

Introduction to NP

Analysis of Algorithms
CSCI 5620

Decision Problem

- Decision problem: One for which the output can be a simple yes or no answer for each input.
- Example:
 - The input is a positive integer C and n positive integers s_1, \dots, s_n .
 - Is there a subset of the integers which add up to C ?

Decision Problem

■ Decision Problem

- Does the graph have a Spanning Tree with total weight K ?

■ Optimization Problem

- What is the MST weight?

■ Decision Problem

- For a given graph, is there a path from A to B of length k ?

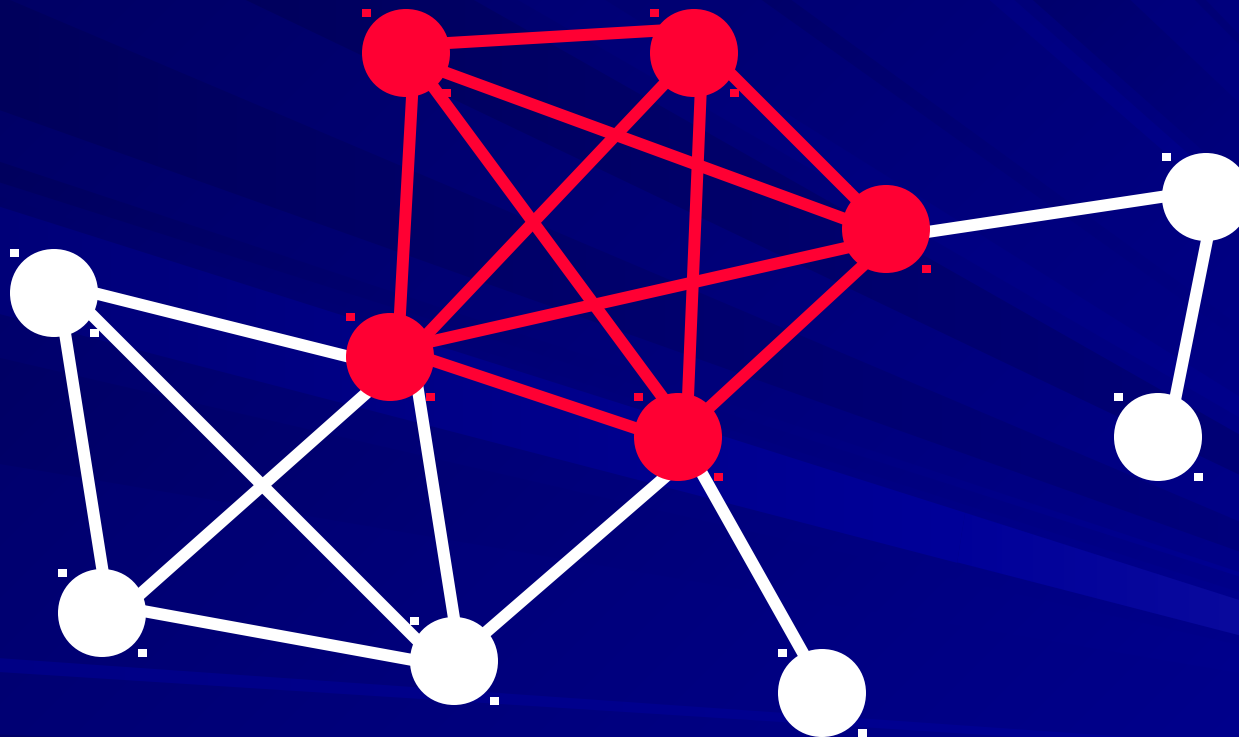
■ Optimization Problem

- What is length of a shortest path from A to B ?

■ Optimization Problems are at least as hard as Decision Problems.

- So if the optimization problem is easy, then the decision problem is easy.

