An Introduction to Integer Programs (IPs) / IP Solvers

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What is a Integer Program?

- Program refers to schedule
- In the Business Community, IP is equivalent to a Decision Problem (Management Science).
- In the CS-IT community, IP is equivalent to “NP complete”

IP Solvers

- Up to 130,000,000 times faster in last 15 years
- Used in Banking, Insurance, Manufacturing, Transportation, Telecommunications and e-commerce applications.
- Coin-OR, Cplex, OSL (IBM), Xpress, Lindo, Excel
- Integratable with most languages including C#, VB.Net, and Java
- Cost $500-$10,000 for commercial

CS-IT Problems

- Easy: Example - Sorting
- Hard: Example - Factoring
  - Some instance can take many CPU years to solve.
- Hardest: NP Complete (Decision Problems)
  - Exponential number of possible solutions.
  - IP solvers can, in many cases, find the optimal solution (considered impossible 10 years ago).
  - TSP, 3-sat, Hamiltonian cycle, graph coloring

Knapsack Problem

- Have a sack that can only hold W weight
- Have n items each with a certain weight and a certain value.
- Which items should we choose to place in the sack to maximize the value.
- Example: Space Shuttle and experiments
- This is an NP-Complete Problem
- Worse case solution with best known algorithm takes exponential time.
- Considered an “easy” hard problem, because running time on average is usually polynomial.

Knapsack Problem

- Used in Encryption
  - Broken in 1982
- Encryption uses hard problems with exponential worst case running time
  - RSA (factoring)

Knapsack Problem

	Textbook Cost	Merit
1	120	8
2	70	6
3	150	10
4	85	4
5	40	9
6	90	7
	budget 220	

Creating a IP for the Textbook Knapsack Problem

- Decision
  - For each textbook, we must decide whether or not to buy that textbook.
- Model for the Decision
  - Binary Variable x_i
    - 1, if we choose to buy textbook i
    - 0, if we choose not to buy textbook i

Textbook IP Continued

- Question: What is cost of textbook one in terms of our variables?
  - If $x_1 = 1$, the cost is 120
  - If $x_1 = 0$, the cost is 0
  - Answer: Cost is $120x_1$

Textbook IP Continued

- Total Cost of Textbooks
  - $120x_1 + 70x_2 + 150x_3 + 85x_4 + 40x_5 + 90x_6 \leq 220$
- Total merit
  - $8x_1 + 6x_2 + 10x_3 + 4x_4 + 9x_5 + 7x_6$

Textbook IP

Max $8x_1 + 6x_2 + 10x_3 + 4x_4 + 9x_5 + 7x_6$

Subject to:

$120x_1 + 70x_2 + 150x_3 + 85x_4 + 40x_5 + 90x_6 \leq 220$

$x_1, x_2, x_3, x_4, x_5, x_6$ are binary

Questions?