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Higgs boson production at the LHC using the q_T subtraction formalism at N^{3}LO QCD

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Abstract

We consider higher-order QCD corrections to Higgs boson production through gluon–gluon fusion in hadron collisions. We use the transverse-momentum (q_T) subtraction method at the next-to-next-to-next-to-leading order (N^{3}LO) to numerically compute arbitrary infrared-safe observables for this class of processes. To cancel the infrared divergences, we exploit the universal behaviour of the associated q_T distributions in the small-q_T region. Our calculation is implemented in a parton level Monte Carlo program that allows the user to apply arbitrary kinematical cuts on the associated jet activity, and to compute the corresponding distributions in the form of bin histograms. We present selected numerical results at the LHC.

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1 Introduction

2 The q_T subtraction formalism at N^{3}LO

This section is devoted to the presentation of the transverse-momentum subtraction formalism at N^{3}LO in perturbative QCD. The method is illustrated in general form and special attention is paid to the case of Higgs boson production through gluon–gluon fusion. The q_T subtraction formalism presented in this section is the extension of the subtraction method originally proposed in Refs. [1, 2, 3].

We consider the inclusive hard-scattering reaction

$$h_1(p_1) + h_2(p_2) \to F(\{q_i\}) + X\,, \tag{1}$$

where h_1 and h_2 are the two hadrons which collide with momenta p_1 and p_2 producing the triggered final-state system F, accompanied by an arbitrary and undetected final state X. The colliding hadrons with centre–of–mass energy $\sqrt{s}$, are treated as massless particles ($s = (p_1+p_2)^2 = 2p_1p_2$). The observed final state F consist of a generic system of non-QCD partons composed by *one* or *more* particles (such as *one* or *more* vector bosons ($\gamma^*, W, Z, \dots$), photons, Higgs particles, Drell–Yan (DY) lepton pairs and so forth) with momenta q_i^μ ($i = 3, 4, 5, \dots$). The total momentum of the system F is denoted by q^μ ($q = \sum_i q_i$) and it can be expressed in terms of the total invariant mass M ($q^2 = M^2$), the transverse momentum $\mathbf{q_T}$ with respect to the direction of the colliding hadrons, and the rapidity y ($2y = \ln(p_2q/p_1q)$) in the centre–of–mass system of the collision. Since F is colourless, the LO partonic Born cross section can be either initiated by $q\bar{q}$ annihilation[†], as in the case of the Drell–Yan process or by gluon–gluon fusion as in the case of the Higgs boson production.

In order to report the principles of the subtraction formalism we first notice that at LO, the transverse momentum $\boldsymbol{q}_T = \sum_i \boldsymbol{q}_{T\,i}$ of the final state system F is identically zero. Therefore, as long $q_T \neq 0$, the N^{n}LO contributions[‡] (with $n = 1, 2, 3$) are given by the N^{n-1}LO contributions to the triggered final state F+jet(s). Consequently, if $q_T \neq 0$ we have:

$$d\sigma^F_{\mathrm{N^nLO}}(q_T \neq 0) \equiv d\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} \quad \text{with} \quad n = 1, 2, 3\,. \tag{2}$$

The last equation implies that if $q_T \neq 0$ the infra–red (IR) divergencies that appear in the computation of $d\sigma^F_{\mathrm{N^nLO}}(q_T \neq 0)$ are those already present in $d\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}}$. These IR singularities can be handled and cancelled with the available subtraction methods at the NLO and NNLO[§]. The only remaining singularities at N^{n}LO are associated with the limit $q_T \to 0$ and we treat them with the transverse momentum subtraction method. Since the small-q_T behaviour of the transverse momentum cross section is well known thorough the resummation program [5] of logarithmically-enhanced contributions to transverse-momentum distributions, we exploit this knowledge to construct the necessary counterterms (CT) in order to subtract the remaining singularity and promote to N^{3}LO Refs. [1, 2, 3]. The precedent strategy is accomplished combining resummed and fixed-order calculations as detailed in [2].

[†]Actually the quarks can be of different flavors (qq') as for the production of a W vector boson.
[‡]The notation N^{n}LO stands for: N^{0}LO=LO, N^{1}LO=NLO, N^{2}LO=NNLO and so forth.
[§]Our present computation uses the *antenna* subtraction method [4] at NNLO.

The sketchy form of the q_T subtraction method [1] for the N^{n}LO cross section is

$$d\sigma^F_{\mathrm{N^nLO}} = \mathcal{H}^F_{\mathrm{N^nLO}} \otimes d\sigma^F_{\mathrm{LO}} + \left[d\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}} - d\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}\right] \quad \text{with} \quad n = 1, 2, 3\ , \tag{3}$$

where $d\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}}$ is the contribution of the counterterm to the N^{n}LO cross section which cancels the divergencies of $d\sigma^{F+\mathrm{jets}}_{\mathrm{N^{n-1}LO}}$ in the limit $q_T \to 0$. The n-order counterterm can be written

$$d\sigma^{F\,\mathrm{CT}}_{\mathrm{N^nLO}} = \Sigma^F_{\mathrm{N^nLO}}(q_T^2/M^2)\ d^2\boldsymbol{q}_T \otimes d\sigma^F_{\mathrm{LO}}\ , \tag{4}$$

where the symbol $\otimes$ manifests convolutions over momentum fractions and sum over flavour indeces of the partons. More precisely, the function $\Sigma^F_{\mathrm{N^nLO}}(q_T^2/M^2)$ is the n-order truncation of the perturbative series in α_S

$$\Sigma^F_{c\bar{c}\leftarrow a_1 a_2}(q_T^2/M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_\mathrm{S}}{\pi}\right)^n \Sigma^{F(n)}_{c\bar{c}\leftarrow a_1 a_2}(q_T^2/M^2)\ , \tag{5}$$

where the labels a_1 and a_2 stands for the partonic channels of the N^{n}LO correction to the Born¶ cross section ($d\sigma^F_{\mathrm{LO}} \equiv d\big[\sigma^{F(0)}_{c\bar{c}}\big]$). The function $\Sigma^F(q_T^2/M^2)$ embodies all the logarithmic terms that are divergent in the limit $q_T \to 0$ reproducing the singular behaviour of $d\sigma^{F+\mathrm{jets}}$ in the small-q_T limit. The counterterm is defined free of terms proportional to $\delta(q_T^2)$ which are all considered in the perturbative factor $\mathcal{H}^F$. The hard coefficient function $\mathcal{H}^F_{\mathrm{N^nLO}}$ that encodes all the infra-red (IR) finite terms of the n-loop contributions, is obtained by the N^{n}LO truncation of the perturbative function

$$\mathcal{H}^F_{c\bar{c}\leftarrow a_1 a_2}(z;\alpha_\mathrm{S}) = \delta_{c\,a_1}\,\delta_{\bar{c}\,a_2}\ \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_\mathrm{S}}{\pi}\right)^n \mathcal{H}^{F(n)}_{c\bar{c}\leftarrow a_1 a_2}(z)\ \ , \tag{6}$$

where $z = M^2/s$. According to the transverse momentum resummation formula (Eq. (10) of Ref. [2]) and using the Fourier transformation between the conjugate variables q_T and b (b is the impact parameter), the perturbative hard function $\mathcal{H}^F$ and the counterterm are obtained by the fixed order truncation of the following identity