Clique Size



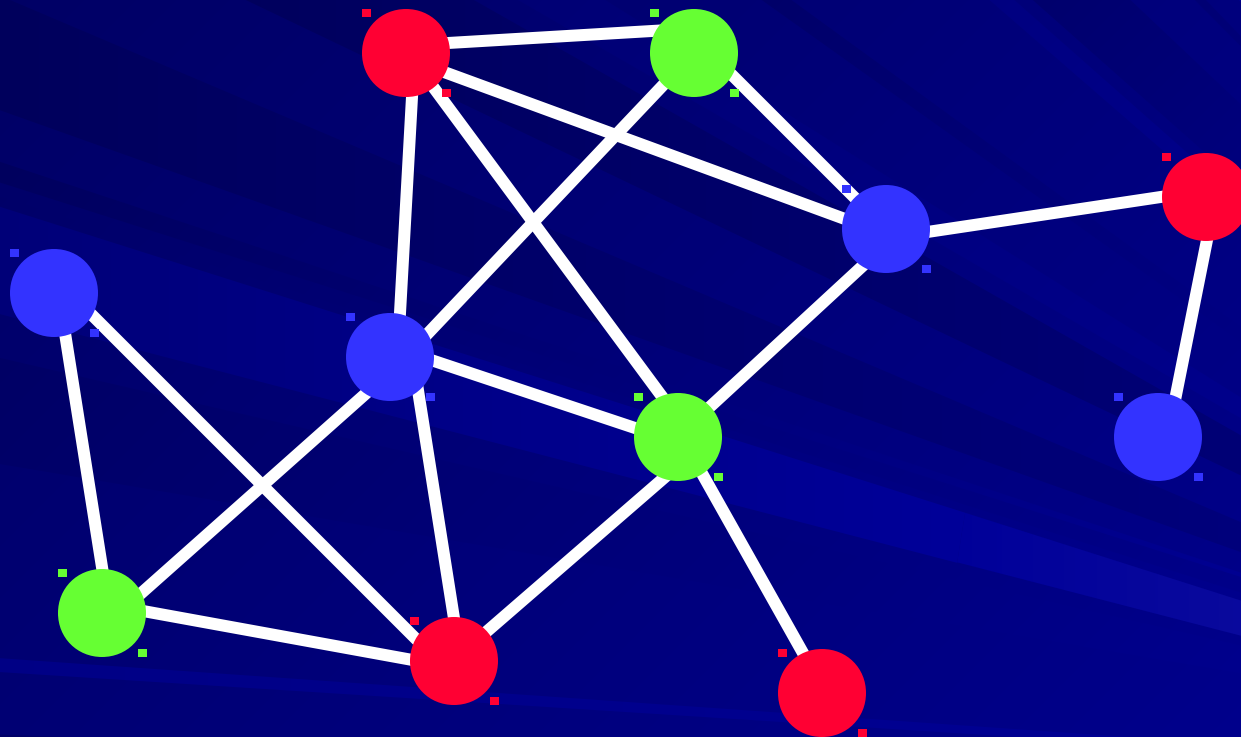
Clique Formulation

$$\max \sum_{i=1}^n w_i x_i,$$

$$\text{s.t. } x_i + x_j \leq 1, \forall (i, j) \in \overline{E},$$

$$x_i \in \{0, 1\}, i = 1, \dots, n.$$

Coloring



Coloring Formulation

Given a set of n colors $A = \{1, 2, \dots, n\}$, the graph coloring problem associated with a graph $G = (V, E)$ can be formulated as the following integer program:

