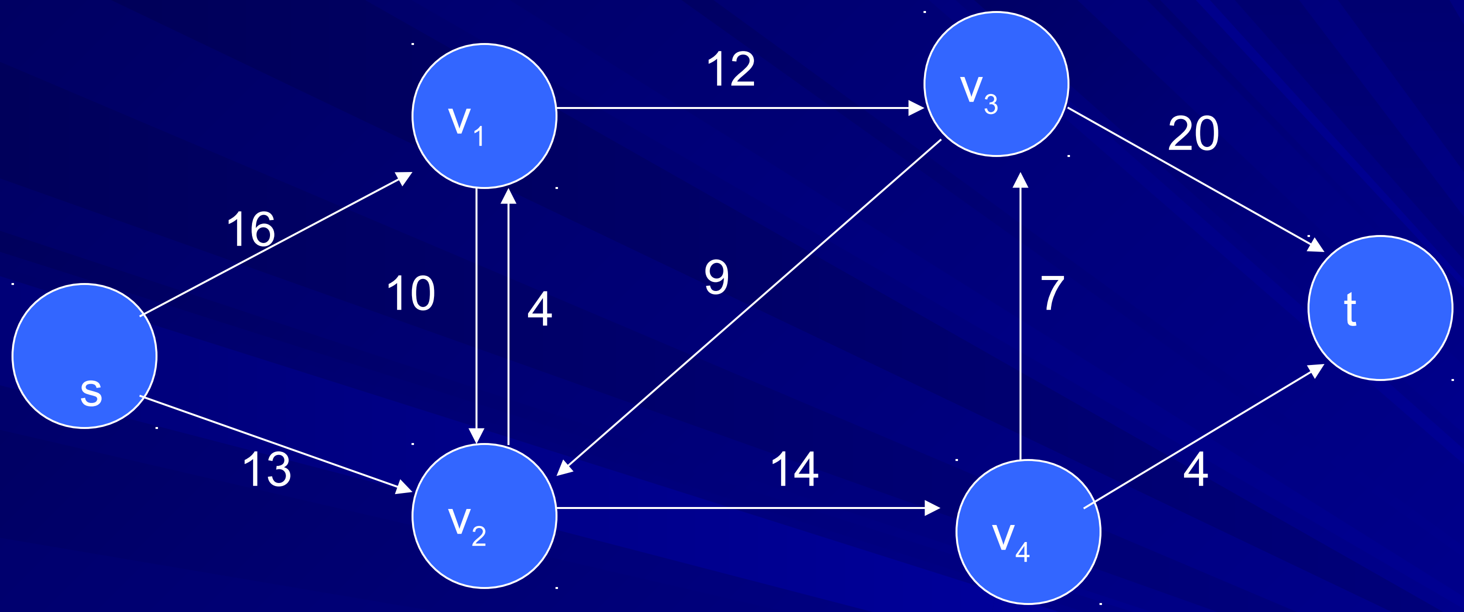
Max-Flow Min Cut

Algorithms

Network



Max Flow Applications

- Data mining.
- Open-pit mining.
- Project selection.
- Airline scheduling.
- Bipartite matching.
- Baseball elimination.
- Image segmentation.
- Network connectivity.