$$\begin{aligned}\big(\Sigma^F_{c\bar{c}\leftarrow a_1 a_2}(q_T^2/M^2) + \mathcal{H}^F_{c\bar{c}\leftarrow a_1 a_2}(\alpha_\mathrm{S}, M)\big) \otimes d\big[\sigma^{F(0)}_{c\bar{c}}\big] &= \frac{M^2}{s}\int_0^\infty db\frac{b}{2} J_0(bq_T)\ S_c(M,b) \\ \times \int_{x_1}^1 \frac{dz_1}{z_1}\int_{x_2}^1 \frac{dz_2}{z_2} d\big[\sigma^{F(0)}_{c\bar{c}}\big]\ f_{a_1/h_1}\left(x_1/z_1, b_0^2/b^2\right) f_{a_2/h_2}&\left(x_2/z_2, b_0^2/b^2\right) \otimes \left[H^F C_1 C_2\right]_{c\bar{c};a_1 a_2}\ ,\end{aligned} \tag{7}$$

where $J_0(bq_T)$ is the 0th-order Bessel function, $f_{c/h}$ corresponds to the distribution of a parton c in a hadron h and $b_0 = 2e^{-\gamma_E}$ ($\gamma_E = 0.5772...$ is the Euler number). The symbolic factor $d\big[\sigma^{F(0)}_{c\bar{c}}\big]$ for the Born cross section $\sigma^{F(0)}_{c\bar{c}}$ denotes

$$d\big[\sigma^{F(0)}_{c\bar{c}}\big] \equiv \frac{d\sigma^{F(0)}_{c\bar{c}}}{d\phi}\ , \tag{8}$$

where ϕ represents the phase–space of the final state system F. The large logarithmic corrections are exponentiated in the Sudakov form factor $S_c(M,b)$ of the quark ($c = q, \bar{q}$) or of the gluon ($c = g$), and it has the following expression:

$$S_c(M,b) = \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left[A_c(\alpha_\mathrm{S}(q^2))\ \ln\frac{M^2}{q^2} + B_c(\alpha_\mathrm{S}(q^2))\right]\right\}\ . \tag{9}$$

¶Notice that at LO the only available configuration is $a_1 = c$ and $a_2 = \bar{c}$, where $c\bar{c}$ is (are) the partonic channel(s) at which the LO cross section is initiated.

The functions A and B in Eq. (9) are perturbative series in $\alpha_{\rm S}$:

$$A_c(\alpha_{\rm S}) \;=\; \sum_{n=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n A_c^{(n)} \;\;, \tag{10}$$

$$B_c(\alpha_{\rm S}) \;=\; \sum_{n=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n B_c^{(n)} \;\;. \tag{11}$$

The structure of the symbolic factor denoted by $\left[H^{F=H}C_1C_2\right]_{c\bar{c};a_1a_2}$ which depends strongly on the initial state channel of the Born subprocess, is explained with detail in Refs. [6, 7]. Here we limit ourselves to the case in which the final state system F is composed by a single Higgs boson

$$\begin{aligned}\left[H^{F=H}C_1C_2\right]_{gg;a_1a_2} \;=\;& H_g^{F=H}(\alpha_{\rm S}(M^2)) \;\Big[\; C_{g\,a_1}(z_1;\alpha_{\rm S}(b_0^2/b^2)) \;\, C_{g\,a_2}(z_2;\alpha_{\rm S}(b_0^2/b^2)) \\ +\;& G_{g\,a_1}(z_1;\alpha_{\rm S}(b_0^2/b^2)) \;\, G_{g\,a_2}(z_2;\alpha_{\rm S}(b_0^2/b^2)) \;\Big] \;\;. \end{aligned}\tag{12}$$

The gluonic hard–collinear coefficient function $C_{g\,a}(z;\alpha_{\rm S})$ $(a = q, \bar{q}, g)$ in the right-hand side of Eq. (12) has the following perturbative structure

$$C_{g\,a}(z;\alpha_{\rm S}) = \delta_{g\,a} \;\; \delta(1-z) + \sum_{n=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n C_{g\,a}^{(n)}(z) \;\;. \tag{13}$$

At variance with Eq. (13), the perturbative expansion of the coefficient functions G_{ga}, which are specific to gluon-initiated processes, starts at $\mathcal{O}(\alpha_{\rm S})$, and we write

$$G_{g\,a}(z;\alpha_{\rm S}) = \frac{\alpha_{\rm S}}{\pi} \; G_{g\,a}^{(1)}(z) \; + \sum_{n=2}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n G_{g\,a}^{(n)}(z) \;\;. \tag{14}$$

The IR finite contribution of the n-loop correction terms to the Born subprocess are embodied in the hard-virtual function

$$H_g^{F=H}(\alpha_{\rm S}(M^2)) = 1 + \sum_{n=1}^{\infty} \left(\frac{\alpha_{\rm S}}{\pi}\right)^n H_g^{F=H\,(n)}(z_1p_1, z_2p_2) \;\;. \tag{15}$$

The right-hand side of Eq. (12) does not depend on the direction of $\mathbf{b}$ and this implies that the $\mathbf{q_T}$ distribution has no azimuthal correlations in the small-q_T region for Higgs boson production [6]. The presence of the $G_{g\,a}(z;\alpha_{\rm S})$ functions in the right–hand side of Eq. (12) is the manifestation of the helicity-flip contributions. Since there are contributions with two $G_{g\,a}(z;\alpha_{\rm S})$ functions, for Higgs boson production there are only double helicity-flip terms; helicity conservation in the hard-process factor for Higgs boson production forbids contributions that are produced by a single helicity-flip. The helicity-flip $G_{g\,a}(z;\alpha_{\rm S})$ functions are absent in processes initiated at the Born level by quark annihilation [6].

We now turn to the discussion of the resummation scheme dependence of the coefficient functions. The resummation formula (7) is invariant under the following 'resummation scheme' transformations [8]:

$$\begin{aligned} H_c^F(\alpha_{\rm S}) \;&\to\; H_c^F(\alpha_{\rm S}) \;\left[\,h(\alpha_{\rm S})\,\right]^{-1} \;, \\ B_c(\alpha_{\rm S}) \;&\to\; B_c(\alpha_{\rm S}) - \beta(\alpha_{\rm S}) \;\frac{d\ln h(\alpha_{\rm S})}{d\ln\alpha_{\rm S}} \;, \\ C_{ab}(\alpha_{\rm S}, z) \;&\to\; C_{ab}(\alpha_{\rm S}, z) \;\left[\,h(\alpha_{\rm S})\,\right]^{1/2} \;. \end{aligned}\tag{16}$$

The invariance can easily be proven by using the following renormalization-group identity:

$$h(\alpha_S(b_0^2/b^2)) = h(\alpha_S(M^2)) \ \exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \ \beta(\alpha_S(q^2)) \ \frac{d\ln h(\alpha_S(q^2))}{d\ln \alpha_S(q^2)} \right\} \quad , \tag{17}$$

which is valid for any perturbative function $h(\alpha_S)$. Notice that Eq. (17) establishes the evolution of the perturbative functions from the scale $q^2 = b_0^2/b^2$ to $q^2 = M^2$. The QCD β function and its corresponding n-order β_n coefficient are defined as

$$\frac{d\ln \alpha_S(\mu^2)}{d\ln \mu^2} = \beta(\alpha_S(\mu^2)) = -\sum_{n=0}^{+\infty} \beta_n \left(\frac{\alpha_S}{\pi}\right)^{n+1} \quad . \tag{18}$$

The explicit expression of the first three coefficients [9, 10], β_0, β_1 and β_2 are

$$\begin{aligned} \beta_0 &= \frac{1}{12}\left(11C_A - 2N_f\right) \quad , \qquad \beta_1 = \frac{1}{24}\left(17C_A^2 - 5C_AN_f - 3C_FN_f\right) \quad , \\ \beta_2 &= \frac{1}{64}\left(\frac{2857}{54}C_A^3 - \frac{1415}{54}C_A^2N_f - \frac{205}{18}C_AC_FN_f + C_F^2N_f + \frac{79}{54}C_AN_f^2 + \frac{11}{9}C_FN_f^2\right) \quad , \end{aligned} \tag{19}$$

where N_f is the number of QCD massless flavours and the $SU(N_c)$ colour factors are $C_A = N_c$ and $C_F = (N_c^2 - 1)/(2N_c)$.

