bitcoin++
Istanbul

@nepetonium
github.com/nepet

Unveiling the Magic of Zero-Knowledge Proofs:

~~From Waldo to~~ Bitcoin's Future

An introduction to the fundamentals of interactive zero-knowledge proofs

ZKPs: From Waldo to 3-Coloring

Today's Journey:

What Are Zero-Knowledge Proofs?

- Building intuition with simple examples
- Interactive Proofs
- The three properties of ZKPs
- Deep dive: 3-Coloring Problem

The Fundamental Question

Can you prove you know something without revealing what you know?

Real-world analogies:

- Proving you're over 21 without showing your exact birthdate
- Proving you have sufficient funds without revealing your balance
- Proving you know a password without typing it

This seems paradoxical, but it's possible!

Where's Waldo?

The Classic Example

The Scenario:

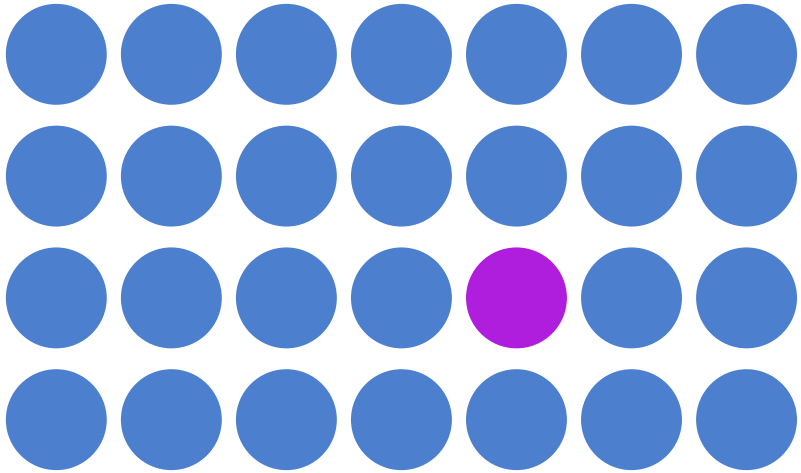
- You have a Where's Waldo book.
- You found Waldo and want to prove it to me.
- But you don't want to show me where Waldo is.

Where's Waldo?

The Classic Example

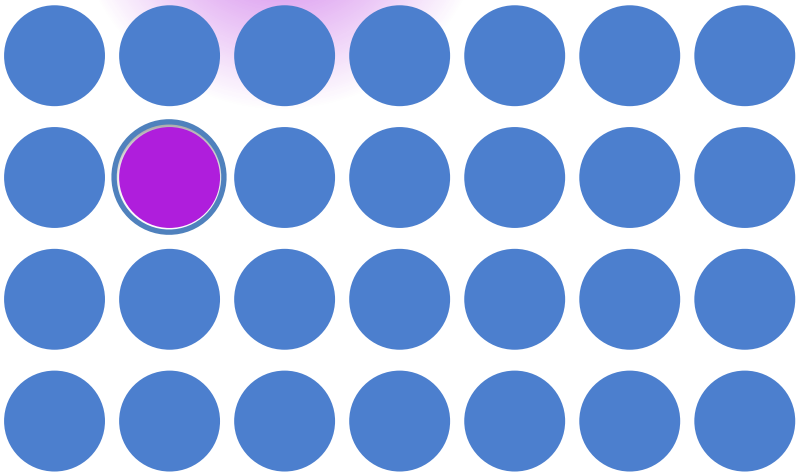
The Zero-Knowledge-Solution:

1. Take a large piece of cardboard
2. Cut a small hole in it
3. Place cardboard over the page
4. Position the hole exactly over Waldo
5. Show me Waldo through the hole



Where's Waldo? - Our first ZKP

The Classic Example



Why This Works?

Analyzing the Waldo Proof

Key Insights:

- **Verification:** I can see Waldo through the hole
- **No Information Leak:** The cardboard hides all context
- **Repeatability:** We can do this for any Waldo puzzle

But this isn’t a formal ZKP:

- **No Interaction:** Verifier can’t issue a challenge
- **Not mathematically verifiable:** Requires physical presence
- **Not simulatable:** can't create fake transcripts

