- $\mathbf{R}(\phi)$ is the rotation matrix, given by

$$\mathbf{R}(\phi) = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

- $\mathbf{t}$ is the translation vector,

$$t = \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

- Δx and Δy are the translations in the Cartesian coordinate system.

Only three parameters, x, y, and ϕ needed to be computed. To determine these parameters, a cost function was defined. This function, denoted as $C(x, y, \phi)$, quantified the alignment error by calculating the 'cost' of deviations for any given set of transformation parameters. The minimization of this cost identified the combination of parameters that achieved the best alignment or overlap of the transformed frame with the reference frame. A lower value of the cost function output indicated a closer match to the target frame, suggesting a better alignment, whereas a higher value signified a less accurate alignment. The equation of the cost function is given in Equation 3:

$$C(\Delta x, \Delta y, \phi) = \sum_{p=1}^{N} \min_{q \in Q} \left(\sqrt{(x_q - x'_p)^2 + (y_q - y'_p)^2} \right) \tag{3}$$

where:

- (x_p, y_p) are the coordinates of the points in the reference frame
- (x'_p, y'_p) are the coordinates of points after having been transformed:

$$x'_p = \Delta x + x_p \cos(\phi) - y_p \sin(\phi)$$
$$y'_p = \Delta y + x_p \sin(\phi) + y_p \cos(\phi)$$

- (x_q, y_q) are the coordinates of points in the target frame.
- N is the number of points in the reference frame, and Q represents all points in the target frame.

This cost function evaluates the alignment of transformed coordinates to target coordinates by calculating the sum of the shortest distances from each point in the