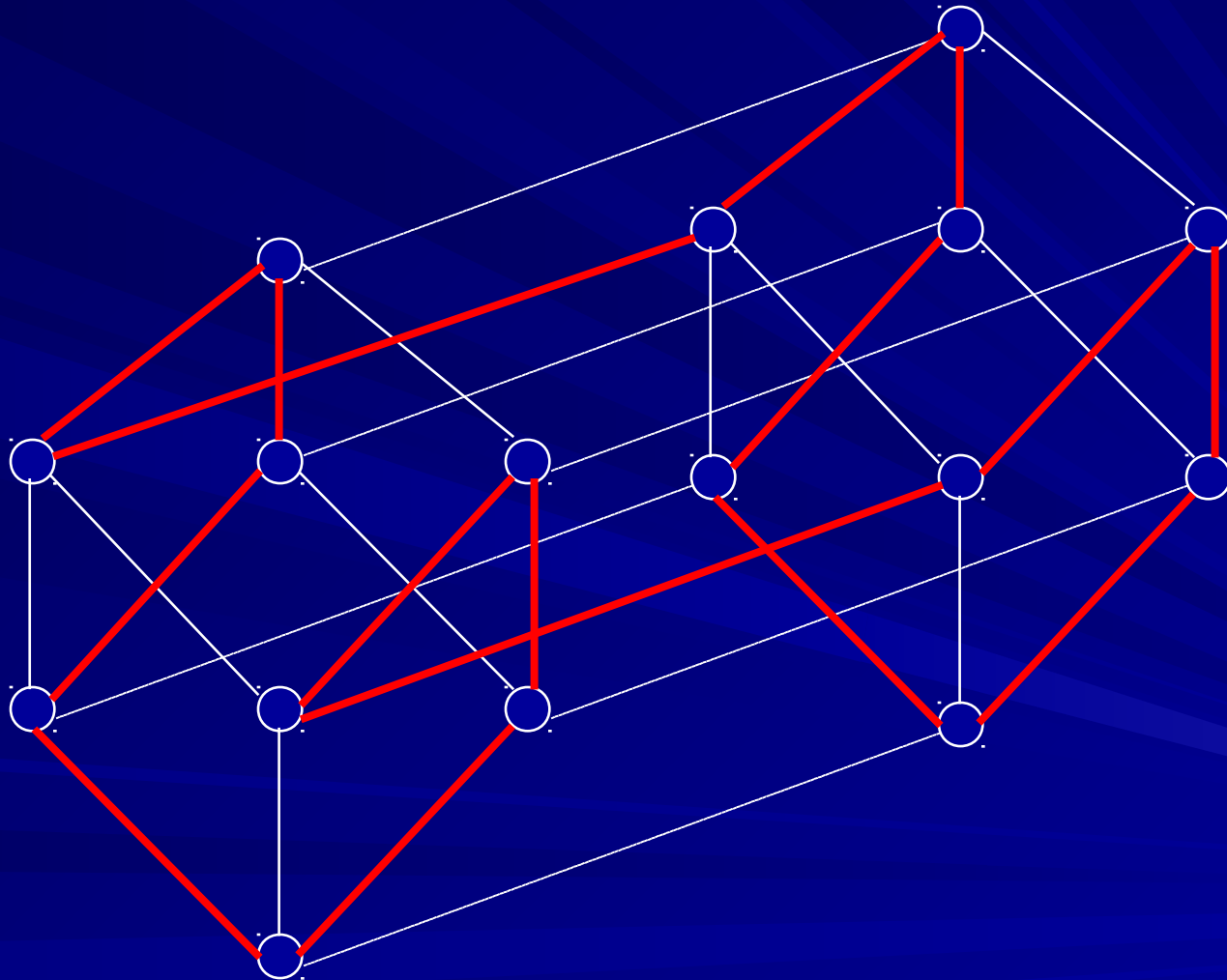
$$\begin{aligned} \text{(P)} \quad & \min \sum_{k=1}^n y_k, \\ \text{s.t.} \quad & \sum_{k=1}^n x_{ik} = 1, \quad \forall i \in V, \end{aligned} \tag{1}$$

$$x_{ik} + x_{jk} \leq 1, \quad \forall (i, j) \in E, \tag{2}$$

$$y_k \geq x_{ik}, \quad \forall i \in V, k = 1, \dots, n, \tag{3}$$

$$x_{ik} \in \{0, 1\}, \quad \forall i \in V, k = 1, \dots, n. \tag{4}$$

Hamiltonian Cycle



NP

- NP “informally” is the class of decision problems whose answer “yes/no” can be verified in polynomial time given a “guess” or “certificate”.
- Example Does G contain a Hamiltonian cycle?
 - What is a guess?
 - Can it be verified in polynomial time?
 - How many total guesses?
 - How many does it take to prove G contains a cycle?
- Example: Is n a composite (not prime)?
 - Does d divide n ? (can be checked in polynomial time)
- Example: Can graph G be colored with k colors?
 - Chromatic Number, smallest number of colors in which G can be colored (optimization problem)

Strings

■ Catenation

- If $s_1 = \text{abac}$ and $s_2 = \text{dade}$, then $s_1 \cdot s_2$ or $s_1 s_2 = \text{abacdade}$.

■ Empty string, $\lambda = \text{"}$

- $s_1 \lambda = \lambda s_1 = \text{abac}$

Alphabet / Words

- For a set A , the set A^* consists of all finite sequences of elements of A .
 - Ex $A=\{0,1\}$
 - $\lambda \in A^*$, $0 \in A^*$, $1 \in A^*$, $01 \in A^*$, $10 \in A^*$
 - $000000000\dots \notin A^*$
- A is called an alphabet, elements of A^* are called words.
- Ex. $A=\{a, b, c, \dots, z\}$, A^* contains all words, such as “discrete”, but also “ababccad”.

Example

- Let $A = \{ab, bc, ba\}$
 - Is $ababab \in A^*$?
 - Is $abc \in A^*$?
 - Is $abba \in A^*$?
 - Is $abbcbaba \in A^*$?
 - Is $bcabbab \in A^*$?
 - Is $abbbcbba \in A^*$?

Regular Expressions

- A **regular expression over A** , is a string constructed from elements of A and symbols $(,), \vee, *, \lambda$.
 - First λ is a regular expression.
 - If $x \in A$, then symbol x is a regular expression.
 - If α and β are regular expression then $(\alpha \vee \beta)$ is regular.
 - If α is a regular expression than $(\alpha)^*$ is a regular expression.

