      | 0.29               | 177.02      |
| G9 <sup>1</sup>        | 255 | 14  | 1.46               | 384.47      | 1.41               | 393.88      | 1.47               | 411.40      | 1.40               | 435.25      | 1.41               | 445.49      |
| G18 <sup>1</sup>       | 300 | 27  | 0.83               | 714.53      | 0.76               | 758.75      | 0.79               | 793.26      | 0.81               | 833.43      | 0.69               | 859.82      |
| c2_4_1 <sup>3</sup>    | 400 | 15  | 0.67               | 1738.73     | 0.61               | 1737.37     | 0.57               | 1715.61     | 0.66               | 1776.34     | 0.53               | 1788.57     |
| c1_4_1 <sup>3</sup>    | 400 | 63  | 0.10               | 1303.45     | 0.06               | 1443.32     | 0.05               | 1528.06     | 0.05               | 1578.59     | 0.06               | 1671.75     |
| G20 <sup>1</sup>       | 420 | 38  | 0.92               | 1954.09     | 0.89               | 2114.32     | 0.91               | 2213.09     | 0.85               | 2272.95     | 0.84               | 2412.72     |
| G16 <sup>1</sup>       | 480 | 37  | 1.17               | 1943.82     | 1.17               | 2093.85     | 1.19               | 2169.48     | 1.18               | 2231.98     | 1.14               | 2349.35     |
| Average                |     |     | 0.30               | 394.28      | 0.29               | 418.46      | 0.28               | 433.25      | 0.29               | 448.19      | 0.27               | 467.42      |

(1) CVRP, (2) OVRP, (3) VRPSD, (4) VRPMPD

composed by 20 instances with 240-483 customers and it was proposed by Golden et al. [80]. In this case 8 instances include route duration constraints (G1-G8).

Tables 6.3-6.4 present the results obtained in the first group of CVRP instances. It can be observed that, except for the instance B-n63-k10, the ILS-RVND managed to find all proven optimal solutions. For the three instances where the optimal solution is not known, the ILS-RVND equaled the BKS of one of them (M-n151-k12) but failed to obtain the BKS for the other two (M-n200-k16 and M-n200-k17). Notice that the computational time spent by ILS-RVND on the instance M-n200-k16 was considerably higher when compared to the instance M-n200-k17. This occurred because the algorithm struggled to generate initial feasible solutions due to the instance tightness ( $(\sum_{i \in V'} d_i)/(mQ) = 0.995$ ).

Table 6.5 contains the results found in the instances of Christofides et al. [31] and a comparison with those reported by Rochat and Taillard [147], Pisinger and Røpke (ALNS 50K) [136], Mester and Bräysy [118], Nagata and Bräysy [127] and Vidal et al. [177]. The performance of the proposed algorithm was quite similar to the one developed by Pisinger and Røpke [136], being successful to equal the BKS in 11 of the 14 instances. The average gap between the Avg. Sols. found by ILS-RVND and the BKSs was 0.26%.

Table 6.6 illustrates a comparison, in terms of average solution, between the results obtained by ILS-RVND and those found by Pisinger and Røpke (ALNS 50K) [136], Nagata and Bräysy [127] and Zachariadis Kiranoudis [187] for the instances of Golden et al. [80].

Table 6.3: Results found for the A and B CVRP instances

| Instance  | $n$ | $v$ | BKS                      | ILS-RVND    |                |         |              |          |
|-----------|-----|-----|--------------------------|-------------|----------------|---------|--------------|----------|
|           |     |     |                          | Best Sol.   | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| A-n32-k5  | 31  | 5   | <sup>a</sup> <b>784</b>  | <b>784</b>  | <b>784.00</b>  | 0.00    | 0.00         | 0.63     |
| A-n33-k5  | 32  | 5   | <sup>a</sup> <b>661</b>  | <b>661</b>  | <b>661.00</b>  | 0.00    | 0.00         | 0.66     |
| A-n33-k6  | 32  | 6   | <sup>a</sup> <b>742</b>  | <b>742</b>  | <b>742.00</b>  | 0.00    | 0.00         | 0.63     |
| A-n34-k5  | 33  | 5   | <sup>a</sup> <b>778</b>  | <b>778</b>  | <b>778.00</b>  | 0.00    | 0.00         | 0.72     |
| A-n36-k5  | 35  | 5   | <sup>a</sup> <b>799</b>  | <b>799</b>  | <b>799.00</b>  | 0.00    | 0.00         | 0.87     |
| A-n37-k5  | 36  | 5   | <sup>a</sup> <b>669</b>  | <b>669</b>  | <b>669.00</b>  | 0.00    | 0.00         | 0.88     |
| A-n37-k6  | 36  | 6   | <sup>a</sup> <b>949</b>  | <b>949</b>  | <b>949.00</b>  | 0.00    | 0.00         | 0.99     |
| A-n38-k5  | 37  | 5   | <sup>a</sup> <b>730</b>  | <b>730</b>  | <b>730.00</b>  | 0.00    | 0.00         | 0.88     |
| A-n39-k5  | 38  | 5   | <sup>a</sup> <b>822</b>  | <b>822</b>  | <b>822.00</b>  | 0.00    | 0.00         | 1.07     |
| A-n39-k6  | 38  | 6   | <sup>a</sup> <b>831</b>  | <b>831</b>  | 831.88         | 0.00    | 0.11         | 0.93     |
| A-n44-k6  | 43  | 6   | <sup>a</sup> <b>937</b>  | <b>937</b>  | <b>937.00</b>  | 0.00    | 0.00         | 1.62     |
| A-n45-k6  | 44  | 6   | <sup>a</sup> <b>944</b>  | <b>944</b>  | 944.24         | 0.00    | 0.03         | 1.32     |
| A-n45-k7  | 44  | 7   | <sup>a</sup> <b>1146</b> | <b>1146</b> | <b>1146.00</b> | 0.00    | 0.00         | 1.40     |
| A-n46-k7  | 45  | 7   | <sup>a</sup> <b>914</b>  | <b>914</b>  | <b>914.00</b>  | 0.00    | 0.00         | 1.14     |
| A-n48-k7  | 47  | 7   | <sup>a</sup> <b>1073</b> | <b>1073</b> | <b>1073.00</b> | 0.00    | 0.00         | 1.37     |
| A-n53-k7  | 52  | 7   | <sup>a</sup> <b>1010</b> | <b>1010</b> | 1010.02        | 0.00    | < 0.01       | 1.76     |
| A-n54-k7  | 53  | 7   | <sup>a</sup> <b>1167</b> | <b>1167</b> | <b>1167.00</b> | 0.00    | 0.00         | 2.04     |
| A-n55-k9  | 54  | 9   | <sup>a</sup> <b>1073</b> | <b>1073</b> | <b>1073.00</b> | 0.00    | 0.00         | 1.70     |
| A-n60-k9  | 59  | 9   | <sup>a</sup> <b>1354</b> | <b>1354</b> | <b>1354.00</b> | 0.00    | 0.00         | 3.01     |
| A-n61-k9  | 60  | 9   | <sup>a</sup> <b>1034</b> | <b>1034</b> | 1034.18        | 0.00    | 0.02         | 2.80     |
| A-n62-k8  | 61  | 8   | <sup>a</sup> <b>1288</b> | <b>1288</b> | 1289.92        | 0.00    | 0.15         | 2.92     |
| A-n63-k10 | 62  | 10  | <sup>a</sup> <b>1314</b> | <b>1314</b> | 1316.82        | 0.00    | 0.21         | 2.92     |
| A-n63-k9  | 62  | 9   | <sup>a</sup> <b>1616</b> | <b>1616</b> | 1618.60        | 0.00    | 0.16         | 2.98     |
| A-n64-k9  | 63  | 9   | <sup>a</sup> <b>1401</b> | <b>1401</b> | 1406.26        | 0.00    | 0.38         | 3.14     |
| A-n65-k9  | 64  | 9   | <sup>a</sup> <b>1174</b> | <b>1174</b> | 1175.80        | 0.00    | 0.15         | 2.63     |
| A-n69-k9  | 68  | 9   | <sup>a</sup> <b>1159</b> | <b>1159</b> | 1159.40        | 0.00    | 0.03         | 3.54     |
| A-n80-k10 | 79  | 10  | <sup>a</sup> <b>1763</b> | <b>1763</b> | 1763.66        | 0.00    | 0.04         | 6.18     |
| B-n31-k5  | 30  | 5   | <sup>a</sup> <b>672</b>  | <b>672</b>  | <b>672.00</b>  | 0.00    | 0.00         | 0.61     |
| B-n34-k5  | 33  | 5   | <sup>a</sup> <b>788</b>  | <b>788</b>  | <b>788.00</b>  | 0.00    | 0.00         | 0.67     |
| B-n35-k5  | 34  | 5   | <sup>a</sup> <b>955</b>  | <b>955</b>  | <b>955.00</b>  | 0.00    | 0.00         | 0.75     |
| B-n38-k6  | 37  | 6   | <sup>a</sup> <b>805</b>  | <b>805</b>  | <b>805.00</b>  | 0.00    | 0.00         | 1.10     |
| B-n39-k5  | 38  | 5   | <sup>a</sup> <b>549</b>  | <b>549</b>  | <b>549.00</b>  | 0.00    | 0.00         | 1.04     |
| B-n41-k6  | 40  | 6   | <sup>a</sup> <b>829</b>  | <b>829</b>  | <b>829.00</b>  | 0.00    | 0.00         | 1.33     |
| B-n43-k6  | 42  | 6   | <sup>a</sup> <b>742</b>  | <b>742</b>  | <b>742.00</b>  | 0.00    | 0.00         | 1.34     |
| B-n44-k7  | 43  | 7   | <sup>a</sup> <b>909</b>  | <b>909</b>  | <b>909.00</b>  | 0.00    | 0.00         | 1.02     |
| B-n45-k5  | 44  | 5   | <sup>a</sup> <b>751</b>  | <b>751</b>  | <b>751.00</b>  | 0.00    | 0.00         | 1.45     |
| B-n45-k6  | 44  | 6   | <sup>a</sup> <b>678</b>  | <b>678</b>  | <b>678.00</b>  | 0.00    | 0.00         | 1.51     |
| B-n50-k7  | 49  | 7   | <sup>a</sup> <b>741</b>  | <b>741</b>  | <b>741.00</b>  | 0.00    | 0.00         | 1.00     |
| B-n50-k8  | 49  | 8   | <sup>a</sup> <b>1312</b> | <b>1312</b> | 1313.40        | 0.00    | 0.11         | 1.73     |
| B-n51-k7  | 50  | 7   | <sup>a</sup> <b>1032</b> | <b>1032</b> | <b>1032.00</b> | 0.00    | 0.00         | 1.55     |
| B-n52-k7  | 51  | 7   | <sup>a</sup> <b>747</b>  | <b>747</b>  | <b>747.00</b>  | 0.00    | 0.00         | 1.64     |
| B-n56-k7  | 55  | 7   | <sup>a</sup> <b>707</b>  | <b>707</b>  | <b>707.00</b>  | 0.00    | 0.00         | 2.02     |
| B-n57-k7  | 56  | 7   | <sup>a</sup> <b>1153</b> | <b>1153</b> | <b>1153.00</b> | 0.00    | 0.00         | 2.75     |
| B-n57-k9  | 56  | 9   | <sup>a</sup> <b>1598</b> | <b>1598</b> | <b>1598.00</b> | 0.00    | 0.00         | 2.42     |
| B-n63-k10 | 62  | 10  | <sup>a</sup> <b>1496</b> | 1497        | 1504.78        | 0.07    | 0.59         | 2.65     |
| B-n64-k9  | 63  | 9   | <sup>a</sup> <b>861</b>  | <b>861</b>  | <b>861.00</b>  | 0.00    | 0.00         | 2.91     |
| B-n66-k9  | 65  | 9   | <sup>a</sup> <b>1316</b> | <b>1316</b> | 1316.24        | 0.00    | 0.02         | 3.26     |
| B-n67-k10 | 66  | 10  | <sup>a</sup> <b>1032</b> | <b>1032</b> | 1033.34        | 0.00    | 0.13         | 3.48     |
| B-n68-k9  | 67  | 9   | <sup>a</sup> <b>1272</b> | <b>1272</b> | 1277.10        | 0.00    | 0.40         | 3.58     |
| B-n78-k10 | 77  | 10  | <sup>a</sup> <b>1221</b> | <b>1221</b> | <b>1221.02</b> | 0.00    | 0.00         | 5.86     |
|           |     |     |                          | Avg.        | 0.00           | 0.05    | 1.93         |          |

<sup>a</sup>: Optimality proved.

It can be seen that the ILS-RVND outperformed the algorithm of Pisinger and R pke [136], but it could not compete with those of Nagata and Br ysy [127], Vidal et al. [177] and Zachariadis and Kiranoudis [187] in terms of average solution quality. Yet, the average gap between the Avg. Sols. found by ILS-RVND and the BKSs was only 1.03%, a value smaller the one obtained by the general heuristic of Pisinger and R pke [136]. On the other hand, in spite of obtaining slightly lower quality solutions, the proposed algorithm



appears to be rather simple than those developed in [127], [177] and [187].

Table 6.4: Results found for the E, F, M and P CVRP instances

| Instance   | $n$ | $v$ | BKS                      | ILS-RVND    |                |         |              |          |
|------------|-----|-----|--------------------------|-------------|----------------|---------|--------------|----------|
|            |     |     |                          | Best Sol.   | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| E-n23-k3   | 22  | 3   | <sup>a</sup> <b>569</b>  | <b>569</b>  | <b>569.00</b>  | 0.00    | 0.00         | 0.25     |
| E-n30-k3   | 29  | 3   | <sup>a</sup> <b>534</b>  | <b>534</b>  | <b>534.00</b>  | 0.00    | 0.00         | 0.45     |
| E-n33-k4   | 32  | 4   | <sup>a</sup> <b>835</b>  | <b>835</b>  | <b>835.00</b>  | 0.00    | 0.00         | 0.63     |
| E-n51-k5   | 50  | 5   | <sup>a</sup> <b>521</b>  | <b>521</b>  | <b>521.00</b>  | 0.00    | 0.00         | 1.98     |
| E-n76-k10  | 75  | 10  | <sup>a</sup> <b>830</b>  | <b>830</b>  | 831.58         | 0.00    | 0.19         | 5.08     |
| E-n76-k14  | 75  | 14  | <sup>a</sup> <b>1021</b> | <b>1021</b> | 1022.12        | 0.00    | 0.11         | 6.79     |
| E-n76-k7   | 75  | 7   | <sup>a</sup> <b>682</b>  | <b>682</b>  | 682.40         | 0.00    | 0.06         | 4.61     |
| E-n76-k8   | 75  | 8   | <sup>a</sup> <b>735</b>  | <b>735</b>  | 735.46         | 0.00    | 0.06         | 4.50     |
| E-n101-k8  | 100 | 8   | <sup>a</sup> <b>815</b>  | <b>815</b>  | 815.50         | 0.00    | 0.06         | 10.98    |
| E-n101-k14 | 100 | 14  | <sup>a</sup> <b>1067</b> | <b>1067</b> | 1073.36        | 0.00    | 0.60         | 10.81    |
| F-n135-k7  | 134 | 7   | <sup>a</sup> <b>1162</b> | <b>1162</b> | 1162.30        | 0.00    | 0.03         | 30.26    |
| F-n45-k4   | 44  | 4   | <sup>a</sup> <b>724</b>  | <b>724</b>  | <b>724.00</b>  | 0.00    | 0.00         | 1.15     |
| F-n72-k4   | 71  | 4   | <sup>a</sup> <b>237</b>  | <b>237</b>  | <b>237.00</b>  | 0.00    | 0.00         | 5.03     |
| M-n101-k10 | 100 | 10  | <sup>a</sup> <b>820</b>  | <b>820</b>  | <b>820.00</b>  | 0.00    | 0.00         | 6.93     |
| M-n121-k7  | 120 | 7   | <sup>a</sup> <b>1034</b> | <b>1034</b> | <b>1034.00</b> | 0.00    | 0.00         | 18.55    |
| M-n151-k12 | 150 | 12  | <sup>b</sup> <b>1015</b> | <b>1015</b> | 1020.56        | 0.00    | 0.55         | 32.88    |
| M-n200-k16 | 199 | 16  | <sup>c</sup> <b>1285</b> | 1290        | 1308.88        | 0.39    | 1.86         | 129.93   |
| M-n200-k17 | 199 | 17  | <sup>b</sup> <b>1275</b> | 1282        | 1290.84        | 0.55    | 1.24         | 73.99    |
| P-n16-k8   | 15  | 8   | <sup>a</sup> <b>450</b>  | <b>450</b>  | <b>450.00</b>  | 0.00    | 0.00         | 0.15     |
| P-n19-k2   | 14  | 2   | <sup>a</sup> <b>212</b>  | <b>212</b>  | <b>212.00</b>  | 0.00    | 0.00         | 0.15     |
| P-n20-k2   | 19  | 2   | <sup>a</sup> <b>216</b>  | <b>216</b>  | <b>216.00</b>  | 0.00    | 0.00         | 0.15     |
| P-n21-k2   | 20  | 2   | <sup>a</sup> <b>211</b>  | <b>211</b>  | <b>211.00</b>  | 0.00    | 0.00         | 0.15     |
| P-n22-k2   | 21  | 2   | <sup>a</sup> <b>216</b>  | <b>216</b>  | <b>216.00</b>  | 0.00    | 0.00         | 0.17     |
| P-n22-k8   | 21  | 8   | <sup>a</sup> <b>603</b>  | <b>603</b>  | <b>603.00</b>  | 0.00    | 0.00         | 0.30     |
| P-n23-k8   | 22  | 8   | <sup>a</sup> <b>529</b>  | <b>529</b>  | <b>529.00</b>  | 0.00    | 0.00         | 0.77     |
| P-n40-k5   | 39  | 5   | <sup>a</sup> <b>458</b>  | <b>458</b>  | <b>458.00</b>  | 0.00    | 0.00         | 1.01     |
| P-n45-k5   | 44  | 5   | <sup>a</sup> <b>510</b>  | <b>510</b>  | <b>510.00</b>  | 0.00    | 0.00         | 1.42     |
| P-n50-k7   | 49  | 7   | <sup>a</sup> <b>554</b>  | <b>554</b>  | <b>554.00</b>  | 0.00    | 0.00         | 1.48     |
| P-n50-k8   | 49  | 8   | <sup>a</sup> <b>631</b>  | <b>631</b>  | 632.90         | 0.00    | 0.30         | 9.61     |
| P-n50-k10  | 49  | 10  | <sup>a</sup> <b>696</b>  | <b>696</b>  | 696.90         | 0.00    | 0.13         | 1.56     |
| P-n51-k10  | 50  | 10  | <sup>a</sup> <b>741</b>  | <b>741</b>  | <b>741.00</b>  | 0.00    | 0.00         | 1.89     |
| P-n55-k7   | 54  | 7   | <sup>a</sup> <b>568</b>  | <b>568</b>  | 568            | 0.00    | 0.00         | 1.17     |
| P-n55-k8   | 54  | 8   | <sup>a</sup> <b>588</b>  | <b>588</b>  | 588            | 0.00    | 0.00         | 1.76     |
| P-n55-k10  | 54  | 10  | <sup>a</sup> <b>694</b>  | <b>694</b>  | 695.20         | 0.00    | 0.17         | 1.94     |
| P-n55-k15  | 54  | 15  | <sup>a</sup> <b>989</b>  | <b>989</b>  | 999.42         | 0.00    | 0.35         | 2.76     |
| P-n60-k10  | 59  | 10  | <sup>a</sup> <b>744</b>  | <b>744</b>  | <b>744.00</b>  | 0.00    | 0.00         | 2.49     |
| P-n60-k15  | 59  | 15  | <sup>a</sup> <b>968</b>  | <b>968</b>  | 968.14         | 0.00    | 0.01         | 2.46     |
| P-n65-k10  | 64  | 10  | <sup>a</sup> <b>792</b>  | <b>792</b>  | <b>792.00</b>  | 0.00    | 0.00         | 2.81     |
| P-n70-k10  | 69  | 10  | <sup>a</sup> <b>827</b>  | <b>827</b>  | 827.52         | 0.00    | 0.06         | 3.82     |
| P-n76-k4   | 75  | 4   | <sup>a</sup> <b>593</b>  | <b>593</b>  | 593.04         | 0.00    | 0.01         | 7.29     |
| P-n76-k5   | 75  | 5   | <sup>a</sup> <b>627</b>  | <b>627</b>  | <b>627.00</b>  | 0.00    | 0.00         | 7.18     |
| P-n101-k4  | 100 | 4   | <sup>a</sup> <b>681</b>  | <b>681</b>  | <b>681.00</b>  | 0.00    | 0.00         | 17.04    |
|            |     |     |                          | Avg.        | 0.02           | 0.13    | 9.65         |          |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Value presented in [63]. <sup>c</sup>: Value obtained in [173].

Table 6.5: Results found for the CVRP instances of Christofides et al. [31]

| Instance | $n$ | $v$ | BKS                 | Rochat and Taillard [147] |      | Pisinger and Ropke [136] |                       | Mester and Bräysy [118] |                       | Nagata and Bräysy (2009) |                       | Vidal et al. [177] |                       | ILS-RVND  |           |         |              |          |
|----------|-----|-----|---------------------|---------------------------|------|--------------------------|-----------------------|-------------------------|-----------------------|--------------------------|-----------------------|--------------------|-----------------------|-----------|-----------|---------|--------------|----------|
|          |     |     |                     | Best Sol.                 | Time | Best Sol.                | Time <sup>1</sup> (s) | Best Sol.               | Time <sup>2</sup> (s) | Best Sol.                | Time <sup>3</sup> (s) | Best Sol.          | Time <sup>4</sup> (s) | Best Sol. | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |     |                     |                           |      |                          |                       |                         |                       |                          |                       |                    |                       |           |           |         |              |          |
| C1       | 50  | 5   | <sup>a</sup> 524.61 | 524.61                    | -    | 524.61                   | 21                    | 524.61                  | 0.2                   | 524.61                   | 4.3                   | 524.61             | 25.8                  | 524.61    | 524.61    | 0.00    | 0.00         | 2.11     |
| C2       | 75  | 10  | 835.26              | 835.26                    | -    | 835.26                   | 36                    | 835.26                  | 5.5                   | 835.26                   | 22.3                  | 835.26             | 57.6                  | 835.26    | 835.73    | 0.00    | 0.06         | 5.49     |
| C3       | 100 | 8   | 826.14              | 826.14                    | -    | 826.14                   | 78                    | 826.14                  | 1.0                   | 826.14                   | 17.1                  | 826.14             | 76.2                  | 826.14    | 826.33    | 0.00    | 0.02         | 12.87    |
| C12      | 100 | 10  | 819.56              | 819.56                    | -    | 819.56                   | 73                    | 819.56                  | 0.2                   | 819.56                   | 8.1                   | 819.56             | 50.4                  | 819.56    | 819.56    | 0.00    | 0.00         | 7.69     |
| C11      | 120 | 7   | 1042.11             | 1042.11                   | -    | 1042.11                  | 113                   | 1042.11                 | 1.1                   | 1042.11                  | 21                    | 1042.11            | 69.0                  | 1042.11   | 1042.11   | 0.00    | 0.00         | 23.58    |
| C4       | 150 | 12  | 1028.42             | 1028.42                   | -    | 1028.42                  | 160                   | 1028.42                 | 10.2                  | 1028.42                  | 75.2                  | 1028.42            | 172.2                 | 1028.42   | 1032.16   | 0.00    | 0.36         | 39.75    |
| C5       | 199 | 17  | 1291.29             | 1291.45                   | -    | 1297.12                  | 219                   | 1291.29                 | 2160.0                | 1291.45                  | 302.1                 | 1291.45            | 356.4                 | 1291.74   | 1305.43   | 0.02    | 1.08         | 92.25    |
| C6       | 50  | 6   | 555.43              | 555.43                    | -    | 555.43                   | 21                    | 555.43                  | 4.2                   | 555.43                   | 5.1                   | 555.43             | 28.8                  | 555.43    | 556.84    | 0.00    | 0.25         | 1.43     |
| C7       | 75  | 11  | 909.68              | 909.68                    | -    | 909.68                   | 36                    | 909.68                  | 0.8                   | 909.68                   | 38.9                  | 909.68             | 65.4                  | 909.68    | 910.70    | 0.00    | 0.11         | 6.24     |
| C8       | 100 | 9   | 865.94              | 865.94                    | -    | 865.94                   | 78                    | 865.94                  | 0.8                   | 865.94                   | 23.5                  | 865.94             | 68.4                  | 865.94    | 866.07    | 0.00    | 0.02         | 10.21    |
| C14      | 100 | 11  | 866.37              | 866.37                    | -    | 866.37                   | 73                    | 866.37                  | 1.7                   | 866.37                   | 13.2                  | 866.37             | 71.4                  | 866.37    | 866.37    | 0.00    | 0.00         | 8.54     |
| C13      | 120 | 11  | 1541.14             | 1541.14                   | -    | 1542.86                  | 113                   | 1541.14                 | 13.5                  | 1541.14                  | 106.9                 | 1541.14            | 169.8                 | 1542.86   | 1544.57   | 0.11    | 0.22         | 18.61    |
| C9       | 150 | 14  | 1162.55             | 1162.55                   | -    | 1163.68                  | 160                   | 1162.55                 | 25.8                  | 1162.55                  | 135.6                 | 1162.55            | 151.8                 | 1163.02   | 1167.78   | 0.04    | 0.45         | 31.65    |
| C10      | 199 | 18  | 1395.85             | 1395.85                   | -    | 1405.88                  | 219                   | 1401.12                 | 52.2                  | 1395.85                  | 390.6                 | 1395.85            | 493.2                 | 1405.47   | 1411.50   | 0.69    | 1.12         | 69.40    |
|          |     |     |                     |                           |      |                          |                       |                         |                       |                          |                       |                    |                       | Avg.      |           | 0.06    | 0.26         | 23.56    |

<sup>a</sup>: Optimality proved.

1: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2: Average of 10 runs on a Pentium IV 2.8 GHz (2444 Mflop/s).

3: Average of 10 runs on an Opteron 2.4 GHz (3485 Mflop/s).

4: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz.

Table 6.6: Results found for the CVRP instances of Golden et al. [80]

| Instance | $n$ | $v$ | BKS                   | Pisinger and Ropke [136] |                       | Nagata and Bräysy [127] |                       | Vidal et al. [177] |                       | Zachariadis and Kiranoudis [187] |                       | ILS-RVND       |                |         |              |          |
|----------|-----|-----|-----------------------|--------------------------|-----------------------|-------------------------|-----------------------|--------------------|-----------------------|----------------------------------|-----------------------|----------------|----------------|---------|--------------|----------|
|          |     |     |                       | Avg. Sol.*               | Time <sup>1</sup> (s) | Avg. Sol.*              | Time <sup>2</sup> (s) | Avg. Sol.*         | Time <sup>3</sup> (s) | Avg. Sol.*                       | Time <sup>4</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| G17      | 240 | 22  | $a, b$ <b>707.76</b>  | 710.59                   | 304                   | 707.78                  | 582.4                 | 708.09             | 423.6                 | 708.94                           | 962.3                 | 708.06         | 709.21         | 0.04    | 0.20         | 176.95   |
| G13      | 252 | 26  | $a, b$ <b>857.19</b>  | 874.24                   | 285                   | 858.42                  | 921.9                 | 859.64             | 561.6                 | 860.44                           | 1189.3                | 861.52         | 865.64         | 0.51    | 0.99         | 171.14   |
| G9       | 255 | 14  | $b$ <b>579.71</b>     | 590.33                   | 437                   | 581.46                  | 1043.3                | 581.79             | 973.2                 | 584.66                           | 929.4                 | 586.90         | 589.54         | 1.24    | 1.69         | 260.16   |
| G18      | 300 | 27  | $a, b$ <b>995.13</b>  | 1007.84                  | 387                   | 995.91                  | 1465.9                | 998.44             | 993.6                 | 997.74                           | 1718.6                | 999.77         | 1003.13        | 0.47    | 0.80         | 489.37   |
| G14      | 320 | 30  | $a, b$ <b>1080.55</b> | 1103.53                  | 393                   | 1080.84                 | 1239.3                | 1082.41            | 847.2                 | 1083.55                          | 1187.4                | 1085.36        | 1092.07        | 0.45    | 1.07         | 344.61   |
| G10      | 323 | 16  | $b$ <b>736.26</b>     | 751.36                   | 616                   | 739.56                  | 1617.5                | 739.86             | 1551.6                | 739.86                           | 1271.4                | 744.07         | 749.16         | 1.06    | 1.75         | 598.85   |
| G19      | 360 | 33  | $b$ <b>1365.60</b>    | 1377.88                  | 449                   | 1366.70                 | 2115.6                | 1367.83            | 1674.6                | 1370.77                          | 1824.2                | 1370.46        | 1376.01        | 0.36    | 0.76         | 711.60   |
| G15      | 396 | 33  | $b$ <b>1337.92</b>    | 1366.23                  | 468                   | 1344.32                 | 1872.2                | 1343.52            | 2349.0                | 1344.41                          | 1658.8                | 1347.42        | 1354.24        | 0.71    | 1.22         | 784.28   |
| G11      | 399 | 18  | $b$ <b>912.84</b>     | 926.57                   | 761                   | 916.27                  | 2337.5                | 916.44             | 2736.6                | 919.52                           | 1392.2                | 922.53         | 928.23         | 1.06    | 1.69         | 1230.89  |
| G20      | 420 | 38  | $b$ <b>1818.32</b>    | 1834.70                  | 488                   | 1821.65                 | 2824.7                | 1822.02            | 2293.8                | 1829.57                          | 1199.3                | 1828.72        | 1836.06        | 0.57    | 0.98         | 1314.44  |
| G16      | 480 | 37  | $b$ <b>1612.50</b>    | 1645.67                  | 549                   | 1622.26                 | 2616.2                | 1621.02            | 3496.2                | 1623.42                          | 1848.5                | 1628.62        | 1635.62        | 1.00    | 1.43         | 1247.34  |
| G12      | 483 | 19  | $b$ <b>1102.69</b>    | 1125.22                  | 911                   | 1108.21                 | 3561.9                | 1106.73            | 5740.2                | 1110.65                          | 1282.3                | 1116.46        | 1121.24        | 1.25    | 1.68         | 2969.83  |
| G5       | 200 | 5   | <b>6460.98</b>        | 6482.49                  | 629                   | <b>6460.98</b>          | 164.7                 | <b>6460.98</b>     | 153.6                 | <b>6460.98</b>                   | 989.6                 | <b>6460.98</b> | <b>6460.98</b> | 0.00    | 0.00         | 112.90   |
| G1       | 240 | 9   | $b$ <b>5623.47</b>    | 5662.57                  | 93                    | 5632.05                 | 3393                  | 5627.00            | 700.8                 | 5637.99                          | 907.7                 | 5654.66        | 5711.19        | 0.55    | 1.56         | 152.09   |
| G6       | 280 | 7   | $c$ <b>8412.88</b>    | 8543.30                  | 876                   | 8413.41                 | 830.3                 | 8412.90            | 502.8                 | 8412.90                          | 1091.6                | 8412.90        | 8412.90        | 0.00    | 0.00         | 333.06   |
| G2       | 320 | 10  | $b$ <b>8404.61</b>    | 8487.94                  | 672                   | 8440.25                 | 1726.2                | 8446.65            | 1245.0                | 8457.92                          | 1249.4                | 8455.11        | 8480.62        | 0.60    | 0.90         | 441.98   |
| G7       | 360 | 9   | $b$ <b>10102.70</b>   | 10265.15                 | 941                   | 10186.93                | 2179.7                | 10157.63           | 1376.4                | 10192.47                         | 1885.5                | 10195.60       | 10228.80       | 0.92    | 1.25         | 681.27   |
| G3       | 400 | 10  | <b>11036.22</b>       | 11052.72                 | 1015                  | <b>11036.22</b>         | 2606.8                | <b>11036.22</b>    | 1679.4                | <b>11036.22</b>                  | 1164.0                | 11038.60       | 11076.20       | 0.02    | 0.36         | 953.88   |
| G8       | 440 | 10  | $b$ <b>11635.30</b>   | 11766.07                 | 1011                  | 11691.54                | 5776.7                | 11646.58           | 2440.2                | 11674.43                         | 1657.4                | 11704.50       | 11842.80       | 0.59    | 1.78         | 1292.34  |
| G4       | 480 | 10  | $a$ <b>13592.88</b>   | 13748.50                 | 1328                  | 13618.55                | 3841.6                | 13624.52           | 2620.2                | 13632.59                         | 1019.0                | 13624.52       | 13665.85       | 0.23    | 0.54         | 1702.90  |
|          |     |     |                       | Avg. Gap (%)             |                       | Avg. Gap (%)            |                       | Avg. Gap (%)       |                       | Avg. Gap (%)                     |                       | Avg.           |                | 0.58    |              | 798.49   |
|          |     |     |                       | 1.34                     |                       | 0.27                    |                       | 0.26               |                       | 0.42                             |                       |                |                |         |              |          |

1: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2: Average of 10 runs on an Opteron 2.4 GHz (3485 Mflop/s)

3: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz.

4: Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).

\*: Average of 10 runs

 $a$ : Found by Nagata and Bräysy [127] $b$ : Found by Vidal et al. [177] $c$ : Found by Mester and Bräysy [118]

### 6.3.1.3 ACVRP

The ILS-RVND algorithm was tested in the ACVRP instances suggested by Fischetti et al. [57]. The capacity of the vehicle is the same ( $Q = 1000$ ) for all these instances and the number of customers varies between 33 and 70. Pessoa et al. [134] also considered the same data set of Fischetti et al. [57] but with different capacities (150, 250 and 500). The instances with  $Q = 150$  are not considered, since they can be easily solved using a Set Partitioning based formulation (most feasible routes contain very few customers and it is practical to perform a complete enumeration), thus leading to a total of 24 instances.

Table 6.7 shows the results found for the ACVRP instances. All the known optimal solutions were consistently found by ILS-RVND. Regarding the two instances where the optimal solutions is not known, the proposed algorithm was capable of improving the BKS in one of them (A071-05f), but the same did not happen with the other one (A071-10f).

Table 6.7: Results found for the ACVRP instances of Fischetti et al. [57] and Pessoa et al. [134]

| Instance | $n$ | $v$ | BKS               | ILS-RVND  |           |         |              |          |
|----------|-----|-----|-------------------|-----------|-----------|---------|--------------|----------|
|          |     |     |                   | Best Sol. | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| A034-02f | 34  | 2   | <sup>a</sup> 1406 | 1406      | 1406.00   | 0.00    | 0.00         | 0.56     |
| A034-04f | 34  | 4   | <sup>a</sup> 1773 | 1773      | 1773.00   | 0.00    | 0.00         | 0.68     |
| A034-08f | 34  | 8   | <sup>a</sup> 2672 | 2672      | 2672.00   | 0.00    | 0.00         | 0.73     |
| A036-03f | 36  | 3   | <sup>a</sup> 1644 | 1644      | 1644.00   | 0.00    | 0.00         | 0.79     |
| A036-05f | 36  | 5   | <sup>a</sup> 2110 | 2110      | 2110.00   | 0.00    | 0.00         | 0.82     |
| A036-10f | 36  | 10  | <sup>a</sup> 3338 | 3338      | 3338.00   | 0.00    | 0.00         | 9.23     |
| A039-03f | 39  | 3   | <sup>a</sup> 1654 | 1654      | 1654.00   | 0.00    | 0.00         | 0.85     |
| A039-06f | 39  | 6   | <sup>a</sup> 2289 | 2289      | 2289.00   | 0.00    | 0.00         | 1.08     |
| A039-12f | 39  | 12  | <sup>a</sup> 3705 | 3705      | 3705.00   | 0.00    | 0.00         | 1.12     |
| A045-03f | 45  | 3   | <sup>a</sup> 1740 | 1740      | 1740.00   | 0.00    | 0.00         | 1.33     |
| A045-06f | 45  | 6   | <sup>a</sup> 2303 | 2303      | 2303.36   | 0.00    | 0.02         | 1.60     |
| A045-11f | 45  | 11  | <sup>a</sup> 3544 | 3544      | 3553.00   | 0.00    | 0.25         | 2.57     |
| A048-03f | 48  | 3   | <sup>a</sup> 1891 | 1891      | 1891.00   | 0.00    | 0.00         | 1.61     |
| A048-05f | 48  | 5   | <sup>a</sup> 2283 | 2283      | 2289.66   | 0.00    | 0.29         | 1.91     |
| A048-10f | 48  | 10  | <sup>a</sup> 3325 | 3325      | 3325.66   | 0.00    | 0.02         | 1.51     |
| A056-03f | 56  | 3   | <sup>a</sup> 1739 | 1739      | 1739.56   | 0.00    | 0.03         | 2.46     |
| A056-05f | 56  | 5   | <sup>a</sup> 2165 | 2165      | 2175.70   | 0.00    | 0.49         | 2.42     |
| A056-10f | 56  | 10  | <sup>a</sup> 3263 | 3263      | 3309.80   | 0.00    | 1.43         | 2.10     |
| A065-03f | 65  | 3   | <sup>a</sup> 1974 | 1974      | 1974.00   | 0.00    | 0.00         | 3.78     |
| A065-06f | 65  | 6   | <sup>a</sup> 2567 | 2567      | 2572.80   | 0.00    | 0.23         | 4.46     |
| A065-12f | 65  | 12  | <sup>a</sup> 3902 | 3902      | 3925.36   | 0.00    | 0.60         | 3.06     |
| A071-03f | 71  | 3   | <sup>a</sup> 2054 | 2054      | 2054.00   | 0.00    | 0.00         | 5.66     |
| A071-05f | 71  | 5   | <sup>b</sup> 2475 | 2457      | 2463.58   | -0.73   | -0.46        | 6.16     |
| A071-10f | 71  | 10  | <sup>b</sup> 3486 | 3489      | 3512.24   | 0.09    | 0.75         | 4.42     |
|          |     |     |                   | Avg.      |           | -0.03   | 0.15         | 2.54     |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Value presented in [134].

#### 6.3.1.4 OVRP

To examine the behavior of the ILS-RVND algorithm when applied to solve the OVRP, use was made of the CVRP A, B, E, F, M, and P instances, but without rounding up the distance matrix. In this set of instances, the maximum number of vehicles is assumed as an input data. The instances of Christofides et al. [31] without and with route durations constraints (see Brandão [21] for more details), as well as another two generated by Fisher [58] were also considered. Finally, the proposed heuristic was also tested in the large-sized instances suggested by Li et al. [105] involving 200-480 customers. For these last two benchmark data sets, the primary objective is to minimize the number of vehicles, whereas the secondary one consists in minimizing the distance traveled.