Understanding Proofs

The Problem Domain

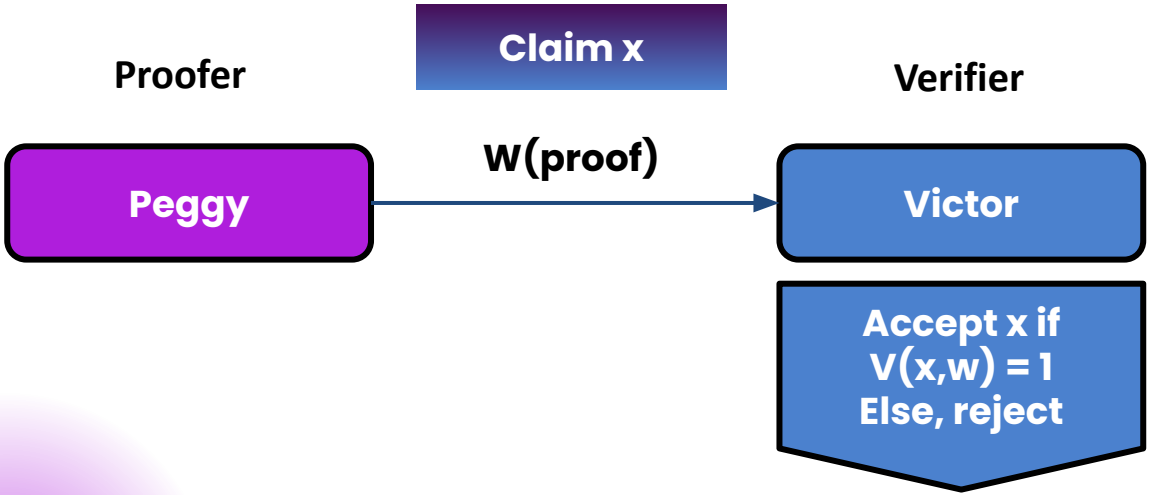
Decision Problems:

- Decision problems have binary answers: Yes, No
- “Is it raining outside of the venue?”
- “Is X a prime number?”
- “Given x and y, is x a multiple of y?”

Defines a whole class of Problems we can solve using computers.

Understanding Proofs

A classical proof



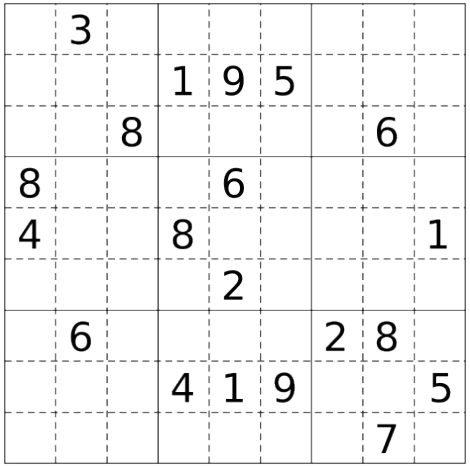
Understanding Proofs

Computational Complexity

NP: Nondeterministic Polynomial Time

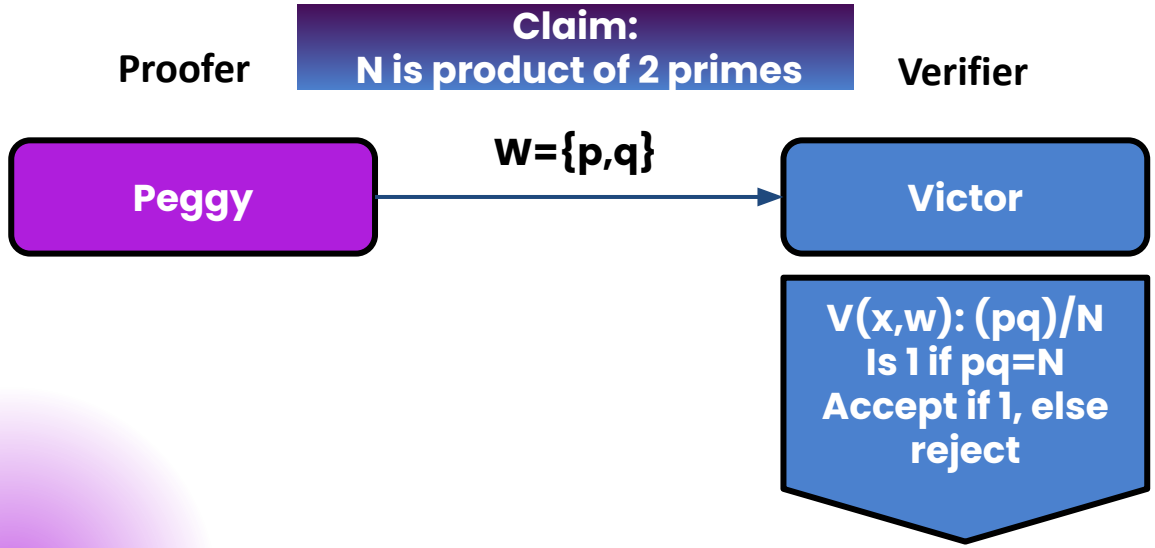
- Can be hard to solve.
- Easy to verify
- Like a Sudoku
- Or factorization of primes

Defines a whole class of Problems we can solve using computers.