The physical origin of the resummation scheme invariance of Eq. (7) is discussed in Ref. [8]. The invariance implies that the hard-virtual factors H_c^F, S_c (more precisely, the function B_c) and C_{ab} are not unambiguously computable order by order in perturbation theory. After choosing a 'resummation scheme', these factors can be unambiguously defined. We rely on the *hard resummation scheme* defined in Ref. [7], which states all the contributions proportional to $\delta(1-z)$ are considered in the hard-virtual functions H_c^F. The precedent condition directly implies that H_c^F is process dependent whereas the collinear C_{ab} functions and the resummation coefficients B_c are independent of the final state system F.

The truncation of Eq. (7) at a given fixed order requieres the explicit use of several resummation coefficients and hard collinear coefficient functions. At NLO, the coefficients $A_g^{(1)}$, $B_g^{(1)}$, $C_{ga}^{(1)}$ $(a = q, \bar{q}, g)$ and $H_g^{H(1)}$ are enough to compute the total cross section and completely differential distributions

$$\begin{aligned} A_g^{(1)} &= C_A \, , \qquad B_g^{(1)} = -\frac{1}{6}\left(11C_A - 2N_f\right) \, , \qquad H_g^{H(1)} = \frac{1}{2}(11 + C_A\pi^2) \, , \\ C_{ga}^{(1)}(z) &= \frac{1}{2}C_F\, z \qquad [a = q, \bar{q}] \, , \qquad C_{gg}^{(1)}(z) = 0 \, . \end{aligned} \tag{20}$$

The coefficients $A_g^{(1)}$ and $B_g^{(1)}$ are process independent, but even more, they are resummation scheme independent. In the hard resummation scheme the collinear functions $C_{ga}^{(1)}$ $(a = q, \bar{q}, g)$ are process independent whereas $H_g^{H(1)}$ depends on final state system $(F = H)$, and both depend on the resummation scheme in such a way to ensure the resummation scheme independence of Eq. (7) at NLO. The computation of the hard-virtual coefficients $H_c^{H(1)}$ requires a particular prescription which is known from some time [11]. The explicit calculations and the results of Ref. [11] show that the NLO hard-virtual coefficient $H_c^{F(1)}$ is explicitly related in a process-independent form to $d\hat{\sigma}_{\rm LO}^F$ and to the IR finite part of the NLO virtual correction to the Born cross section.

At NNLO, the coefficients $A_g^{(2)}$ and $B_g^{(2)}$ are needed,

$$A_g^{(2)} = \frac{1}{2}\, C_A \left[\left(\frac{67}{18} - \frac{\pi^2}{6} \right) C_A - \frac{5}{9} N_f \right] \;, \qquad B_g^{(2)} = \frac{\gamma_{g(1)}}{16} + \beta_0\, C_A\, \zeta_2 \;, \tag{21}$$

where $\gamma_{g(1)}$ is the coefficient of the $\delta(1-z)$ term in the NLO gluon splitting functions [14, 15], which reads

$$\gamma_{g\,(1)} = \left(-\frac{64}{3} - 24\zeta_3 \right) C_A^2 + \frac{16}{3}\, C_A N_f + 4\, C_F N_f \;, \tag{22}$$

and ζ_n is the Riemann zeta-function ($\zeta_2 = \pi^2/6, \zeta_3 = 1.202\ldots, \zeta_4 = \pi^4/90$). The coefficient $A_g^{(2)}$ does not depend on the resummation scheme whereas $B_g^{(2)}$ in Eq. (21) is expressed in the hard resummation scheme and both coefficients are process independent. The collinear functions $C_{ga}^{(2)}$ ($a = q, \bar{q}, g$) in the hard resummation scheme can be extracted from Refs. [7, 12] and they are independent of the final state system F.

The general estructure of the hard-virtual coefficients H_c^F is known only recently [7]. Although this factor is process dependent, in Ref. [7] was shown that it can be directly related in a universal (process-independent) way to the IR finite part of the all-order virtual amplitude of the corresponding partonic subprocess $c\bar{c} \to F$. The precedent relation between H_c^F and the all-order virtual correction to the partonic subprocess $c\bar{c} \to F$ is explicitly known up to the NNLO. In the case of Higgs boson production the hard-virtual factor $H_g^{F=H(2)}$ in the large-m_t limit (in the hard scheme) is [12]

$$\begin{aligned} H_g^{H(2)} &= C_A^2 \left(\frac{3187}{288} + \frac{7}{8} L_t + \frac{157}{72}\pi^2 + \frac{13}{144}\pi^4 - \frac{55}{18}\zeta_3 \right) + C_A\, C_F \left(-\frac{145}{24} - \frac{11}{8} L_t - \frac{3}{4}\pi^2 \right) \\ &+ \frac{9}{4} C_F^2 - \frac{5}{96} C_A - \frac{1}{12} C_F - C_A\, N_f \left(\frac{287}{144} + \frac{5}{36}\pi^2 + \frac{4}{9}\zeta_3 \right) + C_F\, N_f \left(-\frac{41}{24} + \frac{1}{2} L_t + \zeta_3 \right) \;, \end{aligned} \tag{23}$$

where $L_t = \ln(M^2/m_t^2)$. The two-loop scattering amplitude [16] used in the computation of $H_g^{F=H(2)}$ includes corrections to the large-m_t approximation (the evaluation of the corrections uses the expansion parameter $1/m_t^2$). At NNLO, in Eq. (12) (which is proportional to $\delta(q_T^2)$) the first order $G_{ga}^{(1)}$ helicity-flip functions are also needed and they read [6]

$$G_{g\,a}^{(1)}(z) = C_a\, \frac{1-z}{z} \qquad\qquad a = q, \bar{q}, g \,. \tag{24}$$

The first-order functions $G_{ga}^{(1)}$ are resummation-scheme independent and they do not depend on the final state system F.