Tables 6.8-6.9 illustrate the results obtained in the first group of instances. All known optimal solutions were found with the exception instances B-n50-k8, B-n57-k9, M-n200-k16 and P-n60-k15. Furthermore, the result of the instance M-n200-k17 was improved. The overall behavior of the algorithm in this benchmark data set was quite similar to the one verified for the CVRP.

Table 6.10 presents the results found by ILS-RVND in the second and third group of instances and a comparison with those pointed out by Pisinger and Røpke (ALNS 50K) [136], Fleszar et al. [60], Repoussis et al. [146] and Zachariadis and Kiranoudis [186]. Regarding those of Christofides et al. [31] and Fisher [58], ILS-RVND was capable of obtaining the BKS in 11 cases and to improve another 2 solutions, but it failed to find 3 BKSs. Furthermore, ILS-RVND also failed to always obtain solutions with the minimum number of vehicles on instances C7 and C9. The average gap between the Avg. Sols. obtained by ILS-RVND and the BKSs, disregarding those two instances where the average number of vehicles found by the proposed algorithm was larger than those associated to the BKSs, was 0.62%. As for the 8 instances of Li et al. [105], ILS-RVND improved 7 results and the average gap between the Avg. Sols produced by ILS-RVND and the BKSs was 0.19%. Also, the developed algorithm was successful to generate feasible solutions using  $v_{min}$  vehicles in all instances of this group.

Table 6.8: Results found for the A and B OVRP instances

| Instance  | $n$ | $v$ | BKS                         | ILS-RVND       |               |         |              |          |
|-----------|-----|-----|-----------------------------|----------------|---------------|---------|--------------|----------|
|           |     |     |                             | Best Sol.      | Avg. Sol.     | Gap (%) | Avg. Gap (%) | Time (s) |
| A-n32-k5  | 31  | 5   | <sup>a</sup> <b>487.31</b>  | <b>487.31</b>  | <b>487.31</b> | 0.00    | 0.00         | 0.68     |
| A-n33-k5  | 32  | 5   | <sup>a</sup> <b>424.54</b>  | <b>424.54</b>  | <b>424.54</b> | 0.00    | 0.00         | 0.76     |
| A-n33-k6  | 32  | 6   | <sup>a</sup> <b>462.43</b>  | <b>462.43</b>  | <b>462.43</b> | 0.00    | 0.00         | 0.80     |
| A-n34-k5  | 33  | 5   | <sup>a</sup> <b>508.17</b>  | <b>508.17</b>  | <b>508.17</b> | 0.00    | 0.00         | 0.88     |
| A-n36-k5  | 35  | 5   | <sup>a</sup> <b>519.46</b>  | <b>519.46</b>  | <b>519.46</b> | 0.00    | 0.00         | 1.10     |
| A-n37-k5  | 36  | 5   | <sup>a</sup> <b>486.24</b>  | <b>486.24</b>  | <b>486.24</b> | 0.00    | 0.00         | 1.22     |
| A-n37-k6  | 36  | 6   | <sup>a</sup> <b>581.07</b>  | <b>581.07</b>  | <b>581.07</b> | 0.00    | 0.00         | 1.31     |
| A-n38-k5  | 37  | 5   | <sup>a</sup> <b>498.00</b>  | <b>498.00</b>  | <b>498.00</b> | 0.00    | 0.00         | 1.09     |
| A-n39-k5  | 38  | 5   | <sup>a</sup> <b>549.68</b>  | <b>549.68</b>  | <b>549.68</b> | 0.00    | 0.00         | 1.47     |
| A-n39-k6  | 38  | 6   | <sup>a</sup> <b>533.07</b>  | <b>533.07</b>  | <b>533.07</b> | 0.00    | 0.00         | 1.09     |
| A-n44-k6  | 43  | 6   | <sup>a</sup> <b>617.39</b>  | <b>617.39</b>  | <b>617.39</b> | 0.00    | 0.00         | 2.07     |
| A-n45-k6  | 44  | 6   | <sup>a</sup> <b>648.67</b>  | <b>648.67</b>  | 649.26        | 0.00    | 0.09         | 1.72     |
| A-n45-k7  | 44  | 7   | <sup>a</sup> <b>685.16</b>  | <b>685.16</b>  | <b>685.16</b> | 0.00    | 0.00         | 1.64     |
| A-n46-k7  | 45  | 7   | <sup>a</sup> <b>583.54</b>  | <b>583.54</b>  | 584.37        | 0.00    | 0.14         | 1.18     |
| A-n48-k7  | 47  | 7   | <sup>a</sup> <b>669.83</b>  | <b>669.83</b>  | <b>669.83</b> | 0.00    | 0.00         | 1.76     |
| A-n53-k7  | 52  | 7   | <sup>a</sup> <b>655.18</b>  | <b>655.18</b>  | <b>655.18</b> | 0.00    | 0.00         | 2.47     |
| A-n54-k7  | 53  | 7   | <sup>a</sup> <b>709.27</b>  | <b>709.27</b>  | <b>709.27</b> | 0.00    | 0.00         | 2.96     |
| A-n55-k9  | 54  | 9   | <sup>a</sup> <b>669.06</b>  | <b>669.06</b>  | <b>669.06</b> | 0.00    | 0.00         | 2.10     |
| A-n60-k9  | 59  | 9   | <sup>b</sup> <b>798.01</b>  | <b>798.01</b>  | <b>798.01</b> | 0.00    | 0.00         | 3.67     |
| A-n61-k9  | 60  | 9   | <sup>b</sup> <b>678.30</b>  | <b>678.30</b>  | 678.85        | 0.00    | 0.08         | 4.12     |
| A-n62-k8  | 61  | 8   | <sup>a</sup> <b>783.18</b>  | <b>783.18</b>  | 783.45        | 0.00    | 0.03         | 3.42     |
| A-n63-k9  | 62  | 9   | <sup>b</sup> <b>941.53</b>  | <b>941.53</b>  | 941.70        | 0.00    | 0.02         | 4.49     |
| A-n63-k10 | 62  | 10  | <sup>a</sup> <b>778.46</b>  | <b>778.46</b>  | 779.12        | 0.00    | 0.08         | 4.18     |
| A-n64-k9  | 63  | 9   | <sup>a</sup> <b>848.15</b>  | <b>848.15</b>  | 848.39        | 0.00    | 0.03         | 3.99     |
| A-n65-k9  | 64  | 9   | <sup>a</sup> <b>728.59</b>  | <b>728.59</b>  | 728.68        | 0.00    | 0.01         | 3.83     |
| A-n69-k9  | 68  | 9   | <sup>a</sup> <b>757.76</b>  | <b>757.76</b>  | 758.12        | 0.00    | 0.05         | 4.54     |
| A-n80-k10 | 79  | 10  | <sup>a</sup> <b>1067.09</b> | <b>1067.09</b> | 1068.63       | 0.00    | 0.14         | 7.68     |
| B-n31-k5  | 30  | 5   | <sup>a</sup> <b>362.73</b>  | <b>362.73</b>  | <b>362.73</b> | 0.00    | 0.00         | 0.76     |
| B-n34-k5  | 33  | 5   | <sup>a</sup> <b>458.76</b>  | <b>458.76</b>  | <b>458.76</b> | 0.00    | 0.00         | 0.78     |
| B-n35-k5  | 34  | 5   | <sup>a</sup> <b>557.33</b>  | <b>557.33</b>  | <b>557.33</b> | 0.00    | 0.00         | 0.82     |
| B-n38-k6  | 37  | 6   | <sup>a</sup> <b>445.63</b>  | <b>445.63</b>  | <b>445.63</b> | 0.00    | 0.00         | 1.02     |
| B-n39-k5  | 38  | 5   | <sup>a</sup> <b>322.54</b>  | <b>322.54</b>  | <b>322.54</b> | 0.00    | 0.00         | 1.30     |
| B-n41-k6  | 40  | 6   | <sup>a</sup> <b>483.07</b>  | <b>483.07</b>  | <b>483.07</b> | 0.00    | 0.00         | 1.96     |
| B-n43-k6  | 42  | 6   | <sup>a</sup> <b>428.17</b>  | <b>428.17</b>  | <b>428.17</b> | 0.00    | 0.00         | 1.67     |
| B-n44-k7  | 43  | 7   | <sup>a</sup> <b>501.31</b>  | <b>501.31</b>  | <b>501.31</b> | 0.00    | 0.00         | 1.17     |
| B-n45-k5  | 44  | 5   | <sup>a</sup> <b>488.07</b>  | <b>488.07</b>  | <b>488.07</b> | 0.00    | 0.00         | 1.77     |
| B-n45-k6  | 44  | 6   | <sup>a</sup> <b>403.81</b>  | <b>403.81</b>  | 405.18        | 0.00    | 0.34         | 1.92     |
| B-n50-k7  | 49  | 7   | <sup>a</sup> <b>437.15</b>  | <b>437.15</b>  | <b>437.15</b> | 0.00    | 0.00         | 1.67     |
| B-n50-k8  | 49  | 8   | <sup>a</sup> <b>720.79</b>  | <b>720.79</b>  | 722.03        | 0.17    | 0.17         | 2.58     |
| B-n51-k7  | 50  | 7   | <sup>a</sup> <b>625.14</b>  | <b>625.14</b>  | <b>625.14</b> | 0.00    | 0.00         | 1.88     |
| B-n52-k7  | 51  | 7   | <sup>a</sup> <b>441.19</b>  | <b>441.19</b>  | <b>441.19</b> | 0.00    | 0.00         | 1.98     |
| B-n56-k7  | 55  | 7   | <sup>a</sup> <b>420.49</b>  | <b>420.49</b>  | <b>420.49</b> | 0.00    | 0.00         | 2.45     |
| B-n57-k7  | 56  | 7   | <sup>a</sup> <b>646.36</b>  | <b>646.36</b>  | <b>646.36</b> | 0.00    | 0.00         | 4.66     |
| B-n57-k9  | 56  | 9   | <sup>b</sup> <b>869.31</b>  | 869.32         | 869.33        | < 0.01  | < 0.01       | 2.92     |
| B-n63-k10 | 62  | 10  | <sup>a</sup> <b>837.07</b>  | <b>837.07</b>  | <b>837.07</b> | 0.00    | 0.00         | 3.40     |
| B-n64-k9  | 63  | 9   | <sup>a</sup> <b>520.47</b>  | <b>520.47</b>  | 520.53        | 0.00    | 0.01         | 4.39     |
| B-n66-k9  | 65  | 9   | <sup>a</sup> <b>755.27</b>  | <b>755.27</b>  | <b>755.27</b> | 0.00    | 0.00         | 4.82     |
| B-n67-k10 | 66  | 10  | <sup>b</sup> <b>616.54</b>  | <b>616.54</b>  | 618.59        | 0.00    | 0.33         | 4.12     |
| B-n68-k9  | 67  | 9   | <sup>a</sup> <b>701.72</b>  | <b>701.72</b>  | <b>701.72</b> | 0.00    | 0.00         | 5.19     |
| B-n78-k10 | 77  | 10  | <sup>b</sup> <b>722.71</b>  | <b>722.71</b>  | 724.75        | 0.00    | 0.28         | 7.48     |
|           |     |     |                             |                |               | 0.00    | 0.03         | 2.54     |

<sup>a</sup>: Optimality proved.<sup>b</sup>: Optimality proved using the BCP algorithm of Pessoa et al. [134].

Table 6.9: Results found for the E, F, M and P OVRP instances

| Instance   | $n$ | $v$ | BKS                        | ILS-RVND      |               |         |              |          |
|------------|-----|-----|----------------------------|---------------|---------------|---------|--------------|----------|
|            |     |     |                            | Best Sol.     | Avg. Sol.     | Gap (%) | Avg. Gap (%) | Time (s) |
| E-n23-k3   | 22  | 3   | <sup>a</sup> <b>442.98</b> | <b>442.98</b> | <b>442.98</b> | 0.00    | 0.00         | 0.32     |
| E-n30-k3   | 29  | 3   | <sup>a</sup> <b>393.51</b> | <b>393.51</b> | <b>393.51</b> | 0.00    | 0.00         | 0.56     |
| E-n33-k4   | 32  | 4   | <sup>a</sup> <b>511.26</b> | <b>511.26</b> | <b>511.26</b> | 0.00    | 0.00         | 0.82     |
| E-n51-k5   | 50  | 5   | <sup>a</sup> <b>416.06</b> | <b>416.06</b> | <b>416.09</b> | 0.00    | 0.01         | 2.62     |
| E-n76-k7   | 75  | 7   | <sup>a</sup> <b>530.02</b> | <b>530.02</b> | 530.06        | 0.00    | 0.01         | 5.92     |
| E-n76-k8   | 75  | 8   | <sup>a</sup> <b>537.24</b> | <b>537.24</b> | 537.45        | 0.00    | 0.04         | 5.71     |
| E-n76-k10  | 75  | 10  | <sup>a</sup> <b>567.14</b> | <b>567.14</b> | 568.54        | 0.00    | 0.25         | 6.55     |
| E-n76-k14  | 75  | 14  | <sup>a</sup> <b>623.55</b> | <b>623.55</b> | 624.03        | 0.00    | 0.08         | 13.96    |
| E-n101-k8  | 100 | 8   | <sup>a</sup> <b>639.74</b> | <b>639.74</b> | 640.49        | 0.00    | 0.12         | 13.98    |
| E-n101-k14 | 100 | 14  | <sup>a</sup> <b>711.58</b> | <b>711.58</b> | 712.27        | 0.00    | 0.10         | 13.80    |
| F-n45-k4   | 44  | 4   | <sup>a</sup> <b>463.90</b> | <b>463.90</b> | <b>463.90</b> | 0.00    | 0.00         | 1.55     |
| F-n72-k4   | 71  | 4   | <sup>a</sup> <b>177.00</b> | <b>177.00</b> | 177.12        | 0.00    | 0.07         | 6.30     |
| F-n135-k7  | 134 | 7   | <sup>d</sup> <b>769.55</b> | <b>769.55</b> | 770.69        | 0.00    | 0.15         | 41.18    |
| M-n101-k10 | 100 | 10  | <sup>a</sup> <b>534.24</b> | <b>534.24</b> | <b>534.24</b> | 0.00    | 0.00         | 8.85     |
| M-n121-k7  | 120 | 7   | <sup>a</sup> <b>682.12</b> | <b>682.12</b> | 682.21        | 0.00    | 0.01         | 27.79    |
| M-n151-k12 | 150 | 12  | <sup>c</sup> <b>733.13</b> | <b>733.13</b> | 733.43        | 0.00    | 0.04         | 39.71    |
| M-n200-k16 | 199 | 16  | <sup>d</sup> <b>893.39</b> | 898.08        | 927.31        | 0.52    | 3.80         | 101.86   |
| M-n200-k17 | 199 | 17  | <sup>d</sup> <b>869.00</b> | <b>867.89</b> | 870.28        | -0.13   | 0.15         | 89.92    |
| P-n16-k8   | 15  | 8   | <sup>a</sup> <b>235.06</b> | <b>235.06</b> | <b>235.06</b> | 0.00    | 0.00         | 0.16     |
| P-n19-k2   | 18  | 2   | <sup>a</sup> <b>168.57</b> | <b>168.57</b> | <b>168.57</b> | 0.00    | 0.00         | 0.14     |
| P-n20-k2   | 19  | 2   | <sup>a</sup> <b>170.28</b> | <b>170.28</b> | <b>170.28</b> | 0.00    | 0.00         | 0.16     |
| P-n21-k2   | 20  | 2   | <sup>a</sup> <b>163.88</b> | <b>163.88</b> | <b>163.88</b> | 0.00    | 0.00         | 0.16     |
| P-n22-k2   | 21  | 2   | <sup>a</sup> <b>167.19</b> | <b>167.19</b> | <b>167.19</b> | 0.00    | 0.00         | 0.19     |
| P-n22-k8   | 21  | 8   | <sup>a</sup> <b>345.87</b> | <b>345.87</b> | <b>345.87</b> | 0.00    | 0.00         | 0.38     |
| P-n23-k8   | 22  | 8   | <sup>a</sup> <b>302.87</b> | <b>302.87</b> | 303.75        | 0.00    | 0.29         | 4.92     |
| P-n40-k5   | 39  | 5   | <sup>a</sup> <b>349.55</b> | <b>349.55</b> | <b>349.55</b> | 0.00    | 0.00         | 1.17     |
| P-n45-k5   | 44  | 5   | <sup>a</sup> <b>391.81</b> | <b>391.81</b> | <b>391.81</b> | 0.00    | 0.00         | 1.52     |
| P-n50-k7   | 49  | 7   | <sup>a</sup> <b>397.38</b> | <b>397.38</b> | <b>397.38</b> | 0.00    | 0.00         | 1.63     |
| P-n50-k8   | 49  | 8   | <sup>a</sup> <b>436.51</b> | <b>436.51</b> | 436.60        | 0.00    | 0.02         | 9.82     |
| P-n50-k10  | 49  | 10  | <sup>a</sup> <b>440.44</b> | <b>440.44</b> | <b>440.44</b> | 0.00    | 0.00         | 1.88     |
| P-n51-k10  | 50  | 10  | <sup>b</sup> <b>480.78</b> | <b>480.78</b> | 481.55        | 0.00    | 0.16         | 2.34     |
| P-n55-k7   | 54  | 7   | <sup>a</sup> <b>411.58</b> | <b>411.58</b> | <b>411.58</b> | 0.00    | 0.00         | 2.14     |
| P-n55-k8   | 54  | 7   | <sup>a</sup> <b>412.55</b> | <b>412.55</b> | <b>412.55</b> | 0.00    | 0.00         | 2.27     |
| P-n55-k10  | 54  | 10  | <sup>a</sup> <b>444.31</b> | <b>444.31</b> | <b>444.31</b> | 0.00    | 0.00         | 2.18     |
| P-n60-k10  | 59  | 10  | <sup>a</sup> <b>482.09</b> | <b>482.09</b> | 482.36        | 0.00    | 0.06         | 2.79     |
| P-n60-k15  | 59  | 15  | <sup>b</sup> <b>569.43</b> | 569.44        | 569.44        | < 0.01  | < 0.01       | 3.47     |
| P-n65-k10  | 64  | 10  | <sup>a</sup> <b>522.50</b> | <b>522.50</b> | 522.51        | 0.00    | < 0.01       | 3.53     |
| P-n70-k10  | 69  | 10  | <sup>a</sup> <b>552.65</b> | <b>552.65</b> | 553.49        | 0.00    | 0.15         | 5.11     |
| P-n76-k4   | 75  | 4   | <sup>a</sup> <b>522.95</b> | <b>522.95</b> | 524.15        | 0.00    | 0.23         | 8.90     |
| P-n76-k5   | 75  | 5   | <sup>a</sup> <b>525.64</b> | <b>525.64</b> | 525.78        | 0.00    | 0.03         | 9.23     |
| P-n101-k4  | 100 | 4   | <sup>a</sup> <b>621.75</b> | <b>621.75</b> | <b>621.75</b> | 0.00    | 0.00         | 21.57    |
|            |     |     |                            |               |               | 0.01    | 0.14         | 11.13    |

<sup>a</sup>: Optimality proved.<sup>b</sup>: Optimality proved using the BCP algorithm of Pessoa et al. [134].<sup>c</sup>: First found by Pisinger and Røpke [136].<sup>d</sup>: Found by Zachariadis and Kiranoudis [186].

Table 6.10: Results found for the instances of Christofides et al. [31], Fisher [58] and Li et al. [106]

| Instance | $n$ | $v_{min}$ | BKS                 | $v_{best}$ | Pisinger and Ropke [136] |     |                       | Fleszar et al. [60] |     |                       | Repoussis et al. [146] |     |                       | Zachariadis and Kiranoudis [186] |     |                       | ILS-RVND  |     |           |         |              |          |
|----------|-----|-----------|---------------------|------------|--------------------------|-----|-----------------------|---------------------|-----|-----------------------|------------------------|-----|-----------------------|----------------------------------|-----|-----------------------|-----------|-----|-----------|---------|--------------|----------|
|          |     |           |                     |            | Best Sol.                | $v$ | Time <sup>1</sup> (s) | Best Sol.           | $v$ | Time <sup>2</sup> (s) | Best Sol.              | $v$ | Time <sup>3</sup> (s) | Best Sol.                        | $v$ | Time <sup>4</sup> (s) | Best Sol. | $v$ | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |           |                     |            |                          |     |                       |                     |     |                       |                        |     |                       |                                  |     |                       |           |     |           |         |              |          |
| C1       | 50  | 5         | <sup>a</sup> 416.06 | 5          | 416.06                   | 5   | 23                    | 416.06              | 5   | 1.0                   | 416.06                 | 5   | 98                    | 416.06                           | 5   | 25                    | 416.06    | 5   | 416.09    | 0.00    | 0.01         | 2.62     |
| F11      | 71  | 4         | <sup>a</sup> 177.00 | 4          | 177.00                   | 4   | 104                   | 178.66              | 4   | 6.2                   | 177.00                 | 4   | 264                   | 177.00                           | 4   | 93                    | 177.00    | 4   | 177.12    | 0.00    | 0.07         | 6.30     |
| C2       | 75  | 10        | <sup>a</sup> 567.14 | 10         | 567.14                   | 10  | 53                    | 567.14              | 10  | 2.3                   | 567.14                 | 10  | 143                   | 567.14                           | 10  | 68                    | 567.14    | 10  | 568.54    | 0.00    | 0.25         | 6.55     |
| C3       | 100 | 8         | <sup>a</sup> 639.74 | 8          | 641.76                   | 8   | 128                   | 641.40              | 8   | 9.5                   | 639.74                 | 8   | 330                   | 639.74                           | 8   | 103                   | 639.74    | 8   | 640.49    | 0.00    | 0.12         | 13.98    |
| C12      | 100 | 10        | <sup>a</sup> 534.24 | 10         | 534.24                   | 10  | 118                   | 534.40              | 10  | 6.7                   | 534.24                 | 10  | 363                   | 534.24                           | 10  | 39                    | 534.24    | 10  | 534.24    | 0.00    | 0.00         | 8.85     |
| C11      | 120 | 7         | 682.12              | 7          | 682.12                   | 7   | 141                   | 682.12              | 7   | 10.7                  | 682.12                 | 7   | 318                   | 682.12                           | 7   | 85                    | 682.12    | 7   | 682.21    | 0.00    | 0.01         | 27.79    |
| F12      | 134 | 7         | 769.55              | 7          | 770.17                   | 7   | 359                   | 769.66              | 7   | 75.4                  | 769.55                 | 7   | 753                   | 769.55                           | 7   | 30                    | 769.55    | 7   | 770.69    | 0.00    | 0.15         | 41.18    |
| C4       | 150 | 12        | 733.13              | 12         | 733.13                   | 12  | 279                   | 737.82              | 12  | 45.4                  | 733.13                 | 12  | 613                   | 733.13                           | 12  | 190                   | 733.13    | 12  | 733.43    | 0.00    | 0.04         | 39.71    |
| C5       | 199 | 16        | 893.39              | 16         | 896.08                   | 16  | 237                   | 905.96              | 16  | 17.1                  | 893.39                 | 16  | 1272                  | 893.39                           | 16  | 355                   | 898.08    | 16  | 927.31    | 0.52    | 3.80         | 101.86   |
| C6       | 50  | 5         | 412.96              | 6          | 412.96                   | 6   | 31                    | 412.96              | 6   | 75.8                  | 412.96                 | 6   | 215                   | 412.96                           | 6   | -                     | 412.96    | 6   | 412.96    | 0.00    | 0.00         | 1.64     |
| C7       | 75  | 10        | 583.19              | 10         | 583.19                   | 10  | 33                    | 596.47              | 10  | 22.3                  | 584.15                 | 10  | 367                   | 584.15                           | 10  | -                     | 584.15    | 10  | 588.07*   | 0.16    | 0.16         | 10.48    |
| C8       | 100 | 8         | 644.63              | 9          | 645.16                   | 9   | 114                   | 644.63              | 9   | 587.6                 | 644.63                 | 9   | 510                   | 644.63                           | 9   | -                     | 644.63    | 9   | 645.67    | 0.00    | 0.16         | 9.48     |
| C14      | 100 | 10        | 591.87              | 11         | 591.87                   | 11  | 75                    | 591.87              | 11  | 389                   | 591.87                 | 11  | 411                   | 591.87                           | 11  | -                     | 591.87    | 11  | 591.87    | 0.00    | 0.00         | 9.76     |
| C13      | 120 | 7         | 904.04              | 11         | 909.80                   | 11  | 116                   | 904.94              | 11  | 1820.1                | 910.26                 | 11  | 890                   | 910.26                           | 11  | -                     | 899.16    | 11  | 928.00    | -0.54   | 2.65         | 19.50    |
| C9       | 150 | 12        | 757.84              | 13         | 757.84                   | 13  | 185                   | 760.06              | 13  | 1094.1                | 764.56                 | 13  | 933                   | 764.56                           | 13  | -                     | 759.36    | 13  | 759.27*   | 0.20    | 33.57        | 33.57    |
| C10      | 199 | 16        | 875.67              | 17         | 875.67                   | 17  | 224                   | 875.67              | 17  | 1252.4                | 888.46                 | 17  | 1678                  | 888.46                           | 17  | -                     | 875.24    | 17  | 887.92    | -0.05   | 1.40         | 58.64    |
| Avg.     |     |           |                     |            |                          |     |                       |                     |     |                       |                        |     |                       |                                  |     |                       |           |     | 0.02      | 0.62    | 24.50        |          |
| O1       | 200 | 5         | 6018.52             | 5          | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 452                   | 6018.52                          | 5   | 612                   | 6018.52   | 5   | 6018.52   | 0.00    | 0.00         | 160.27   |
| O2       | 240 | 9         | 4557.38             | 9          | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 613                   | 4557.38                          | 9   | 774                   | 4547.67   | 9   | 4567.72   | -0.21   | 0.23         | 172.18   |
| O3       | 280 | 7         | 7731.00             | 7          | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 736                   | 7731.00                          | 7   | 681                   | 7721.16   | 7   | 7730.30   | -0.13   | -0.01        | 335.04   |
| O4       | 320 | 10        | 7253.20             | 10         | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 833                   | 7253.20                          | 10  | 957                   | 7220.19   | 10  | 7248.29   | -0.46   | -0.07        | 490.66   |
| O5       | 360 | 8         | 9193.15             | 8          | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 1365                  | 9193.15                          | 8   | 1491                  | 9225.78   | 8   | 9289.39   | 0.35    | 1.05         | 996.76   |
| O6       | 400 | 9         | 9793.72             | 9          | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 1213                  | 9793.72                          | 9   | 1070                  | 9771.31   | 9   | 9809.87   | -0.23   | 0.16         | 1207.13  |
| O7       | 440 | 10        | 10347.70            | 10         | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 1547                  | 10347.70                         | 10  | 1257                  | 10325.70  | 10  | 10365.80  | -0.21   | 0.17         | 1567.80  |
| O8       | 480 | 10        | 12415.36            | 10         | -                        | -   | -                     | -                   | -   | -                     | -                      | -   | 1653                  | 12415.36                         | 10  | 1512                  | 12389.40  | 10  | 12412.60  | -0.21   | -0.02        | 1816.19  |
| Avg.     |     |           |                     |            |                          |     |                       |                     |     |                       |                        |     |                       |                                  |     |                       |           |     | -0.14     | 0.19    | 843.25       |          |

<sup>a</sup>: Optimality proved.

1.: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2.: Best run on a Pentium M 2.0 GHz (1738 Mflop/s).

3.: Best run on a Pentium IV 2.8 GHz (2444 Mflop/s).

4.: Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).



### 6.3.1.5 VRPSPD

The same three set of VRPSPD instances used in Subsection 3.4.1 were also adopted to evaluate the performance of the ILS-RVND. In this case the Montané and Galvão's instances [121] involving 400 customers were considered, as well as those of Salhi and Nagy that include route duration constraints (CMT6X-CMT10X, CMT6Y-CMT10Y, CMT13X-CMT14X, CMT13Y-CMT14Y).

Table 6.11 shows the results found in the benchmark instances of Dethloff [48] and a comparison with those reported by Gajpal and Abad [64], Zachariadis et al. [190] and Subramanian et al. [157]. All optimal solutions were found and in most cases the Avg. Sol. was equal to the Best Sol.

Table 6.12 contains the results obtained on the set of instances of Salhi and Nagy [149]. The same works mentioned in Table 6.11 are considered for comparison. It can be verified that the ILS-RVND equaled 20 BKSs and it was found capable of improving another 3. The Avg. Sols. appear to be highly consistent with an exception of instances CMT11X and CMT11Y in which the Avg. Gaps were more than 1.50%.

Table 6.13 presents the results found by ILS-RNVD in the instances of Montané and Galvão [121] and also those reported by Souza et al. [153], Zachariadis et al. [188] and Subramanian et al. [157]. The ILS-RVND managed to find 10 BKSs and to improve another 3 results. It is noteworthy to mention that the proposed algorithm had a satisfactory performance in the large-sized instances, always producing, on average, competitive results.

The average gap between the Avg. Sols. generated by ILS-RVND and the BKSs for the first, second and third group of instances was, respectively, 0.02%, 0.28% and 0.26%.

Table 6.11: Results found for the VRPSPD instances of Dethloff et al. [48]

| Instance | $n$ | $v$ | BKS                         | Gajpal and<br>Abad [64] |                          | Zachariadis<br>et al. [190] |                          | Subramanian<br>et al. [157] |                          | ILS-RVND       |                |            |                 |             |
|----------|-----|-----|-----------------------------|-------------------------|--------------------------|-----------------------------|--------------------------|-----------------------------|--------------------------|----------------|----------------|------------|-----------------|-------------|
|          |     |     |                             | Best<br>Sol.            | Time <sup>1</sup><br>(s) | Best<br>Sol.                | Time <sup>2</sup><br>(s) | Best<br>Sol.                | Time <sup>3</sup><br>(s) | Best<br>Sol.   | Avg.<br>Sol.   | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| SCA3-0   | 50  | 4   | <sup>a</sup> <b>635.62</b>  | <b>635.62</b>           | 6.0                      | <b>635.62</b>               | 2.5                      | <b>635.62</b>               | 2.31                     | <b>635.62</b>  | 636.05         | 0.00       | 0.07            | 1.67        |
| SCA3-1   | 50  | 4   | <sup>a</sup> <b>697.84</b>  | <b>697.84</b>           | 6.0                      | <b>697.84</b>               | 2.5                      | <b>697.84</b>               | 2.28                     | <b>697.84</b>  | <b>697.84</b>  | 0.00       | 0.00            | 1.65        |
| SCA3-2   | 50  | 4   | <sup>a</sup> <b>659.34</b>  | <b>659.34</b>           | 6.0                      | <b>659.34</b>               | 2.9                      | <b>659.34</b>               | 2.14                     | <b>659.34</b>  | <b>659.34</b>  | 0.00       | 0.00            | 1.87        |
| SCA3-3   | 50  | 4   | <sup>a</sup> <b>680.04</b>  | <b>680.04</b>           | 6.1                      | <b>680.04</b>               | 2.3                      | <b>680.04</b>               | 2.49                     | <b>680.04</b>  | <b>680.04</b>  | 0.00       | 0.00            | 1.76        |
| SCA3-4   | 50  | 4   | <sup>a</sup> <b>690.50</b>  | <b>690.50</b>           | 5.7                      | <b>690.50</b>               | 2.9                      | <b>690.50</b>               | 2.18                     | <b>690.50</b>  | <b>690.50</b>  | 0.00       | 0.00            | 1.66        |
| SCA3-5   | 50  | 4   | <sup>a</sup> <b>659.90</b>  | <b>659.90</b>           | 5.1                      | <b>659.90</b>               | 3.0                      | <b>659.90</b>               | 2.23                     | <b>659.91</b>  | <b>659.91</b>  | 0.00       | 0.00            | 1.86        |
| SCA3-6   | 50  | 4   | <sup>a</sup> <b>651.09</b>  | <b>651.09</b>           | 6.1                      | <b>651.09</b>               | 3.1                      | <b>651.09</b>               | 2.51                     | <b>651.09</b>  | <b>651.09</b>  | 0.00       | 0.00            | 1.99        |
| SCA3-7   | 50  | 4   | <sup>a</sup> <b>659.17</b>  | <b>659.17</b>           | 6.8                      | <b>659.17</b>               | 2.8                      | <b>659.17</b>               | 2.49                     | <b>659.17</b>  | 659.86         | 0.00       | 0.11            | 1.63        |
| SCA3-8   | 50  | 4   | <sup>a</sup> <b>719.48</b>  | <b>719.47</b>           | 5.4                      | <b>719.47</b>               | 3.5                      | <b>719.48</b>               | 2.26                     | <b>719.48</b>  | 719.48         | 0.00       | 0.00            | 1.85        |
| SCA3-9   | 50  | 4   | <sup>a</sup> <b>681.00</b>  | <b>681.00</b>           | 6.0                      | <b>681.00</b>               | 4.7                      | <b>681.00</b>               | 1.90                     | <b>681.00</b>  | <b>681.00</b>  | 0.00       | 0.00            | 1.70        |
| SCA8-0   | 50  | 9   | <sup>a</sup> <b>961.50</b>  | <b>961.50</b>           | 11.0                     | <b>961.50</b>               | 2.7                      | <b>961.50</b>               | 3.37                     | <b>961.50</b>  | <b>961.50</b>  | 0.00       | 0.00            | 2.19        |
| SCA8-1   | 50  | 9   | <sup>a</sup> <b>1049.65</b> | <b>1049.65</b>          | 11.5                     | <b>1049.65</b>              | 3.8                      | <b>1049.65</b>              | 2.89                     | <b>1049.65</b> | <b>1049.65</b> | 0.00       | 0.00            | 2.27        |
| SCA8-2   | 50  | 9   | <sup>a</sup> <b>1039.64</b> | <b>1042.69</b>          | 11.9                     | <b>1039.64</b>              | 3.9                      | <b>1039.64</b>              | 2.38                     | <b>1039.64</b> | 1040.52        | 0.00       | 0.08            | 2.20        |
| SCA8-3   | 50  | 9   | <sup>a</sup> <b>983.34</b>  | <b>983.34</b>           | 11.3                     | <b>983.34</b>               | 2.6                      | <b>983.34</b>               | 2.98                     | <b>983.34</b>  | <b>983.34</b>  | 0.00       | 0.00            | 2.05        |
| SCA8-4   | 50  | 9   | <sup>a</sup> <b>1065.49</b> | <b>1065.49</b>          | 11.1                     | <b>1065.49</b>              | 2.4                      | <b>1065.49</b>              | 2.81                     | <b>1065.49</b> | <b>1065.49</b> | 0.00       | 0.00            | 1.87        |
| SCA8-5   | 50  | 9   | <sup>a</sup> <b>1027.08</b> | <b>1027.08</b>          | 11.3                     | <b>1027.08</b>              | 3.4                      | <b>1027.08</b>              | 3.31                     | <b>1027.08</b> | <b>1027.08</b> | 0.00       | 0.00            | 2.34        |
| SCA8-6   | 50  | 9   | <sup>a</sup> <b>971.82</b>  | <b>971.82</b>           | 12.0                     | <b>971.82</b>               | 2.7                      | <b>971.82</b>               | 3.51                     | <b>971.82</b>  | 971.86         | 0.00       | < 0.01          | 2.16        |
| SCA8-7   | 50  | 10  | <sup>a</sup> <b>1051.28</b> | <b>1052.17</b>          | 12.5                     | <b>1051.28</b>              | 5.1                      | <b>1051.28</b>              | 3.12                     | <b>1051.28</b> | 1052.74        | 0.00       | 0.14            | 2.18        |
| SCA8-8   | 50  | 9   | <sup>a</sup> <b>1071.18</b> | <b>1071.18</b>          | 11.0                     | <b>1071.18</b>              | 3.6                      | <b>1071.18</b>              | 2.92                     | <b>1071.18</b> | <b>1071.18</b> | 0.00       | 0.00            | 1.81        |
| SCA8-9   | 50  | 9   | <sup>a</sup> <b>1060.50</b> | <b>1060.50</b>          | 11.5                     | <b>1060.50</b>              | 4.8                      | <b>1060.50</b>              | 2.18                     | <b>1060.50</b> | <b>1060.50</b> | 0.00       | 0.00            | 2.10        |
| CON3-0   | 50  | 4   | <sup>a</sup> <b>616.52</b>  | <b>616.52</b>           | 8.3                      | <b>616.52</b>               | 4.7                      | <b>616.52</b>               | 3.12                     | <b>616.52</b>  | <b>616.52</b>  | 0.00       | 0.00            | 2.42        |
| CON3-1   | 50  | 4   | <sup>a</sup> <b>554.47</b>  | <b>554.47</b>           | 7.1                      | <b>554.47</b>               | 2.2                      | <b>554.47</b>               | 2.83                     | <b>554.47</b>  | <b>554.47</b>  | 0.00       | 0.00            | 1.82        |
| CON3-2   | 50  | 4   | <sup>a</sup> <b>518.00</b>  | <b>518.00</b>           | 6.9                      | <b>518.00</b>               | 3.1                      | <b>518.00</b>               | 2.77                     | <b>518.01</b>  | 519.19         | 0.00       | 0.23            | 2.08        |
| CON3-3   | 50  | 4   | <sup>a</sup> <b>591.19</b>  | <b>591.19</b>           | 7.2                      | <b>591.19</b>               | 3.2                      | <b>591.19</b>               | 2.34                     | <b>591.19</b>  | <b>591.19</b>  | 0.00       | 0.00            | 2.02        |
| CON3-4   | 50  | 4   | <sup>a</sup> <b>588.79</b>  | <b>588.79</b>           | 6.0                      | <b>588.79</b>               | 2.3                      | <b>588.79</b>               | 2.63                     | <b>588.79</b>  | 588.82         | 0.00       | < 0.01          | 2.06        |
| CON3-5   | 50  | 4   | <sup>a</sup> <b>563.70</b>  | <b>563.70</b>           | 6.9                      | <b>563.70</b>               | 3.7                      | <b>563.70</b>               | 2.69                     | <b>563.70</b>  | <b>563.70</b>  | 0.00       | 0.00            | 2.26        |
| CON3-6   | 50  | 4   | <sup>a</sup> <b>499.05</b>  | <b>499.05</b>           | 7.3                      | <b>499.05</b>               | 3.7                      | <b>499.05</b>               | 2.75                     | <b>499.05</b>  | <b>499.05</b>  | 0.00       | 0.00            | 2.13        |
| CON3-7   | 50  | 4   | <sup>a</sup> <b>576.48</b>  | <b>576.48</b>           | 7.0                      | <b>576.48</b>               | 1.9                      | <b>576.48</b>               | 2.75                     | <b>576.48</b>  | <b>576.48</b>  | 0.00       | 0.00            | 2.37        |
| CON3-8   | 50  | 4   | <sup>a</sup> <b>523.05</b>  | <b>523.05</b>           | 7.4                      | <b>523.05</b>               | 3.8                      | <b>523.05</b>               | 2.46                     | <b>523.05</b>  | <b>523.05</b>  | 0.00       | 0.00            | 1.96        |
| CON3-9   | 50  | 4   | <sup>a</sup> <b>578.25</b>  | <b>578.25</b>           | 6.8                      | <b>578.25</b>               | 2.2                      | <b>578.25</b>               | 3.37                     | <b>578.25</b>  | 578.80         | 0.00       | 0.09            | 2.07        |
| CON8-0   | 50  | 9   | <sup>a</sup> <b>857.17</b>  | <b>857.17</b>           | 12.3                     | <b>857.17</b>               | 4.4                      | <b>857.17</b>               | 3.65                     | <b>857.17</b>  | <b>857.17</b>  | 0.00       | 0.00            | 2.53        |
| CON8-1   | 50  | 9   | <sup>a</sup> <b>740.85</b>  | <b>740.85</b>           | 12.0                     | <b>740.85</b>               | 3.3                      | <b>740.85</b>               | 3.02                     | <b>740.85</b>  | <b>740.85</b>  | 0.00       | 0.00            | 1.94        |
| CON8-2   | 50  | 9   | <sup>a</sup> <b>712.89</b>  | <b>712.89</b>           | 13.0                     | <b>712.89</b>               | 2.7                      | <b>712.89</b>               | 3.08                     | <b>712.89</b>  | 712.90         | 0.00       | < 0.01          | 2.09        |
| CON8-3   | 50  | 9   | <sup>a</sup> <b>811.07</b>  | <b>811.07</b>           | 13.9                     | <b>811.07</b>               | 2.8                      | <b>811.07</b>               | 3.99                     | <b>811.07</b>  | <b>811.07</b>  | 0.00       | 0.00            | 2.13        |
| CON8-4   | 50  | 9   | <sup>a</sup> <b>772.25</b>  | <b>772.25</b>           | 11.9                     | <b>772.25</b>               | 2.8                      | <b>772.25</b>               | 3.69                     | <b>772.25</b>  | <b>772.25</b>  | 0.00       | 0.00            | 2.12        |
| CON8-5   | 50  | 9   | <sup>a</sup> <b>754.88</b>  | <b>754.88</b>           | 12.4                     | <b>754.88</b>               | 5.7                      | <b>754.88</b>               | 4.18                     | <b>754.88</b>  | 754.92         | 0.00       | 0.01            | 2.06        |
| CON8-6   | 50  | 9   | <sup>a</sup> <b>678.92</b>  | <b>678.92</b>           | 12.4                     | <b>678.92</b>               | 3.4                      | <b>678.92</b>               | 4.09                     | <b>678.92</b>  | 679.02         | 0.00       | 0.01            | 2.24        |
| CON8-7   | 50  | 9   | <sup>a</sup> <b>811.96</b>  | <b>811.96</b>           | 13.0                     | <b>811.96</b>               | 2.5                      | <b>811.96</b>               | 4.03                     | <b>811.96</b>  | <b>811.96</b>  | 0.00       | 0.00            | 2.10        |
| CON8-8   | 50  | 9   | <sup>a</sup> <b>767.53</b>  | <b>767.53</b>           | 12.5                     | <b>767.53</b>               | 3.2                      | <b>767.53</b>               | 3.42                     | <b>767.53</b>  | <b>767.53</b>  | 0.00       | 0.00            | 2.16        |
| CON8-9   | 50  | 9   | <sup>a</sup> <b>809.00</b>  | <b>809.00</b>           | 12.9                     | <b>809.00</b>               | 3.8                      | <b>809.00</b>               | 3.48                     | <b>809.00</b>  | <b>809.00</b>  | 0.00       | 0.00            | 2.23        |
|          |     |     |                             |                         |                          |                             |                          |                             |                          |                | Avg.           | 0.00       | 0.02            | 2.04        |

<sup>a</sup>: Optimality proved.<sup>1</sup>: Best run on a Xeon 2.4 GHz.<sup>2</sup>: Best run on a T5500 1.66 GHz.<sup>3</sup>: Best run on a Cluster with 32 SMP nodes, where each node consists of two Intel Xeon 2.66 GHz (wall clock).