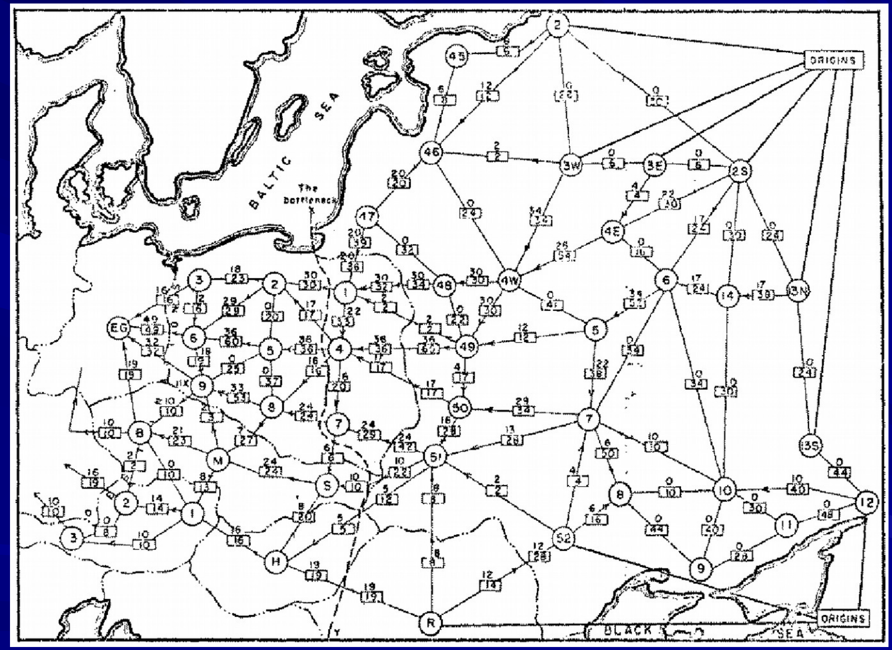
– Network reliability.
– Distributed computing.
– Egalitarian stable matching.
– Security of statistical data.
– Network intrusion detection.
– Multi-camera scene reconstruction

Soviet Rail Network, 1955

"The Soviet rail system also roused the interest of the Americans, and again it inspired fundamental research in optimization."
-- Schrijver

G. Danzig* 1951…First soln…

Again formulted by Harris in 1955 for the US Airforce ("unclassified in 1999")
What were they looking for?



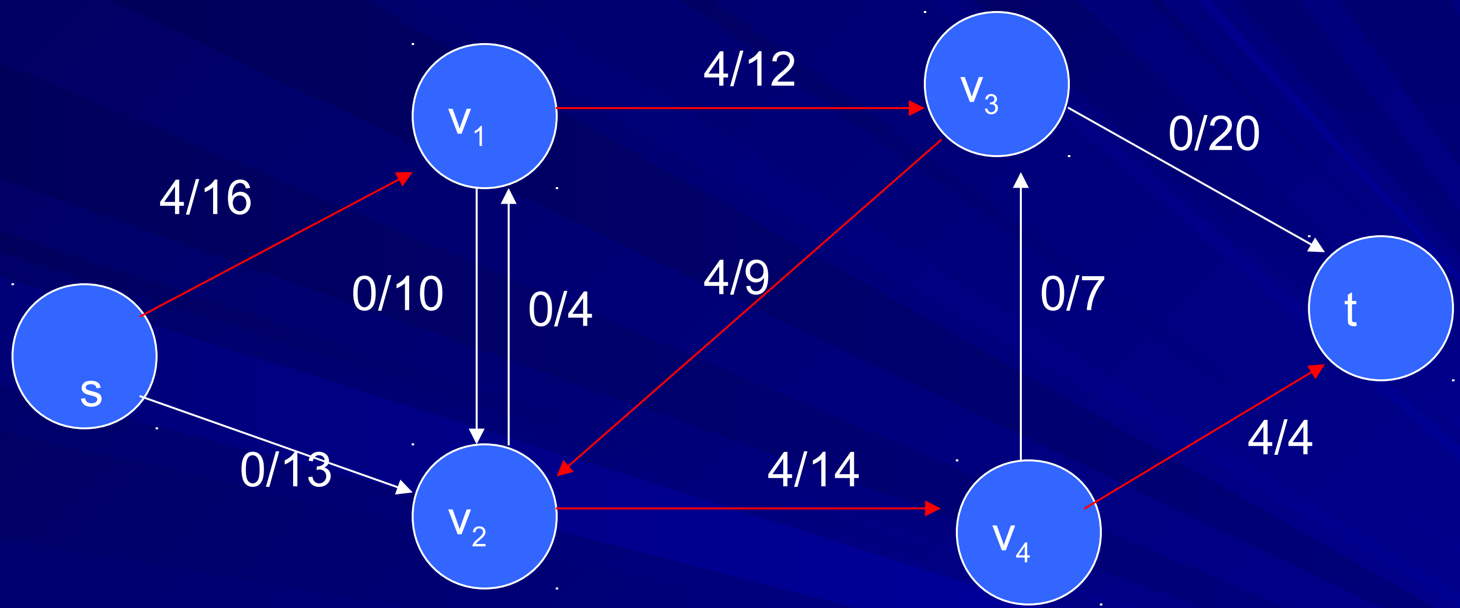
Reference: *On the history of the transportation and maximum flow problems.* Alexander Schrijver in Math Programming, 91: 3, 2002.