Understanding Proofs

A classical decision proof

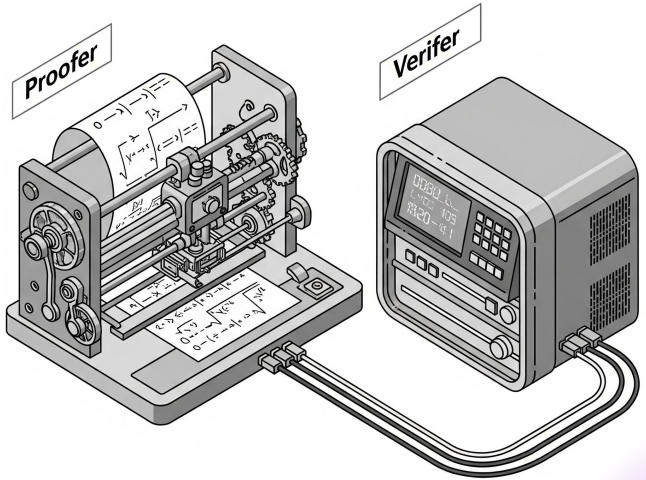


Understanding Proofs

Summary

For a decision problem Q(x) in NP:

- **Complete:** If Q(x)=YES, than there is a proof w such that V(x,w) = 1
- **Sound:** If Q(x) = No, than for all proofs w, V(x,w) = 0



Interactive Proofs

Interaction and Randomness

Scenario

- You can see colors.
- Everybody else can't.
- How can you proof that you do see colors?

Setup

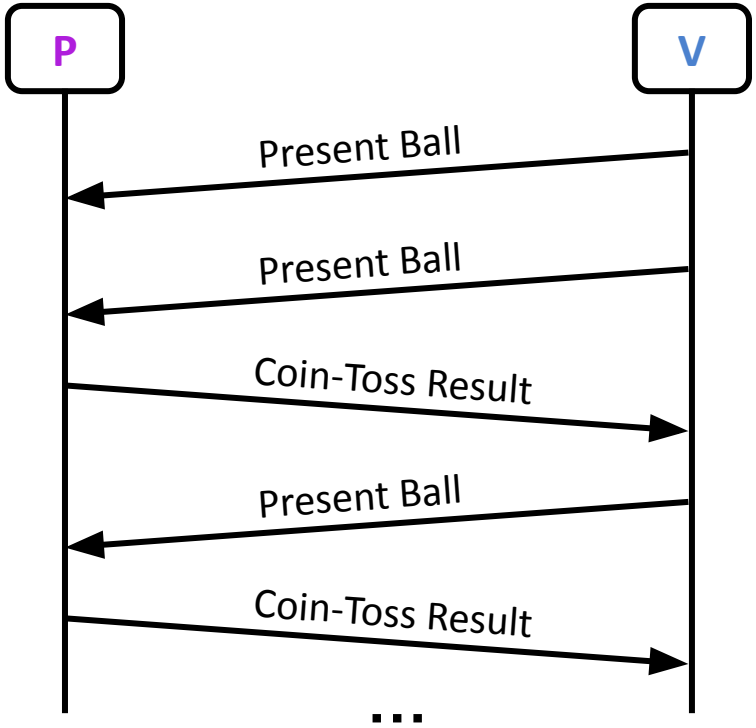
- Claim is: "I can see colors" or "I can see more than you"
- You are the proofer
- Some representative of the others is the verifier
- You have two balls of different color

Interactive Proofs

Interaction and Randomness

Proof System (Protocol)

1. V takes the balls and reveals one to you
2. V tosses a coin in private and shows you a ball based on the result:
   a. Heads: show other ball
   b. Tails: show same ball
3. P tells V the result of the coin toss.
4. Repeat



Interactive Proofs

Summary

- Instead of proving a statement directly we proved that we know something the other party don't
- We don't proof a 100% but with a certain confidence
- For this we introduced repeated interaction and randomness

- **Complete:** If Q(x)=YES, than there is a proof w such that V(x,w) = 1
- **Sound:** If Q(x) = No, than for all proofs w, V(x,w) = 0 except with negl. Probability
- Negl < 1/polynomial(|x|)
- Example: (½)^k where k is no rounds

Zero -Knowledge-Proofs

Let's put it all together

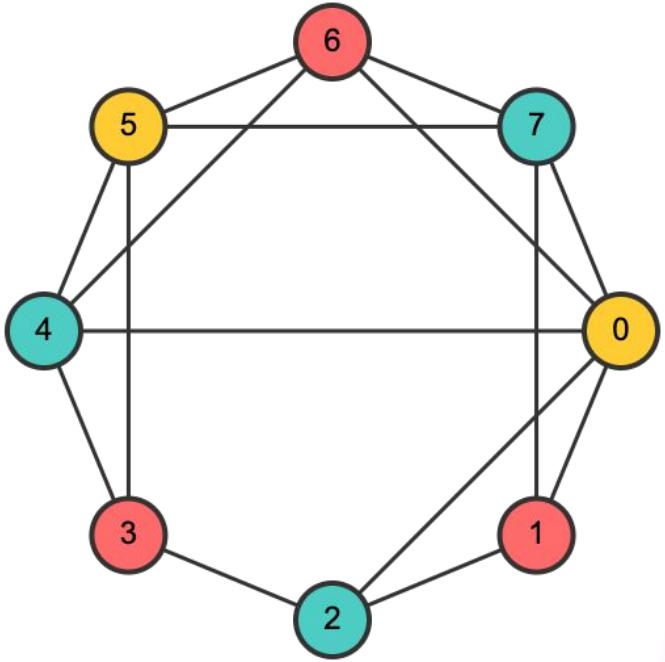
- **Complete:** If Q(x)=YES, than there is a proof w such that V(x,w) = 1
- **Sound:** If Q(x) = No, than for all proofs w, V(x,w) = 0 except with negligible Probability
- **Zero-Knowledge:** If Q(x) = YES, than V does not learn anything other than the fact that the statement is true