Table 6.12: Results found for the VRPSPD instances of Salhi and Nagy [149]

| Instance | $n$ | $v$ | BKS                        | Gajpal and<br>Abad [64] |                          | Zachariadis<br>et al. [188] |                          | Subramanian<br>et al. [157] |                          | ILS-RVND       |               |            |                 |             |
|----------|-----|-----|----------------------------|-------------------------|--------------------------|-----------------------------|--------------------------|-----------------------------|--------------------------|----------------|---------------|------------|-----------------|-------------|
|          |     |     |                            | Best<br>Sol.            | Time <sup>1</sup><br>(s) | Best<br>Sol.                | Time <sup>2</sup><br>(s) | Best<br>Sol.                | Time <sup>3</sup><br>(s) | Best<br>Sol.   | Avg.<br>Sol.  | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| CMT1X    | 50  | 3   | <sup>a</sup> <b>466.77</b> | <b>466.77</b>           | 5.00                     | 469.80                      | 2.1                      | <b>466.77</b>               | 2.3                      | <b>466.77</b>  | <b>466.77</b> | 0.00       | 0.00            | 2.25        |
| CMT1Y    | 50  | 3   | <sup>a</sup> <b>466.77</b> | <b>466.77</b>           | 5.00                     | 469.80                      | 3.8                      | <b>466.77</b>               | 2.3                      | <b>466.77</b>  | <b>466.77</b> | 0.00       | 0.00            | 2.25        |
| CMT2X    | 75  | 6   | <b>684.21</b>              | <b>684.21</b>           | 41.25                    | <b>684.21</b>               | 5.4                      | <b>684.21</b>               | 6.4                      | <b>684.21</b>  | 685.45        | 0.00       | 0.18            | 6.74        |
| CMT2Y    | 75  | 6   | <b>684.21</b>              | 684.94                  | 22.25                    | <b>684.21</b>               | 6.8                      | <b>684.21</b>               | 6.4                      | <b>684.21</b>  | 685.25        | 0.00       | 0.15            | 6.73        |
| CMT3X    | 100 | 5   | <sup>a</sup> <b>721.27</b> | 721.40                  | 377.50                   | <b>721.27</b>               | 11.9                     | <b>721.27</b>               | 12.1                     | <b>721.27</b>  | 721.99        | 0.00       | 0.10            | 16.59       |
| CMT3Y    | 100 | 5   | <sup>a</sup> <b>721.27</b> | 721.40                  | 43.75                    | <b>721.27</b>               | 11                       | <b>721.27</b>               | 12.3                     | <b>721.27</b>  | 721.95        | 0.00       | 0.09            | 16.63       |
| CMT12X   | 100 | 5   | <b>662.22</b>              | 663.01                  | 36.25                    | <b>662.22</b>               | 9.3                      | <b>662.22</b>               | 10.3                     | <b>662.22</b>  | 663.82        | 0.00       | 0.24            | 13.39       |
| CMT12Y   | 100 | 5   | <b>662.22</b>              | 663.50                  | 39.25                    | <b>662.22</b>               | 4.8                      | <b>662.22</b>               | 10.8                     | <b>662.22</b>  | 664.23        | 0.00       | 0.31            | 13.51       |
| CMT11X   | 120 | 4   | <b>833.92</b>              | 839.66                  | 57.25                    | <b>833.92</b>               | 21.2                     | <b>833.92</b>               | 18.9                     | 836.02         | 846.79        | 0.25       | 1.54            | 44.63       |
| CMT11Y   | 120 | 4   | <b>833.92</b>              | 840.19                  | 52.75                    | <b>833.92</b>               | 14.4                     | <b>833.92</b>               | 19.0                     | 835.53         | 847.51        | 0.19       | 1.63            | 45.22       |
| CMT4X    | 150 | 7   | <b>852.46</b>              | 854.12                  | 131.75                   | <b>852.46</b>               | 29.6                     | <b>852.46</b>               | 30.9                     | <b>852.46</b>  | 853.70        | 0.00       | 0.15            | 56.40       |
| CMT4Y    | 150 | 7   | <b>852.46</b>              | 855.76                  | 140.25                   | <b>852.46</b>               | 27.4                     | <b>852.46</b>               | 31.6                     | <b>852.46</b>  | 854.05        | 0.00       | 0.19            | 55.98       |
| CMT5X    | 199 | 10  | <b>1029.25</b>             | 1034.87                 | 377.50                   | 1030.55                     | 62.8                     | <b>1029.25</b>              | 71.5                     | <b>1029.25</b> | 1029.44       | 0.00       | 0.74            | 137.41      |
| CMT5Y    | 199 | 10  | <b>1029.25</b>             | 1037.34                 | 393.50                   | 1030.55                     | 47.7                     | <b>1029.25</b>              | 69.6                     | <b>1029.25</b> | 1033.38       | 0.00       | 0.40            | 138.68      |
|          |     |     |                            |                         |                          |                             |                          |                             |                          |                |               |            |                 |             |
| CMT6X    | 50  | 7   | <b>555.43</b>              | <b>555.43</b>           | 14.00                    | -                           | -                        | -                           | -                        | <b>555.43</b>  | 556.89        | 0.00       | 0.26            | 1.74        |
| CMT6Y    | 50  | 7   | <b>555.43</b>              | <b>555.43</b>           | 13.75                    | -                           | -                        | -                           | -                        | <b>555.43</b>  | 556.99        | 0.00       | 0.28            | 1.72        |
| CMT7X    | 75  | 13  | <b>900.12</b>              | <b>900.12</b>           | 47.75                    | -                           | -                        | -                           | -                        | 900.54         | 901.20        | 0.05       | 0.12            | 6.45        |
| CMT7Y    | 75  | 13  | <b>900.54</b>              | <b>900.54</b>           | 46.25                    | -                           | -                        | -                           | -                        | <b>900.12</b>  | 901.16        | -0.05      | 0.07            | 6.36        |
| CMT8X    | 100 | 10  | <b>865.50</b>              | <b>865.50</b>           | 80.75                    | -                           | -                        | -                           | -                        | <b>865.50</b>  | <b>865.50</b> | 0.00       | 0.00            | 11.71       |
| CMT8Y    | 100 | 10  | <b>865.50</b>              | <b>865.50</b>           | 77.75                    | -                           | -                        | -                           | -                        | <b>865.50</b>  | <b>865.50</b> | 0.00       | 0.00            | 11.61       |
| CMT14X   | 100 | 11  | <b>821.75</b>              | <b>821.75</b>           | 78.50                    | -                           | -                        | -                           | -                        | <b>821.75</b>  | <b>821.75</b> | 0.00       | 0.00            | 9.09        |
| CMT14Y   | 100 | 11  | <b>821.75</b>              | <b>821.75</b>           | 74.75                    | -                           | -                        | -                           | -                        | <b>821.75</b>  | <b>821.75</b> | 0.00       | 0.00            | 9.08        |
| CMT13X   | 120 | 12  | <b>1542.86</b>             | <b>1542.86</b>          | 160.25                   | -                           | -                        | -                           | -                        | <b>1542.86</b> | 1544.24       | 0.00       | 0.09            | 22.09       |
| CMT13Y   | 120 | 12  | <b>1542.86</b>             | <b>1542.86</b>          | 160.25                   | -                           | -                        | -                           | -                        | <b>1542.86</b> | 1544.29       | 0.00       | 0.09            | 22.45       |
| CMT9X    | 150 | 16  | <b>1161.54</b>             | <b>1161.54</b>          | 300.00                   | -                           | -                        | -                           | -                        | 1163.02        | 1166.30       | 0.13       | 0.41            | 37.13       |
| CMT9Y    | 150 | 16  | <b>1161.54</b>             | <b>1161.54</b>          | 291.75                   | -                           | -                        | -                           | -                        | 1161.88        | 1165.48       | 0.03       | 0.34            | 37.30       |
| CMT10X   | 199 | 20  | <b>1386.29</b>             | <b>1386.29</b>          | 773.50                   | -                           | -                        | -                           | -                        | <b>1383.97</b> | 1395.64       | -0.17      | 0.67            | 86.86       |
| CMT10Y   | 199 | 20  | <b>1395.04</b>             | <b>1395.04</b>          | 757.50                   | -                           | -                        | -                           | -                        | <b>1381.71</b> | 1394.89       | -0.96      | -0.01           | 85.59       |
|          |     |     |                            |                         |                          |                             |                          |                             |                          |                |               | Avg.       | -0.02           | 32.34       |

<sup>a</sup>: Optimality proved.<sup>b</sup>: Average of 10 runs considering the following instances: CMT1X, CMT1Y, CMT2X, CMT2Y, CMT3X, CMT3Y, CMT12X, CMT12Y, CMT11X, CMT11Y, CMT4X, CMT4Y, CMT5X and CMT5Y.<sup>1</sup>: Best run on a Xeon 2.4 GHz (1978 Mflop/s). <sup>2</sup>: Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).<sup>3</sup>: Average of 50 runs on a cluster with 32 SMP nodes, where each consists of two Intel Xeon 2.66 GHz (wall clock).

Table 6.13: Results found for the VRPSD instances of Montané and Galvão [121]

| Instance | n   | v  | BKS                         | Souza<br>et al. [153] |                          | Zachariadis<br>et al. [188] |                          | Subramanian<br>et al. [161] |                          | ILS-RVND       |              |            |                 |             |
|----------|-----|----|-----------------------------|-----------------------|--------------------------|-----------------------------|--------------------------|-----------------------------|--------------------------|----------------|--------------|------------|-----------------|-------------|
|          |     |    |                             | Best<br>Sol.          | Time <sup>1</sup><br>(s) | Best<br>Sol.                | Time <sup>2</sup><br>(s) | Best<br>Sol.                | Time <sup>3</sup><br>(s) | Best<br>Sol.   | Avg.<br>Sol. | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
|          |     |    |                             |                       |                          |                             |                          |                             |                          |                |              |            |                 |             |
| r101     | 100 | 12 | <sup>a</sup> <b>1009.95</b> | 35.65                 | <b>1009.95</b>           | 28.7                        | <b>1009.95</b>           | 15.81                       | <b>1009.95</b>           | 1013.16        | 0.00         | 0.32       | 15.47           |             |
| r201     | 100 | 3  | <sup>a</sup> <b>666.20</b>  | 39.62                 | <b>666.20</b>            | 31.4                        | <b>666.20</b>            | 15.95                       | <b>666.20</b>            | <b>666.20</b>  | 0.00         | 0.00       | 20.25           |             |
| c101     | 100 | 16 | <sup>a</sup> <b>1220.18</b> | 18.34                 | <b>1220.18</b>           | 18.5                        | <b>1220.18</b>           | 10.39                       | <b>1220.18</b>           | 1220.95        | 0.00         | 0.06       | 12.02           |             |
| c201     | 100 | 5  | <sup>a</sup> <b>662.07</b>  | 16.62                 | <b>662.07</b>            | 23.5                        | <b>662.07</b>            | 8.83                        | <b>662.07</b>            | <b>662.07</b>  | 0.00         | 0.00       | 11.73           |             |
| rc101    | 100 | 10 | <sup>a</sup> <b>1059.32</b> | 12.79                 | <b>1059.32</b>           | 23.8                        | <b>1059.32</b>           | 11.07                       | <b>1059.32</b>           | <b>1059.32</b> | 0.00         | 0.00       | 15.74           |             |
| rc201    | 100 | 3  | <sup>a</sup> <b>672.92</b>  | 24.03                 | <b>672.92</b>            | 21.2                        | <b>672.92</b>            | 7.28                        | <b>672.92</b>            | <b>672.92</b>  | 0.00         | 0.00       | 15.18           |             |
| r1_2.1   | 200 | 23 | <b>3357.64</b>              | 175.81                | 3375.19                  | 84.6                        | 3360.02                  | 66.21                       | 3363.66                  | 3382.49        | 0.18         | 0.74       | 121.623         |             |
| r2_2.1   | 200 | 5  | <sup>a</sup> <b>1665.58</b> | 103.44                | <b>1665.58</b>           | 72.7                        | <b>1665.58</b>           | 45.3                        | <b>1665.58</b>           | 1665.94        | 0.00         | 0.02       | 139.83          |             |
| c1_2.1   | 200 | 28 | <b>3629.89</b>              | 3636.74               | 3641.89                  | 57.0                        | <b>3629.89</b>           | 87.38                       | 3637.42                  | 3641.39        | 0.21         | 0.32       | 107.652         |             |
| c2_2.1   | 200 | 9  | <b>1726.59</b>              | 127.81                | 1726.73                  | 67.3                        | <b>1726.59</b>           | 65.01                       | <b>1726.59</b>           | 1727.72        | 0.00         | 0.07       | 117.89          |             |
| rc1_2.1  | 200 | 23 | <b>3306.00</b>              | 3312.92               | 299.3                    | 83.4                        | <b>3306.00</b>           | 71.71                       | 3309.83                  | 3328.22        | 0.12         | 0.67       | 116.661         |             |
| rc2_2.1  | 200 | 5  | <sup>a</sup> <b>1560.00</b> | 77.48                 | 1560.00                  | 74.4                        | <b>1560.00</b>           | 44.71                       | <b>1560.00</b>           | <b>1560.00</b> | 0.00         | 0.00       | 136.68          |             |
| r1_4.1   | 400 | 54 | <sup>b</sup> <b>9605.75</b> | 9627.43               | 9628.31                  | 421.5                       | 9668.18                  | 481.61                      | <b>9595.75</b>           | 9642.55        | -0.10        | 0.38       | 1095.17         |             |
| r2_4.1   | 400 | 10 | <b>3551.38</b>              | 3582.08               | 3560.73                  | 352.0                       | <b>3551.38</b>           | 459.15                      | 3552.05                  | 3568.33        | 0.02         | 0.48       | 1365.81         |             |
| c1_4.1   | 400 | 63 | <b>11098.21</b>             | 1510.44               | 11125.14                 | 384.6                       | 11099.54                 | 546.22                      | <b>11086.30</b>          | 11107.70       | -0.11        | 0.09       | 963.16          |             |
| c2_4.1   | 400 | 15 | <b>3546.10</b>              | 3596.37               | 3549.20                  | 341.1                       | <b>3546.10</b>           | 488.56                      | 3550.45                  | 3569.64        | 0.12         | 0.66       | 1156.61         |             |
| rc1_4.1  | 400 | 52 | <b>9520.06</b>              | 9535.46               | 2244.18                  | 412.7                       | 9536.77                  | 513.38                      | 9526.52                  | 9558.51        | 0.07         | 0.40       | 1266.56         |             |
| rc2_4.1  | 400 | 11 | <b>3403.70</b>              | 3422.11               | 3414.90                  | 264.7                       | <b>3403.70</b>           | 422.61                      | <b>3403.70</b>           | 3418.76        | 0.00         | 0.44       | 1132.05         |             |
|          |     |    |                             |                       |                          |                             |                          |                             |                          | Avg.           | 0.03         | 0.26       | 433.89          |             |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Best run on an Intel Core 2 Duo 1.66 GHz (2791 Mflop/s).<sup>2</sup>: Average of 10 runs T5500 1.66 GHz ((2791 Mflop/s)).<sup>3</sup>: Average of 50 runs on a cluster with 32 SMP nodes, where each node consists of two Intel Xeon 2.66 GHz (wall clock).<sup>b</sup>: Found by Subramanian et al. [157].

### 6.3.1.6 VRPMPD

The set composed of 21 VRPMPD instances suggested by Salhi and Nagy [149] (see Subsection 3.4.2) plus another 21 of the same authors that include route duration constraints (CMT6H-CMT10H, CMT6Q-CMT10Q, CMT6T-CMT10T CMT13H-CMT14H, CMT13Q-CMT14Q, CMT13T-CMT14T) were used to test the proposed algorithm.

Table 6.14 illustrates the results found by ILS-RVND on the VRPMPD instances and a comparison with those reported by Røpke and Pisinger (6R - normal learning) [148] and Gajpal and Abad [64]. ILS-RVND obtained the BKS in 24 instances and it managed to improve the result of another 10. The developed algorithm outperformed both the algorithms of Røpke and Pisinger [148] and Gajpal and Abad [64] in terms of solution quality. The average gap between the Avg. Sols. obtained by ILS-RVND and the BKSs was 0.09%.

### 6.3.1.7 MDVRP

Two MDVRP benchmark instances proposed by Cordeau et al. [35] were used to test the ILS-RVND algorithm. The first one, known as old, contains 23 instances involving 50-360 customers, where 12 of them have route duration constraints (p13, p14, p16, p17, p19, p20, p08, p09, p10, p11, p22 and p23). The second set, known as new, contains 10 instances involving 48-288 customers, where all of them have route duration constraints.

Tables 6.15 and 6.16 present a comparison, in terms of average solution, between the results found by ILS-RVND and those determined by Pisinger and Røpke (ALNS 50K) [136] and Vidal et al. [177]. The latter clearly outperformed the first two in terms of solution quality. The average gap between the Avg. Sols. found by ILS-RVND and the BKSs for the old and new benchmark sets was respectively 0.47% and 0.66%.

### 6.3.1.8 MDVRPMPD

The proposed algorithm was tested in the set of 33 MDVRPMPD instances proposed by Salhi and Nagy [149], where 12 of them (all with 249 customers) include route duration constraints.

Table 6.17 presents the results found by ILS-RVND and those pointed out by Røpke and Pisinger (6R - no learning) [148]. With respect to the solution quality, the developed algorithm clearly had a better performance, equaling 17 BKSs and improving the result of another 14. The average gap between the Avg. Sols. and the BKSs was 0.23%.

Table 6.14: Results found for the VRPMPD instances of Salhi and Nagy [149]

| Instance | $n$ | $v$ | BKS                        | Røpke and<br>Pisinger [148] |                          | Gajpal and<br>Abad [64] |                          | ILS-RVND                   |               |            |                 |             |
|----------|-----|-----|----------------------------|-----------------------------|--------------------------|-------------------------|--------------------------|----------------------------|---------------|------------|-----------------|-------------|
|          |     |     |                            | Best<br>Sol.                | Time <sup>1</sup><br>(s) | Best<br>Sol.            | Time <sup>2</sup><br>(s) | Best<br>Sol.               | Avg.<br>Sol.  | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| CMT1H    | 50  | 4   | <sup>a</sup> <b>465.02</b> | <b>465</b>                  | 51                       | <b>465.02</b>           | 5.6                      | <b>465.02</b>              | 465.04        | 0.00       | < 0.01          | 2.40        |
| CMT1Q    | 50  | 6   | <sup>a</sup> <b>489.74</b> | <b>490</b>                  | 41                       | <b>489.74</b>           | 6.0                      | <b>489.74</b>              | <b>489.74</b> | 0.00       | 0.00            | 1.91        |
| CMT1T    | 50  | 7   | <sup>a</sup> <b>520.06</b> | <b>520</b>                  | 34                       | <b>520.06</b>           | 7.0                      | <b>520.06</b>              | <b>520.06</b> | 0.00       | 0.00            | 1.86        |
| CMT2H    | 75  | 5   | <b>662.63</b>              | <b>663</b>                  | 78                       | <b>662.63</b>           | 22.0                     | <b>662.63</b>              | 662.70        | 0.00       | 0.01            | 6.67        |
| CMT2Q    | 75  | 7   | <b>732.76</b>              | <b>733</b>                  | 65                       | <b>732.76</b>           | 26.2                     | <b>731.26</b>              | 732.44        | -0.20      | -0.04           | 6.11        |
| CMT2T    | 75  | 9   | <sup>a</sup> <b>782.77</b> | <b>783</b>                  | 57                       | <b>782.77</b>           | 26.0                     | <b>782.77</b>              | 783.68        | 0.00       | 0.12            | 6.25        |
| CMT3H    | 100 | 3   | <b>701.31</b>              | <b>701</b>                  | 186                      | <b>701.31</b>           | 35.6                     | <sup>a</sup> <b>700.94</b> | 700.95        | -0.05      | -0.05           | 19.63       |
| CMT3Q    | 100 | 4   | <sup>a</sup> <b>747.15</b> | <b>747</b>                  | 128                      | <b>747.15</b>           | 39.8                     | <b>747.15</b>              | <b>747.15</b> | 0.00       | 0.00            | 15.24       |
| CMT3T    | 100 | 5   | <sup>a</sup> <b>798.07</b> | <b>798</b>                  | 109                      | <b>798.07</b>           | 42.6                     | <b>798.07</b>              | 798.19        | 0.00       | 0.01            | 15.77       |
| CMT12H   | 100 | 6   | <sup>a</sup> <b>629.37</b> | <b>629</b>                  | 150                      | <b>629.37</b>           | 32.8                     | <b>629.37</b>              | <b>629.37</b> | 0.00       | 0.00            | 16.04       |
| CMT12Q   | 100 | 8   | <sup>a</sup> <b>729.46</b> | <b>729</b>                  | 108                      | <b>729.46</b>           | 42.0                     | <b>729.25</b>              | 729.38        | -0.03      | -0.01           | 15.71       |
| CMT12T   | 100 | 9   | <sup>a</sup> <b>787.52</b> | <b>788</b>                  | 96                       | <b>787.52</b>           | 52.0                     | <b>787.52</b>              | <b>787.52</b> | 0.00       | 0.00            | 9.85        |
| CMT11H   | 120 | 4   | <b>818</b>                 | <b>818</b>                  | 303                      | 820.35                  | 45.8                     | <b>818.05</b>              | 818.11        | 0.01       | 0.01            | 34.06       |
| CMT11Q   | 120 | 6   | <sup>a</sup> <b>939.36</b> | <b>939</b>                  | 196                      | <b>939.36</b>           | 66.2                     | <b>939.36</b>              | <b>939.36</b> | 0.00       | 0.00            | 28.96       |
| CMT11T   | 120 | 7   | <b>998.8</b>               | 1000                        | 164                      | <b>998.80</b>           | 70.2                     | <b>998.80</b>              | 998.83        | 0.00       | < 0.01          | 25.06       |
| CMT4H    | 150 | 6   | <b>829</b>                 | <b>829</b>                  | 345                      | 831.39                  | 125.4                    | <b>829.42</b>              | 835.59        | 0.05       | 0.79            | 62.63       |
| CMT4Q    | 150 | 9   | <b>913.93</b>              | 918                         | 244                      | <b>913.93</b>           | 153.0                    | 915.27                     | 915.59        | 0.15       | 0.18            | 46.98       |
| CMT4T    | 150 | 11  | <b>990.39</b>              | 1000                        | 212                      | <b>990.39</b>           | 166.8                    | <b>990.39</b>              | 992.05        | 0.00       | 0.17            | 43.82       |
| CMT5H    | 200 | 9   | <b>992.37</b>              | 983                         | 514                      | <b>992.37</b>           | 351.4                    | <b>978.74</b>              | 982.32        | -1.37      | -1.01           | 132.02      |
| CMT5Q    | 200 | 12  | <sup>b</sup> <b>1118</b>   | 1119                        | 381                      | 1134.72                 | 451.8                    | <b>1109.43</b>             | 1114.09       | -0.77      | -0.35           | 136.89      |
| CMT5T    | 200 | 15  | <b>1227</b>                | <b>1227</b>                 | 333                      | 1232.08                 | 460.8                    | <b>1222.29</b>             | 1226.82       | -0.38      | -0.01           | 107.71      |
|          |     |     |                            |                             |                          |                         |                          |                            |               |            |                 |             |
| CMT6H    | 50  | 7   | <b>555.43</b>              | <b>555</b>                  | 31                       | <b>555.43</b>           | 13.0                     | <b>555.43</b>              | 557.24        | 0.00       | 0.33            | 1.79        |
| CMT6Q    | 50  | 7   | <b>555.43</b>              | <b>555</b>                  | 30                       | <b>555.43</b>           | 12.8                     | <b>555.43</b>              | 557.06        | 0.00       | 0.29            | 1.78        |
| CMT6T    | 50  | 7   | <b>555.43</b>              | <b>555</b>                  | 31                       | <b>555.43</b>           | 11.6                     | <b>555.43</b>              | 556.93        | 0.00       | 0.27            | 1.75        |
| CMT7H    | 75  | 13  | <b>900</b>                 | <b>900</b>                  | 54                       | 900.84                  | 50.0                     | 900.84                     | 901.10        | 0.09       | 0.12            | 6.30        |
| CMT7Q    | 75  | 14  | <b>900.69</b>              | <b>901</b>                  | 53                       | <b>900.69</b>           | 46.8                     | <b>900.69</b>              | 902.86        | 0.00       | 0.24            | 7.19        |
| CMT7T    | 75  | 14  | <b>903.05</b>              | <b>903</b>                  | 52                       | <b>903.05</b>           | 39.0                     | <b>903.05</b>              | 903.12        | 0.00       | 0.01            | 7.26        |
| CMT8H    | 100 | 10  | <b>865.50</b>              | <b>866</b>                  | 95                       | <b>865.50</b>           | 85.6                     | <b>865.50</b>              | <b>865.50</b> | 0.00       | 0.00            | 12.04       |
| CMT8Q    | 100 | 10  | <b>865.50</b>              | <b>866</b>                  | 93                       | <b>865.50</b>           | 74.4                     | <b>865.50</b>              | <b>865.50</b> | 0.00       | 0.00            | 11.70       |
| CMT8T    | 100 | 10  | <b>865.54</b>              | <b>866</b>                  | 95                       | <b>865.54</b>           | 65.6                     | <b>865.54</b>              | 865.55        | 0.00       | < 0.01          | 11.95       |
| CMT14H   | 100 | 11  | <b>821.75</b>              | <b>822</b>                  | 89                       | <b>821.75</b>           | 81.6                     | <b>821.75</b>              | <b>821.75</b> | 0.00       | 0.00            | 9.22        |
| CMT14Q   | 100 | 11  | <b>821.75</b>              | <b>822</b>                  | 85                       | <b>821.75</b>           | 72.4                     | <b>821.75</b>              | <b>821.75</b> | 0.00       | 0.00            | 9.12        |
| CMT14T   | 100 | 11  | <b>826.77</b>              | <b>827</b>                  | 86                       | <b>826.77</b>           | 64.6                     | <b>826.77</b>              | <b>826.77</b> | 0.00       | 0.00            | 10.31       |
| CMT13H   | 120 | 12  | <b>1542.86</b>             | <b>1543</b>                 | 125                      | <b>1542.86</b>          | 164.2                    | <b>1542.86</b>             | 1544.24       | 0.00       | 0.09            | 22.57       |
| CMT13Q   | 120 | 12  | <b>1542.97</b>             | <b>1543</b>                 | 120                      | <b>1542.97</b>          | 157.8                    | <b>1542.86</b>             | 1544.23       | -0.01      | 0.08            | 22.46       |
| CMT13T   | 120 | 12  | <b>1542.97</b>             | 1544                        | 127                      | <b>1542.97</b>          | 152.8                    | <b>1541.25</b>             | 1544.21       | -0.11      | 0.08            | 22.81       |
| CMT9H    | 150 | 16  | <sup>b</sup> <b>1161</b>   | 1166                        | 177                      | <b>1161.63</b>          | 306.4                    | <b>1161.31</b>             | 1166.06       | 0.03       | 0.44            | 37.12       |
| CMT9Q    | 150 | 16  | <b>1161.51</b>             | 1162                        | 171                      | <b>1161.51</b>          | 289.6                    | 1161.76                    | 1166.35       | 0.02       | 0.42            | 37.66       |
| CMT9T    | 150 | 16  | <b>1162.68</b>             | 1164                        | 178                      | <b>1162.68</b>          | 261.0                    | 1164.05                    | 1167.44       | 0.12       | 0.41            | 37.31       |
| CMT10H   | 199 | 20  | <b>1383.78</b>             | 1393                        | 296                      | <b>1383.78</b>          | 791.0                    | <b>1379.83</b>             | 1389.41       | -0.29      | 0.41            | 88.14       |
| CMT10Q   | 199 | 20  | <b>1386.54</b>             | 1389                        | 288                      | <b>1386.54</b>          | 730.2                    | <b>1384.10</b>             | 1391.99       | -0.18      | 0.39            | 85.89       |
| CMT10T   | 199 | 20  | <sup>b</sup> <b>1395</b>   | 1402                        | 291                      | <b>1400.22</b>          | 658.6                    | <b>1390.04</b>             | 1401.34       | -0.36      | 0.45            | 82.05       |
| Avg.     |     |     |                            |                             |                          |                         |                          |                            |               | -0.08      | 0.09            | 30.05       |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Found by Røpke and Pisinger [148] using another version of their algorithm.<sup>1</sup>: Average of 10 runs on a Pentium IV 1.5 GHz (1311 Mflop/s). <sup>2</sup>: Best run on a Xeon 2.4 GHz (1978 Mflop/s).

Table 6.15: Results found for the old MDVRP instances of Cordeau et al. [35]

| Instance     | $n$ | $v$ | $ G $ | BKS                         | Pisinger and Røpke [136] |                       | Vidal et al. [177] |                       | ILS-RVND       |                |         |              |          |
|--------------|-----|-----|-------|-----------------------------|--------------------------|-----------------------|--------------------|-----------------------|----------------|----------------|---------|--------------|----------|
|              |     |     |       |                             | Avg.* Sol.               | Time <sup>1</sup> (s) | Avg.* Sol.         | Time <sup>2</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| p01          | 50  | 4   | 4     | <sup>a</sup> <b>576.87</b>  | <b>576.87</b>            | 29                    | <b>576.87</b>      | 13.8                  | <b>576.87</b>  | <b>576.87</b>  | 0.00    | 0.00         | 3.98     |
| p02          | 50  | 2   | 4     | <sup>a</sup> <b>473.53</b>  | <b>473.53</b>            | 28                    | <b>473.53</b>      | 12.6                  | <b>473.53</b>  | <b>473.53</b>  | 0.00    | 0.00         | 3.12     |
| p03          | 75  | 3   | 2     | <sup>c</sup> <b>641.19</b>  | <b>641.19</b>            | 64                    | <b>641.19</b>      | 25.8                  | <b>641.19</b>  | <b>641.19</b>  | 0.00    | 0.00         | 8.91     |
| p12          | 80  | 5   | 2     | <sup>b</sup> <b>1318.95</b> | 1319.13                  | 75                    | <b>1318.95</b>     | 31.2                  | <b>1318.95</b> | 1318.96        | 0.00    | < 0.01       | 8.27     |
| p04          | 100 | 8   | 2     | <sup>d</sup> <b>1001.04</b> | 1006.09                  | 88                    | 1001.23            | 116.4                 | <b>1001.04</b> | 1004.53        | 0.18    | 0.53         | 20.63    |
| p05          | 100 | 5   | 2     | <sup>c</sup> <b>750.03</b>  | 752.34                   | 120                   | <b>750.03</b>      | 63.6                  | <b>750.03</b>  | 751.26         | 0.00    | 0.16         | 20.67    |
| p06          | 100 | 6   | 3     | <sup>b</sup> <b>876.50</b>  | 883.01                   | 93                    | <b>876.50</b>      | 68.4                  | <b>876.50</b>  | 878.40         | 0.00    | 0.22         | 20.94    |
| p07          | 100 | 4   | 4     | <sup>d</sup> <b>881.97</b>  | 889.36                   | 88                    | 884.43             | 93.0                  | <b>881.97</b>  | 883.61         | 0.00    | 0.19         | 18.42    |
| p15          | 160 | 5   | 4     | <sup>c</sup> <b>2505.42</b> | 2519.64                  | 253                   | <b>2505.42</b>     | 115.2                 | <b>2505.42</b> | <b>2505.42</b> | 0.00    | 0.00         | 75.62    |
| p18          | 240 | 5   | 6     | <sup>c</sup> <b>3702.85</b> | 3736.53                  | 419                   | <b>3702.85</b>     | 271.2                 | <b>3702.85</b> | 3714.48        | 0.00    | 0.31         | 367.91   |
| p21          | 360 | 5   | 9     | <sup>c</sup> <b>5474.84</b> | 5501.58                  | 582                   | 5476.41            | 600.0                 | <b>5474.84</b> | 5500.75        | 0.00    | 0.47         | 1656.68  |
|              |     |     |       |                             |                          |                       |                    |                       |                |                |         |              |          |
| p13          | 80  | 5   | 2     | <sup>b</sup> <b>1318.95</b> | <b>1318.95</b>           | 60                    | <b>1318.95</b>     | 34.2                  | <b>1318.95</b> | 1318.96        | 0.00    | < 0.01       | 7.74     |
| p14          | 80  | 5   | 2     | <sup>c</sup> <b>1360.12</b> | <b>1360.12</b>           | 58                    | <b>1360.12</b>     | 33.0                  | <b>1360.12</b> | <b>1360.12</b> | 0.00    | 0.00         | 49.60    |
| p16          | 160 | 5   | 4     | <sup>b</sup> <b>2572.23</b> | 2573.95                  | 188                   | <b>2572.23</b>     | 118.2                 | <b>2572.23</b> | 2573.94        | 0.00    | 0.07         | 33.65    |
| p17          | 160 | 5   | 4     | <sup>c</sup> <b>2709.09</b> | <b>2709.09</b>           | 179                   | <b>2709.09</b>     | 128.4                 | <b>2709.09</b> | 2734.93        | 0.00    | 0.95         | 58.99    |
| p19          | 240 | 5   | 6     | <sup>b</sup> <b>3827.06</b> | 3838.76                  | 315                   | <b>3827.06</b>     | 252.0                 | <b>3827.06</b> | 3838.39        | 0.00    | 0.30         | 103.08   |
| p20          | 240 | 5   | 6     | <sup>c</sup> <b>4058.07</b> | 4064.76                  | 300                   | <b>4058.07</b>     | 262.2                 | <b>4058.07</b> | 4110.74        | 0.00    | 1.30         | 135.98   |
| p08          | 249 | 14  | 2     | <sup>e</sup> <b>4372.78</b> | 4421.03                  | 333                   | 4397.42            | 600.0                 | 4403.47        | 4425.42        | 0.70    | 1.20         | 440.78   |
| p09          | 249 | 12  | 3     | <sup>e</sup> <b>3858.66</b> | 3892.50                  | 361                   | 3868.59            | 570.0                 | 3877.15        | 3900.27        | 0.48    | 1.08         | 484.65   |
| p10          | 249 | 8   | 4     | <sup>e</sup> <b>3631.11</b> | 3666.85                  | 363                   | 3636.08            | 589.2                 | 3634.16        | 3664.73        | 0.08    | 0.93         | 488.37   |
| p11          | 249 | 6   | 5     | <sup>d</sup> <b>3546.06</b> | 3573.23                  | 357                   | 3548.25            | 428.4                 | <b>3546.06</b> | 3563.52        | 0.00    | 0.49         | 423.68   |
| p22          | 360 | 5   | 9     | <sup>c</sup> <b>5702.16</b> | 5722.19                  | 462                   | <b>5702.16</b>     | 600.0                 | 5714.45        | 5737.82        | 0.22    | 0.63         | 352.18   |
| p23          | 360 | 5   | 9     | <sup>d</sup> <b>6078.75</b> | 6092.66                  | 443                   | <b>6078.75</b>     | 600.0                 | 6112.46        | 6189.67        | 0.55    | 1.82         | 393.60   |
|              |     |     |       |                             |                          |                       |                    |                       |                |                |         |              |          |
| Avg. Gap (%) |     |     |       |                             | 0.40                     | Avg. Gap (%)          | 0.07               |                       | Avg.           | 0.10           |         | 0.46         | 225.11   |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).<sup>2</sup>: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz. \*: Average of 10 runs.<sup>b</sup>: First found by Renaud et al. [145]. <sup>c</sup>: First found by Cordeau et al. [35].<sup>d</sup>: First found by Pisinger and Røpke [136]. <sup>e</sup>: First found by Vidal et al. [177].