year	discoverer(s)	bound
1951	Dantzig [11]	$O(n^2mU)$
1956	Ford & Fulkerson [17]	$O(nmU)$
1970	Dinitz [13] Edmonds & Karp [15]	$O(nm^2)$
1970	Dinitz [13]	$O(n^2m)$
1972	Edmonds & Karp [15] Dinitz [14]	$O(m^2 \log U)$
1973	Dinitz [14] Gabow [19]	$O(nm \log U)$
1974	Karzanov [36]	$O(n^3)$
1977	Cherkassky [9]	$O(n^2m^{1/2})$
1980	Galil & Naamad [20]	$O(nm \log^2 n)$
1983	Sleator & Tarjan [46]	$O(nm \log n)$
1986	Goldberg & Tarjan [26]	$O(nm \log(n^2/m))$
1987	Ahuja & Orlin [2]	$O(nm + n^2 \log U)$
1987	Ahuja et al. [3]	$O(nm \log(n\sqrt{\log U}/m))$
1989	Cheriyan & Hagerup [7]	$E(nm + n^2 \log^2 n)$
1990	Cheriyan et al. [8]	$O(n^3/\log n)$
1990	Alon [4]	$O(nm + n^{8/3} \log n)$
1992	King et al. [37]	$O(nm + n^{2+\epsilon})$
1993	Phillips & Westbrook [44]	$O(nm(\log_{m/n} n + \log^{2+\epsilon} n))$
1994	King et al. [38]	$O(nm \log_{m/(n \log n)} n)$
1997	Goldberg & Rao [24]	$O(\min(n^{2/3}, m^{1/2})m \log(n^2/m) \log U)$

Ford-Fulkerson(G,s,t)

- F = 0
- While there is augmenting path from s to t,P
  - F=f+f_P
  - Subtract f_p from all capacities of edges on P