Zero-Knowledge-Proof

For the 3-Coloring Problem

Scenario

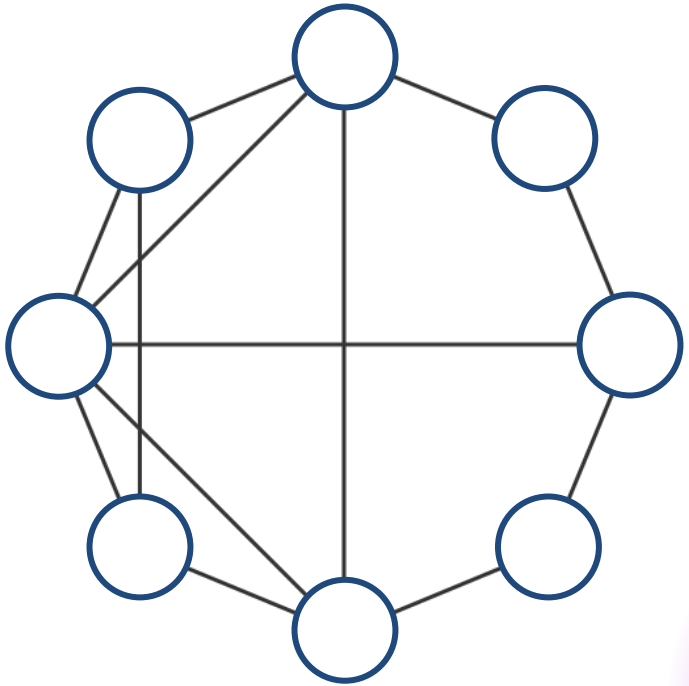
- You have a graph
- You have 3 colors
- The nodes of the graph shall be colored
- Can the whole graph be colored such, that no two adjacent nodes have the same color?
- Claim is: “Graph is 3-colorable”



Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v1

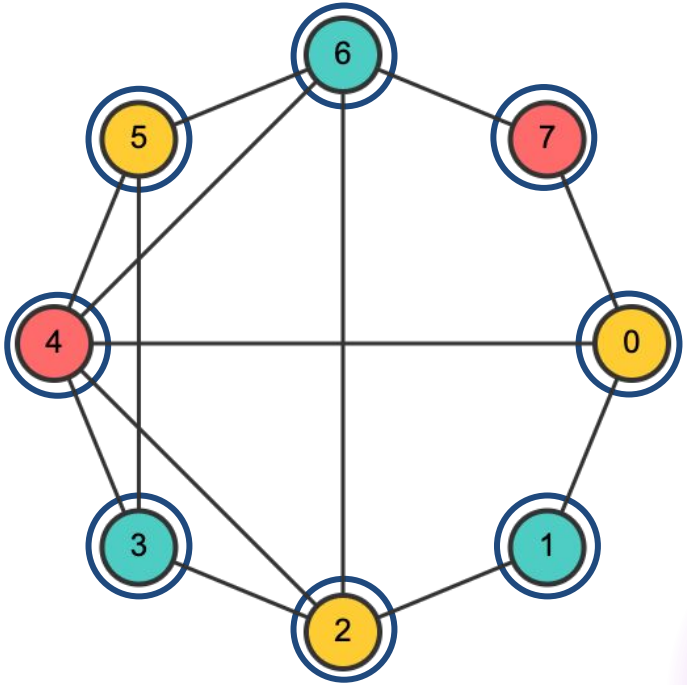


Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v1

1. Prover: Send Verifier our solution as proof

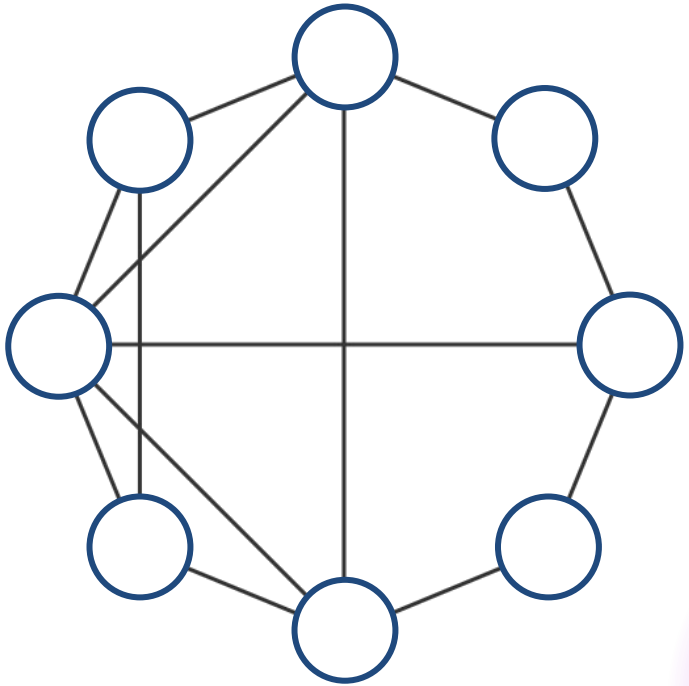


Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v2

1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send tuple

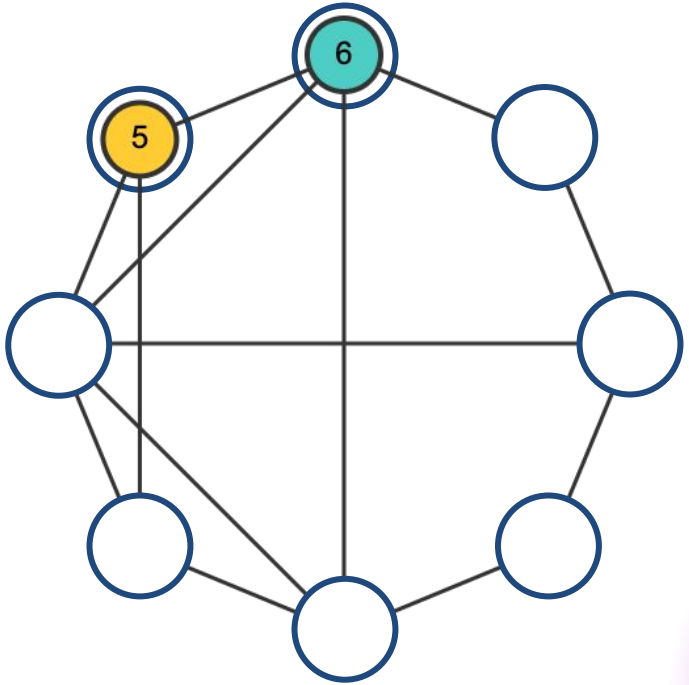


Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v2

1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send tuple
3. Verifier: Check that c_n1 != c_n2
4. Repeat



Soundness error is (1-1/|E|)^k, E is edges, k is rounds

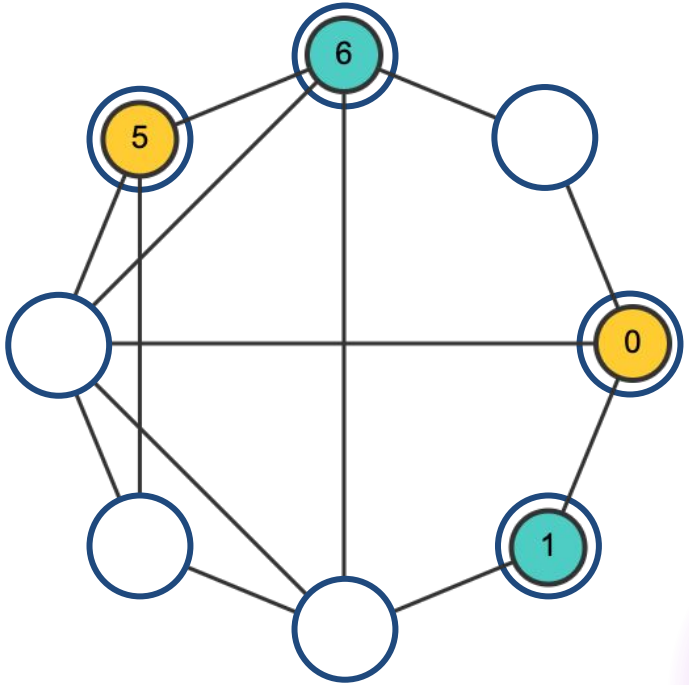
(1-1/|E|)^(n|E|) approx e^(-n) is negligible

Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v2

1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send tuple
3. Verifier: Check that c_n1 != c_n2
4. Repeat

