STT 465, Midterm

Your name:__________________________________

Instructor: Gustavo de los Campos

- The midterm is strictly individual. You are allowed to use a calculator and up to one sheet with notes.
- The midterm has 4 questions, answer the questions in the space provided. If you need extra space you can use the back of each page.
- The exams will be collected at 11:40 (no extra time).
- If you need quantiles ask me and I will let you get that info using my computer.

1. Are the following statements TRUE/FALSE (remember, TRUE implies that it always holds, otherwise, consider the statement as FALSE).

1.1. $E(a + bX + Y) = a + bE(X) + E(Y)$	
1.2. $E[X^2] = Var(X) + E[X]^2$	
1.3. $E(X) = \int E(X\|Z)p(Z)dZ$	
1.4. $Var(X + Y) = Var(X) + Var(Y)$	
1.5. If X and Y are independent =>$Cov(X + bY, Y) = bVar(Y)$	
1.6. If X and Y are independent => $Var(X + bY) = Var(X) + b^2Var(Y)$	
1.7. $p(X, Y) = p(X)p(Y)$	
1.8. If X and Y are IID => $p(X = k_1, Y = k_2) = p(X = k_2, Y = k_1)$	
1.9. If X and Y are independent => $p(X = k_1, Y = k_2) = p(X = k_2, Y = k_1)$	
1.10. If X and Y are exchangeable $p(X, Y) = \int p(X\|Z)p(Y\|Z)p(Z)dZ$ for some $p(Z)$	

2. X and Y are two Bernoulli Random Variables that, conditional in another Bernoulli random variable, Z, are independent and identically distributed

   $p(Y=1|Z=0)=p(X=1|Z=0)=1/4$ and
   $p(Y=1|Z=1)=p(X=1|Z=1)=3/4$

2.1. Using the above information fill the white cells of the tables below

	Z=0		
	Y=0	Y=1	
X=0			
X=1			
			1

	Z=0		
	Y=0	Y=1	
X=0			
X=1			
			1

Note: The tables are needed to answer most of the items that follow. If you cannot complete the two tables I can provide you help with a penalty of 20% of the points of the question.

2.2. If p(Z=1)=2/3, then P(X=0)= ____________________

2.3. P(Z=1|X=0,Y=1)=_____________________________

2.4. Are X and Y exchangeable? Why Yes?/Why Not?

2.5. Are X and Y independent? Why Yes?/Why Not?

3. A random sample of **20** observations was collected. The sum and sum of squares of the data were **73** and **311**, respectively.

3.1. Write the likelihood function assuming that the random variable follows a Poisson distribution. Remember, the Poisson distribution has the following form: $p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$ with $E(X) = Var(X) = \lambda$

3.2. Derive the Maximum Likelihood estimator.

3.3. Using the data provided compute and report the ML estimate and an approximate 95% Frequentist Confidence Interval.

3.2. Derive the posterior distribution of λ, assuming a Poisson likelihood and a gamma prior for λ with shape and rate parameters equal to $\alpha = 4$ and $\beta = 1$, respectively. Recall that the density function of a gamma distributed random variable X with shape and rate parameters α and β is $p(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$. The mean and variance of this distribution are: $E(x) = \frac{\alpha}{\beta}$ and $V(x) = \frac{\alpha}{\beta^2}$ respectively.

3.3. Report the Posterior mean, posterior standard deviation and a 95% credibility region.