At N^{3}LO, the numerical implementation of Eq. (7) requieres the following ingredients: $A_g^{(3)}$, $B_g^{(3)}$, $C_{ga}^{(3)}$ ($a = q, \bar{q}, g$) and $H_g^{H(3)}$. The coefficient $A_c^{(3)}$ [20] reads

$$\begin{aligned} A_c^{(3)} \; &= \frac{1}{4} C_c \Bigg\{ C_A^2 \left(\frac{245}{24} - \frac{67}{9}\zeta_2 + \frac{11}{6}\zeta_3 + \frac{11}{5}\zeta_2^2 \right) + C_F n_f \left(-\frac{55}{24} + 2\zeta_3 \right) \\ &+ \; C_A n_f \left(-\frac{209}{108} + \frac{10}{9}\zeta_2 - \frac{7}{3}\zeta_3 \right) - \frac{1}{27} n_f^2 \Bigg\} + \frac{1}{8}\beta_0 C_c \Bigg(C_A \left(\frac{808}{27} - 28\zeta_3 \right) - \frac{224}{54} n_f \Bigg) \,. \end{aligned} \tag{25}$$

The explicit expression of the $B_c^{(3)}$ $(a = q, g)$ coefficients in the hard scheme can be computed from Refs. [17, 18]. In the particular case of the gluon channel we have

$$\begin{aligned}
B_g^{(3)} &= -\frac{2133}{64} + \frac{3029}{576}N_f - \frac{349}{1728}N_f^2 + \frac{109}{6}\pi^2 - \frac{283}{144}\pi^2 N_f + \frac{5}{108}\pi^2 N_f^2 - \frac{253}{160}\pi^4 + \frac{23}{240}\pi^4 N_f \\
&- \frac{843}{8}\zeta_3 + 2\zeta_3 N_f + \frac{1}{6}\zeta_3 N_f^2 + \frac{9}{4}\pi^2\zeta_3 + \frac{135}{2}\zeta_5 \,. \qquad (26)
\end{aligned}$$

The analytical form of the function $\Sigma^{F(3)}$ in Eq. (7) is completely known and the subtraction of the divergencies can be performed at N^{3}LO without any approximation. Nevertheless, the general structure of the coefficient $\mathcal{H}^{F\,(3)}$ (which is proportional to $\delta(q_T^2)$) is not known[‖] in analytic form for any hard-scattering process. Nonetheless, within our subtraction formalism, $\mathcal{H}^{F\,(3)}$ can be determined for any hard-scattering process whose corresponding total cross section is known at N^{3}LO. This point is discussed in detail in Sect. 2.1.

At N^{3}LO the universal relation between $H_c^{F(3)}$ and the third-order virtual correction to the partonic subprocess $c\bar{c} \to F$ presents one missing ingredient: the *single* coefficient (of *soft* origin) that appear in the finite part of the structure of the IR singularities of the third-order virtual amplitude of the corresponding partonic subprocess $c\bar{c} \to F$. Although a general prescription to compute analytically the hard-virtual coefficient $H_g^{H(3)}$ is not completely known, using the corresponding hard-virtual factor $C_{gg\to H}^{\mathrm{th}(3)}$ [19] for threshold resummation and the exponential equation that relates threshold and q_T resummation hard-virtual coefficients (Eq. (81) of Ref. [7]), we compute the following approximated expression

$$\begin{aligned}
\widetilde{H}_g^{H(3)} = C_A^3 &\left(-\frac{15649\zeta_3}{432} - \frac{121\pi^2\zeta_3}{432} + \frac{3\zeta_3^2}{2} + \frac{869\zeta_5}{144} + \frac{215131}{5184} + \frac{16151\pi^2}{7776} - \frac{961\pi^4}{15552} + \frac{\pi^6}{810}\right) \\
&+ C_A^2\left(\frac{605\zeta_3}{72} + \frac{55\pi^2\zeta_3}{36} + \frac{737\pi^2}{432} + \frac{167\pi^4}{432} + \frac{\pi^6}{72}\right) \\
&+ C_A\left(\frac{19\pi^2 L_t}{48} - \frac{55\pi^2\zeta_3}{8} - \frac{\pi^6}{480} + \frac{133\pi^4}{72} + \frac{11399\pi^2}{864}\right) \\
&+ N_f^2\left(\frac{43C_A\zeta_3}{108} - \frac{19\pi^4 C_A}{3240} - \frac{133\pi^2 C_A}{1944} + \frac{2515C_A}{1728} - \frac{7C_F\zeta_3}{6}\right. \\
&\left. + \frac{4481C_F}{2592} - \frac{\pi^4 C_F}{3240} - \frac{23\pi^2 C_F}{432}\right) \\
&+ N_f\left(\frac{101C_A^2\zeta_5}{72} - \frac{97}{216}\pi^2 C_A^2\zeta_3 + \frac{29C_A^2\zeta_3}{8} + \frac{1849\pi^4 C_A^2}{38880} - \frac{35\pi^2 C_A^2}{243} - \frac{98059C_A^2}{5184}\right. \\
&+ \frac{5C_A C_F\zeta_5}{2} + \frac{13C_A C_F\zeta_3}{2} + \frac{1}{2}\pi^2 C_A C_F\zeta_3 - \frac{63991C_A C_F}{5184} + \frac{11\pi^4 C_A C_F}{6480} - \frac{71}{216}\pi^2 C_A C_F \\
&+ \frac{1}{9}\pi^2 C_A \mathrm{Lt} - \frac{5}{36}\pi^2 C_A\zeta_3 - \frac{55C_A\zeta_3}{36} - \frac{5\pi^4 C_A}{54} - \frac{1409\pi^2 C_A}{864} \\
&\left. -5C_F^2\zeta_5 + \frac{37C_F^2\zeta_3}{12} + \frac{19C_F^2}{18}\right) \,. \qquad (27)
\end{aligned}$$

The $\widetilde{H}_g^{H(3)}$ coefficient in Eq. (27) will be used in the numerical computations in the present paper.

[‖]The functions $\mathcal{H}^{F\,(3)}$ depend on $H_c^{H(3)}$, $C_{ga}^{(3)}$ and $G_{ga}^{(2)}$ which are not completely known.

Finally the collinear functions $C_{ga}^{(3)}$ ($a = q, \bar{q}, g$) and the second-order helicity-flip functions $G_{ga}^{(2)}$ (which are universal) are not known.

2.1 The Higgs boson total cross section at N^{3}LO

We start this section with some observations related to the hard-scattering function $\mathcal{H}^F_{c\bar{c}\leftarrow ab}$. This function is resummation-scheme independent, but it depends on the specific hard-scattering subprocess $c + \bar{c} \to F$. The coefficients $\mathcal{H}^{F\,(n)}_{c\bar{c}\leftarrow ab}$ of the perturbative expansion in Eq. (6) can be determined by performing a customary perturbative calculation of the q_T distribution in the limit $q_T \to 0$. In right-hand side of Eq. (7), within our subtraction method, the function $\mathcal{H}^F$ controls the strict perturbative normalization of the corresponding total cross section (i.e. the integral of the total q_T distribution). This unitary-related property can be exploited to determine the coefficients $\mathcal{H}^{F\,(n)}_{c\bar{c}\leftarrow ab}$ from the perturbative calculation of the total cross section. At the partonic level, the integral of the total q_T distribution in Eq. (3) results in the total cross section $\hat{\sigma}^{\rm tot}_{F\,ab}$,

$$\hat{\sigma}^{\rm tot}_{F\,ab}(M, \hat{s}; \alpha_{\rm S}(\mu_R^2), \mu_R^2, \mu_F^2) = \int_0^\infty dq_T^2 \, \frac{d\hat{\sigma}_{F\,ab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_{\rm S}(\mu_R^2), \mu_R^2, \mu_F^2) \;. \tag{28}$$

Since the the hard-scattering function $\mathcal{H}^F_{c\bar{c}\leftarrow ab}$ is simply proportional to $\delta(q_T^2)$, we evaluate the q_T spectrum on right-hand side of Eq. (3) according to the following decomposition

$$\hat{\sigma}^{\rm tot}_{F\,ab} = \frac{M^2}{\hat{s}} \, \mathcal{H}^F_{ab} + \int_0^\infty dq_T^2 \, \frac{d\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{dq_T^2} \quad , \tag{29}$$

where $d\hat{\sigma}^{\rm (fin.)}_{F\,ab}$ is directly related to the quantity in square bracket in the right-hand side of Eq. (3)

$$\frac{d\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{dq_T^2} \equiv \left[\frac{d\hat{\sigma}^{F+{\rm jets}}_{ab}}{dq_T^2} - \frac{d\hat{\sigma}^{F\;{\rm CT}}_{ab}}{dq_T^2} \right] \;. \tag{30}$$