Regular Expressions

- Ex. Let $A=\{0,1\}$
- 0^* is a regular expression.
- $(01)^*$ is a regular expression
- $(0\vee 1)^*$ is a regular expression
- $1(0\vee 1)^*$ is a regular expression
- $(011)^*$ is a regular expression
- $0 \vee 1^*$ is a regular expression
- $(0$ is not a regular expression

Regular Expressions

- Give the regular expression for the set of strings over $\{a,b\}$ in which all the a's precede the b's which in turn precedes the c's.

Regular Set

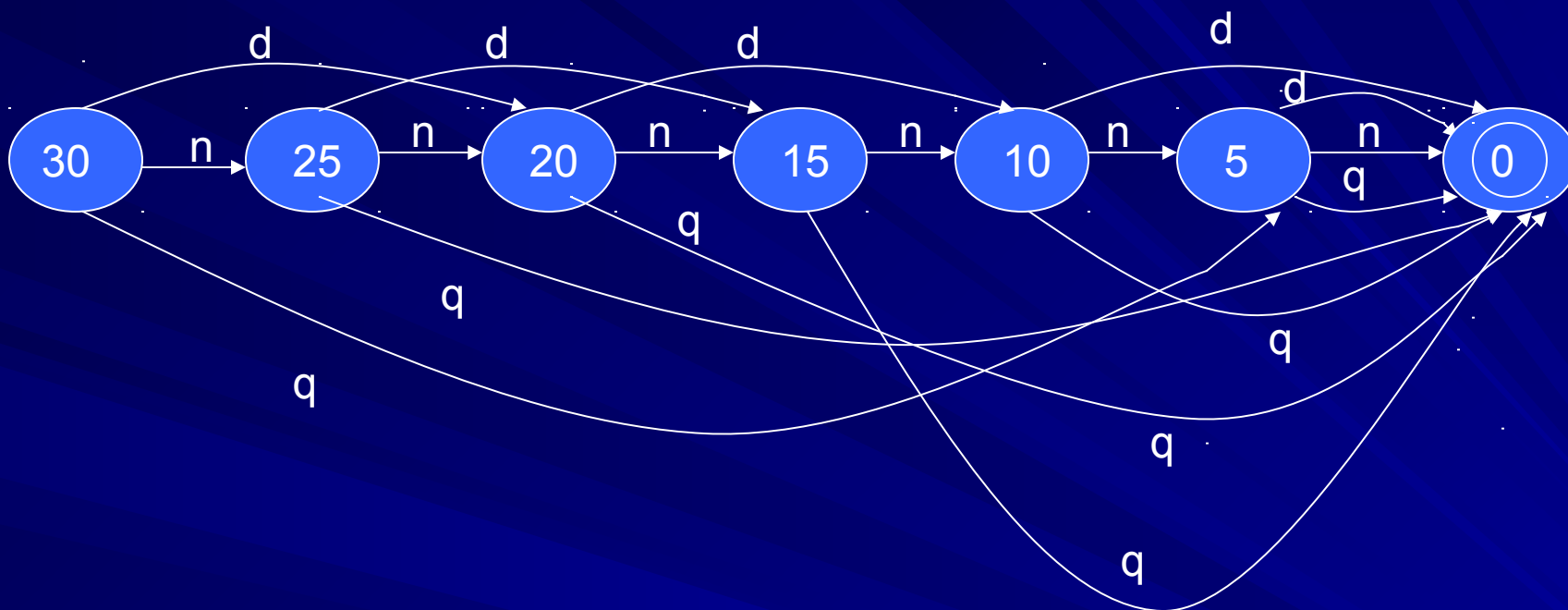
- For each regular expression, there is a corresponding subset of A^* , called a **regular set**.
- Example Let $A = \{a, b\}$
 - For a^* , the regular set Y contains
 - $\lambda, a, aa, aaa, aaaa, aaaaa$
 - $Y = \{s, s = aaa...a \text{ (repeated } n \text{ times for } n \geq 0)\}$
 - $aaaaaaa... \notin Y$

Language

- Let Σ be an alphabet.
- A **language** over Σ is a subset of Σ^* .
- Computer Language
 - Contains words (if, then , else, etc)
- English Language
 - $\Sigma = \{a,b,c,d,....z\}$
 - Subset of Σ^*

Deterministic Finite Automata (DFA)