Table 6.16: Results found for the new MDVRP instances of Cordeau et al. [35]

| Instance     | $n$ | $v$ | $ G $ | BKS                         | Pisinger and Røpke [136] |                       | Vidal et al. [177] |                       | ILS-RVND       |                |         |              |          |
|--------------|-----|-----|-------|-----------------------------|--------------------------|-----------------------|--------------------|-----------------------|----------------|----------------|---------|--------------|----------|
|              |     |     |       |                             | Avg.* Sol.               | Time <sup>1</sup> (s) | Avg.* Sol.         | Time <sup>2</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| pr01         | 48  | 2   | 4     | <sup>b</sup> <b>861.32</b>  | <b>861.32</b>            | 30                    | <b>861.32</b>      | 10.2                  | <b>861.32</b>  | <b>861.32</b>  | 0.00    | 0.00         | 3.17     |
| pr07         | 72  | 3   | 6     | <sup>b</sup> <b>1089.56</b> | 1089.56                  | 58                    | <b>1089.56</b>     | 20.4                  | <b>1089.56</b> | <b>1089.56</b> | 0.00    | 0.00         | 9.28     |
| pr02         | 96  | 4   | 4     | <sup>c</sup> <b>1307.34</b> | 1308.17                  | 103                   | <b>1307.34</b>     | 45.6                  | <b>1307.34</b> | 1308.64        | 0.00    | 0.10         | 22.81    |
| pr03         | 144 | 6   | 4     | <sup>d</sup> <b>1803.80</b> | 1810.66                  | 214                   | 1803.80            | 114.6                 | 1803.81        | 1807.50        | 0.00    | 0.21         | 74.64    |
| pr08         | 144 | 6   | 6     | <sup>c</sup> <b>1664.85</b> | 1675.74                  | 207                   | 1665.05            | 123.0                 | <b>1664.85</b> | 1670.19        | 0.00    | 0.32         | 67.14    |
| pr04         | 192 | 8   | 4     | <sup>d</sup> <b>2058.31</b> | 2073.16                  | 296                   | 2059.36            | 313.2                 | 2058.47        | 2075.57        | 0.01    | 0.84         | 179.79   |
| pr09         | 216 | 9   | 6     | <sup>d</sup> <b>2133.20</b> | 2144.84                  | 350                   | 2134.17            | 366.0                 | 2133.64        | 2151.52        | 0.02    | 0.86         | 307.68   |
| pr05         | 240 | 10  | 4     | <sup>d</sup> <b>2331.20</b> | 2350.31                  | 372                   | 2340.29            | 573.6                 | 2347.37        | 2359.94        | 0.69    | 1.23         | 439.88   |
| pr06         | 288 | 12  | 4     | <sup>d</sup> <b>2676.30</b> | 2695.74                  | 465                   | 2681.93            | 600.0                 | 2696.03        | 2711.99        | 0.74    | 1.33         | 908.23   |
| pr10         | 288 | 12  | 6     | <sup>d</sup> <b>2868.26</b> | 2905.43                  | 455                   | 2886.59            | 600.0                 | 2890.68        | 2916.15        | 0.78    | 1.67         | 908.94   |
|              |     |     |       |                             |                          |                       |                    |                       |                |                |         |              |          |
| Avg. Gap (%) |     |     |       |                             | 0.52                     | Avg. Gap (%)          | 0.13               |                       | Avg.           | 0.22           |         | 0.66         | 292.16   |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).<sup>2</sup>: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz. \*: Average of 10 runs.<sup>b</sup>: First found by Cordeau et al. [35]. <sup>c</sup>: First found by Pisinger and Røpke [136]. <sup>d</sup>: First found by Vidal et al. [177].

Table 6.17: Results found for the MDVRPMPD instances of Salhi and Nagy [149]

| Instance | $n$ | $v$ | $ G $ | BKS                      | Røpke and<br>Pisinger [148] |              |                          | ILS-RVND                   |               |            |                 |             |
|----------|-----|-----|-------|--------------------------|-----------------------------|--------------|--------------------------|----------------------------|---------------|------------|-----------------|-------------|
|          |     |     |       |                          | Best<br>Sol.                | Avg.<br>Sol. | Time <sup>1</sup><br>(s) | Best<br>Sol.               | Avg.<br>Sol.  | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| GJ01H    | 50  | 4   | 4     | <b>499</b>               | <b>499</b>                  | <b>499</b>   | 40                       | <b>499.12</b>              | <b>499.12</b> | 0.02       | 0.02            | 5.71        |
| GJ01Q    | 50  | 4   | 4     | <b>528</b>               | <b>528</b>                  | <b>528</b>   | 36                       | <b>528.30</b>              | <b>528.30</b> | 0.06       | 0.06            | 4.81        |
| GJ01T    | 50  | 4   | 4     | <b>569</b>               | <b>569</b>                  | <b>569</b>   | 34                       | <b>569.43</b>              | <b>569.43</b> | 0.08       | 0.08            | 4.72        |
| GJ02H    | 50  | 4   | 2     | <b>440</b>               | <b>440</b>                  | <b>440</b>   | 51                       | <sup>a</sup> <b>440.00</b> | <b>440.00</b> | 0.00       | 0.00            | 4.55        |
| GJ02Q    | 50  | 4   | 2     | <b>450</b>               | <b>450</b>                  | 451          | 43                       | <sup>a</sup> <b>449.72</b> | <b>449.72</b> | -0.06      | -0.06           | 4.24        |
| GJ02T    | 50  | 4   | 2     | <b>464</b>               | <b>464</b>                  | <b>464</b>   | 37                       | <sup>a</sup> <b>464.13</b> | <b>464.13</b> | 0.03       | 0.03            | 3.91        |
| GJ03H    | 75  | 5   | 3     | <b>581</b>               | <b>581</b>                  | 583          | 81                       | <u><b>579.45</b></u>       | 579.59        | -0.27      | -0.24           | 14.82       |
| GJ03Q    | 75  | 5   | 3     | <b>605</b>               | <b>605</b>                  | 608          | 71                       | <b>605.25</b>              | <b>605.25</b> | 0.04       | 0.04            | 13.62       |
| GJ03T    | 75  | 5   | 3     | <b>624</b>               | <b>624</b>                  | 626          | 65                       | <b>624.44</b>              | 625.00        | 0.07       | 0.16            | 11.85       |
| GJ04H    | 100 | 2   | 8     | <b>790</b>               | <b>790</b>                  | 797          | 112                      | <b>789.19</b>              | 789.49        | -0.10      | -0.06           | 29.62       |
| GJ04Q    | 100 | 2   | 8     | <b>875</b>               | <b>875</b>                  | 876          | 94                       | <b>874.78</b>              | 875.37        | -0.03      | 0.04            | 26.23       |
| GJ04T    | 100 | 2   | 8     | <b>962</b>               | <b>962</b>                  | 969          | 85                       | <b>962.25</b>              | 968.30        | 0.03       | 0.65            | 23.61       |
| GJ05H    | 100 | 2   | 5     | <b>678</b>               | <b>678</b>                  | 680          | 168                      | <b>676.81</b>              | 677.11        | -0.18      | -0.13           | 30.52       |
| GJ05Q    | 100 | 2   | 5     | <sup>b</sup> <b>700</b>  | 702                         | 705          | 133                      | <sup>a</sup> <b>700.15</b> | <b>700.15</b> | 0.02       | 0.02            | 26.99       |
| GJ05T    | 100 | 2   | 5     | <b>733</b>               | <b>733</b>                  | 738          | 118                      | <b>733.17</b>              | 733.74        | 0.02       | 0.10            | 24.99       |
| GJ06H    | 100 | 3   | 6     | <sup>b</sup> <b>745</b>  | 747                         | 751          | 116                      | <b>742.18</b>              | 743.05        | -0.38      | -0.26           | 30.84       |
| GJ06Q    | 100 | 3   | 6     | <b>794</b>               | <b>794</b>                  | 800          | 100                      | <b>793.85</b>              | 794.13        | -0.02      | 0.02            | 27.52       |
| GJ06T    | 100 | 3   | 6     | <b>851</b>               | <b>851</b>                  | 853          | 90                       | <b>850.82</b>              | 851.61        | -0.02      | 0.07            | 26.60       |
| GJ07H    | 100 | 4   | 4     | <b>733</b>               | <b>733</b>                  | 734          | 117                      | <b>732.73</b>              | 732.88        | -0.04      | -0.02           | 28.76       |
| GJ07Q    | 100 | 4   | 4     | <sup>b</sup> <b>802</b>  | 803                         | 807          | 94                       | <b>802.20</b>              | 803.76        | 0.02       | 0.22            | 24.46       |
| GJ07T    | 100 | 4   | 4     | <sup>b</sup> <b>854</b>  | 855                         | 862          | 88                       | <b>853.54</b>              | 853.80        | -0.05      | -0.02           | 23.79       |
| GJ08H    | 249 | 2   | 14    | <b>3327</b>              | <b>3327</b>                 | 3373         | 581                      | 3348.39                    | 3373.40       | 0.64       | 1.39            | 464.27      |
| GJ08Q    | 249 | 2   | 14    | <sup>b</sup> <b>3762</b> | 3774                        | 3810         | 479                      | 3773.99                    | 3811.91       | 0.32       | 1.33            | 450.30      |
| GJ08T    | 249 | 2   | 14    | <b>4134</b>              | <b>4134</b>                 | 4170         | 431                      | <u><b>4121.68</b></u>      | 4151.68       | -0.30      | 0.43            | 463.77      |
| GJ09H    | 249 | 3   | 12    | <sup>b</sup> <b>3005</b> | 3006                        | 3028         | 646                      | <u><b>3003.66</b></u>      | 3029.16       | -0.04      | 0.80            | 532.14      |
| GJ09Q    | 249 | 3   | 12    | <b>3355</b>              | <b>3355</b>                 | 3393         | 535                      | <u><b>3351.66</b></u>      | 3386.62       | -0.10      | 0.94            | 510.96      |
| GJ09T    | 249 | 3   | 12    | <b>3677</b>              | <b>3677</b>                 | 3718         | 492                      | <u><b>3667.55</b></u>      | 3695.26       | -0.26      | 0.50            | 525.20      |
| GJ010H   | 249 | 4   | 8     | <sup>b</sup> <b>2927</b> | 2930                        | 2963         | 644                      | <b>2907.94</b>             | 2931.95       | -0.65      | 0.17            | 512.72      |
| GJ010Q   | 249 | 4   | 8     | <sup>b</sup> <b>3242</b> | 3245                        | 3267         | 513                      | <u><b>3232.64</b></u>      | 3250.22       | -0.29      | 0.25            | 495.35      |
| GJ010T   | 249 | 4   | 8     | <b>3485</b>              | <b>3485</b>                 | 3524         | 472                      | <u><b>3478.80</b></u>      | 3502.61       | -0.18      | 0.51            | 492.28      |
| GJ011H   | 249 | 5   | 6     | <sup>b</sup> <b>2855</b> | 2880                        | 2905         | 609                      | <u><b>2846.03</b></u>      | 2871.93       | -0.31      | 0.59            | 477.51      |
| GJ011Q   | 249 | 5   | 6     | <sup>b</sup> <b>3155</b> | 3165                        | 3192         | 511                      | <u><b>3140.15</b></u>      | 3159.68       | -0.47      | 0.15            | 455.25      |
| GJ011T   | 249 | 5   | 6     | <b>3390</b>              | <b>3390</b>                 | 3421         | 469                      | <u><b>3376.54</b></u>      | 3388.91       | -0.40      | -0.03           | 452.56      |
|          |     |     |       |                          | Avg.                        | Gap (%)      | 0.66                     | Avg.                       | -0.08         | 0.23       | 188.62          |             |

\*: Average of 10 runs.

<sup>b</sup>: Found by Røpke and Pisinger [148] using another version of their algorithm.<sup>1</sup>: Average of 10 runs on a Pentium IV 1.5 GHz (1311 Mflop/s).

### 6.3.2 HFVRP

The developed heuristic was tested in well-known HFVRP instances, namely those proposed by Golden et al. [76] and by Taillard [164]. The latter introduced dependent costs and established a limit for the number of vehicles of each type. Table 6.18 describes the characteristics of these instances. The number of executions of the ILS-RVND on each instance was 30.

A more detailed description of the results obtained by ILS-RVND for the HFVRP instances can be found in [133].



Table 6.18: HFVRP Instances

[illegible]

### 6.3.2.1 Parameter tuning

Four instances with varying number of customers (20 - 100) and vehicles were selected for tuning the values of the parameters. For each of these test-problems the ILS-RVND was executed 10 times in all of their respective HFVRP variants. Two values of *MaxIter* were tested, specifically 400 and 450. For each of these, ten values of  $\beta$  were evaluated.

Table 6.19 contains the results of the average gap and the average CPU time for the tests involving *MaxIter* = 400, while those obtained for *MaxIter* = 450 are presented in Table 6.20. The selected configuration was  $\beta = 5$  and *MaxIter* = 400 and these values were chosen using the same criterion employed for the VRP with homogeneous fleet.

Table 6.19: Results of the parameter tuning for *MaxIter* = 400

| $\beta$ | Instance        |             |                 |             |                 |             |                 |             | Avg. |       |
|---------|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|------|-------|
|         | 04              |             | 13              |             | 17              |             | 20              |             |      |       |
|         | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) |      |       |
| 1       | 2.13            | 1.68        | 0.51            | 14.43       | 0.64            | 29.61       | 0.78            | 55.19       | 1.02 | 25.23 |
| 2       | 0.50            | 2.03        | 0.49            | 17.49       | 0.60            | 32.56       | 0.67            | 60.32       | 0.56 | 28.10 |
| 3       | 0.28            | 2.40        | 0.42            | 20.32       | 0.56            | 35.44       | 0.65            | 65.43       | 0.48 | 30.90 |
| 4       | 0.48            | 2.71        | 0.33            | 22.87       | 0.54            | 38.12       | 0.64            | 70.29       | 0.50 | 33.49 |
| 5       | 0.01            | 3.02        | 0.30            | 25.42       | 0.51            | 41.00       | 0.58            | 75.01       | 0.35 | 36.11 |
| 6       | 0.00            | 3.38        | 0.31            | 28.18       | 0.46            | 43.83       | 0.58            | 79.81       | 0.34 | 38.80 |
| 7       | 0.23            | 3.67        | 0.28            | 30.71       | 0.47            | 46.38       | 0.53            | 84.48       | 0.38 | 41.31 |
| 8       | 0.23            | 3.94        | 0.26            | 33.33       | 0.48            | 49.23       | 0.52            | 89.44       | 0.37 | 43.98 |
| 9       | 0.00            | 4.30        | 0.29            | 35.94       | 0.48            | 51.73       | 0.49            | 94.27       | 0.32 | 46.56 |
| 10      | 0.00            | 4.61        | 0.31            | 38.26       | 0.48            | 54.22       | 0.51            | 98.00       | 0.33 | 48.77 |

Table 6.20: Results of the parameter tuning for *MaxIter* = 450

| $\beta$ | Instance        |             |                 |             |                 |             |                 |             | Avg. |       |
|---------|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|------|-------|
|         | 04              |             | 13              |             | 17              |             | 20              |             |      |       |
|         | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) |      |       |
| 1       | 1.69            | 1.95        | 0.50            | 16.51       | 0.55            | 33.90       | 0.66            | 62.94       | 0.85 | 28.82 |
| 2       | 0.71            | 2.32        | 0.46            | 19.79       | 0.60            | 37.05       | 0.62            | 68.53       | 0.60 | 31.92 |
| 3       | 0.50            | 2.73        | 0.42            | 22.98       | 0.53            | 40.58       | 0.61            | 74.60       | 0.51 | 35.22 |
| 4       | 0.01            | 3.06        | 0.35            | 26.28       | 0.49            | 43.85       | 0.58            | 80.38       | 0.36 | 38.39 |
| 5       | 0.01            | 3.47        | 0.36            | 29.08       | 0.50            | 47.02       | 0.58            | 86.19       | 0.36 | 41.44 |
| 6       | 0.01            | 3.86        | 0.29            | 32.13       | 0.45            | 49.82       | 0.61            | 91.57       | 0.34 | 44.34 |
| 7       | 0.01            | 4.20        | 0.30            | 35.05       | 0.44            | 53.18       | 0.53            | 97.23       | 0.32 | 47.42 |
| 8       | 0.00            | 4.54        | 0.29            | 37.98       | 0.45            | 56.10       | 0.54            | 102.13      | 0.32 | 50.19 |
| 9       | 0.00            | 4.91        | 0.27            | 40.44       | 0.39            | 59.05       | 0.53            | 107.34      | 0.30 | 52.93 |
| 10      | 0.00            | 5.31        | 0.25            | 43.56       | 0.37            | 62.07       | 0.45            | 112.42      | 0.27 | 55.84 |

### 6.3.2.2 HVRPFD

To the knowledge of the author the HVRPFD was only examined by Baldacci and Mingozzi [10] and Li et al. [107]. From Table 6.21 it can be verified that ILS-RVND found all proven optimal solutions and improved the result of one instance. The average gap between the Avg. Sols found by the developed algorithm and the BKSs was 0.27%.

### 6.3.2.3 HVRPD

Table 6.22 presents a comparison between the results found by ILS-RVND and the best heuristics proposed in the literature, namely those of Taillard [164], Tarantilis et al. [169], Li et al. [106] and Prins (SMA-D2) [141]. As in the previous variant all proven optimal solutions were found by ILS-RVND. In the only instance where the optimality was not proved, ILS-RVND was unsuccessful to obtain the best solution pointed out by Taillard [164]. The average gap between the Avg. Sols found by the proposed algorithm and the BKSs was 0.22%.

### 6.3.2.4 FSMFD

In Table 6.23 a comparison is performed between the results found by ILS-RVND and those of Choi and Tcha [29], Prins (SMA-U1) [141] and Imran et al. [90]. The ILS-RVND failed to equal the results one instance but it was capable of improving the results of another one. When individually comparing the ILS-RVND with each one of these algorithms, one can verify that the ILS-RVND produced, on average, superior results in terms of best solutions and the average gap between the Avg. Sols. and the BKSs was 0.09%.

### 6.3.2.5 FSMF

Table 6.24 illustrates the results obtained by ILS-RVND for the FSMF. These results are compared with those of Choi and Tcha [29], Brandão [22], Prins (SMA-D1) [141], Imran et al. [90] and Liu et al. [111]. It can be seen that the proposed algorithm equaled the results of 10 instances and improved the result of another one. However, it failed to obtain the best known solutions in another 3. The average gap between the Avg. Sols. found by ILS-RVND and the BKSs was 0.23%.

### 6.3.2.6 FSMD

The best results obtained in the literature for the FSMD using heuristic approaches were reported by Choi and Tcha [29], Brandão [22], Prins (SMA-D1) [141], Imran et al. [90] and Liu et al. [111]. These results along with those found by ILS-RVND are presented in Table 6.25. In this variant the optimal solutions were determined by Baldacci and Mingozzi [10] for all instances. From Table 6.25 it can be observed that ILS-RVND was capable of finding all optimal solutions, except for one instance. The average gap between the Avg. Sols. obtained by the proposed algorithm and the BKSs was 0.17%.

Table 6.21: Results for the HVRPFD instances of Taillard [164]

| Instance | $n$ | BKS                         | Li et al. [107] |                       | ILS-RVND        |           |         |              |               |
|----------|-----|-----------------------------|-----------------|-----------------------|-----------------|-----------|---------|--------------|---------------|
|          |     |                             | Best Sol.       | Time <sup>1</sup> (s) | Best Sol.       | Avg. Sol. | Gap (%) | Avg. Gap (%) | Avg. Time (s) |
| 13       | 50  | <b>3185.09<sup>a</sup></b>  | <b>3185.09</b>  | 110                   | <b>3185.09</b>  | 3189.17   | 0.00    | 0.13         | 19.04         |
| 14       | 50  | <b>10107.53<sup>a</sup></b> | <b>10107.53</b> | 34                    | <b>10107.53</b> | 10107.94  | 0.00    | < 0.01       | 11.28         |
| 15       | 50  | <b>3065.29<sup>a</sup></b>  | <b>3065.29</b>  | 46                    | <b>3065.29</b>  | 3065.34   | 0.00    | < 0.01       | 12.48         |
| 16       | 50  | <b>3265.41<sup>a</sup></b>  | 3278.96         | 99                    | <b>3265.41</b>  | 3278.06   | 0.00    | 0.39         | 12.22         |
| 17       | 75  | <b>2076.96<sup>a</sup></b>  | <b>2076.96</b>  | 148                   | <b>2076.96</b>  | 2083.19   | 0.00    | 0.30         | 29.59         |
| 18       | 75  | <b>3743.58<sup>a</sup></b>  | <b>3743.58</b>  | 119                   | <b>3743.58</b>  | 3758.84   | 0.00    | 0.41         | 36.38         |
| 19       | 100 | <b>10420.34</b>             | <b>10420.34</b> | 287                   | <b>10420.34</b> | 10421.39  | 0.00    | 0.01         | 73.66         |
| 20       | 100 | <b>4806.69</b>              | 4834.17         | 200                   | <b>4788.49</b>  | 4839.53   | -0.38   | 0.68         | 68.46         |
|          |     |                             | Avg.            |                       | Avg.            |           | Avg.    |              | 32.89         |
|          |     |                             |                 |                       | -0.05           |           | 0.24    |              |               |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Best run on an Intel 2.2 GHz (the model was not provided by the authors).

Table 6.22: Results for the HVRPD instances of Taillard [164]

| Instance | $n$ | BKS                         | Taillard [164] |                       | Tarantilis et al. [169] |                       | Li et al. [106] |                       | Prins [141]    |                       | ILS-RVND       |           |         |              |          |
|----------|-----|-----------------------------|----------------|-----------------------|-------------------------|-----------------------|-----------------|-----------------------|----------------|-----------------------|----------------|-----------|---------|--------------|----------|
|          |     |                             | Best Sol.      | Time <sup>1</sup> (s) | Best Sol.               | Time <sup>2</sup> (s) | Best Sol.       | Time <sup>3</sup> (s) | Best Sol.      | Time <sup>4</sup> (s) | Best Sol.      | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| 13       | 50  | <sup>a</sup> <b>1517.84</b> | 1518.05        | 473                   | 1519.96                 | 843                   | <b>1517.84</b>  | 358                   | <b>1517.84</b> | 33.2                  | <b>1517.84</b> | 1518.58   | 0.00    | 0.05         | 19.29    |
| 14       | 50  | <sup>a</sup> <b>607.53</b>  | 615.64         | 575                   | 611.39                  | 387                   | <b>607.53</b>   | 141                   | <b>607.53</b>  | 37.6                  | <b>607.53</b>  | 607.64    | 0.00    | 0.02         | 11.20    |
| 15       | 50  | <sup>a</sup> <b>1015.29</b> | 1016.86        | 335                   | <b>1015.29</b>          | 368                   | <b>1015.29</b>  | 166                   | <b>1015.29</b> | 6.6                   | <b>1015.29</b> | 1015.33   | 0.00    | < 0.01       | 12.56    |
| 16       | 50  | <sup>a</sup> <b>1144.94</b> | 1154.05        | 350                   | 1145.52                 | 341                   | <b>1144.94</b>  | 188                   | <b>1144.94</b> | 7.5                   | <b>1144.94</b> | 1145.04   | 0.00    | 0.01         | 12.29    |
| 17       | 75  | <sup>a</sup> <b>1061.96</b> | 1071.79        | 2245                  | 1071.01                 | 363                   | <b>1061.96</b>  | 216                   | 1065.85        | 81.5                  | <b>1061.96</b> | 1065.27   | 0.00    | 0.31         | 29.92    |
| 18       | 75  | <sup>a</sup> <b>1823.58</b> | 1870.16        | 2876                  | 1846.35                 | 971                   | <b>1823.58</b>  | 366                   | <b>1823.58</b> | 190.6                 | <b>1823.58</b> | 1832.52   | 0.00    | 0.49         | 38.34    |
| 19       | 100 | <b>1117.51</b>              | <b>1117.51</b> | 5833                  | 1123.83                 | 428                   | 1120.34         | 404                   | 1120.34        | 177.8                 | 1120.34        | 1120.34   | 0.25    | 0.25         | 67.72    |
| 20       | 100 | <sup>a</sup> <b>1534.17</b> | 1559.77        | 3402                  | 1556.35                 | 1156                  | <b>1534.17</b>  | 447                   | <b>1534.17</b> | 223.3                 | <b>1534.17</b> | 1544.08   | 0.00    | 0.65         | 63.77    |
|          |     |                             |                |                       |                         |                       |                 |                       |                |                       |                | Avg.      | 0.03    | 0.22         | 31.89    |

<sup>a</sup>: Optimal solution.

<sup>1</sup>: Best run on a Sun Sparc 10 workstation with 50 MHz (27 Mflop/s).

<sup>2</sup>: Best run on a Pentium II 400 MHz (262 Mflop/s).

<sup>3</sup>: Best run on a AMD Athlon 1.0 GHz (1168 Mflop/s).

<sup>4</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s).

Table 6.23: Results for the FSMFD instances of Golden et al. [76]

| Instance | n   | BKS                  | Choi and Tcha [29] |                       | Prins [141] |                       | Inrran et al. [90] |                       | ILS-RVND  |           |         |              |          |
|----------|-----|----------------------|--------------------|-----------------------|-------------|-----------------------|--------------------|-----------------------|-----------|-----------|---------|--------------|----------|
|          |     |                      | Best Sol.          | Time <sup>1</sup> (s) | Best Sol.   | Time <sup>2</sup> (s) | Best Sol.          | Time <sup>3</sup> (s) | Best Sol. | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |                      |                    |                       |             |                       |                    |                       |           |           |         |              |          |
| 3        | 20  | <sup>a</sup> 1144.22 | 1144.22            | 0.25                  | 1144.22     | 0.01                  | 1144.22            | 19                    | 1144.22   | 1144.22   | 0.00    | 0.00         | 4.05     |
| 4        | 20  | <sup>a</sup> 6437.33 | 6437.33            | 0.45                  | 6437.33     | 0.07                  | 6437.33            | 17                    | 6437.33   | 6437.66   | 0.00    | 0.01         | 3.03     |
| 5        | 20  | <sup>a</sup> 1322.26 | 1322.26            | 0.19                  | 1322.26     | 0.02                  | 1322.26            | 24                    | 1322.26   | 1322.26   | 0.00    | 0.00         | 4.85     |
| 6        | 20  | <sup>a</sup> 6516.47 | 6516.47            | 0.41                  | 6516.47     | 0.07                  | 6516.47            | 21                    | 6516.47   | 6516.47   | 0.00    | 0.00         | 3.01     |
| 13       | 50  | <sup>a</sup> 2964.65 | 2964.65            | 3.95                  | 2964.65     | 0.32                  | 2964.65            | 328                   | 2964.65   | 2971.32   | 0.00    | 0.22         | 27.44    |
| 14       | 50  | <sup>a</sup> 9126.9  | 9126.9             | 51.7                  | 9126.9      | 8.9                   | 9126.9             | 250                   | 9126.90   | 9126.91   | 0.00    | < 0.01       | 11.66    |
| 15       | 50  | <sup>a</sup> 2634.96 | 2634.96            | 4.36                  | 2635.21     | 1.04                  | 2634.96            | 275                   | 2634.96   | 2635.02   | 0.00    | < 0.01       | 13.83    |
| 16       | 50  | <sup>a</sup> 3168.92 | 3168.92            | 5.98                  | 3169.14     | 13.05                 | 3168.95            | 313                   | 3168.92   | 3170.81   | 0.00    | 0.06         | 18.20    |
| 17       | 75  | <sup>a</sup> 2004.48 | 2023.61            | 68.11                 | 2004.48     | 23.92                 | 2004.48            | 641                   | 2004.48   | 2012.23   | 0.00    | 0.39         | 43.68    |
| 18       | 75  | <sup>a</sup> 3147.99 | 3147.99            | 18.78                 | 3153.16     | 24.85                 | 3153.67            | 835                   | 3149.63   | 3158.24   | 0.05    | 0.33         | 47.80    |
| 19       | 100 | <sup>a</sup> 8661.81 | 8664.29            | 905.2                 | 8664.67     | 163.25                | 8666.57            | 1411                  | 8661.81   | 8664.81   | 0.00    | 0.03         | 59.13    |
| 20       | 100 | 4153.11              | 4154.49            | 53.02                 | 4154.49     | 41.25                 | 4164.85            | 1460                  | 4153.02   | 4155.90   | 0.00    | 0.07         | 59.07    |
| Avg.     |     |                      |                    |                       |             |                       |                    |                       |           | Avg.      | 0.00    | 0.09         | 24.65    |

<sup>a</sup>: Optimal solution.<sup>1</sup>: Best run on a Sun Sparc 10 workstation with 50 MHz (27 Mflop/s).<sup>2</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s).<sup>3</sup>: Average of 10 runs on a Pentium M 1.7 GHz (1477 Mflop/s).

Table 6.24: Results for the FSMF instances of Golden et al. [76]

| Instance | $n$ | BKS                         | Choi and Tcha [29] |                       | Brandão [22]   |                       | Prins [141]    |                       | Imran et al. [90] |                       | Liu et al. [111] |                       | ILS-RVND       |                |         |              |          |
|----------|-----|-----------------------------|--------------------|-----------------------|----------------|-----------------------|----------------|-----------------------|-------------------|-----------------------|------------------|-----------------------|----------------|----------------|---------|--------------|----------|
|          |     |                             | Best Sol.          | Time <sup>1</sup> (s) | Best Sol.      | Time <sup>2</sup> (s) | Best Sol.      | Time <sup>3</sup> (s) | Best Sol.         | Time <sup>4</sup> (s) | Best Sol.        | Time <sup>5</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |                             |                    |                       |                |                       |                |                       |                   |                       |                  |                       |                |                |         |              |          |
| 3        | 20  | <sup>a</sup> <b>961.03</b>  | <b>961.03</b>      | 0                     | <b>961.03</b>  | 21                    | <b>961.03</b>  | 0.04                  | <b>961.03</b>     | 21                    | <b>961.03</b>    | 0                     | <b>961.03</b>  | 961.10         | 0.00    | 0.01         | 4.91     |
| 4        | 20  | <sup>a</sup> <b>6437.33</b> | <b>6437.33</b>     | 1                     | <b>6437.33</b> | 22                    | <b>6437.33</b> | 0.03                  | <b>6437.33</b>    | 18                    | <b>6437.33</b>   | 0                     | <b>6437.33</b> | 6437.63        | 0.00    | < 0.01       | 3.16     |
| 5        | 20  | <sup>a</sup> <b>1007.05</b> | <b>1007.05</b>     | 1                     | <b>1007.05</b> | 20                    | <b>1007.05</b> | 0.09                  | <b>1007.05</b>    | 13                    | <b>1007.05</b>   | 2                     | <b>1007.05</b> | <b>1007.05</b> | 0.00    | 0.00         | 5.88     |
| 6        | 20  | <sup>a</sup> <b>6516.47</b> | <b>6516.47</b>     | 0                     | <b>6516.47</b> | 25                    | <b>6516.47</b> | 0.08                  | <b>6516.47</b>    | 22                    | <b>6516.47</b>   | 0                     | <b>6516.47</b> | <b>6516.47</b> | 0.00    | 0.00         | 3.07     |
| 13       | 50  | <sup>a</sup> <b>2406.36</b> | <b>2406.36</b>     | 10                    | <b>2406.36</b> | 145                   | <b>2406.36</b> | 17.12                 | <b>2406.36</b>    | 252                   | <b>2406.36</b>   | 91                    | 2408.41        | 2419.38        | 0.09    | 0.54         | 30.29    |
| 14       | 50  | <sup>a</sup> <b>9119.03</b> | <b>9119.03</b>     | 51                    | <b>9119.03</b> | 220                   | <b>9119.03</b> | 19.66                 | <b>9119.03</b>    | 274                   | <b>9119.03</b>   | 42                    | <b>9119.03</b> | <b>9119.03</b> | 0.00    | 0.00         | 11.89    |
| 15       | 50  | <sup>a</sup> <b>2586.37</b> | <b>2586.37</b>     | 10                    | 2586.84        | 110                   | <b>2586.37</b> | 25.1                  | <b>2586.37</b>    | 303                   | <b>2586.37</b>   | 48                    | <b>2586.37</b> | 2586.80        | 0.00    | 0.02         | 20.24    |
| 16       | 50  | <sup>a</sup> <b>2720.43</b> | <b>2720.43</b>     | 11                    | 2728.14        | 111                   | 2728.14        | 16.37                 | <b>2720.43</b>    | 253                   | 2724.22          | 107                   | <b>2720.43</b> | 2737.59        | 0.00    | 0.63         | 20.67    |
| 17       | 75  | <sup>a</sup> <b>1734.53</b> | 1744.83            | 207                   | 1736.09        | 322                   | 1736.09        | 52.22                 | 1741.95           | 745                   | <b>1734.53</b>   | 109                   | <b>1734.53</b> | 1748.06        | 0.00    | 0.78         | 52.49    |
| 18       | 75  | <sup>a</sup> <b>2369.65</b> | 2371.49            | 70                    | 2376.89        | 267                   | <b>2369.65</b> | 36.92                 | <b>2369.65</b>    | 897                   | <b>2369.65</b>   | 197                   | 2371.48        | 2380.98        | 0.08    | 0.48         | 55.35    |
| 19       | 100 | <sup>a</sup> <b>8661.81</b> | 8664.29            | 1179                  | 8667.26        | 438                   | 8665.12        | 169.93                | 8665.05           | 1613                  | 8662.94          | 778                   | 8662.86        | 8665.31        | 0.01    | 0.04         | 63.92    |
| 20       | 100 | <b>4038.46</b>              | 4039.49            | 264                   | 4048.09        | 601                   | 4044.78        | 172.73                | 4044.68           | 1595                  | 4038.46          | 1004                  | <b>4037.90</b> | 4051.11        | -0.01   | 0.31         | 93.88    |
|          |     |                             |                    |                       |                |                       |                |                       |                   |                       |                  |                       | Avg.           |                | 0.01    | 0.23         | 30.48    |

<sup>a</sup>: Optimal solution.

1.: Best run on a Pentium IV 2.6 GHz (2266 Mflop/s).

2.: Best run on a Pentium M 1.4 GHz (1564 Mflop/s).

3.: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s).

4.: Average of 10 runs on a Pentium M 1.7 GHz (1477 Mflop/s).

5.: Best run on a Pentium IV 3.0 GHz (3181 Mflop/s).

Table 6.25: Results for the FSMD instances of Golden et al. [76]

| Instance | $n$ | BKS                  | Choi and Tcha [29] |                       | Brandão [22] |                       | Prins [141] |                       | Imran et al. [90] |                       | Liu et al. [111] |                       | ILS-RVND  |           |         |              |          |
|----------|-----|----------------------|--------------------|-----------------------|--------------|-----------------------|-------------|-----------------------|-------------------|-----------------------|------------------|-----------------------|-----------|-----------|---------|--------------|----------|
|          |     |                      | Best Sol.          | Time <sup>1</sup> (s) | Best Sol.    | Time <sup>2</sup> (s) | Best Sol.   | Time <sup>3</sup> (s) | Best Sol.         | Time <sup>4</sup> (s) | Best Sol.        | Time <sup>5</sup> (s) | Best Sol. | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |                      |                    |                       |              |                       |             |                       |                   |                       |                  |                       |           |           |         |              |          |
| 3        | 20  | <sup>a</sup> 623.22  | 623.22             | 0.19                  | -            | -                     | -           | -                     | -                 | -                     | -                | -                     | 623.22    | 623.22    | 0.00    | 0.00         | 4.58     |
| 4        | 20  | <sup>a</sup> 387.18  | 387.18             | 0.44                  | -            | -                     | -           | -                     | -                 | -                     | -                | -                     | 387.18    | 387.18    | 0.00    | 0.00         | 2.85     |
| 5        | 20  | <sup>a</sup> 742.87  | 742.87             | 0.23                  | -            | -                     | -           | -                     | -                 | -                     | -                | -                     | 742.87    | 742.87    | 0.00    | 0.00         | 5.53     |
| 6        | 20  | <sup>a</sup> 415.03  | 415.03             | 0.92                  | -            | -                     | -           | -                     | -                 | -                     | -                | -                     | 415.03    | 415.03    | 0.00    | 0.00         | 3.37     |
| 13       | 50  | <sup>a</sup> 1491.86 | 1491.86            | 4.11                  | 1491.86      | 101                   | 1491.86     | 3.45                  | 1491.86           | 310                   | 1491.86          | 117                   | 1491.86   | 1495.61   | 0.00    | 0.25         | 31.62    |
| 14       | 50  | <sup>a</sup> 603.21  | 603.21             | 20.41                 | 603.21       | 135                   | 603.21      | 0.86                  | 603.21            | 161                   | 603.21           | 26                    | 603.21    | 603.21    | 0.00    | 0.00         | 14.66    |
| 15       | 50  | <sup>a</sup> 999.82  | 999.82             | 4.61                  | 999.82       | 137                   | 999.82      | 9.14                  | 999.82            | 218                   | 999.82           | 37                    | 999.82    | 1001.70   | 0.00    | 0.19         | 15.33    |
| 16       | 50  | <sup>a</sup> 1131.00 | 1131.00            | 3.36                  | 1131.00      | 95                    | 1131.00     | 13.00                 | 1131.00           | 239                   | 1131.00          | 54                    | 1131.00   | 1134.52   | 0.00    | 0.31         | 17.77    |
| 17       | 75  | <sup>a</sup> 1038.60 | 1038.60            | 69.38                 | 1038.60      | 312                   | 1038.60     | 9.53                  | 1038.60           | 509                   | 1038.60          | 153                   | 1038.60   | 1041.12   | 0.00    | 0.24         | 49.18    |
| 18       | 75  | <sup>a</sup> 1800.80 | 1801.40            | 48.06                 | 1801.40      | 269                   | 1800.80     | 18.92                 | 1800.80           | 606                   | 1801.40          | 394                   | 1800.80   | 1804.07   | 0.00    | 0.18         | 53.88    |
| 19       | 100 | <sup>a</sup> 1105.44 | 1105.44            | 182.86                | 1105.44      | 839                   | 1105.44     | 52.31                 | 1105.44           | 1058                  | 1105.44          | 479                   | 1105.44   | 1108.21   | 0.00    | 0.25         | 77.84    |
| 20       | 100 | <sup>a</sup> 1530.43 | 1530.43            | 98.14                 | 1531.83      | 469                   | 1535.12     | 104.41                | 1533.24           | 1147                  | 1534.37          | 826                   | 1530.52   | 1540.32   | 0.01    | 0.65         | 88.02    |
|          |     |                      |                    |                       |              |                       |             |                       |                   |                       |                  |                       | Avg.      |           | 0.00    | 0.17         | 30.39    |

<sup>a</sup>: Optimal solution.

