NOTES ON
SUPERCONDUCTIVITY

ALESSANDRO GORI
github/nepero27178

Last compiled: August 20, 2024

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 4.0 International" license.

Contents

1 SUPERCONDUCTIVITY AND THERMODYNAMICS 5
- 1.1 Macroscopic theory of superconductivity 6
  - 1.1.1 The land where electrons do not collide 6
  - 1.1.2 The land where magnetic fields are not welcome 8
- 1.2 Thermodynamics 10
  - 1.2.1 The critical field 10
  - 1.2.2 First order transition at non-zero field 12
  - 1.2.3 Second order transition at zero field 14

2 GINZBURG–LANDAU THEORY OF SUPERCONDUCTIVITY 17
- 2.1 Ginzburg-Landau theory for classical spins 17
  - 2.1.1 Homogeneous magnetization in absence of fields 19
  - 2.1.2 Breaking of symmetry through field coupling 19
- 2.2 Symmetry breking in superconductors 21
- 2.3 Fluctuations of the complex order parameter 23
  - 2.3.1 Magnitude fluctuations 24
  - 2.3.2 Phase fluctuations 25
- 2.4 Superconductivity 26
  - 2.4.1 Free energy expansion and Ginzburg-Landau equations 27
  - 2.4.2 The critical field 28
- 2.5 Spontaneous symmetry breaking in superconductors 28
  - 2.5.1 The Goldstone boson of superconductivity 29
  - 2.5.2 The Anderson-Higgs mechanism 32

3 CONVENTIONAL SUPERCONDUCTORS 35
- 3.1 Superconductors of type I and type II 35
  - 3.1.1 Type I 37
  - 3.1.2 Type II 38
- 3.2 Magnetic properties of superconductors 39
  - 3.2.1 Flux quantization 39
  - 3.2.2 Nucleation field 41
  - 3.2.3 The mixed phase 44
- 3.3 The Abrikosov vortex impurity 46
  - 3.3.1 Impurities: the Abrikosov vortex 46
  - 3.3.2 Phase diagram of Abrikosov superconductors 48
- 3.4 The Abrikosov lattice 52
  - 3.4.1 Interaction of two static fluxons with same charge 52
  - 3.4.2 Interaction of two static fluxons with opposite charge 56
  - 3.4.3 Interaction of two moving fluxons with same charge 56
  - 3.4.4 Many fluxons with the same charge: the lattice ground state 56

4 COOPER PAIRS: HOW ELECTRONS FORM BOUND STATES 61
- 4.1 What if electrons attract? 62

4.1.1 Bound states 63
4.1.2 Adding particles on top of the Fermi sphere 66
4.1.3 Bound states, considering statistics 68
4.1.4 (In)Stability of the Fermi Sea 72
4.2 How the phonon pairing works 73
4.2.1 Born-Oppenheimer quantization 73
4.2.2 Electron-phonon interactions 75
4.2.3 The effective interaction hamiltonian: Schrieffer-Wolff transformation 76
4.2.4 Effective form of the phonon-assisted interaction 78
4.2.5 A few assumptions, and Cooper pairs 80
5 THE BCS THEORY 85
5.1 The BCS hamiltonian 85
5.2 The Importance of Being Gapped, a Trivial Comedy for Superconducting People 85

5 THE BCS THEORY

5.1 The BCS hamiltonian 85
5.2 The Importance of Being Gapped,
a Trivial Comedy for Superconducting People 85

It is time to develop formally and completely the theory by Bardeen, Cooper and Schrieffer. We saw in the last chapter how electrons form Cooper pairs, because apparently in superconductors charges flow coupled. The question now is: why is the coupling of electrons (a rather weak one, also) necessary for the exotic phenomena of superconductivity, like resistanceless flow of charge and Meissner effect?

5.1 THE BCS HAMILTONIAN

From last chapter we know that the phonon-mediated effective hamiltonian is given by

$$\hat{H} = \sum_{\mathbf{k}} \epsilon_{\mathbf{k}} \left(\hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}_{\mathbf{k}\uparrow} + \hat{c}^{\dagger}_{\mathbf{k}\downarrow} \hat{c}_{\mathbf{k}\downarrow} \right) + \sum_{\mathbf{k}\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'} \left(\hat{c}^{\dagger}_{\mathbf{k}\uparrow} \hat{c}_{\mathbf{k}'\uparrow} \right) \left(\hat{c}^{\dagger}_{-\mathbf{k}\downarrow} \hat{c}_{-\mathbf{k}'\downarrow} \right)$$

5.2 THE IMPORTANCE OF BEING GAPPED,

A TRIVIAL COMEDY FOR SUPERCONDUCTING PEOPLE

BIBLIOGRAPHY

[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer. "Theory of Superconductivity". In: *Phys. Rev.* 108 (5 Dec. 1957), pp. 1175–1204. DOI: 10.1103/PhysRev.108.1175. URL: https://link.aps.org/doi/10.1103/PhysRev.108.1175.

[2] G. Grosso and G. Pastori Parravicini. *Solid State Physics*. Elsevier Science, 2000. ISBN: 9780080481029. URL: https://books.google.it/books?id=L5RrQbbvWn8C.

[3] G. Giuliani and G. Vignale. *Quantum Theory of the Electron Liquid*. Cambridge University Press, 2008. ISBN: 9781139471589. URL: https://books.google.it/books?id=FFydOyv_r78C.

[4] D. L. Goodstein. *States of Matter*. Dover Books on Physics. Dover Publications, 2014. ISBN: 9780486795515. URL: https://books.google.it/books?id=OfEIBAAAQBAJ.

[5] M. Tinkham. *Introduction to Superconductivity*. Dover Books on Physics Series. Dover Publications, 2004. ISBN: 9780486134727. URL: https://books.google.it/books?id=VpUk3NfwDIkC.

[6] K. Huang. *Statistical Mechanics, 2nd Ed*. Wiley India Pvt. Limited, 2008. ISBN: 9788126518494. URL: https://books.google.it/books?id=ZHl8HLk-K3AC.

[7] A. Altland and B. D. Simons. *Condensed Matter Field Theory*. Cambridge books online. Cambridge University Press, 2010. ISBN: 9780521769754. URL: https://books.google.it/books?id=GpF0Pgo8CqAC.

[8] F. London, H. London, and Frederick Alexander Lindemann. "The electromagnetic equations of the supraconductor". In: *Proceedings of the Royal Society of London. Series A - Mathematical and Physical Sciences* 149.866 (1935), pp. 71–88. DOI: 10.1098/rspa.1935.0048. URL: https://royalsocietypublishing.org/doi/abs/10.1098/rspa.1935.0048.

[9] A. A. Abrikosov. "On the Magnetic properties of superconductors of the second group". In: *Sov. Phys. JETP* 5 (1957), pp. 1174–1182.

[10] V. G. Kogan and R. Prozorov. "Interaction between moving Abrikosov vortices in type-II superconductors". In: *Phys. Rev. B* 102 (2 July 2020), p. 024506. DOI: 10.1103/PhysRevB.102.024506. URL: https://link.aps.org/doi/10.1103/PhysRevB.102.024506.

[11] J Simmendinger et al. "Bound and stable vortex–antivortex pairs in high-Tc superconductors". In: *New Journal of Physics* 22.12 (Dec. 2020), p. 123035. DOI: 10.1088/1367-2630/abd123. URL: https://dx.doi.org/10.1088/1367-2630/abd123.