CSCI 5620 Homework 1 Due September 4, 2015 by midnight

You can work with other students to discuss the problems, I recommend doing so, but DO NOT LOOK AT OTHER STUDENTS WRITEUP. WRITE THE SOLUTION IN YOUR OWN STYLE. Each person has there own unique style, so it is easy for me to detect when students are coping work. First warning 0 on homework, second warning "F" in course! Every year there is at least two students who receive the first warning.

Prove 1-9 using induction

Fibonacci numbers, 0, 1, 1, 2, 3, 5, 8, 13, 21,...
Recurrence $f_0 = 0$, $f_1 = 1$, $f_i = f_{i-1} + f_{i-2}$ for $i \geq 2$

1) $f_n = f_0 + f_1 + ... + f_{n-2} + 1$ for $n \geq 2$ by induction.

2) Prove that f_{3n} is even.

3) Prove that $1 \cdot 1! + 2 \cdot 2! + ... + n \cdot n! = (n+1)! - 1$ for $n \geq 1$

4) 3.2-6 pg 57 of textbook
Prove Prove that the ith Fibonacci number satisfies the equality

$$f_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$$

where

$$\phi^i = \frac{1 + \sqrt{5}}{2}$$

$$\hat{\phi}^i = \frac{1 - \sqrt{5}}{2}$$

5) Prove that $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{n}} \geq \sqrt{n}$ for $n \geq 1$.

(i.e this diverges Bonus 5 points each, show the sum with $\frac{1}{n}$ diverges also but the sum with $\frac{1}{n^2}$ converges **using induction only!**).

6) Prove that $1 + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{n^2} < 2 - \frac{1}{n}$ for $n \geq 2$.

7) Prove that 21 divides $4^{n+1} + 5^{2n-1}$ for $n \geq 1$

8) Show $2^{2^n} - 1$ is divisible by at least n distinct primes for $n \geq 1$. We can conclude from this there is an infinite number of prime numbers. See the additional bonus (15 points) for a much stronger result, the Prime Density Function, an important result in computer science algorithms.

9) Use induction to show that given a set of $n + 1$ positive integers, none exceeding 2n, there is a least one integer in this set that divides another integer in the set.

10) Use the definition of big oh, to show that $3n^2 + 2n + 7 \in O(n^3)$, i.e you will need to find a c and k and a do small proof.

11) 3-3 a) Rank by order of growth only. Prove each step using limits (see Quiz 1 part 3). Do not include a function if you have no proof, leave it out.

$(\sqrt{2})^{lg n}$, $n^2, n!$, $(lg\, n)!$, $(\frac{3}{2})^n$, n^3, $lg^2\, n$, $lg\,(n!)$, 2^{2^n}, $n^{\frac{1}{lg\, n}}$, $ln\, ln\, n$,
$n2^n$, $n^{lg\, lg\, n}$, $ln\, n$, 1, $2^{lg\, n}$, $(lg\, n)^{lg\, n}$, e^n, $4^{lg\, n}$, $(n+1)!$, $\sqrt{lg\, n}$, $2^{2 lg\, n}$, n, 2^n, $n lg\, n$, $2^{2^{n+1}}$

12) 2.2-3 in textbook. Consider linear search.

Input: A sequence of n numbers in an array A, i.e. $A =< a_1, a_2, ..., a_n >$ and a value v

Output: An index i such that $v = A[i]$ or -1 if v does not appear in A.

How many elements of the input sequence need to be checked on average, assuming that the element being searched is equally likely to be any element in the array? How about in the worst case? What are the average-case and worst-case running times of linear search in Θ notation?

13)

1 a) Write recursive code to find the nth Fibonacci number. (attach the code)

b) What is the largest number that can be calculated in under a minute ?

c) What is the recurrence relation?

d) Bonus, solve the recuurence relation 1c (5 points)

2a) Write code to solve $pow(x, n) = x^n$ recursively, output the number of steps also. Assuming x is a double and n a positive integer.

Using: pow(x,n)= pow(x,n-1)*x

2b) How many steps to find 1^{1000}

2c) What is the recurrence relation for 2a?

2d) Solve the relation in 2c

3 a) Can we do pow faster?

Write code to solve pow(x,n) using pow (x, $\lfloor \frac{n}{2} \rfloor$) * pow (x, $\lceil \frac{n}{2} \rceil$)

3b) How many steps to find 1^{1000}

3c) What is the recurrence relation for 2a?

3d) Solve the relation in 3c

4a How can we fix 3?

Solution we are calculating the same item twice.

$$pow(x, n) = \begin{cases} pow(x, \frac{n}{2})^2 & \text{if } n \text{ is even} \\ pow(x, \frac{n}{2})^2 * x & \text{if } n \text{ is odd} \end{cases}$$

4b) code 4a

4c How many steps does the code for 1^{1000}

4d What is the recurrence relation

4e Solve relation in 4d