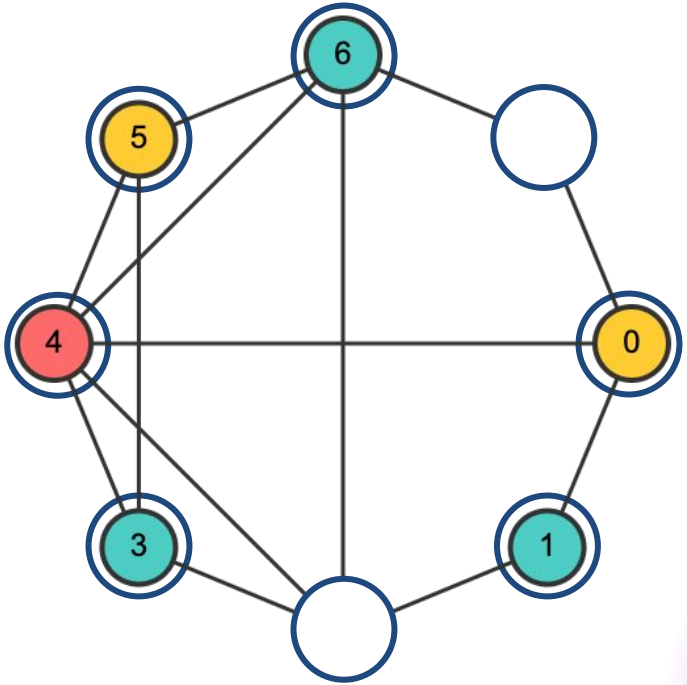


Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol v2

1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send tuple
3. Verifier: Check that c_n1 != c_n2
4. Repeat

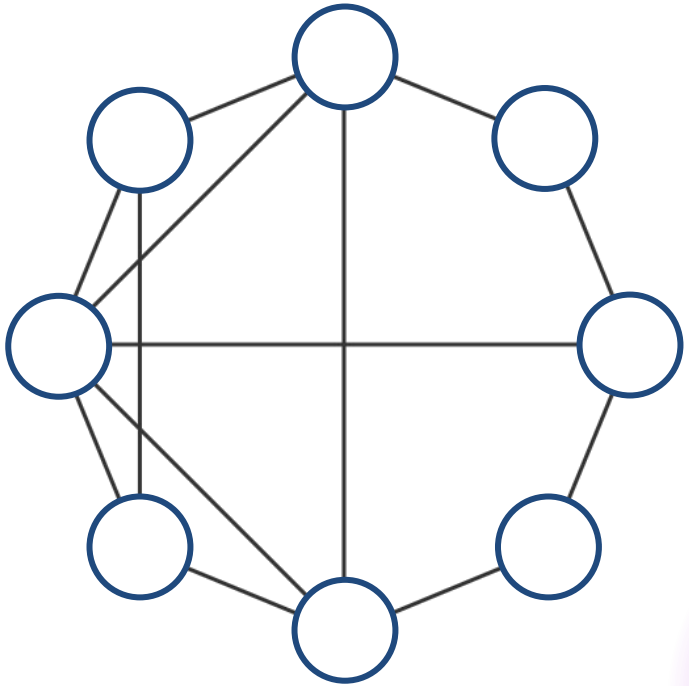


Zero-Knowledge-Proof

For the 3-Coloring Problem

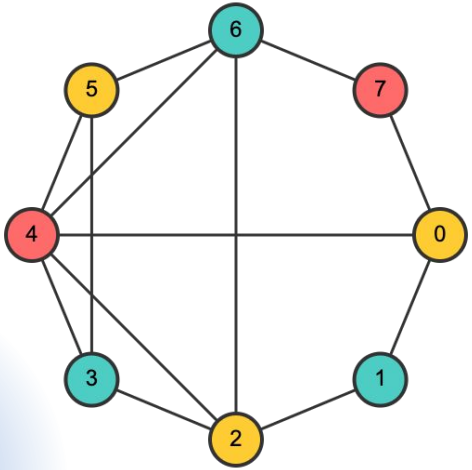
Protocol v3

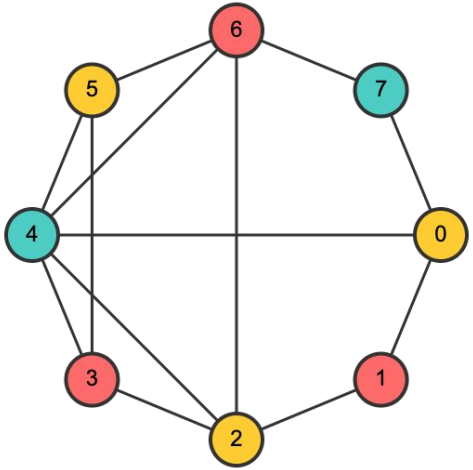
1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send a permutation of the tuple
3. Verifier: Check that c_n1 != c_n2
4. Repeat

