Exercise Week 35

FYS-STK4155 Nadia Ørning

1. Show that $\frac{\delta(a^T x)}{\delta x} = a^T$:

$$\frac{\delta(a^T x)}{\delta x} = \frac{\delta}{\delta x}\left(\sum_i a_i^T x_i\right) = \sum_i \frac{\delta}{\delta x} a_i^T x_i = \sum_i a_i^T \cdot 1 = a^T$$

Show that $\frac{\delta(a^T A a)}{\delta \hat{a}} = a^T(A + A^T)$:

$$\frac{\delta(a^T A a)}{\delta a} = \overset{\text{Jacobian}}{J} = \left(\frac{\delta(a^T A a)}{\delta a_0} \dots \frac{\delta(a^T A a)}{\delta a_{n-1}}\right) = \left(\frac{\delta\left(\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_i A_{ij} a_j\right)}{\delta a_0} \dots \frac{\delta\left(\sum_{i=0}^{n-1}\sum_{j=0}^{n-1} a_i A_{ij} a_j\right)}{\delta a_{n-1}}\right)$$

$$= \left(\frac{\delta}{\delta a_0}\left[a_0(A_{00}a_0 + A_{01}a_1 + \dots + A_{0(n-1)}a_{n-1}) + \dots + a_{n-1}(A_{(n-1)0}a_0 + \dots + A_{(n-1)(n-1)}a_{n-1})\right] \dots\right)$$

$$= \dots = \left(a_0 A_{00} + \sum_{i=0}^{n-1} a_i A_{1i} + \sum_{j=0}^{n-1} a_j A_{1j} \dots a_{n-1}A_{(n-1)(n-1)} + \sum_{i=0}^{n-1} a_i A_{(n-1)i} + \sum_{j=0}^{n-1} a_j A_{(n-1)j}\right)$$

↳ a lot to write

$$= \left(\sum_{i=0}^{n-1} a_i A_{1i} + \sum_{j=0}^{n-1} a_j A_{1j} \dots \sum_{i=0}^{n-1} a_i A_{(n-1)i} + \sum_{j=0}^{n-1} a_j A_{(n-1)j}\right)$$

Like in example 3 in lecture notes, these sums are identified as:

$$= a^T(A + A^T)$$

Show that $\frac{\delta (x-As)^T(x-As)}{\delta s} = -2(x-As)^TA$:

$$\frac{\delta(x-As)^T(x-As)}{\delta s} = \frac{\delta(x-As)^T}{\delta s}(x-As) + \frac{\delta(x-As)}{\delta s}(x-As)^T$$

$$= \frac{\delta(x^T - A^T s^T)}{\delta s}(x-As) + \frac{\delta(x-As)}{\delta s}(x-As)^T$$

$$= \frac{\delta(-As)^T}{\delta s}(x-As) + \frac{\delta(-As)}{\delta s}(x-As)^T$$

$$= -A^T(x-As) + (-A)(x-As)^T = -2(x-As)^TA$$

Find the second derivative:

$$\frac{\delta^2(x-As)^T(x-As)}{\delta s^2} = \frac{\delta}{\delta s}\left(-2(x-As)^TA\right) = -2\frac{\delta}{\delta s}\left(x^TA - (As)^TA\right)$$

$$= -2\frac{\delta}{\delta s}\left(-s^TA^TA\right) = 2A^TA$$