The relation in Eq. (29) is valid order by order in QCD perturbation theory. Once the perturbative coefficients of the fixed-order expansions of $\hat{\sigma}^{\rm tot}_{F\,ab}$, $\mathcal{H}^F_{ab}$ and $d\hat{\sigma}^{\rm (fin.)}_{F\,ab}/dq_T^2$ are all known, the relation (29) has to be regarded as an identity, which can explicitly be checked. Note, however, that since the fixed-order truncation $d\hat{\sigma}^{\rm (fin.)}_{F\,ab}/dq_T^2$ does not contain any contributions proportional to $\delta(q_T^2)$, $\left[d\hat{\sigma}^{\rm (fin.)}_{F\,ab}/dq_T^2\right]_{\rm NLO}$ does not explicitly depend on the coefficient $\mathcal{H}^{F\,(1)}_{ab}$. Analogously, $\left[d\hat{\sigma}^{\rm (fin.)}_{F\,ab}/dq_T^2\right]_{\rm NNLO}$ does not explicitly depend on the coefficient $\mathcal{H}^{F\,(2)}_{ab}$, and so forth. Therefore, Eq. (29) can be used to determine the N$^{\rm n}$LO coefficient $\mathcal{H}^{F\,(n)}_{ab}$ from the knowledge of $\hat{\sigma}^{\rm tot}_{F\,ab}$ at N$^{\rm n}$LO and of $d\hat{\sigma}^{\rm (fin.)}_{F\,ab}/dq_T^2$ at N$^{\rm n}$LO, without the need of explicitly computing the small-q_T behaviour of the spectrum $d\hat{\sigma}_{F\,ab}/dq_T^2$ at N$^{\rm n}$LO. For example, at N^{3}LO Eq. (29) reads

$$\begin{aligned}
&\left(\frac{\alpha_{\rm S}}{\pi}\right)^3 \frac{M^2}{\hat{s}} \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_{\rm S}, M)\, \mathcal{H}^{F\,(3)}_{c\bar{c}\leftarrow ab}\left(\frac{M^2}{\hat{s}}; \frac{M^2}{\mu_R^2}, \frac{M^2}{\mu_F^2}, \frac{M^2}{Q^2}\right) \\
&= \left\{ \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^3LO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm NNLO} \right\} - \int_0^\infty dq_T^2 \left\{ \left[\frac{d\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{dq_T^2}\right]_{\rm N^3LO} - \left[\frac{d\hat{\sigma}^{\rm (fin.)}_{F\,ab}}{dq_T^2}\right]_{\rm NNLO} \right\} \;,
\end{aligned} \tag{31}$$

where $\alpha_S = \alpha_S(\mu_R^2)$ and we have used

$$\left[\hat{\sigma}^{\rm tot}_{F\,ab}(M,\hat{s};\alpha_S)\right]_{\rm LO} = \delta(1-M^2/\hat{s})\ \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_S,M)\ \delta_{ca}\ \delta_{\bar{c}b}\ . \tag{32}$$

The generalization* at any order n ($n>1$) is [2]

$$\begin{aligned}\left(\frac{\alpha_S}{\pi}\right)^n \frac{M^2}{\hat{s}} \sum_c \sigma^{(0)}_{c\bar{c},\,F}(\alpha_S,M)\ \mathcal{H}^{F\,(n)}_{c\bar{c}\leftarrow ab} \ &= \ \left\{\left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^nLO} - \left[\hat{\sigma}^{\rm tot}_{F\,ab}\right]_{\rm N^{n-1}LO}\right\} \\ &- \ \int_0^\infty dq_T^2 \left\{\left[\frac{d\hat{\sigma}^{(\rm fin.)}_{F\,ab}}{dq_T^2}\right]_{\rm N^nLO} - \left[\frac{d\hat{\sigma}^{(\rm fin.)}_{F\,ab}}{dq_T^2}\right]_{\rm N^{n-1}LO}\right\}.\end{aligned} \tag{33}$$

If all the components in right-hand side of Eq. (33) are known analytically (as it was the case at NNLO in Refs. [12, 13]) the function $\mathcal{H}^F_{ab}$ can be extracted exactly in analytical form. At NLO the extraction of the function $\mathcal{H}^{F\,(1)}_{ab}$ is straightforward for Drell-Yan and Higgs boson production. The function $\mathcal{H}^{F\,(2)}_{ab}$ at NNLO (for Higgs ($F=H$) boson production [12] and Drell-Yan ($F=DY$) [13]) can be obtained with a dedicated analytical computation using Eq. (33) for $n=2$. Since for Higgs boson production, the transverse momentum cross section H+jet at NNLO is not known analytically, Eq. (31) can be used only numerically in the computation of the function $\mathcal{H}^{F\,(3)}_{ab}$.

Instead of compute the entire third order function $\mathcal{H}^{H\,(3)}_{ab}$ numerically (which is not recommended) we first report all its ingredients with the aim of reduce the numerical extraction to only a few components. With this purpose, using Eqs. (7) and (12) we define

$$\mathcal{H}^H_{gg\leftarrow ab}(z;\alpha_S) \equiv H^H_g(\alpha_S)\int_0^1 dz_1 \int_0^1 dz_2\, \delta(z-z_1z_2)\Big[C_{g\,a}(z_1;\alpha_S)C_{g\,b}(z_2;\alpha_S)+G_{g\,a}(z_1;\alpha_S)G_{g\,b}(z_2;\alpha_S)\Big]\,. \tag{34}$$

There are only two differences between Eqs. (12) and (34). The first difference is due to the fact that the function $\mathcal{H}^H$ depends on the energy fraction z, since the right-hand side of Eq. (34) involves a convolution integral over the momentum fractions z_1 and z_2. The second difference regards the scale of α_S: in the functions $H^H(\alpha_S), C(\alpha_S)$ and $G(\alpha_S)$ on the right-hand side of Eq. (34), the argument of α_S is set to the same value (this common scale is not explicitly denoted in Eq. (34)). Owing to this feature, the process-dependent function $\mathcal{H}^H_{gg\leftarrow ab}$ is unambiguously defined (i.e., it is independent of the specification of the resummation scheme) [8]. The perturbative expansion of the function $\mathcal{H}^H$ directly follows from Eqs. (13)–(15) and for the first-order and second-order contributions we have

$$\mathcal{H}^{H(1)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H(1)}_g + \delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\ \ , \tag{35}$$

$$\begin{aligned}\mathcal{H}^{H(2)}_{gg\leftarrow ab}(z) = \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H(2)}_g + \delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z) + H^{H(1)}_g\left(\delta_{g\,a}\,C^{(1)}_{g\,b}(z) + \delta_{g\,b}\,C^{(1)}_{g\,a}(z)\right) \\ + \left(C^{(1)}_{g\,a}\otimes C^{(1)}_{g\,b}\right)(z) + \left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z)\ \ .\end{aligned} \tag{36}$$

*At LO, where only the Born subprocess is available, $\left[\frac{d\hat{\sigma}^{(\rm fin.)}_{F\,ab}}{dq_T^2}\right]_{\rm LO}$ is identically zero by definition.