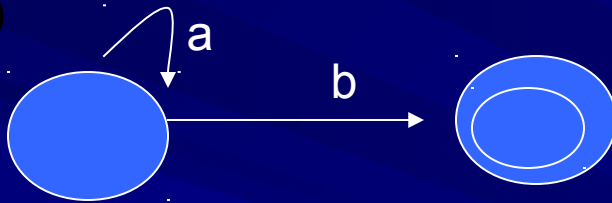
- Finite State Machine
- Vending machine
 - Simple newspaper vending machine
 - 30 cents
 - No change given
 - Takes Nickels, dimes, and quarters
 - No memory used



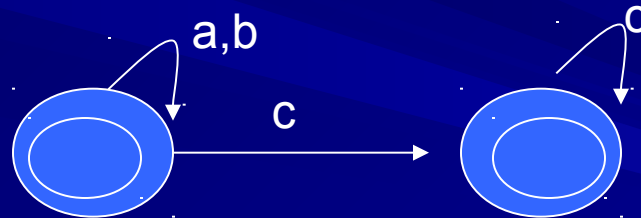
Regular expressions

■ $A = \{a, b, c\}$

■ a^*b



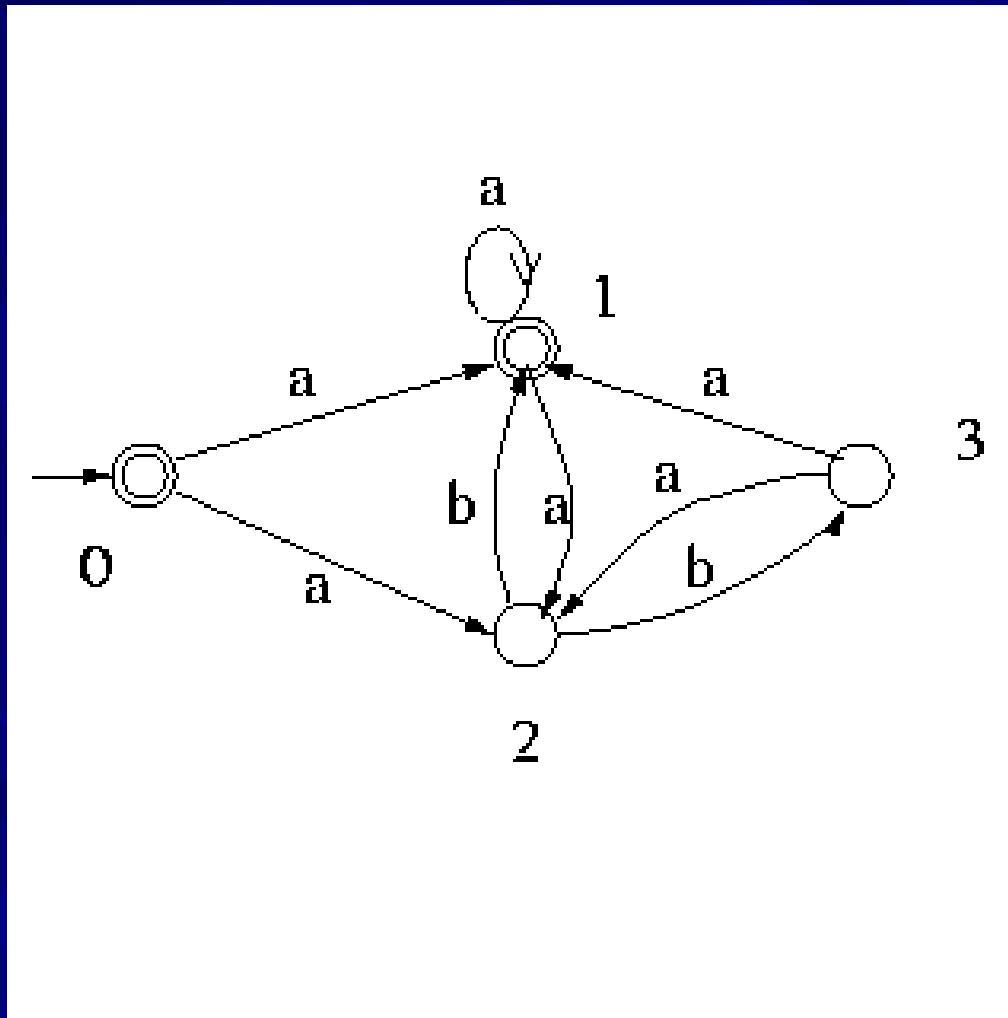
■ $(a|b)^*c^*$



Regular Language

- Equivalent to A regular set
- Accepted by a DFA (deterministic finite automation), NFA (nondeterministic finite automation).

NFA example



DFA example