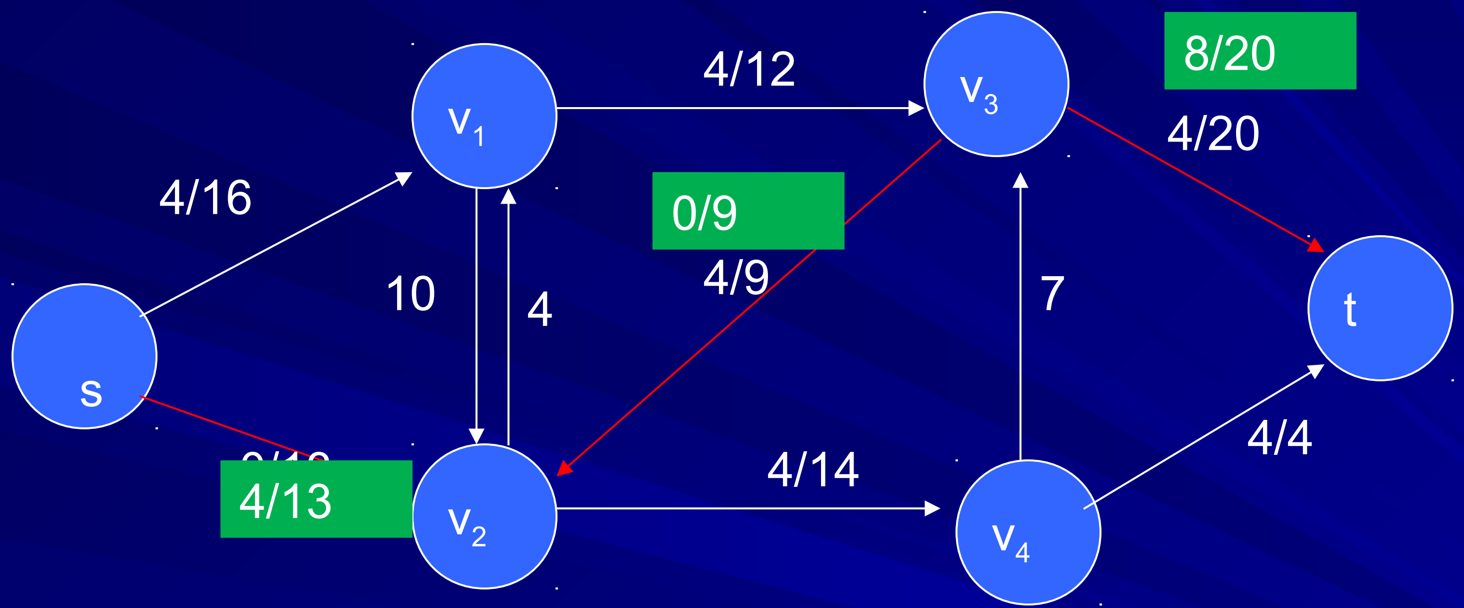
Network



Flow of path is 4

F=4

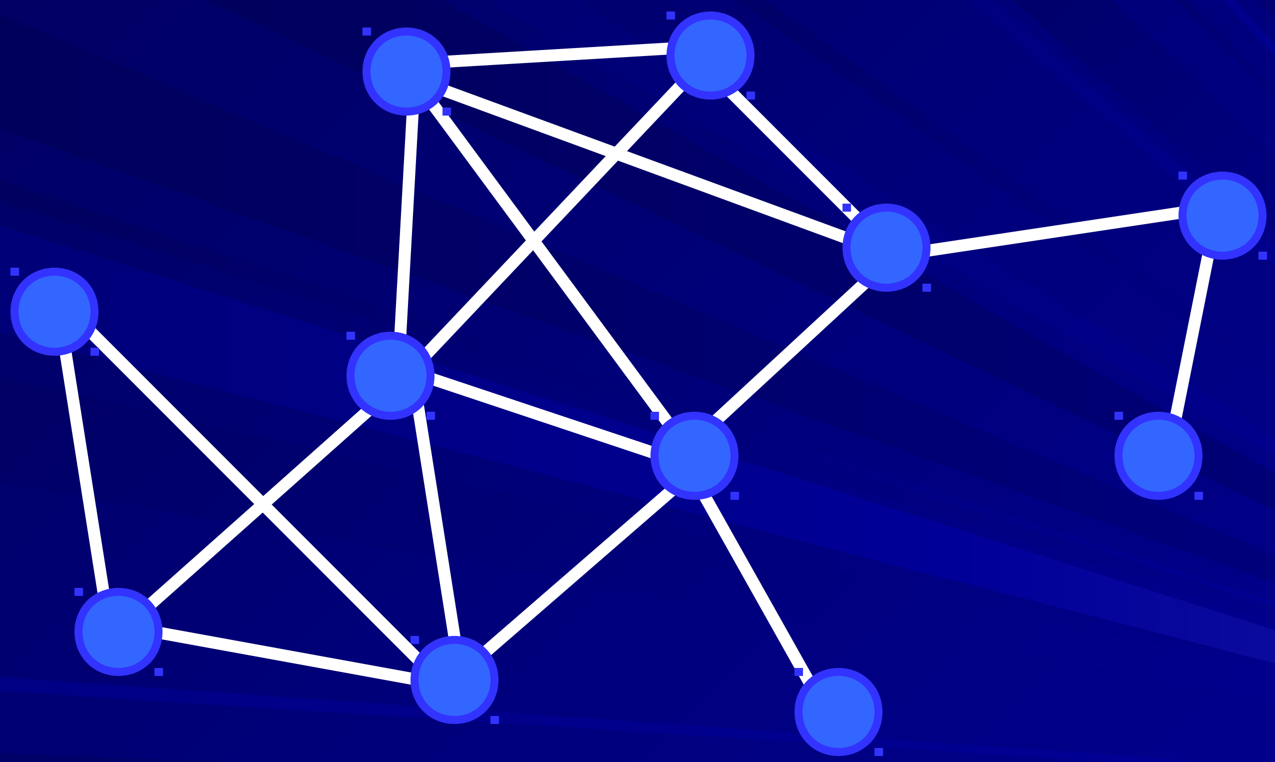
Network



Flow of path is 4,
Total =8

F=4

Matching



Maximal Matching

- Maximal Matching
  - k edges sharing no common vertices.
  - No other edges from G can be added