1: Best run on a Pentium IV 2.6 GHz (2266 Mflop/s).

2: Best run on a Pentium M 1.4 GHz (1564 Mflop/s).

3: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s).

4: Average of 10 runs on a Pentium M 1.7 GHz (1477 Mflop/s).

5: Best run on a Pentium IV 3.0 GHz (3181 Mflop/s).

### 6.3.3 TSPMPD

The ILS-RVND was tested on the benchmark instances generated by Mosheiov [122]. This set of instances consists of 100 test-problems involving 20-500 customers. The algorithm was executed 50 times for each instance.

#### 6.3.3.1 Parameter tuning

Since there is only a single vehicle involved in the TSPMPD, a different approach from the one adopted in the previous variants was employed to calibrate the value of  $MaxIterILS$ . This time an expression given by Eq. 6.5 computes the value of  $MaxIterILS$  only in terms of the number of customers  $n$ . A non-negative parameter  $\alpha$  is used to adjust the level of influence of  $n$  in the value of  $MaxIterILS$ .

$$MaxIterILS = \alpha \times n \quad (6.5)$$

Tables 6.26 and 6.27 present the results of the parameter tuning for  $MaxIter = 50$  and  $MaxIter = 75$ , respectively. For each case, four values of  $\alpha$  were tested. Ten instances of different sizes were considered. Apparently, all configurations lead to competitive results in terms of solution quality. The chosen parameters were  $MaxIter = 50$  and  $\alpha = 1/5$ , because the correspondent Avg. Gap was only 0.07% worse than the “heaviest” configuration ( $MaxIterILS = 75$  and  $\alpha = 1/3$ ), but the average computational time was approximately 50% smaller.

Table 6.26: Results of the parameter tuning for  $MaxIter = 50$  (TSPMPD)

| Instance | $n$ | $\alpha$        |             |                 |             |                 |             |                 |             |
|----------|-----|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
|          |     | 1/6             |             | 1/5             |             | 1/4             |             | 1/3             |             |
|          |     | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) |
| n20mosA  | 20  | 0.00            | 0.01        | 0.00            | 0.01        | 0.00            | 0.01        | 0.00            | 0.02        |
| n30mosA  | 30  | 0.00            | 0.04        | 0.00            | 0.05        | 0.00            | 0.07        | 0.00            | 0.09        |
| n40mosA  | 40  | 0.00            | 0.15        | 0.00            | 0.17        | 0.00            | 0.23        | 0.00            | 0.32        |
| n50mosA  | 50  | 0.00            | 0.32        | 0.00            | 0.36        | 0.00            | 0.47        | 0.00            | 0.63        |
| n60mosA  | 60  | 0.00            | 0.54        | 0.00            | 0.65        | 0.00            | 0.83        | 0.00            | 1.09        |
| n100mosA | 100 | 0.02            | 4.90        | 0.02            | 5.75        | 0.00            | 6.96        | 0.00            | 8.99        |
| n200mosA | 200 | 0.38            | 66.74       | 0.25            | 76.94       | 0.25            | 92.49       | 0.12            | 116.87      |
| n300mosA | 300 | 0.15            | 286.30      | 0.10            | 323.85      | 0.02            | 397.83      | -0.04           | 499.79      |
| n400mosA | 400 | -0.41           | 1067.08     | -0.52           | 1189.65     | -0.60           | 1310.76     | -0.69           | 1767.85     |
| n500mosA | 500 | -0.50           | 2980.21     | -0.66           | 3257.70     | -0.76           | 3775.85     | -0.85           | 4546.10     |
| Avg.     |     | -0.04           | 440.63      | -0.08           | 485.51      | -0.11           | 558.55      | -0.15           | 694.17      |

#### 6.3.3.2 Results

Table 6.28 shows the results for the small sized instances, that is, those involving 20-60 customers. A comparison is performed with those reported by Hernández-Pérez and Salazar-González [85] ([webpages.ull.es/users/hhperez/PDsite/TS2004t2.sol](http://webpages.ull.es/users/hhperez/PDsite/TS2004t2.sol)). It can be observed that both the algorithms found all known optimal solutions.



Table 6.27: Results of the parameter tuning for  $MaxIter = 75$  (TSPMPD)

| Instance | $n$ | $\alpha$        |             |                 |             |                 |             |                 |             |
|----------|-----|-----------------|-------------|-----------------|-------------|-----------------|-------------|-----------------|-------------|
|          |     | 1/6             |             | 1/5             |             | 1/4             |             | 1/3             |             |
|          |     | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) | Avg.<br>Gap (%) | Time<br>(s) |
| n20mosA  | 20  | 0.00            | 0.02        | 0.00            | 0.02        | 0.00            | 0.03        | 0.00            | 0.0         |
| n30mosA  | 30  | 0.00            | 0.07        | 0.00            | 0.08        | 0.00            | 0.11        | 0.00            | 0.1         |
| n40mosA  | 40  | 0.00            | 0.23        | 0.00            | 0.27        | 0.00            | 0.32        | 0.00            | 0.5         |
| n50mosA  | 50  | 0.00            | 0.45        | 0.00            | 0.51        | 0.00            | 0.67        | 0.00            | 0.9         |
| n60mosA  | 60  | 0.00            | 0.76        | 0.00            | 0.93        | 0.00            | 1.18        | 0.00            | 1.6         |
| n100mosA | 100 | 0.01            | 6.93        | 0.00            | 8.35        | 0.00            | 10.41       | 0.00            | 13.4        |
| n200mosA | 200 | 0.29            | 96.02       | 0.17            | 113.38      | 0.12            | 134.88      | 0.06            | 165.6       |
| n300mosA | 300 | 0.07            | 417.64      | 0.06            | 493.66      | -0.02           | 580.61      | -0.03           | 728.3       |
| n400mosA | 400 | -0.52           | 1429.72     | -0.57           | 1672.15     | -0.62           | 2060.60     | -0.68           | 2642.8      |
| n500mosA | 500 | -0.64           | 3957.70     | -0.71           | 4271.90     | -0.77           | 4899.42     | -0.82           | 6470.2      |
| Avg.     |     | -0.08           | 590.95      | -0.10           | 656.13      | -0.13           | 768.82      | -0.15           | 1002.34     |

Table 6.29 contains the results found by ILS-RNVD and those found by Hernández-Pérez and Salazar-González [85] ([webpages.ull.es/users/hhperez/PDsite/TS2004t3.sol](http://webpages.ull.es/users/hhperez/PDsite/TS2004t3.sol)) for the medium/large sized instances (200-500 customers). The proposed algorithm equaled the results of 6 BKSs and improve those of the remaining 44 instances. The average gap between the Avg. Sols and the BKSs was -0.46%.

Table 6.28: Results found for the TSPMPD instances (20-60 customers) of Mosheiov [122]

| Instance | $n$ | BKS                      | Hernández-Pérez and<br>Salazar-González [85] |                          | ILS-RVND     |              |            |                 |             |
|----------|-----|--------------------------|--|--------------------------|--------------|--------------|------------|-----------------|-------------|
|          |     |                          | Best<br>Sol.                                 | Time <sup>1</sup><br>(s) | Best<br>Sol. | Avg.<br>Sol. | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| n20mosA  | 20  | <sup>a</sup> <b>3816</b> | <b>3816</b>                                  | 0.06                     | <b>3816</b>  | 3816.00      | 0.00       | 0.00            | 0.01        |
| n20mosB  | 20  | <sup>a</sup> <b>3942</b> | <b>3942</b>                                  | 0.06                     | <b>3942</b>  | 3942.00      | 0.00       | 0.00            | 0.01        |
| n20mosC  | 20  | <sup>a</sup> <b>4048</b> | <b>4048</b>                                  | 0.05                     | <b>4048</b>  | 4048.00      | 0.00       | 0.00            | 0.01        |
| n20mosD  | 20  | <sup>a</sup> <b>4151</b> | <b>4151</b>                                  | 0.11                     | <b>4151</b>  | 4151.00      | 0.00       | 0.00            | 0.01        |
| n20mosE  | 20  | <sup>a</sup> <b>4387</b> | <b>4387</b>                                  | 0.11                     | <b>4387</b>  | 4387.00      | 0.00       | 0.00            | 0.01        |
| n20mosF  | 20  | <sup>a</sup> <b>4262</b> | <b>4262</b>                                  | 0.10                     | <b>4262</b>  | 4262.00      | 0.00       | 0.00            | 0.01        |
| n20mosG  | 20  | <sup>a</sup> <b>4248</b> | <b>4248</b>                                  | 0.11                     | <b>4248</b>  | 4248.00      | 0.00       | 0.00            | 0.01        |
| n20mosH  | 20  | <sup>a</sup> <b>4057</b> | <b>4057</b>                                  | 0.05                     | <b>4057</b>  | 4057.00      | 0.00       | 0.00            | 0.01        |
| n20mosI  | 20  | <sup>a</sup> <b>4064</b> | <b>4064</b>                                  | 0.06                     | <b>4064</b>  | 4064.00      | 0.00       | 0.00            | 0.01        |
| n20mosJ  | 20  | <sup>a</sup> <b>3793</b> | <b>3793</b>                                  | 0.06                     | <b>3793</b>  | 3793.00      | 0.00       | 0.00            | 0.01        |
| n30mosA  | 30  | <sup>a</sup> <b>4686</b> | <b>4686</b>                                  | 0.11                     | <b>4686</b>  | 4686.00      | 0.00       | 0.00            | 0.05        |
| n30mosB  | 30  | <sup>a</sup> <b>4805</b> | <b>4805</b>                                  | 0.06                     | <b>4805</b>  | 4805.00      | 0.00       | 0.00            | 0.05        |
| n30mosC  | 30  | <sup>a</sup> <b>4503</b> | <b>4503</b>                                  | 0.06                     | <b>4503</b>  | 4503.00      | 0.00       | 0.00            | 0.06        |
| n30mosD  | 30  | <sup>a</sup> <b>5047</b> | <b>5047</b>                                  | 0.11                     | <b>5047</b>  | 5047.00      | 0.00       | 0.00            | 0.07        |
| n30mosE  | 30  | <sup>a</sup> <b>4929</b> | <b>4929</b>                                  | 0.16                     | <b>4929</b>  | 4929.00      | 0.00       | 0.00            | 0.06        |
| n30mosF  | 30  | <sup>a</sup> <b>4500</b> | <b>4500</b>                                  | 0.11                     | <b>4500</b>  | 4500.00      | 0.00       | 0.00            | 0.05        |
| n30mosG  | 30  | <sup>a</sup> <b>4948</b> | <b>4948</b>                                  | 0.10                     | <b>4948</b>  | 4948.00      | 0.00       | 0.00            | 0.05        |
| n30mosH  | 30  | <sup>a</sup> <b>4602</b> | <b>4602</b>                                  | 0.16                     | <b>4602</b>  | 4602.00      | 0.00       | 0.00            | 0.07        |
| n30mosI  | 30  | <sup>a</sup> <b>4390</b> | <b>4390</b>                                  | 0.05                     | <b>4390</b>  | 4390.00      | 0.00       | 0.00            | 0.06        |
| n30mosJ  | 30  | <sup>a</sup> <b>4469</b> | <b>4469</b>                                  | 0.05                     | <b>4469</b>  | 4469.00      | 0.00       | 0.00            | 0.06        |
| n40mosA  | 40  | <sup>a</sup> <b>5192</b> | <b>5192</b>                                  | 0.11                     | <b>5192</b>  | 5192.00      | 0.00       | 0.00            | 0.18        |
| n40mosB  | 40  | <sup>a</sup> <b>5416</b> | <b>5416</b>                                  | 0.11                     | <b>5416</b>  | 5416.00      | 0.00       | 0.00            | 0.19        |
| n40mosC  | 40  | <sup>a</sup> <b>5079</b> | <b>5079</b>                                  | 0.28                     | <b>5079</b>  | 5079.00      | 0.00       | 0.00            | 0.16        |
| n40mosD  | 40  | <sup>a</sup> <b>5640</b> | <b>5640</b>                                  | 0.11                     | <b>5640</b>  | 5640.00      | 0.00       | 0.00            | 0.19        |
| n40mosE  | 40  | <sup>a</sup> <b>5364</b> | <b>5364</b>                                  | 0.11                     | <b>5364</b>  | 5364.00      | 0.00       | 0.00            | 0.19        |
| n40mosF  | 40  | <sup>a</sup> <b>5187</b> | <b>5187</b>                                  | 0.44                     | <b>5187</b>  | 5187.00      | 0.00       | 0.00            | 0.16        |
| n40mosG  | 40  | <sup>a</sup> <b>5424</b> | <b>5424</b>                                  | 0.11                     | <b>5424</b>  | 5424.00      | 0.00       | 0.00            | 0.17        |
| n40mosH  | 40  | <sup>a</sup> <b>5010</b> | <b>5010</b>                                  | 0.44                     | <b>5010</b>  | 5010.00      | 0.00       | 0.00            | 0.18        |
| n40mosI  | 40  | <sup>a</sup> <b>5065</b> | <b>5065</b>                                  | 0.22                     | <b>5065</b>  | 5065.00      | 0.00       | 0.00            | 0.18        |
| n40mosJ  | 40  | <sup>a</sup> <b>5068</b> | <b>5068</b>                                  | 0.44                     | <b>5068</b>  | 5068.00      | 0.00       | 0.00            | 0.18        |
| n50mosA  | 50  | <sup>a</sup> <b>5826</b> | <b>5826</b>                                  | 0.39                     | <b>5826</b>  | 5826.00      | 0.00       | 0.00            | 0.37        |
| n50mosB  | 50  | <sup>a</sup> <b>6080</b> | <b>6080</b>                                  | 0.55                     | <b>6080</b>  | 6080.00      | 0.00       | 0.00            | 0.37        |
| n50mosC  | 50  | <sup>a</sup> <b>6203</b> | <b>6203</b>                                  | 0.77                     | <b>6203</b>  | 6203.00      | 0.00       | 0.00            | 0.41        |
| n50mosD  | 50  | <sup>a</sup> <b>6239</b> | <b>6239</b>                                  | 0.27                     | <b>6239</b>  | 6239.00      | 0.00       | 0.00            | 0.42        |
| n50mosE  | 50  | <sup>a</sup> <b>6200</b> | <b>6200</b>                                  | 0.44                     | <b>6200</b>  | 6200.00      | 0.00       | 0.00            | 0.45        |
| n50mosF  | 50  | <sup>a</sup> <b>5537</b> | <b>5537</b>                                  | 0.16                     | <b>5537</b>  | 5537.00      | 0.00       | 0.00            | 0.37        |
| n50mosG  | 50  | <sup>a</sup> <b>5938</b> | <b>5938</b>                                  | 0.38                     | <b>5938</b>  | 5938.00      | 0.00       | 0.00            | 0.37        |
| n50mosH  | 50  | <sup>a</sup> <b>5820</b> | <b>5820</b>                                  | 0.50                     | <b>5820</b>  | 5820.00      | 0.00       | 0.00            | 0.36        |
| n50mosI  | 50  | <sup>a</sup> <b>5625</b> | <b>5625</b>                                  | 0.38                     | <b>5625</b>  | 5625.00      | 0.00       | 0.00            | 0.35        |
| n50mosJ  | 50  | <sup>a</sup> <b>5969</b> | <b>5969</b>                                  | 0.27                     | <b>5969</b>  | 5969.00      | 0.00       | 0.00            | 0.39        |
| n60mosA  | 60  | <sup>a</sup> <b>6279</b> | <b>6279</b>                                  | 1.48                     | <b>6279</b>  | 6279.00      | 0.00       | 0.00            | 0.67        |
| n60mosB  | 60  | <sup>a</sup> <b>6561</b> | <b>6561</b>                                  | 0.71                     | <b>6561</b>  | 6561.00      | 0.00       | 0.00            | 0.83        |
| n60mosC  | 60  | <sup>a</sup> <b>6295</b> | <b>6295</b>                                  | 0.76                     | <b>6295</b>  | 6295.00      | 0.00       | 0.00            | 0.71        |
| n60mosD  | 60  | <sup>a</sup> <b>6774</b> | <b>6774</b>                                  | 0.77                     | <b>6774</b>  | 6774.00      | 0.00       | 0.00            | 0.76        |
| n60mosE  | 60  | <sup>a</sup> <b>6628</b> | <b>6628</b>                                  | 2.53                     | <b>6628</b>  | 6642.04      | 0.00       | 0.21            | 0.80        |
| n60mosF  | 60  | <sup>a</sup> <b>6246</b> | <b>6246</b>                                  | 1.98                     | <b>6246</b>  | 6246.00      | 0.00       | 0.00            | 0.87        |
| n60mosG  | 60  | <sup>a</sup> <b>6417</b> | <b>6417</b>                                  | 0.44                     | <b>6417</b>  | 6417.00      | 0.00       | 0.00            | 0.85        |
| n60mosH  | 60  | <sup>a</sup> <b>6204</b> | <b>6204</b>                                  | 0.49                     | <b>6204</b>  | 6204.00      | 0.00       | 0.00            | 0.83        |
| n60mosI  | 60  | <sup>a</sup> <b>6072</b> | <b>6072</b>                                  | 0.71                     | <b>6072</b>  | 6072.00      | 0.00       | 0.00            | 0.72        |
| n60mosJ  | 60  | <sup>a</sup> <b>6651</b> | <b>6651</b>                                  | 0.71                     | <b>6651</b>  | 6651.00      | 0.00       | 0.00            | 0.85        |
|          |     |                          |  |                          |              |              | Avg.       | 0.00            | < 0.01      |
|          |     |                          |  |                          |              |              |            |                 | 0.28        |

<sup>a</sup>: Optimal Solution.<sup>1</sup>: Best run on an AMD 1.33 GHz (1554 Mflop/s)

Table 6.29: Results found for the TSPMPD instances (100-500 customers) of Mosheiov [122]

| Instance | $n$ | BKS                      | Hernández-Pérez and Salazar-González [85] |                       | ILS-RVND                 |           |         |              |          |
|----------|-----|--------------------------|---|-----------------------|--------------------------|-----------|---------|--------------|----------|
|          |     |                          | Best Sol.                                 | Time <sup>1</sup> (s) | Best Sol.                | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| n100mosA | 100 | <sup>a</sup> <b>7860</b> | <b>7860</b>                               | 1.98                  | <b>7860</b>              | 7860.72   | 0.00    | 0.01         | 5.66     |
| n100mosB | 100 | <sup>a</sup> <b>7843</b> | <b>7843</b>                               | 1.42                  | <b>7843</b>              | 7843.00   | 0.00    | 0.00         | 5.38     |
| n100mosC | 100 | <sup>a</sup> <b>8244</b> | <b>8244</b>                               | 3.95                  | <b>8244</b>              | 8244.86   | 0.00    | 0.01         | 6.01     |
| n100mosD | 100 | <sup>a</sup> <b>8100</b> | <b>8100</b>                               | 3.18                  | <b>8100</b>              | 8100.80   | 0.00    | 0.01         | 6.12     |
| n100mosE | 100 | <b>8105</b>              | <b>8105</b>                               | 2.58                  | <sup>a</sup> <b>8100</b> | 8103.56   | -0.06   | -0.02        | 6.03     |
| n100mosF | 100 | <b>7941</b>              | <b>7941</b>                               | 1.43                  | <sup>a</sup> <b>7918</b> | 7918.04   | -0.29   | -0.29        | 6.17     |
| n100mosG | 100 | <sup>a</sup> <b>8080</b> | <b>8080</b>                               | 1.86                  | <b>8080</b>              | 8080.00   | 0.00    | 0.00         | 6.31     |
| n100mosH | 100 | <b>7923</b>              | <b>7923</b>                               | 1.48                  | <sup>a</sup> <b>7909</b> | 7909.32   | -0.18   | -0.17        | 5.80     |
| n100mosI | 100 | <b>8080</b>              | <b>8080</b>                               | 1.26                  | <sup>a</sup> <b>8071</b> | 8071.00   | -0.11   | -0.11        | 6.32     |
| n100mosJ | 100 | <b>7872</b>              | <b>7872</b>                               | 3.13                  | <sup>a</sup> <b>7850</b> | 7850.00   | -0.28   | -0.28        | 5.52     |
| n200mosA | 200 | <b>10715</b>             | <b>10715</b>                              | 18.40                 | <b>10715</b>             | 10748.70  | 0.00    | 0.31         | 82.37    |
| n200mosB | 200 | <b>10976</b>             | <b>10976</b>                              | 17.69                 | <b>10930</b>             | 10952.70  | -0.42   | -0.21        | 82.59    |
| n200mosC | 200 | <b>10652</b>             | <b>10652</b>                              | 21.81                 | <b>10601</b>             | 10608.20  | -0.48   | -0.41        | 89.83    |
| n200mosD | 200 | <b>11125</b>             | <b>11125</b>                              | 30.86                 | <b>10934</b>             | 10958.40  | -1.72   | -1.50        | 85.75    |
| n200mosE | 200 | <b>10541</b>             | <b>10541</b>                              | 19.39                 | <b>10540</b>             | 10543.50  | -0.01   | 0.02         | 78.75    |
| n200mosF | 200 | <b>10854</b>             | <b>10854</b>                              | 10.82                 | <b>10816</b>             | 10824.30  | -0.35   | -0.27        | 80.47    |
| n200mosG | 200 | <b>10844</b>             | <b>10844</b>                              | 13.89                 | <b>10759</b>             | 10776.50  | -0.78   | -0.62        | 71.01    |
| n200mosH | 200 | <b>11345</b>             | <b>11345</b>                              | 12.91                 | <b>11231</b>             | 11247.50  | -1.00   | -0.86        | 87.22    |
| n200mosI | 200 | <b>10835</b>             | <b>10835</b>                              | 9.95                  | <b>10824</b>             | 10835.30  | -0.10   | 0.00         | 78.85    |
| n200mosJ | 200 | <b>10648</b>             | <b>10648</b>                              | 9.22                  | <b>10593</b>             | 10604.40  | -0.52   | -0.41        | 75.98    |
| n300mosA | 300 | <b>12889</b>             | <b>12889</b>                              | 58.00                 | <b>12872</b>             | 12901.40  | -0.13   | 0.10         | 357.18   |
| n300mosB | 300 | <b>13176</b>             | <b>13176</b>                              | 45.97                 | <b>13041</b>             | 13093.50  | -1.02   | -0.63        | 393.81   |
| n300mosC | 300 | <b>13053</b>             | <b>13053</b>                              | 86.12                 | <b>12947</b>             | 12974.40  | -0.81   | -0.60        | 351.62   |
| n300mosD | 300 | <b>13151</b>             | <b>13151</b>                              | 27.58                 | <b>13131</b>             | 13190.10  | -0.15   | 0.30         | 350.00   |
| n300mosE | 300 | <b>12882</b>             | <b>12882</b>                              | 24.82                 | <b>12723</b>             | 12771.90  | -1.23   | -0.85        | 366.02   |
| n300mosF | 300 | <b>13138</b>             | <b>13138</b>                              | 47.46                 | <b>13122</b>             | 13155.60  | -0.12   | 0.13         | 372.93   |
| n300mosG | 300 | <b>13157</b>             | <b>13157</b>                              | 39.93                 | <b>12933</b>             | 12955.10  | -1.70   | -1.53        | 353.74   |
| n300mosH | 300 | <b>13357</b>             | <b>13357</b>                              | 30.16                 | <b>13258</b>             | 13314.80  | -0.74   | -0.32        | 367.32   |
| n300mosI | 300 | <b>13194</b>             | <b>13194</b>                              | 80.46                 | <b>13071</b>             | 13121.00  | -0.93   | -0.55        | 353.67   |
| n300mosJ | 300 | <b>13304</b>             | <b>13304</b>                              | 89.97                 | <b>13300</b>             | 13340.80  | -0.03   | 0.28         | 345.73   |
| n400mosA | 400 | <b>14890</b>             | <b>14890</b>                              | 112.49                | <b>14779</b>             | 14817.20  | -0.75   | -0.49        | 1365.47  |
| n400mosB | 400 | <b>15177</b>             | <b>15177</b>                              | 91.12                 | <b>14919</b>             | 14959.10  | -1.70   | -1.44        | 1230.52  |
| n400mosC | 400 | <b>15121</b>             | <b>15121</b>                              | 168.95                | <b>14911</b>             | 14967.00  | -1.39   | -1.02        | 1307.10  |
| n400mosD | 400 | <b>15291</b>             | <b>15291</b>                              | 66.46                 | <b>15179</b>             | 15252.40  | -0.73   | -0.25        | 1204.40  |
| n400mosE | 400 | <b>14937</b>             | <b>14937</b>                              | 75.85                 | <b>14791</b>             | 14855.40  | -0.98   | -0.55        | 1143.59  |
| n400mosF | 400 | <b>15191</b>             | <b>15191</b>                              | 155.88                | <b>15061</b>             | 15104.70  | -0.86   | -0.57        | 1180.64  |
| n400mosG | 400 | <b>15015</b>             | <b>15015</b>                              | 57.50                 | <b>14863</b>             | 14925.50  | -1.01   | -0.60        | 1109.28  |
| n400mosH | 400 | <b>15196</b>             | <b>15196</b>                              | 107.49                | <b>14922</b>             | 14978.40  | -1.80   | -1.43        | 1062.90  |
| n400mosI | 400 | <b>14989</b>             | <b>14989</b>                              | 277.04                | <b>14769</b>             | 14867.50  | -1.47   | -0.81        | 1209.04  |
| n400mosJ | 400 | <b>15134</b>             | <b>15134</b>                              | 73.54                 | <b>15050</b>             | 15090.60  | -0.56   | -0.29        | 1191.84  |
| n500mosA | 500 | <b>16555</b>             | <b>16555</b>                              | 54.87                 | <b>16503</b>             | 16552.20  | -0.31   | -0.02        | 3161.28  |
| n500mosB | 500 | <b>16672</b>             | <b>16672</b>                              | 109.74                | <b>16572</b>             | 16625.80  | -0.60   | -0.28        | 3373.15  |
| n500mosC | 500 | <b>16859</b>             | <b>16859</b>                              | 133.08                | <b>16720</b>             | 16777.70  | -0.82   | -0.48        | 3474.14  |
| n500mosD | 500 | <b>17093</b>             | <b>17093</b>                              | 426.61                | <b>16776</b>             | 16842.20  | -1.85   | -1.47        | 3552.75  |
| n500mosE | 500 | <b>16738</b>             | <b>16738</b>                              | 152.96                | <b>16560</b>             | 16613.50  | -1.06   | -0.74        | 2993.31  |
| n500mosF | 500 | <b>16991</b>             | <b>16991</b>                              | 80.03                 | <b>16711</b>             | 16808.70  | -1.65   | -1.07        | 3292.06  |
| n500mosG | 500 | <b>16708</b>             | <b>16708</b>                              | 76.18                 | <b>16638</b>             | 16686.10  | -0.42   | -0.13        | 3186.70  |
| n500mosH | 500 | <b>16987</b>             | <b>16987</b>                              | 39.55                 | <b>16665</b>             | 16710.70  | -1.90   | -1.63        | 3353.80  |
| n500mosI | 500 | <b>16668</b>             | <b>16668</b>                              | 118.69                | <b>16470</b>             | 16528.80  | -1.19   | -0.84        | 2969.25  |
| n500mosJ | 500 | <b>16941</b>             | <b>16941</b>                              | 109.30                | <b>16798</b>             | 16829.90  | -0.84   | -0.66        | 3294.63  |
| Avg.     |     |                          |   |                       |                          |           | -0.70   | -0.46        | 982.80   |

<sup>a</sup>: Optimality proved using the BC approach presented in Chapter 3.<sup>1</sup>: Best run on an AMD 1.33 GHz (1554 Mflop/s)

## 6.4 Concluding remarks

This chapter presented a heuristic algorithm, called ILS-RVND, for a large class of VRPs. The developed algorithm has a simple structure and relies on relatively very few parameters. Extensive computational experiments proved the efficiency of the proposed solution approach, especially in terms of flexibility and solution quality, as can be observed in Table 6.30. In this table, **Avg. Gap** corresponds to the average gap between the average solutions and the BKSs, **#Instances** is the number of instances of a particular benchmark, **#Improvements** denotes the number of solutions improved and **#Ties** represents the number of ties. It can be observed that ILS-RVND was found capable of improving the result of 98 instances and to equal the result of another 464. Furthermore, it is possible to verify that the Avg. Gap was always smaller than 0.66%, except in the CVRP instances of Golden et al. [80], where the Avg. Gap was 1.03%.

Table 6.30: Summary of ILS-RVND results

| Variant               | #Instances  | #Improvements                                  | #Ties   | Avg. Gap (%)  | Avg. Time (s)  |
|-----------------------|---|--|---|---|--|
| CVRP <sup>1</sup>     | <sup>a</sup> 93, <sup>b</sup> 14, <sup>c</sup> 20 | <sup>a</sup> 0, <sup>b</sup> 0, <sup>c</sup> 0 | <sup>a</sup> 90, <sup>b</sup> 10, <sup>c</sup> 1  | <sup>a</sup> 0.09, <sup>b</sup> 0.26, <sup>c</sup> 1.03 | <sup>a</sup> 5.50, <sup>b</sup> 23.56, <sup>c</sup> 798.49 |
| ACVRP <sup>1</sup>    | <sup>d</sup> 24                                   | <sup>d</sup> 1                                 | <sup>d</sup> 23                                   | <sup>d</sup> 0.15                                       | <sup>d</sup> 2.54  |
| OVRP <sup>1</sup>     | <sup>a</sup> 93, <sup>e</sup> 16, <sup>f</sup> 8  | <sup>a</sup> 1, <sup>e</sup> 2, <sup>f</sup> 6 | <sup>a</sup> 88, <sup>e</sup> 12, <sup>f</sup> 1  | <sup>a</sup> 0.09, <sup>e</sup> 0.62, <sup>f</sup> 0.19 | <sup>a</sup> 9.31, <sup>e</sup> 24.50, <sup>f</sup> 843.25 |
| VRPSPD <sup>1</sup>   | <sup>g</sup> 40, <sup>h</sup> 28, <sup>i</sup> 18 | <sup>g</sup> 0, <sup>h</sup> 3, <sup>i</sup> 2 | <sup>g</sup> 40, <sup>h</sup> 20, <sup>i</sup> 10 | <sup>g</sup> 0.02, <sup>h</sup> 0.28, <sup>i</sup> 0.26 | <sup>g</sup> 2.04, <sup>h</sup> 12.60, <sup>i</sup> 433.89 |
| VRPMPD <sup>1</sup>   | <sup>h</sup> 42                                   | <sup>h</sup> 12                                | <sup>h</sup> 26                                   | <sup>h</sup> 0.09                                       | <sup>h</sup> 30.05   |
| MDVRP <sup>1</sup>    | <sup>j</sup> 23, <sup>k</sup> 10                  | <sup>j</sup> 0, <sup>k</sup> 0                 | <sup>j</sup> 16, <sup>k</sup> 4                   | <sup>j</sup> 0.47, <sup>k</sup> 0.66                    | <sup>j</sup> 225.11, <sup>k</sup> 292.16                   |
| MDVRPMPD <sup>1</sup> | <sup>h</sup> 33                                   | <sup>h</sup> 14                                | <sup>h</sup> 17                                   | <sup>h</sup> 0.23                                       | <sup>h</sup> 188.62  |
| HFVRP <sup>2</sup>    | <sup>l</sup> 16, <sup>m</sup> 36                  | <sup>l</sup> 1, <sup>m</sup> 2                 | <sup>l</sup> 14, <sup>m</sup> 29                  | <sup>l</sup> 0.23, <sup>m</sup> 0.16                    | <sup>l</sup> 32.39, <sup>m</sup> 28.51                     |
| TSPMPD <sup>1</sup>   | <sup>n</sup> 60, <sup>o</sup> 60                  | <sup>n</sup> 0, <sup>o</sup> 44                | <sup>n</sup> 60, <sup>o</sup> 6                   | <sup>n</sup> < 0.01, <sup>o</sup> -0.46                 | <sup>n</sup> 0.28, <sup>o</sup> 982.80                     |
| Total                 | 634   | 87   | 467   |   |  |

<sup>a</sup>: A, B, E, F, M, P set; <sup>b</sup>: Christofides et al. [31]; <sup>c</sup>: Golden et al. [80].

<sup>d</sup>: Fischetti et al. [57] and Pessoa et al. [134].

<sup>e</sup>: Christofides et al. [31] and Fisher [58]; <sup>f</sup>: Li et al. [105].

<sup>g</sup>: Dethloff [48]; <sup>h</sup>: Salhi and Nagy [149]; <sup>i</sup>: Montané and Galvão [121].

<sup>j</sup>: Cordeau et al. (old) [35]; <sup>k</sup>: Cordeau et al. (new) [35].

<sup>l</sup>: Taillard [164]; <sup>m</sup>: Golden et al. [76].

<sup>n</sup>: Mosheiov (small) [122]; <sup>o</sup>: Mosheiov (large) [122].

<sup>1</sup>: Core 2 Quad 2.4 GHz (single thread).

<sup>2</sup>: Core i7 2.93 GHz (single thread).

# Chapter 7

## A Hybrid Algorithm for General Vehicle Routing Problems

This chapter presents a unified hybrid heuristic algorithm for a large class of Vehicle Routing Problems. The developed approach consists of a combination of the ILS-RVND heuristic described in Chapter 6 and a Set Partitioning (SP) formulation. A sequence of SP models, with columns corresponding to routes found by ILS-RVND, are solved, not necessarily to optimality, by means of a Mixed Integer Programming (MIP) solver, that may interact with the ILS-RVND heuristic during its execution. The contents of this chapter were partially published in [158].

The ILS-RVND structure was slightly modified in order to store routes during its execution. Every time a local search is performed, the routes associated to the local optimal solution  $s$  may be added to a pool of routes (*RoutePool*). The method decides whether to add or not such routes based on the average number of customers per route ( $n/v$ ) and on the deviation between the current best solution  $s^*$  and  $s$  (see Subsection 7.2). If this deviation, given by  $(f(s) - f(s^*)) / f(s^*)$ , where  $f(\cdot)$  is the cost function, is smaller than a threshold value (*tolerance*) then the routes of  $s$  are added to *RoutePool*. Of course, if  $s_0$  is provided then the procedure that generates an initial solution is skipped.

### 7.1 A Set Partitioning approach

Let  $\mathcal{R}$  be the set of all possible routes of all vehicle types,  $\mathcal{R}_i \subseteq \mathcal{R}$  be the subset of routes that contain customer  $i \in V'$ . Define  $y_j$  as the binary variable associated to a route  $j \in \mathcal{R}$ , and  $c_j$  as its cost.

Consider the following basic SP formulation F1:

$$\text{Min } \sum_{j \in \mathcal{R}} c_j y_j \quad (7.1)$$

$$\text{s.t. } \sum_{j \in \mathcal{R}_i} y_j = 1 \quad \forall i \in V' \quad (7.2)$$

$$y_j \in \{0, 1\} \quad \forall j \in \mathcal{R}. \quad (7.3)$$

The objective function (7.1) minimizes the sum of the costs by choosing the best combination of routes. Constraints (7.2) state that a single route from the subset  $\mathcal{R}_i$  visits customer  $i \in V'$ . Since the enumeration of set  $\mathcal{R}$  is an impractical task, ILS-RVND-SP only considers a subset of this set, usually limited to a few thousand routes. Formulation F1 is mainly suitable for variants such as the Fleet Size and Mix because the number of vehicles of each type is not predefined. Let  $\mathcal{R}_u \subseteq \mathcal{R}$  be the set of routes associated with vehicle type  $u \in M$  or with depot  $u \in G$  and let  $m_u$  be an upper bound on the number of vehicles of a given type or available at a given depot. In order to deal with HVRPs or with MDVRPs one can add the following constraints:

$$\sum_{j \in \mathcal{R}_u} y_j \leq m_u \quad \forall u \in M (\text{or } \forall u \in G). \quad (7.4)$$

Let  $v$  be the number of vehicles. For the remaining variants, one must include the constraint that ensure that the number of routes in the solution is equal to the number of vehicles available, i.e.,

$$\sum_{j \in \mathcal{R}} y_j = v. \quad (7.5)$$

It is important to mention that there are some instances of VRPs with homogeneous fleet that do not specify the number of vehicles, but ILS-RVND-SP fixes this value by using the number of vehicles of the best current solution. Although the solution space is reduced, this helps the problem to be solved more efficiently.

The pseudocode of the SP procedure is illustrated in Alg. 7.1. Input parameter *MaxSPTTime* corresponds to the time limit imposed to the MIP solver, whereas *MaxRootGap* is the maximum gap allowed between the Lower Bound (LB) and the Upper Bound (UB) after solving the root node. It is assumed that the MIP solver uses a branch-and-bound or a branch-and-cut procedure. The algorithm starts by verifying if the number of vehicles should be minimized (e.g. OVRP) and if the number of vehicles of  $s^*$  is larger than the estimated lower bound on the number of vehicles ( $v_{min} = \lceil (\sum_{i \in V'} d_i) / Q \rceil$ ). If so, solution  $s^*$  is stored in  $s'$  and the number of vehicles is decreased by one unit (lines 2-3). Next, the *SP\_Model* is created (line 4) according to the VRP variant and the *Cutoff* value is initialized (line 5). The SP problem is given to a MIP solver (line 6),

which calls ILS-RVND whenever an incumbent solution is found (Procedure Incumbent-Callback). If the solution  $s^*$  is improved in the IncumbentCallback, the *Cutoff* value is updated (lines 19-21), but  $s^*$  is not given back to the solver since it may contain a route that does not belong to the subset of routes  $\mathcal{R}$  of the SP model. The solver is interrupted if: (i) the optimal solution is found; (ii)  $LB > Cutoff$  (iii) *MaxSPTime* is exceeded; (iv) *MaxRootGap* is exceeded. If the fleet is unlimited (e.g. FSM) and the solver has been interrupt due to (iii) or (iv) then the SP model is updated by adding constraints that enforce the fleet composition to be the same as in  $s^*$  (lines 7-9). This is most likely to happen when fixed costs are considered. The addition of these constraints may prevent the algorithm to find better solutions but it makes the problem much more computationally tractable in an acceptable time. Finally, if the primary objective is to minimize the number of vehicles and the solution  $s^*$  is infeasible, then the number of vehicles is incremented by one unit,  $s^*$  is restored and the MIP solver is called again (lines 10-13).