In Eq. (36) and in the following, the symbol $\otimes$ denotes the convolution integral (i.e., we define $(g \otimes h)(z) \equiv \int_0^1 dz_1 \int_0^1 dz_2\, \delta(z - z_1 z_2)\, g(z_1)\, h(z_2))$. The new third-order contribution is

$$\begin{aligned}
\mathcal{H}^{H(3)}_{gg\leftarrow ab}(z) = {} & \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\, H^{H(3)}_g + \delta_{g\,a}\, C^{(3)}_{g\,b}(z) + \delta_{g\,b}\, C^{(3)}_{g\,a}(z) + H^{H(1)}_g \left(\delta_{g\,a}\, C^{(2)}_{g\,b}(z) + \delta_{g\,b}\, C^{(2)}_{g\,a}(z)\right) \\
& + H^{H(2)}_g \left(\delta_{g\,a}\, C^{(1)}_{g\,b}(z) + \delta_{g\,b}\, C^{(1)}_{g\,a}(z)\right) + \left(C^{(1)}_{g\,a} \otimes C^{(2)}_{g\,b}\right)(z) + \left(C^{(2)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) \\
& + \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z) + H^{H(1)}_g \left(C^{(1)}_{g\,a} \otimes C^{(1)}_{g\,b}\right)(z) \\
& + H^{H(1)}_g \left(G^{(1)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z)\,. \qquad (37)
\end{aligned}$$

As stated in Sec. 2, the second-order helicity-flip functions $G^{(2)}_{g\,a}(z)$ and the third-order collinear functions $C^{(3)}_{g\,a}(z)$ are not known. In addition, the third-order hard-virtual coefficient $H^{H(3)}_g$ is not completely known. The missing terms in Eq. (27), for the final expression of $H^{H(3)}_g$, have a *soft* origin. Following notation of Ref. [7], these missing terms are proportional to the unknown $\delta^{(2)}_{q_T}$ factor and we can perform a subsequent decomposition for the third order hard-virtual coefficient defined in Eq. (27)

$$\widetilde{H}^{H(3)}_g \equiv H^{H(3)}_g - H^{H(3)}_g(\delta^{(2)}_{q_T})\,. \qquad (38)$$

Therefore the numerical extraction is constrained to the functions $G^{(2)}_{g\,a}(z)$, $C^{(3)}_{g\,a}(z)$ and $H^{H(3)}_g(\delta^{(2)}_{q_T})$. A naïve numerical implementation of Eq. (31) results in the following approximation

$$\begin{aligned}
C_{N3}\;\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z) \;\leftarrow\; & \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\, H^{H(3)}_g(\delta^{(2)}_{q_T}) + \delta_{g\,a}\, C^{(3)}_{g\,b}(z) + \delta_{g\,b}\, C^{(3)}_{g\,a}(z) \\
& + \left(G^{(1)}_{g\,a} \otimes G^{(2)}_{g\,b}\right)(z) + \left(G^{(2)}_{g\,a} \otimes G^{(1)}_{g\,b}\right)(z)\;\;, \qquad (39)
\end{aligned}$$

where the third-order numerical coefficient C_{N3} (which is proportional to $\delta(1-z)$) embodies the numerical extraction of the hard-virtual coefficient $H^{H(3)}_g(\delta^{(2)}_{q_T})$ (with no approximation) *plus* the numerical reduction (approximation) of a function of the variable z to a numerical term proportional to $\delta(1-z)$. This numerical method to approximate numerically unknown terms in the hard-virtual function $\mathcal{H}^{H}_{gg\leftarrow ab}(z)$ is not new. It was first used in Ref. [2] in order to estimate numerically the second order function $\mathcal{H}^{H(2)}_{gg\leftarrow ab}(z)$ at NNLO, providing a reasonable approximation†. Notice that Eq. (39) allows to recover the total cross section (at N^{3}LO in this case) with no approximation. After integration over the transverse momentum q_T, Eq. (28) provides the same total integral (numerically in this case) that in the full analytical case. Even more, for IR observables at fixed order which verify that the *back-to-back* kinematical region ($q_T = 0$) is located in a single phase space point (e.g the q_T distribution‡), we consider our fixed order result as with no approximation, i.e, the integral of the unknown terms (which have $q_T = 0$) are located in one point of the exclusive differential distributions.

In Appendix 4 we perform a detailed numerical study, of the proposed approximation in Eq. (39), but at one order less (NNLO), showing that even for the rapidity distribution, the approximated result differs less than 1% in kinematical regions where the bulk of the cross section takes place.

†In Appendix 4 we show that at NNLO, the approximation proposed in Eq. (86) of Ref. [2] reproduces the exact result within 1% precision in differential predictions.

‡The same consideration applies considering the Higgs boson decay into diphotons for the angular separation $\Delta\Phi_{\gamma\gamma}$ between the two photons, etc.

2.2 The numerical computation of C_{N3}

Now we turn the discussion to the numerical extraction of the coefficient C_{N3} using Eqs. (37), (27) and (31). In the numerical studies performed in this Section and in the differential predictions presented in Section 3, we consider the Higgs boson production in pp collisions at the centre–of–mass energy $\sqrt{s} = 13$ TeV. We use the Higgs boson mass $M_H = 125$ GeV and the vacuum expectation value $v = 246.2$ GeV. The mass of the top quark is taken $m_t = 173.2$ GeV. We use the central member of the `PDF4LHC15` parton distribution functions (PDFs) [21] as implemented in the `LHAPDF` framework [23] and the associated strong coupling constant α_S. Note that we systematically employ the same order of the PDFs (NNLO) for the LO,NLO,NNLO and N^{3}LO results presented in this paper. The default factorisation and renormalisation scales are chosen accordingly to $\mu \equiv \mu_R = \mu_F = M_H/2$. The theoretical uncertainty is estimated by varying the default scale choice by a factor $\{1/2, 2\}$.

The numerical implementation of the q_T subtraction formalism of Eq. (3) makes use of the cross section $d\hat{\sigma}^{H+\text{jets}}_{ab}$ up to NNLO, which is implemented in the parton-level Monte Carlo generator `NNLOJET` [22]. This program provides the necessary infrastructure for the antenna subtraction [4] of hadron collider processes at NNLO and performs the integration of all contributing subprocesses of the type $d\hat{\sigma}^{H+\text{jets}}_{ab}$ at this order. The counter term and the hard function $\mathcal{H}^H$ are encoded in a new Monte Carlo generator `HN3LO` up to the third order in the strong coupling constant.