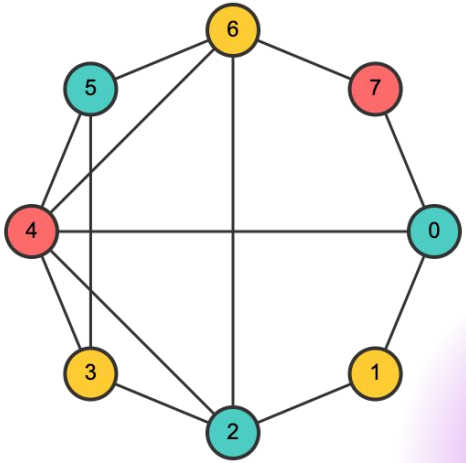


Zero-Knowledge-Proof

For the 3-Coloring Problem







Zero-Knowledge-Proof

For the 3-Coloring Problem

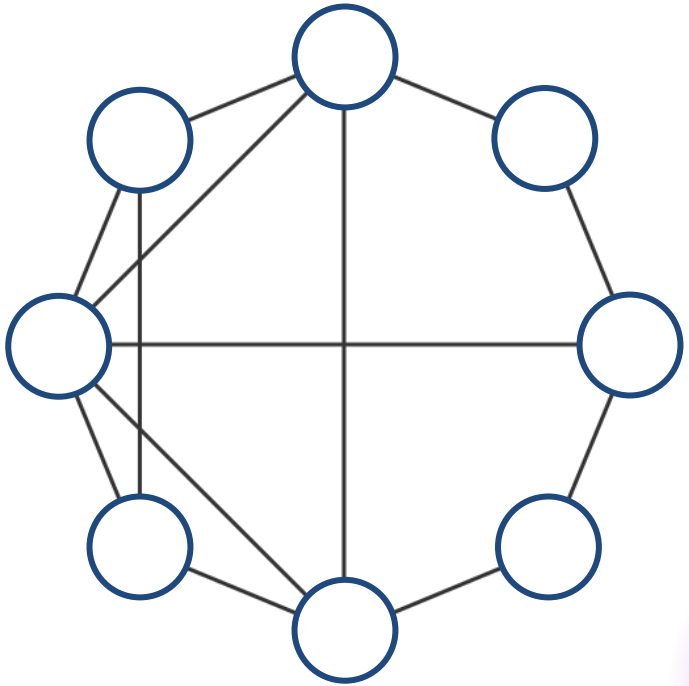
https://nepet.github.io/btcpp-3-color-zkp/

Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol 3

1. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
2. Prover: Send a permutation of the tuple
3. Verifier: Check that c_n1 != c_n2
4. Repeat

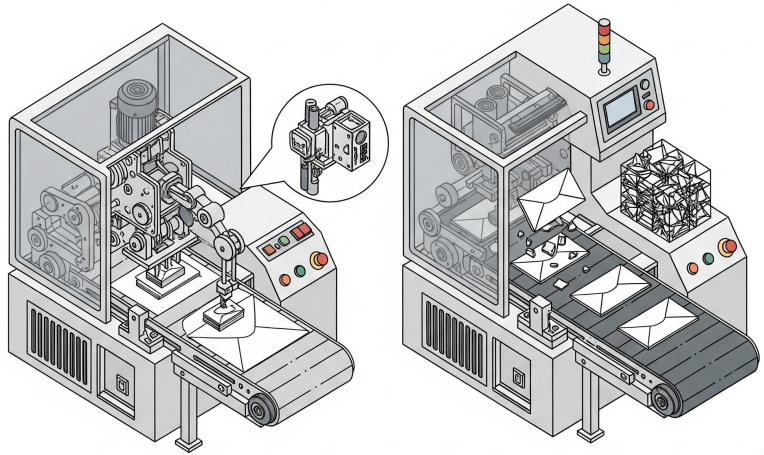


Zero-Knowledge-Proof

For the 3-Coloring Problem

Commitment

1. Prover: Randomly permute the 3-colors
2. Prover: Calculate for every node
   COM(id, c, r) = SHA256(id || c || r)
3. Prover: Send commitments
4. Verifier: When checking c_n1 != c_n2, also verify commitments for n1 and n2