---

**Algorithm 7.1** SP
 

---

```

1: Procedure SP( $s^*$ , RoutePool, MaxSPTime, MaxRootGap,  $v$ );
2: if  $v$  must be minimized and  $v > v_{min}$  then
3:    $v \leftarrow v - 1$ ;  $s' \leftarrow s^*$ ;
4:  $SP\_Model \leftarrow \text{CreateSetPartitioningModel}(\text{RoutePool}, v)$ ;
5:  $Cutoff \leftarrow f(s^*)$ ; {Only if  $v = v_{min}$ . Otherwise,  $Cutoff \leftarrow \infty$ }
6:  $s^* \leftarrow \text{MIPSolver}(SP\_Model, s^*, Cutoff, \text{MaxSPTime}, \text{MaxRootGap}, \text{IncumbentCallback}(s^*))$ ;
7: if Unlimited fleet and ( $\text{Time} > \text{MaxSPTime}$  or  $\text{RootGap} > \text{MaxRootGap}$ ) then
8:   Update  $SP\_Model$  {Fixing the fleet};  $\text{MaxRootGap} \leftarrow \infty$ ;
9:    $s^* \leftarrow \text{MIPSolver}(SP\_Model, s^*, Cutoff, \text{MaxSPTime}, \text{MaxRootGap}, \text{IncumbentCallback}(s^*))$ ;
10: if  $v$  must be minimized and  $s^*$  is infeasible then
11:    $s^* \leftarrow s'$ ;  $v \leftarrow v + 1$ ;
12:   Update  $SP\_Model$  {Increasing one vehicle};
13:    $s^* \leftarrow \text{MIPSolver}(SP\_Model, s^*, Cutoff, \text{MaxSPTime}, \text{MaxRootGap}, \text{IncumbentCallback}(s^*))$ ;
14: return  $s^*$ ;
15: end SP.
16: Procedure IncumbentCallback( $s^*$ )
17:  $s \leftarrow$  Incumbent Solution
18:  $s \leftarrow \text{ILS-RVND}(1, s, \text{NULL})$ 
19: if  $f(s) < f(s^*)$  then
20:    $s^* \leftarrow s$ 
21:    $Cutoff \leftarrow f(s)$ 
22: end IncumbentCallback

```

---

## 7.2 The ILS-RVND-SP algorithm

One of the challenges of designing a unified hybrid solution approach is to ensure that the MIP model is computationally tractable, regardless of the instance. For example, a SP model that exceeds the time limit only to solve its linear relaxation (e.g. due to an excessive number of routes) is not a suitable improving mechanism. On the other hand, a SP model that contains relatively few routes, is easily solved, but seldom finds improved solutions. Hence, it is necessary that the SP models generated throughout the

algorithm find a balance between computational tractability and improvement potential. Experiments carried out in many instances with distinct characteristics indicated that the following simple pieces of data are crucial for estimating the dimension (number of routes) of a properly balanced SP model: (i) number of customers; (ii) the average number of customers per route; and (iii) the vehicle fleet (homogeneous or heterogeneous). The latter has been already discussed in Subsection 7.1.

With respect to (i), two strategies for ILS-RVND-SP were developed. The first one, called ILS-RVND-SP-a, is executed when the number of customers is less or equal to a parameter  $\mathcal{N}$ . The idea of ILS-RVND-SP-a is straightforward: the SP procedure is run only once at the end of the algorithm, after the ILS-RVND heuristic. The second one, called ILS-RVND-SP-b, is executed when  $n > \mathcal{N}$ . In this case, the SP procedure is called after each iteration of the ILS-RVND heuristic. Both strategies are completely independent, as well as some of their parameters, namely:  $MaxIter$ -a,  $MaxIter$ -b,  $MaxIterILS$ -a,  $MaxIterILS$ -b,  $TDev$ -a,  $TDev$ -b. The parameter  $TDev$  is described next.

With respect to (ii), the following is done. Let  $\mathcal{A}$  be a parameter. It has been observed that when the ratio between the number of customers and the number of vehicles is smaller than  $\mathcal{A} = 11$ , the SP models tend to become harder. In such cases, one only adds the routes of a solution to the SP model if its deviation when compared to the incumbent solution is smaller than a given threshold  $TDev$ . However, this parameter is difficult to tune, especially in ILS-RVND-SP-b. To overcome this issue, a reactive approach that dynamically adjusts its value throughout the execution of the algorithm was implemented, as will be further explained.

The pseudocode of ILS-RVND-SP and ILS-RVND-SP-a are depicted in Algs. 7.2 and 7.3, respectively, and their explanation will be omitted since they are quite simple.

---

**Algorithm 7.2** ILS-RVND-SP

---

```

1: Procedure ILS-RVND-SP( $MaxIter$ -a,  $MaxIter$ -b,  $MaxIterILS$ -a,  $MaxIterILS$ -b,  $TDev$ -a,
    $TDev$ -b,  $MaxSPTime$ ,  $MaxRootGap$ )
2: LoadData();
3: if  $v$  was not defined then
4:    $v \leftarrow \text{EstimateTheNumberOfVehicles}(\text{seed})$ ;
5:  $RoutePool \leftarrow \text{NULL}$ ;
6: if  $n \leq \mathcal{N}$  then
7:    $s^* \leftarrow \text{ILS-RVND-SP-a}(MaxIter$ -a,  $MaxIterILS$ -a,  $RoutePool$ ,  $v$ ,  $TDev$ -a,  $MaxSPTime$ ,
      $MaxRootGap$ );
8: else
9:    $s^* \leftarrow \text{ILS-RVND-SP-b}(MaxIter$ -b,  $MaxIterILS$ -b,  $RoutePool$ ,  $v$ ,  $TDev$ -b,  $MaxSPTime$ ,
      $MaxRootGap$ );
10: return  $s^*$ ;
11: end ILS-RVND-SP.

```

---

Alg. 7.4 shows the pseudocode of ILS-RVND-SP-b. Firstly, *tolerance* (threshold deviation) is set to a given value according to average number of customers per route (lines 2-5). In the main loop (lines 7-24), the ILS-RVND heuristic is executed with a single



iteration (line 8) and the SP procedure is repeatedly called while there is any improvement over the best current solution (lines 10-21). When no improvement is observed, the non-permanent routes (short-term memory) are removed from *RoutePool* (line 16). After each call to the SP procedure, the algorithm may update the value of *tolerance*, in case  $n/v < \mathcal{A}$ , according to the following conditions. If the SP model is solved at the root node, meaning that the problem is easy, then *tolerance* is increased by one tenth of *TDev-b* (lines 17-18). If the time limit is exceeded then *tolerance* is decreased by one tenth of *TDev-b* (lines 19-20). If there is any improvement at the end of a given iteration, the incumbent solution  $s^*$  is updated and the associated routes are permanently added (long-term memory) to *RoutePool* (lines 22-24).

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**Algorithm 7.3** ILS-RVND-SP-a

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```

1: Procedure ILS-RVND-SP-a(MaxIter-a, MaxIterILS-a, RoutePool, v, TDev-a, MaxSPTime,
   MaxRootGap);
2: if  $n/v < \mathcal{A}$  then
3:   tolerance  $\leftarrow$  TDev-a;
4: else
5:   tolerance  $\leftarrow$  1;
6:  $s^* \leftarrow$  ILS-RVND(MaxIter-a, MaxIterILS-a, NULL, RoutePool, v, tolerance);
7:  $s^* \leftarrow$  SP( $s^*$ , RoutePool, MaxSPTime, MaxRootGap, v);
8: return  $s^*$ ;
9: end ILS-RVND-SP-a.

```

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### 7.3 Computational results

The algorithm ILS-RVND-SP was coded in C++ and the tests were executed in an Intel® Core™ i7 with 2.93 GHz and 8 GB of RAM running under Linux 64 bits. CPLEX 12.2 was used as a MIP solver. The computational experiments were carried out using a single thread. Table 7.1 shows the values of the parameters used by ILS-RVND-SP, which were calibrated after preliminary experiments. The most crucial parameters are  $\mathcal{N}$  and  $\mathcal{A}$ . The values adopted for the remaining ones are not so critical, which is reflected in the round numbers chosen.

Table 7.1: Values of the parameters used by ILS-RVND-SP

| Parameter            | Value  |
|----------------------|--|
| $\mathcal{N}$        | 150  |
| $\mathcal{A}$        | 11   |
| <i>MaxIter-a</i>     | 50   |
| <i>MaxIter-b</i>     | 100  |
| <i>MaxIterILS-a</i>  | $n + 0, 5 \times v$  |
| <i>MaxIterILS-b</i>  | 2000   |
| <i>Tdev-a</i>        | 0.05   |
| <i>Tdev-b</i>        | 0.005  |
| $\gamma$             | random value of the set $\{0.00, 0.05, 0.10, \dots, 1, 70\}$ |
| <i>MaxRootGap</i>    | 0.02   |
| <i>MaxSPTime</i> (s) | 60   |

In the following tables, **Instance** denotes the test-problem,  **$n$**  is the number of cus-

tomers,  $|\mathbf{G}|$  is the number of depots,  $v$  is the number of vehicles available per depot, **BKS** represents the Best Known Solution (BKS) reported in the literature, **Best Sol.**, **Avg. Sol.** and **Time (s)** indicate, respectively, the best solution, the average solution and the average computational time in seconds associated to the correspondent work, **Gap** denotes the gap, given by  $100 \times ((z_{\text{ILS-RVND-SP}} - z_{\text{BKS}})/z_{\text{BKS}})$ , between the best solution found by ILS-RVND-SP and the BKS, **Avg. Gap** corresponds to the gap between the average solution found by ILS-RVND-SP and the best known solution. The BKSs are highlighted in boldface and the improved solutions are underlined.

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**Algorithm 7.4** ILS-RVND-SP-b
 

---