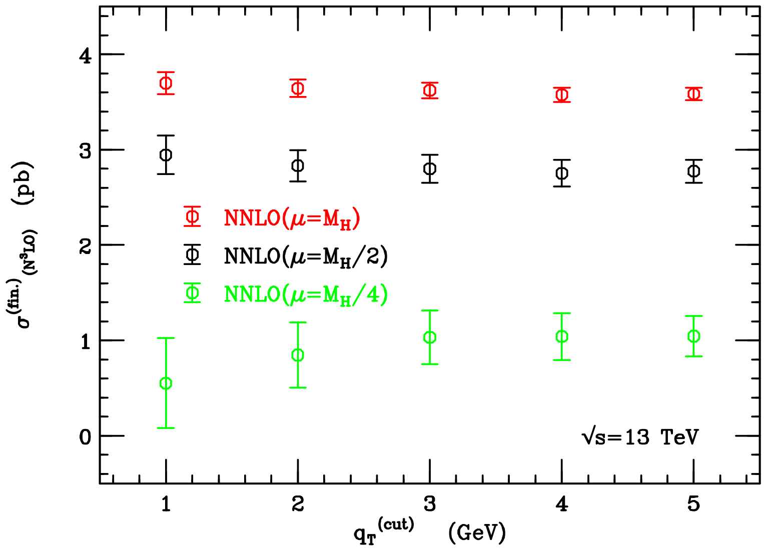

Figure 1: *The q_T integrated finite contribution to the cross section of Eq. (30) at N^{3}LO exclusively (i.e N^{3}LO-NNLO) between q_T^{cut} and ∞, for three different scales $\mu = \mu_R = \mu_F$.*

In the numerical implementation of Eq. (31), the lower bound of the q_T integral ($q_T = 0$) has to be replaced with a suitable choice of the q_T^{cut}. In Fig. 1 we report the $\sigma^{(\text{fin.})}_{F\,ab}$ (i.e $\hat{\sigma}^{(\text{fin.})}_{F\,ab}$ convoluted with the parton distribution functions) cross section at N^{3}LO exclusively[§]. Using Eq. (37) with Eq. (31) and the value of the resulting integral $\sigma^{(\text{fin.})}_{F\,ab}(q_T^{\text{cut}} = 1 \text{ GeV})$ in Fig. 1, it is possible to

[§]With N^{3}LO exclusively we understand the finite cross section between curly brackets in Eq. (31), i.e $\left[d\sigma^{(\text{fin.})}_{F\,ab}\right]_{\text{N}^3\text{LO}} - \left[d\sigma^{(\text{fin.})}_{F\,ab}\right]_{\text{NNLO}}$.

obtain the q_T integrated cross section of the unknown terms in the right hand side of Eq. (39) and consequently C_{N3}.

Notice that in Fig. 2, the C_{N3} predicted value for each scale (in black points) is independent of that choice (within the uncertainties), in complete agreement with Eq. (31). The scale independence of C_{N3} is not related with the used ansatz of Eq. (39). The terms in the right-hand side of Eq. (31) are all scale independent. Notice that the relation between C_{N3} and $\widetilde{H}_g^{H(3)}$ is univocal defined in Eq. (37). The solid red central line in Fig. 2, is the weight average calculated with

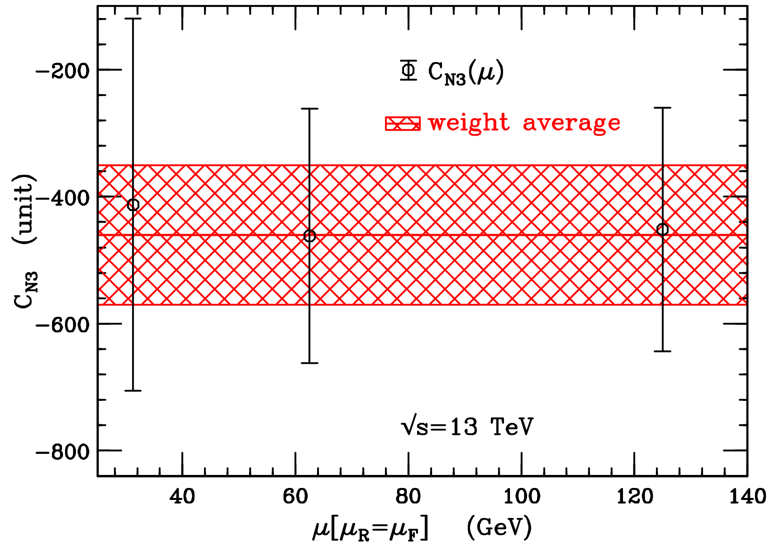

Figure 2: *The numerically computed C_{N3} coefficient as defined in Eq. (39) (black points). The red band is calculated with the weight average of the three particular C_{N3} values for each scale as detailed in the text.*

the three different black points corresponding to each scale choice, whereas the red band is the predicted uncertainty. The value of the resulting coefficient is $C_{N3} = -460 \pm 110$.

The numerically calculated C_{N3} coefficient allows to predict the total cross section at N^{3}LO with the transverse momentum subtraction method (as well completely differential distributions). In Fig. 3 we compare the fully analytical N^{3}LO Higgs boson total cross section [24] (black points) and our best estimation (green points) for three different scales. The blue squared points constitute our best approximation without the use of the C_{N3} coefficient (i.e $C_{N3} = 0$), that can be consider as the prediction of the q_T subtraction method in the case in which the total cross section is unknown (e.g for Drell-Yan at N^{3}LO). The uncertainty bands in the q_T subtraction prediction correspond to the statistical errors of the numerical computation and are mainly due to the finite contribution of Eq. (30) at N^{3}LO exclusively.

3 Exclusive predictions at the LHC

In the following we present numerical results for Higgs boson production at the LHC. The setup of the calculation was stated in Sec. 2.2.

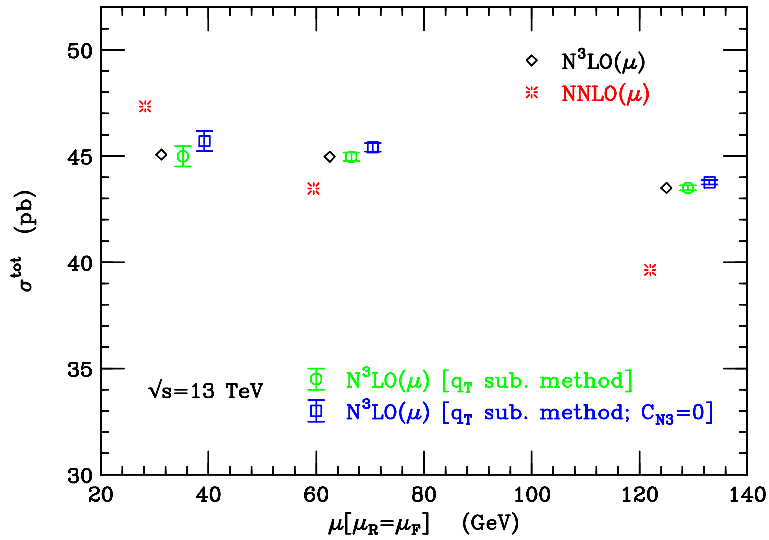

Figure 3: *Total Higgs boson cross section at N^{3}LO, as predicted by the q_T subtraction formalism (green points), compared with the analytical Higgs boson total cross section at N^{3}LO of Ref. [24] (black diamonds). The q_T subtraction prediction at N^{3}LO with the C_{N3} numerical coefficient fixed to zero is shown with blue squared points. The NNLO analytical Higgs boson cross section is presented in pink star points. All the cross sections are shown for three different scales: $\mu \equiv \mu_R = \mu_F = \{1/4, 1/2, 1\}M_H$. The uncertainty bands in the q_T subtraction predictions are calculated with the customary propagation of statistical uncertainties.*

In Fig. 4 we show the rapidity distribution of the Higgs boson at LO (dashed black band), NLO (blue solid band), NNLO (green solid band) and N^{3}LO (magenta solid band). The central scale ($\mu = M_H/2$) is shown with a solid line while the variation of the scales by a factor $\{1/2, 2\}$ produce the band. It is interesting to notice that from LO to NNLO, the central scale $\mu = M_H/2$ lies between the prediction with $\mu = M_H/4$ (upper band) and $\mu = M_H$ (lower band) for the rapidity distribution in Fig. 4. At N^{3}LO the central scale $\mu = M_H/2$ (magenta solid line) almost coincides with the limit of the upper band for $\mu = M_H/4$. The precedent fact is the differential manifestation of what was shown in Fig. 3 for the total cross section: for $\mu = M_H/4$ and $\mu = M_H/2$ the N^{3}LO cross sections are in accord at the 0.2% level. The Figs. 3 and 4 show the reduction in the size of the variation of the scales at N^{3}LO, in the total cross section and in differential distributions, respectively. The impact of the N^{3}LO corrections on the NNLO result is almost independent on the rapidity y_H in the entire rapidity range $|y_H| \leq 4$. The total cross section (for the central scale) increases by about 3.4% when going from NNLO to N^{3}LO.

