Task 3 Analysis

Moritz Jasper Techen, Paul Kang, Bailiang Li, Max Spanning

December 5, 2024

Paramter Name	*Value*
Resolution	128
Number of Iterations	10, 100, 1000, 10000, 50000
Values for the Dirichlet Boundary Condition on left and right side	0,0
Values for the Neumann Boundary Condition	0
(x,y) center coordinates for the Gaussian Source	(0.5,0.5)
Deviation of the Gaussian Source	0.1

Table 1: Parameters used for Solving the Poisson Equation ($-\Delta\vec{u} = \vec{b}, \vec{b} =$ source function) via Finite Discretization with the Jacobi Matrix Method

1 Discussion of Converged Solution Plot

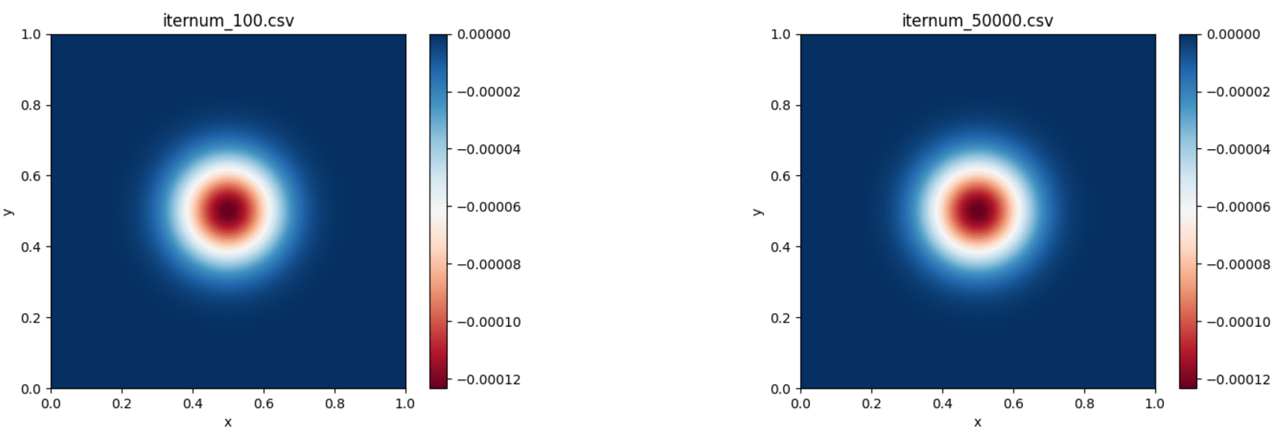

Figure 1: Converged Solution Heatmap after 100 and 50000 Iterations

A couple of key characteristics can be observed from Figure 1 that show signs of correct behavior. The circular behavior of the figure above can be immediately noticed and is to be expected because the form of the Poisson equation is reminiscent of the simple 2D formula of a circle: $x^2 + y^2 = r^2$. Other immediate physical characteristics is that the solution is centered at $(0.5, 0.5)$, which is the center that was set for the Gaussian source function used for this task. Examining the edges of the heatmap reveal that the boundary conditions have also been fulfilled. Along $x = 0, 1$, the values are the Dirichlet values assigned. Looking along the edges at $y = 0, 1$ show that the values at the point and around it are a constant value, which fulfills the Neumann boundary condition as well. The peak of solution vector also grows negative, and is to be expected as the negative of the laplacian operator was used in tandem with the positive values of the Gaussian source function, and therefore is expected to behave inversely with the said function.

2 Discussion of Residual Norm Plots

$$\text{residual} = |A\vec{u} - \vec{b}| \tag{1}$$

Figure 2: Formula for Calculation of Residuals

In order to examine the error in the converged solution by number of iterations, the 2-norm and infinite normt (inf-norm) produced by the residuals were examined. The residual values were obtained by obtained using the equation provided in Figure 2. In this case, the A matrix used was the matrix with constants that approximate the laplacian operator acting on $\vec{u}$, which is the solution vector of interest. 2 norms were used on the residual vector to examine behavior of the average error and its maximum possible error of the finite discretization method over increasing N iterations (refer to Table 1 for specific parameters used). Examining the plots for the residuals show extremely fast convergence from 100 to 1000 iterations. The residual values between 1000 iterations and beyond only differ by a negligible magnitude of 10^{-15}. This is especially evident in Figure 1, and comparing the produced nearly identical heatmaps of the solution between 100 and 50000 iterations. Examination of both figures shows that perhaps resolution may have more of an impact on convergence than the number of iterations needed.

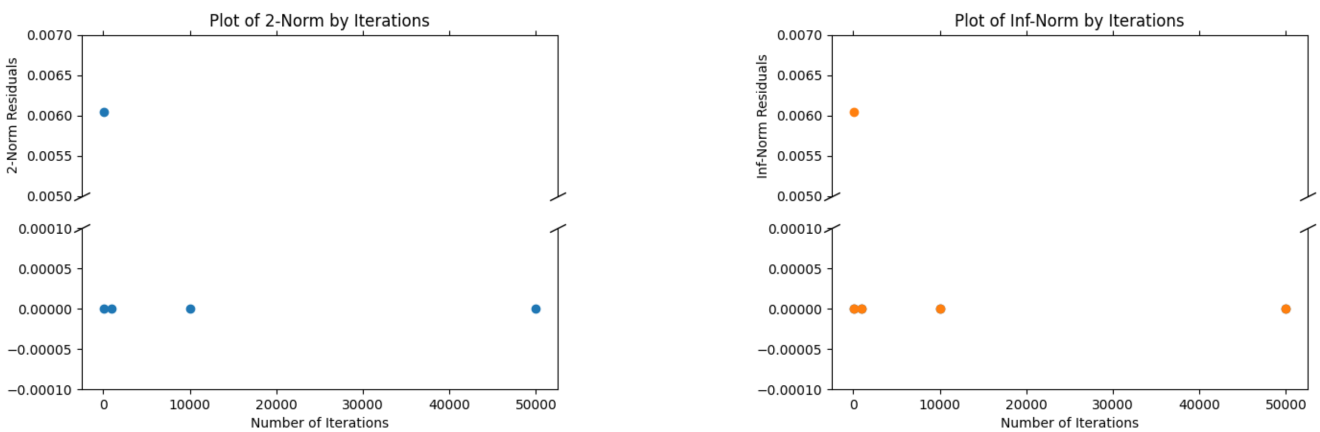

Figure 3: Plots of the Residual Norms by Number of Iterations (Left: 2-norm, Right: inf-norm)