```

1: Procedure ILS-RVND-SP-b(MaxIter-b, MaxIterILS-b, RoutePool, v, TDev-b, MaxSPTime,
   MaxRootGap);
2: if  $n/v < \mathcal{A}$  then
3:   tolerance  $\leftarrow TDev$ -b;
4: else
5:   tolerance  $\leftarrow 1$ ;
6: iter  $\leftarrow 0$ ; s*  $\leftarrow \emptyset$ ; s0  $\leftarrow NULL$ ;
7: while iter < MaxIter-b do
8:   s  $\leftarrow$  ILS-RVND(1, MaxIterILS-b, s0, RoutePool, v, tolerance);
9:   improvement  $\leftarrow$  true;
10:  while improvement do
11:    s'  $\leftarrow$  SP(s, RoutePool, MaxSPTime, MaxRootGap, v);
12:    if  $f(s') < f(s)$  then
13:      s  $\leftarrow s'$ ;
14:    else
15:      improvement  $\leftarrow$  false;
16:      Remove non-permanent routes from RoutePool;
17:      if  $n/v < \mathcal{A}$  and Time > MaxSPTime then
18:        tolerance  $\leftarrow tolerance - 0.1 \times TDev$ -b;
19:      if  $n/v < \mathcal{A}$  and Problem solved at the root node then
20:        tolerance  $\leftarrow tolerance + 0.1 \times TDev$ -b;
21:      iter  $\leftarrow iter + 1$ ;
22:      if  $f(s) < f(s^*)$  or s* is empty then
23:        s*  $\leftarrow s$ ;
24:        Add routes associated to s* permanently to the pool;
25:  return s*;
26: end ILS-RVND-SP-b.

```

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### 7.3.1 CVRP

The developed algorithm was tested in the instances of the A, B, E, M, P series and all known optimal solutions were easily determined. Table 7.2 only shows the results obtained in the three open instances of the M-series, namely: M-n151-k12, M-n200-k16 and M-n200-k17. ILS-RVND-SP was found capable of improving the result of the second one and to equal the BKSs of the first and third ones. Table 7.3 contains the results found in the instances of Christofides et al. [31] and a comparison with those reported by Rochat and Taillard [147], Pisinger and Røpke (ALNS 50K) [136], Mester and Bräysy [118], Nagata and Bräysy [127] and Vidal et al. [177]. ILS-RVND-SP was successful to

equal the BKS in 13 of the 14 instances and the average gap between the Avg. Sols. found by ILS-RVND-SP and the BKSs was 0.08%. Table 7.4 illustrates a comparison, in terms of average solution, between the results obtained by ILS-RVND-SP and those found by Pisinger and Røpke [136] (ALNS 50K), Nagata and Bräysy [127] and Zachariadis and Kiranoudis [187] for the instances of Golden et al. [80]. It can be seen that the ILS-RVND outperformed the algorithm of Pisinger and Røpke [136], but it could not compete with those of Nagata and Bräysy [127], Vidal et al. [177] in terms of average solution quality. Yet, the average gap between the Avg. Sols. found by ILS-RVND and the BKSs was only 0.55%, a value smaller the one obtained by the general heuristic of [136]. On the other hand, in spite of obtaining slightly lower quality solutions, the proposed algorithm appears to be simpler than those developed in [127], [177] and [187].

Table 7.2: Results found for the open instances of the M-series

| Instance   | $n$ | $v$ | BKS         | ILS-RVND-SP |           |         |              |          |
|------------|-----|-----|-------------|-------------|-----------|---------|--------------|----------|
|            |     |     |             | Best Sol.   | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| M-n151-k12 | 150 | 12  | <b>1015</b> | <b>1015</b> | 1015.5    | 0.00    | 0.05         | 37.12    |
| M-n200-k16 | 199 | 16  | <b>1285</b> | <b>1278</b> | 1285.8    | -0.54   | 0.06         | 772.01   |
| M-n200-k17 | 199 | 17  | <b>1275</b> | <b>1275</b> | 1279.9    | 0.00    | 0.38         | 513.00   |
|            |     |     |             | Avg.        | -0.18     |         | 0.17         | 440.71   |

### 7.3.2 ACVRP

ILS-RVND-SP was tested in the ACVRP instances suggested by Fischetti et al. [57] and extended by Pessoa et al. [134]. Table 7.5 shows the results obtained for the ACVRP instances. All known optimal solutions were found by ILS-RVND-SP. Regarding the two instances where the optimal solutions is not known, the proposed algorithm was capable of improving the BKS in one of them and to equal the best result in the other one.

### 7.3.3 OVRP

Table 7.6 presents the results found by ILS-RVND-SP in the set of instances of Christofides et al. [31] / Fisher [58] and in the set of instances of Li et al. [105], as well as a comparison with those pointed out by Pisinger and Røpke [136] (ALNS 50K), Fleszar et al. [60], Repoussis et al. [146] and Zachariadis and Kiranoudis [186]. Regarding those of Christofides et al. [31] / Fisher [58], ILS-RVND-SP was capable of obtaining the BKS in 12 cases and to improve another 3 solutions, but it failed to find 1 BKS. Furthermore, ILS-RVND-SP also failed to always obtain solutions with the minimum number of vehicles on instance C7. The average gap between the Avg. Sols. obtained by ILS-RVND-SP and the BKSs, disregarding instance C7, was 0.06%. As for the 8 instances of Li et al. [105], ILS-RVND-SP improved all solutions and the average gap between the Avg. Sols. produced by ILS-RVND-SP and the BKSs was -0.08%.

Table 7.3: Results found for the CVRP instances of Christofides et al. [31]

| Instance | $n$ | $v$ | BKS                 | Rochat and Taillard |      | Pisinger and Ropke |                       | Mester and Bräysy |                       | Nagata and Bräysy |                       | Vidal et al. |                       | ILS-RVND-SP |           |         |              |          |
|----------|-----|-----|---------------------|---------------------|------|--------------------|-----------------------|-------------------|-----------------------|-------------------|-----------------------|--------------|-----------------------|-------------|-----------|---------|--------------|----------|
|          |     |     |                     | Best Sol.           | Time | Best Sol.          | Time <sup>1</sup> (s) | Best Sol.         | Time <sup>2</sup> (s) | Best Sol.         | Time <sup>3</sup> (s) | Best Sol.    | Time <sup>4</sup> (s) | Best Sol.   | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
|          |     |     |                     |                     |      |                    |                       |                   |                       |                   |                       |              |                       |             |           |         |              |          |
| C1       | 50  | 5   | <sup>a</sup> 524.61 | 524.61              | -    | 524.61             | 21                    | 524.61            | 0.2                   | 524.61            | 4.3                   | 524.61       | 25.8                  | 524.61      | 524.61    | 0.00    | 0.00         | 1.48     |
| C2       | 75  | 10  | 835.26              | 835.26              | -    | 835.26             | 36                    | 835.26            | 5.5                   | 835.26            | 22.3                  | 835.26       | 57.6                  | 835.26      | 835.26    | 0.00    | 0.00         | 13.52    |
| C3       | 100 | 8   | 826.14              | 826.14              | -    | 826.14             | 78                    | 826.14            | 1.0                   | 826.14            | 17.1                  | 826.14       | 76.2                  | 826.14      | 826.14    | 0.00    | 0.00         | 12.49    |
| C12      | 100 | 10  | 819.56              | 819.56              | -    | 819.56             | 73                    | 819.56            | 0.2                   | 819.56            | 8.1                   | 819.56       | 50.4                  | 819.56      | 819.56    | 0.00    | 0.00         | 5.23     |
| C11      | 120 | 7   | 1042.11             | 1042.11             | -    | 1042.11            | 113                   | 1042.11           | 1.1                   | 1042.11           | 21.0                  | 1042.11      | 69.0                  | 1042.11     | 1042.11   | 0.00    | 0.00         | 20.66    |
| C4       | 150 | 12  | 1028.42             | 1028.42             | -    | 1028.42            | 160                   | 1028.42           | 10.2                  | 1028.42           | 75.2                  | 1028.42      | 172.2                 | 1028.42     | 1028.73   | 0.00    | 0.03         | 53.48    |
| C5       | 199 | 17  | 1291.29             | 1291.45             | -    | 1297.12            | 219                   | 1291.29           | 2160.0                | 1291.45           | 302.1                 | 1291.45      | 356.4                 | 1291.45     | 1293.18   | < 0.01  | 0.13         | 625.17   |
| C6       | 50  | 6   | 555.43              | 555.43              | -    | 555.43             | 21                    | 555.43            | 4.2                   | 555.43            | 5.1                   | 555.43       | 28.8                  | 555.43      | 556.49    | 0.00    | 0.19         | 0.93     |
| C7       | 75  | 11  | 909.68              | 909.68              | -    | 909.68             | 36                    | 909.68            | 0.8                   | 909.68            | 38.9                  | 909.68       | 65.4                  | 909.68      | 910.00    | 0.00    | 0.03         | 5.05     |
| C8       | 100 | 9   | 865.94              | 865.94              | -    | 865.94             | 78                    | 865.94            | 0.8                   | 865.94            | 23.5                  | 865.94       | 68.4                  | 865.94      | 865.94    | 0.00    | 0.00         | 7.61     |
| C14      | 100 | 11  | 866.37              | 866.37              | -    | 866.37             | 73                    | 866.37            | 1.7                   | 866.37            | 13.2                  | 866.37       | 71.4                  | 866.37      | 866.37    | 0.00    | 0.00         | 7.12     |
| C13      | 120 | 11  | 1541.14             | 1541.14             | -    | 1542.86            | 113                   | 1541.14           | 13.5                  | 1541.14           | 106.9                 | 1541.14      | 169.8                 | 1541.14     | 1544.07   | 0.00    | 0.19         | 80.24    |
| C9       | 150 | 14  | 1162.55             | 1162.55             | -    | 1163.68            | 160                   | 1162.55           | 25.8                  | 1162.55           | 135.6                 | 1162.55      | 151.8                 | 1162.55     | 1164.11   | 0.00    | 0.13         | 82.50    |
| C10      | 199 | 18  | 1395.85             | 1395.85             | -    | 1405.88            | 219                   | 1401.12           | 52.2                  | 1395.85           | 390.6                 | 1395.85      | 493.2                 | 1395.85     | 1402.03   | 0.00    | 0.44         | 496.07   |
|          |     |     |                     |                     |      |                    |                       |                   |                       |                   |                       |              | Avg:                  |             | 0.00      | 0.08    | 100.83       |          |

<sup>a</sup>: Optimality proved.

1: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2: Average of 10 runs on a Pentium IV 2.8 GHz (2444 Mflop/s).

3: Average of 10 runs on an Opteron 2.4 GHz (3485 Mflop/s).

4: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz.

Table 7.4: Results found for the CVRP instances of Golden et al. [80]

| Instance | $n$ | $v$ | BKS                   | Pisinger and Røpke |                       | Nagata and Bräysy |                       | Vidal et al.    |                       | Zachariadis and Kiranoudis |                       | ILS-RVND-SP     |           |         |              |          |
|----------|-----|-----|-----------------------|--------------------|-----------------------|-------------------|-----------------------|-----------------|-----------------------|----------------------------|-----------------------|-----------------|-----------|---------|--------------|----------|
|          |     |     |                       | Avg. Sol.*         | Time <sup>1</sup> (s) | Avg. Sol.*        | Time <sup>2</sup> (s) | Avg. Sol.*      | Time <sup>3</sup> (s) | Avg. Sol.*                 | Time <sup>4</sup> (s) | Best Sol.       | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| G17      | 240 | 22  | $a, b$ <b>707.76</b>  | 710.59             | 304                   | 707.78            | 582.4                 | 708.09          | 423.6                 | 708.94                     | 962.3                 | <b>707.76</b>   | 707.81    | 0.00    | 0.01         | 937.59   |
| G13      | 252 | 26  | $a, b$ <b>857.19</b>  | 874.24             | 285                   | 858.42            | 921.9                 | 859.64          | 561.6                 | 860.44                     | 1189.3                | <b>857.19</b>   | 860.00    | 0.00    | 0.33         | 910.35   |
| G9       | 255 | 14  | $b$ <b>579.71</b>     | 590.33             | 437                   | 581.46            | 1043.3                | 581.79          | 973.2                 | 584.66                     | 929.4                 | 583.24          | 585.21    | 0.61    | 0.95         | 1720.76  |
| G18      | 300 | 27  | $a, b$ <b>995.13</b>  | 1007.84            | 387                   | 995.91            | 1465.9                | 998.44          | 993.6                 | 997.74                     | 1718.6                | 995.65          | 997.85    | 0.05    | 0.27         | 2297.62  |
| G14      | 320 | 30  | $a, b$ <b>1080.55</b> | 1103.53            | 393                   | 1080.84           | 1239.3                | 1082.41         | 847.2                 | 1083.55                    | 1187.4                | <b>1080.55</b>  | 1082.15   | 0.00    | 0.15         | 1513.32  |
| G10      | 323 | 16  | $a, b$ <b>736.26</b>  | 751.36             | 616                   | 739.56            | 1617.5                | 739.86          | 1551.6                | 739.86                     | 1271.4                | 741.96          | 744.17    | 0.77    | 1.07         | 3229.35  |
| G19      | 360 | 33  | $a, b$ <b>1365.60</b> | 1377.88            | 449                   | 1366.70           | 2115.6                | 1367.83         | 1674.6                | 1370.77                    | 1824.2                | 1366.29         | 1367.25   | 0.05    | 0.12         | 2917.31  |
| G15      | 396 | 33  | $a, b$ <b>1337.92</b> | 1366.23            | 468                   | 1344.32           | 1872.2                | 1343.52         | 2349.0                | 1344.41                    | 1658.8                | 1347.13         | 1349.23   | 0.69    | 0.85         | 3265.68  |
| G11      | 399 | 18  | $a, b$ <b>912.84</b>  | 926.57             | 761                   | 916.27            | 2337.5                | 916.44          | 2736.6                | 919.52                     | 1392.2                | 921.46          | 922.93    | 0.94    | 1.11         | 5978.97  |
| G20      | 420 | 38  | $a, b$ <b>1818.32</b> | 1834.70            | 488                   | 1821.65           | 2824.7                | 1822.02         | 2293.8                | 1829.57                    | 1199.3                | 1821.16         | 1823.52   | 0.16    | 0.29         | 4997.31  |
| G16      | 480 | 37  | $a, b$ <b>1612.50</b> | 1645.67            | 549                   | 1622.26           | 2616.2                | 1621.02         | 3496.2                | 1623.42                    | 1848.5                | 1624.55         | 1627.76   | 0.75    | 0.95         | 4835.12  |
| G12      | 483 | 19  | $a, b$ <b>1102.69</b> | 1125.22            | 911                   | 1108.21           | 3561.9                | 1106.73         | 5740.2                | 1110.65                    | 1282.3                | 1113.30         | 1116.52   | 0.96    | 1.25         | 10410.70 |
| G5       | 200 | 5   | <b>6460.98</b>        | 6482.49            | 629                   | <b>6460.98</b>    | 164.7                 | <b>6460.98</b>  | 153.6                 | <b>6460.98</b>             | 989.6                 | <b>6460.98</b>  | 6460.98   | 0.00    | 0.00         | 978.69   |
| G1       | 240 | 9   | $b$ <b>5623.47</b>    | 5662.57            | 93                    | 5632.05           | 3393                  | 5627.00         | 700.8                 | 5637.99                    | 907.7                 | 5657.74         | 5671.65   | 0.61    | 0.86         | 994.59   |
| G6       | 280 | 7   | $c$ <b>8412.88</b>    | 8543.30            | 876                   | 8413.41           | 830.3                 | 8412.90         | 502.8                 | 8412.90                    | 1091.6                | 8412.90         | 8412.90   | 0.00    | 0.00         | 2455.56  |
| G2       | 320 | 10  | $b$ <b>8404.61</b>    | 8487.94            | 672                   | 8440.25           | 1726.2                | 8446.65         | 1245.0                | 8457.92                    | 1249.4                | 8447.92         | 8449.82   | 0.52    | 0.54         | 2659.68  |
| G7       | 360 | 9   | $b$ <b>10102.70</b>   | 10265.15           | 941                   | 10186.93          | 2179.7                | 10157.63        | 1376.4                | 10192.47                   | 1885.5                | 10195.58        | 10195.60  | 0.92    | 0.92         | 4410.84  |
| G3       | 400 | 10  | <b>11036.22</b>       | 11052.72           | 1015                  | <b>11036.22</b>   | 2606.8                | <b>11036.22</b> | 1679.4                | <b>11036.22</b>            | 1164.0                | <b>11036.22</b> | 11036.22  | 0.00    | 0.00         | 6064.98  |
| G8       | 440 | 10  | $b$ <b>11635.30</b>   | 11766.07           | 1011                  | 11691.54          | 5776.7                | 11646.58        | 2440.2                | 11674.43                   | 1657.4                | 11710.47        | 11774.40  | 0.65    | 1.20         | 7541.75  |
| G4       | 480 | 10  | $a$ <b>13592.88</b>   | 13748.50           | 1328                  | 13618.55          | 3841.6                | 13624.52        | 2620.2                | 13632.59                   | 1019.0                | 13624.53        | 13624.53  | 0.23    | 0.23         | 10644.50 |
|          |     |     |                       | Avg. Gap (%)       | 1.34                  | Avg. Gap (%)      | 0.27                  | Avg. Gap (%)    | 0.26                  | Avg. Gap (%)               | 0.42                  | Avg.            | Avg.      | 0.40    | 0.55         | 3938.23  |

1; Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2; Average of 10 runs on an Opteron 2.4 GHz (3485 Mflop/s)

3; Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz.

4; Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).

\*: Average of 10 runs

 $a$ ; Found by Nagata and Bräysy [127] $b$ ; Found by Vidal et al. [177] $c$ ; Found by Mester and Bräysy [118]

Table 7.5: Results found for the ACVRP instances of Fischetti et al. [57] and Pessoa et al. [134]

| Instance | $n$ | $v$ | BKS                      | ILS-RVND-SP |           |         |              |          |
|----------|-----|-----|--------------------------|-------------|-----------|---------|--------------|----------|
|          |     |     |                          | Best Sol.   | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| A034-02f | 34  | 2   | <sup>a</sup> <b>1406</b> | <b>1406</b> | 1406.00   | 0.00    | 0.00         | 0.57     |
| A034-04f | 34  | 4   | <sup>a</sup> <b>1773</b> | <b>1773</b> | 1773.00   | 0.00    | 0.00         | 0.47     |
| A034-08f | 34  | 8   | <sup>a</sup> <b>2672</b> | <b>2672</b> | 2672.00   | 0.00    | 0.00         | 0.47     |
| A036-03f | 36  | 3   | <sup>a</sup> <b>1644</b> | <b>1644</b> | 1644.00   | 0.00    | 0.00         | 0.64     |
| A036-05f | 36  | 5   | <sup>a</sup> <b>2110</b> | <b>2110</b> | 2110.00   | 0.00    | 0.00         | 0.61     |
| A036-10f | 36  | 10  | <sup>a</sup> <b>3338</b> | <b>3338</b> | 3338.00   | 0.00    | 0.00         | 5.67     |
| A039-03f | 39  | 3   | <sup>a</sup> <b>1654</b> | <b>1654</b> | 1654.00   | 0.00    | 0.00         | 0.69     |
| A039-06f | 39  | 6   | <sup>a</sup> <b>2289</b> | <b>2289</b> | 2289.00   | 0.00    | 0.00         | 0.60     |
| A039-12f | 39  | 12  | <sup>a</sup> <b>3705</b> | <b>3705</b> | 3705.00   | 0.00    | 0.00         | 0.77     |
| A045-03f | 45  | 3   | <sup>a</sup> <b>1740</b> | <b>1740</b> | 1740.00   | 0.00    | 0.00         | 1.06     |
| A045-06f | 45  | 6   | <sup>a</sup> <b>2303</b> | <b>2303</b> | 2303.00   | 0.00    | 0.00         | 0.88     |
| A045-11f | 45  | 11  | <sup>a</sup> <b>3544</b> | <b>3544</b> | 3544.00   | 0.00    | 0.00         | 2.46     |
| A048-03f | 48  | 3   | <sup>a</sup> <b>1891</b> | <b>1891</b> | 1891.00   | 0.00    | 0.00         | 1.26     |
| A048-05f | 48  | 5   | <sup>a</sup> <b>2283</b> | <b>2283</b> | 2289.50   | 0.00    | 0.28         | 1.79     |
| A048-10f | 48  | 10  | <sup>a</sup> <b>3325</b> | <b>3325</b> | 3325.60   | 0.00    | 0.02         | 1.29     |
| A056-03f | 56  | 3   | <sup>a</sup> <b>1739</b> | <b>1739</b> | 1740.00   | 0.00    | 0.06         | 2.09     |
| A056-05f | 56  | 5   | <sup>a</sup> <b>2165</b> | <b>2165</b> | 2165.00   | 0.00    | 0.00         | 3.53     |
| A056-10f | 56  | 10  | <sup>a</sup> <b>3263</b> | <b>3263</b> | 3264.50   | 0.00    | 0.05         | 2.26     |
| A065-03f | 65  | 3   | <sup>a</sup> <b>1974</b> | <b>1974</b> | 1974.00   | 0.00    | 0.00         | 3.08     |
| A065-06f | 65  | 6   | <sup>a</sup> <b>2567</b> | <b>2567</b> | 2571.70   | 0.00    | 0.18         | 3.30     |
| A065-12f | 65  | 12  | <sup>a</sup> <b>3902</b> | <b>3902</b> | 3904.90   | 0.00    | 0.07         | 3.41     |
| A071-03f | 71  | 3   | <sup>a</sup> <b>2054</b> | <b>2054</b> | 2054.00   | 0.00    | 0.00         | 4.46     |
| A071-05f | 71  | 5   | <sup>b</sup> <b>2475</b> | <b>2457</b> | 2457.90   | -0.73   | -0.69        | 6.72     |
| A071-10f | 71  | 10  | <sup>b</sup> <b>3486</b> | <b>3486</b> | 3492.90   | 0.00    | 0.20         | 5.61     |
|          |     |     |                          | Avg.        | -0.03     | 0.01    | 2.24         |          |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Value presented by Pessoa et al. [134].

### 7.3.4 VRPSPD

Table 7.7 contains the results obtained in the set of instances of Salhi and Nagy [149] and a comparison with those reported by Gajpal and Abad [64], Zachariadis et al. [190] and Subramanian et al. [157]. It can be verified that the ILS-RVND-SP equaled 21 BKSs and improved another 5. Table 7.8 presents the results found in the instances of Montané and Galvão [121] and also those reported by Souza et al. [153], Zachariadis and Kiranoudis [188] and Subramanian et al. [157]. ILS-RVND-SP found 12 BKSs and improved another 6 results. It is noteworthy to mention that the proposed algorithm had a satisfactory performance in the large size instances, always producing, on average, competitive results. The average gap between the Avg. Sols produced by ILS-RVND-SP and the BKSs in the first and second group of instances was 0.12% and -0.07% respectively.

Table 7.6: Results found for the OVRP instances of Christofides et al. [31], Fisher [58] and Li et al. [106]

| Instance   | $n$ | $v_{min}$ | BKS                 | Pisinger and Ropke |                   |        | Fleszar et al. |        |                   | Repoussis et al. |      |                   | Zachariadis and Kiranoudis |      |                   | ILS-RVND-SP |         |           |         |          |
|------------|-----|-----------|---------------------|--------------------|-------------------|--------|----------------|--------|-------------------|------------------|------|-------------------|----------------------------|------|-------------------|-------------|---------|-----------|---------|----------|
|            |     |           |                     | $v_{best}$         | Time <sup>1</sup> |        | Best Sol.      | $v$    | Time <sup>2</sup> | Best Sol.        | $v$  | Time <sup>3</sup> | Best Sol.                  | $v$  | Time <sup>4</sup> | Best Sol.   | $v$     | Avg. Sol. | Gap (%) | Time (s) |
|            |     |           |                     |                    |                   |        |                |        |                   |                  |      |                   |                            |      |                   |             |         |           |         |          |
| C1         | 50  | 5         | <sup>a</sup> 416.06 | 5                  | 23                | 416.06 | 5              | 1.0    | 416.06            | 5                | 98   | 416.06            | 5                          | 28   | 416.06            | 5           | 416.06  | 0.00      | 1.78    |          |
| F11        | 71  | 4         | <sup>a</sup> 177.00 | 4                  | 104               | 178.66 | 4              | 6.2    | 177.00            | 4                | 264  | 177.00            | 4                          | 132  | 177.00            | 4           | 177.21  | 0.00      | 0.12    |          |
| C2         | 75  | 10        | <sup>a</sup> 567.14 | 10                 | 53                | 567.14 | 10             | 2.3    | 567.14            | 10               | 143  | 567.14            | 10                         | 72   | 567.14            | 10          | 567.14  | 0.00      | 6.44    |          |
| C3         | 100 | 8         | <sup>a</sup> 639.74 | 8                  | 128               | 641.76 | 8              | 9.5    | 639.74            | 8                | 330  | 639.74            | 8                          | 97   | 639.74            | 8           | 639.81  | 0.00      | 0.01    |          |
| C12        | 100 | 10        | <sup>a</sup> 534.24 | 10                 | 118               | 534.40 | 10             | 6.7    | 534.24            | 10               | 363  | 534.24            | 10                         | 47   | 534.24            | 10          | 534.24  | 0.00      | 0.00    |          |
| C11        | 120 | 7         | 682.12              | 7                  | 141               | 682.12 | 7              | 10.7   | 682.12            | 7                | 318  | 682.12            | 7                          | 76   | 682.12            | 7           | 682.12  | 0.00      | 0.00    |          |
| F12        | 134 | 7         | 769.55              | 7                  | 359               | 769.66 | 7              | 75.4   | 769.55            | 7                | 753  | 769.55            | 7                          | 278  | 769.55            | 7           | 770.00  | 0.00      | 0.06    |          |
| C4         | 150 | 12        | 733.13              | 12                 | 279               | 737.82 | 12             | 45.4   | 733.13            | 12               | 613  | 733.13            | 12                         | 204  | 733.13            | 12          | 733.13  | 0.00      | 0.00    |          |
| C5         | 199 | 16        | 893.39              | 16                 | 237               | 905.96 | 16             | 17.1   | 894.11            | 16               | 1272 | 893.39            | 16                         | 332  | 883.15            | 16          | 895.55  | -1.11     | 0.24    |          |
| C6         | 50  | 5         | 412.96              | 6                  | 31                | 412.96 | 6              | 75.8   | 412.96            | 6                | 215  | 412.96            | 6                          | -    | 412.96            | 6           | 412.96  | 0.00      | 0.00    |          |
| C7         | 75  | 10        | 583.19              | 10                 | 33                | 596.47 | 10             | 22.3   | 584.15            | 10               | 367  | 583.19            | 10                         | -    | 583.19            | 10          | 582.07  | 0.00      | 7.29    |          |
| C8         | 100 | 8         | 644.63              | 9                  | 114               | 644.63 | 9              | 587.6  | 644.63            | 9                | 510  | 644.63            | 9                          | -    | 644.63            | 9           | 644.95  | 0.00      | 0.05    |          |
| C14        | 100 | 10        | 591.87              | 11                 | 75                | 591.87 | 11             | 389    | 591.87            | 11               | 411  | 591.87            | 11                         | -    | 591.87            | 11          | 591.87  | 0.00      | 0.00    |          |
| C13        | 120 | 7         | 904.04              | 11                 | 116               | 904.94 | 11             | 1820.1 | 910.26            | 11               | 890  | 899.16            | 11                         | -    | 899.16            | 11          | 904.02  | -0.54     | 0.00    |          |
| C9         | 150 | 12        | 757.84              | 13                 | 185               | 760.06 | 13             | 1094.1 | 764.56            | 13               | 933  | 757.91            | 13                         | -    | 757.91            | 13          | 759.38  | 0.01      | 0.20    |          |
| C10        | 199 | 16        | 875.67              | 17                 | 224               | 875.67 | 17             | 1252.4 | 888.46            | 17               | 1678 | 874.71            | 17                         | -    | 874.71            | 17          | 877.68  | -0.11     | 0.23    |          |
| Avg. -0.11 |     |           |                     |                    |                   |        |                |        |                   |                  |      |                   |                            |      |                   |             | 143.44  | 0.06      | 143.44  |          |
| O1         | 200 | 5         | 6018.52             | 5                  | -                 | -      | -              | -      | 6018.52           | 5                | 452  | 6018.52           | 5                          | 612  | 6018.52           | 5           | 6018.52 | 0.00      | 0.00    |          |
| O2         | 240 | 9         | 4557.38             | 9                  | -                 | -      | -              | -      | 4557.38           | 9                | 613  | 4557.38           | 9                          | 774  | 4544.46           | 9           | 4551.74 | -0.28     | -0.12   |          |
| O3         | 280 | 7         | 7731.00             | 7                  | -                 | -      | -              | -      | 7731.00           | 7                | 736  | 7731.00           | 7                          | 681  | 7721.16           | 7           | 7728.77 | -0.13     | -0.03   |          |
| O4         | 320 | 10        | 7253.20             | 10                 | -                 | -      | -              | -      | 7253.20           | 10               | 833  | 7253.20           | 10                         | 957  | 7215.48           | 10          | 7229.56 | -0.52     | -0.33   |          |
| O5         | 360 | 8         | 9193.15             | 8                  | -                 | -      | -              | -      | 9193.15           | 8                | 1365 | 9193.15           | 8                          | 1491 | 9180.93           | 8           | 9205.01 | -0.13     | 0.13    |          |
| O6         | 400 | 9         | 9793.72             | 9                  | -                 | -      | -              | -      | 9793.72           | 9                | 1213 | 9793.72           | 9                          | 1070 | 9773.83           | 9           | 9784.52 | -0.20     | -0.09   |          |
| O7         | 440 | 10        | 10347.70            | 10                 | -                 | -      | -              | -      | 10347.70          | 10               | 1547 | 10347.70          | 10                         | 1257 | 10326.57          | 10          | 10342.1 | -0.20     | -0.05   |          |
| O8         | 480 | 10        | 12415.36            | 10                 | -                 | -      | -              | -      | 12415.36          | 10               | 1653 | 12389.43          | 10                         | 1512 | 12389.43          | 10          | 12393.4 | -0.21     | -0.18   |          |
| Avg. -0.21 |     |           |                     |                    |                   |        |                |        |                   |                  |      |                   |                            |      |                   |             | 3844.03 | -0.08     | 3844.03 |          |

<sup>a</sup>: Optimality proved.

1: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

2: Best run on a Pentium M 2.0 GHz (1738 Mflop/s).

3: Best run on a Pentium IV 2.8 GHz (2444 Mflop/s).

4: Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).

Table 7.7: Results found for the VRPSPD instances of Salhi and [149]

| Instance | $n$ | $v$ | BKS                        | Gajpal and Abad |                       | Zachariadis et al. |                       | Subramanian et al. |                       | ILS-RVND-SP           |           |         |              |          |
|----------|-----|-----|----------------------------|-----------------|-----------------------|--------------------|-----------------------|--------------------|-----------------------|-----------------------|-----------|---------|--------------|----------|
|          |     |     |                            | Best Sol.       | Time <sup>1</sup> (s) | Best Sol.          | Time <sup>2</sup> (s) | Best Sol.          | Time <sup>3</sup> (s) | Best Sol.             | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| CMT1X    | 50  | 3   | <sup>a</sup> <b>466.77</b> | <b>466.77</b>   | 5.00                  | 469.80             | 2.1                   | <b>466.77</b>      | 2.3                   | <b>466.77</b>         | 466.77    | 0.00    | 0.00         | 2.08     |
| CMT1Y    | 50  | 3   | <sup>a</sup> <b>466.77</b> | <b>466.77</b>   | 5.00                  | 469.80             | 3.8                   | <b>466.77</b>      | 2.3                   | <b>466.77</b>         | 466.77    | 0.00    | 0.00         | 1.97     |
| CMT2X    | 75  | 6   | <b>684.21</b>              | <b>684.21</b>   | 41.25                 | <b>684.21</b>      | 5.4                   | <b>684.21</b>      | 6.4                   | <b>684.21</b>         | 684.78    | 0.00    | 0.08         | 12.79    |
| CMT2Y    | 75  | 6   | <b>684.21</b>              | 684.94          | 22.25                 | <b>684.21</b>      | 6.8                   | <b>684.21</b>      | 6.4                   | <b>684.21</b>         | 684.59    | 0.00    | 0.06         | 10.83    |
| CMT3X    | 100 | 5   | <sup>a</sup> <b>721.27</b> | 721.40          | 377.50                | <b>721.27</b>      | 11.9                  | <b>721.27</b>      | 12.1                  | <b>721.27</b>         | 721.46    | 0.00    | 0.03         | 17.69    |
| CMT3Y    | 100 | 5   | <sup>a</sup> <b>721.27</b> | 721.40          | 43.75                 | <b>721.27</b>      | 11                    | <b>721.27</b>      | 12.3                  | <b>721.27</b>         | 721.50    | 0.00    | 0.03         | 17.61    |
| CMT12X   | 100 | 5   | <b>662.22</b>              | 663.01          | 36.25                 | <b>662.22</b>      | 9.3                   | <b>662.22</b>      | 10.3                  | <b>662.22</b>         | 663.44    | 0.00    | 0.18         | 9.07     |
| CMT12Y   | 100 | 5   | <b>662.22</b>              | 663.50          | 39.25                 | <b>662.22</b>      | 4.8                   | <b>662.22</b>      | 10.8                  | <b>662.22</b>         | 663.12    | 0.00    | 0.14         | 9.34     |
| CMT11X   | 120 | 4   | <b>833.92</b>              | 839.66          | 57.25                 | <b>833.92</b>      | 21.2                  | <b>833.92</b>      | 18.9                  | 846.23                | 848.65    | 1.48    | 1.77         | 51.82    |
| CMT11Y   | 120 | 4   | <b>833.92</b>              | 840.19          | 52.75                 | <b>833.92</b>      | 14.4                  | <b>833.92</b>      | 19.0                  | 846.23                | 848.74    | 1.48    | 1.78         | 48.63    |
| CMT4X    | 150 | 7   | <b>852.46</b>              | 854.12          | 131.75                | <b>852.46</b>      | 29.6                  | <b>852.46</b>      | 30.9                  | <b>852.46</b>         | 853.02    | 0.00    | 0.07         | 98.03    |
| CMT4Y    | 150 | 7   | <b>852.46</b>              | 855.76          | 140.25                | <b>852.46</b>      | 27.4                  | <b>852.46</b>      | 31.6                  | <b>852.46</b>         | 852.73    | 0.00    | 0.03         | 80.63    |
| CMT5X    | 199 | 10  | <b>1029.25</b>             | 1034.87         | 377.50                | 1030.55            | 62.8                  | <b>1029.25</b>     | 71.5                  | <b>1029.25</b>        | 1029.52   | 0.00    | 0.03         | 1786.74  |
| CMT5Y    | 199 | 10  | <b>1029.25</b>             | 1037.34         | 393.50                | 1030.55            | 47.7                  | <b>1029.25</b>     | 69.6                  | <b>1029.25</b>        | 1029.25   | 0.00    | 0.00         | 1726.18  |
| CMT6X    | 50  | 7   | <b>555.43</b>              | <b>555.43</b>   | 14.00                 | -                  | -                     | -                  | -                     | <b>555.43</b>         | 557.35    | 0.00    | 0.35         | 1.04     |
| CMT6Y    | 50  | 7   | <b>555.43</b>              | <b>555.43</b>   | 13.75                 | -                  | -                     | -                  | -                     | <b>555.43</b>         | 557.10    | 0.00    | 0.30         | 1.08     |
| CMT7X    | 75  | 13  | <b>900.12</b>              | <b>900.12</b>   | 47.75                 | -                  | -                     | -                  | -                     | <b>900.12</b>         | 901.02    | 0.00    | 0.10         | 4.55     |
| CMT7Y    | 75  | 13  | <b>900.54</b>              | <b>900.54</b>   | 46.25                 | -                  | -                     | -                  | -                     | <u><b>900.12</b></u>  | 901.08    | -0.05   | 0.06         | 4.87     |
| CMT8X    | 100 | 10  | <b>865.50</b>              | <b>865.50</b>   | 80.75                 | -                  | -                     | -                  | -                     | <b>865.50</b>         | 865.50    | 0.00    | 0.00         | 7.36     |
| CMT8Y    | 100 | 10  | <b>865.50</b>              | <b>865.50</b>   | 77.75                 | -                  | -                     | -                  | -                     | <b>865.50</b>         | 865.50    | 0.00    | 0.00         | 7.74     |
| CMT14X   | 100 | 11  | <b>821.75</b>              | <b>821.75</b>   | 78.50                 | -                  | -                     | -                  | -                     | <b>821.75</b>         | 821.75    | 0.00    | 0.00         | 5.42     |
| CMT14Y   | 100 | 11  | <b>821.75</b>              | <b>821.75</b>   | 74.75                 | -                  | -                     | -                  | -                     | <b>821.75</b>         | 821.75    | 0.00    | 0.00         | 5.48     |
| CMT13X   | 120 | 12  | <b>1542.86</b>             | <b>1542.86</b>  | 160.25                | -                  | -                     | -                  | -                     | <b>1542.86</b>        | 1543.54   | -0.03   | 0.04         | 68.72    |
| CMT13Y   | 120 | 12  | <b>1542.86</b>             | <b>1542.86</b>  | 160.25                | -                  | -                     | -                  | -                     | <b>1542.86</b>        | 1544.42   | 0.00    | 0.10         | 73.49    |
| CMT9X    | 150 | 16  | <b>1161.54</b>             | <b>1161.54</b>  | 300.00                | -                  | -                     | -                  | -                     | <u><b>1160.68</b></u> | 1161.77   | -0.07   | 0.02         | 64.43    |
| CMT9Y    | 150 | 16  | <b>1161.54</b>             | <b>1161.54</b>  | 291.75                | -                  | -                     | -                  | -                     | <u><b>1160.68</b></u> | 1162.59   | -0.07   | 0.09         | 80.86    |
| CMT10X   | 199 | 20  | <b>1386.29</b>             | <b>1386.29</b>  | 773.50                | -                  | -                     | -                  | -                     | <u><b>1373.40</b></u> | 1379.19   | -0.93   | -0.51        | 552.81   |
| CMT10Y   | 199 | 20  | <b>1395.04</b>             | <b>1395.04</b>  | 757.50                | -                  | -                     | -                  | -                     | <u><b>1373.40</b></u> | 1377.03   | -1.55   | -1.29        | 547.39   |
|          |     |     |                            |                 |                       |                    |                       |                    |                       |                       | Avg.      | 0.01    | 0.12         | 189.24   |

<sup>a</sup>: Optimality proved.<sup>1</sup>: Best run on a Xeon 2.4 GHz (1978 Mflop/s). <sup>2</sup>: Average of 10 runs on a T5500 1.66 GHz (2791 Mflop/s).<sup>3</sup>: Average of 50 runs on a cluster with 32 SMP nodes, where each consists of two Intel Xeon 2.66 GHz (wall clock).

### 7.3.5 VRPMPD

Table 7.9 illustrates the results found by ILS-RVND-SP in the VRPMPD instances of Salhi and Nagy [149] and a comparison with those reported by Røpke and Pisinger [148] (6R - normal learning) and Gajpal and Abad [64]. ILS-RVND-SP obtained the BKS in 25 instances and it managed to improve the result of another 12. The developed algorithm outperformed both the algorithms of Røpke and Pisinger [148] and Gajpal and Abad [64] in terms of solution quality. The average gap between the Avg. Sols. obtained by ILS-RVND-SP and the BKSs was -0.06%.



Table 7.8: Results found for the VRPSPD instances of Montané and Galvão [121]

| Instance | n   | v  | BKS                  | Souza<br>et al. |                          | Zachariadis<br>et al. |                          | Subramanian<br>et al. |                          | ILS-RVND-SP  |              |            |                 |             |
|----------|-----|----|----------------------|-----------------|--------------------------|-----------------------|--------------------------|-----------------------|--------------------------|--------------|--------------|------------|-----------------|-------------|
|          |     |    |                      | Best<br>Sol.    | Time <sup>1</sup><br>(s) | Best<br>Sol.          | Time <sup>2</sup><br>(s) | Best<br>Sol.          | Time <sup>3</sup><br>(s) | Best<br>Sol. | Avg.<br>Sol. | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
|          |     |    |                      |                 |                          |                       |                          |                       |                          |              |              |            |                 |             |
| r101     | 100 | 12 | <sup>a</sup> 1009.95 | 1009.95         | 35.7                     | 1009.95               | 28.7                     | 1009.95               | 15.8                     | 1009.95      | 1010.08      | 0.00       | 0.01            | 65.42       |
| r201     | 100 | 3  | <sup>a</sup> 666.20  | 666.20          | 39.6                     | 666.20                | 31.4                     | 666.20                | 16.0                     | 666.20       | 666.20       | 0.00       | 0.00            | 15.71       |
| c101     | 100 | 16 | <sup>a</sup> 1220.18 | 1220.18         | 18.3                     | 1220.18               | 18.5                     | 1220.18               | 10.4                     | 1220.18      | 1220.43      | 0.00       | 0.02            | 12.93       |
| c201     | 100 | 5  | <sup>a</sup> 662.07  | 662.07          | 16.6                     | 662.07                | 23.5                     | 662.07                | 8.8                      | 662.07       | 662.07       | 0.00       | 0.00            | 9.77        |
| rc101    | 100 | 10 | <sup>a</sup> 1059.32 | 1059.32         | 12.8                     | 1059.32               | 23.8                     | 1059.32               | 11.1                     | 1059.32      | 1059.32      | 0.00       | 0.00            | 16.89       |
| rc201    | 100 | 3  | <sup>a</sup> 672.92  | 672.92          | 24.0                     | 672.92                | 21.2                     | 672.92                | 7.3                      | 672.92       | 672.92       | 0.00       | 0.00            | 11.42       |
| r1.2.1   | 200 | 23 | 3357.64              | 3357.64         | 175.8                    | 3375.19               | 84.6                     | 3360.02               | 66.2                     | 3353.80      | 3355.04      | -0.11      | -0.08           | 1142.05     |
| r2.2.1   | 200 | 5  | <sup>a</sup> 1665.58 | 1665.58         | 103.4                    | 1665.58               | 72.7                     | 1665.58               | 45.3                     | 1665.58      | 1665.58      | 0.00       | 0.00            | 1425.88     |
| c1.2.1   | 200 | 28 | 3629.89              | 3636.74         | 117.6                    | 3641.89               | 57.0                     | 3629.89               | 87.4                     | 3628.51      | 3636.53      | -0.04      | 0.18            | 2874.50     |
| c2.2.1   | 200 | 9  | 1726.59              | 1726.59         | 127.8                    | 1726.73               | 67.3                     | 1726.59               | 65.0                     | 1726.59      | 1726.59      | 0.00       | 0.00            | 1365.93     |
| rc1.2.1  | 200 | 23 | 3306.00              | 3312.92         | 299.3                    | 3316.94               | 83.4                     | 3306.00               | 71.7                     | 3303.70      | 3306.73      | -0.07      | 0.02            | 1293.53     |
| rc2.2.1  | 200 | 5  | <sup>a</sup> 1560.00 | 1560.00         | 77.5                     | 1560.00               | 74.4                     | 1560.00               | 44.7                     | 1560.00      | 1560.00      | 0.00       | 0.00            | 1361.87     |
| r1.4.1   | 400 | 54 | <sup>b</sup> 9605.75 | 9627.43         | 2928.3                   | 9668.18               | 421.5                    | 9618.97               | 481.6                    | 9519.45      | 9539.56      | -0.90      | -0.69           | 9177.90     |
| r2.4.1   | 400 | 10 | 3551.38              | 3582.08         | 768.6                    | 3560.73               | 352.0                    | 3551.38               | 459.2                    | 3546.49      | 3549.49      | -0.14      | -0.05           | 9086.79     |
| c1.4.1   | 400 | 63 | 11098.21             | 11098.21        | 1510.4                   | 11125.14              | 384.6                    | 11099.54              | 546.2                    | 11047.19     | 11075.60     | -0.46      | -0.20           | 8016.83     |
| c2.4.1   | 400 | 15 | 3546.10              | 3596.37         | 569.0                    | 3549.20               | 341.1                    | 3546.10               | 488.6                    | 3539.50      | 3543.65      | -0.19      | -0.07           | 10691.30    |
| rc1.4.1  | 400 | 52 | 9520.06              | 9535.46         | 2244.2                   | 9520.06               | 412.7                    | 9536.77               | 513.4                    | 9447.53      | 9478.12      | -0.76      | -0.44           | 10867.10    |
| rc2.4.1  | 400 | 11 | 3403.70              | 3422.11         | 3306.8                   | 3414.90               | 264.7                    | 3403.70               | 422.6                    | 3403.70      | 3403.70      | 0.00       | 0.00            | 8326.18     |
|          |     |    |                      |                 |                          |                       |                          |                       |                          | Avg.         |              | 0.15       | -0.07           | 3653.44     |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Best run on an Intel Core 2 Duo 1.66 GHz (2791 Mflop/s).<sup>2</sup>: Average of 10 runs T5500 1.66 GHz ((2791 Mflop/s)).<sup>3</sup>: Average of 50 runs on a cluster with 32 SMP nodes, where each node consists of two Intel Xeon 2.66 GHz (wall clock).<sup>b</sup>: Found by Subramanian et al. [157].

Table 7.9: Results found for the VRPMPD instances of Salhi and Nagy [149]

| Instance | $n$ | $v$ | BKS                        | Røpke and<br>Pisinger |                          | Gajpal and<br>Abad |                          | ILS-RVND-SP           |              |            |                 |             |
|----------|-----|-----|----------------------------|-----------------------|--------------------------|--------------------|--------------------------|-----------------------|--------------|------------|-----------------|-------------|
|          |     |     |                            | Best<br>Sol.          | Time <sup>1</sup><br>(s) | Best<br>Sol.       | Time <sup>2</sup><br>(s) | Best<br>Sol.          | Avg.<br>Sol. | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| CMT1H    | 50  | 4   | <sup>a</sup> <b>465.02</b> | <b>465</b>            | 51                       | <b>465.02</b>      | 5.6                      | <b>465.02</b>         | 465.03       | 0.00       | 0.00            | 2.07        |
| CMT1Q    | 50  | 6   | <sup>a</sup> <b>489.74</b> | <b>490</b>            | 41                       | <b>489.74</b>      | 6.0                      | <b>489.74</b>         | 489.74       | 0.00       | 0.00            | 1.52        |
| CMT1T    | 50  | 7   | <sup>a</sup> <b>520.06</b> | <b>520</b>            | 34                       | <b>520.06</b>      | 7.0                      | <b>520.06</b>         | 520.06       | 0.00       | 0.00            | 1.60        |
| CMT2H    | 75  | 5   | <b>662.63</b>              | <b>663</b>            | 78                       | <b>662.63</b>      | 22.0                     | <b>662.63</b>         | 662.63       | 0.00       | 0.00            | 5.16        |
| CMT2Q    | 75  | 7   | <b>732.76</b>              | <b>733</b>            | 65                       | <b>732.76</b>      | 26.2                     | <u><b>731.26</b></u>  | 731.40       | -0.20      | -0.19           | 8.03        |
| CMT2T    | 75  | 9   | <sup>a</sup> <b>782.77</b> | <b>783</b>            | 57                       | <b>782.77</b>      | 26.0                     | <b>782.77</b>         | 782.77       | 0.00       | 0.00            | 7.56        |
| CMT3H    | 100 | 3   | <sup>a</sup> <b>700.94</b> | <b>701</b>            | 186                      | <b>701.31</b>      | 35.6                     | <b>700.94</b>         | 700.94       | 0.00       | 0.00            | 17.65       |
| CMT3Q    | 100 | 4   | <sup>a</sup> <b>747.15</b> | <b>747</b>            | 128                      | <b>747.15</b>      | 39.8                     | <b>747.15</b>         | 747.15       | 0.00       | 0.00            | 9.70        |
| CMT3T    | 100 | 5   | <sup>a</sup> <b>798.07</b> | <b>798</b>            | 109                      | <b>798.07</b>      | 42.6                     | <b>798.07</b>         | 798.07       | 0.00       | 0.00            | 28.76       |
| CMT12H   | 100 | 6   | <sup>a</sup> <b>629.37</b> | <b>629</b>            | 150                      | <b>629.37</b>      | 32.8                     | <b>629.37</b>         | 629.37       | 0.00       | 0.00            | 13.93       |
| CMT12Q   | 100 | 8   | <sup>a</sup> <b>729.46</b> | <b>729</b>            | 108                      | <b>729.46</b>      | 42.0                     | <b>729.25</b>         | 729.25       | -0.03      | -0.03           | 17.37       |
| CMT12T   | 100 | 9   | <sup>a</sup> <b>787.52</b> | <b>788</b>            | 96                       | <b>787.52</b>      | 52.0                     | <b>787.52</b>         | 787.52       | 0.00       | 0.00            | 6.79        |
| CMT11H   | 120 | 4   | <b>818</b>                 | <b>818</b>            | 303                      | 820.35             | 45.8                     | <b>818.05</b>         | 818.06       | 0.01       | 0.01            | 63.18       |
| CMT11Q   | 120 | 6   | <sup>a</sup> <b>939.36</b> | <b>939</b>            | 196                      | <b>939.36</b>      | 66.2                     | <b>939.36</b>         | 939.36       | 0.00       | 0.00            | 20.35       |
| CMT11T   | 120 | 7   | <b>998.8</b>               | 1000                  | 164                      | <b>998.80</b>      | 70.2                     | <b>998.80</b>         | 998.81       | 0.00       | 0.00            | 19.91       |
| CMT4H    | 150 | 6   | <b>829</b>                 | <b>829</b>            | 345                      | 831.39             | 125.4                    | <b>828.12</b>         | 831.59       | -0.11      | 0.31            | 80.24       |
| CMT4Q    | 150 | 9   | <b>913.93</b>              | 918                   | 244                      | <b>913.93</b>      | 153.0                    | 915.27                | 915.27       | 0.15       | 0.15            | 58.92       |
| CMT4T    | 150 | 11  | <b>990.39</b>              | 1000                  | 212                      | <b>990.39</b>      | 166.8                    | <b>990.39</b>         | 990.39       | 0.00       | 0.00            | 50.42       |
| CMT5H    | 200 | 9   | <b>992.37</b>              | 983                   | 514                      | <b>992.37</b>      | 351.4                    | <u><b>978.736</b></u> | 978.736      | -1.37      | -1.37           | 1531.73     |
| CMT5Q    | 200 | 12  | <sup>b</sup> <b>1118</b>   | 1119                  | 381                      | 1134.72            | 451.8                    | <u><b>1104.87</b></u> | 1105.79      | -1.17      | -1.09           | 1627.78     |
| CMT5T    | 200 | 15  | <b>1227</b>                | <b>1227</b>           | 333                      | 1232.08            | 460.8                    | <u><b>1218.77</b></u> | 1220.24      | -0.67      | -0.55           | 1802.81     |