Acknowledgements.

4 Appendix: NNLO total cross section and rapidity distribution

The aim of the present section is quantify the uncertainty of the approximation proposed in Eq. (39). Since at NNLO all the ingredients of the q_T subtraction formalism are known, it is possible

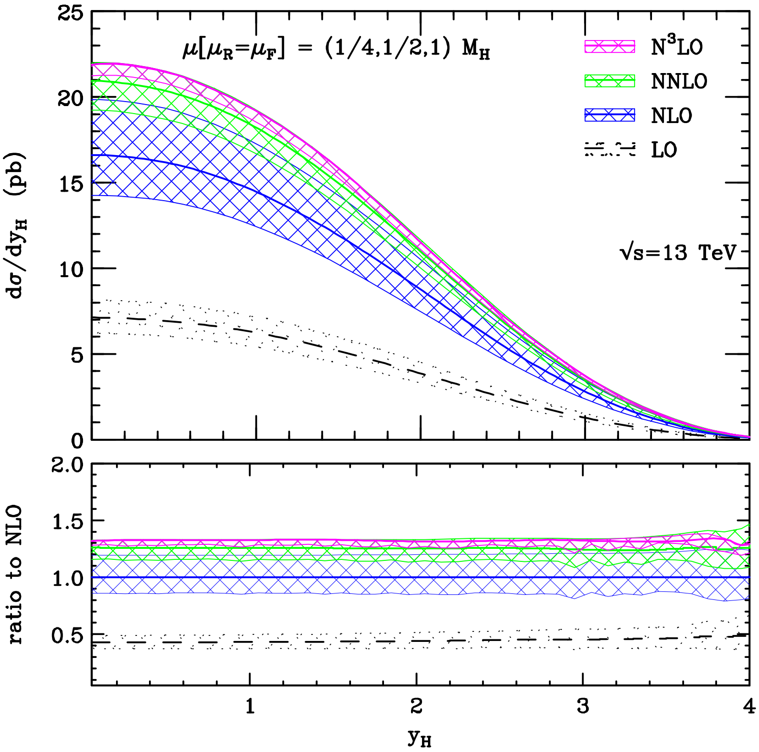

Figure 4: *Rapidity distribution of the Higgs boson as predicted by the q_T subtraction formalism up to N^{3}LO. The scale variation bands of the LO, NLO, NNLO and N^{3}LO(C_{N3}) results are as follows: LO (black dashed), NLO (blue solid), NNLO (green solid) and N^{3}LO(C_{N3}) (magenta solid). The central scale ($\mu = M_H/2$) at each perturbative order, is shown with solid lines respecting the colour of the corresponding perturbative order. In the lower panel, the ratio to the NLO prediction is shown.*

to evaluate the deviation from the exact result. For the present quantitative exercise we take as known the collinear functions $C^{(1)}_{g\,a}$ and the hard-virtual factor $H^{H(1)}_g$ in Eq. (36). The collinear functions $C^{(2)}_{g\,a}$ and the first order helicity-flip functions $G^{(1)}_{g\,a}$ are regarded as unknown. The hard-virtual factor $H^{H(2)}_g$ is divided in two contributions as in Eq. (39)

$$\widetilde{H}^{H(2)}_g \equiv H^{H(2)}_g - H^{H(2)}_g(\delta^{(1)}_{q_T}) \,, \tag{40}$$

where $H^{H(2)}_g(\delta^{(1)}_{q_T})$ is considered as unknown. The unknown functions of the variable z in Eq. (36) are approximated with a single numerical coefficient C_N proportional to $\delta(1-z)$

$$\begin{aligned} C_N \;\; \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z) \;\leftarrow\;\; & \delta_{g\,a}\,\delta_{g\,b}\,\delta(1-z)\,H^{H(2)}_g(\delta^{(1)}_{q_T}) \\ & +\delta_{g\,a}\,C^{(2)}_{g\,b}(z) + \delta_{g\,b}\,C^{(2)}_{g\,a}(z) + \left(G^{(1)}_{g\,a}\otimes G^{(1)}_{g\,b}\right)(z) \;\; . \end{aligned} \tag{41}$$

In Fig. 5 we show the rapidity distribution at NNLO computed with the exact q_T subtraction (green band) and the NNLO prediction using the C_N coefficient (red, blue and black points). In the lower panel of Fig. 5 we show the ratio to the exact NNLO result, i.e we present the ratio for each scale. As expected, the approximation presents its best behaviour at central rapidity and for $|y_H| \leq 3.5$ it is at the *sub percent* level.

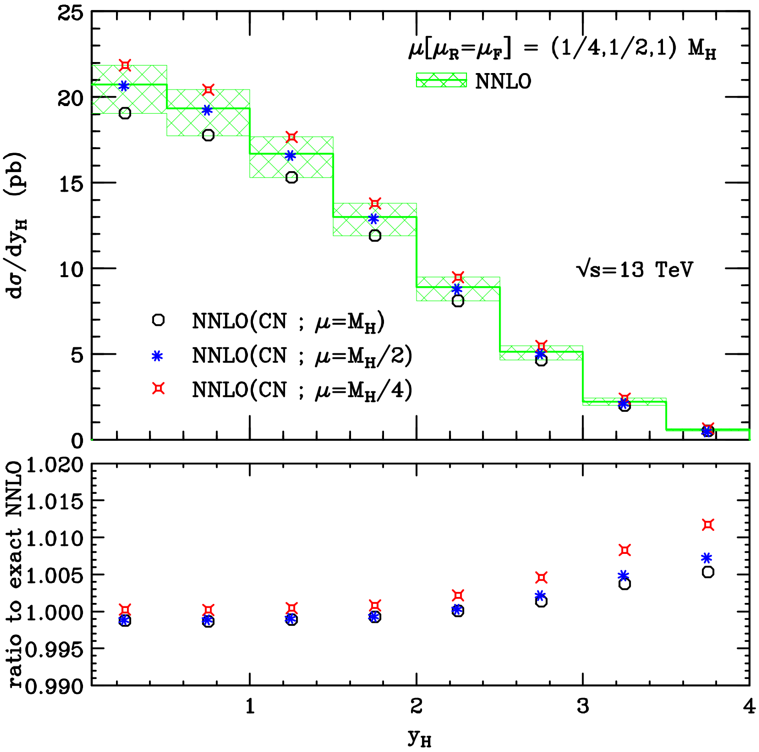

Figure 5: *Rapidity distribution at NNLO as predicted by the q_T subtraction formalism (solid green band) compared with the prediction using the C_N numerical coefficient (red, blue and black points). In the lower panel we show the ratio to the exact NNLO result.*

5 Appendix: Necessary convolutions at N^{3}LO

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