(1) Reduce 2-CNF-SAT to an efficiently solvable problem on a directed graph.

Ans: We have to show that 2-CNF-SAT is solvable in linear time. Each sub-expression or clause in such formula contains at most two literals.

If the clause is viewed as $\bar{u} \Rightarrow v$ and $\bar{v} \Rightarrow u$, we construct a graph such that if a_1, a_2 ... are variables of the formula there are two vertices a_i and $\bar{a}_i$ (complement) that is in V for each a.

The formula is satisfiable if and only if no pair of complementary literals are in the same strongly connected component of G.

If there is a path from a node u to v and from v to u, it implies that both the node must have same value (in truth assignment). Therefore, if there is a path from u to v and from v to u, following algorithm rejects the value.

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for each a in V
    if there is path from u to ū and from ū to u
                    (where u & ū are possible values of a_i)
        reject
    else
        accept
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Thus having such a formula a 2-CNF-SAT is a problem that can be solvable in polynomial time, thus it is polynomial time decidable.

② 34.5-1

The subgraph-isomorphism problem takes two undirected graphs G_1 and G_2, and it asks whether G_1 is isomorphic to a subgraph of G_2. Show that the subgraph-isomorphism problem is NP-complete.

Ans: We have two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$. By taking above problem into consideration we consider a subgraph $G = (V, E)$ in G_2 that is isomorphic to G_1.

In graph G, V is a subset of V_2 and E is a subset of E_2, such that $|V| = |V_1|$ and $|E| = |E_1|$. For the graphs G and G_1 to be isomorphic there should exist a one-to-one function f from V_1 to V such that

$$(u, v) \in E_1 \quad \text{iff} \quad (f(u), f(v)) \in E_2$$

First, we show that subgraph-isomorphism is in NP. We have to ensure what was described in previous paragraph. i.e

There exists a sub-graph $G = (V, E)$, function f from V_1 to V and $|V| = |V_1|$, $|E| = |E_1|$, $V \subseteq V_2$, $E \subseteq E_2$ and the function f correctly maps all edges in E_1 to all the edges in E.

Since there are $O(n^2)$ such pairs (between $G \times G_1$), the check requires polynomial time. This proves that subgraph-isomorphism is in NP.

→ We now show that subgraph-isomorphism in NP-hard. We show this by using subgraph-isomorphism to solve the known NP-complete problem Clique.

We create a clique of size k. This clique is isomorphic another clique of same size. Using this fact we can create a subgraph isomorphism problem in order to find a subgraph in G_2 that is isomorphic to G_1. In G_2 we have a completely connected graph with k vertices.

To do this we take $G_2 = (V_2, E_2)$ graph and construct a completely connected graph of size k as our second graph, $G_1 = (V_1, E_1)$ such that $|V_1| = k$ and E_1 contains the edges for the completely connect graph G_1 (k-clique). It requires polynomial time to create G_1 and G_2.

If there exists a clique of size k in G_2, then subgraph isomorphism should determine that that graph is equivalent to G_1. If not, there would not be such graph equivalent to G_1. Based on this fact subgraph isomorphism is NP-hard and hence NP-complete

3 4 5 Used programming (Shown in lab, and code uploaded to D2L) as substitute for other 3-problems