| CMT6H    | 50  | 7   | <b>555.43</b>              | <b>555</b>            | 31                       | <b>555.43</b>      | 13.0                     | <b>555.43</b>         | 557.35       | 0.00       | 0.35            | 1.08        |
| CMT6Q    | 50  | 7   | <b>555.43</b>              | <b>555</b>            | 30                       | <b>555.43</b>      | 12.8                     | <b>555.43</b>         | 557.15       | 0.00       | 0.31            | 1.08        |
| CMT6T    | 50  | 7   | <b>555.43</b>              | <b>555</b>            | 31                       | <b>555.43</b>      | 11.6                     | <b>555.43</b>         | 556.64       | 0.00       | 0.22            | 1.15        |
| CMT7H    | 75  | 13  | <b>900</b>                 | <b>900</b>            | 54                       | 900.84             | 50.0                     | <b>900.54</b>         | 900.84       | 0.06       | 0.09            | 4.47        |
| CMT7Q    | 75  | 14  | <b>900.69</b>              | <b>901</b>            | 53                       | <b>900.69</b>      | 46.8                     | <b>900.69</b>         | 902.62       | 0.00       | 0.21            | 4.90        |
| CMT7T    | 75  | 14  | <b>903.05</b>              | <b>903</b>            | 52                       | <b>903.05</b>      | 39.0                     | <b>903.05</b>         | 903.05       | 0.00       | 0.00            | 4.77        |
| CMT8H    | 100 | 10  | <b>865.50</b>              | <b>866</b>            | 95                       | <b>865.50</b>      | 85.6                     | <b>865.50</b>         | 865.50       | 0.00       | 0.00            | 7.78        |
| CMT8Q    | 100 | 10  | <b>865.50</b>              | <b>866</b>            | 93                       | <b>865.50</b>      | 74.4                     | <b>865.50</b>         | 865.50       | 0.00       | 0.00            | 7.50        |
| CMT8T    | 100 | 10  | <b>865.54</b>              | <b>866</b>            | 95                       | <b>865.54</b>      | 65.6                     | <b>865.54</b>         | 865.54       | 0.00       | 0.00            | 7.18        |
| CMT14H   | 100 | 11  | <b>821.75</b>              | <b>822</b>            | 89                       | <b>821.75</b>      | 81.6                     | <b>821.75</b>         | 821.75       | 0.00       | 0.00            | 5.37        |
| CMT14Q   | 100 | 11  | <b>821.75</b>              | <b>822</b>            | 85                       | <b>821.75</b>      | 72.4                     | <b>821.75</b>         | 821.75       | 0.00       | 0.00            | 5.47        |
| CMT14T   | 100 | 11  | <b>826.77</b>              | <b>827</b>            | 86                       | <b>826.77</b>      | 64.6                     | <b>826.77</b>         | 826.77       | 0.00       | 0.00            | 6.30        |
| CMT13H   | 120 | 12  | <b>1542.86</b>             | <b>1543</b>           | 125                      | <b>1542.86</b>     | 164.2                    | <b>1542.86</b>        | 1544.54      | 0.00       | 0.11            | 73.82       |
| CMT13Q   | 120 | 12  | <b>1542.97</b>             | <b>1543</b>           | 120                      | <b>1542.97</b>     | 157.8                    | <u><b>1542.86</b></u> | 1544.05      | -0.01      | 0.07            | 69.87       |
| CMT13T   | 120 | 12  | <b>1542.97</b>             | 1544                  | 127                      | <b>1542.97</b>     | 152.8                    | <u><b>1542.86</b></u> | 1544.11      | -0.01      | 0.07            | 73.59       |
| CMT9H    | 150 | 16  | <sup>b</sup> <b>1161</b>   | 1166                  | 177                      | <b>1161.63</b>     | 306.4                    | <b>1160.68</b>        | 1162.17      | -0.03      | 0.10            | 77.95       |
| CMT9Q    | 150 | 16  | <b>1161.51</b>             | 1162                  | 171                      | <b>1161.51</b>     | 289.6                    | <u><b>1161.24</b></u> | 1161.69      | -0.02      | 0.02            | 80.64       |
| CMT9T    | 150 | 16  | <b>1162.68</b>             | 1164                  | 178                      | <b>1162.68</b>     | 261.0                    | <u><b>1162.55</b></u> | 1164.37      | -0.01      | 0.15            | 83.29       |
| CMT10H   | 199 | 20  | <b>1383.78</b>             | 1393                  | 296                      | <b>1383.78</b>     | 791.0                    | <u><b>1372.52</b></u> | 1377.23      | -0.81      | -0.47           | 550.45      |
| CMT10Q   | 199 | 20  | <b>1386.54</b>             | 1389                  | 288                      | <b>1386.54</b>     | 730.2                    | <u><b>1374.18</b></u> | 1379.47      | -0.89      | -0.51           | 537.66      |
| CMT10T   | 199 | 20  | <sup>b</sup> <b>1395</b>   | 1402                  | 291                      | <b>1400.22</b>     | 658.6                    | <u><b>1381.04</b></u> | 1388.17      | -1.00      | -0.49           | 501.65      |
| Avg.     |     |     |                            |                       |                          |                    |                          |                       |              | -0.15      | -0.06           | 178.13      |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Found by Røpke and Pisinger [148] using another version of their algorithm.

<sup>1</sup>: Average of 10 runs on a Pentium IV 1.5 GHz (1311 Mflop/s). <sup>2</sup>: Best run on a Xeon 2.4 GHz (1978 Mflop/s).

### 7.3.6 MDVRP

Tables 7.10 and 7.11 present a comparison, in terms of average solution, between the results found by ILS-RVND-SP and those determined by Pisinger and Røpke [136] (ALNS 50K) and Vidal et al. [177] in the old and new set of instances of [35], respectively. The latter two clearly outperformed the first one in terms of solution quality. The average gap between the Avg. Sols. found by ILS-RVND-SP and the BKSs for the old and new benchmark sets was, respectively, 0.04% and 0.15%.

Table 7.10: Results found for the old MDVRP instances of Cordeau et al. [35]

| Instance     | $n$ | $v$ | $ G $ | BKS                         | Pisinger and Røpke |                       | Vidal et al.   |                       | ILS-RVND-SP    |                |         |              |          |
|--------------|-----|-----|-------|-----------------------------|--------------------|-----------------------|----------------|-----------------------|----------------|----------------|---------|--------------|----------|
|              |     |     |       |                             | Avg.* Sol.         | Time <sup>1</sup> (s) | Avg.* Sol.     | Time <sup>2</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| p01          | 50  | 4   | 4     | <sup>a</sup> <b>576.87</b>  | <b>576.87</b>      | 29                    | <b>576.87</b>  | 13.8                  | <b>576.87</b>  | <b>576.87</b>  | 0.00    | 0.00         | 2.80     |
| p02          | 50  | 2   | 4     | <sup>a</sup> <b>473.53</b>  | <b>473.53</b>      | 28                    | <b>473.53</b>  | 12.6                  | <b>473.53</b>  | <b>473.53</b>  | 0.00    | 0.00         | 2.27     |
| p03          | 75  | 3   | 2     | <sup>c</sup> <b>641.19</b>  | <b>641.19</b>      | 64                    | <b>641.19</b>  | 25.8                  | <b>641.19</b>  | <b>641.19</b>  | 0.00    | 0.00         | 7.25     |
| p12          | 80  | 5   | 2     | <sup>b</sup> <b>1318.95</b> | 1319.13            | 75                    | <b>1318.95</b> | 31.2                  | <b>1318.95</b> | <b>1318.95</b> | 0.00    | 0.00         | 6.14     |
| p04          | 100 | 8   | 2     | <sup>d</sup> <b>1001.04</b> | 1006.09            | 88                    | 1001.23        | 116.4                 | <b>1001.04</b> | <b>1001.04</b> | 0.00    | 0.00         | 51.76    |
| p05          | 100 | 5   | 2     | <sup>c</sup> <b>750.03</b>  | 752.34             | 120                   | <b>750.03</b>  | 63.6                  | <b>750.03</b>  | 750.21         | 0.00    | 0.02         | 31.54    |
| p06          | 100 | 6   | 3     | <sup>b</sup> <b>876.50</b>  | 883.01             | 93                    | <b>876.50</b>  | 68.4                  | <b>876.50</b>  | <b>876.50</b>  | 0.00    | 0.00         | 25.70    |
| p07          | 100 | 4   | 4     | <sup>d</sup> <b>881.97</b>  | 889.36             | 88                    | 884.43         | 93.0                  | <b>881.97</b>  | <b>881.97</b>  | 0.00    | 0.00         | 21.88    |
| p15          | 160 | 5   | 4     | <sup>c</sup> <b>2505.42</b> | 2519.64            | 253                   | <b>2505.42</b> | 115.2                 | <b>2505.42</b> | <b>2505.42</b> | 0.00    | 0.00         | 48.59    |
| p18          | 240 | 5   | 6     | <sup>c</sup> <b>3702.85</b> | 3736.53            | 419                   | <b>3702.85</b> | 271.2                 | <b>3702.85</b> | <b>3702.85</b> | 0.00    | 0.00         | 1019.76  |
| p21          | 360 | 5   | 9     | <sup>c</sup> <b>5474.84</b> | 5501.58            | 582                   | 5476.41        | 600.0                 | <b>5474.84</b> | <b>5474.84</b> | 0.00    | 0.00         | 2544.57  |
| <hr/>        |     |     |       |                             |                    |                       |                |                       |                |                |         |              |          |
| p13          | 80  | 5   | 2     | <sup>b</sup> <b>1318.95</b> | <b>1318.95</b>     | 60                    | <b>1318.95</b> | 34.2                  | <b>1318.95</b> | <b>1318.95</b> | 0.00    | 0.00         | 3.06     |
| p14          | 80  | 5   | 2     | <sup>c</sup> <b>1360.12</b> | <b>1360.12</b>     | 58                    | <b>1360.12</b> | 33.0                  | <b>1360.12</b> | <b>1360.12</b> | 0.00    | 0.00         | 19.11    |
| p16          | 160 | 5   | 4     | <sup>b</sup> <b>2572.23</b> | 2573.95            | 188                   | <b>2572.23</b> | 118.2                 | <b>2572.23</b> | <b>2572.23</b> | 0.00    | 0.00         | 247.77   |
| p17          | 160 | 5   | 4     | <sup>c</sup> <b>2709.09</b> | <b>2709.09</b>     | 179                   | <b>2709.09</b> | 128.4                 | <b>2709.09</b> | 2710.21        | 0.00    | 0.04         | 1448.47  |
| p19          | 240 | 5   | 6     | <sup>b</sup> <b>3827.06</b> | 3838.76            | 315                   | <b>3827.06</b> | 252.0                 | <b>3827.06</b> | 3827.55        | 0.00    | 0.01         | 1214.57  |
| p20          | 240 | 5   | 6     | <sup>c</sup> <b>4058.07</b> | 4064.76            | 300                   | <b>4058.07</b> | 262.2                 | <b>4058.07</b> | <b>4058.07</b> | 0.00    | 0.00         | 544.80   |
| p08          | 249 | 14  | 2     | <sup>e</sup> <b>4372.78</b> | 4421.03            | 333                   | 4397.42        | 600.0                 | 4379.46        | 4393.70        | 0.15    | 0.48         | 1244.57  |
| p09          | 249 | 12  | 3     | <sup>e</sup> <b>3858.66</b> | 3892.50            | 361                   | 3868.59        | 570.0                 | 3859.54        | 3864.22        | 0.02    | 0.14         | 1431.88  |
| p10          | 249 | 8   | 4     | <sup>e</sup> <b>3631.11</b> | 3666.85            | 363                   | 3636.08        | 589.2                 | 3631.37        | 3634.72        | 0.01    | 0.10         | 1422.66  |
| p11          | 249 | 6   | 5     | <sup>d</sup> <b>3546.06</b> | 3573.23            | 357                   | 3548.25        | 428.4                 | <b>3546.06</b> | 3546.15        | 0.00    | 0.00         | 1217.35  |
| p22          | 360 | 5   | 9     | <sup>c</sup> <b>5702.16</b> | 5722.19            | 462                   | <b>5702.16</b> | 600.0                 | <b>5702.15</b> | 5705.84        | 0.00    | 0.06         | 846.01   |
| p23          | 360 | 5   | 9     | <sup>d</sup> <b>6078.75</b> | 6092.66            | 443                   | <b>6078.75</b> | 600.0                 | <b>6078.75</b> | <b>6078.75</b> | 0.00    | 0.00         | 1019.15  |
| <hr/>        |     |     |       |                             |                    |                       |                |                       |                |                |         |              |          |
| Avg. Gap (%) |     |     |       |                             | 0.40               | Avg. Gap (%)          |                | 0.07                  | Avg.           |                | 0.01    | 0.04         | 627.03   |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

<sup>2</sup>: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz. \*: Average of 10 runs.

<sup>b</sup>: First found by Renaud et al. [145]. <sup>c</sup>: First found by Cordeau et al. [35].

<sup>d</sup>: First found by Pisinger and Røpke [136]. <sup>e</sup>: First found by Vidal et al. [177].

Table 7.11: Results found for the new MDVRP instances of Cordeau et al. [35]

| Instance     | $n$ | $v$ | $ G $ | BKS                         | Pisinger and Røpke |                       | Vidal et al.   |                       | ILS-RVND-SP    |           |         |              |          |
|--------------|-----|-----|-------|-----------------------------|--------------------|-----------------------|----------------|-----------------------|----------------|-----------|---------|--------------|----------|
|              |     |     |       |                             | Avg.* Sol.         | Time <sup>1</sup> (s) | Avg.* Sol.     | Time <sup>2</sup> (s) | Best Sol.      | Avg. Sol. | Gap (%) | Avg. Gap (%) | Time (s) |
| pr01         | 48  | 2   | 4     | <sup>b</sup> <b>861.32</b>  | <b>861.32</b>      | 30                    | <b>861.32</b>  | 10.2                  | <b>861.32</b>  | 861.32    | 0.00    | 0.00         | 1.24     |
| pr07         | 72  | 3   | 6     | <sup>b</sup> <b>1089.56</b> | 1089.56            | 58                    | <b>1089.56</b> | 20.4                  | <b>1089.56</b> | 1089.56   | 0.00    | 0.00         | 3.87     |
| pr02         | 96  | 4   | 4     | <sup>c</sup> <b>1307.34</b> | 1308.17            | 103                   | <b>1307.34</b> | 45.6                  | <b>1307.34</b> | 1308.53   | 0.00    | 0.09         | 12.39    |
| pr03         | 144 | 6   | 4     | <sup>d</sup> <b>1803.80</b> | 1810.66            | 214                   | 1803.80        | 114.6                 | <b>1803.81</b> | 1804.09   | 0.00    | 0.02         | 55.04    |
| pr08         | 144 | 6   | 6     | <sup>c</sup> <b>1664.85</b> | 1675.74            | 207                   | 1665.05        | 123.0                 | <b>1664.85</b> | 1665.08   | 0.00    | 0.01         | 393.98   |
| pr04         | 192 | 8   | 4     | <sup>d</sup> <b>2058.31</b> | 2073.16            | 296                   | 2059.36        | 313.2                 | <b>2058.31</b> | 2060.93   | 0.00    | 0.13         | 779.30   |
| pr09         | 216 | 9   | 6     | <sup>d</sup> <b>2133.20</b> | 2144.84            | 350                   | 2134.17        | 366.0                 | <b>2133.20</b> | 2135.37   | 0.00    | 0.10         | 1070.41  |
| pr05         | 240 | 10  | 4     | <sup>d</sup> <b>2331.20</b> | 2350.31            | 372                   | 2340.29        | 573.6                 | <b>2331.20</b> | 2338.12   | 0.00    | 0.30         | 1337.10  |
| pr06         | 288 | 12  | 4     | <sup>d</sup> <b>2676.30</b> | 2695.74            | 465                   | 2681.93        | 600.0                 | 2680.77        | 2685.23   | 0.17    | 0.33         | 2297.66  |
| pr10         | 288 | 12  | 6     | <sup>d</sup> <b>2868.26</b> | 2905.43            | 455                   | 2886.59        | 600.0                 | 2874.28        | 2882.41   | 0.21    | 0.49         | 3009.53  |
| <hr/>        |     |     |       |                             |                    |                       |                |                       |                |           |         |              |          |
| Avg. Gap (%) |     |     |       |                             | 0.52               | Avg. Gap (%)          |                | 0.13                  | Avg.           |           | 0.04    | 0.15         | 896.05   |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Average of 10 runs on a Pentium IV 3.0 GHz (3181 Mflop/s).

<sup>2</sup>: Average of 10 runs on an Opteron 2.4 GHz scaled for a Pentium IV 3.0 GHz. \*: Average of 10 runs.

<sup>b</sup>: First found by Cordeau et al. [35]. <sup>c</sup>: First found by Pisinger and Røpke [136]. <sup>d</sup>: First found by Vidal et al. [177].

### 7.3.7 MDVRPMPD

Table 7.12 presents the results found by ILS-RVND-SP and those pointed out by Røpke and Pisinger [148] (6R - no learning) in the set of instances of Salhi and Nagy [149]. With respect to the solution quality, the developed algorithm clearly had a better performance,

equaling 17 BKSs and improving the result of another 16. The average gap between the Avg. Sols. and the BKSs was -0.10%.

Table 7.12: Results found for the MDVRPMPD instances of Salhi and Nagy [149]

| Instance | $n$ | $v$ | $ G $ | BKS         | Røpke and<br>Pisinger |               |                          | ILS-RVND-SP    |               |            |                 |             |
|----------|-----|-----|-------|-------------|-----------------------|---------------|--------------------------|----------------|---------------|------------|-----------------|-------------|
|          |     |     |       |             | Best<br>Sol.          | Avg.<br>Sol.* | Time <sup>1</sup><br>(s) | Best<br>Sol.   | Avg.<br>Sol.  | Gap<br>(%) | Avg.<br>Gap (%) | Time<br>(s) |
| GJ01Q    | 50  | 4   | 4     | <b>528</b>  | <b>528</b>            | <b>528</b>    | 36                       | <b>528.30</b>  | <b>528.30</b> | 0.06       | 0.06            | 3.12        |
| GJ01T    | 50  | 4   | 4     | <b>569</b>  | <b>569</b>            | <b>569</b>    | 34                       | <b>569.43</b>  | <b>569.43</b> | 0.08       | 0.08            | 2.98        |
| GJ02H    | 75  | 4   | 2     | <b>440</b>  | <b>440</b>            | <b>440</b>    | 51                       | <b>440.00</b>  | <b>440.00</b> | 0.00       | 0.00            | 2.95        |
| GJ02Q    | 75  | 4   | 2     | <b>450</b>  | <b>450</b>            | 451           | 43                       | <b>449.72</b>  | <b>449.72</b> | -0.06      | -0.06           | 2.60        |
| GJ02T    | 75  | 4   | 2     | <b>464</b>  | <b>464</b>            | <b>464</b>    | 37                       | <b>464.13</b>  | <b>464.13</b> | 0.03       | 0.03            | 2.50        |
| GJ03H    | 100 | 5   | 3     | <b>581</b>  | 581                   | 583           | 81                       | <b>579.45</b>  | <b>579.45</b> | -0.27      | -0.27           | 9.54        |
| GJ03Q    | 100 | 5   | 3     | <b>605</b>  | <b>605</b>            | 608           | 71                       | <b>605.25</b>  | <b>605.25</b> | 0.04       | 0.04            | 8.95        |
| GJ03T    | 100 | 5   | 3     | <b>624</b>  | <b>624</b>            | 626           | 65                       | <b>624.44</b>  | <b>624.44</b> | 0.07       | 0.07            | 10.94       |
| GJ04H    | 100 | 2   | 8     | <b>790</b>  | <b>790</b>            | 797           | 112                      | <b>789.19</b>  | 789.30        | -0.10      | -0.09           | 31.69       |
| GJ04Q    | 100 | 2   | 8     | <b>875</b>  | <b>875</b>            | 876           | 94                       | <b>874.78</b>  | 874.79        | -0.03      | -0.02           | 41.89       |
| GJ04T    | 100 | 2   | 8     | <b>962</b>  | <b>962</b>            | 969           | 85                       | <b>962.25</b>  | <b>962.65</b> | 0.03       | 0.07            | 37.95       |
| GJ05H    | 100 | 2   | 5     | <b>678</b>  | <b>678</b>            | 680           | 168                      | <b>676.81</b>  | 676.91        | -0.18      | -0.16           | 22.24       |
| GJ05Q    | 100 | 2   | 5     | <b>700</b>  | 702                   | 705           | 133                      | <b>700.15</b>  | <b>700.15</b> | 0.02       | 0.02            | 19.55       |
| GJ05T    | 100 | 2   | 5     | <b>733</b>  | <b>733</b>            | 738           | 118                      | <b>733.17</b>  | 733.18        | 0.02       | 0.02            | 39.31       |
| GJ06H    | 100 | 3   | 6     | <b>745</b>  | 747                   | 751           | 116                      | <b>742.18</b>  | <b>742.18</b> | -0.38      | -0.38           | 32.28       |
| GJ06Q    | 100 | 3   | 6     | <b>794</b>  | <b>794</b>            | 800           | 100                      | <b>793.85</b>  | 793.87        | -0.02      | -0.02           | 25.00       |
| GJ06T    | 100 | 3   | 6     | <b>851</b>  | <b>851</b>            | 853           | 90                       | <b>850.82</b>  | <b>850.82</b> | -0.02      | -0.02           | 27.19       |
| GJ07H    | 100 | 4   | 4     | <b>733</b>  | <b>733</b>            | 734           | 117                      | <b>732.73</b>  | <b>732.73</b> | -0.04      | -0.04           | 24.66       |
| GJ07Q    | 100 | 4   | 4     | <b>802</b>  | 803                   | 807           | 94                       | <b>801.91</b>  | 801.94        | -0.01      | -0.01           | 38.58       |
| GJ07T    | 100 | 4   | 4     | <b>854</b>  | 855                   | 862           | 88                       | <b>853.54</b>  | <b>853.54</b> | -0.05      | -0.05           | 20.64       |
| GJ08H    | 249 | 2   | 14    | <b>3327</b> | <b>3327</b>           | 3373          | 581                      | <b>3320.39</b> | 3342.91       | -0.20      | 0.48            | 1435.21     |
| GJ08Q    | 249 | 2   | 14    | <b>3762</b> | 3774                  | 3810          | 479                      | <b>3745.18</b> | 3769.01       | -0.45      | 0.19            | 1288.57     |
| GJ08T    | 249 | 2   | 14    | <b>4134</b> | 4134                  | 4170          | 431                      | <b>4110.78</b> | 4120.27       | -0.56      | -0.33           | 1272.63     |
| GJ09H    | 249 | 3   | 12    | <b>3005</b> | 3006                  | 3028          | 646                      | <b>2990.92</b> | 3005.52       | -0.47      | 0.02            | 1478.35     |
| GJ09Q    | 249 | 3   | 12    | <b>3355</b> | <b>3355</b>           | 3393          | 535                      | <b>3351.18</b> | 3361.23       | -0.11      | 0.19            | 1362.18     |
| GJ09T    | 249 | 3   | 12    | <b>3677</b> | <b>3677</b>           | 3718          | 492                      | <b>3656.03</b> | 3661.62       | -0.57      | -0.42           | 1316.84     |
| GJ010H   | 249 | 4   | 8     | <b>2927</b> | 2930                  | 2963          | 644                      | <b>2894.71</b> | 2905.23       | -1.10      | -0.74           | 1452.52     |
| GJ010Q   | 249 | 4   | 8     | <b>3242</b> | 3245                  | 3267          | 513                      | <b>3220.79</b> | 3226.79       | -0.65      | -0.47           | 1315.68     |
| GJ010T   | 249 | 4   | 8     | <b>3485</b> | <b>3485</b>           | 3524          | 472                      | <b>3470.70</b> | 3477.99       | -0.41      | -0.20           | 1281.92     |
| GJ011H   | 249 | 5   | 6     | <b>2855</b> | 2880                  | 2905          | 609                      | <b>2842.79</b> | 2845.71       | -0.43      | -0.33           | 1357.58     |
| GJ011Q   | 249 | 5   | 6     | <b>3155</b> | 3165                  | 3192          | 511                      | <b>3138.64</b> | 3143.33       | -0.52      | -0.37           | 1267.12     |
| GJ011T   | 249 | 5   | 6     | <b>3390</b> | <b>3390</b>           | 3421          | 469                      | <b>3360.48</b> | 3367.63       | -0.87      | -0.66           | 1181.56     |
|          |     |     |       |             | Avg. Gap (%)          |               | 0.66                     | Avg.           |               | -0.22      | -0.10           | 497.54      |

\*: Average of 10 runs.

<sup>b</sup>: Found by Røpke and Pisinger [148] using another version of their algorithm.

<sup>1</sup>: Average of 10 runs on a Pentium IV 1.5 GHz (1311 Mflop/s).

### 7.3.8 HFVRP

Table 7.13 illustrates a comparison between ILS-RVND-SP and the algorithms of Li et al. [107] and Penna et al. [133] in the HVRP instances of Taillard [164] with fixed and dependent costs (HVRPFD). It can be verified that ILS-RVND-SP found all proven optimal solutions and improved the result of one instance. The average gap between the Avg. Sols found by the developed algorithm and the BKSs was 0.19%.

Table 7.14 presents a comparison between the results found by ILS-RVND-SP and those of Prins [141] (SMA-D2), Li et al. [106] and Penna et al. [133] in the HVRP instances of [164] only with dependent costs (HVRPD). As in the previous variant all proven optimal solutions were found by ILS-RVND-SP. In the only instance where the optimality was not proved, ILS-RVND-SP was unsuccessful to obtain the best solution

Table 7.13: Results found for the HVRPFD instances of Taillard [164]

| Instance | $n$ | BKS                          | Li et al.       |                       | Penna et al.    |                       | ILS-RVND-SP     |                 |         |              |          |
|----------|-----|------------------------------|-----------------|-----------------------|-----------------|-----------------------|-----------------|-----------------|---------|--------------|----------|
|          |     |                              | Best Sol.       | Time <sup>1</sup> (s) | Best Sol.       | Time <sup>2</sup> (s) | Best Sol.       | Avg. Sol.       | Gap (%) | Avg. Gap (%) | Time (s) |
| 13       | 50  | <sup>a</sup> <b>3185.09</b>  | <b>3185.09</b>  | 110                   | <b>3185.09</b>  | 19.04                 | <b>3185.09</b>  | <b>3185.09</b>  | 0.00    | 0.00         | 4.48     |
| 14       | 50  | <sup>a</sup> <b>10107.53</b> | <b>10107.53</b> | 34                    | <b>10107.53</b> | 11.28                 | <b>10107.53</b> | 10109.74        | 0.00    | 0.02         | 2.24     |
| 15       | 50  | <sup>a</sup> <b>3065.29</b>  | <b>3065.29</b>  | 46                    | <b>3065.29</b>  | 12.48                 | <b>3065.29</b>  | <b>3065.29</b>  | 0.00    | 0.00         | 4.47     |
| 16       | 50  | <sup>a</sup> <b>3265.41</b>  | 3278.96         | 99                    | <b>3265.41</b>  | 12.22                 | <b>3265.41</b>  | 3277.52         | 0.00    | 0.37         | 3.68     |
| 17       | 75  | <sup>a</sup> <b>2076.96</b>  | <b>2076.96</b>  | 148                   | <b>2076.96</b>  | 29.59                 | <b>2076.96</b>  | 2079.38         | 0.00    | 0.12         | 13.9     |
| 18       | 75  | <sup>a</sup> <b>3743.58</b>  | <b>3743.58</b>  | 119                   | <b>3743.58</b>  | 36.38                 | <b>3743.58</b>  | 3752.54         | 0.00    | 0.24         | 57.77    |
| 19       | 100 | <b>10420.34</b>              | <b>10420.34</b> | 287                   | <b>10420.34</b> | 73.66                 | <b>10420.34</b> | <b>10420.34</b> | 0.00    | 0.00         | 27.40    |
| 20       | 100 | <b>4788.49</b>               | 4834.17         | 200                   | <b>4788.49</b>  | 68.46                 | <b>4761.26</b>  | 4826.98         | -0.57   | 0.80         | 26.85    |
| Avg.     |     |                              |                 |                       |                 |                       |                 |                 | -0.07   | 0.19         | 17.60    |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Best run on an Intel 2.2 GHz (the model was not provided by the authors).

<sup>2</sup>: Average of 30 runs on an i7 2.93 GHz (5839 Mflop/s).

pointed out by Taillard [164]. The average gap between the Avg. Sols found by the proposed algorithm and the BKSs was 0.12%.

Table 7.14: Results found for the HVRPD instances of [164]

| Instance | $n$ | BKS                         | Li et al.      |                       | Prins          |                       | Penna et al.   |                       | ILS-RVND-SP    |                |         |              |          |
|----------|-----|-----------------------------|----------------|-----------------------|----------------|-----------------------|----------------|-----------------------|----------------|----------------|---------|--------------|----------|
|          |     |                             | Best Sol.      | Time <sup>1</sup> (s) | Best Sol.      | Time <sup>2</sup> (s) | Best Sol.      | Time <sup>3</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s) |
| 13       | 50  | <sup>a</sup> <b>1517.84</b> | <b>1517.84</b> | 358                   | <b>1517.84</b> | 33.2                  | <b>1517.84</b> | 19.29                 | <b>1517.84</b> | <b>1517.84</b> | 0.00    | 0.00         | 3.61     |
| 14       | 50  | <sup>a</sup> <b>607.53</b>  | <b>607.53</b>  | 141                   | <b>607.53</b>  | 37.6                  | <b>607.53</b>  | 11.20                 | <b>607.53</b>  | 608.41         | 0.00    | 0.14         | 2.50     |
| 15       | 50  | <sup>a</sup> <b>1015.29</b> | <b>1015.29</b> | 166                   | <b>1015.29</b> | 6.6                   | <b>1015.29</b> | 12.56                 | <b>1015.29</b> | <b>1015.29</b> | 0.00    | 0.00         | 3.07     |
| 16       | 50  | <sup>a</sup> <b>1144.94</b> | <b>1144.94</b> | 188                   | <b>1144.94</b> | 7.5                   | <b>1144.94</b> | 12.29                 | <b>1144.94</b> | <b>1144.94</b> | 0.00    | 0.00         | 2.69     |
| 17       | 75  | <sup>a</sup> <b>1061.96</b> | <b>1061.96</b> | 216                   | 1065.85        | 81.5                  | <b>1061.96</b> | 29.92                 | <b>1061.96</b> | 1065.20        | 0.00    | 0.31         | 7.65     |
| 18       | 75  | <sup>a</sup> <b>1823.58</b> | <b>1823.58</b> | 366                   | <b>1823.58</b> | 190.6                 | <b>1823.58</b> | 38.34                 | <b>1823.58</b> | 1826.93        | 0.00    | 0.18         | 7.69     |
| 19       | 100 | <sup>b</sup> <b>1117.51</b> | 1120.34        | 404                   | 1120.34        | 177.8                 | 1120.34        | 67.72                 | 1120.34        | 1120.41        | 0.25    | 0.26         | 16.92    |
| 20       | 100 | <sup>a</sup> <b>1534.17</b> | <b>1534.17</b> | 447                   | <b>1534.17</b> | 223.3                 | <b>1534.17</b> | 63.77                 | <b>1534.17</b> | 1534.65        | 0.00    | 0.03         | 16.56    |
| Avg.     |     |                             |                |                       |                |                       |                |                       | 0.03           | 0.12           | 7.59    |              |          |

<sup>a</sup>: Optimality proved. <sup>b</sup>: Found by Taillard [164]. <sup>1</sup>: Best run on an Athlon 1.0 GHz (1168 Mflop/s).

<sup>2</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s). <sup>3</sup>: Average of 30 runs on an i7 2.93 GHz (5839 Mflop/s).

Table 7.15 shows the results obtained by ILS-RVND-SP and those of Choi and Tcha [29], Prins (SMA-U1) [141] and Penna et al. [133] in the FSM instances of [76] with fixed and dependent costs (FSMFD). ILS-RVND-SP was capable of finding all BKSs. When individually comparing ILS-RVND-SP with each one of these algorithms, one can verify that the ILS-RVND-SP produced, on average, superior results in terms of best solutions and the average gap between the Avg. Sols. and the BKSs was 0.01%.

Table 7.16 contains the results found by ILS-RVND-SP in the FSM instances of Golden et al. [76] only with fixed costs (FSMF). A comparison is performed with those of Choi and Tcha [29], Prins [141] (SMA-D1) and Penna et al. [133]. ILS-RVND-SP managed to determine the BSKs of 10 instances, besides improving the result of another one. The average gap between the Avg. Sols. found by ILS-RVND-SP and the BKSs was 0.07%.

The best heuristic results obtained in the literature in the FSM instances of Golden et al. [76] with only dependent costs (FSMD) were reported by Choi and Tcha [29], Prins [141] (SMA-D1) and Penna et al. [133]. Table 7.17 illustrates a comparison between the

Table 7.15: Results found for the FSMFD instances of [76]

| Instance | $n$ | BKS                         | Choi and Tcha  |                       | Prins          |                       | Penna et al.   |                       | ILS-RVND-SP    |                |         |              |            |
|----------|-----|-----------------------------|----------------|-----------------------|----------------|-----------------------|----------------|-----------------------|----------------|----------------|---------|--------------|------------|
|          |     |                             | Best Sol.      | Time <sup>1</sup> (s) | Best Sol.      | Time <sup>2</sup> (s) | Best Sol.      | Time <sup>3</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s)   |
| 3        | 20  | <sup>a</sup> <b>1144.22</b> | <b>1144.22</b> | 0.25                  | <b>1144.22</b> | 0.01                  | <b>1144.22</b> | 4.05                  | <b>1144.22</b> | <b>1144.22</b> | 0.00    | 0.00         | 0.78       |
| 4        | 20  | <sup>a</sup> <b>6437.33</b> | <b>6437.33</b> | 0.45                  | <b>6437.33</b> | 0.07                  | <b>6437.33</b> | 3.03                  | <b>6437.33</b> | <b>6437.33</b> | 0.00    | 0.00         | 0.48       |
| 5        | 20  | <sup>a</sup> <b>1322.26</b> | <b>1322.26</b> | 0.19                  | <b>1322.26</b> | 0.02                  | <b>1322.26</b> | 4.85                  | <b>1322.26</b> | <b>1322.26</b> | 0.00    | 0.00         | 0.79       |
| 6        | 20  | <sup>a</sup> <b>6516.47</b> | <b>6516.47</b> | 0.41                  | <b>6516.47</b> | 0.07                  | <b>6516.47</b> | 3.01                  | <b>6516.47</b> | 6516.48        | 0.00    | < 0.01       | 0.46       |
| 13       | 50  | <sup>a</sup> <b>2964.65</b> | <b>2964.65</b> | 3.95                  | <b>2964.65</b> | 0.32                  | <b>2964.65</b> | 27.44                 | <b>2964.65</b> | <b>2964.65</b> | 0.00    | 0.00         | 7.49       |
| 14       | 50  | <sup>a</sup> <b>9126.9</b>  | <b>9126.9</b>  | 51.7                  | <b>9126.9</b>  | 8.90                  | <b>9126.90</b> | 11.66                 | <b>9126.90</b> | 9127.01        | 0.00    | < 0.01       | 2.18       |
| 15       | 50  | <sup>a</sup> <b>2634.96</b> | <b>2634.96</b> | 4.36                  | 2635.21        | 1.04                  | <b>2634.96</b> | 13.83                 | <b>2634.96</b> | <b>2634.96</b> | 0.00    | 0.00         | 2.98       |
| 16       | 50  | <sup>a</sup> <b>3168.92</b> | <b>3168.92</b> | 5.98                  | 3169.14        | 13.05                 | <b>3168.92</b> | 18.20                 | <b>3168.92</b> | <b>3168.92</b> | 0.00    | 0.00         | 13.06      |
| 17       | 75  | <sup>a</sup> <b>2004.48</b> | 2023.61        | 68.11                 | <b>2004.48</b> | 23.92                 | <b>2004.48</b> | 43.68                 | <b>2004.48</b> | <b>2004.48</b> | 0.00    | 0.00         | 9.76       |
| 18       | 75  | <sup>a</sup> <b>3147.99</b> | <b>3147.99</b> | 18.78                 | 3153.16        | 24.85                 | 3149.63        | 47.80                 | <b>3147.99</b> | 3149.72        | 0.00    | 0.05         | 8.30       |
| 19       | 100 | <sup>a</sup> <b>8661.81</b> | 8664.29        | 905.2                 | 8664.67        | 163.25                | <b>8661.81</b> | 59.13                 | <b>8661.81</b> | 8661.94        | 0.00    | < 0.01       | 49.07      |
| 20       | 100 | <b>4153.02</b>              | 4154.49        | 53.02                 | 4154.49        | 41.25                 | <b>4153.02</b> | 59.07                 | <b>4153.02</b> | 4153.31        | 0.00    | 0.01         | 76.51      |
|          |     |                             |                |                       |                |                       |                |                       |                |                | Avg.    | 0.00         | 0.01 14.32 |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Pentium IV 2.6 GHz (2266 Mflop/s).<sup>2</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s). <sup>3</sup>: Average of 30 runs on an i7 2.93 GHz (5839 Mflop/s).

Table 7.16: Results found for the FSMF instances of Golden et al. [76]

| Instance | $n$ | BKS                         | Choi and Tcha  |                       | Prins          |                       | Penna et al.   |                       | ILS-RVND-SP    |                |         |              |            |
|----------|-----|-----------------------------|----------------|-----------------------|----------------|-----------------------|----------------|-----------------------|----------------|----------------|---------|--------------|------------|
|          |     |                             | Best Sol.      | Time <sup>1</sup> (s) | Best Sol.      | Time <sup>2</sup> (s) | Best Sol.      | Time <sup>3</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s)   |
| 3        | 20  | <sup>a</sup> <b>961.03</b>  | <b>961.03</b>  | 0                     | <b>961.03</b>  | 0.04                  | <b>961.03</b>  | 4.91                  | <b>961.03</b>  | <b>961.03</b>  | 0.00    | 0.00         | 0.76       |
| 4        | 20  | <sup>a</sup> <b>6437.33</b> | <b>6437.33</b> | 1                     | <b>6437.33</b> | 0.03                  | <b>6437.33</b> | 3.16                  | <b>6437.33</b> | <b>6437.33</b> | 0.00    | 0.00         | 0.53       |
| 5        | 20  | <sup>a</sup> <b>1007.05</b> | <b>1007.05</b> | 1                     | <b>1007.05</b> | 0.09                  | <b>1007.05</b> | 5.88                  | <b>1007.05</b> | 1007.83        | 0.00    | 0.08         | 0.77       |
| 6        | 20  | <sup>a</sup> <b>6516.47</b> | <b>6516.47</b> | 0                     | <b>6516.47</b> | 0.08                  | <b>6516.47</b> | 3.07                  | <b>6516.47</b> | 6516.51        | 0.00    | < 0.01       | 0.57       |
| 13       | 50  | <sup>a</sup> <b>2406.36</b> | <b>2406.36</b> | 10                    | <b>2406.36</b> | 17.12                 | 2408.41        | 30.29                 | <b>2406.36</b> | 2406.57        | 0.00    | 0.01         | 4.43       |
| 14       | 50  | <sup>a</sup> <b>9119.03</b> | <b>9119.03</b> | 51                    | <b>9119.03</b> | 19.66                 | <b>9119.03</b> | 11.89                 | <b>9119.03</b> | 9119.16        | 0.00    | < 0.01       | 2.31       |
| 15       | 50  | <sup>a</sup> <b>2586.37</b> | <b>2586.37</b> | 10                    | <b>2586.37</b> | 25.10                 | <b>2586.37</b> | 20.24                 | <b>2586.37</b> | <b>2586.37</b> | 0.00    | 0.00         | 10.37      |
| 16       | 50  | <sup>a</sup> <b>2720.43</b> | <b>2720.43</b> | 11                    | 2729.08        | 16.37                 | <b>2720.43</b> | 20.67                 | <b>2720.43</b> | 2727.76        | 0.00    | 0.27         | 5.93       |
| 17       | 75  | <sup>a</sup> <b>1734.53</b> | 1744.83        | 207                   | 1746.09        | 52.22                 | <b>1734.53</b> | 52.49                 | <b>1734.53</b> | 1741.37        | 0.00    | 0.39         | 18.61      |
| 18       | 75  | <sup>a</sup> <b>2369.65</b> | 2371.49        | 70                    | <b>2369.65</b> | 36.92                 | 2371.48        | 55.35                 | <b>2369.65</b> | 2371.01        | 0.00    | 0.06         | 22.40      |
| 19       | 100 | <sup>a</sup> <b>8661.81</b> | 8664.29        | 1179                  | 8665.12        | 169.93                | 8662.86        | 63.92                 | <b>8661.81</b> | 8662.46        | 0.00    | 0.01         | 55.86      |
| 20       | 100 | <b>4037.90</b>              | 4039.49        | 264                   | 4044.78        | 172.73                | <b>4037.90</b> | 93.88                 | <b>4029.74</b> | 4037.44        | -0.20   | -0.01        | 91.30      |
|          |     |                             |                |                       |                |                       |                |                       |                |                | Avg.    | -0.02        | 0.07 17.82 |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Pentium IV 2.6 GHz (2266 Mflop/s).<sup>2</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s). <sup>3</sup>: Average of 30 runs on an i7 2.93 GHz (5839 Mflop/s).

results presented in such works and those obtained by ILS-RVND-SP. It can be verified that all optimal solutions were found and the average gap between the Avg. Sols. obtained by ILS-RVND-SP and the BKSs was 0.03%.

## 7.4 Concluding remarks

This chapter presented an algorithm that hybridizes an Iterated Local Search based heuristic and a Set Partitioning formulation. Its design favored the flexibility, allowing its application in the solution of several VRP variants. Moreover, the developed hybrid approach is relatively simple and easy to implement. The key aspect of the proposed methodology is the interaction between a solver and a metaheuristic approach while solving a given MIP model. This idea can be employed to efficiently solve a large class of combinatorial optimization problems.

Table 7.17: Results found for the FSMD instances of Golden et al. [76]

| Instance | $n$ | BKS                         | Choi and Tcha  |                       | Prins          |                       | Penna et al.   |                       | ILS-RVND-SP    |                |         |              |           |
|----------|-----|-----------------------------|----------------|-----------------------|----------------|-----------------------|----------------|-----------------------|----------------|----------------|---------|--------------|-----------|
|          |     |                             | Best Sol.      | Time <sup>1</sup> (s) | Best Sol.      | Time <sup>2</sup> (s) | Best Sol.      | Time <sup>3</sup> (s) | Best Sol.      | Avg. Sol.      | Gap (%) | Avg. Gap (%) | Time (s)  |
| 3        | 20  | <sup>a</sup> <b>623.22</b>  | <b>623.22</b>  | 0.19                  | -              | -                     | <b>623.22</b>  | 4.58                  | <b>623.22</b>  | <b>623.22</b>  | 0.00    | 0.00         | 0.69      |
| 4        | 20  | <sup>a</sup> <b>387.18</b>  | <b>387.18</b>  | 0.44                  | -              | -                     | <b>387.18</b>  | 2.85                  | <b>387.18</b>  | <b>387.18</b>  | 0.00    | 0.00         | 0.41      |
| 5        | 20  | <sup>a</sup> <b>742.87</b>  | <b>742.87</b>  | 0.23                  | -              | -                     | <b>742.87</b>  | 5.53                  | <b>742.87</b>  | <b>742.87</b>  | 0.00    | 0.00         | 0.80      |
| 6        | 20  | <sup>a</sup> <b>415.03</b>  | <b>415.03</b>  | 0.92                  | -              | -                     | <b>415.03</b>  | 3.37                  | <b>415.03</b>  | <b>415.03</b>  | 0.00    | 0.00         | 0.46      |
| 13       | 50  | <sup>a</sup> <b>1491.86</b> | <b>1491.86</b> | 4.11                  | <b>1491.86</b> | 3.45                  | <b>1491.86</b> | 31.62                 | <b>1491.86</b> | <b>1491.86</b> | 0.00    | 0.00         | 5.27      |
| 14       | 50  | <sup>a</sup> <b>603.21</b>  | <b>603.21</b>  | 20.41                 | <b>603.21</b>  | 0.86                  | <b>603.21</b>  | 14.66                 | <b>603.21</b>  | <b>603.21</b>  | 0.00    | 0.00         | 2.97      |
| 15       | 50  | <sup>a</sup> <b>999.82</b>  | <b>999.82</b>  | 4.61                  | <b>999.82</b>  | 9.14                  | <b>999.82</b>  | 15.33                 | <b>999.82</b>  | <b>999.82</b>  | 0.00    | 0.00         | 3.05      |
| 16       | 50  | <sup>a</sup> <b>1131.00</b> | <b>1131.00</b> | 3.36                  | <b>1131.00</b> | 13.00                 | <b>1131.00</b> | 17.77                 | <b>1131.00</b> | 1131.56        | 0.00    | 0.05         | 3.25      |
| 17       | 75  | <sup>a</sup> <b>1038.60</b> | <b>1038.60</b> | 69.38                 | <b>1038.60</b> | 9.53                  | <b>1038.60</b> | 49.18                 | <b>1038.60</b> | 1039.36        | 0.00    | 0.07         | 9.32      |
| 18       | 75  | <sup>a</sup> <b>1800.80</b> | 1801.40        | 48.06                 | <b>1800.80</b> | 18.92                 | <b>1800.80</b> | 53.88                 | <b>1800.80</b> | 1801.43        | 0.00    | 0.03         | 9.11      |
| 19       | 100 | <sup>a</sup> <b>1105.44</b> | <b>1105.44</b> | 182.86                | <b>1105.44</b> | 52.31                 | <b>1105.44</b> | 77.84                 | <b>1105.44</b> | 1105.51        | 0.00    | 0.01         | 18.4      |
| 20       | 100 | <sup>a</sup> <b>1530.43</b> | <b>1530.43</b> | 98.14                 | 1535.12        | 104.41                | 1530.52        | 88.02                 | <b>1530.43</b> | 1533.07        | 0.00    | 0.17         | 21.92     |
|          |     |                             |                |                       |                |                       |                |                       |                |                | Avg.    | 0.00         | 0.03 6.30 |

<sup>a</sup>: Optimality proved. <sup>1</sup>: Pentium IV 2.6 GHz (2266 Mflop/s).

<sup>2</sup>: Best run on a Pentium IV M 1.8 GHz (1564 Mflop/s). <sup>3</sup>: Average of 30 runs on an i7 2.93 GHz (5839 Mflop/s).

<sup>4</sup>: Average of 10 runs considering the following instances: 13, 14, 15, 16, 17, 18, 19 and 20.

The ILS-RVND-SP algorithm was evaluated in hundreds of well-known instances of the variants considered in this work, with up to 480 customers. The same parameter tuning was adopted and the results obtained were quite competitive with those found by heuristics devoted to specific variants. Table 7.18 shows the summary of the results found by ILS-RVND-SP. In this table **Avg. Gap** corresponds to the average gap between the average solutions and the BKSs, **#Instances** is the number of instances of a particular benchmark, **#Improvements** denotes the number of solutions improved and **#Ties** represents the number of ties. It can be seen that 54 new best solutions were found and that the Avg. Gap was always smaller than 0.55%.

Table 7.18: Summary of ILS-RVND-SP results

| Variant              | #Instances  | #Improvements                                  | #Ties  | Avg. Gap (%)  | Avg. Time (s)  |
|----------------------|---|--|--|---|--|
| CVRP <sup>1</sup>    | <sup>a</sup> 93, <sup>b</sup> 14, <sup>c</sup> 20 | <sup>a</sup> 1, <sup>b</sup> 0, <sup>c</sup> 0 | <sup>a</sup> 92, <sup>b</sup> 13, <sup>c</sup> 5 | <sup>a</sup> 0.03, <sup>b</sup> 0.08, <sup>c</sup> 0.55 | <sup>a</sup> 17.41 <sup>b</sup> 100.83, <sup>c</sup> 3938.23 |
| ACVRP <sup>1</sup>   | <sup>d</sup> 24                                   | <sup>d</sup> 1                                 | <sup>d</sup> 23                                  | <sup>d</sup> 0.01                                       | <sup>d</sup> 2.24  |
| OVRP <sup>1</sup>    | <sup>e</sup> 16, <sup>f</sup> 8                   | <sup>e</sup> 3, <sup>f</sup> 7                 | <sup>e</sup> 12, <sup>f</sup> 1                  | <sup>e</sup> 0.06, <sup>f</sup> -0.08                   | <sup>e</sup> 143.44, <sup>f</sup> 3844.03                    |
| VRSPD <sup>1</sup>   | <sup>g</sup> 28, <sup>h</sup> 18                  | <sup>g</sup> 5, <sup>h</sup> 7                 | <sup>g</sup> 21, <sup>h</sup> 11                 | <sup>g</sup> 0.12, <sup>h</sup> -0.07                   | <sup>g</sup> 189.24, <sup>h</sup> 3653.44                    |
| VRMPD <sup>1</sup>   | <sup>g</sup> 42                                   | <sup>g</sup> 12                                | <sup>g</sup> 29                                  | <sup>g</sup> -0.06                                      | <sup>g</sup> 178.13  |
| MDVRP <sup>1</sup>   | <sup>i</sup> 23, <sup>j</sup> 10                  | <sup>i</sup> 0, <sup>j</sup> 0                 | <sup>i</sup> 20, <sup>j</sup> 8                  | <sup>i</sup> 0.04, <sup>j</sup> 0.15                    | <sup>i</sup> 627.03, <sup>j</sup> 896.05                     |
| MDVRMPD <sup>1</sup> | <sup>g</sup> 33                                   | <sup>g</sup> 16                                | <sup>g</sup> 17                                  | <sup>g</sup> -0.10                                      | <sup>g</sup> 497.54  |
| HFVRP <sup>1</sup>   | <sup>k</sup> 16, <sup>l</sup> 36                  | <sup>k</sup> 1, <sup>l</sup> 1                 | <sup>k</sup> 14, <sup>l</sup> 35                 | <sup>k</sup> 0.16, <sup>l</sup> 0.04                    | <sup>k</sup> 11.10, <sup>l</sup> 12.81                       |
| Total                | 381   | 54   | 301  |   |  |

<sup>a</sup>: A, B, E, F, M, P set; <sup>b</sup>: Christofides et al. [31]; <sup>c</sup>: Golden et al. [80].

<sup>d</sup>: Fischetti et al. [57] and Pessoa et al. [134].

<sup>e</sup>: Christofides et al. [31] and Fisher [58]; <sup>f</sup>: Li et al. [105].

<sup>g</sup>: Salhi and Nagy [149]; <sup>h</sup>: Montané and Galvão [121];

<sup>i</sup>: Cordeau et al. (old) [35]; <sup>j</sup>: Cordeau et al. (new) [35].

<sup>k</sup>: Taillard [164]; <sup>l</sup>: Golden et al. [76].

<sup>1</sup>: Core i7 2.93 GHz (single thread).

# Chapter 8

## Concluding Remarks and Future Work

This thesis dealt with heuristic, exact and hybrid approaches for VRPs. One of the objectives was to present the state-of-the-art of the VRPs considered here i.e., CVRP, ACVRP, OVRP, VRPSPD, VRPMPD, TSPMPD, MDVRP, MDVRPMPD and HFVRP. An extensive literature review was carried out, focusing on describing the main contributions of each work. By observing the substantial number of publications, one can verify that this is indeed an area of intense and continuous research in the fields of CO and OR.

The present work also dealt with MIP flow formulations for the VRPSPD/VRPMPD. Two versions of two-commodity flow formulations (an undirected and a directed) were tested within a BC scheme, using cuts from the CVRPSEP library [114], and their results were compared with the one-commodity flow formulation of Dell’Amico et al. [44]. The optimal solutions of 30 VRPSPD open problems were proved. The three formulations were also tested in benchmark instances of the VRPMPD, which is a particular case of the VRPSPD, and were able to prove the optimality of 7 open problems. Furthermore, new lower bounds were produced for both VRPSPD and VRPMPD instances with up to 200 customers. In addition, although it has been shown that the one-commodity flow formulation produces a stronger linear relaxation, the two-commodity flow formulations have found, on average, better lower bounds in the VRPSPD instances. As for the VRPMPD, the lower bounds were, on average, quite similar, but with a slight superiority of the one-commodity formulation.

A BC algorithm with a lazy separation scheme was developed for the VRPSPD, VRPMPD and MDVRPMPD. This approach relies on a formulation only over the edge variables and the constraints that ensure that the capacity is not exceeded in the middle of the route and those that ensure that a route starts and ends at the same depot are separated in a lazy fashion. The results obtained in the VRPSPD/VRPMPD instances using this BC outperformed those found using the flow formulations in most cases, where a total of 59 optimal solutions were found for instances with up to 200 customers. As for the MDVRPMPD, the first LBs were presented for this variant and 4 optimal solutions



were found for instances with up to 100 customers.

The third exact approach proposed for the VRPSPD/VRPMPD consists of a BCP algorithm that combines the CVRP cuts used in the BCs (rounded capacity, multistar and comb) with column generation. The BCP is mostly based on the one developed by Fukasawa et al. [63] for the CVRP. The original column generation module was replaced by a dynamic programming based algorithm that is capable of considering both delivery and pickup demands. The main motivation of this method was to improve the lower bounds of the instances with relatively large number of vehicles produced by the BCs. As a result, 4 new optimal solutions were found and some lower bounds were improved.

An ILS based heuristic algorithm, called ILS-RVND, was developed to solve a large class of VRPs. The proposed approach is quite simple and computational experiments showed that the algorithm was capable of producing, on average, competitive results, regardless of the variant. The ILS-RVND heuristic was tested in 628 instances with up to 500 customers and it managed to equal the BKS in 461 cases and to improve 87 results. For every group of instances, the average gap between the average solutions obtained by the ILS-RVND and the BKSs was always smaller than 1.03%.

Finally, a hybrid algorithm, called ILS-RVND-SP, was proposed to solve a large class of VRPs where an exact procedure, based on a SP formulation, was incorporated into the ILS-RVND heuristic. While ILS-RVND seeks an equilibrium among solution quality, speed, simplicity and flexibility, ILS-RVND-SP gives preference to the solution quality at the expense of the speed, but still holding all the flexibility. The ILS-RVND-SP algorithm was evaluated in 378 instances with up to 480 customers, where 297 results were equaled and 54 were improved.

As for future work, the following lines of research are suggested: (i) development of efficient local search limitation strategies in order to decrease the computational effort without compromising the solution quality, which in turn can allow ILS-RVND to be competitively applied to solve very-large sized VRPs; (ii) extension of the range of application of ILS-RVND and ILS-RVND-SP to other VRP variants that may include time windows, backhauls, site/time dependence constraints and so on; (iii) application of the ideas contained in the proposed heuristic and hybrid algorithms to efficiently solve other COs like scheduling on single/multiple machines or clustering problems; (iv) investigation of alternative forms of hybridization between heuristic and exact approaches for VRPs.

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