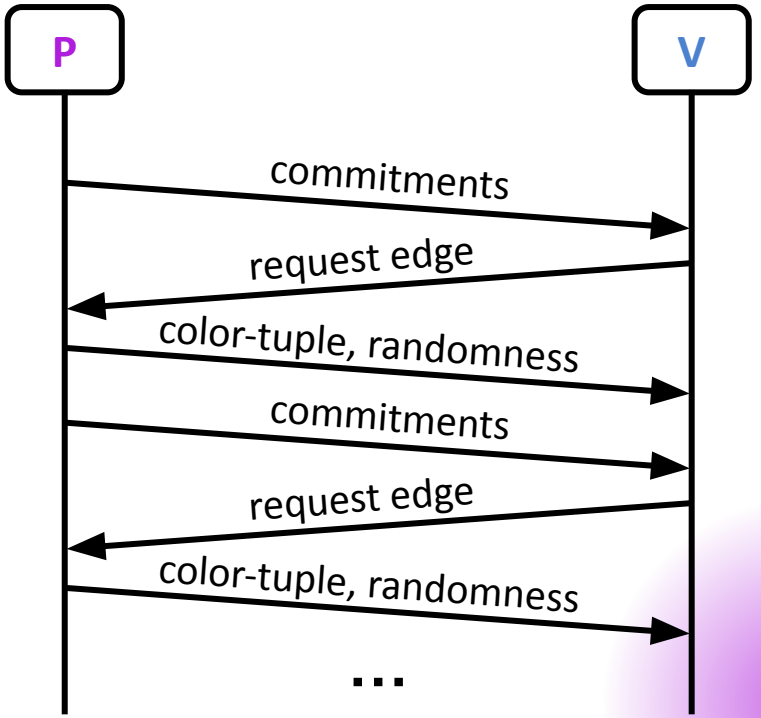


Zero-Knowledge-Proof

For the 3-Coloring Problem

Protocol 4

1. Prover: Permute solution
2. Prover: Send commitment
3. Verifier: Roll a n-sided dice and ask to reveal the corresponding edge as a sample
4. Prover: tuple and randomness
5. Verifier: Check that c_n1 != c_n2 + coms n1 and n2
6. Repeat



Zero-Knowledge-Proof

For the 3-Coloring Problem

https://nepet.github.io/btcpp-3-color-zkp/

Zero-Knowledge-Proof

For Every Problem in NP

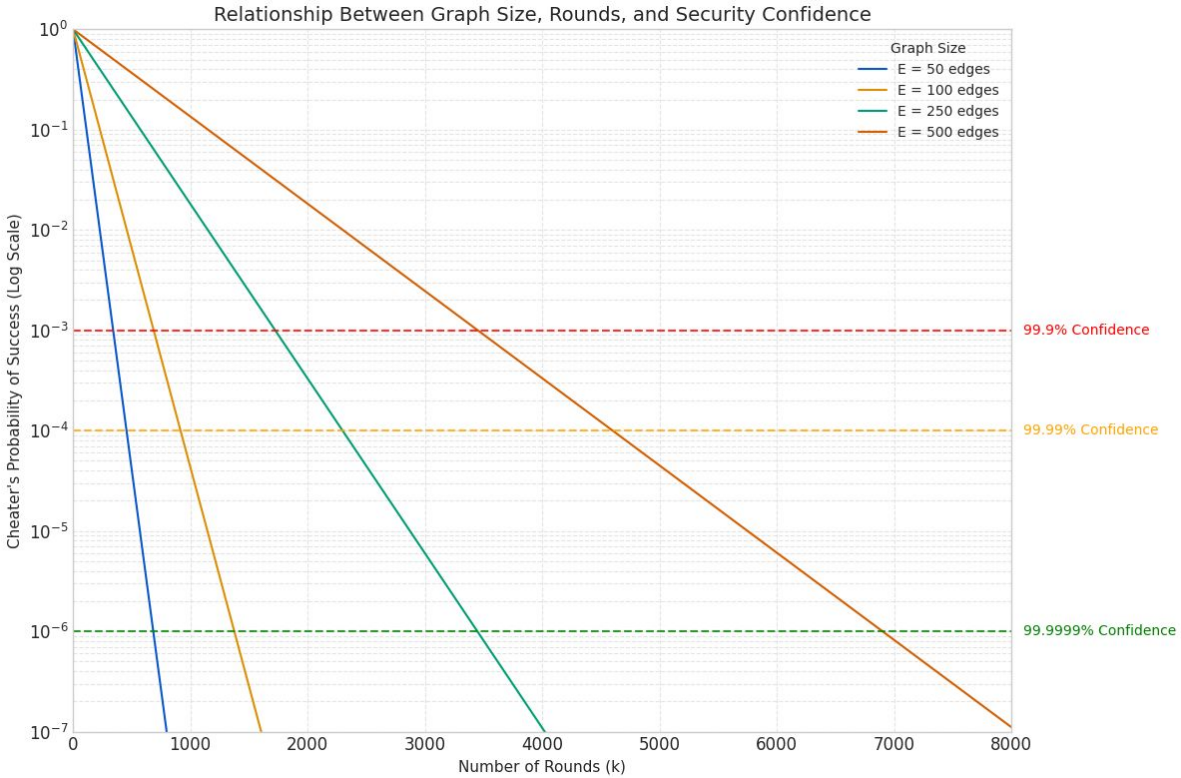
- We constructed a perfect ZKP
- No information is leaked whatsoever
- But it is impractical for a lot of Applications
- Huge amount of round-trips, Huge amount of data
- But Proof by Oded Goldreich, Silvio Micali et.al in 1986 laid the foundation
- Core of the proof is 3-Coloring

bitcoin++
Istanbul

twitter: @nepetonium
github.com/nepet

Thank You
Teşekkürler

https://github.com/nepet/btcpp-3-color-zkp


Relationship Between Graph Size, Rounds, and Security Confidence
Graph Size
E = 50 edges
E = 100 edges
E = 250 edges
E = 500 edges
99.9% Confidence
99.99% Confidence
99.9999% Confidence
Cheater's Probability of Success (Log Scale)
Number of Rounds (k)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
1000
2000
3000
4000
5000
6000
7000
8000