Introduction To Optimization

Optimization

- Taught in many departments under many names
- Used in almost every aspect of our lives from TVs to Cars to Phones to McDonalds to the Military to AI to Operating Systems to Cryptography to Compilers
- http://www.informs.org/Press/Timeline03f.htm

Optimization

- Business (Design Problems)
  - Economics
  - Industrial Engineering
  - Management Science
  - Operations Research
- Computer Science (NP-Complete)
  - As of now, any problem that is in NP-Complete has exponential running time in the worst case with the current best algorithm.
  - Used indirectly and directly
    - Resume Builder
  - Algorithms , TSP, Knapsack, 3-Sat, Network Routing, Project Management, Shortest Path, Network Flows, etc

Optimization

- Mathematics
  - Underlying algorithms built on the work of Mathematicians and Computer Science Theorists
  - Example Game Theory (Movie A Beautiful Mind)
- Military
  - Started in WWII with radar
  - Moved to business
- Science
  - Chemistry, Chemical Engineering

Optimization

- Linear Programs (variables are Reals)
  - Ellipsoid Algorithms solves the worst case running time.
  - Simplex most common (average case faster than Ellipsoid, but worst case is exponential)
- Integer Programs (variables are ints)
  - NP-Complete
  - Branch and Bound (relax the problem, solve the LP, if a variable in not integer, branch)
  - Approximation Algorithms
  - Heuristic Algorithms

Examples of Problem Types

- **Allocation problems** :allocate resources in order to optimize some measure of success (for example - minimize cost or maximize profit) .
- **Sequencing problems :** to schedule multistage operations in the most efficient manner, example- drill holes for mounting components on a circuit board (traveling salesman problem)
- **Routing problem :** to plan the movement of goods in an efficient manner (trucking industry)
- **Scheduling problem** optimal planning production to meet customer demand within capacity and supply constraints (scheduling workers closest to their preferred times)

Examples of Problem Types

- **Project management problems** : plan tasks to be accomplished so that the project is completed on time (building a skyscraper, developing software)
- **Network Problems** : link location in a efficient manner (computer network, telephone networks)
- **Knapsack problem** choosing items to take on a back-packing trip

- Farmer Brown is planning his planting for the coming year. He expects to raise two crops: potatoes and wheat. He has 100 acres of land available for planting and will be able to devote 160 days of labor to his crops. He expects an acre of wheat to require four days of labor, while an acre of potatoes requires only one day.
- He has $1,100 that he can use for the start-up costs of planting and cultivating. It costs $10 an acre to plant and cultivate potatoes, while the corresponding costs for an acre of wheat are $20.
- If Brown expects a revenue of $40 per acre for potatoes and $120 an acre for wheat, how many acres of each should he plant in order to achieve the maximum possible revenue?

Formulation

- p -> number acres of potatoes
- w -> number of acres of wheat
- max 40p+120w (revenue)
- p + w <=100 (land)
- p + 4w <=160 (labor)
- 10p + 20w <=1100 (Money)
- p,w >=0

Solution

- Graphical Solution (2 variables)
  - Graph the constraints
  - Optimum will be a “corner point”
- Simplex
  - Pivot from one “corner point” to another until optimum solution is reached

Another Example: Game Theory

- 2-PLAYER ZERO-SUM GAMES: Player A hides a nickel or quarter. Player B guesses. If B gets it right, he gets the coin. If wrong, he has to pay A the value of the coin.
  - How should A play to maximize the payoff?
- payoff matrix

	Player N	B Q
N	-5	5
Q	25	-25

	Player N	B Q
N	-5	5
Q	25	-25

- Let $x1$ =% of times A picks N, $x2$=% times A picks Q
- $x1+x2=1$
- If B picks N, A gets?
  - $-5x1+25x2$ (-5 A picked N also, or 25 if A picked Q)
- If B picks Q, A get
  - $5x1-25x2$
- Max v
  - $x1 + x2 = 1$
  - $-5x1+25x2 >= v$
  - $5x1-25x2 >= v$
- Best Strategy, [5/6, 1/6]